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Effective multi-Higgs couplings to gluons

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Abstract: Standard-Model Higgs bosons are dominantly produced via the gluon-fusion mechanism $gg \to H$ at the LHC, i.e. in a loop-mediated process with top loops providing the dominant contribution. For the measured Higgs boson mass of $\sim 125\,\mathrm{GeV}$ the limit of heavy top quarks provides a reliable approximation as long as the relative QCD corrections are scaled with the full mass-dependent LO cross section. In this limit the Higgs coupling to gluons can be described by an effective Lagrangian. The same approach can also be applied to the coupling of more than one Higgs boson to gluons. We will derive the effective Lagrangian for multi-Higgs couplings to gluons up to N^{4}LO thus extending previous results for more than one Higgs boson. Moreover we discuss gluonic Higgs couplings up to NNLO, if several heavy quarks contribute.

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1 Introduction

The discovery of a resonance with 125 GeV mass [1, 2] that is compatible with the Standard-Model (SM) Higgs boson [3] marked a milestone in particle physics. The existence of the Higgs boson is inherently related to the mechanism of spontaneous symmetry breaking [4–8] while preserving the full gauge symmetry and the renormalizability of the SM [9, 10]. The dominant production process of the Higgs boson at the LHC is the loop-induced gluon-fusion process mediated by top-quark loops and to a lesser extent bottom- and charm-quark loops [11]. The QCD corrections are known up to N^{3}LO in the limit of heavy top quarks [12–27], while the full quark mass dependence is only known up to NLO [28–31]. At NNLO subleading terms in the large top mass expansion [32–35] and leading contributions to the top+bottom interference [36] are known. The limit of heavy top quarks has also been adopted for threshold-resummed calculations [37–48], while the inclusion of finite quark-mass effects in the resummation has been considered recently [49–51]. It has been shown that the limit of heavy top quarks $m_t^2 \gg M_H^2$ provides a reasonable approximation to the calculation of the gluon-fusion cross section with full mass dependence as long as the relative QCD corrections are scaled with the fully massive LO cross section [28, 37, 38]. In the heavy-top-quark limit the calculation of the gluon-fusion cross section can be simplified by starting from an effective Lagrangian describing the Higgs coupling to gluons after integrating out the top contribution [52–54]. The same approach has also been applied to Higgs pair production via gluon fusion, $gg \to HH$, at NLO [55], NNLO [56–58] as well as to threshold resummation up to NNLL [59, 60]. It has been shown that finite mass effects amount to about 5% in the single Higgs case and 15% for Higgs boson pairs [61–65].

In this letter we will derive the effective Lagrangian for multi-Higgs couplings to gluons to N^{4}LO for arbitrary numbers of external Higgs bosons thus extending previous work beyond the single-Higgs case. In section 2 we will discuss and present the effective Lagrangian for the SM Higgs boson up to N^{4}LO, while section 3 will extend this analysis to an arbitrary number of heavy quarks contributing to the gluonic Higgs coupling up to NNLO. In section 4 we will conclude.

2 Standard-model Higgs bosons

The starting point for the derivation of the effective Lagrangian in the heavy-top-quark limit is the low-energy limit of the top-quark contributions to the Wilson coefficient of the gluonic field-strength operator $\hat{G}^{a\mu\nu}\hat{G}^a_{\mu\nu}$, where $\hat{G}^{a\mu\nu}$ denotes the ($\overline{\text{MS}}$-subtracted) gluonic operator of colour-SU(3) in the low-energy limit with 6 active flavours,[1]

$$\mathcal{L}_g = -\frac{1-\Pi_t}{4}\hat{G}^{a\mu\nu}\hat{G}^a_{\mu\nu} \tag{2.1}$$

The Wilson coefficient Π_t denotes the gauge-invariant vacuum polarization function of the gluon that is determined by the top-quark contribution to the gluon self-energy and the two-point-function parts of the external vertices attached to the gluons. This boils down to the inverse top-quark contribution to the strong coupling constant so that Π_t is related to the decoupling relation between the strong coupling constant in an (N_F+1)- and N_F-flavour theory ($N_F = 5$),

$$\alpha_s^{(N_F)}(\mu_R^2) = \zeta_{\alpha_s}\, \alpha_s^{(N_F+1)}(\mu_R^2)\,, \qquad \zeta_{\alpha_s} = 1 + \sum_n D_n \left(\frac{\alpha_s^{(N_F+1)}(\mu_R^2)}{\pi}\right)^n \tag{2.2}$$

with the perturbative coefficients up to fourth order [66–69] $[L_t = \log(\mu_R^2/\overline{m_t}^2(\mu_R^2))]$

$$\begin{aligned}
D_1 &= -\frac{1}{6}L_t \qquad\qquad D_2 = \frac{11}{72} - \frac{11}{24}L_t + \frac{1}{36}L_t^2 \\
D_3 &= \frac{564731}{124416} - \frac{82043}{27648}\zeta_3 - \frac{2633}{31104}N_F - \frac{955-67N_F}{576}L_t + \frac{53-16N_F}{576}L_t^2 - \frac{1}{216}L_t^3 \\
D_4 &= \frac{291716893}{6123600} - \frac{121}{4320}\log^5 2 + \frac{3031309}{1306368}\log^4 2 + \frac{121}{432}\zeta_2\log^3 2 - \frac{3031309}{217728}\zeta_2\log^2 2 \\
&\quad + \frac{2057}{576}\zeta_4\log 2 + \frac{1389}{256}\zeta_5 - \frac{76940219}{2177280}\zeta_4 - \frac{2362581983}{87091200}\zeta_3 + \frac{3031309}{54432}a_4 + \frac{121}{36}a_5 \\
&\quad - \frac{151369}{2177280}X_0 + N_F\left(-\frac{4770941}{2239488} + \frac{685}{124416}\log^4 2 - \frac{685}{20736}\zeta_2\log^2 2 + \frac{3645913}{995328}\zeta_3\right. \\
&\quad \left. - \frac{541549}{165888}\zeta_4 + \frac{115}{576}\zeta_5 + \frac{685}{5184}a_4\right) + N_F^2\left(-\frac{271883}{4478976} + \frac{167}{5184}\zeta_3\right) \\
&\quad - \left[\frac{7391699}{746496} + \frac{2529743}{165888}\zeta_3 + N_F\left(\frac{110341}{373248} - \frac{110779}{82944}\zeta_3\right) - N_F^2\frac{6865}{186624}\right]L_t \\
&\quad + \left(\frac{2177}{3456} - N_F\frac{1483}{10368} - N_F^2\frac{77}{20736}\right)L_t^2 - \left(\frac{1883}{10368} + N_F\frac{127}{5184} - \frac{N_F^2}{324}\right)L_t^3 + \frac{L_t^4}{1296}
\end{aligned} \tag{2.3}$$

where $\overline{m_t}^2(\mu_R^2))$ denotes the $\overline{\text{MS}}$ top mass at the renormalization scale μ_R. The constants used in this expression are given by $a_n = Li_n(1/2)$ and $X_0 = 1.8088795462\ldots$ (only known numerically) [70]. The decoupling coefficient contains one-particle-reducible contributions and the Wilson coefficient of the Lagrangian eq. (2.1) is obtained from the inverse,

$$\Pi_t = 1 - \frac{1}{\zeta_{\alpha_s}} = \sum_n C_n \left(\frac{\alpha_s^{(N_F+1)}}{\pi}\right)^n \tag{2.4}$$

[1] The same ansatz has also been used in the derivation of the effective Hgg coupling in refs. [37, 38].

with the perturbative coefficients up to fifth order

$$
\begin{aligned}
C_1 &= -\frac{1}{6}L_t \qquad\qquad\qquad\qquad C_2 = \frac{11}{72} - \frac{11}{24}L_t \\
C_3 &= \frac{564731}{124416} - \frac{82043}{27648}\zeta_3 - \frac{2633}{31104}N_F - \frac{2777-201N_F}{1728}L_t - \frac{35+16N_F}{576}L_t^2 \\
C_4 &= \frac{1166295847}{24494400} - \frac{121}{4320}\log^5 2 + \frac{3031309}{1306368}\log^4 2 + \frac{121}{432}\zeta_2\log^3 2 - \frac{3031309}{217728}\zeta_2\log^2 2 \\
&\quad + \frac{2057}{576}\zeta_4\log 2 + \frac{1389}{256}\zeta_5 - \frac{76940219}{2177280}\zeta_4 - \frac{2362581983}{87091200}\zeta_3 + \frac{3031309}{54432}a_4 + \frac{121}{36}a_5 \\
&\quad - \frac{151369}{2177280}X_0 + N_F\left(-\frac{4770941}{2239488} + \frac{685}{124416}\log^4 2 - \frac{685}{20736}\zeta_2\log^2 2 + \frac{3645913}{995328}\zeta_3\right. \\
&\quad \left. -\frac{541549}{165888}\zeta_4 + \frac{115}{576}\zeta_5 + \frac{685}{5184}a_4\right) + N_F^2\left(-\frac{271883}{4478976} + \frac{167}{5184}\zeta_3\right) \\
&\quad + \left[\frac{2875235}{248832} - \frac{897943}{55296}\zeta_3 - N_F\left(\frac{40291}{124416} - \frac{110779}{82944}\zeta_3\right) + N_F^2\frac{6865}{186624}\right]L_t \\
&\quad - \left(\frac{1333}{10368} + N_F\frac{1081}{10368} + N_F^2\frac{77}{20736}\right)L_t^2 - \left(\frac{1697}{10368} + N_F\frac{175}{5184} - N_F^2\frac{1}{324}\right)L_t^3 \\
C_5 &= C_{50} + \left(-\frac{685}{10368}N_F^2 a_4 - \frac{11679301}{435456}N_F a_4 + \frac{93970579}{217728}a_4 - \frac{121}{72}N_F a_5 + \frac{3751}{144}a_5\right. \\
&\quad + \frac{121}{8640}N_F\log^5 2 - \frac{3751}{17280}\log^5 2 - \frac{685}{248832}N_F^2\log^4 2 - \frac{11679301}{10450944}N_F\log^4 2 \\
&\quad + \frac{93970579}{5225472}\log^4 2 - \frac{121}{864}N_F\zeta_2\log^3 2 + \frac{3751}{1728}\zeta_2\log^3 2 + \frac{685}{41472}N_F^2\zeta_2\log^2 2 \\
&\quad + \frac{11679301}{1741824}N_F\zeta_2\log^2 2 - \frac{93970579}{870912}\zeta_2\log^2 2 - \frac{2057}{1152}N_F\zeta_4\log 2 + \frac{63767}{2304}\zeta_4\log 2 \\
&\quad - \frac{211}{10368}N_F^3\zeta_3 + \frac{270407}{8957952}N_F^3 - \frac{4091305}{1990656}N_F^2\zeta_3 + \frac{576757}{331776}N_F^2\zeta_4 + \frac{115}{2304}N_F^2\zeta_5 \\
&\quad + \frac{48073}{165888}N_F^2 + \frac{151369}{4354560}N_F X_0 + \frac{12171659669}{232243200}N_F\zeta_3 - \frac{608462731}{69672960}N_F\zeta_4 \\
&\quad - \frac{313489}{41472}N_F\zeta_5 - \frac{75861299783}{3135283200}N_F - \frac{4692439}{8709120}X_0 - \frac{4660543511}{19353600}\zeta_3 \\
&\quad \left. - \frac{4674213853}{17418240}\zeta_4 + \frac{807193}{10368}\zeta_5 + \frac{846138861149}{3135283200}\right)L_t + \left(-\frac{481}{62208}N_F^3 - \frac{28297}{110592}N_F^2\zeta_3\right. \\
&\quad \left. + \frac{373637}{746496}N_F^2 + \frac{2985893}{331776}N_F\zeta_3 - \frac{47813}{4608}N_F - \frac{26296585}{442368}\zeta_3 + \frac{143939741}{1990656}\right)L_t^2 \\
&\quad + \left(\frac{77}{124416}N_F^3 + \frac{175}{27648}N_F^2 - \frac{5855}{124416}N_F - \frac{130201}{124416}\right)L_t^3 \\
&\quad + \left(-\frac{1}{2592}N_F^3 + \frac{47}{4608}N_F^2 - \frac{317}{6912}N_F - \frac{51383}{165888}\right)L_t^4
\end{aligned} \tag{2.5}
$$

where the logarithms of the coefficient C_5 have been reconstructed from the result of refs. [68, 69] including the recent five-loop result of the QCD beta function [71] (partly confirmed by [72]). The constant C_{50} is irrelevant for our derivation of the effective Lagrangian for gluonic Higgs couplings. Note that the highest powers of the logarithmic L_t terms disappeared in this expression as required by the proper RG-evolution of the one-particle-irreducible part Π_t. Using the low-energy theorem for a light Higgs boson [52–54] the effective top-quark contribution to the Lagrangian of eq. (2.1) is related to the couplings of external Higgs bosons in the heavy-top-quark limit by the replacement[2] $\overline{m_t}(\mu_R^2) \to \overline{m_t}(\mu_R^2)(1+H/v)$, i.e.

$$L_t \to \bar{L}_t = L_t - 2\log\left(1+\frac{H}{v}\right) \qquad \text{and} \qquad \Pi_t \to \bar{\Pi}_t \tag{2.6}$$

where H denotes the physical Higgs field, v the vacuum expectation value and $\bar{\Pi}_t$ the contribution to the Wilson coefficient with the shifted top-quark mass.[3] Based on this replacement it is obvious that only the logarithmic L_t terms of Π_t are relevant for the effective gluonic Higgs couplings. The object $\bar{\Pi}_t$ is expressed in terms of the (N_F+1)-flavour coupling $\alpha_s^{(N_F+1)}$. To derive the low-energy Lagrangian in the N_F-flavour theory we have to transform the (N_F+1)-flavour coupling into the N_F-flavour one by means of the relation [66–69]

$$\begin{aligned}\alpha_s^{(N_F+1)}(\mu_R^2) = \alpha_s^{(N_F)}(\mu_R^2)\Bigg\{&1+\frac{\alpha_s^{(N_F)}(\mu_R^2)}{\pi}\frac{L_t}{6}+\left(\frac{\alpha_s^{(N_F)}(\mu_R^2)}{\pi}\right)^2\left[-\frac{11}{72}+\frac{11}{24}L_t+\frac{L_t^2}{36}\right]\\ &+\left(\frac{\alpha_s^{(N_F)}(\mu_R^2)}{\pi}\right)^3\Bigg[-\frac{564731}{124416}+\frac{82043}{27648}\zeta_3+\frac{2633}{31104}N_F\\ &+\left(\frac{2645}{1728}-\frac{67}{576}N_F\right)L_t+\left(\frac{167}{576}+\frac{N_F}{36}\right)L_t^2+\frac{L_t^3}{216}\Bigg]+\mathcal{O}(\alpha_s^4)\Bigg\}\end{aligned} \tag{2.7}$$

derived from inverting eq. (2.2). For the proper low-energy limit the gluonic field-strength operator is expressed in terms of the one with $N_F = 5$ active flavours which leads to a global factor ζ_{α_s} so that the kinetic term of the gluons is properly normalized in the low-energy limit.[4] In this way we arrive at the low-energy Lagrangian in terms of the top $\overline{\rm MS}$ mass. The effective N^{4}LO Lagrangian for (multi-)Higgs couplings to gluons reads finally

$$\begin{aligned}\mathcal{L}_{\rm eff} = \frac{\alpha_s}{12\pi}\Bigg\{&(1+\delta)\log\left(1+\frac{H}{v}\right)-\frac{\eta}{2}\log^2\left(1+\frac{H}{v}\right)\\ &+\frac{\rho}{3}\log^3\left(1+\frac{H}{v}\right)-\frac{\sigma}{4}\log^4\left(1+\frac{H}{v}\right)\Bigg\}G^{a\mu\nu}G^a_{\mu\nu}\end{aligned} \tag{2.8}$$

[2]In the case of an extended Higgs sector with several scalar Higgs bosons coupling to the top quark the replacement $\overline{m_t}(\mu_R^2) \to \overline{m_t}(\mu_R^2)(1+\sum_i c_i H_i/v)$ has to be implemented, where c_i are the top quark Yukawa couplings normalized to the SM coupling. This results in the correspondence $H/v \leftrightarrow \sum_i c_i H_i/v$ for all subsequent steps.

[3]Note that diagrammatically for the single-Higgs case this expression coincides with the replacement $\frac{1}{\not p - m_t} \to \frac{1}{\not p - m_t}\frac{m_t}{v}\frac{1}{\not p - m_t}$ of the top-quark propagators inside the gluonic correlation functions up to 4th order in the gluon fields at the point where m_t is either the unrenormalized or the pure $\overline{\rm MS}$ mass [28].

[4]Diagrammatically this step corresponds to adding the external $\overline{\rm MS}$-renormalized self-energies and two-point-function contributions to the vertices involving top quarks at vanishing external momentum.

with the QCD corrections up to N^{4}LO

$$
\begin{aligned}
\delta &= \delta_1 \frac{\alpha_s}{\pi} + \delta_2 \left(\frac{\alpha_s}{\pi}\right)^2 + \delta_3 \left(\frac{\alpha_s}{\pi}\right)^3 + \delta_4 \left(\frac{\alpha_s}{\pi}\right)^4 + \mathcal{O}(\alpha_s^5) \\
\eta &= \eta_2 \left(\frac{\alpha_s}{\pi}\right)^2 + \eta_3 \left(\frac{\alpha_s}{\pi}\right)^3 + \eta_4 \left(\frac{\alpha_s}{\pi}\right)^4 + \mathcal{O}(\alpha_s^5) \\
\rho &= \rho_3 \left(\frac{\alpha_s}{\pi}\right)^3 + \rho_4 \left(\frac{\alpha_s}{\pi}\right)^4 + \mathcal{O}(\alpha_s^5) \\
\sigma &= \sigma_4 \left(\frac{\alpha_s}{\pi}\right)^4 + \mathcal{O}(\alpha_s^5)
\end{aligned}
\tag{2.9}
$$

The explicit perturbative coefficients are given by

$$
\begin{aligned}
\delta_1 &= \frac{11}{4} \qquad\qquad \delta_2 = \frac{2777}{288} + \frac{19}{16} L_t + N_F \left(\frac{L_t}{3} - \frac{67}{96}\right) \\
\delta_3 &= \frac{897943}{9216}\zeta_3 - \frac{2892659}{41472} + \frac{209}{64} L_t^2 + \frac{1733}{288} L_t \\
&\quad + N_F \left(\frac{40291}{20736} - \frac{110779}{13824}\zeta_3 + \frac{23}{32} L_t^2 + \frac{55}{54} L_t\right) + N_F^2 \left(-\frac{L_t^2}{18} + \frac{77}{1728} L_t - \frac{6865}{31104}\right) \\
\delta_4 &= -\frac{121}{1440} N_F \log^5 2 + \frac{3751}{2880} \log^5 2 + \frac{685}{41472} N_F^2 \log^4 2 + \frac{11679301}{1741824} N_F \log^4 2 \\
&\quad - \frac{93970579}{870912} \log^4 2 + \frac{121}{144} N_F \zeta_2 \log^3 2 - \frac{3751}{288} \zeta_2 \log^3 2 - \frac{685}{6912} N_F^2 \zeta_2 \log^2 2 \\
&\quad - \frac{11679301}{290304} N_F \zeta_2 \log^2 2 + \frac{93970579}{145152} \zeta_2 \log^2 2 + \frac{2057}{192} N_F \zeta_4 \log 2 - \frac{63767}{384} \zeta_4 \log 2 \\
&\quad + \frac{685}{1728} N_F^2 a_4 + \frac{11679301}{72576} N_F a_4 - \frac{93970579}{36288} a_4 + \frac{121}{12} N_F a_5 - \frac{3751}{24} a_5 + \frac{211}{1728} N_F^3 \zeta_3 \\
&\quad - \frac{270407}{1492992} N_F^3 + \frac{4091305}{331776} N_F^2 \zeta_3 - \frac{576757}{55296} N_F^2 \zeta_4 - \frac{115}{384} N_F^2 \zeta_5 - \frac{48073}{27648} N_F^2 \\
&\quad - \frac{151369}{725760} N_F X_0 - \frac{12171659669}{38707200} N_F \zeta_3 + \frac{608462731}{11612160} N_F \zeta_4 + \frac{313489}{6912} N_F \zeta_5 \\
&\quad + \frac{76094378783}{522547200} N_F + \frac{4692439}{1451520} X_0 + \frac{28121193841}{19353600} \zeta_3 + \frac{4674213853}{2903040} \zeta_4 - \frac{807193}{1728} \zeta_5 \\
&\quad - \frac{854201072999}{522547200} + \left(\frac{481}{5184} N_F^3 + \frac{28297}{9216} N_F^2 \zeta_3 - \frac{21139}{3456} N_F^2 - \frac{32257}{288} N_F \zeta_3\right. \\
&\quad \left. + \frac{5160073}{41472} N_F + \frac{9364157}{12288} \zeta_3 - \frac{49187545}{55296}\right) L_t + \left(-\frac{77}{6912} N_F^3 - \frac{1267}{13824} N_F^2 + \frac{4139}{2304} N_F\right. \\
&\quad \left. + \frac{8401}{384}\right) L_t^2 + \left(\frac{1}{108} N_F^3 - \frac{157}{576} N_F^2 + \frac{275}{192} N_F + \frac{2299}{256}\right) L_t^3
\end{aligned}
\tag{2.10}
$$

and

$$
\begin{aligned}
\eta_2 &= \frac{35}{24} + \frac{2}{3} N_F \\
\eta_3 &= \frac{1333}{432} + \frac{589}{48} L_t + N_F \left(\frac{1081}{432} + \frac{191}{72} L_t\right) + N_F^2 \left(\frac{77}{864} - \frac{2}{9} L_t\right)
\end{aligned}
$$

$$
\begin{aligned}
\eta_4 =& \frac{481}{2592}N_F^3 + N_F^2\left(\frac{28297}{4608}\zeta_3 - \frac{373637}{31104}\right) + N_F\left(\frac{429965}{1728} - \frac{2985893}{13824}\zeta_3\right)\\
&+ \frac{26296585}{18432}\zeta_3 - \frac{143976701}{82944} + \left(-\frac{77}{1728}N_F^3 - \frac{1421}{3456}N_F^2 + \frac{9073}{1728}N_F + \frac{45059}{576}\right)L_t\\
&+ \left(\frac{N_F^3}{18} - \frac{455}{288}N_F^2 + \frac{63}{8}N_F + \frac{6479}{128}\right)L_t^2\\
\rho_3 =& \frac{1697}{144} + \frac{175}{72}N_F - \frac{2}{9}N_F^2\\
\rho_4 =& \frac{130201}{1728} + \frac{18259}{192}L_t + N_F\left(\frac{5855}{1728} + \frac{2077}{144}L_t\right) - N_F^2\left(\frac{175}{384} + \frac{439}{144}L_t\right)\\
&+ N_F^3\left(\frac{L_t}{9} - \frac{77}{1728}\right)\\
\sigma_4 =& \frac{51383}{864} + \frac{317}{36}N_F - \frac{47}{24}N_F^2 + \frac{2}{27}N_F^3
\end{aligned}
\tag{2.11}
$$

where $G^a_{\mu\nu}$ denotes the gluon field strength tensor and α_s the strong coupling constant with $N_F = 5$ active flavours. Note that in accordance with the RG-evolution the coefficients δ_1, η_2, ρ_3 and σ_4 are free of L_t terms. Numerically we obtain for $N_F = 5$ light flavours

$$
\begin{aligned}
\delta_1 &= 2.75 & \delta_2 &= 6.1528 + 2.8542L_t\\
\delta_3 &= 3.4043 + 12.2240L_t + 5.4705L_t^2\\
\delta_4 &= 36.0373 - 73.5997L_t + 27.1760L_t^2 + 10.4851L_t^3\\
\eta_2 &= 4.7917 & \eta_3 &= 17.8252 + 19.9792L_t\\
\eta_4 &= -167.5239 + 88.6311L_t + 57.4401L_t^2 & \rho_3 &= 18.3819\\
\rho_4 &= 75.3261 + 104.8906L_t & \sigma_4 &= 63.7998
\end{aligned}
\tag{2.12}
$$

If the running $\overline{\text{MS}}$ top mass is replaced by the top pole mass M_t [73–78][5] [i.e. $L_t = \log(\mu_R^2/M_t^2)$ is used everywhere],

$$
\begin{aligned}
\overline{m_t}(\mu_R^2) = M_t \Bigg\{ 1 &- \left(\frac{4}{3} + \log\frac{\mu_R^2}{M_t^2}\right)\frac{\alpha_s^{(N_F)}(\mu_R^2)}{\pi} + \Bigg[-\frac{3019}{288} - 2\zeta_2 - \frac{2}{3}\zeta_2\log 2 + \frac{\zeta_3}{6}\\
&-\frac{461}{72}\log\frac{\mu_R^2}{M_t^2} - \frac{23}{24}\log^2\frac{\mu_R^2}{M_t^2} + N_F\left(\frac{71}{144} + \frac{\zeta_2}{3} + \frac{13}{36}\log\frac{\mu_R^2}{M_t^2} + \frac{1}{12}\log^2\frac{\mu_R^2}{M_t^2}\right)\\
&-\frac{4}{3}\sum_{1\le i\le N_F}\Delta\left(\frac{M_i}{M_t}\right)\Bigg]\left(\frac{\alpha_s^{(N_F)}(\mu_R^2)}{\pi}\right)^2\Bigg\} + \mathcal{O}(\alpha_s^3)
\end{aligned}
\tag{2.13}
$$

where the mass-dependent term involving the light flavours can (for $0 \le x \le 1$) be approximated by

$$
\Delta(x) = \frac{\pi^2}{8}\,x - 0.579\,x^2 + 0.230\,x^3 \tag{2.14}
$$

[5] Note that the low-energy strong coupling constant with $N_F = 5$ active flavours is used in this relation.

the QCD corrections are formally different from the $\overline{\mathrm{MS}}$ case above only for the coefficients δ_3, δ_4 and η_4,

$$
\begin{aligned}
\delta_3 = {} & \frac{897943}{9216}\zeta_3 - \frac{2761331}{41472} + \frac{209}{64}L_t^2 + \frac{2417}{288}L_t \\
& + N_F\left(\frac{58723}{20736} - \frac{110779}{13824}\zeta_3 + \frac{23}{32}L_t^2 + \frac{91}{54}L_t\right) + N_F^2\left(-\frac{L_t^2}{18} + \frac{77}{1728}L_t - \frac{6865}{31104}\right) \\
\delta_4 = {} & -\frac{121}{1440}N_F\log^5 2 + \frac{3751}{2880}\log^5 2 + \frac{685}{41472}N_F^2\log^4 2 + \frac{11679301}{1741824}N_F\log^4 2 \\
& - \frac{93970579}{870912}\log^4 2 + \frac{121}{144}N_F\zeta_2\log^3 2 - \frac{3751}{288}\zeta_2\log^3 2 - \frac{685}{6912}N_F^2\zeta_2\log^2 2 \\
& - \frac{11679301}{290304}N_F\zeta_2\log^2 2 + \frac{93970579}{145152}\zeta_2\log^2 2 + \frac{4}{9}N_F\zeta_2\log 2 + \frac{19}{12}\zeta_2\log 2 \\
& + \frac{2057}{192}N_F\zeta_4\log 2 - \frac{63767}{384}\zeta_4\log 2 + \frac{685}{1728}N_F^2 a_4 + \frac{11679301}{72576}N_F a_4 - \frac{93970579}{36288}a_4 \\
& + \frac{121}{12}N_F a_5 - \frac{3751}{24}a_5 + \frac{211}{1728}N_F^3\zeta_3 - \frac{270407}{1492992}N_F^3 - \frac{2}{9}N_F^2\zeta_2 + \frac{4091305}{331776}N_F^2\zeta_3 \\
& - \frac{576757}{55296}N_F^2\zeta_4 - \frac{115}{384}N_F^2\zeta_5 - \frac{161627}{82944}N_F^2 - \frac{151369}{725760}N_F X_0 + \frac{13}{24}N_F\zeta_2 + \frac{19}{4}\zeta_2 \\
& - \frac{12175960469}{38707200}N_F\zeta_3 + \frac{608462731}{11612160}N_F\zeta_4 + \frac{313489}{6912}N_F\zeta_5 + \frac{80863176383}{522547200}N_F \\
& + \frac{4692439}{1451520}X_0 + \frac{28113533041}{19353600}\zeta_3 + \frac{4674213853}{2903040}\zeta_4 - \frac{807193}{1728}\zeta_5 - \frac{831703495799}{522547200} \\
& + \left(\frac{481}{5184}N_F^3 + \frac{28297}{9216}N_F^2\zeta_3 - \frac{22687}{3456}N_F^2 - \frac{32257}{288}N_F\zeta_3 + \frac{5581849}{41472}N_F + \frac{9364157}{12288}\zeta_3\right. \\
& \left. - \frac{46543033}{55296}\right)L_t + \left(-\frac{77}{6912}N_F^3 - \frac{5107}{13824}N_F^2 + \frac{12547}{2304}N_F + \frac{14747}{384}\right)L_t^2 \\
& + \left(\frac{1}{108}N_F^3 - \frac{157}{576}N_F^2 + \frac{275}{192}N_F + \frac{2299}{256}\right)L_t^3 + \frac{4}{3}\left(\frac{2}{3}N_F + \frac{19}{8}\right)\sum_{1\le i\le N_F}\Delta\left(\frac{M_i}{M_t}\right) \\
\eta_4 = {} & \frac{481}{2592}N_F^3 + N_F^2\left(\frac{28297}{4608}\zeta_3 - \frac{392069}{31104}\right) + N_F\left(\frac{442189}{1728} - \frac{2985893}{13824}\zeta_3\right) \\
& + \frac{26296585}{18432}\zeta_3 - \frac{141262589}{82944} + \left(-\frac{77}{1728}N_F^3 - \frac{2957}{3456}N_F^2 + \frac{18241}{1728}N_F + \frac{59195}{576}\right)L_t \\
& + \left(\frac{N_F^3}{18} - \frac{455}{288}N_F^2 + \frac{63}{8}N_F + \frac{6479}{128}\right)L_t^2
\end{aligned} \tag{2.15}
$$

For the on-shell top-quark mass we obtain numerically for $N_F = 5$ light flavours

$$
\begin{aligned}
\delta_3 &= 11.0154 + 17.9323L_t + 5.4705L_t^2 \\
\delta_4 &= 125.7997 + 13.8777L_t + 55.0041L_t^2 + 10.4851L_t^3 + 7.6111\sum_{1\le i\le N_F}\Delta\left(\frac{M_i}{M_t}\right) \\
\eta_4 &= -114.2461 + 128.5894L_t + 57.4401L_t^2
\end{aligned} \tag{2.16}
$$

The explicit expansion of the Lagrangian of eq. (2.8) in powers of the Higgs field results in

$$
\mathcal{L}_{\mathrm{eff}} = \frac{\alpha_s}{12\pi}\left\{\sum_{n=1}^{\infty}\Delta_n\frac{(-1)^{n-1}}{n}\left(\frac{H}{v}\right)^n\right\}G^{a\mu\nu}G^a_{\mu\nu} \tag{2.17}
$$

with the QCD corrections up to N^{4}LO

$$
\begin{aligned}
\Delta_1 &= 1+\delta_1\frac{\alpha_s}{\pi}+\delta_2\left(\frac{\alpha_s}{\pi}\right)^2+\delta_3\left(\frac{\alpha_s}{\pi}\right)^3+\delta_4\left(\frac{\alpha_s}{\pi}\right)^4+\mathcal{O}(\alpha_s^5)\\
\Delta_2 &= 1+\delta_1\frac{\alpha_s}{\pi}+(\delta_2+\eta_2)\left(\frac{\alpha_s}{\pi}\right)^2+(\delta_3+\eta_3)\left(\frac{\alpha_s}{\pi}\right)^3+(\delta_4+\eta_4)\left(\frac{\alpha_s}{\pi}\right)^4+\mathcal{O}(\alpha_s^5)\\
\Delta_3 &= 1+\delta_1\frac{\alpha_s}{\pi}+\left(\delta_2+\frac{3}{2}\eta_2\right)\left(\frac{\alpha_s}{\pi}\right)^2+\left(\delta_3+\frac{3}{2}\eta_3+\rho_3\right)\left(\frac{\alpha_s}{\pi}\right)^3\\
&\quad+\left(\delta_4+\frac{3}{2}\eta_4+\rho_4\right)\left(\frac{\alpha_s}{\pi}\right)^4+\mathcal{O}(\alpha_s^5)\\
\Delta_4 &= 1+\delta_1\frac{\alpha_s}{\pi}+\left(\delta_2+\frac{11}{6}\eta_2\right)\left(\frac{\alpha_s}{\pi}\right)^2+\left(\delta_3+\frac{11}{6}\eta_3+2\rho_3\right)\left(\frac{\alpha_s}{\pi}\right)^3\\
&\quad+\left(\delta_4+\frac{11}{6}\eta_4+2\rho_4+\sigma_4\right)\left(\frac{\alpha_s}{\pi}\right)^4+\mathcal{O}(\alpha_s^5)\\
\Delta_5 &= 1+\delta_1\frac{\alpha_s}{\pi}+\left(\delta_2+\frac{25}{12}\eta_2\right)\left(\frac{\alpha_s}{\pi}\right)^2+\left(\delta_3+\frac{25}{12}\eta_3+\frac{35}{12}\rho_3\right)\left(\frac{\alpha_s}{\pi}\right)^3\\
&\quad+\left(\delta_4+\frac{25}{12}\eta_4+\frac{35}{12}\rho_4+\frac{5}{2}\sigma_4\right)\left(\frac{\alpha_s}{\pi}\right)^4+\mathcal{O}(\alpha_s^5)
\end{aligned}
\tag{2.18}
$$

for up to five external Higgs bosons. It should be noted that the coefficients δ_{1-4} of the single-Higgs term Δ_1 agree with previous results up to N^{4}LO [37, 38, 67–69, 79, 80], while the coefficient η_2 of the double-Higgs contribution Δ_2 agrees with the explicit diagrammatic calculation of ref. [58].

Connecting our approach to derive the effective Lagrangian to the method of refs. [67–69] for the single-Higgs case we can easily derive their final relation,

$$
C_H = -\frac{1}{4}\zeta_{\alpha_s}\, g_t\partial_{m_t}\frac{1}{\zeta_{\alpha_s}} = \frac{1}{2v}\frac{m_t^2\partial}{\partial(m_t^2)}\log\zeta_{\alpha_s} \tag{2.19}
$$

with $g_t = m_t/v$, $\partial_{m_t} = \partial/\partial m_t$ and C_H denoting the full coefficient in front of the operator $G^{a\mu\nu}G^a_{\mu\nu}H$. This expression agrees with refs. [67–69]. For the double-Higgs case we arrive at

$$
C_{HH} = \frac{1}{8}\zeta_{\alpha_s}\, g_t^2\partial_{m_t}^2\frac{1}{\zeta_{\alpha_s}} = \frac{1}{4v^2}\left\{\left(\frac{m_t\partial_{m_t}\zeta_{\alpha_s}}{\zeta_{\alpha_s}}\right)^2-\frac{m_t^2\partial_{m_t}^2\zeta_{\alpha_s}}{2\zeta_{\alpha_s}}\right\} \tag{2.20}
$$

where C_{HH} denotes the coefficient in front of the operator $G^{a\mu\nu}G^a_{\mu\nu}H^2$.

A final comment addresses the removal of one-particle-reducible contributions in eq. (2.4): this corresponds to the removal of one-particle-reducible diagrams of the type shown in figure 1 after attaching external Higgs bosons according to eq. (2.6). We have checked this correspondence explicitly for Higgs boson pair production in the heavy-top-quark limit at NLO [55].

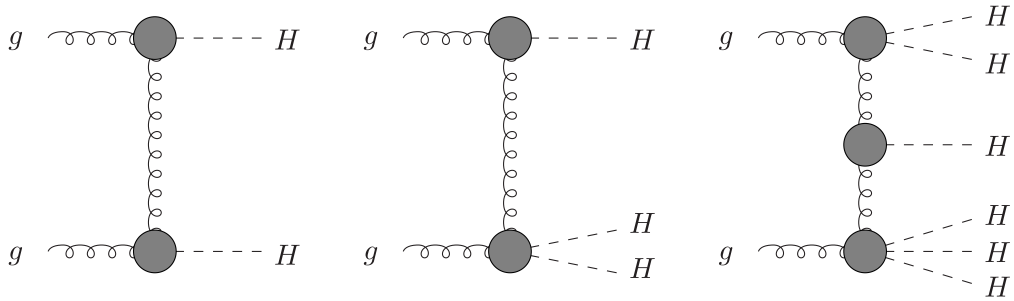

Figure 1. Typical one-particle-reducible Feynman diagrams for multi-Higgs boson production.

3 Several heavy quarks

Starting from the expression of the effective single-Higgs coupling to gluons of ref. [81] with N_H heavy quarks contributing we can reconstruct the corresponding logarithmic parts of the function Π_Q,

$$\mathcal{L}_g = -\frac{1-\Pi_Q}{4}\hat{G}^{a\mu\nu}\hat{G}^a_{\mu\nu}$$
$$\Pi_Q = \sum_n C_n \left(\frac{\alpha_s^{(N_F+N_H)}}{\pi}\right)^n \tag{3.1}$$

with the perturbative coefficients up to third order

$$C_1 = -\frac{N_H}{6}L_Q \tag{3.2}$$
$$C_2 = N_H\left[\frac{11}{72} - \frac{11}{24}L_Q\right]$$
$$C_3 = C_{30} - N_H\left(\frac{1877}{1152} - \frac{77}{3456}N_H - \frac{67}{576}N_F\right)L_Q - N_H\left(\frac{19}{192} - \frac{11}{288}N_H + \frac{N_F}{36}\right)L_Q^2$$

where $\hat{G}^{a\mu\nu}$ denotes the gluonic field-strength operator of colour-SU(3) in the low-energy limit with $N_F + N_H$ active flavours. The logarithm is defined as

$$L_Q = \frac{1}{N_H}\sum_{i=1}^{N_H}\log\left(\frac{\mu_R^2}{M_i^2}\right) \tag{3.3}$$

For the derivation of the effective Lagrangian for the gluonic Higgs coupling the constant C_{30} is irrelevant. Performing the replacement[6]

$$L_Q \to \bar{L}_Q = L_Q - 2\log\left(1+\frac{H}{v}\right) \qquad \text{and} \qquad \Pi_Q \to \bar{\Pi}_Q \tag{3.4}$$

[6]Here we assume SM-type couplings of the heavy quarks to the Higgs boson as e.g. for a sequential 4th fermion generation. For the case of different couplings and N_S scalar Higgs bosons this shift has to be replaced by $\log(1+H/v) \to \sum_{i=1}^{N_H}\log\left(1+\sum_{j=1}^{N_S}c_{ij}H_j/v\right)/N_H$ in all subsequent steps, where the factors c_{ij} denote the Higgs Yukawa couplings normalized to the SM-Higgs coupling.

and decoupling the heavy quarks from the strong coupling constant α_s by

$$\alpha_s^{(N_F+N_H)}(\mu_R^2) = \alpha_s^{(N_F)}(\mu_R^2)\left\{1 + \frac{\alpha_s^{(N_F)}(\mu_R^2)}{\pi} N_H \frac{L_Q}{6} + \left(\frac{\alpha_s^{(N_F)}(\mu_R^2)}{\pi}\right)^2 N_H \left[-\frac{11}{72} + \frac{11}{24} L_Q + N_H \frac{L_Q^2}{36}\right]\right\} + \mathcal{O}(\alpha_s^4) \tag{3.5}$$

and from the gluon-field-strength operator we arrive at the effective Lagrangian for the gluonic Higgs couplings up to NNLO

$$\mathcal{L}_{\text{eff}} = N_H \frac{\alpha_s}{12\pi}\left\{(1+\delta)\log\left(1+\frac{H}{v}\right) - \frac{\eta}{2}\log^2\left(1+\frac{H}{v}\right)\right\} G^{a\mu\nu} G^a_{\mu\nu} \tag{3.6}$$

with the QCD corrections up to NNLO

$$\begin{aligned} \delta &= \delta_1 \frac{\alpha_s}{\pi} + \delta_2 \left(\frac{\alpha_s}{\pi}\right)^2 + \mathcal{O}(\alpha_s^3) \\ \eta &= \eta_2 \left(\frac{\alpha_s}{\pi}\right)^2 + \mathcal{O}(\alpha_s^3) \end{aligned} \tag{3.7}$$

The explicit perturbative coefficients read

$$\begin{aligned} \delta_1 &= \frac{11}{4} \\ \delta_2 &= \frac{1877}{192} - \frac{77}{576} N_H + \frac{19}{16} L_Q + N_F \left(\frac{L_Q}{3} - \frac{67}{96}\right) \\ \eta_2 &= \frac{19}{8} - \frac{11}{12} N_H + \frac{2}{3} N_F \end{aligned} \tag{3.8}$$

The result for δ_2 in the single-Higgs case agrees with the results of refs. [81, 82]. The NNLO results for more than one external Higgs boson are new.

4 Conclusions

In this work we have derived effective (multi-)Higgs couplings to gluons after integrating out all heavy quarks mediating these couplings. The effective Lagrangians can be used for the computation of the production of one or several Higgs bosons in gluon fusion at hadron colliders in the limit of heavy quarks. In the SM we have extended the effective Lagrangian for double-Higgs couplings to gluons to N^{4}LO and derived for the first time the N^{4}LO Lagrangian for more than two SM Higgs bosons. In the second part we extended the analysis to the case of several heavy quarks coupling to the Higgs bosons up to NNLO. We reproduced the existing NNLO results for the single-Higgs case. We have derived these effective Lagrangians from their connection to the decoupling relations of the strong coupling constant.

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