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The non-SUSY AdS_6 and AdS_7 fixed points are brane-jet unstable

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Abstract: In six- and seven-dimensional gauged supergravity, each scalar potential has one supersymmetric and one non-supersymmetric fixed points. The non-supersymmetric AdS⁷ fixed point is perturbatively unstable. On the other hand, the non-supersymmetric $AdS₆$ fixed point is known to be perturbatively stable. In this note we examine the newly proposed non-perturbative decay channel, called brane-jet instabilities of the AdS_6 and AdS⁷ vacua. We find that when they are uplifted to massive type IIA and elevendimensional supergravity, respectively, the non-supersymmetric AdS_6 and AdS_7 vacua are both brane-jet unstable, in fond of the weak gravity conjecture.

Keywords: AdS-CFT Correspondence, Superstring Vacua

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Contents

1 Introduction and conclusions

The AdS/CFT correspondence, [\[1\]](#page-10-0), has provided a framework to study quantum field theories in various dimensions with various amount of supersymmetry through their gravitational duals. When it comes to the non-supersymmetric quantum field theories, even though there are several known perturbatively stable, [\[2–](#page-10-1)[4\]](#page-10-2), non-supersymmetric AdS vacua, due to the limited control over the non-supersymmetry, not much was able to be investigated.^{[1](#page-1-1)} Furthermore, recently, as a stronger version of the weak gravity conjecture, [\[6\]](#page-10-3), a conjecture on non-supersymmetric AdS vacua was suggested: there are no stable non-supersymmetric AdS vacua from string and M-theory, [\[7\]](#page-10-4). In support of testing the conjecture, a new non-perturbative decay channel called brane-jet instability was proposed by Bena, Pilch and Warner in [\[8\]](#page-10-5). This examines the force acting on the probe branes and if the force is repulsive, the vacuum is determined to be unstable. In [\[8\]](#page-10-5), the authors showed the only known perturbatively stable non-supersymmetric AdS_4 vacuum, [\[9,](#page-10-6) [10\]](#page-10-7), among the AdS vacua of four-, [\[11\]](#page-10-8), and five-, [\[12\]](#page-10-9), dimensional maximal gauged supergravity is, in fact, brane-jet unstable. See also [\[13\]](#page-10-10) for the brane-jet stability from the D2-brane theories.

There is a closely related channel of non-perturbative instability from instantons. The condition for nucleation of a bubble in Euclidean AdS is analyzed in [\[14\]](#page-10-11). It is given by the competition of tension and charge of the particles. Once the bubble is created, it reaches the boundary of AdS in finite Lorentzian time and destabilizes the AdS spacetime.^{[2](#page-1-2)} This idea was recently extended to branes in string theory in [\[15\]](#page-10-12). See e.g., [\[16–](#page-10-13)[19\]](#page-10-14) also for studies of instability of AdS and instantons.

¹There is a recent search of curious $5d$ non-supersymmetric CFTs in [\[5\]](#page-10-15).

²We would like to thank Gabriele Lo Monaco and collaborators of $[15]$ for comments on this.

The purpose of this note is to examine the brane-jet instability of AdS vacua of sixand seven-dimensional gauged supergravity in $[20]$ and in $[21-24]$ $[21-24]$, respectively. In six- and seven-dimensional gauged supergravity, each scalar potential has one supersymmetric and one non-supersymmetric fixed points.

In seven dimensions, minimal gauged supergravity, [\[21,](#page-11-0) [22\]](#page-11-2), is a subsector of maximal gauged supergravity, [\[23,](#page-11-3) [24\]](#page-11-1). As we identify the scalar fields to a scalar field, the maximal theory reduces to the minimal theory. The scalar potentials of the theories have a pair of supersymmetric and non-supersymmetric fixed points. The non-supersymmetric fixed point is known to be perturbatively stable in the minimal theory, [\[22\]](#page-11-2), but not stable in the maximal theory, [\[24\]](#page-11-1). Maximal and minimal theories commonly uplift to elevendimensional supergravity, [\[25](#page-11-4)[–27\]](#page-11-5) and [\[28\]](#page-11-6), but the minimal theory also uplifts to massive type IIA supergravity, $[29]$. We will examine the brane-jet stability of the AdS₇ fixed points when they are uplifted to eleven-dimensional supergravity.

In $F(4)$ gauged supergravity in six dimensions, [\[20\]](#page-10-16), there are also a pair of supersymmetric and non-supersymmetric fixed points. The non-supersymmetric $AdS₆$ fixed point is known to be perturbatively stable, [\[20\]](#page-10-16). $F(4)$ gauged supergravity is a consistent truncation of massive type IIA supergravity, [\[30\]](#page-11-8) and also of type IIB supergravity, [\[31–](#page-11-9)[33\]](#page-11-10). We will examine the brane-jet stability of the $AdS₆$ fixed points when they are uplifted to massive type IIA supergravity.

Indeed we show that when they are uplifted to massive type IIA and eleven-dimensional supergravity, respectively, the non-supersymmetric AdS_6 and AdS_7 fixed points are both brane-jet unstable in favor of the conjecture on non-supersymmetric vacua in [\[7\]](#page-10-4).

It would be interesting to consider the alternative uplifts of the AdS_6 and AdS_7 fixed points to type IIB, [\[31](#page-11-9)[–33\]](#page-11-10) from [\[34,](#page-11-11) [35\]](#page-11-12), and massive type IIA supergravity, [\[29\]](#page-11-7) from [\[36\]](#page-11-13), respectively. Indeed, the instabilities of AdS_7 solutions in massive type IIA supergravity are already examined in [\[15,](#page-10-12) [37,](#page-11-14) [38\]](#page-11-15).[3](#page-2-2)

In section [2](#page-2-0) and 3, we test the brane-jet instabilities of AdS fixed points from sixand seven-dimensional gauged supergravity, respectively. In an appendix, we present the calculation of potentials of the fluxes for supersymmetric flows and show that the probe brane potentials vanish over the whole flows identically.

2 The AdS_6 fixed points

2.1 Solutions in massive type IIA supergravity

We consider the scalar-gravity action of $F(4)$ gauged supergravity, [\[20\]](#page-10-16), in the conventions of [\[30\]](#page-11-8),

$$
e^{-1}\mathcal{L} = R - \frac{1}{2}\partial_{\mu}\phi\partial^{\mu}\phi - g^{2}\left(\frac{2}{9}e^{\frac{3}{\sqrt{2}}\phi} - \frac{8}{3}e^{\frac{1}{\sqrt{2}}\phi} - 2e^{-\frac{1}{\sqrt{2}}\phi}\right). \tag{2.1}
$$

There are supersymmetric and non-supersymmetric fixed points of the scalar potential at $e^{-\frac{1}{2\sqrt{2}}\phi} = 1$ and $e^{-\frac{1}{2\sqrt{2}}\phi} = 1/3^{1/4}$, respectively.

³Some massless solutions considered in [\[15\]](#page-10-12) would be obtained from dimensional reduction of the AdS₇ solutions of eleven-dimensional supergravity we study in this work.

We consider the domain wall background,

$$
ds_6^2 = e^{2A} ds_{1,4}^2 + dr^2, \qquad (2.2)
$$

where $A = r/l$ at the AdS₆ fixed points. The radius of AdS₆ is given by l.

We employ the uplift formula to massive type IIA supergravity, $[39]$, in $[30]$. In Einstein frame, the metric, the dilaton, and the four-form flux are non-trivial and are given, respectively, by, [\[40\]](#page-12-1),

$$
ds^2 = X^{1/8} \sin^{1/12} \xi \left(\Delta^{3/8} ds_6^2 + \frac{2}{g^2} \Delta^{3/8} X^2 d\xi^2 + \frac{1}{2g^2} \frac{\cos^2 \xi}{\Delta^{5/8} X} ds_{S^3}^2 \right),
$$
 (2.3)

$$
e^{\Phi} = \frac{\Delta^{1/4}}{X^{5/4} \sin^{5/6} \xi},
$$
\n(2.4)

$$
F_{(4)} = -\frac{\sqrt{2}}{6} \frac{U \sin^{1/3} \xi \cos^3 \xi}{g^3 \Delta^2} d\xi \wedge \text{vol}_{S^3} - \sqrt{2} \frac{\sin^{4/3} \xi \cos^4 \xi}{g^3 \Delta^2 X^3} dX \wedge \text{vol}_{S^3},\tag{2.5}
$$

where we define

$$
X = e^{-\frac{\phi}{2\sqrt{2}}},
$$

\n
$$
\Delta = X \cos^2 \xi + X^{-3} \sin^2 \xi,
$$

\n
$$
U = X^{-6} \sin^2 \xi - 3X^2 \cos^2 \xi + 4X^{-2} \cos^2 \xi - 6X^{-2}.
$$
\n(2.6)

The metric and the volume form of the three-sphere are given, respectively, by

$$
ds_{S^3}^2 = \sum_{I=1}^3 (\sigma^I)^2 ,
$$

\n
$$
\text{vol}_{S^3} = \sigma_1 \wedge \sigma_2 \wedge \sigma_3 ,
$$
\n(2.7)

where σ^I are SU(2) left-invariant one-forms,

$$
d\sigma^I = -\frac{1}{2} \epsilon_{IJK} \sigma^J \wedge \sigma^K . \qquad (2.8)
$$

We may introduce explicit $SU(2)$ left-invariant one-forms,

$$
\sigma^{1} = -\sin \alpha_{2} \cos \alpha_{3} d\alpha_{1} + \sin \alpha_{3} d\alpha_{2},
$$

\n
$$
\sigma^{2} = \sin \alpha_{2} \sin \alpha_{3} d\alpha_{1} + \cos \alpha_{3} d\alpha_{2},
$$

\n
$$
\sigma^{3} = \cos \alpha_{2} d\alpha_{1} + d\alpha_{3}.
$$
\n(2.9)

Then the metric and the volume form are

$$
ds_{S^3}^2 = d\alpha_1^2 + d\alpha_2^2 + d\alpha_3^2 + 2\cos\alpha_2 d\alpha_1 d\alpha_3,
$$

\n
$$
\text{vol}_{S^3} = \sin\alpha_2 d\alpha_1 d\alpha_2 d\alpha_3.
$$
\n(2.10)

2.2 D4-brane probes

The uplift formula for the six-form flux is given by, [\[30\]](#page-11-8),

$$
F_{(6)} = e^{\Phi/2} * F_{(4)} = -\frac{\sqrt{2}g}{3} U \text{vol}_6 + \frac{4\sqrt{2}}{g} \frac{\sin\xi\cos\xi}{X} * dX \wedge d\xi + \cdots
$$
 (2.11)

At the AdS_6 fixed points, it gives

$$
F_{(6)} = -\frac{\sqrt{2}g}{3} U e^{5A} dx_0 \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge dx_4 \wedge dr,
$$
 (2.12)

where

$$
A = \frac{r}{l}, \qquad U = U(\xi), \qquad X = \text{constant}, \tag{2.13}
$$

and l is the radius of AdS_6 . Thus we obtain that the five-form potential is

$$
C_{(5)} = \frac{\sqrt{2}g}{3} \frac{l}{5} U e^{5A} dx_0 \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge dx_4 , \qquad (2.14)
$$

where we use $\partial_r U = 0$, $\partial_{\xi} U = 0$ at the fixed points. U is so-called geometric scalar potential.[4](#page-4-1)

We partition the spacetime coordinates,

$$
x^{a} = \{x_{0}, x_{1}, x_{2}, x_{3}, x_{4}\}, \qquad y^{m} = \{r, \xi, \alpha_{1}, \alpha_{2}, \alpha_{3}\}, \tag{2.15}
$$

and choose the static gauge,

$$
x_0 = t = \eta^0
$$
, $x^a = \eta^a$, $y^m = y^m(t)$, (2.16)

where η^a are the worldvolume coordinates. The pull-back of the metric is

$$
\tilde{G}_{ab} = G_{\mu\nu} \frac{\partial x^{\mu}}{\partial \eta^{a}} \frac{\partial x^{\nu}}{\partial \eta^{b}}.
$$
\n(2.17)

Now we study the worldvolume action of the D4-branes which is given by a sum of DBI and WZ terms. If the probe branes move slowly, the worldvolume action in Einstein frame is

$$
S = -e^{\Phi/4} \int d^5 \eta \sqrt{-\det(\tilde{G})} - \int \tilde{C}_{(5)}
$$

=
$$
-\frac{\Delta^{1/16}}{X^{5/16} \sin^{5/24} \xi} \int d^5 \eta \Big(e^{5A} \Delta^{15/16} X^{5/16} \sin^{5/24} \xi - \frac{1}{2} e^{3A} \Delta^{9/16} X^{3/16} \sin^{1/8} \xi G_{mn} \dot{y}^m \dot{y}^n + \cdots \Big) - \int \frac{\sqrt{2}g}{3} \frac{l}{5} e^{5A} U \, dx_0 \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge dx_4, \qquad (2.18)
$$

where $\tilde{C}_{(5)}$ $\tilde{C}_{(5)}$ $\tilde{C}_{(5)}$ is the pull-back of the five-form potential.⁵ Then the worldvolume action reduces to

$$
S = \int d^5 \eta \left(K - V \right) , \qquad (2.19)
$$

⁴For the supersymmetric flows we can calculate the five-form potential over the whole flow. See appendix [A.1.](#page-8-1)

⁵The sign of the $\tilde{C}_{(5)}$ term is determined by the orientation of our solution. There is an overall sign choice for the supersymmetry equations in $(A.2)$ and it is interelated to the sign choice.

Figure 1. he probe brane potentials of the supersymmetric and non-supersymmetric fixed points at $X = 1$ and $X = 1/3^{1/4}$, respectively.

where the kinetic and the potential terms are

$$
K = \frac{1}{2}e^{3A} \frac{\Delta^{5/8}}{X^{1/8}\sin^{1/12}\xi} G_{mn} \dot{y}^m \dot{y}^n + \cdots ,
$$

\n
$$
V = e^{5A} \left(\Delta + \frac{\sqrt{2}g}{3} \frac{l}{5} U \right) .
$$
\n(2.20)

The final probe brane potential is quite simple. From the probe brane potential, we test the brane-jet instabilities of the supersymmetric and non-supersymmetric AdS_6 fixed points. We set $g = \frac{3\sqrt{2}}{2}$ $\frac{\sqrt{2}}{2}$ for $l = 1$. The plots of the brane potential over the hemisphere, $0 \leq \xi \leq \pi$, are given in figure [1.](#page-5-2) We conclude that the non-supersymmetric AdS₆ fixed point is not stable.

3 The AdS_7 fixed points

3.1 Solutions in eleven-dimensional supergravity

We consider the minimal scalar-gravity action of seven-dimensional gauged supergravity, $[21, 22]$ $[21, 22]$ $[21, 22]$ and $[23, 24]$ $[23, 24]$ $[23, 24]$, in the conventions of $[41]$,

$$
e^{-1}\mathcal{L} = R - 20\partial_{\mu}\lambda\partial^{\mu}\lambda + g^{2}\left(4X^{2} + 4X^{-3} - \frac{1}{2}X^{-8}\right),
$$
\n(3.1)

where $X = e^{2\lambda}$. There are supersymmetric and non-supersymmetric fixed points of the scalar potential at $X = 1$ and $X = 1/2^{1/5}$, respectively.

We consider the domain wall background,

$$
ds_7^2 = e^{2A} ds_{1,5}^2 + dr^2, \qquad (3.2)
$$

where $A = r/l$ at the AdS₇ fixed points. The radius of AdS₇ is given by l.

We employ the uplift formula to eleven-dimensional supergravity, [\[42\]](#page-12-3), in [\[41\]](#page-12-2). The metric and the seven-form flux are given by,

$$
ds^{2} = \Delta^{1/3} ds_{7}^{2} + \frac{1}{g^{2}} \Delta^{-2/3} \left(X_{0}^{-1} d\mu_{0}^{2} + \sum_{i=1}^{2} X_{i}^{-1} \left(d\mu_{i}^{2} + \mu_{i}^{2} d\phi_{i}^{2} \right) \right), \tag{3.3}
$$

$$
F_{(7)} = U \text{vol}_7 + \frac{1}{2g} \sum_{\alpha=0}^{2} X_{\alpha}^{-1} *_{7} dX_{\alpha} \wedge d(\mu_{\alpha}^{2}), \qquad (3.4)
$$

where vol₇ and $*$ ₇ are volume form and Hodge dual on ds_7^2 . We define^{[6](#page-6-1)}

$$
X = X_1 = X_2 = e^{2\lambda}, \qquad X_0 = (X_1 X_2)^{-2},
$$

\n
$$
\Delta = \sum_{\alpha=0}^{2} X_{\alpha} \mu_{\alpha}^{2},
$$

\n
$$
U = 2g \sum_{\alpha=0}^{2} (X_{\alpha}^{2} \mu_{\alpha}^{2} - \Delta X_{\alpha}) + g \Delta X_{0},
$$

\n
$$
1 = \sum_{\alpha=0}^{2} \mu_{\alpha}^{2}.
$$
\n(3.5)

We introduce explicit coordinates,

$$
\mu_0 = \cos \alpha \,, \qquad \mu_1 = \sin \alpha \cos \beta \,, \qquad \mu_2 = \sin \alpha \cos \beta \,. \tag{3.6}
$$

3.2 M5-brane probes

At the $AdS₇$ fixed points, the seven-form flux is

$$
F_{(7)} = U e^{6A} dx_0 \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge dx_4 \wedge dx_5 \wedge dr,
$$
\n(3.7)

where

$$
A = \frac{r}{l}, \qquad U = U(\alpha), \qquad X = \text{constant}, \tag{3.8}
$$

and l is the radius of AdS₇. Thus we obtain that the six-form potential is

$$
C_{(6)} = \frac{l}{6} U e^{6A} dx_0 \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge dx_4 \wedge dx_5, \qquad (3.9)
$$

where we use $\partial_r U = 0$, $\partial_\alpha U = 0$ at the fixed points. U is so-called geometric scalar potential.[7](#page-6-2)

We partition the spacetime coordinates,

$$
x^{a} = \{x_{0}, x_{1}, x_{2}, x_{3}, x_{4}, x_{5}\}, \qquad y^{m} = \{r, \alpha, \beta, \phi_{1}, \phi_{2}\}, \qquad (3.10)
$$

 6 For the scalar fields in [\[41\]](#page-12-2), $X_1 = e^{-\frac{1}{\sqrt{2}}\varphi_1 - \frac{1}{\sqrt{10}}\varphi_2}$ and $X_2 = e^{\frac{1}{\sqrt{2}}\varphi_1 - \frac{1}{\sqrt{10}}\varphi_2}$, we set $\varphi_1 = 0$ and $\varphi_2 = \frac{1}{\sqrt{2}}$ $-2\sqrt{10\lambda}$.

⁷For the supersymmetric flows we can calculate the six-form potential over the whole flow. See appendix [A.2.](#page-9-0)

and choose the static gauge,

$$
x_0 = t = \eta^0
$$
, $x^a = \eta^a$, $y^m = y^m(t)$, (3.11)

where η^a are the worldvolume coordinates. The pull-back of the metric is

$$
\tilde{G}_{ab} = G_{\mu\nu} \frac{\partial x^{\mu}}{\partial \eta^a} \frac{\partial x^{\nu}}{\partial \eta^b}.
$$
\n(3.12)

Now we study the worldvolume action of the M5-branes which is given by a sum of DBI and WZ terms. If the probe branes move slowly, the worldvolume action is

$$
S = -\int d^6\eta \sqrt{-\det(\tilde{G})} - \int \tilde{C}_{(6)}
$$

=
$$
- \int d^6\eta \left(e^{6A} \Delta - \frac{1}{2} e^{4A} \Delta^{2/3} G_{mn} \dot{y}^m \dot{y}^n + \cdots \right)
$$

$$
- \int \frac{l}{6} U e^{6A} dx_0 \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge dx_4 \wedge dx_5 , \qquad (3.13)
$$

where $\tilde{C}_{(6)}$ is the pull-back of the six-form potential.^{[8](#page-7-0)} Then the worldvolume action reduces to

$$
S = \int d^6 \eta \left(K - V \right) , \qquad (3.14)
$$

where the kinetic and the potential terms are

$$
K = \frac{1}{2}e^{4A}\Delta^{2/3}G_{mn}\dot{y}^m\dot{y}^n + \cdots,
$$

$$
V = e^{6A}\left(\Delta + \frac{l}{6}U\right).
$$
 (3.15)

The final probe brane potential is quite simple. From the probe brane potential, we test the brane-jet instabilities of the supersymmetric and non-supersymmetric AdS_7 fixed points. We set $g = 2$ for $l = 1$. The plots are given in figure [2.](#page-8-3) We conclude that the nonsupersymmetric $AdS₇$ fixed point is *not* stable.

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⁸The sign of the $\tilde{C}_{(6)}$ term is determined by the orientation of our solution. There is an overall sign choice for the supersymmetry equations in $(A.10)$ and it is interelated to the sign choice.

Figure 2. The probe brane potentials of the supersymmetric and non-supersymmetric fixed points at $X = 1$ and $X = 1/2^{1/5}$, respectively.

A Potentials of the fluxes for supersymmetric flows

For the flows to supersymmetric fixed points, we can derive the potentials of the fluxes not just at the fixed point but over the whole flow. In the appendix we present the derivations.

A.1 Flows from AdS_6

We consider the domain wall background, [\[43\]](#page-12-4),

$$
ds_6^2 = e^{2A} ds_{1,4}^2 + dr^2.
$$
 (A.1)

The supersymmetry equations are given by

$$
\phi' = g \left(e^{-\frac{\phi}{2\sqrt{2}}} - e^{\frac{3\phi}{2\sqrt{2}}} \right) ,
$$

\n
$$
A' = \frac{g}{2\sqrt{2}} \left(e^{-\frac{\phi}{2\sqrt{2}}} + \frac{1}{3} e^{\frac{3\phi}{2\sqrt{2}}} \right) .
$$
\n(A.2)

The uplift formula for the six-form flux is given by, [\[30\]](#page-11-8),

$$
F_{(6)} = e^{\Phi/2} * F_{(4)} = -\frac{\sqrt{2}g}{3} U \text{vol}_6 + \frac{4\sqrt{2}}{g} \frac{\sin\xi\cos\xi}{X} * dX \wedge d\xi.
$$
 (A.3)

For the domain wall solutions, the six-form flux is

$$
F_{(6)} = \omega_r dx_0 \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge dx_4 \wedge dr + \omega_\xi dx_0 \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge dx_4 \wedge d\xi, \qquad (A.4)
$$

where

$$
\omega_r = -\frac{\sqrt{2}g}{3} e^{5A} U,
$$

$$
\omega_{\xi} = \frac{4\sqrt{2}}{g} \frac{e^{5A} X' \sin \xi \cos \xi}{X}.
$$
 (A.5)

Employing the supersymmetry equations, $(A.2)$, they satisfy a relation,^{[9](#page-9-2)}

$$
\frac{\partial \omega_{\xi}}{\partial r} = \frac{\partial \omega_{r}}{\partial \xi} \,. \tag{A.6}
$$

Then we obtain that the five-form potential is

$$
C_{(5)} = -e^{5A} \Delta dx_0 \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge dx_4.
$$
 (A.7)

If we employ this five-form potential to compute the probe brane potential, it vanishes identically over the whole flow,

$$
V = e^{5A} (\Delta - \Delta) = 0.
$$
 (A.8)

A.2 Flows from AdS₇

We consider the domain wall background, [\[46\]](#page-12-5),

$$
ds_7^2 = e^{2A} ds_{1,5}^2 + dr^2.
$$
 (A.9)

The supersymmetry equations are given by

$$
\lambda' = \frac{2}{5}e^{-8\lambda} - \frac{2}{5}e^{2\lambda},
$$

\n
$$
A' = \frac{1}{5}e^{-8\lambda} + \frac{4}{5}e^{2\lambda}.
$$
\n(A.10)

The uplift formula for the seven-form flux is given by, [\[41\]](#page-12-2),

$$
F_{(7)} = U \text{vol}_7 + \frac{1}{2g} \sum_{\alpha=0}^{2} X_{\alpha}^{-1} *_{7} dX_{\alpha} \wedge d(\mu_{\alpha}^{2}). \tag{A.11}
$$

From an analogous calculation of the previous subsection, we obtain that the six-form potential is

$$
C_{(6)} = -e^{6A} \Delta dx_0 \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge dx_4 \wedge dx_5.
$$
 (A.12)

If we employ this six-form potential to compute the probe brane potential, it vanishes identically over the whole flow,

$$
V = e^{6A} \left(\Delta - \Delta \right) = 0. \tag{A.13}
$$

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⁹This is an analogous calculation of (3.13) , (3.14) , (3.28) , (3.29) from [\[44\]](#page-12-6) and (8) from [\[45\]](#page-12-7).

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