Higher derivative three-form gauge theories and their supersymmetric extension

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ABSTRACT: We investigate three-form gauge theories with higher derivative interactions and their supersymmetric extensions in four space-time dimensions. For the bosonic three-form gauge theories, we show that derivatives on the field strength of the 3-form gauge field yield a tachyon as far as the Lagrangian contains a quadratic kinetic term, while such the term with opposite sign gives rise to a ghost. We confirm that there is neither a tachyon nor a ghost when all higher derivative terms are given by functions of the field strength. For this ghost/tachyon-free Lagrangian, we determine the boundary term necessary for the consistency between the equation of motion and energy-momentum tensor. For supersymmetric extensions, we present ghost/tachyon-free higher derivative interactions of arbitrary order of the field strength and corresponding boundary terms as well.

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1   Introduction

3-form gauge theories in four-dimensional (4D) spacetime have been studied extensively in the past decades. They were first considered in the context of quantum chromodynamics (QCD) to describe a long-range confinement force between quarks [1, 2]. Furthermore, a 3-form gauge field can be regarded to provide an effective description of a Chern-Simons 3-form in Yang-Mills theories [2, 3], in particular in the context of the U(1) problem [4, 5] and the strong CP problem [6–8]. In cosmology, a 3-form gauge field was used for dynamical neutralization of the cosmological constant [9–16], quintessence [17], inflationary models [18–23]. A relation between the 3-form gauge theories and condensed matter physics was also discussed, see e.g. [24]. A supersymmetric (SUSY) extension of the 3-form gauge fields was first formulated in ref. [25]. The SUSY 3-form gauge fields naturally appear in superstring theory and M-theory, therefore they were studied extensively with various applications: supergravity (SUGRA) [26–28] (see refs. [29, 30] as a review), Stückelberg coupling [31–34], topological coupling [33, 35, 36], coupling with a membrane [37–39], alternative formulation of old-minimal SUGRA [37, 40–42], gaugino condensation in SUSY Yang-Mills theories [43, 44], the cosmological constant problem [41], SUSY breaking [41, 45], string effective theories [42, 46–48], and inflationary models [35, 49]. Complex 3-form gauge theories were also considered in refs. [38, 39, 42].

One of the characterizations of p-form gauge fields is their couplings to extended objects. As 1-form and 2-form gauge fields can be electrically coupled to a particle and string, respectively in 4D spacetime, a 3-form gauge field can be electrically coupled to a membrane [1]. Since membranes and 3-form gauge fields naturally arise in string theory and M-theory as fundamental degrees of freedom, 3-form gauge fields in 4D spacetime appear as 4D compactification of these theories. Another characteristic feature of the 3-form gauge field is its coupling to scalar fields. Since a field strength of the 3-form gauge field is a 4-form, the 3-form gauge field can be topologically coupled to a pseudo-scalar field [4, 6, 18, 50]. It was pointed out that the topological coupling generates a potential (e.g., mass term) for the pseudo-scalar field while preserving a shift symmetry of the pseudo-scalar field in an action [17, 18]. This mechanism was applied to inflationary models [20, 22, 23, 51], quintessence [17], the strong CP problem [6–8] and so on.

Apart from many applications, the 3-form gauge field would have some theoretical and fundamental subtleties which we have to discuss carefully, compared with other p-form gauge theories. The 3-from gauge field with canonical (quadratic derivative) kinetic term has no dynamical degrees of freedom, and hence it is classically dual to a constant term, analogous to an electromagnetic field in 2D spacetime. Nevertheless, there are several merits to consider a 3-form gauge field itself rather than merely a constant. In fact, it was shown that a 3-form gauge field is inequivalent to a constant at a quantum level [52].

In contrast to other p-form gauge theories, Lagrangians of the 3-form gauge theories generally should include boundary terms [11, 12]. If the boundary term were missing, the functional variation of the 3-form gauge field at the boundary would not vanish, and consequently the energy-momentum tensor of the 3-form gauge field would be inconsistent with its equation of motion (EOM) [13, 14]. This situation is the same with the θ-term

	Bosonic	SUSY
Instability	−	− (cf. chiralsuperfield [53])
2nd (bulk)	[1]	[25]
2nd (boundary)	[11]	[31, 42]
H.D. (bulk)	Ghost/tachyon-free [6, 8]	Ghost/tachyon-free (4th order) [35]
		Ghost/tachyon (all order, chiral) [54]
		Ghost/tachyon-free (all order) −
H.D. (boundary)	−	−

Table 1. Higher derivative Lagrangians of 3-form gauge theories. "2nd" and "H.D." imply second order canonical term and higher derivative term, respectively. "−" implies that it has not been done and will be done in this paper. The reason why we list chiral superfield is that 3-form gauge theories can be formulated in terms of a chiral superfield.

in 2D electromagnetism, where the θ-term is needed if one considers a non-trivial field strength in the bulk.

As mentioned above, 3-form gauge theories are often considered to describe infrared (low-energy or long-range) effective theories, such as a Chern-Simons 3-form in QCD, and compactifications of string theory and M-theory. Since effective theories inevitably include nonrenormalizable interactions which depend on an ultraviolet cutoff parameter, it is natural to consider higher derivative corrections to the 3-form gauge fields. Since the Lagrangian should be gauge invariant, the nonrenormalizable interactions may be described in terms of the field strength of the 3-form gauge field rather than the 3-form gauge field itself. In particular, higher derivative corrections in 3-form gauge theories provide a pseudo-scalar field with a non-trivial potential [6, 8, 23] such as a cosine-type potential [6]. This is in contrast to the case of the quadratic derivative term (kinetic term) giving rise to only a mass term for the pseudo-scalar field.

Construction of consistent higher derivative 3-form gauge theories consists of three procedures for both bosonic and SUSY cases:

(1) identifying unstable modes such as ghosts and tachyons,

(2) constructing bulk Lagrangians free from unstable modes,

(3) determining boundary Lagrangian corresponding to the bulk.

Which have already been done and which have not been are summarized in table 1.

(1) In general, higher derivative interactions give rise to additional degrees of freedom, which cause instabilities. For higher derivative theories of a scalar field ϕ, such an instability is known as the Ostrogradsky's ghost [55, 56]. A sufficient condition for the absence of such a ghost is that the Lagrangians should be written by a function of the first order derivative (such as $\partial\phi$), but not by the higher order derivatives than the first order (such as $\partial\partial\phi$). However, in the case of the 3-form gauge theories, the existence or absence of such unwanted degrees of freedom is not known in general.

(2) The examples of the higher derivative Lagrangians without unstable modes is known in bosonic 3-form gauge theories. One of sufficient conditions free from unstable modes is that the Lagrangian consists of arbitrary function of the field strength of a 3-form gauge field [6, 8, 23]. However, this is not a necessary condition; even when there exist derivative terms of the field strength of a 3-form gauge field [57–59], there are no unstable modes, if the canonical kinetic term is absent or has the wrong sign [60]. On the other hand, in contrast to the bosonic case, the only known example of a higher derivative term free from unstable modes in the SUSY 3-form gauge theories is a four derivative term [35].

One of the most characteristic features of the SUSY 3-form gauge fields, in contrast to non-SUSY cases, is that there exist dynamical degrees of freedom of bosons and fermions even at on-shell, which are superpartners of the 3-form gauge field. In higher derivative theories, one should be careful with unstable modes originated from higher derivative terms of the dynamical degrees of freedom.

When we construct higher derivative extensions of SUSY 3-form gauge theories, we can use higher derivative Lagrangians for a chiral superfield because the field strength of the 3-form gauge field can be expressed in terms of a chiral superfield. This is because a 3-form gauge field can be embedded into a vector component of a real superfield, and its field strength can be embedded into an auxiliary field component of a chiral superfield which is defined by the real superfield [25]. These are analogous to a vector superfield and a chiral superfield (gaugino superfield) for a SUSY electromagnetism, respectively. A quartic order term of the field strength was described in ref. [35] as mentioned above. Although higher order terms of an auxiliary field were considered in ref. [54], they contain the Ostrogradsky's ghost instability in general. The most general ghost-free higher derivative terms for SUSY 3-form fields are still missing.

This is in contrast to the cases of chiral matter and vector superfields. For the chiral superfields, ghost-free higher derivative Lagrangians were systematically given in refs. [61–64], which were later generalized in ref. [65]. Before their constructions, it has been known that higher derivative terms in SUSY theories often encounter the so-called auxiliary field problem [66–70]: the action contains the terms with spacetime derivatives on auxiliary fields. In this case, the auxiliary terms cannot be eliminated by their EOM, which is the case for a Wess-Zumino term [71, 72] and Skyrme-like models [73, 74]. This auxiliary field problem usually comes up together with the higher derivative ghosts [53], although such the ghost can be removed by introducing a non-dynamical gauge field (the ghostbuster mechanism) [75, 76]. The higher derivative terms free from these problems given in refs. [61–65] were applied to many topics such as low-energy effective theories [77–80], SUGRA [63, 81], SUSY extension [62, 82] of Galileons [83], ghost condensation [61, 64], the Dirac-Born-Infeld (DBI) inflation [84], flattening of the inflaton potential [85, 86], baby Skyrme models [65, 87–92], Skyrme-like models [93–95], solitons [65, 89, 96, 97], nonlinear realizations [98], SUSY breaking in modulated vacua [99, 100], and a formulation of a liber-

ated SUGRA [101]. For the vector superfield case, ghost-free higher derivative actions were considered in the context of a correction to a scalar potential in SUGRA [102], SUSY Euler-Heisenberg model [53, 81, 103], nonlinearly self-dual actions [104, 105], and the DBI action [106, 107]. The most general ghost-free action of an arbitrary order of the field strength was achieved in ref. [108]. Later, higher derivative theories were applied to the Fayet-Iliopoulos term without gauged R-symmetry [109–111], and inflationary models [112, 113].

(3) The boundary terms are also needed corresponding to higher derivative terms for the consistency between EOM and the energy-momentum tensor. However, in both the bosonic and SUSY cases, they have not been explicitly presented.

In this paper, we give all constructions of 3-form gauge theories missing in table 1. First, in the bosonic case, we show that higher derivative terms given by derivatives on the field strength may cause a tachyon as far as the canonical kinetic term exists. If such higher derivative terms are absent, there are no additional degrees of freedom. We further argue that there are no additional degrees of freedom if the higher derivative terms are given by functions of the field strength but not of derivatives of the field strength. All the previously known examples in refs. [6, 8, 19, 22, 23] fall into this class.

Second, we give the most general higher derivative Lagrangian including an arbitrary order of the field strength without tachyons as well as ghosts in the SUSY case. Since the field strength of the 3-form gauge field can be embedded into the auxiliary field of a chiral superfield, we can construct a ghost-free higher derivative system of 3-form gauge fields by a Lagrangian with an arbitrary function of the auxiliary field. This Lagrangian is obtained by choosing ghost-free sector of the Lagrangian in ref. [54]. We show that this Lagrangian is ghost-free and tachyon-free in the bosonic sector.

Third, we determine the boundary terms for the higher derivative Lagrangians given by arbitrary functions of the field strength in both the bosonic and SUSY cases. These boundary terms are determined by requiring the condition that a functional variation of the Lagrangian vanishes at the boundary.

This paper is organized as follows. In section 2, we first review a 3-form gauge theory with a quadratic kinetic term and the role of the boundary term. We then argue that a Lagrangian with derivatives on the field strength gives rise to a tachyon by using an example. We then discuss a tachyon-free higher derivative Lagrangian, and its the boundary term. We further confirm that our boundary term gives us an energy-momentum tensor consistent with EOM. In section 3, we discuss SUSY extension of ghost/tachyon-free higher derivative 3-form gauge theories. First, we review 3-form gauge theories with 4D $\mathcal{N} = 1$ global SUSY with a quadratic kinetic term and the corresponding boundary term. Second, we propose the ghost/tachyon-free Lagrangian for SUSY 3-form gauge theories. We then determine the boundary term for the ghost/tachyon-free Lagrangian by requiring that a superspace functional variation at a boundary should vanish. Section 4 is devoted to a summary and discussion. In appendix A, we summarize our notation. In appendix B, we review a dual formulation of a 3-form gauge theory. In appendix C, we also review a dual

formulation of a 3-form gauge theory in SUSY field theories. In appendix D, we discuss an auxiliary field method for the ghost/tachyon-free Lagrangian proposed in section 3.3.

We use the notation of the textbook [114].

2 3-form gauge theories

In this section, we consider 3-form gauge theories in 4D. First, we review a 3-form gauge theory with a canonical kinetic term with a corresponding boundary term. Second, we argue that a higher derivative term given by a derivative of the field strength of the 3-form gauge field gives rise to a tachyon in the presence of the canonical kinetic term. We also show that higher derivative Lagrangians are tachyon-free if the higher derivative terms are given by a function of the field strength but not of a function of the derivatives of the field strength. Finally, we consider the necessity of the boundary term for the higher derivative term, and we specify a higher derivative extension of the boundary term.

2.1 3-form gauge theory with canonical kinetic term

Here, we review a 3-form gauge field, its field strength, and gauge invariant Lagrangian with a canonical kinetic term with a boundary term. This review part is mainly based on refs. [11–13].

2.1.1 3-form gauge field

A 3-form gauge field is a third-rank antisymmetric tensor field which is transformed by a 2-form antisymmetric tensor local parameter ξ_{mn} as follows:

$$\delta_3 C_{mnp} = \partial_m \xi_{np} + \partial_n \xi_{pm} + \partial_p \xi_{mn}. \tag{2.1}$$

Here, δ_3 denotes an infinitesimal gauge transformation of the 3-form gauge field. The field strength of the 3-form gauge field is introduced as follows:

$$F_{mnpq} = \partial_m C_{npq} - \partial_n C_{mpq} + \partial_p C_{mnq} - \partial_q C_{mnp}. \tag{2.2}$$

The field strength is invariant under the gauge transformation of the 3-form:

$$\delta_3 F_{mnpq} = 0. \tag{2.3}$$

Note that the field strength can be written by using totally anti-symmetric tensor ϵ_{mnpq}:

$$F_{mnpq} = -\frac{1}{4!} \epsilon_{mnpq} \epsilon^{rstu} F_{rstu} \tag{2.4}$$

because F_{mnpq} is totally antisymmetric tensor in 4D. It is convenient to define the Hodge dual of the field strength F as follows:

$$F := \frac{1}{4!} \epsilon^{mnpq} F_{mnpq}. \tag{2.5}$$

We can write F_{mnpq} in terms of F as

$$F_{mnpq} = -\epsilon_{mnpq} F. \tag{2.6}$$

2.1.2   Lagrangian with canonical kinetic term: bulk part

In the following, we give a Lagrangian with a quadratic derivative term. As we will explain below, the Lagrangian gives us a constant term. As a merit, we can describe the cosmological constant in terms of a gauge symmetry [11–13, 15]. A quadratic derivative Lagrangian of the 3-form gauge field is given by

$$\mathcal{L}_{\text{kin.}} = -\frac{1}{2 \cdot 4!} F^{mnpq} F_{mnpq} + \frac{1}{3!} \partial^m (C^{npq} F_{mnpq}). \tag{2.7}$$

The first term is a quadratic derivative term, which we call the canonical kinetic term. The second term is a boundary term corresponding to the kinetic term, which is necessary for the consistency as described below. We impose the gauge invariant boundary condition for the 3-form gauge field:

$$F|_{\text{bound.}} = -c, \tag{2.8}$$

where c is a real constant, and the minus sign is just a convention. The symbol $|_{\text{bound.}}$ denotes the value at the boundary. We further impose that the functional variation of the field strength at the boundary is zero:

$$\delta F|_{\text{bound.}} = 0, \tag{2.9}$$

which we will use to discuss the boundary term. Note that the Lagrangian in eq. (2.7) can be rewritten by using F as

$$\mathcal{L}_{\text{kin.}} = +\frac{1}{2} F^2 - \frac{1}{3!} \partial^m (C^{npq} \epsilon_{mnpq} F). \tag{2.10}$$

The Lagrangian written in term of F will be useful when we consider higher derivative extensions.

The variation of the Lagrangian in eq. (2.7) by the 3-form gauge field gives us the EOM of the 3-form gauge field:

$$\partial^m F_{mnpq} = 0, \quad \text{or equivalently} \quad \partial^m F = 0. \tag{2.11}$$

This can be solved as

$$F_{mnpq} = \epsilon_{mnpq} c, \tag{2.12}$$

where the constant c is determined by the boundary condition in eq. (2.8). We will use them below.

2.1.3   Lagrangian with canonical kinetic term: boundary part

Now, we review a role of the boundary term. The boundary term is necessary in order that the variation of the kinetic term at the boundary vanishes: the variation of the Lagrangian by the 3-form gauge field is

$$\begin{aligned} \delta\mathcal{L}_{\text{kin.}} &= -\frac{1}{3!} (\partial^m \delta C^{npq}) F_{mnpq} + \frac{1}{3!} \partial^m (\delta C^{npq} F_{mnpq}) \\ &= -\frac{1}{3!} \partial^m (\delta C^{npq} F_{mnpq}) + \frac{1}{3!} \partial^m (\delta C^{npq} F_{mnpq}) + \frac{1}{3!} \delta C^{npq} \partial^m F_{mnpq}. \end{aligned} \tag{2.13}$$

Here, we used that the variation of the field strength at the boundary is zero by the condition in eq. (2.9). The right hand side of the second line shows that the variation of the kinetic term at the boundary $-\frac{1}{3!}\partial^m(\delta C^{npq}F_{mnpq})$ is canceled by that of the boundary term $+\frac{1}{3!}\partial^m(\delta C^{npq}F_{mnpq})$. If the boundary term were not introduced, the variation of the Lagrangian would not vanish at the boundary.

The boundary term is also needed for the consistency between the energy-momentum tensor and the EOM. If boundary term were absent, the energy-momentum tensor and the EOM would not be compatible with each other [13, 14]. This can be seen as follows. If the boundary term were absent, the Lagrangian would be written as

$$\mathcal{L}_{\text{kin.,bulk}} := -\frac{1}{2 \cdot 4!}F^{mnpq}F_{mnpq}. \tag{2.14}$$

The energy-momentum tensor of this Lagrangian is

$$T^{mn} = \frac{1}{3!}F^{mpqr}F^n{}_{pqr} - \frac{1}{2 \cdot 4!}\eta^{mn}F^{pqrs}F_{pqrs} = -\eta^{mn}F^2 + \eta^{mn}\frac{1}{2}F^2 = -\frac{1}{2}\eta^{mn}F^2. \tag{2.15}$$

Note that the energy-momentum tensor is a local quantity, and is the same whether boundary term is included or not before substituting the solution of EOM of the 3-form gauge field.

The variation of the Lagrangian leads to the same EOM and its solution which is also independent of whether the boundary term is included or not. Let us substitute the solution into the energy-momentum tensor and the Lagrangian. The energy-momentum tensor is proportional to a constant:

$$T^{mn} = -\frac{1}{2}\eta^{mn}F^2 = -\frac{1}{2}\eta^{mn}c^2. \tag{2.16}$$

The on-shell Lagrangian is also merely a constant:

$$\mathcal{L}_{\text{kin.,bulk}} = -\frac{1}{2 \cdot 4!}\epsilon^{mnpq}\epsilon_{mnpq}c^2 = +\frac{1}{2}c^2. \tag{2.17}$$

We can also derive the energy-momentum tensor of the on-shell Lagrangian in eq. (2.17). However, the Lagrangian gives the energy-momentum tensor which is not equal to the one in eq. (2.16):

$$T^{mn} = +\frac{1}{2}\eta^{mn}c^2. \tag{2.18}$$

Therefore, if the boundary term were absent, the EOM is not consistent with energy-momentum tensor unless the constant c is equal to zero.[1]

The boundary term resolves this problem [13, 14]. We again consider the Lagrangian $\mathcal{L}_{\text{kin.}}$ in eq. (2.7). In the presence of the boundary term, the Lagrangian after substituting the solution of the EOM is changed as follows:

$$\mathcal{L}_{\text{kin.}} = -\frac{1}{2}c^2. \tag{2.19}$$

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[1]Historically, the 3-form gauge field was used to consider the cosmological constant problem in ref. [10]. However, it was pointed out that the discussion led to wrong sign for the cosmological constant since the boundary term for the 3-form was not included [14, 16].

Then the energy-momentum tensor is given by

$$T^{mn} = -\frac{1}{2}\eta^{mn}c^2. \qquad (2.20)$$

Therefore, the solution of the equation of motion is consistent with the energy-momentum tensor.

2.2 Higher derivative term causing tachyon and/or ghost in 3-form gauge theories

Here, we show that higher derivative terms in the form of derivatives on the field strength in 3-form gauge theories may cause a tachyon as far as the canonical kinetic term exists. We show that the tachyon can also be a ghost depending on parameters.

We see that the field strength can become dynamical and tachyonic if higher derivative terms in the form of derivatives of the field strength of a 3-form gauge field are present, and if the canonical kinetic term exists. As an example, we consider the following Lagrangian with a term $\partial^m F\partial_m F$:

$$\mathcal{L}_{\partial F} = +\frac{1}{2}F^2 + \frac{\alpha}{2}\partial^m F\partial_m F - \frac{1}{3!}\partial_m(\epsilon^{mnpq}C_{npq}F), \qquad (2.21)$$

where α is an arbitrary parameter with mass dimension -2. The first term is the canonical kinetic term, and the second term is a higher derivative term which includes a derivative on the field strength $\partial_m F$. The third term is the corresponding boundary term. Note that the boundary term for the second term is not needed because derivatives on the field strength at the boundary are zero by the boundary condition in eq. (2.9).

To see that there is a tachyonic mode, we rewrite the Lagrangian by introducing new fields F' and q:

$$\mathcal{L}'_{\partial F} = +\frac{1}{2}F'^2 + \frac{\alpha}{2}\partial^m F'\partial_m F' - \frac{1}{3!}\partial_m(\epsilon^{mnpq}C_{npq}F') + q\left(F' - \frac{1}{3!}\epsilon^{mnpq}\partial_m C_{npq}\right). \qquad (2.22)$$

Here, F' is a pseudo-scalar field independent of the 3-form gauge field. We assume that the boundary condition for F' is the same as F: $F'|_{\text{bound.}} = -c$. q is a Lagrange's multiplier field whose EOM gives us the original Lagrangian in eq. (2.21). Vanishing of the variation of C_{mnp} at the boundary requires the boundary condition for q: $q|_{\text{bound.}} = -F'|_{\text{bound.}} = c$. The EOM of C_{mnp} implies that q is a local constant, which is equal to c by the boundary condition. By substituting the solution into eq. (2.22), and redefining F' as $F'' = F' + c$, we obtain

$$\mathcal{L}'_{\partial F} = +\frac{1}{2}F''^2 + \frac{\alpha}{2}\partial^m F''\partial_m F'' - \frac{1}{2}c^2. \qquad (2.23)$$

The first term $+\frac{1}{2}F''^2$ becomes a tachyon mass term. The origin of the tachyon mass term is the canonical kinetic term with the correct sign. Furthermore, the second term is the kinetic term for F''. Thus, the Lagrangian in eq. (2.22) contains a dynamical tachyon. If the sign of α is positive, the field F'' is a ghost as well.

Note that there is no tachyon, if the canonical kinetic term in the original Lagrangian has a wrong sign $-\frac{1}{2}F^2$. Furthermore, if the parameter α is negative in addition to the kinetic term with the wrong sign, there is neither a ghost nor a tachyon [60].

Let us make a comment on a relation to SUSY Lagrangian. The Lagrangian in eq. (2.21) is obtained from the SUSY Lagrangian in eq. (3.44) given in ref. [53] by regarding the imaginary part of the auxiliary field as the field strength of the 3-form gauge field and truncating all fields other than the field strength.

2.3 Ghost/tachyon-free higher derivative 3-form gauge theories

Here, we consider a ghost/tachyon-free higher derivative extension of the 3-form gauge theory. In the previous subsection, we have seen that the derivatives on the field strength can cause a tachyon. Thus, we only consider a higher derivative Lagrangian with a function of the field strength but not of derivatives on the field strength. Such higher derivative Lagrangians were previously considered in refs. [6, 8, 23]. In this subsection, we confirm that there are no tachyons as well as ghosts in this higher derivative Lagrangian in contrast to the previous subsection. We determine the boundary term for the ghost/tachyon-free higher derivative term. We show that the boundary terms are also needed for the higher derivative interactions of the 3-form gauge field. We determine it by requiring that the variation of the Lagrangian at the boundary should vanish.

2.3.1 Ghost/tachyon-free higher derivative Lagrangian: bulk part

First of all, we confirm that a higher derivative term given by a function of the field strength is ghost/tachyon-free. We consider the following Lagrangian whose bulk part was discussed in refs. [6, 8]:

$$\mathcal{L}_{\mathrm{HD}} = G(F) + \mathcal{L}_{\mathrm{HD,bound.}} \tag{2.24}$$

where $G(F)$ is an arbitrary real function of F, and $\mathcal{L}_{\mathrm{HD,bound.}}$ is the boundary term which we will determine below. One can assume that $G(F)$ includes the canonical kinetic term: $G(F) = +\frac{1}{2}F^2 + \cdots$. We impose the same boundary condition on F as eqs. (2.8) and (2.9):

$$F|_{\mathrm{bound.}} = c, \tag{2.25}$$

and

$$\delta F|_{\mathrm{bound.}} = 0. \tag{2.26}$$

In this Lagrangian in eq. (2.24), we can see that there are no additional dynamical degrees of freedom. The EOM of the 3-form gauge field is

$$0 = G''(F)\partial^m F_{mnpq}. \tag{2.27}$$

The solutions of the EOM are $\partial^m F_{mnpq} = 0$ or $G''(F) = 0$. For $\partial^m F_{mnpq} = 0$, F is a constant, which is determined by the boundary condition. For $G''(F) = 0$, the value of F depends on $G(F)$, and this value does not always satisfy the boundary condition. Thus, we focus on the former solution $\partial^m F_{mnpq} = 0$. The EOM can be solved as

$$F_{mnpq} = c\epsilon_{mnpq}, \tag{2.28}$$

where c is a constant determined by the boundary condition in eq. (2.25). For this solution, there are no additional degrees of freedom in the higher derivative Lagrangian since the EOM is not changed from the case of the canonical kinetic term. Thus, there are no tachyons as well as ghosts in eq. (2.24).

2.3.2 Ghost/tachyon-free higher derivative Lagrangian: boundary part

Next, we see that the boundary term is needed for the vanishing of the variation of the Lagrangian at the boundary and for the consistency between EOM and the energy-momentum tensor. We show that the ghost/tachyon-free Lagrangian of the 3-form gauge field without the corresponding boundary term also gives rise to an inconsistency between the EOM and the energy-momentum tensor.

We see the inconsistency between the energy-momentum tensor and the EOM of the Lagrangian in eq. (2.24). On one hand, the energy-momentum tensor of the Lagrangian in eq. (2.24) can be calculated as

$$T^{mn} = \eta^{mn}(-FG'(F) + G(F)). \tag{2.29}$$

On the other hand, substituting the solution in eq. (2.28) into the energy-momentum tensor in eq. (2.29), we obtain

$$T^{mn} = \eta^{mn}(cG'(-c) + G(-c)). \tag{2.30}$$

If the boundary term $\mathcal{L}_{\text{HD,bound.}}$ were absent in the Lagrangian in eq. (2.24), the Lagrangian would be

$$\mathcal{L}_{\text{HD,bulk}} := G(F), \tag{2.31}$$

and the on-shell Lagrangian would be

$$\mathcal{L}_{\text{HD,bulk}} = G(-c). \tag{2.32}$$

We can see that the energy-momentum tensor which is calculated by the on-shell Lagrangian in eq. (2.32), $T^{mn} = \eta^{mn}G(-c)$, is inconsistent with the one in eq. (2.30).

Therefore, we need the boundary term which gives us the consistent energy-momentum tensor. We can find the boundary term by the variational principle. The variation of the Lagrangian by the 3-form gauge field is

$$\delta\mathcal{L}_{\text{HD}} = \frac{1}{3!}G'(F)\epsilon^{mnpq}\partial_m\delta C_{npq} + \delta\mathcal{L}_{\text{HD,bound.}}. \tag{2.33}$$

By the partial integration, we obtain the variation at the boundary:

$$\delta\mathcal{L}_{\text{HD}} = \frac{1}{3!}\partial_m(\epsilon^{mnpq}G'(F)\delta C_{npq}) + \delta\mathcal{L}_{\text{HD,bound.}} + \cdots, \tag{2.34}$$

where the ellipsis $\cdots$ denotes the variation in the bulk. To cancel the variation at the boundary, we propose the boundary term corresponding to the higher derivative term:

$$\mathcal{L}_{\text{HD,bound.}} = -\frac{1}{3!}\partial_m(\epsilon^{mnpq}G'(F)C_{npq}). \tag{2.35}$$

Note that boundary term given in eq. (2.7) is naturally included into the boundary term in eq. (2.35) by choosing $G(F) = +\frac{1}{2}F^2$. To confirm whether the boundary term is consistent or not, we consider the EOM and the energy-momentum tensor. We start with the following Lagrangian:

$$\mathcal{L}_{\text{HD}} = \mathcal{L}_{\text{HD, bulk}} + \mathcal{L}_{\text{HD, bound.}} = G(F) - \frac{1}{3!}\partial_m(\epsilon^{mnpq}G'(F)C_{npq}). \tag{2.36}$$

The EOM is the same as eq. (2.27), but the on-shell Lagrangian is modified as

$$\mathcal{L}_{\mathrm{HD}} = G(-c) + cG'(-c), \tag{2.37}$$

which leads to the energy-momentum tensor which is consistent with eq. (2.29):

$$T^{mn} = \eta^{mn}(cG'(-c) + G(-c)). \tag{2.38}$$

Therefore, the Lagrangian which includes the boundary term in eq. (2.36) gives us the consistent energy-momentum tensor.

3 SUSY 3-form gauge theories

In this section, we give the most general ghost/tachyon-free SUSY Lagrangian of a 3-from gauge field of an arbitrary order of the field strength. In section 3.1, we review a formulation of a Lagrangian with the quadratic kinetic term in SUSY 3-form gauge theories. We also review a boundary term for the quadratic kinetic term. In section 3.2 we give an example of SUSY higher derivative Lagrangian of 3-form gauge field containing a ghost as well as tachyon. In section 3.3, we give the most general ghost/tachyon-free Lagrangian and its boundary term.

In this section, we use superspace to formulate manifestly SUSY theories. Superspace is spanned by spacetime coordinate (x^m) and fermionic coordinate given by Grassmann variables ($\theta^\alpha, \bar{\theta}_{\dot{\alpha}}$). Here, undotted and dotted Greek letters $\alpha, \beta, \ldots$ and $\dot{\alpha}, \dot{\beta}, \ldots$ denote undotted and dotted spinors, respectively.

3.1 SUSY 3-form gauge theory with canonical kinetic term

In this subsection, we review formulations of Lagrangians with quadratic kinetic terms in SUSY 3-form gauge theories. This subsection is essentially based on refs. [25, 31, 42].

3.1.1 3-form prepotential

Here we explain how to embed a 3-form gauge field into a superfield. In superspace, fields are embedded into superfields. A 3-form gauge field C_{mnp} is embedded into a real superfield X ($X^\dagger = X$) [25]:

$$C_{mnp} = \frac{\sqrt{2}}{8} \epsilon_{mnpq} (\bar{\sigma}^q)^{\dot{\alpha}\alpha} [D_\alpha, \bar{D}_{\dot{\alpha}}] X|. \tag{3.1}$$

Here, the vertical bar "|" denotes $\theta = \bar{\theta} = 0$ projection in superspace. The derivatives D_α and $\bar{D}_{\dot{\alpha}}$ are SUSY covariant spinor derivatives. Following ref. [29], we call X "3-form prepotential" in this paper. An infinitesimal superfield gauge transformation of the 3-form prepotential is given by

$$\delta_{3,\mathrm{SUSY}} X = \frac{1}{2i} (D^\alpha \Upsilon_\alpha - \bar{D}_{\dot{\alpha}} \bar{\Upsilon}^{\dot{\alpha}}), \tag{3.2}$$

where Υ_α is a chiral superfield $\bar{D}_{\dot{\alpha}} \Upsilon_\alpha = 0$. Here, $\delta_{3,\mathrm{SUSY}}$ denotes the infinitesimal gauge transformation of the 3-form prepotential. Since $L := \frac{1}{2i}(D^\alpha \Upsilon_\alpha - \bar{D}_{\dot{\alpha}} \bar{\Upsilon}^{\dot{\alpha}})$ is a real linear

superfield satisfying $D^\alpha D_\alpha L = \bar{D}_{\dot\alpha}\bar{D}^{\dot\alpha}L = 0$, the gauge transformation can be simply rewritten by the real linear superfield as

$$\delta_{3,\text{SUSY}}X = L. \tag{3.3}$$

The gauge transformation of the bosonic gauge field is included in this superfield gauge transformation:

$$\begin{aligned}
\delta_{3,\text{SUSY}}C_{mnp} &= \frac{\sqrt{2}}{8}\epsilon_{mnpq}(\bar{\sigma}^q)^{\dot\alpha\alpha}[D_\alpha, \bar{D}_{\dot\alpha}]\delta_{3,\text{SUSY}}X| \\
&= \frac{\sqrt{2}}{8}\epsilon_{mnpq}(\bar{\sigma}^q)^{\dot\alpha\alpha}[D_\alpha, \bar{D}_{\dot\alpha}]\frac{1}{2i}(D^\gamma\Upsilon_\gamma - \bar{D}_{\dot\gamma}\bar{\Upsilon}^{\dot\gamma})| \\
&= \partial_m\lambda_{np} + \partial_n\lambda_{pm} + \partial_p\lambda_{mn},
\end{aligned} \tag{3.4}$$

where the gauge parameter λ_{mn} is embedded into the chiral superfield Υ_α as

$$\lambda_{mn} = \frac{\sqrt{2}}{2i}\left((\sigma_{mn})_\alpha{}^\beta D^\alpha\Upsilon_\beta - (\bar{\sigma}_{mn})^{\dot\alpha}{}_{\dot\beta}\bar{D}_{\dot\alpha}\bar{\Upsilon}^{\dot\beta}\right)|. \tag{3.5}$$

Note that a prepotential for a 1-form (vector superfield) is also a real superfield, but the gauge transformation law of the 1-form prepotential is different from that of the 3-form prepotential.

The field strength of the 3-form gauge field is embedded into a chiral superfield Y (and its Hermitian conjugate), which is given by the 3-form prepotential as follows:

$$Y := -\frac{1}{4}\bar{D}^2X. \tag{3.6}$$

Here, the second order spinor derivative $\bar{D}^2 := \bar{D}_{\dot\alpha}\bar{D}^{\dot\alpha}$ acts on X as a chiral projection $\bar{D}_{\dot\alpha}\bar{D}^2 = 0$. Therefore, Y is a chiral superfield: $\bar{D}_{\dot\alpha}Y = 0$. We will call Y "4-form field strength superfield" in this paper. Note that Y is gauge invariant: $\delta_{3,\text{SUSY}}Y = -\frac{1}{4}\bar{D}^2\delta_{3,\text{SUSY}}X = -\frac{1}{4}\bar{D}^2L = 0$. The bosonic field strength F_{mnpq} is embedded into the imaginary part of the auxiliary field of Y:

$$F_{mnpq} = \frac{\sqrt{2}i}{8}\epsilon_{mnpq}(D^2Y - \bar{D}^2\bar{Y})|, \tag{3.7}$$

where $D^2 := D^\alpha D_\alpha$. This is equivalently written by using F as

$$F = -\frac{\sqrt{2}i}{8}(D^2Y - \bar{D}^2\bar{Y})|. \tag{3.8}$$

In SUSY theories, the 3-form gauge field has dynamical superpartners. Since the 3-form gauge field has one off-shell degree of freedom, there are also off-shell fermionic degrees of freedom as well as additional bosonic ones. The superpartners of the 3-form gauge field can be found as components of the chiral superfield Y. They are one complex scalar y (two bosons), one Weyl fermion χ_α (four fermions), and one real auxiliary scalar field H (one boson). We define the components as follows:

$$y := Y|, \quad \chi_\alpha := \frac{1}{\sqrt{2}}D_\alpha Y|, \quad H := -\frac{\sqrt{2}}{8}(D^2Y + \bar{D}^2\bar{Y})|. \tag{3.9}$$

Note that the dynamical degrees of freedom are y and two of χ_α, thus the on-shell bosonic degrees of freedom are also equal to the fermionic ones. It is convenient to define the following complex auxiliary field $\mathcal{F}$:

$$\mathcal{F} := -\frac{1}{4}D^2 Y| = \frac{1}{\sqrt{2}}(H - iF). \tag{3.10}$$

We impose the following boundary conditions for the component fields. One is the boundary condition for F, which is the same as the previous one:

$$F|_{\text{bound.}} = -c. \tag{3.11}$$

Other boundary conditions are imposed so that SUSY and gauge invariance are preserved at the boundary [31]:

$$\delta y|_{\text{bound.}} = 0, \quad \delta \chi_\alpha|_{\text{bound.}} = 0, \quad \delta H|_{\text{bound.}} = 0, \quad \text{and} \quad \delta F|_{\text{bound.}} = 0. \tag{3.12}$$

Here, δ denotes variations of fields. In the superfield language, the boundary conditions can be written as

$$\bar{D}^2 \delta X||_{\text{bound.}} = 0, \quad D_\alpha \bar{D}^2 \delta X||_{\text{bound.}} = 0, \quad D^2 \bar{D}^2 \delta X||_{\text{bound.}} = 0, \tag{3.13}$$

and their Hermitian conjugates. Here, $||_{\text{bound.}}$ denotes the $\theta = \bar{\theta} = 0$ projection at the boundary. The other boundary conditions for the higher order spinor derivatives on $\bar{D}^2 \delta X$ and $D^2 \delta X$ are also assumed to be zero. The examples used later are

$$[D_\alpha, \bar{D}_{\dot\alpha}]\bar{D}^2 \delta X||_{\text{bound.}} = 0, \quad \bar{D}^2 D_\alpha \bar{D}^2 \delta X||_{\text{bound.}} = 0, \tag{3.14}$$

and so on. Meanwhile, as in the non-SUSY case, we do not impose a specific boundary condition for the gauge field C_{mnp}, since the gauge field is not gauge invariant.

3.1.2 SUSY Lagrangian with canonical kinetic term: bulk part

The Lagrangian with a canonical kinetic term is given by

$$\mathcal{L}_{\text{kin.,SUSY}} = -\frac{1}{8}\int d^2\theta \bar{D}^2 Y \bar{Y} - \frac{1}{8}\int d^2\bar{\theta} D^2 Y \bar{Y} + \mathcal{L}_{\text{kin.,SUSY,bound.}}$$
$$= -\partial^m y \partial_m \bar{y} - i\bar{\chi}_{\dot\alpha}(\bar{\sigma}^m)^{\dot\alpha\alpha}\partial_m \chi_\alpha + \frac{1}{2}H^2 - \frac{1}{2 \cdot 4!}F^{mnpq}F_{mnpq} + \mathcal{L}_{\text{kin.,SUSY,bound.}} \tag{3.15}$$

Here $\int d^2\theta = -\frac{1}{4}D^2|$ is the F-type integration. We use $-\frac{1}{8}(\int d^2\theta \bar{D}^2 + \int d^2\bar{\theta} D^2)$ for the D-type integration instead of the conventional $\int d^4\theta$ in order to fix the definition of the D-type integration. Our choice for the D-type integration may have a natural extension to Poincaré and conformal SUGRA [114–120]. Note that we have neglected the boundary terms which do not depend on the field strength F_{mnpq}. Such boundary terms are not relevant to our discussion. As in the non-SUSY case, we derive the EOM for the 3-form gauge field in the SUSY case. This can be evaluated by the variation of the 3-form prepotential in the bulk:

$$0 = -\frac{1}{4}(\bar{D}^2 \bar{Y} + D^2 Y). \tag{3.16}$$

The $\theta = \bar{\theta} = 0$ component of the EOM leads to the EOM for the real auxiliary field H:

$$H = 0. \tag{3.17}$$

The EOM for F can be found in the $[D_\alpha, \bar{D}_{\dot\alpha}]$ component of the EOM in eq. (3.16):

$$0 = -\frac{1}{4}[D_\alpha, \bar{D}_{\dot\alpha}](\bar{D}^2\bar{Y} + D^2Y)| = -\frac{i}{2}(\sigma^m)_{\alpha\dot\alpha}\partial_m(\bar{D}^2\bar{Y} - D^2Y)| = 2\sqrt{2}(\sigma^m)_{\alpha\dot\alpha}\partial_m F, \tag{3.18}$$

where we have used $[D_\alpha, \bar{D}_{\dot\alpha}](\text{chiral}) = +2i\partial_{\alpha\dot\alpha}(\text{chiral})$ and $[D_\alpha, \bar{D}_{\dot\alpha}](\text{anti-chiral}) = -2i\partial_{\alpha\dot\alpha}(\text{anti-chiral})$. Here, $\partial_{\alpha\dot\alpha}$ is the spinor representation of the spacetime derivative: $\partial_{\alpha\dot\alpha} = (\sigma^m)_{\alpha\dot\alpha}\partial_m$. Thus, F is equal to a constant, which is determined by the boundary condition for F in eq. (3.11):

$$F = -c. \tag{3.19}$$

We can rewrite the solution to the EOM in terms of the complex auxiliary field $\mathcal{F}$ as

$$\mathcal{F} = -\frac{1}{4}D^2Y| = i\frac{c}{\sqrt{2}}. \tag{3.20}$$

Since the 4-form field strength superfield is described by the chiral superfield, the kinetic term $Y\bar{Y}$ can be generalized to a Kähler potential $K(Y, \bar{Y})$. Further, the 4-form field strength superfield can have a superpotential. Therefore, the Lagrangian with quadratic derivative terms is given by

$$\mathcal{L}_{KW} = \left(-\frac{1}{8}\int d^2\theta\bar{D}^2 K(Y, \bar{Y}) + \int d^2\theta W(Y) + \text{h.c.}\right) + \mathcal{L}_{KW,\text{bound.}} \tag{3.21}$$

Here, $\mathcal{L}_{KW,\text{bound.}}$ is the boundary term for the above Lagrangian, which we will determine below. The EOM for the field strength can be obtained in the same way as the case of the Lagrangian in eq. (3.15).

3.1.3 SUSY Lagrangian with canonical kinetic term: boundary part

In the variation of the Lagrangian in eq. (3.15) by the 3-form prepotential, we have neglected the variation at the boundary, which we should discuss carefully. Therefore, we consider the boundary term for the canonical kinetic term [31, 42]. The term $\mathcal{L}_{\text{kin.,SUSY,bound.}}$ in eq. (3.15) is a boundary term corresponding to the canonical kinetic term. As in the non-SUSY case, there is generally a non-trivial boundary condition for the field strength of the 3-form. The boundary term is obtained by either the variational principle [31] or a dual formulation [42]. Since we can straightforwardly obtain the boundary term by the variational principle in terms of the 3-form prepotential only, we use the former option here. The latter option is summarized in appendix C.

In the following discussion, we consider the variation of the Lagrangian by the 3-form prepotential, and find the term proportional to $\partial_m\delta C_{npq}$, which gives a nontrivial variation at the boundary as in the non-SUSY case in section 2.1. We then introduce a boundary term which cancels the variation at the boundary.

The variation of the kinetic term in eq. (3.15) by the 3-form prepotential is

$$\delta\mathcal{L}_{\text{kin., SUSY}} = \frac{1}{32}\left(\int d^2\theta\bar{D}^2(\bar{Y}\bar{D}^2\delta X) + \int d^2\theta\bar{D}^2(YD^2\delta X)\right.$$
$$\left. + \int d^2\bar{\theta}D^2(\bar{Y}\bar{D}^2\delta X) + \int d^2\bar{\theta}D^2(YD^2\delta X)\right) + \delta\mathcal{L}_{\text{kin.,SUSY,bound.}}.$$

(3.22)

Here, we will show that the second and the third terms in the right hand side are equal to the fourth and the first terms, respectively. For example, we consider the third term $\int d^2\bar{\theta}D^2(\bar{Y}\bar{D}^2\delta X)$. By using the identity

$$D^2\bar{D}^2 - \bar{D}^2D^2 = -4i\partial^{\dot{\alpha}\alpha}[D_\alpha, \bar{D}_{\dot{\alpha}}],$$

(3.23)

this term is equal to

$$\int d^2\bar{\theta}D^2(\bar{Y}\bar{D}^2\delta X) = -\frac{1}{4}\bar{D}^2D^2(\bar{Y}\bar{D}^2\delta X)|$$
$$= -\frac{1}{4}D^2\bar{D}^2(\bar{Y}\bar{D}^2\delta X)| - i\partial^{\dot{\alpha}\alpha}[D_\alpha, \bar{D}_{\dot{\alpha}}](\bar{Y}\bar{D}^2\delta X)|.$$

(3.24)

The difference between the first and the third terms in eq. (3.22) is the total derivative term $-i\partial^{\dot{\alpha}\alpha}([D_\alpha, \bar{D}_{\dot{\alpha}}](\bar{Y}\bar{D}^2\delta X)|)$. The total derivative is equal to zero because the variation $\delta Y|$, $D_\alpha\delta Y|$, and $\partial_m\delta Y|$ are assumed to be zero in eq. (3.12) to preserve SUSY at the boundary. Therefore, eq. (3.22) can be rewritten as

$$\delta\mathcal{L}_{\text{kin., SUSY}} = \frac{1}{16}\left(\int d^2\theta\bar{D}^2(\bar{Y}\bar{D}^2\delta X) + \int d^2\bar{\theta}D^2(YD^2\delta X)\right) + \delta\mathcal{L}_{\text{kin.,SUSY,bound.}}.$$ (3.25)

By using $D_\alpha D^2 = 0$, we can further rewrite the variation as

$$\delta\mathcal{L}_{\text{kin., SUSY}} = \frac{1}{16}\left(\int d^2\theta(\bar{D}^2\bar{Y})\bar{D}^2\delta X + \int d^2\bar{\theta}(D^2Y)D^2\delta X\right) + \delta\mathcal{L}_{\text{kin.,SUSY,bound.}}.$$ (3.26)

For later use, we define a chiral superfield

$$T := -\frac{1}{4}\bar{D}^2\bar{Y},$$

(3.27)

whose $\theta = \bar{\theta} = 0$ component is

$$T| = -\frac{1}{4}\bar{D}^2\bar{Y}| = \frac{1}{\sqrt{2}}(H + iF).$$

(3.28)

Using the chiral superfield, the variation is manifestly written by the product of the chiral superfield T and $\bar{D}^2\delta X$. This structure will be useful in the higher derivative case discussed later.

In this variation of the Lagrangian, we show that there is a term proportional to $\partial_m\delta C_{npq}$ which gives rise to the variation of the 3-form at the boundary. This can be seen by the component expansion:

$$\delta\mathcal{L}_{\text{kin., SUSY}} = -\frac{1}{4}\left(\int d^2\theta T\bar{D}^2\delta X + \int d^2\bar{\theta}\bar{T}D^2\delta X\right) + \delta\mathcal{L}_{\text{kin.,SUSY,bound.}}$$
$$= \frac{1}{16}iT_I(D^2\bar{D}^2 - \bar{D}^2D^2)\delta X| + \delta\mathcal{L}_{\text{kin.,SUSY,bound.}} + \cdots,$$

(3.29)

where $T_I = \frac{1}{2i}(T - \bar{T})$ is the imaginary part of T, and the ellipsis $\cdots$ means the terms which are not related to the variation of the 3-form. The first term can be calculated as

$$\frac{1}{16}iT_I(D^2\bar{D}^2 - \bar{D}^2D^2)\delta X| = \frac{1}{16}iT_I(-4i)\partial^{\dot{\alpha}\alpha}[D_\alpha, \bar{D}_{\dot{\alpha}}]\delta X| = \frac{1}{3!}F\epsilon^{mnpq}\partial_m\delta C_{npq}. \quad (3.30)$$

Therefore, the variation gives us the following boundary term

$$\frac{1}{16}\partial^{\dot{\alpha}\alpha}(iT_I(-4i)[D_\alpha, \bar{D}_{\dot{\alpha}}]\delta X)| = \frac{1}{3!}\partial_m(F\epsilon^{mnpq}\delta C_{npq}). \quad (3.31)$$

To cancel the variation at the boundary, we determine the boundary term $\mathcal{L}_{\text{kin.,SUSY,bound.}}$ as follows:

$$\mathcal{L}_{\text{kin.,SUSY,bound.}} = \frac{i}{4}\left(\int d^2\theta\bar{D}^2 - \int d^2\bar{\theta}D^2\right)T_IX. \quad (3.32)$$

In fact, this Lagrangian is a boundary term which cancels eq. (3.30) since the identity of the spinor derivatives gives us

$$\frac{i}{4}\left(\int d^2\theta\bar{D}^2 - \int d^2D^2\right)T_IX = -\frac{i}{16}(D^2\bar{D}^2 - \bar{D}^2D^2)T_IX| = -\frac{1}{4}\partial^{\dot{\alpha}\alpha}[D_\alpha, \bar{D}_{\dot{\alpha}}]T_IX|. \quad (3.33)$$

In the Wess-Zumino (WZ) gauge [29] where

$$X| = D_\alpha X| = \bar{D}_{\dot{\alpha}}X| = 0, \quad (3.34)$$

the boundary term is

$$\mathcal{L}_{\text{kin.,SUSY,bound.}} = -\frac{1}{3!}\partial_m(F\epsilon^{mnpq}C_{npq}). \quad (3.35)$$

Thus, the Lagrangian in eq. (3.32) gives correct boundary term.

For convenience, we refer to the following θ-integration given in ref. [42]

$$-\frac{1}{4}\cdot\frac{1}{2i}\left(\int d^2\theta\bar{D}^2 - \int d^2\bar{\theta}D^2\right) \quad (3.36)$$

as an I(maginary)-type integration in this paper. This integration is convenient to describe boundary terms such as the term in eq. (3.32).

Of course, the boundary term is also needed in the case where the Lagrangian is written by a Kähler potential and a superpotential in eq. (3.21). The boundary term is found in the same way as the previous case. The difference is merely the choice of the chiral superfield T. We will see the boundary term more precisely. The variation of the Lagrangian in eq. (3.21) by the 3-form gauge field is

$$\delta\mathcal{L}_{KW} = \frac{1}{32}\left(\int d^2\theta\bar{D}^2 + \int d^2\bar{\theta}D^2\right)\left(\frac{\partial K}{\partial Y}\bar{D}^2\delta X + \frac{\partial K}{\partial\bar{Y}}D^2\delta X\right)$$
$$- \frac{1}{4}\int d^2\theta\frac{\partial W}{\partial Y}\bar{D}^2\delta X - \frac{1}{4}\int d^2\theta\frac{\partial\bar{W}}{\partial\bar{Y}}D^2\delta X + \delta\mathcal{L}_{KW,\text{bound.}}. \quad (3.37)$$

We can further rewrite the variation as

$$\delta\mathcal{L}_{KW} = -\frac{1}{4}\int d^2\theta\left(-\frac{1}{4}\bar{D}^2\frac{\partial K}{\partial Y} + \frac{\partial W}{\partial Y}\right)\bar{D}^2\delta X - \frac{1}{4}\int d^2\bar{\theta}\left(-\frac{1}{4}D^2\frac{\partial K}{\partial \bar{Y}} + \frac{\partial \bar{W}}{\partial \bar{Y}}\right)D^2\delta X$$
$$+ \delta\mathcal{L}_{KW,\text{bound.}}\,, \tag{3.38}$$

because the following equation holds from the boundary conditions in eqs. (3.12) and (3.14):

$$\partial^{\dot{\alpha}\alpha}[D_\alpha, \bar{D}_{\dot{\alpha}}]\frac{\partial K}{\partial Y}\bar{D}^2\delta X| = 0. \tag{3.39}$$

If we define the chiral superfield T_{KW} as

$$T_{KW} := -\frac{1}{4}\bar{D}^2\frac{\partial K}{\partial Y} + \frac{\partial W}{\partial Y}, \tag{3.40}$$

the variation can be simply written as

$$\delta\mathcal{L}_{KW} = -\frac{1}{4}\int d^2\theta T_{KW}\bar{D}^2\delta X - \frac{1}{4}\int d^2\bar{\theta}\bar{T}_{KW}D^2\delta X + \delta\mathcal{L}_{KW,\text{bound.}}. \tag{3.41}$$

Since the variation has the same structure as eq. (3.25), we can repeat the same procedure as the previous case. The variation at the boundary is

$$\delta\mathcal{L}_{KW}|_{\text{bound.}} = \frac{1}{16}(D^2\bar{D}^2 - \bar{D}^2D^2)iT_{I,KW}\delta X + \mathcal{L}_{KW,\text{bound.}}$$
$$= \frac{1}{4}\partial^{\dot{\alpha}\alpha}[D_\alpha, \bar{D}_{\dot{\alpha}}]T_{I,KW}\delta X + \delta\mathcal{L}_{KW,\text{bound.}}, \tag{3.42}$$

where $T_{I,KW} = \frac{1}{2i}(T_{KW} - \bar{T}_{KW})$ is the imaginary part of T_{KW}. Therefore, we introduce the following boundary term by using the I-type integration as follows:

$$\mathcal{L}_{KW,\text{bound.}} = \frac{i}{4}\left(\int d^2\theta\bar{D}^2 - \int d^2\bar{\theta}D^2\right)T_{I,KW}X. \tag{3.43}$$

Thus, we have determined the boundary term for the Lagrangian with the Kähler potential and the superpotential.

3.2 Higher derivative term causing ghost and tachyon in SUSY 3-form gauge theories

In this subsection, we consider higher derivative Lagrangians which may cause tachyons as well as ghosts in SUSY 3-form gauge theories. The following discussion is an extension of the discussion in ref. [53] for higher derivative chiral superfields.

We can expect that higher derivative Lagrangians with arbitrary order of the auxiliary field can be obtained by choosing an arbitrary function of D^2Y and its Hermitian conjugate. However, a naively constructed higher derivative Lagrangian gives rise to a ghost.

We will explain this more concretely. For example, we may naively consider the following higher derivative Lagrangian [53]

$$\mathcal{L}_{\partial F,\text{SUSY}} = -\frac{1}{8}\left(\int d^2\theta\bar{D}^2Y\bar{Y} + \frac{\alpha}{16}\int d^2\theta\bar{D}^2(\bar{D}^2\bar{Y}D^2Y) + \text{h.c.}\right) + \mathcal{L}_{\partial F,\text{SUSY,bound.}}, \tag{3.44}$$

where $\mathcal{L}_{\partial F,\text{SUSY,bound.}}$ is the boundary term for $\mathcal{L}_{\partial F,\text{SUSY}}$, which is not relevant in this discussion. We will show that the Lagrangian contains an Ostrogradsky's ghost as well as a tachyon. We can see these unstable modes by the component expression. The bosonic sector of the component Lagrangian is

$$\mathcal{L}_{\partial F,\text{SUSY}} = -\partial^m y \partial_m \bar{y} + |\mathcal{F}|^2 + \alpha(|\Box y|^2 - \partial^m \mathcal{F} \partial_m \bar{\mathcal{F}}). \qquad (3.45)$$

Here, we have abbreviated the boundary term. We briefly show that there is a ghost from the higher derivative term $|\Box y|^2$. This can be seen by an auxiliary field method. The higher derivative term $|\Box y|^2$ can be rewritten by adding a new complex scalar field (auxiliary field) ψ as

$$\mathcal{L}'_{\partial F,\text{SUSY}} = -\partial^m y \partial_m \bar{y} + |\mathcal{F}|^2 + \alpha(-|\psi|^2 - \partial^m \psi \partial_m \bar{y} - \partial^m \bar{\psi} \partial_m y - \partial^m \mathcal{F} \partial_m \bar{\mathcal{F}}). \qquad (3.46)$$

The EOM for ψ gives us the original Lagrangian in eq. (3.45). Instead, we obtain one negative eigenvalue by diagonalizing quadratic derivative terms. The eigenmode of the negative eigenvalue corresponds to a ghost.

We also show that there are also dynamical tachyons. The tachyons are the real auxiliary field H and the field strength F. These fields are now dynamical because of the higher derivative term $-\alpha \partial^m \mathcal{F} \partial_m \bar{\mathcal{F}} = -\frac{\alpha}{2}(\partial^m H \partial_m H + \partial^m F \partial_m F)$. The terms are tachyonic because the term $|\mathcal{F}|^2 = \frac{1}{2}(H^2 + F^2)$ in eq. (3.46) becomes mass terms with wrong signs. The origin of the wrong signs are the canonical kinetic term, but not the sign of the coefficient of the higher derivative term α. Therefore, the tachyons exist as far as we include the canonical kinetic term. The presence of the tachyons has the same structure as the bosonic model in section 2.2.

The ghosts and tachyons can be more simply seen by a SUSY auxiliary method [53] than the above discussion. In superspace, we can rewrite the Lagrangian in eq. (3.44) by adding Ψ and Σ as

$$
\begin{aligned}
\mathcal{L}'_{\partial F,\text{SUSY}} &= \left(-\frac{1}{8} \int d^2\theta \bar{D}^2 Y \bar{Y} - \frac{\alpha}{8} \int d^2\theta \bar{D}^2 \Psi \bar{\Psi} + \int d^2\theta \Sigma \left(\Psi + \frac{1}{4} \bar{D}^2 \bar{Y} \right) + \text{h.c.} \right) \\
&\quad + \mathcal{L}'_{\partial F,\text{SUSY,bound.}} \\
&= \left(-\frac{1}{8} \int d^2\theta \bar{D}^2 (|Y - \Sigma|^2 - \Sigma\bar{\Sigma}) - \frac{\alpha}{8} \int d^2\theta \bar{D}^2 \Psi \bar{\Psi} + \int d^2\theta \Sigma \Psi + \text{h.c.} \right) \\
&\quad + \mathcal{L}'_{\partial F,\text{SUSY,bound.}} \cdot
\end{aligned}
\qquad (3.47)
$$

Thus, dynamical superfields are $Y - \Sigma$, Σ and Ψ. By seeing the sign, Σ becomes a ghost. Furthermore, Ψ becomes a tachyon. This can be seen by solving EOM for the auxiliary fields of Σ and Ψ, the on-shell scalar potential V is

$$V = -|\psi|^2 + \frac{1}{\alpha}|\sigma|^2. \qquad (3.48)$$

Here, $\psi = \Psi|$ and $\sigma = \Sigma|$. Thus, the chiral superfield Σ can be regarded as an independent dynamical field. Therefore, we cannot use the Lagrangian in eq. (3.44) to construct ghost/tachyon-free Lagrangians.

One may wonder whether there are also a ghost or tachyons in eq. (3.44) or not when we put $\alpha = 0$. Of course, there should not be such a ghost or tachyons. This can be seen as follows. We can use the above auxiliary method even in the case that higher derivative terms are absent $\alpha = 0$. The EOM of Ψ leads to $\Sigma = 0$ if $\alpha = 0$. Thus, Σ is not a dynamical field. Furthermore, Ψ drops out of the Lagrangian. Therefore, there is neither a ghost nor a tachyon in this case as expected.

We make a comment on the sign of the kinetic term in eq. (3.44). In section 2.2, we have mentioned the sign of the kinetic term. If the canonical kinetic term of the field strength has a wrong sign, there is no tachyon. However, the kinetic term of y has also the wrong sign by SUSY. Therefore, y becomes a ghost in this case.

3.3 SUSY ghost/tachyon-free higher derivative 3-form gauge theories

In this subsection, we give the most general ghost/tachyon-free Lagrangian of SUSY 3-form gauge theories. First, we present a ghost/tachyon-free Lagrangian at an arbitrary order of the field strength, and then we specify the boundary term for the ghost/tachyon-free Lagrangian.

3.3.1 SUSY ghost/tachyon-free higher derivative Lagrangian: bulk part

The previously known ghost/tachyon-free Lagrangian of 3-form gauge theories is at most the fourth order of the auxiliary field, *i. e.* the fourth order of the field strength of the 3-form gauge field. Here, we construct the most general ghost/tachyon-free higher derivative Lagrangian of an arbitrary order of the field strength. To this end, we use the fact that the field strength of the 3-form gauge field is embedded into a chiral superfield Y. Therefore, ghost/tachyon-free Lagrangian for the SUSY 3-form gauge field can be formulated by that of the chiral superfield [61].

We present the following Lagrangian:

$$
\begin{aligned}
&\mathcal{L}_{\text{HD,SUSY}} \\
&= -\frac{1}{8 \cdot 16} \left(\int d^2\theta \bar{D}^2 \Lambda(Y, \bar{Y}, \partial_m Y, \partial_m \bar{Y}, D^2 Y, \bar{D}^2 \bar{Y})(D^\alpha Y)(D_\alpha Y)(\bar{D}_{\dot\alpha}\bar{Y})(\bar{D}^{\dot\alpha}\bar{Y}) + \text{h.c.} \right) \\
&\quad + \mathcal{L}_{\text{HD,SUSY,bound.}}
\end{aligned}
$$

(3.49)

Here, $\mathcal{L}_{\text{HD,SUSY,bound.}}$ is the boundary term, which we will discuss later. This Lagrangian is a natural extension of ghost-free Lagrangian of a chiral superfield [61, 65]. Previously known ghost-free Lagrangian is that Λ is a function of Y, $\bar{Y}$ [61], $\partial_m Y$, and $\partial_m \bar{Y}$ [65]. One new point is that Λ in eq. (3.49) can also be a function of $D^2 Y$ and $\bar{D}^2 \bar{Y}$. The Lagrangian presented in ref. [54] is more relaxed and consequently contains an Ostrogradsky's ghost. This can be shown by the auxiliary method in ref. [103].

We prove that the Lagrangian in eq. (3.49) is ghost/tachyon-free. One way to show it is to calculate the component expression of the Lagrangian. Another way is using the auxiliary method, which we summarize in appendix D. Here, we use the former way. We show that there are no ghosts/tachyons in the purely bosonic sector. If there are no ghosts in the bosonic sector, there should be no ghosts/tachyons in the fermionic sector because of SUSY.

The bosonic sector of the component Lagrangian is

$$
\begin{aligned}
\mathcal{L}_{\text{HD,SUSY,boson}} = {} & \Lambda(y, \bar{y}, \partial_m y, \partial_m \bar{y}, -2\sqrt{2}(H - iF), -2\sqrt{2}(H + iF)) \\
& \times \left(\frac{1}{4}(H^2 + F^2)^2 - \partial^n y \partial_n \bar{y}(H^2 + F^2) + (\partial^n y \partial_n y)(\partial^p \bar{y} \partial_p \bar{y}) \right) \quad (3.50) \\
& + \mathcal{L}_{\text{HD,SUSY,bound.}}.
\end{aligned}
$$

Since there are no $\partial\partial y$ terms in the bosonic sector of the Lagrangian, we conclude that there are no ghosts in the bosonic sector. Because there are no ∂F terms, the field strength is not dynamical, and there is also no tachyonic mode which is discussed in section 2.2 and 3.2. By SUSY transformation, we conclude that there should be no fermionic ghosts in the Lagrangian as well.

Some comments are in order. One comment is that the ghost/tachyon-free Lagrangian in eq. (3.49) can be extended into a system with matter chiral superfield (ordinary chiral superfield). That is, we can relax the assumption that a chiral superfield Y is related to a real superfield $Y = -\frac{1}{4}\bar{D}^2 X$. In this case, we generally do not need the boundary terms which we will discuss later for the 3-form gauge theories.

Another comment is that we can straightforwardly extend the ghost/tachyon-free Lagrangian into a case of multicomponent chiral superfields Φ^i $(i = 1, \ldots, n)$ with couplings of a Kähler potential $K(\Phi^i, \bar{\Phi}^{i^*})$ and a superpotential $W(\Phi^i)$:

$$
\begin{aligned}
\mathcal{L}_{\text{HD,multi}} = {} & -\frac{1}{8} \int d^2\theta \bar{D}^2 K(\Phi^i, \bar{\Phi}^{i^*}) + \int d^2\theta W(\Phi^i) \\
& -\frac{1}{8 \cdot 16} \int d^2\theta \bar{D}^2 \Lambda_{ijk^*l^*}(D^\alpha \Phi^i)(D_\alpha \Phi^j)(\bar{D}_{\dot{\alpha}} \bar{\Phi}^{k^*})(\bar{D}_{\dot{\alpha}} \bar{\Phi}^{l^*}) + \text{h.c.}, \quad (3.51)
\end{aligned}
$$

where $\Lambda_{ijk^*l^*}$ is a tensor $\Lambda_{ijk^*l^*} = \Lambda_{ijk^*l^*}(\Phi^i, \bar{\Phi}^{i^*}, \partial_m \Phi^i, \partial_m \bar{\Phi}^{i^*}, D^2 \Phi^i, \bar{D}^2 \bar{\Phi}^{i^*})$. If we consider multicomponent 4-form field strength superfields as chiral superfields, we need the corresponding boundary term. We can obtain the boundary term by the same procedure as the following discussion.

3.3.2 SUSY ghost/tachyon-free higher derivative Lagrangian: boundary part

Now, we specify the boundary term which corresponds to the higher derivative Lagrangian in eq. (3.49). As mentioned in section 2.3, the boundary term which corresponds to the higher derivative term is needed. To specify the boundary term, we determine the chiral superfield for the boundary term T_{HD}. This chiral superfield is a generalization of T and T_{KW} in the cases in which the Lagrangians are given by the canonical kinetic term in eq. (3.15) and by a Kähler potential as well as a superpotential in eq. (3.21), respectively.

We consider the variation of the Lagrangian in eq. (3.49) by the 3-form prepotential X:

$$
\begin{aligned}
\delta\mathcal{L}_{\text{HD,SUSY}} = {} & -\frac{1}{128} \Bigg(\int d^2\theta \bar{D}^2 \Bigg(\frac{\partial \Lambda}{\partial Y} \left(-\frac{1}{4} \bar{D}^2 \delta X \right) |D_\alpha Y|^4 + \frac{\partial \Lambda}{\partial \bar{Y}} \left(-\frac{1}{4} D^2 \delta X \right) |D_\alpha Y|^4 \\
& + \frac{\partial \Lambda}{\partial \partial_m Y} \left(-\frac{1}{4} \partial_m \bar{D}^2 \delta X \right) |D_\alpha Y|^4 + \frac{\partial \Lambda}{\partial \partial_m \bar{Y}} \left(-\frac{1}{4} \partial_m D^2 \delta X \right) |D_\alpha Y|^4 \\
& + \frac{\partial \Lambda}{\partial D^2 Y} \left(-\frac{1}{4} D^2 \bar{D}^2 \delta X \right) |D_\alpha Y|^4 + \frac{\partial \Lambda}{\partial \bar{D}^2 \bar{Y}} \left(-\frac{1}{4} \bar{D}^2 D^2 \delta X \right) |D_\alpha Y|^4
\end{aligned}
$$

$$+ 2\Lambda(D^\alpha Y)\left(-\frac{1}{4}D_\alpha\bar{D}^2\delta X\right)(\bar{D}_{\dot\alpha}\bar{Y})(\bar{D}^{\dot\alpha}\bar{Y})$$

$$+ 2\Lambda(D^\alpha Y)(D_\alpha Y)(\bar{D}_{\dot\alpha}\bar{Y})\left(-\frac{1}{4}\bar{D}^{\dot\alpha}D^2\delta X\right)\Bigg) + \text{h.c.}\Bigg)$$

$$+ \delta\mathcal{L}_{\text{HD,SUSY,bound.}}, \tag{3.52}$$

where $|D_\alpha Y|^4 := (D^\alpha Y)(D_\alpha Y)(\bar{D}_{\dot\alpha}\bar{Y})(\bar{D}^{\dot\alpha}\bar{Y})$.

We will use partial integrations for each spinor or vector derivative terms to find the chiral superfield T_{HD}. Unlike the cases of T and T_{KW}, there are spinor or vector derivatives on the variation terms such as $\bar{D}_{\dot\alpha}\bar{D}^2\delta X$, $\partial_m\bar{D}^2\delta X$ and $D^2\bar{D}^2\delta X$. Naively, we would ignore the boundary term for each of the partial integrations. However, we should be careful when the 3-form gauge field is included because the total derivative terms may give δC_{mnp} at the boundary. Therefore, we explicitly execute the partial integration.

For example, we show the following relation

$$-\frac{1}{128}\int d^2\theta\bar{D}^2\frac{\partial\Lambda}{\partial D^2Y}\left(-\frac{1}{4}D^2\bar{D}^2\delta X\right)|D_\alpha Y|^4$$

$$= -\frac{1}{128}\int d^2\theta\bar{D}^2\left(D^2\frac{\partial\Lambda}{\partial D^2Y}|D_\alpha Y|^4\right)\left(-\frac{1}{4}\bar{D}^2\delta X\right) \tag{3.53}$$

which may be the most complicated term in eq. (3.52) to derive the relation. To show this relation, we change the integration $\int d^2\theta\bar{D}^2$ to $\int d^2\bar{\theta}D^2$. The change gives us the following total derivative term:

$$-\frac{1}{128}\int d^2\theta\bar{D}^2\frac{\partial\Lambda}{\partial D^2Y}\left(-\frac{1}{4}D^2\bar{D}^2\delta X\right)|D_\alpha Y|^4$$

$$= -\frac{1}{128}(-4i)\partial^{\dot\beta\beta}\left([D_\beta,\bar{D}_{\dot\beta}]\frac{\partial\Lambda}{\partial D^2Y}\left(-\frac{1}{4}D^2\bar{D}^2\delta X\right)|D_\alpha Y|^4\right)| \tag{3.54}$$

$$- \frac{1}{128}\int d^2\bar{\theta}D^2\frac{\partial\Lambda}{\partial D^2Y}|D_\alpha Y|^4\left(-\frac{1}{4}D^2\bar{D}^2\delta X\right).$$

Note that we can safely execute the partial integration for the term $\int d^2\bar{\theta}D^2\ldots$ in the right hand side by using the identity $D^2D_\alpha = 0$. We now consider the total derivative term in eq. (3.54). In the total derivative, there are terms proportional to $D^2\bar{D}^2\delta X||_{\text{bound.}}$, $\bar{D}_{\dot\beta}D^2\bar{D}^2\delta X||_{\text{bound.}}$, and $D_\beta\bar{D}_{\dot\beta}D^2\bar{D}^2\delta X||_{\text{bound.}}$. By the boundary conditions in eqs. (3.12) and (3.14), we conclude that the boundary term is equal to zero. Therefore, eq. (3.54) can be rewritten as

$$-\frac{1}{128}\int d^2\theta\bar{D}^2\frac{\partial\Lambda}{\partial D^2Y}\left(-\frac{1}{4}D^2\bar{D}^2\delta X\right)|D_\alpha Y|^4$$

$$= -\frac{1}{128}\int d^2\bar{\theta}D^2\left(D^2\frac{\partial\Lambda}{\partial D^2Y}|D_\alpha Y|^4\right)\left(-\frac{1}{4}\bar{D}^2\delta X\right). \tag{3.55}$$

We again change the integration $\int d^2\bar{\theta}D^2$ to $\int d^2\theta\bar{D}^2$ as

$$
\begin{aligned}
-\frac{1}{128}\int d^2\theta\bar{D}^2 &\frac{\partial\Lambda}{\partial D^2Y}\left(-\frac{1}{4}D^2\bar{D}^2\delta X\right)|D_\alpha Y|^4\\
&= -\frac{1}{128}\int d^2\theta\bar{D}^2\left(D^2\frac{\partial\Lambda}{\partial D^2Y}|D_\alpha Y|^4\right)\left(-\frac{1}{4}\bar{D}^2\delta X\right)\\
&\quad -\frac{1}{128}(+4i)\partial^{\dot{\beta}\beta}\left([D_\beta,\bar{D}_{\dot{\beta}}]\left(D^2\frac{\partial\Lambda}{\partial D^2Y}|D_\alpha Y|^4\right)\left(-\frac{1}{4}\bar{D}^2\delta X\right)\right)|.
\end{aligned}
\tag{3.56}
$$

The total derivative term does not have the term including δC_{mnp} at the boundary, and this total derivative term is zero. By using the identity $\bar{D}_{\dot{\beta}}\bar{D}^2 = 0$, we finally obtain

$$
\begin{aligned}
-\frac{1}{128}\int d^2\theta\bar{D}^2 &\frac{\partial\Lambda}{\partial D^2Y}\left(-\frac{1}{4}D^2\bar{D}^2\delta X\right)|D_\alpha Y|^4\\
&= -\frac{1}{128}\int d^2\theta\left(\bar{D}^2D^2\frac{\partial\Lambda}{\partial D^2Y}|D_\alpha Y|^4\right)\left(-\frac{1}{4}\bar{D}^2\delta X\right).
\end{aligned}
\tag{3.57}
$$

As a consequence, boundary terms given by the partial integrations are equal to zero. Thus, the variation of the higher derivative Lagrangian can be written by using the following chiral superfield T_{HD}

$$
\begin{aligned}
T_{\mathrm{HD}} = -\frac{1}{4}\bar{D}^2\cdot\frac{1}{16}\Bigg(&\frac{\partial\Lambda}{\partial Y}|D_\alpha Y|^4 - \partial_m\left(\frac{\partial\Lambda}{\partial\partial_m Y}|D_\alpha Y|^4\right)\\
&+ D^2\left(\frac{\partial\Lambda}{\partial D^2Y}|D_\alpha Y|^4\right) - 2D^\alpha(\Lambda(D_\alpha Y)(\bar{D}_{\dot{\alpha}}\bar{Y})(\bar{D}^{\dot{\alpha}}\bar{Y}))\Bigg)
\end{aligned}
\tag{3.58}
$$

as

$$
\delta\mathcal{L}_{\mathrm{HD,SUSY}} = \left(\int d^2\theta T_{\mathrm{HD}}\left(-\frac{1}{4}\bar{D}^2\delta X\right) + \mathrm{h.c.}\right) + \delta\mathcal{L}_{\mathrm{HD,SUSY,bound}}.
\tag{3.59}
$$

The variation has the same structure as the previous quadratic derivative case in eq. (3.41). The superspace integration leads to the boundary term with δC_{mnp}:

$$
\begin{aligned}
\delta\mathcal{L}_{\mathrm{HD,SUSY}} &= iT_{I,\mathrm{HD}}\left(\frac{1}{16}(D^2\bar{D}^2 - \bar{D}^2D^2)\delta X\right)| + \delta\mathcal{L}_{\mathrm{HD,SUSY,bound}}.\cdots\\
&= iT_{I,\mathrm{HD}}(-4i)\left(\frac{1}{16}\partial^{\dot{\alpha}\alpha}[D_\alpha,\bar{D}_{\dot{\alpha}}]\delta X\right)| + \delta\mathcal{L}_{\mathrm{HD,SUSY,bound}}. + \cdots\\
&= +\frac{\sqrt{2}}{3!}\partial_m\left(T_{I,\mathrm{HD}}|\epsilon^{mnpq}\delta C_{npq}\right) + \delta\mathcal{L}_{\mathrm{HD,SUSY,bound}}. + \cdots.
\end{aligned}
\tag{3.60}
$$

Here, the ellipsis $\cdots$ denotes the terms that do not contribute to the boundary terms, and $T_{I,\mathrm{HD}} = \frac{1}{2i}(T_{\mathrm{HD}} - \bar{T}_{\mathrm{HD}})$. To see the variation more concretely, we express T_{HD} in terms of the component fields. For simplicity, we focus on the bosonic term of $T_{\mathrm{HD}}|$. In the WZ gauge in eq. (3.34), the bosonic term of T_{HD} can be calculated as

$$
\begin{aligned}
T_{\mathrm{HD}}| = 2\Lambda|\mathcal{F}|^2\bar{\mathcal{F}} &- 2\Lambda\bar{\mathcal{F}}\partial^m y\partial_m\bar{y}\\
&- 4\frac{\partial\Lambda}{\partial D^2Y}\left(|\mathcal{F}|^4 - 2\partial^m y\partial_m\bar{y}|\mathcal{F}|^2 + (\partial^m y\partial_m y)(\partial^n\bar{y}\partial_n\bar{y})\right)\\
&+ (\text{fermions}).
\end{aligned}
\tag{3.61}
$$

Therefore, the imaginary part $T_{I,\text{HD}}$ is

$$T_{I,\text{HD}}| = \frac{1}{\sqrt{2}}\Lambda(H^2 + F^2)F - \sqrt{2}\Lambda(\partial^m y \partial_m \bar{y})F$$
$$- 4\left(\text{Im}\frac{\partial\Lambda}{\partial D^2 Y}\right)\left(\frac{1}{4}(H^2 + F^2)^2 - \partial^m y \partial_m \bar{y}(H^2 + F^2) + (\partial^m y \partial_m y)(\partial^n \bar{y} \partial_n \bar{y})\right)$$
$$+ \text{(fermions)}. \tag{3.62}$$

We now consider the boundary term which cancels the variation at the boundary in eq. (3.60). Such boundary term is given by the I-type integral as follows:

$$\mathcal{L}_{\text{HD,SUSY,bound.}} = \frac{i}{4}\left(\int d^2\theta \bar{D}^2 - \int d^2\bar{\theta} D^2\right) T_{I,\text{HD}} X. \tag{3.63}$$

In the WZ gauge, the boundary term is expressed as

$$\mathcal{L}_{\text{HD,SUSY,bound.}} = -\frac{\sqrt{2}}{3!}\partial_m\left(T_{I,\text{HD}}|\epsilon^{mnpq}\delta C_{npq}\right). \tag{3.64}$$

Therefore, this boundary term precisely cancels the variation in eq. (3.60).

By adding a Kähler potential and a superpotential, we obtain the most general Lagrangian of the 3-form gauge field with an arbitrary order of the field strength:

$$\mathcal{L}_{KW,\text{HD,SUSY}}$$
$$= -\frac{1}{8}\int d^2\theta \bar{D}^2 K(Y,\bar{Y}) + \int d^2\theta W(Y)$$
$$- \frac{1}{8\cdot 16}\int d^2\theta \bar{D}^2 \Lambda(Y,\bar{Y},\partial_m Y,\partial_m \bar{Y},D^2 Y,\bar{D}^2\bar{Y})(D^\alpha Y)(D_\alpha Y)(\bar{D}_{\dot{\alpha}}\bar{Y})(\bar{D}^{\dot{\alpha}}\bar{Y}) \tag{3.65}$$
$$+ \frac{i}{4}\int d^2\theta \bar{D}^2(T_{I,KW} + T_{I,\text{HD}})X + \text{h.c.}$$

3.3.3 EOM for 3-form gauge field

Before closing this section, we write down the general EOM for the 3-form gauge field, which should be useful for applications. We consider the most general Lagrangian in eq. (3.65). The EOM for the 3-form prepotential can be derived by the variation of the Lagrangian in the bulk:

$$0 = -\frac{1}{4}\bar{D}^2\frac{\partial K}{\partial Y} + \frac{\partial W}{\partial Y} - \frac{1}{4\cdot 16}\bar{D}^2\frac{\partial\Lambda}{\partial Y}|D_\alpha Y|^4 + \frac{1}{4\cdot 16}\bar{D}^2\partial_m\frac{\partial\Lambda}{\partial\partial_m Y}|D_\alpha Y|^4$$
$$- \frac{1}{4\cdot 16}\bar{D}^2 D^2\frac{\partial\Lambda}{\partial D^2 Y}|D_\alpha Y|^4 + \frac{1}{2\cdot 16}\bar{D}^2 D^\alpha\Lambda(D_\alpha Y)(\bar{D}_{\dot{\alpha}}\bar{Y})(\bar{D}^{\dot{\alpha}}\bar{Y}) + \text{h.c.} \tag{3.66}$$

As we have seen in section 3.1, the $\theta = \bar{\theta} = 0$ component of this EOM corresponds to the EOM for the the auxiliary field H. In order to obtain the EOM for the 3-form gauge field, we consider a derivative $[D_\beta, \bar{D}_{\dot{\beta}}]$ on the both hand sides of eq. (3.66):

$$0 = -\frac{1}{4}(+2i)\partial_{\beta\dot{\beta}}\left(\bar{D}^2\frac{\partial K}{\partial Y} - 4\frac{\partial W}{\partial Y} + \frac{1}{16}\bar{D}^2\frac{\partial\Lambda}{\partial Y}|D_\alpha Y|^4 - \frac{1}{16}\bar{D}^2\partial_m\frac{\partial\Lambda}{\partial\partial_m Y}|D_\alpha Y|^4\right.$$
$$\left. + \frac{1}{16}\bar{D}^2 D^2\frac{\partial\Lambda}{\partial D^2 Y}|D_\alpha Y|^4 - \frac{1}{8}\bar{D}^2 D^\alpha\Lambda(D_\alpha Y)(\bar{D}_{\dot{\alpha}}\bar{Y})(\bar{D}^{\dot{\alpha}}\bar{Y})\right) + \text{h.c.}$$
$$\tag{3.67}$$

The EOM can be solved as follows:

$$k = \frac{i}{4}\left(\bar{D}^2\frac{\partial K}{\partial Y} - 4\frac{\partial W}{\partial Y} + \frac{1}{16}\bar{D}^2\frac{\partial\Lambda}{\partial Y}|D_\alpha Y|^4 - \frac{1}{16}\bar{D}^2\partial_m\frac{\partial\Lambda}{\partial\partial_m Y}|D_\alpha Y|^4\right.$$
$$\left. + \frac{1}{16}\bar{D}^2 D^2\frac{\partial\Lambda}{\partial D^2 Y}|D_\alpha Y|^4 - \frac{1}{8}\bar{D}^2 D^\alpha\Lambda(D_\alpha Y)(\bar{D}_{\dot\alpha}\bar{Y})(\bar{D}^{\dot\alpha}\bar{Y})\right) + \text{h.c.,} \tag{3.68}$$

where k is a constant which will be determined by the boundary conditions. By using eqs. (3.66) and (3.68), we obtain the following solution:

$$ik = -\frac{1}{4}\bar{D}^2\frac{\partial K}{\partial Y} + \frac{\partial W}{\partial Y} - \frac{1}{4\cdot 16}\bar{D}^2\frac{\partial\Lambda}{\partial Y}|D_\alpha Y|^4 + \frac{1}{4\cdot 16}\bar{D}^2\partial_m\frac{\partial\Lambda}{\partial\partial_m Y}|D_\alpha Y|^4$$
$$- \frac{1}{4\cdot 16}\bar{D}^2 D^2\frac{\partial\Lambda}{\partial D^2 Y}|D_\alpha Y|^4 + \frac{1}{2\cdot 16}\bar{D}^2 D^\alpha\Lambda(D_\alpha Y)(\bar{D}_{\dot\alpha}\bar{Y})(\bar{D}^{\dot\alpha}\bar{Y}). \tag{3.69}$$

The bosonic sector of the $\theta = \bar{\theta} = 0$ component of the above solution is expressed as

$$ik = \frac{\partial^2 K}{\partial Y\partial\bar{Y}}|\bar{\mathcal{F}} + \frac{\partial W}{\partial Y}|$$
$$- 4\frac{\partial\Lambda}{\partial D^2 Y}|(|\mathcal{F}|^4 - 2|\mathcal{F}|^2(\partial^m y\partial_m\bar{y}) + (\partial^m y\partial_m y)(\partial^n\bar{y}\partial_n\bar{y})) \tag{3.70}$$
$$+ 2\Lambda(|\mathcal{F}|^2\bar{\mathcal{F}} - \bar{\mathcal{F}}\partial^m y\partial_m\bar{y}).$$

To solve this equation, we need to give a concrete model of K, W, and Λ. We will discuss such models and their solutions elsewhere.

4 Summary and discussion

We have considered higher derivative extensions of 3-form gauge theories in the both bosonic and SUSY cases. For the bosonic case, we have shown that higher derivative terms given by derivatives on the field strength causes a tachyon as long as the canonical kinetic term exists. We have also argued that the tachyon can also be a ghost, depending on models and parameters. We have shown that there is neither a tachyon nor a ghost if the higher derivative terms are given by functions of the field strength but not of the derivative of the field strength. This is because the EOM is not changed from the case of the canonical kinetic term. We have confirmed that previously known higher derivative Lagrangians [6, 8] fall into this class, Then we have specified the boundary term which corresponds to the ghost/tachyon-free higher derivative Lagrangian of an arbitrary order of the field strength. For the SUSY case, we have shown that a naive higher derivative extension of the SUSY 3-form gauge theory may cause a tachyon as well as a ghost, as long as the canonical kinetic term exists. Then we have presented the most general ghost/tachyon-free Lagrangian of an arbitrary order of the field strength, the corresponding boundary term, and EOM for the 3-form gauge field.

There can be several extensions and applications of our work. One may apply the higher derivative theory in section 2.3 to the cosmological constant problem. Since the Lagrangian gives us a more general constant term given by $G(-c) + cG'(-c)$ than that

of the canonical kinetic term $-\frac{1}{2}c^2$, it may give a correction to the application to the cosmological constant problem.

We can extend the higher derivative Lagrangians to include a topological coupling between a 3-form gauge field and a pseudo-scalar field, which will give us the potential of the pseudo-scalar field. The general discussion between higher derivative terms and potentials is known for bosonic case in ref. [6], and so the SUSY extension is possible by using our new ghost/tachyon-free Lagrangian. Since coupling gives us a potential for the pseudo-scalar field, it should be useful for inflationary models.

To apply the higher derivative Lagrangian to cosmology, we should embed the Lagrangian into SUGRA. It may be straightforward since 3-form gauge theories in SUGRA are known in refs. [28, 42].

In this paper, we have shown no ghost in the bosonic sector of the SUSY case. We expect that there is no fermionic ghost in eq. (3.65) by SUSY transformations. However, a concrete explanation of whether fermionic ghosts are absent or not is an open question. We may discuss it along the line of refs. [121, 122].

It would also be interesting to consider solutions of the EOM in eq. (3.69). In the case where Λ is a constant, it is known that there is a non-trivial solution of the auxiliary field (so-called a non-canonical branch), where the canonical kinetic term for the boson in the chiral superfield vanishes at on-shell. Our higher derivative term would deform this solution.

Higher derivative chiral superfield Lagrangians give several BPS equations for BPS solitons [65, 89]. It is interesting to discuss whether our SUSY higher derivative Lagrangian of the 3-form gauge field admit any BPS equations and their soliton solutions.

Acknowledgements

A   Notation

In this appendix, we summarize our notation. We use Wess-Bagger's notation in ref. [114]. The Minkowski metric η_{mn} and the totally anti-symmetric tensor ϵ_{mnpq} are given by

$$\eta_{mn} = (-1, 1, 1, 1), \quad \epsilon_{0123} = -\epsilon^{0123} = 1, \tag{A.1}$$

respectively. Here, the Roman letters beginning with $m, n, p, \ldots$ denote indices of vectors and tensors.

In the following, we summarize our notation of spinors. The Greek letters beginning with $\alpha, \beta, \ldots$ denote undotted spinor indices, while dotted Greek letters beginning with $\dot{\alpha}, \dot{\beta}, \ldots$ denote dotted spinor indices. The undotted and dotted spinors are related to each other by the Hermitian conjugate denoted by $\dagger$. The Hermitian conjugate of a spinor χ_α, $\bar{\chi}_{\dot{\alpha}}$ are defined by

$$(\chi_\alpha)^\dagger = \bar{\chi}_{\dot{\alpha}}, \quad (\bar{\chi}_{\dot{\alpha}})^\dagger = \chi_\alpha. \tag{A.2}$$

The Hermitian conjugate of a product of spinors χ_α and ψ_α is defined by

$$(\chi_\alpha \psi_\beta)^\dagger = \bar{\psi}_{\dot{\beta}} \bar{\chi}_{\dot{\alpha}}. \tag{A.3}$$

Spinor indices are raised and lowered by the following totally antisymmetric tensors $\epsilon_{\alpha\beta}$, $\epsilon^{\alpha\beta}$, $\epsilon_{\dot{\alpha}\dot{\beta}}$, and $\epsilon^{\dot{\alpha}\dot{\beta}}$ with the following normalizations

$$\epsilon^{12} = -\epsilon^{21} = -\epsilon_{12} = \epsilon_{21} = 1, \quad \epsilon^{\dot{1}\dot{2}} = -\epsilon^{\dot{2}\dot{1}} = -\epsilon_{\dot{1}\dot{2}} = \epsilon_{\dot{2}\dot{1}} = 1 \tag{A.4}$$

as

$$\chi^\alpha = \epsilon^{\alpha\beta}\chi_\beta, \quad \chi_\alpha = \epsilon_{\alpha\beta}\chi^\beta, \quad \bar{\chi}^{\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}}\bar{\chi}_{\dot{\beta}}, \quad \bar{\chi}_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}}\bar{\chi}^{\dot{\beta}}. \tag{A.5}$$

The anti-symmetric tensors satisfy

$$\epsilon^{\alpha\beta}\epsilon_{\beta\gamma} = \delta^\alpha_\gamma, \quad \epsilon^{\dot{\alpha}\dot{\beta}}\epsilon_{\dot{\beta}\dot{\gamma}} = \delta^{\dot{\alpha}}_{\dot{\gamma}}, \tag{A.6}$$

where δ^α_β and $\delta^{\dot{\alpha}}_{\dot{\beta}}$ are the Kronecker's delta for undotted and dotted spinors, respectively. The contraction of spinors are

$$\psi^\alpha \chi_\alpha = \epsilon^{\alpha\beta}\psi_\beta \chi_\alpha, \quad \bar{\psi}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}}\bar{\psi}_{\dot{\beta}}\bar{\chi}^{\dot{\alpha}}. \tag{A.7}$$

A vector is represented by a tensor product of spinors. The relations between vectors and spinors are given by the 4D Pauli matrices $(\sigma_m)_{\alpha\dot{\alpha}}$ defined by

$$(\sigma_m)_{\alpha\dot{\beta}} = (\sigma_0, \sigma_1, \sigma_2, \sigma_3)_{\alpha\dot{\beta}} = \left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right). \tag{A.8}$$

For example, the spacetime derivative ∂_m are represented by using spinors as

$$\partial_{\alpha\dot{\alpha}} = (\sigma^m)_{\alpha\dot{\alpha}}\partial_m. \tag{A.9}$$

The relation of Hermitian conjugates of the Pauli matrices $(\bar{\sigma}_m)^{\dot{\alpha}\beta}$ to $(\sigma_m)_{\alpha\dot{\beta}}$ is

$$(\bar{\sigma}_m)^{\dot{\alpha}\beta} = (\sigma_m)^{\beta\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\gamma}}\epsilon^{\beta\delta}(\sigma_m)_{\delta\dot{\gamma}}. \tag{A.10}$$

The Pauli matrices satisfy the following relation:

$$(\sigma_m)_{\alpha\dot{\beta}}(\bar{\sigma}_n)^{\dot{\beta}\gamma} + (\sigma_n)_{\alpha\dot{\beta}}(\bar{\sigma}_m)^{\dot{\beta}\gamma} = -2\eta_{mn}\delta^\gamma_\alpha, \quad (\bar{\sigma}_m)^{\dot{\alpha}\beta}(\sigma_n)_{\beta\dot{\gamma}} + (\bar{\sigma}_n)^{\dot{\alpha}\beta}(\sigma_m)_{\beta\dot{\gamma}} = -2\eta_{mn}\delta^{\dot{\alpha}}_{\dot{\gamma}}. \tag{A.11}$$

The matrices σ_{mn} and $\bar{\sigma}_{mn}$ are defined by

$$
\begin{aligned}
(\sigma_{mn})_\alpha{}^\beta &= \frac{1}{4}((\sigma_m)_{\alpha\dot{\gamma}}(\bar{\sigma}_n)^{\dot{\gamma}\beta} - (\sigma_n)_{\alpha\dot{\gamma}}(\bar{\sigma}_m)^{\dot{\gamma}\beta}), \\
(\bar{\sigma}_{mn})^{\dot{\alpha}}{}_{\dot{\beta}} &= \frac{1}{4}((\bar{\sigma}_m)^{\dot{\alpha}\gamma}(\sigma_n)_{\gamma\dot{\beta}} - (\sigma_n)^{\dot{\alpha}\gamma}(\bar{\sigma}_m)_{\gamma\dot{\beta}}).
\end{aligned}
\tag{A.12}
$$

To formulate SUSY theories, we have used SUSY covariant spinor derivatives. The definition of the spinor derivatives are

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i\bar{\theta}^{\dot{\beta}}(\sigma_m)_{\alpha\dot{\beta}}\frac{\partial}{\partial x^m}, \quad \bar{D}^{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\theta}_{\dot{\alpha}}} - i\theta_\beta(\bar{\sigma}_m)^{\dot{\alpha}\beta}\frac{\partial}{\partial x^m}. \tag{A.13}$$

B   Duality transformation of bosonic 3-form gauge theory

In this appendix, we review the duality transformation of the 3-form. The 3-form is classically dual to a constant. This can be shown as follows. We consider the following first order Lagrangian instead of the Lagrangian in eq. (2.7):

$$\mathcal{L}'_{\text{kin.}} = -\frac{1}{2 \cdot 4!}F'^{mnpq}F'_{mnpq} + \frac{1}{3!}\partial^m(C^{npq}F'_{mnpq}) + \frac{1}{4!}q\epsilon^{mnpq}(F'_{mnpq} - 3\partial_m C_{npq}). \tag{B.1}$$

Here, F'_{mnpq} is a 4-form field which is independent of the 3-form gauge field C_{mnp}, but we assume the boundary condition for the 4-form:

$$F' = -c, \tag{B.2}$$

where $F' = \frac{1}{4!}\epsilon^{mnpq}F'_{mnpq}$. In eq. (B.1), q is a pseudo-scalar field which is regarded as a Lagrange's multiplier. The vanishing of the variation by the 3-form gauge field at the boundary requires the boundary condition for q:

$$q|_{\text{bound.}} = c. \tag{B.3}$$

The EOM of q gives $\epsilon^{mnpq}F'_{mnpq} = 4\epsilon^{mnpq}\partial_m C_{npq}$, and we obtain the original Lagrangian in eq. (2.7). On the other hand, the EOM for the 4-form field F'_{mnpq} gives

$$F'_{mnpq} = q\epsilon_{mnpq}, \quad \text{or equilvalently} \quad q = -F'. \tag{B.4}$$

Substituting the solution to the Lagrangian in eq. (B.1), we obtain

$$\begin{aligned}\mathcal{L}'_{\text{kin.}} &= -\frac{1}{2}q^2 + \frac{1}{3!}\partial^m(C^{npq}\epsilon_{mnpq}q) - \frac{1}{3!}q\epsilon^{mnpq}\partial_m C_{npq} \\ &= -\frac{1}{2}q^2 + \frac{1}{3!}C^{npq}\epsilon_{mnpq}\partial^m q.\end{aligned} \tag{B.5}$$

The EOM of C_{mnp} gives

$$\partial_m q = 0. \tag{B.6}$$

Therefore, q is local constant, which is equal to c by the boundary condition. Therefore, the Lagrangian is equal to a constant.

$$\mathcal{L}'_{\text{kin.}} = -\frac{1}{2}c^2. \tag{B.7}$$

The reverse of the duality transformation is possible, and the boundary term in eq. (2.7) is naturally understood by this transformation. We start with the following Lagrangian:

$$\mathcal{L}_{\text{const.}} = -\frac{1}{2}c^2, \tag{B.8}$$

where c is a real constant. The first-order Lagrangian is

$$\mathcal{L}'_{\text{const.}} = -\frac{1}{2}q^2 + \frac{1}{3!}\epsilon^{mnpq}C_{npq}\partial_m q, \tag{B.9}$$

where we assume the boundary conditions for q as $q|_{\text{bound.}} = c$. The Lagrangian can be constructed as follows. The constant term can be considered as a closed 0-form. A closed 0-form f is defined by the condition $\partial_m f = 0$. Thus, we introduce the condition as a solution of the Lagrange's multiplier field. In 4D, the Lagrange's multiplier can be a 3-form, and this condition can be imposed by the term $\epsilon^{mnpq}C_{npq}\partial_m q$. In this case, there is a gauge symmetry of C_{mnp} given in eq. (2.1).

The EOM for q is

$$q = -\frac{1}{4!}\epsilon^{mnpq}\partial_m C_{npq} = -\frac{1}{4!}\epsilon^{mnpq}F_{mnpq} = -F. \tag{B.10}$$

Substituting this solution into eq. (B.9), we obtain

$$\begin{aligned}
\mathcal{L}'_{\text{const.}} &= -\frac{1}{2}F^2 - \frac{1}{3!}\epsilon^{mnpq}C_{npq}\partial_m F = +\frac{1}{2}F^2 - \frac{1}{3!}\partial_m(\epsilon^{mnpq}C_{npq}F) \\
&= -\frac{1}{2\cdot 4!}F^{mnpq}F_{mnpq} + \frac{1}{3!}\partial_m(C_{npq}F^{mnpq}).
\end{aligned} \tag{B.11}$$

Thus, we have the Lagrangian in eq. (2.7).

C   Duality transformation of SUSY 3-form gauge theory

In this appendix, we review a duality transformation of SUSY 3-form gauge theories [42]. A 3-from field in SUSY theories can be dualized into a chiral superfield Φ which has a linear superpotential $ir\Phi$ with a real constant r. The boundary term for the 3-form gauge theories can be straightforwardly obtained by this duality procedure. Here, we only consider the case of a canonical kinetic term, although this discussion can be generalized to the case in which the Lagrangian is given by a Kähler potential and a superpotential.

We consider a dual transformation from a single chiral superfield Φ with a linear superpotential to a 3-form field system. We start with the following Lagrangian:

$$\mathcal{L}_{\text{kin.,chiral}} = -\frac{1}{8}\int d^2\theta \bar{D}^2\Phi\bar{\Phi} + \int d^2\theta\, ir\Phi + \text{h.c.}, \tag{C.1}$$

where r is a real constant.

Now we dualize the Lagrangian by considering the following Lagrangian with a chiral superfield Q and a real superfield X:

$$\mathcal{L}'_{\text{kin.,chiral}} = -\frac{1}{8}\int d^2\theta \bar{D}^2\Phi\bar{\Phi} + \int d^2\theta\, Q\Phi + \frac{1}{8}\int d^2\theta \bar{D}^2 X(Q+\bar{Q}) + \text{h.c.}, \tag{C.2}$$

where we assume the boundary condition $Q||_{\text{bound.}} = ir$. Note that X has a gauge symmetry $X \to X + L$, where L is a linear superfield $D^2L = \bar{D}^2L = 0$. The gauge transformation implies that X is a 3-form prepotential. The EOM of X gives us $Q + \bar{Q} = 0$, i.e. Q is a

pure imaginary constant, and Q is equal to ir by the boundary condition. Substituting this solution into eq. (C.2), we obtain the original Lagrangian in eq. (C.1).

Instead, the EOM for the chiral superfield Q relates the original chiral superfield Φ with the real superfield X:

$$\Phi = -\frac{1}{4}\bar{D}^2 X =: Y, \tag{C.3}$$

where Y can be identified with a 4-form field strength superfield since Y is related to the real superfield X. The EOM for the chiral superfield Φ leads to the relation between Y and Q:

$$-\frac{1}{4}\bar{D}^2\bar{\Phi} = -\frac{1}{4}\bar{D}^2\bar{Y} = -Q. \tag{C.4}$$

The solution gives us the boundary condition for $D^2 Y$:

$$-\frac{1}{4}D^2 Y||_{\text{bound.}} = -\bar{Q}||_{\text{bound.}} = ir. \tag{C.5}$$

Eliminating the chiral superfield Φ, we obtain the Lagrangian

$$\mathcal{L}'_{\text{kin.,chiral}} = -\frac{1}{8}\int d^2\theta \bar{D}^2 Y\bar{Y} + \frac{i}{4}\int d^2\theta \bar{D}^2 X T_I + \text{h.c.}, \tag{C.6}$$

Here T_I is a imaginary part of the chiral superfield $T := -\frac{1}{4}\bar{D}^2\bar{Y}$. The second term in the right hand side in eq. (C.6) is the boundary term which is equal to the one in eq. (3.32). Note that to derive the Lagrangian in eq. (C.6), we have used the following calculation:

$$\begin{aligned}
\int d^2\theta Q\Phi + \frac{1}{8}\int d^2\theta \bar{D}^2 X(Q + \bar{Q}) &= -\frac{1}{4}\int d^2\theta \bar{D}^2 XQ + \frac{1}{8}\int d^2\theta \bar{D}^2 X(Q + \bar{Q}) \\
&= \frac{i}{4}\int d^2\theta \bar{D}^2 X \frac{1}{2i}\left(-\frac{1}{4}\bar{D}^2\bar{Y} + \frac{1}{4}D^2 Y\right).
\end{aligned} \tag{C.7}$$

The Lagrangian in eq. (C.6) is the same as the one in eq. (3.15), and thus the Lagrangian with a linear superpotential in eq. (C.1) can be dualized into a Lagrangian for a 3-form gauge field including the boundary term.

D   Ghost/tachyon-free Lagrangian and auxiliary field method

Here, we consider an auxiliary field method for the ghost/tachyon-free Lagrangian proposed in section 3.3. In section 3.3, we have shown that the Lagrangian in eq. (3.65) is ghost/tachyon-free by using component expression. We can also show it by using the auxiliary field method in section 3.2.

The Lagrangian with auxiliary superfields is given as follows:

$$\begin{aligned}
\mathcal{L}'_{\text{HD,SUSY}} = &\left(\frac{1}{8 \cdot 16}\int d^2\theta \bar{D}^2 \Lambda(Y, \bar{Y}, \partial_m Y, \partial_m \bar{Y}, \bar{\Psi}, \Psi)(D^\alpha Y)(D_\alpha Y)(\bar{D}_{\dot{\alpha}}\bar{Y})(\bar{D}^{\dot{\alpha}}\bar{Y})\right. \\
&\left. + \int d^2\theta \Sigma\left(\Psi + \frac{1}{4}\bar{D}^2\bar{Y}\right) + \text{h.c.}\right) + \mathcal{L}'_{\text{HD,SUSY,bound.}}.
\end{aligned} \tag{D.1}$$

Here, $\mathcal{L}'_{\text{HD,SUSY,bound.}}$ is the boundary term for this Lagrangian, which is not relevant to the following discussion. In this Lagrangian, the superfield Σ is not an independent dynamical

superfield as far as fermions are set to be zero in the vacuum. This can be seen by the EOM for the chiral superfield Ψ. The EOM for Ψ is

$$0 = -\tfrac{1}{4}\bar{D}^2 \left(\frac{\partial \Lambda}{\partial \Psi} (D^\alpha Y)(D_\alpha Y)(\bar{D}_{\dot\alpha}\bar{Y})(\bar{D}^{\dot\alpha}\bar{Y}) \right) + \Sigma. \tag{D.2}$$

The EOM leads to $\Sigma| = 0$ around the vacuum where $D_\alpha Y| = 0$. Thus, $\Sigma|$ has no dynamical degrees of freedom. Therefore, $\Sigma|$ does not give rise to a ghost. As long as SUSY is preserved, the fermionic component $D_\alpha \Sigma|$ is also not dynamical, and so there is no fermionic ghost as well.

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