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Non-singular string cosmology via α' corrections

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ABSTRACT: In string theory, an important challenge is to show if the big-bang singularity could be resolved by the higher derivative α' corrections. In this work, based on the Hohm-Zwiebach action, we construct a series of non-singular non-perturbative cosmological solutions with the complete α' corrections, for the bosonic gravi-dilaton system. In the perturbative regime, these solutions exactly match the perturbative results given in literature. Our results show that the big-bang singularity indeed could be smoothed out by the higher derivative α' corrections.

KEYWORDS: Bosonic Strings, Nonperturbative Effects, Spacetime Singularities, String Duality

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1 Introduction

The big-bang singularity appears in the Einstein gravity as the initial singularity. Nevertheless, in string cosmology, the situation is somewhat different. For $D = d+1$ dimensional spacetime, in a cosmological context, where all the fields only depend on time, the string effective action possesses a “scale-factor duality” [1–5], which turns out to be a particular case of $O(d, d)$ symmetry.

The scale-factor duality had been first observed in the tree level¹ gravi-dilaton effective theory, for cosmological background. It shows that the equations of motion (EOM) with the FLRW-like background is invariant under the transformation between the scale factor and its inverse, $a(t) \longleftrightarrow 1/a(t)$. This duality is absent in the Einstein gravity since the dilaton plays a central role in the transformation. There are two main differences between T-duality and scale-factor duality. The first is that the scale-factor duality does not require compactified backgrounds. Furthermore, the scale-factor duality is a property of classical fields, in contrast to that T-duality is manifested by the energy levels of the quantum string. The combination of time-reversal and the scale-factor duality leads to a remarkable pre-big-bang cosmology [6–9]. It implies that there exists a long evolution in the region of pre-big-bang. Pre- and post-big-bang scenarios are disconnected by the big-bang singularity.

As the universe approaches the big-bang singularity, the growth of the string coupling $g_s = \exp(2\phi)$ and the Hubble parameter $H(t)$ makes the perturbative theory break down. In such non-perturbative regions, two kinds of corrections should be included: (1) higher derivative α' corrections at the string curvature scale $H(t) \sim 1/\sqrt{\alpha'}$, and (2) the quantum loop corrections at the strong coupling regime $g_s \sim 1$. The first expansion represents “stringy” effects and has no correspondence in point particle. The second one is similar to the loop expansion in quantum field theory, dedicated to quantum effects.

It is natural to expect that these corrections could resolve the big-bang singularity in the non-perturbative regime. However, the difficulty is that little is known for specific

¹In this paper, “tree level” means the theory is truncated to the lowest order in both α' expansion and loop expansion.

information of these two kinds of corrections. As for the quantum loop corrections, a lot of effective dilaton potentials were proposed and indeed, the big-bang singularity could be smoothed out by these phenomenological models [9, 10]. A comprehensive treatment and a large number of references are referred to [11].

On the other hand, progresses on this problem for the α' corrections are not satisfactory. The main reason is that, the higher order α' corrections in general are expected to change the EOM from second order differential equations to higher order differential equations. Thus the α' corrections cannot be implemented simply by adding some phenomenological effective terms. For the first order α' correction, it is well known that the higher than second order derivatives can be eliminated from the EOM by field redefinitions [12]. By using this property and carefully designed final states, in ref. [13], the authors *numerically* verified that the singularity could be smoothed out, at the price of losing the scale-factor duality. In [14], by assuming the heterotic string admits non-singular constant curvature solutions in the Einstein frame, an $O(d, d)$ violating first order α' correction was chosen. The EOM, as expected, are fourth order differential equations in terms of the scale factor $a(t)$. A special simple effective dilaton potential was also introduced to enable a non-singular evolution from an early-time de-Sitter phase to a late-time Minkowski space-time. On the other hand, as showed in ref. [15], the perturbative solutions obtained order by order always suffer the big-bang singularity. Since the truncation of the α' corrections typically causes various pathologies, analytical non-perturbative analyses of the full stringy effects are desirable. Some other relevant discussions can be found in refs. [16–18].

To this end, let us focus on the recent remarkable developments on classifying all the α' corrections. In 1990s, Meissner and Veneziano showed that, to the first order in α' , when all fields only depend on time, the classical string effective action has an explicit $O(d, d)$ symmetry [1, 19]. Sen proved this is true to all orders in α' and for configurations independent of m coordinates, the symmetry is $O(m, m)$ [2, 3]. This is also confirmed in ref. [20] from the perspective of σ model expansion. It turns out that to the first order in α' , the $O(d, d)$ matrix can maintain the standard form in terms of α' corrected fields, for both time dependent [19] or single space dependent configurations [21]. One can be easily convinced that this is also true for all orders in α' , from the derivations in [19, 21]. Based on this assumption, Hohm and Zwiebach [15, 22, 23] showed that, for cosmological configurations, the α' corrections to all orders, can be put into incredibly simple patterns. The dilaton appears trivially and only first order time derivatives need to be included. This seminal progress makes it possible to conduct non-perturbative analyses on the stringy effects. They subsequently proved that non-perturbative de-Sitter (dS) vacua are possible in bosonic string theory. The analogy in the Einstein frame is then studied in [24]. In our recent work [21], we showed that for fields dependent on a single space coordinate, similar story occurs and non-perturbative Anti-de-Sitter (AdS) vacua are also acceptable. Furthermore, we proposed a conjecture that the non-perturbative AdS and dS vacua might not be able to coexist in bosonic string theory.

Based on the Hohm-Zwiebach action, the purpose of this paper is to construct non-perturbative non-singular cosmological solutions. One may question this is not viable since only the first two orders in α' expansion have been determined and there are infinitely many

unknown coefficients. To answer this question, let us recall the methodology adopted to study the loop corrections. Since the loop corrections do not change the order of the differential equations, one may implement the loop corrections phenomenologically by a (non-local) dilaton potential, which of course must respect some general conditions, say, $O(d, d)$ and general coordinate covariance. A large number of such examples are summarized in ref. [11]. So, in the similar pattern, for the case concerned here, we could make some ansatz for the coefficients of the higher orders and solve the equations of motion (EOM). It turns out that, even constructing phenomenological solutions is difficult, since the EOM are nonlinear and the cosmological solutions are constrained by two conditions: (a) In the perturbative regime, namely, as $\alpha' \rightarrow 0$ or $|t| \rightarrow \infty$, the solution must reduce to the perturbative vacua. Particularly, the first two orders, which are already known, of the perturbative solution should be exactly matched. (b) The solution is supposed to be regular everywhere. Indeed, the solutions we construct in this work do respect these constraints. As expected, these solutions are obviously non-perturbative since they are defined in the whole regime $t \in (-\infty, \infty)$, in sharp contrast to the perturbative solution which is defined only in the perturbative regime $\alpha' \rightarrow 0$ or $|t| \rightarrow \infty$. Thus the pre-big-bang and post-big-bang are smoothly connected in these solutions.

The reminder of this paper is outlined as follows. In section 2, we briefly review the results of string cosmology. In section 3, we construct consistent non-singular solutions. Section 4 is the conclusion.

2 A brief review of string cosmology

It is of help to review some relevant results of string cosmology for later convenience. A comprehensive treatment is referred to [11] and references therein. We start with the tree level string effective action without matter sources. The action in $D = d + 1$ dimensional spacetime is

$$I_0 = \int d^D x \sqrt{-g} e^{-2\phi} \left[R + 4(\partial_\mu \phi)^2 - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} \right], \quad (2.1)$$

where ϕ is the physical dilaton, $g_{\mu\nu}$ is the string metric, and $H_{\mu\nu\rho} = 3\partial_{[\mu} b_{\nu\rho]}$ is the field strength of the anti-symmetric Kalb-Ramond $b_{\mu\nu}$ field. For simplicity, we set the anti-symmetric Kalb-Ramond field $b_{\mu\nu} = 0$. The equations of motion (EOM) are given by

$$\begin{aligned} R_{\mu\nu} + 2\nabla_\mu \nabla_\nu \phi &= 0, \\ \nabla^2 \phi - 2(\partial_\mu \phi)^2 &= 0. \end{aligned} \quad (2.2)$$

It is convenient for following discussions to introduce the $O(d, d)$ invariant dilaton field:

$$\Phi = 2\phi - \ln \sqrt{-g}. \quad (2.3)$$

For the FLRW background,

$$ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j, \quad (2.4)$$

I	$\dot{a}_+(t) > 0$, expansion	$\ddot{a}_+(t) < 0$, decelerated	$\dot{H}_+ < 0$, decreasing curvature	post-big bang
II	$\dot{a}_-(t) < 0$, contraction	$\ddot{a}_-(t) > 0$, decelerated	$\dot{H}_- > 0$, decreasing curvature	post-big bang
III	$\dot{a}_+(-t) < 0$, contraction	$\ddot{a}_+(-t) < 0$, accelerated	$\dot{H}_+ < 0$, increasing curvature	pre-big bang
IV	$\dot{a}_-(-t) > 0$, expansion	$\ddot{a}_-(-t) > 0$, accelerated	$\dot{H}_- > 0$, increasing curvature	pre-big bang

Table 1. The properties of the string cosmological solutions at the leading order.

the EOM (2.2) become

$$\begin{aligned}
 2\ddot{\Phi} - \dot{\Phi}^2 - dH^2 &= 0, \\
 -dH^2 + \ddot{\Phi} &= 0, \\
 \dot{H} - \dot{\Phi}H &= 0,
 \end{aligned}
 \tag{2.5}$$

where the Hubble parameter is defined as $H(t) = \frac{\dot{a}}{a} = \frac{d}{dt} \log a(t)$. The EOM are invariant under the scale-factor duality:

$$a(t) \longleftrightarrow a(t)^{-1}, \quad H \rightarrow -H \quad \Phi(t) \longleftrightarrow \Phi(t).
 \tag{2.6}$$

The system is also invariant under time reversal $t \rightarrow -t$. The dual solutions hence are

$$ds_{\pm}^2 = -dt^2 + \left| \frac{t}{t_0} \right|^{\pm 2/\sqrt{d}} \delta_{ij} dx^i dx^j, \quad \Phi = -\ln \left| \frac{t}{t_0} \right|,
 \tag{2.7}$$

with the following notations:

$$a_{\pm}(t) = \left| \frac{t}{t_0} \right|^{\pm 1/\sqrt{d}}, \quad H_{\pm}(t) = \frac{\dot{a}_{\pm}}{a_{\pm}} = \pm \frac{1}{\sqrt{d}|t|}.
 \tag{2.8}$$

The properties of the solutions are summarized in table 1.

Note that deceleration occurs when $\text{sign } \dot{a} = -\text{sign } \ddot{a}$, and acceleration occurs when $\text{sign } \dot{a} = \text{sign } \ddot{a}$. When H^2 is growing with time, the curvature is increasing, otherwise the curvature is decreasing. Moreover, when $H > 0$, the universe is expanding, otherwise the universe is contracting. All these solutions share the curvature singularity located at $|t| \rightarrow 0$ as illustrated in figure 1.

3 Non-singular string cosmology via α' corrections

It turns out that for FLRW metric (2.4), the action (2.1) can be recast in an $O(d, d)$ covariant form. To this end, it is convenient to choose the gauge $b_{0i} = 0$ and write the fields in the form

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 \\ 0 & G_{ij}(t) \end{pmatrix}, \quad b_{\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & B_{ij}(t) \end{pmatrix},
 \tag{3.1}$$

where G_{ij} and B_{ij} are $d \times d$ matrices representing the spatial part of the tensors. The action can be rewritten as

$$I_0 = \int dt e^{-\Phi} \left[-\dot{\Phi}^2 - \frac{1}{8} \text{Tr} \left(\dot{\mathcal{S}}^2 \right) \right],
 \tag{3.2}$$

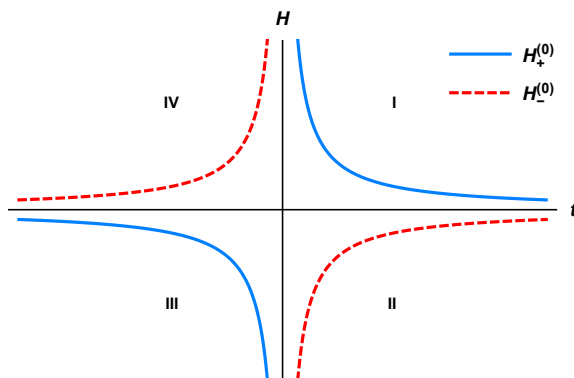


Figure 1. The evolutions of the Hubble parameters of four solutions (we set $d = 3$ in this plot).

with

$$M = \begin{pmatrix} G^{-1} & -G^{-1}B \\ BG^{-1} & G - BG^{-1}B \end{pmatrix}, \quad \mathcal{S} = \eta M = \begin{pmatrix} BG^{-1} & G - BG^{-1}B \\ G^{-1} & -G^{-1}B \end{pmatrix}, \quad (3.3)$$

where η is the invariant metric of the $O(d, d)$ group

$$\eta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}. \quad (3.4)$$

Noticing M is symmetric and then $\mathcal{S} = \mathcal{S}^{-1}$, this action is manifestly invariant under the $O(d, d)$ transformations

$$\Phi \longrightarrow \Phi, \quad \mathcal{S} \longrightarrow \tilde{\mathcal{S}} = \Omega^T \mathcal{S} \Omega, \quad (3.5)$$

where Ω is a constant matrix, satisfying

$$\Omega^T \eta \Omega = \eta. \quad (3.6)$$

For vanishing Kalb-Ramond field $B = 0$ and the FLRW metric (2.4), $G_{ij} = \delta_{ij} a^2(t)$, the matrix \mathcal{S} becomes

$$\mathcal{S} = \begin{pmatrix} 0 & a^2(t) \\ a^{-2}(t) & 0 \end{pmatrix}. \quad (3.7)$$

Choosing $\Omega = \eta$ in (3.5), we have a new inequivalent solution

$$\tilde{\mathcal{S}} = \begin{pmatrix} 0 & a^{-2}(t) \\ a^2(t) & 0 \end{pmatrix}, \quad (3.8)$$

which is precisely the scale-factor duality. We thus see that the scale-factor duality does belong to the more general $O(d, d)$ symmetry.

When higher derivative terms are introduced to the action, the standard $O(d, d)$ matrix (3.3) receives higher order α' corrections. To the first order in α' , these corrections can be absorbed into the field redefinitions to keep the standard $O(d, d)$ matrix unchanged [19].

The derivations in ref. [19] make it reliable to assume this also happens for all orders in α' . With this assumption, Hohm and Zwiebach [15, 22, 23] showed that all orders in α' are classified by even powers of $\dot{\mathcal{S}}$ only:

$$I = \int d^D x \sqrt{-g} e^{-2\phi} \left(R + 4(\partial\phi)^2 - \frac{1}{12}H^2 + \frac{1}{4}\alpha' (R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} + \dots) + \alpha'^2(\dots) + \dots \right), \tag{3.9}$$

$$= \int dt e^{-\Phi} \left(-\dot{\Phi}^2 + \sum_{k=1}^{\infty} (\alpha')^{k-1} c_k \text{tr} \left(\dot{\mathcal{S}}^{2k} \right) \right). \tag{3.10}$$

Eq. (3.9) is the classical action for the general background with all α' corrections included. In literature, only the zeroth order and first order in α' are unambiguously determined, while orders higher than one are still out of reach. Eq. (3.10), the Hohm-Zwiebach action, is the action in the FLRW background (2.4) with $B = 0$, where $c_1 = -\frac{1}{8}$ to recover eq. (3.2), $c_2 = \frac{1}{64}$ for the bosonic string theory [15] and $c_{k \geq 3}$ are yet unknown constants. The EOM of eq. (3.10) are

$$\begin{aligned} \ddot{\Phi} + \frac{1}{2}Hf(H) &= 0, \\ \frac{d}{dt} (e^{-\Phi} f(H)) &= 0, \\ \dot{\Phi}^2 + g(H) &= 0. \end{aligned} \tag{3.11}$$

where

$$\begin{aligned} H(t) &= \frac{\dot{a}(t)}{a(t)}, \\ f(H) &= d \sum_{k=1}^{\infty} (-\alpha')^{k-1} 2^{2(k+1)} k c_k H^{2k-1} = -2dH - 2d\alpha' H^3 + \mathcal{O}(\alpha'^2), \\ g(H) &= d \sum_{k=1}^{\infty} (-\alpha')^{k-1} 2^{2k+1} (2k-1) c_k H^{2k} = -dH^2 - \frac{3}{2}d\alpha' H^4 + \mathcal{O}(\alpha'^2). \end{aligned} \tag{3.12}$$

It is easy to check that $g'(H) = Hf'(H)$ and $g(H) = Hf(H) - \int_0^H f(x)dx$. With the surprising simplification of the Hohm-Zwiebach action, the non-perturbative EOM are still second order differential equations, even after including all the α' corrections!

It turns out that the perturbative solution of these EOM inevitably has the big-bang singularity in every order, as shown in ref. [15]. Therefore, the possible non-singular solutions must be non-perturbative. The main purpose of this paper is to construct such non-singular non-perturbative solutions. One may doubt this is not possible since there are infinitely many unknown coefficients $c_{k \geq 3}$. To answer this question, let us recall the methodology adopted to study the loop corrections. Since the loop corrections do not change the order of the differential equations, one may implement the loop corrections phenomenologically by a (non-local) dilaton potential, which of course must maintain the

$O(d, d)$ symmetry and general coordinate covariance. A large number of such examples are summarized in the book [11]. For the case studied here, due to the Hohm-Zwiebach action, the higher order corrections do not change the order of the differential equations in the EOM, too. Thus, one can assume values for $c_{k \geq 3}$ and solve the EOM, at least phenomenologically. Two constraints must be respected by such cosmological solutions:

- a. As $\alpha' \rightarrow 0$ or $|t| \rightarrow \infty$, the solutions must exactly match the zeroth and first orders in α' of the perturbative results.
- b. The constructed solution is anticipated to be regular everywhere.

However, it is far from easy to look for such solutions. As an illustration, referring to eq. (3.12), one can make an ansatz for $f(H)$, whose first two terms of the expansion in α' agree with the perturbative results (easy). Then we have $g(H) = Hf(H) - \int_0^H f(x) dx$ (might be solvable). The insurmountable barrier is to solve $H(t)$ and $\Phi(t)$ by substituting $f(H)$ and $g(H)$ into the *nonlinear* EOM.

After amount of trial and error, we find a class of solutions of the EOM (3.11):

$$\begin{aligned} \Phi(t) &= \log \left(-\frac{\sqrt{\alpha'}}{\sqrt{32}d^{3/2}} f(t) \right), \\ H(t) &= \frac{\sqrt{\alpha'}}{2\sqrt{2}d^{3/2}t^2} \left(\frac{2^{\frac{5-6n}{2-4n}} d^{\frac{3-4n}{2-4n}} t}{\sqrt{\alpha'}} \right)^{2n} \left(\frac{4^{\frac{n}{1-2n}} d^{\frac{n}{1-2n}}}{\left(\frac{2^{\frac{5-6n}{2-4n}} d^{\frac{3-4n}{2-4n}} t}{\sqrt{\alpha'}} \right)^{2n} + 1} \right)^{-\frac{1}{2n}} \times \\ &\quad \left(\left(\frac{2^{\frac{5-6n}{2-4n}} d^{\frac{3-4n}{2-4n}} t}{\sqrt{\alpha'}} \right)^{2n} - 2n + 1 \right) \left(\left(\frac{2^{\frac{5-6n}{2-4n}} d^{\frac{3-4n}{2-4n}} t}{\sqrt{\alpha'}} \right)^{2n} + 1 \right)^{-2}, \end{aligned} \quad (3.13)$$

where n are positive integers. $f(t)$ and $g(t)$ are given by

$$\begin{aligned} f(t) &= -4\sqrt{2}d^{3/2} \left(\frac{4^{\frac{n}{1-2n}} d^{\frac{n}{1-2n}}}{\left(\frac{2^{\frac{5-6n}{2-4n}} d^{\frac{3-4n}{2-4n}} t}{\sqrt{\alpha'}} \right)^{2n} + \alpha'^n} \right)^{\frac{1}{2n}}, \\ g(t) &= -df(t)^2 - f(t)^2 \left(\left[\left(2\sqrt{d} \right)^{-\frac{2n}{2n-1}} - \left(\frac{\sqrt{\alpha'}}{\sqrt{32}d^{3/2}} f(t) \right)^{2n} \right]^{\frac{2n-1}{n}} - d \right). \end{aligned} \quad (3.14)$$

It is worth noting that $-H(t)$ is also a solution simply from the scale-factor duality:

$$H \rightarrow -H, \quad \Phi \rightarrow \Phi, \quad f(t) \rightarrow -f(t), \quad g(t) \rightarrow g(t). \quad (3.15)$$

In these solutions, the power n is determined by the particular value c_2 of various string theories. To match $c_2 = 1/64$ for the bosonic string theory we are concerned here,² we set

²For heterotic string, $c_2 = \frac{1}{128}$ and we also have $n = 1$. For type II strings, $c_2 = 0$ and we need to set $n \geq 2$. But one needs to first prove the Hohm-Zwiebach action for these string theories.

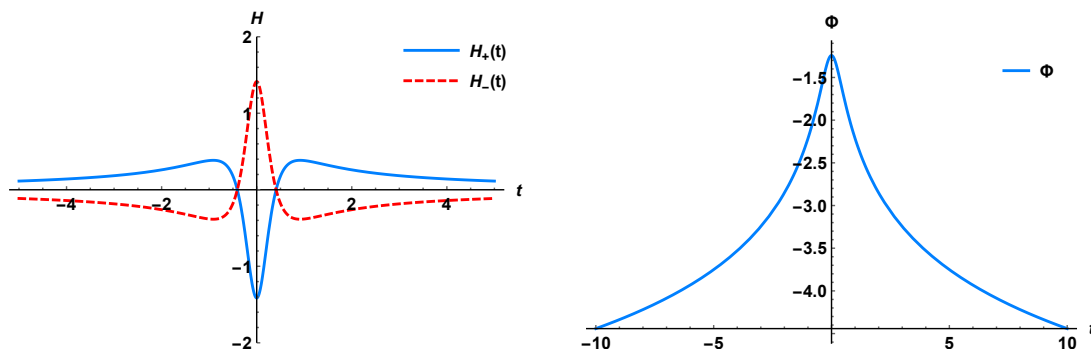


Figure 2. The left panel shows the non-perturbative Hubble parameters. Blue solid line describes $H_+(t)$ and red dashed line denotes $H_-(t)$. The right panel shows the $O(d, d)$ dilaton evolution along the time. We set $d = 3$, $\alpha' = 1$ and $n = 1$ in these plots.

$n = 1$ and the solutions are

$$\begin{aligned}
 H_{\pm}(t) &= \mp \frac{\sqrt{2}(\alpha' - 2dt^2)}{(\alpha' + 2dt^2)^{3/2}}, \\
 \Phi(t) &= \log\left(\frac{1}{2}\sqrt{\frac{\alpha'}{d(\alpha' + 2dt^2)}}\right), \\
 f_{\pm}(t) &= \mp \frac{2\sqrt{2}d}{\sqrt{\alpha' + 2dt^2}}, \\
 g(t) &= -\frac{4d^2t^2}{(\alpha' + 2dt^2)^2},
 \end{aligned} \tag{3.16}$$

as plotted in figure 2.

We now check the consistency of these solutions. To simplify the notation, we consider H_+ only and suppress the symbol $+$. It is obvious that, for fixed finite α' , the solutions are regular everywhere in $t \in (-\infty, \infty)$. The big-bang singularity is indeed smoothed out. In the perturbative regime, $|t| \rightarrow \infty$ (or equivalently $\alpha' \rightarrow 0$), the solution is expanded as

$$H(t) = \frac{1}{\sqrt{dt}} - \frac{5}{4} \frac{1}{d^{3/2}t^3} \alpha' + \mathcal{O}(\alpha'^2), \tag{3.17}$$

$$\begin{aligned}
 \Phi(t) &= -\frac{1}{2} \log\left(8d^2 \cdot \frac{t^2}{\alpha'}\right) - \frac{1}{4d} \cdot \frac{\alpha'}{t^2} + \mathcal{O}\left(\frac{\alpha'^2}{t^4}\right) \\
 &= -\log\left|\frac{t}{t_0}\right| - 2d \frac{t_0^2}{t^2} + \mathcal{O}\left(\frac{t_0^4}{t^4}\right), \quad t_0^2 \equiv \frac{\alpha'}{8d^2},
 \end{aligned} \tag{3.18}$$

$$\begin{aligned}
 f(H(t)) &= -\frac{2\sqrt{d}}{t} + \frac{\alpha'}{2\sqrt{dt}^3} + \mathcal{O}(\alpha'^2) \\
 &= -2d \left(\frac{1}{\sqrt{dt}} - \frac{5}{4} \frac{1}{d^{3/2}t^3} \alpha' + \dots\right) - 2d\alpha' \left(\frac{1}{\sqrt{dt}} - \frac{5}{4} \frac{1}{d^{3/2}t^3} \alpha' + \dots\right)^3 + \mathcal{O}(\alpha'^2) \\
 &= -2dH - 2d\alpha' H^3 + \mathcal{O}(\alpha'^2),
 \end{aligned} \tag{3.19}$$

$$\begin{aligned}
 g(H(t)) &= -\frac{1}{t^2} + \frac{\alpha'}{dt^4} + \mathcal{O}(\alpha'^2) \\
 &= -d \left(\frac{1}{\sqrt{dt}} - \frac{5}{4} \frac{1}{d^{3/2}t^3} \alpha' + \dots \right)^2 - \frac{3}{2} d\alpha' \left(\frac{1}{\sqrt{dt}} - \frac{5}{4} \frac{1}{d^{3/2}t^3} \alpha' + \dots \right)^4 + \mathcal{O}(\alpha'^2) \\
 &= -dH^2 - \frac{3}{2} d\alpha' H^4 + \mathcal{O}(\alpha'^2), \tag{3.20}
 \end{aligned}$$

It is ready to see that the first term of the Hubble parameter (3.17) or the dilaton (3.18) matches the perturbative result of the tree level string cosmology eq. (2.8) or (2.7), respectively. The second term of the Hubble parameter or the dilaton completely agrees with that calculated perturbatively in [15], where we adopt $c_1 = -\frac{1}{8}$ and $c_2 = \frac{1}{64}$ for the bosonic string theory. Moreover, we see that the singularity $t = 0$ in the perturbative solution is an artifact of the truncation.

In eq. (3.19) and eq. (3.20), we used eq. (3.17) to replace t by the Hubble parameter H . Comparing with the perturbative results eq. (3.12), we find complete agreement.

On the other hand, we have known that the big-bang singularity also could be resolved by the loop corrections implemented by phenomenological (non-local) dilaton potentials. It is of interest to take a look at the similarities and differences between these two kinds of corrections. To this end, a non-local dilaton potential

$$V(\Phi(t)) = -V_0 e^{4\Phi(t)}, \quad V_0 > 0, \tag{3.21}$$

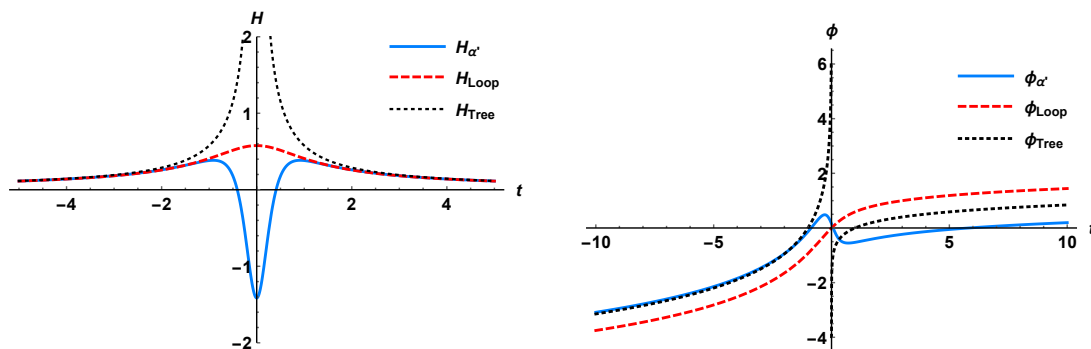
is added into the tree level action (2.1). The solution is [9, 10]:

$$H_{\text{Loop}}(t) = \left(t_0 \sqrt{d} \sqrt{\frac{t^2}{t_0^2} + 1} \right)^{-1}, \quad \Phi_{\text{Loop}}(t) = -\frac{1}{2} \log \left[\sqrt{V_0} t_0 \left(1 + \frac{t^2}{t_0^2} \right) \right], \tag{3.22}$$

where t_0 is an integration constant. Since the $O(d, d)$ symmetry is not broken by the potential, a scale-factor dual solution also exists, namely $H_{\text{Loop}}(t) \rightarrow -H_{\text{Loop}}(t)$, $\Phi_{\text{Loop}}(t) \rightarrow \Phi_{\text{Loop}}(t)$. We plot the non-perturbative α' -corrected, non-perturbative loop corrected and tree level perturbative solutions in figure 3. Note the right panel in figure 3 is the physical dilaton $\phi = \Phi/2 + 1/4 \log |g|$ rather than the $O(d, d)$ dilaton Φ . One can see that the α' corrections, much stronger than the loop corrections, leads to a contraction phase around $t = 0$. We would like to note that for the loop and α' corrected solutions with vanishing Kalb-Ramond field, the physical dilaton monotonically grows as $t \rightarrow \infty$. This is the property of the perturbative solutions which we need to match. However, considering the loop corrected solutions, ref. [25] showed that the stabilization of string coupling could be obtained by introducing the non-trivial Kalb-Ramond field. We therefore expect this stabilization mechanism also applies to the α' corrected solutions.

Finally, let us consider the Hubble parameter (3.16) in the Einstein frame. The relation between the string frame and the Einstein frame is given by

$$g_{\mu\nu}^E = \exp \left(-\frac{4\phi}{d-1} \right) g_{\mu\nu}. \tag{3.23}$$



Blue solid line:	Hubble parameter (left panel) and physical dilaton (right panel) with all α' corrections.
Red dashed line:	Hubble parameter (left panel) and physical dilaton (right panel) with loop corrections.
Black dotted line:	Tree level Hubble parameter (left panel) and physical dilaton (right panel).

Figure 3. The Hubble parameters and physical dilatons computed with various corrections.

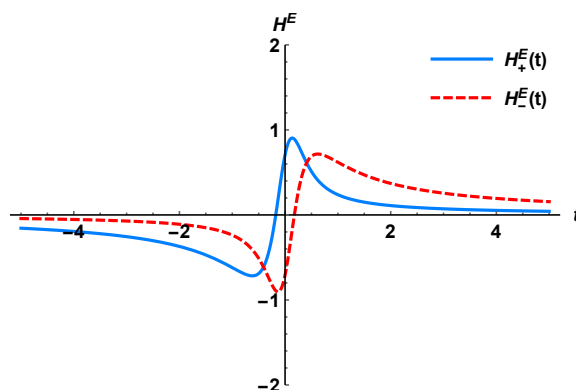


Figure 4. Hubble parameters in the Einstein frame.

Therefore, we have

$$\begin{aligned}
 H_{\pm}^E(t) &= \frac{\dot{a}_E(t)}{a_E(t)} = -\frac{1}{d-1} (\dot{\Phi} + H_{\pm}) \\
 &= \frac{2dt \left(\alpha'^{3/2} + 2\sqrt{\alpha'} dt^2 \right) \pm \sqrt{2(\alpha' + 2dt^2)} \left(\alpha'^{3/2} - 2\sqrt{\alpha'} dt^2 \right)}{\sqrt{\alpha'}(d-1)(\alpha' + 2dt^2)^2}, \quad (3.24)
 \end{aligned}$$

which are also regular in $t \in (-\infty, \infty)$, as plotted in figure 4.

4 Conclusions

In this paper, we constructed consistent non-perturbative non-singular cosmological solutions with all higher-derivative α' corrections included. This becomes possible because of the classification on the higher derivative terms by the Hohm-Zwiebach action. Though the construction is phenomenological, our solutions do confirm that the big-bang singularity could be resolved by α' corrections, in a non-perturbative way. As an outset in this

direction, we addressed gravi-dilaton system only. It would be of importance to include the time dependent Kalb-Ramond field or matter sources in the future work. By these extensions, we expect more realistic evolutions can be achieved.

In the last section, we compared the influences on the evolution by the α' correction and loop corrections. Both corrections are able to resolve the singularity. It is conceivable in the complete quantum gravity regime, their combination leads to a regular evolution. It is of interest to find (phenomenological) solutions with both kinds of corrections and some new features might arise.

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