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Holographic chiral induced W-gravities

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Abstract: We study boundary conditions for 3-dimensional higher spin gravity that admit asymptotic symmetry algebras expected of 2-dimensional induced higher spin theories in the light-cone gauge. For the higher spin theory based on $sl(3,\mathbb{R}) \oplus sl(3,\mathbb{R})$ algebra, our boundary conditions give rise to one copy of classical W_3 and a copy of $sl(3,\mathbb{R})$ or $su(1,2)$ Kac-Moody as the asymptotic symmetry algebra. We propose that the higher spin theories with these boundary conditions describe appropriate chiral induced W-gravity theories on the boundary. We also consider boundary conditions of spin-3 higher spin gravity that admit a $u(1) \oplus u(1)$ current algebra.

Keywords: Higher Spin Gravity, 2D Gravity, Conformal and W Symmetry, Chern-Simons Theories

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Contents

1 Introduction

A conformal field theory (CFT) in a background space-time with flat metric $\eta_{\mu\nu}$ admits a spin-2 current namely the energy-momentum tensor $T_{\mu\nu}$ which is symmetric, traceless and conserved ($\partial^\mu T_{\mu\nu} = 0$). In two dimensions it has two independent components $T_{++}(x^+)$ and $T_{--}(x^-)$ where $x^\pm = t \pm x$ are the null coordinates. Classically one can couple the CFT to an arbitrary background metric making it a gravitational theory as well. Such a gravity coupled to matter in two dimensions plays an important role as the world-sheet theory of string theories. Classically this theory would be diffeomorphism and Weyl invariant though quantum mechanically these symmetries may become anomalous. Demanding that the Weyl symmetry is not anomalous constrains the matter content or the possible metric backgrounds. For instance, for a string propagating in the flat n-dimensional space-time the world-sheet theory being Weyl invariant at the quantum level means that the string should propagate in critical dimensions ($n = 26$) and one can gauge fix the world-sheet metric completely. Gauge fixing the world-sheet metric to the flat metric leaves one with a 2d CFT in flat background whose symmetries are two commuting copies of Virasoro algebra generated by the modes of the conserved stress-energy tensor of the CFT. However, away from the critical dimension the 2d metric cannot be gauged away because of the Weyl anomaly — leaving one degree of freedom in the metric. Long ago, in a seminal paper [1], Polyakov addressed the problem of quantizing this theory.

When one integrates over the matter sector one obtains a non-local theory [2] of the metric referred to as the induced gravity theory. The induced gravity theory is diffeomorphism invariant but not Weyl invariant as expected. One can use the diffeomorphisms to gauge fix the metric down to one independent component. There are two standard gauge choices used in the literature:

- the conformal gauge: $ds^2 = -e^{\phi(x^+,x^-)}dx^+dx^-$
- the light-cone gauge: $ds^2 = -dx^+dx^- + F(x^+,x^-)\,(dx^+)^2$

The induced gravity theory becomes local in either of these gauge choices. In the conformal gauge it is known to reduce to the Liouville theory. In the light-cone gauge the induced gravity theory is called the chiral induced gravity (CIG) theory. Polyakov examined this CIG and uncovered an $sl(2,\mathbb{R})$ current algebra worth of symmetries of it which in turn led to the determination of all correlation functions in that theory [1].

On the other hand holography through the celebrated AdS/CFT correspondence has been a powerful tool to study CFTs. The simplest non-trivial manifestation of this duality is that the global symmetry algebra of the CFT is identical to the asymptotic symmetry algebra of Brown-Henneaux [3] with the Dirichlet boundary conditions for Einstein gravity in one higher dimension with a negative cosmological constant (AdS-gravity). A natural question is whether one can generalize AdS/CFT to include gravity on the CFT side. To generalize the CFT to include gravity one requires to consider boundary conditions that are not Dirichlet type.

To begin addressing this question, in an earlier paper [4] (see also [5]) we proposed a set of boundary conditions for AdS_3 gravity which admitted the same symmetry algebra as Polyakov's 2d chiral induced gravity [1]. We therefore proposed that the AdS_3 gravity with those boundary conditions should provide the bulk description of the CIG. On the other hand boundary conditions for AdS_3 gravity that allow the boundary metric to be in the conformal gauge were considered in [6]. The boundary conditions of [4, 6] were further generalized and studied in [5] which provided further evidence for the proposed duality. We will refer to the boundary conditions of [4] as Chiral Induced Gravity (CIG) boundary conditions. The asymptotic symmetry algebra of the CIG boundary conditions consists of one copy of the Virasoro algebra and one copy of $sl(2,\mathbb{R})$ Kac-Moody algebra with level k given by $c/6$. Furthermore, the Ward identity (called the Virasoro Ward identity [1]) for the partition function of the CIG theory emerges from the bulk equations of motion with the CIG boundary conditions.

The CIG theories provide some important examples of 2-dimensional field theories that admit only one copy of Virasoro algebra. Some recent examples of such theories have been defined by [7, 8] via holography.

In this paper we seek to extend the results of [4, 5] on the holographic description of the CIG theories to the two-dimensional chiral induced W-gravity theories.

2d chiral induced W-gravity. A given conformal field theory in 2d flat space-time can admit more conserved currents than the energy-momentum tensor $T_{\mu\nu}$. The corresponding symmetry algebra should include the two commuting copies of Virasoro algebra. One such enhancement of symmetry algebra involves extending each copy of the Virasoro algebra to a W_N algebra first discovered by Zamolodchikov [9]. A CFT with $W_N \oplus W_N$ symmetry (referred to as a WCFT) will have conserved currents given by completely symmetric and traceless rank-s tensors $\mathcal{W}^{(s)}_{\mu_1\cdots\mu_s}$ with the spin s ranging from 2 to N (here $\mathcal{W}^{(2)}_{\mu\nu} = T_{\mu\nu}$). The two independent components of such a traceless spin-s current are $\mathcal{W}^{(s)}_{+\cdots+}(x^+)$ and $\mathcal{W}^{(s)}_{-\cdots-}(x^-)$ in light-cone coordinates. One can again couple such a CFT to background spin-s gauge fields resulting in a higher spin extension of the 2d gravity referred to as the W-gravity theory (see [10] for a review). Again integrating out the WCFT field content will generically induce a dynamical theory for these higher spin fields which is the induced W-gravity.

Following Polyakov's work [1] and the discovery of W-symmetries (see [10, 11] for a review) people studied the induced W-gravity theories in a particular light-cone gauge.[1] In this gauge the background spin-s gauge field is coupled only to one of the two independent components of the corresponding W-current, say $\mathcal{W}^{(s)}_{-\cdots-}(x^-)$. This is achieved by considering the action

$$S_{\mathrm{W-gravity}} = S_{\mathrm{WCFT}} + \int d^2x \sum_{s=2}^{N} \mu^{(s)}_{+\cdots+}\mathcal{W}^{(s)}_{-\cdots-} \tag{1.1}$$

before integrating out the WCFT fields, where S_{WCFT} denotes the action of the WCFT in 2d flat space-time. After integrating out the WCFT field content the resulting theory of $\mu^{(s)}_{+\cdots+}$ fields is dubbed the chiral induced W-gravity (CIWG). It was shown in [12] that these CIWG theories are expected to have $sl(n,\mathbb{R})$-type current algebra symmetries generalizing the $sl(2,\mathbb{R})$ current algebra symmetry of the CIG of Polyakov. The Ward identities of CIWG (generalizing the Virasoro Ward identities of 2d CIG) have also been known for quite some time. For details of how these symmetries and Ward identities emerge see, for instance, [12, 14]. It is an interesting question to ask if CIWG theories also admit holographic descriptions.

In this paper we generalize the results of [4, 5] towards describing chiral induced W-gravities (CIWG) holographically. It is natural to expect that the bulk theory should be a higher spin theory with one higher spin gauge field corresponding to each higher spin field in the induced W-gravity theory of interest. Such 3d theories have a description in terms of the difference of two Chern-Simons theories each with gauge algebra $sl(n,\mathbb{R})$ [16]. We therefore expect that this 3d higher spin gauge theory admits a set of boundary conditions that can describe a suitable chiral induced W-gravity with spins ranging from 2 to n.

We, in particular, provide and study a set of boundary conditions for the case of $n = 3$ and compute their asymptotic symmetry algebras. We verify that these boundary

[1] See, for instance, [13–15] and references therein.

conditions give rise to the W_3-Ward identities of the CIWG. We find that, in this case, the higher spin theory with our boundary conditions admits one copy of classical W_3 algebra and an $sl(3,\mathbb{R})$ (or an $su(1,2)$) current algebra as its asymptotic symmetry algebra. As a byproduct we also provide a generalization of the boundary conditions of [7] to this higher spin theory and compute the corresponding symmetry algebra.

The rest of the paper is organized as follows. In section 2 we briefly review the results of [4, 5] and translate them into the $sl(2,\mathbb{R}) \oplus sl(2,\mathbb{R})$ Chern-Simons formalism of AdS_3 gravity. In section 3 we generalize the results of section 2 to the higher spin theory with spin 2 and spin 3 fields in the $sl(3,\mathbb{R}) \oplus sl(3,\mathbb{R})$ Chern-Simons formalism. In section 4 we compute the asymptotic symmetry algebras. In the last section we provide some comments, discuss open issues and conclude. The appendices contain our conventions and some details of the calculations used in the main text.

2 CIG boundary conditions

In an earlier paper [4] with S. Avery we proposed a set of boundary conditions for AdS_3 gravity (in the metric formalism)

$$
\begin{aligned}
S_{d=3} = &-\frac{1}{16\pi G}\int_{\mathcal{M}} d^3x\,\sqrt{|g|}\left(R+\frac{2}{l^2}\right) - \frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^2x\,\sqrt{|h|}\,K \\
&+\frac{1}{8\pi G}\int_{\partial\mathcal{M}} d^2x\,\frac{1}{l}\sqrt{|h|} + \cdots
\end{aligned}
\tag{2.1}
$$

where the dots represent any additional boundary terms one may add to make the variational problem well-defined when using non-Dirichlet boundary conditions. This admitted an $sl(2,\mathbb{R})$ current algebra as their asymptotic symmetry algebra. The motivation was to provide a holographic description of the induced gravity studied in a light-cone gauge by Polyakov [1]. Here we begin with a brief review of those results with clarifications and generalizations. Let us consider the following boundary conditions [4, 5] for the metric

$$
\begin{aligned}
g_{rr} &= \frac{l^2}{r^2} + \mathcal{O}(r^{-4}), & g_{r+} &= \mathcal{O}(r^{-1}), \quad g_{r-} = \mathcal{O}(r^{-3}), \\
g_{+-} &= -\frac{r^2}{2} + \mathcal{O}(r^0), & g_{--} &= \mathcal{O}(r^0), \\
g_{++} &= r^2 F(x^+, x^-) + \mathcal{O}(r^0), & &
\end{aligned}
\tag{2.2}
$$

where x^+, x^- are the boundary coordinates and r is the radial coordinate with the asymptotic boundary at $r^{-1} = 0$. One can write a general non-linear solution of AdS_3 gravity in Fefferman-Graham coordinates [17] as:

$$
ds^2 = l^2\frac{dr^2}{r^2} + r^2\left[g^{(0)}_{ab} + \frac{l^2}{r^2}\,g^{(2)}_{ab} + \frac{l^4}{r^4}g^{(4)}_{ab}\right] dx^a dx^b. \tag{2.3}
$$

Therefore, the full set of non-linear solutions consistent with our boundary conditions is obtained when

$$\begin{aligned}
&g^{(0)}_{++} = F(x^+,x^-), \qquad g^{(0)}_{+-} = -\frac{1}{2}, \qquad g^{(0)}_{--} = 0,\\
&g^{(2)}_{++} = \kappa(x^+,x^-), \qquad g^{(2)}_{+-} = \sigma(x^+,x^-), \quad g^{(2)}_{--} = \tilde{\kappa}(x^+,x^-),\\
&g^{(4)}_{ab} = \frac{1}{4} g^{(2)}_{ac}\, g^{cd}_{(0)}\, g^{(2)}_{db}\,,
\end{aligned} \tag{2.4}$$

where in the last line $g^{cd}_{(0)}$ is $g^{(0)}_{cd}$ inverse. Imposing the equations of motion $R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} - \frac{1}{l^2} g_{\mu\nu} = 0$ one finds that these equations are satisfied for $\mu,\nu = +,-$. Then the remaining three equations coming from $(\mu,\nu) = (r,r),(r,+),(r,-)$ impose the following relations:

$$\begin{aligned}
\sigma(x^+,x^-) &= \frac{1}{2}[\partial_-^2 F - 2\,\tilde{\kappa}\, F]\\
\kappa(x^+,x^-) &= \kappa_0(x^+) + \frac{1}{2}[\partial_+\partial_- F + 2\,\tilde{\kappa}\, F^2 - F\,\partial_-^2 F - \frac{1}{2}(\partial_- F)^2]
\end{aligned} \tag{2.5}$$

and

$$2\,(\partial_+ + 2\,\partial_- F + F\,\partial_-)\,\tilde{\kappa} = \partial_-^3 F\,. \tag{2.6}$$

This last equation may be recognized as the Virasoro Ward identity of Polyakov [1] expected from the 2d CIG. This Ward identity is integrable. To find the solution, inspired by Polyakov, let us parametrize $F = -\frac{\partial_+ f}{\partial_- f}$. With this parametrization one can show that the above constraint (2.6) can be cast into the following form:

$$(\partial_- f\,\partial_+ - \partial_+ f\,\partial_-)\Big[4\,(\partial_- f)^{-2}\,\tilde{\kappa} - (\partial_- f)^{-4}\,[3\,(\partial_-^2 f)^2 - 2\,\partial_- f(\partial_-^3 f)]\Big] = 0 \tag{2.7}$$

For an arbitrary $f(x^+,x^-)$ the general solution to this equation is

$$\tilde{\kappa}(x^+,x^-) = \frac{1}{4} G[f](\partial_- f)^2 + \frac{1}{4}(\partial_- f)^{-2}\,[3\,(\partial_-^2 f)^2 - 2\,\partial_- f(\partial_-^3 f)] \tag{2.8}$$

where $G[f]$ is an arbitrary functional of $f(x^+,x^-)$. The second term in the solution may be recognized as the Schwarzian derivative of f with respect to x^-.

Along with this solution (2.8) for $\tilde{\kappa}$ the configurations in (2.3)–(2.5) provide the most general set of solutions consistent with the boundary conditions in (2.2).

The AdS_3 gravity with the boundary conditions (2.2) should provide a holographic description of the 2d CIG with F playing the role of its dynamical field. However, the classical solutions of the 2d CIG should correspond to bulk solutions with $\tilde{\kappa}$ either vanishing or an appropriate non-zero constant. In the latter case one needs to add additional boundary terms to the action (2.1), see [4, 7]. When $\tilde{\kappa} = 0$ one gets solutions that are appropriate to asymptotically Poincare AdS_3, whereas $\tilde{\kappa} = -1/4$ corresponds to the solutions considered in [4].[2]

[2]Let us note that these solutions also include those of [7] when one takes $\tilde{\kappa} = \Delta$ and $\partial_- F = 0$.

The asymptotic symmetries of the configurations in (2.3)–(2.5) are generated by the following vector field:

$$\xi = -\frac{1}{2}[\lambda'_\kappa(x^+) + \partial_-\lambda(x^+,x^-) + \mathcal{O}(r^{-4})]\, r\,\partial_r + [\lambda_\kappa(x^+) + \frac{l^2}{2r^2}\partial_-^2\lambda(x^+,x^-) + \mathcal{O}(r^{-4})]\,\partial_+$$
$$+[\lambda(x^+,x^-) + \frac{l^2}{2r^2}(\lambda''_\kappa(x^+) + 2\,F\,\partial_-^2\lambda(x^+,x^-) + \partial_+\partial_-\lambda(x^+,x^-)) + \mathcal{O}(r^{-4})]\,\partial_- \,. \qquad (2.9)$$

To compare the answers with [4] we consider the case of $\tilde{\kappa} = -1/4$. Then the Virasoro Ward identity can be solved to get $F = f(x^+) + g(x^+)\,e^{i\,x^-} + \bar{g}(x^+)\,e^{-i\,x^-}$ which in turn implies

$$\kappa = \kappa_0(x^+) - \frac{1}{4}f^2 + \frac{1}{2}(e^{2ix^-}g^2 + e^{-2ix^-}\bar{g}^2) + \frac{i}{2}(e^{ix^-}g' - e^{-ix^-}\bar{g}')\,. \qquad (2.10)$$

At the same time keeping $\tilde{\kappa}$ fixed under the asymptotic diffeomorphisms requires that

$$\lambda(x^+,x^-) = \lambda_f(x^+) + e^{i\,x^-}\,\lambda_g(x^+) + e^{-i\,x^-}\,\bar{\lambda}_{\bar{g}}(x^+)\,. \qquad (2.11)$$

These induce the following transformations on $f(x^+)$, $g(x^+)$, $\bar{g}(x^+)$ and $\kappa_0(x^+)$:

$$\delta_\xi f(x^+) = \lambda'_f + 2i\,(\bar{g}\,\lambda_g - \bar{\lambda}_{\bar{g}}\,g) + (f\lambda_\kappa)', \qquad \delta_\xi g = \lambda'_g + i\,(f\,\lambda_g - g\,\lambda_f) + (g\lambda_\kappa)',$$
$$\delta_\xi \bar{g} = \bar{\lambda}'_{\bar{g}} - i\,(f\,\bar{\lambda}_{\bar{g}} - \bar{g}\,\lambda_f) + (\bar{g}\lambda_\kappa)', \qquad \delta_\xi\kappa_0 = \lambda_\kappa\,\kappa'_0 + 2\,\kappa_0\,\partial_+\lambda_\kappa - \frac{1}{2}\partial_+^3\lambda_\kappa \qquad (2.12)$$

These expressions can be summarized as

$$\begin{aligned}\delta_\xi J^a &= \partial_+\lambda^a - i\,f^a{}_{bc}J^b\,\lambda^c + \partial_+(J^a\,\lambda)\\ &= \partial_+(\lambda^a + J^a\,\lambda) - i\,f^a{}_{bc}J^b\,(\lambda^c + J^c\,\lambda)\end{aligned} \qquad (2.13)$$

where $\{J^{(-1)}, J^{(0)}, J^{(1)}\} = \{\bar{g}, f, g\}$ and $\{\lambda^{(-1)}, \lambda^{(0)}, \lambda^{(1)}\} = \{\bar{\lambda}_{\bar{g}}, \lambda_f, \lambda_g\}$ and $\lambda = \lambda_\kappa$. The numbers $f^a{}_{bc}$ here are the structure constants of the $sl(2,\mathbb{R})$ algebra $[L_m, L_n] = (m - n)\,L_{m+n}$ for $m, n = -1, 0, 1$ written as $[L_b, L_c] = f^a{}_{bc}L_a$.

The charges for these symmetries were shown to be integrable and finite, and computed in [4]. In the next section we would like to recover these results from the first order formalism of AdS_3 gravity as a warm up for generalization to the higher spin case.

2.1 Holographic CIG in the first order formalism

It is well-known [18, 19] that the AdS_3 gravity in the Hilbert-Palatini formulation can be recast as a Chern-Simons gauge theory with[3]

$$S[A,\tilde{A}] = \frac{k}{4\pi}\int \mathrm{tr}\Big(A\wedge A + \frac{2}{3}A\wedge A\wedge A\Big) - \frac{k}{4\pi}\int \mathrm{tr}\Big(\tilde{A}\wedge\tilde{A} + \frac{2}{3}\tilde{A}\wedge\tilde{A}\wedge\tilde{A}\Big) \qquad (2.14)$$

up to boundary terms, where the gauge group is $\mathrm{SL}(2,\mathbb{R})$. These are related to vielbeins and spin connections through $A^a = \omega^a + \frac{1}{l}e^a$ and $\tilde{A}^a = \omega^a - \frac{1}{l}e^a$. The equations of motion

[3] As is standard the symbol tr is understood to be $\frac{1}{2\mathrm{Tr}L_0^2}$Tr where Tr is the ordinary matrix trace.

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are $F = dA + A \wedge A = 0$ and $\tilde{F} = d\tilde{A} + \tilde{A} \wedge \tilde{A} = 0$. See appendix A for details on the most general solutions to these flatness conditions.

Next we will write the solutions of AdS_3 gravity consistent with (2.2) in the CS language. For this we simply specialize the flat connections given in appendix A to

$$\begin{aligned} A &= b^{-1}\partial_r b\, dr + b^{-1}[(L_1 + a_+^{(-)}\, L_{-1} + a_+^{(0)}\, L_0)\, dx^+ + (a_-^{(-)}\, L_{-1})\, dx^-]\, b \\ \tilde{A} &= b\, \partial_r b^{-1}\, dr + b\, [(\tilde{a}_+^{(0)}\, L_0 + \tilde{a}_+^{(+)}\, L_1 + \tilde{a}_+^{(-)}\, L_{-1})\, dx^+ + (\tilde{a}_-^{(+)}\, L_1 - L_{-1})\, dx^-]\, b^{-1} \end{aligned} \tag{2.15}$$

where $b = e^{L_0 \ln \frac{r}{l}}$ and all the functions are taken to depend on both the boundary coordinates (x^+, x^-). The equations of motion impose the following conditions:

$$\begin{aligned} a_-^{(-)} &= \frac{1}{2}\partial_- a_+^{(0)}, & a_+^{(-)} &= -\,\kappa_0(x^+) + \frac{1}{4}(a_+^{(0)})^2 + \frac{1}{2}\partial_+ a_+^{(0)} \\ \tilde{a}_+^{(0)} &= -\,\partial_- \tilde{a}_+^{(-)}, & \tilde{a}_+^{(+)} &= -\,\tilde{a}_+^{(-)}\, \tilde{a}_-^{(+)} - \frac{1}{2}\partial_- \tilde{a}_+^{(0)}, \end{aligned} \tag{2.16}$$

and

$$(\partial_+ + 2\, \partial_- \tilde{a}_+^{(-)} + \tilde{a}_+^{(-)}\, \partial_-)\, \tilde{a}_-^{(+)} = \frac{1}{2}\partial_-^3 \tilde{a}_+^{(-)} \tag{2.17}$$

The last equation is again the Virasoro Ward identity and it can be solved as before. To obtain the metric in the FG gauge we need to impose $a_+^{(0)} = \tilde{a}_+^{(0)}$. With this choice it is easy to see that the metric obtained matches exactly with the solution given above in (2.3)–(2.5) with $F = \tilde{a}_+^{(-)}$ and $\tilde{\kappa} = \tilde{a}_-^{(+)}$.

To be able to define a variational principle that admits a fluctuating F, we add the following boundary action:

$$S_{bdy.} = -\frac{k}{4\pi}\int d^2x\, tr(L_0\, [a_+, a_-]) - \frac{k}{4\pi}\int d^2x\, tr(L_0\, [\tilde{a}_+, \tilde{a}_-] - 2\, \tilde{\kappa}_0\, L_1\, \tilde{a}_+) \tag{2.18}$$

where $\tilde{\kappa}_0$ is a constant. Then it is easy to see that the variation of the full action gives

$$\delta S_{total} = \frac{k}{2\pi}\int d^2x\, (\tilde{\kappa} - \tilde{\kappa}_0)\, \delta F. \tag{2.19}$$

In showing this we have to use all the constraints in (2.16) coming from the equations of motion except the Virasoro Ward identity. Therefore, we again have two ways to impose the variational principle $\delta S = 0$: (i) $\delta F = 0$ and (ii) $\tilde{\kappa} = \tilde{\kappa}_0$. The former is the usual Brown-Henneaux [3] type Dirichlet boundary condition. We therefore consider the latter.

2.2 Residual gauge transformations

To analyze the asymptotic symmetries in the CS language we seek the residual gauge transformations that leave $\tilde{\kappa}$ fixed, and the above flat connections form-invariant.

The gauge transformations act as $\delta_\Lambda A = d\Lambda + [A, \Lambda]$ which in turn act as $\delta_\lambda a = d\lambda + [a, \Lambda]$ where $A = b^{-1}\, a\, b + b^{-1}\, db$ with $\Lambda = b^{-1}\, \lambda\, b$ (and similarly on the right sector gauge field $\tilde{a}$ with parameter labeled $\tilde{\lambda}$). The resulting gauge parameters are

$$\begin{aligned} \lambda &= \lambda^{(-)}(x^+, x^-)\, L_{-1} + [a_+^{(0)}\, \lambda^{(+)}(x^+) - \partial_+ \lambda^{(+)}(x^+)]\, L_0 + \lambda^{(+)}(x^+)\, L_1 \\ \tilde{\lambda} &= \tilde{\lambda}^{(-)}\, L_{-1} - \partial_- \tilde{\lambda}^{(-)}\, L_0 - \left[\tilde{a}_-^{(+)}\, \tilde{\lambda}^{(-)} - \frac{1}{2}\partial_-^2\, \tilde{\lambda}^{(-)}\right] L_1, \end{aligned} \tag{2.20}$$

which induce the variations

$$\begin{aligned}\delta_\lambda a_+^{(0)} &= 2\,[\lambda^{(-)} - a_+^{(-)}\,\lambda^{(+)}] - \partial_+[\partial_+\lambda^{(+)} - a_+^{(0)}\,\lambda^{(+)}] \\ \delta_\lambda a_+^{(-)} &= \partial_+\lambda^{(-)} + a_+^{(0)}\,\lambda^{(-)} + a_+^{(-)}\,[\partial_+\lambda^{(+)} - a_+^{(0)}\lambda^{(+)}\,]\end{aligned} \tag{2.21}$$

and

$$\begin{aligned}\delta_{\tilde\lambda}\tilde a_+^{(-)} &= (\partial_+ + \tilde a_+^{(-)}\,\partial_- - \partial_-\tilde a_+^{(-)})\,\tilde\lambda^{(-)} \\ \delta_{\tilde\lambda}\tilde a_-^{(+)} &= -\tilde\lambda^{(-)}\,\partial_-\tilde a_-^{(+)} - 2\,\tilde a_-^{(+)}\,\partial_-\tilde\lambda^{(-)} + \frac{1}{2}\,\partial_-^3\,\tilde\lambda^{(-)}\end{aligned} \tag{2.22}$$

respectively. In the global case when we hold $\tilde a_-^{(+)}$ fixed at $-1/4$ we find that $\tilde\lambda^{(-)} = \lambda_f + e^{i\,x^-}\,\lambda_g + e^{-i\,x^-}\,\bar\lambda_{\bar g}$. When we make a gauge transformation to ensure that we remain in the FG gauge for the metric we need to impose

$$(\delta_\lambda a_+^{(0)} - \delta_{\tilde\lambda}\tilde a_+^{(0)})\Big|_{a_+^{(0)}=\tilde a_+^{(0)}} = 0 \tag{2.23}$$

We find that this condition drastically reduces the number of independent residual gauge parameters down to four functions of x^+. In particular, the function $\lambda^{(-)}(x^+, x^-)$ is determined to be

$$\begin{aligned}\lambda^{(-)}(x^+, x^-) = &-\frac{1}{4}\lambda^{(+)}(\bar g\,e^{-i\,x^-} - g\,e^{i\,x^-})^2 - \,\kappa_0\,\lambda^{(+)} + \frac{1}{2}\partial_+^2\lambda^{(+)} \\ &+\frac{i}{2}(g\,e^{i\,x^-} - \bar g\,e^{-i\,x^-})\partial_+\lambda^{(+)} + \frac{1}{2}\Big[(\lambda_g\,e^{i\,x^-} + \bar\lambda_{\bar g}\,e^{-i\,x^-})\,f \\ &-(g\,e^{i\,x^-} + \bar g\,e^{-i\,x^-})\,\lambda_f - i(\partial_+\lambda_g\,e^{i\,x^-} - \partial_+\bar\lambda_{\bar g}\,e^{-i\,x^-})\Big]\end{aligned} \tag{2.24}$$

These induce the following transformations:

$$\begin{aligned}&\delta_\lambda f = \lambda_f' + 2i\,(\bar g\,\lambda_g - \bar\lambda_{\bar g}\,g), \quad \delta_\lambda g = \lambda_g' + i\,(f\,\lambda_g - g\,\lambda_f), \quad \delta_\lambda\bar g = \bar\lambda_{\bar g}' - i\,(f\,\bar\lambda_{\bar g} - \bar g\,\lambda_f), \\ &\delta\kappa_0 = \lambda^{(+)}\,\kappa_0' + 2\kappa_0\,\partial_+\lambda^{(+)} - \frac{1}{2}\partial_+^3\lambda^{(+)}\end{aligned} \tag{2.25}$$

We could have obtained this result starting with the left sector a to be $a = [L_1 - \kappa_0(x^+)\,L_{-1}]\,dx^+$.

Now, comparing this result with (2.12) one finds that $\{\lambda_f, \lambda_g, \bar\lambda_{\bar g}, \lambda^{(+)}\}$ are not quite the parameters in (2.12) that correspond to the asymptotic symmetry vector fields of [4]. For this it turns out that we have to redefine the gauge parameters

$$\{\lambda_f, \lambda_g, \bar\lambda_{\bar g}, \lambda\} \to \{\lambda_f + f\,\lambda, \lambda_g + g\,\lambda, \bar\lambda_{\bar g} + \bar g\,\lambda\} \tag{2.26}$$

Then the transformations read

$$\begin{aligned}&\delta_{\tilde\lambda} f = \lambda_f' + 2i\,(\bar g\,\lambda_g - \bar\lambda_{\bar g}\,g) + (f\lambda^{(+)})', \qquad \delta_{\tilde\lambda} g = \lambda_g' + i\,(f\,\lambda_g - g\,\lambda_f) + (g\lambda^{(+)})', \\ &\delta_{\tilde\lambda}\bar g = \bar\lambda_{\bar g}' - i\,(f\,\bar\lambda_{\bar g} - \bar g\,\lambda_f) + (\bar g\lambda^{(+)})', \qquad \delta\kappa_0 = \lambda^{(+)}\,\kappa_0' + 2\kappa_0\,\partial_+\lambda^{(+)} - \frac{1}{2}\partial_+^3\lambda^{(+)}\end{aligned} \tag{2.27}$$

which match exactly with those in (2.12). If we compute the charges and the algebra of these symmetries it can be seen that they match exactly with those of [4]. In the next section we turn to generalization to higher spin theories.

3 Chiral boundary conditions for $\mathrm{SL}(3,\mathbb{R})$ higher spin theory

In this section we are interested in proposing boundary conditions for higher spin theories such that they holographically describe appropriate chiral induced W-gravity theories. To demonstrate the result we restrict to the simplest higher spin theory in three dimensions that contains a spin-2 field and a spin-3 field. In the first order formalism the theory is formulated on the same lines as AdS_3 gravity but with the gauge group replaced by $\mathrm{SL}(3,\mathbb{R})$ (or $\mathrm{SU}(1,2)$) [16, 20]. The Dirichlet boundary conditions of this theory were considered by Campoleoni et al. in [20] where it was shown that the asymptotic symmetry algebra is two commuting copies of classical W_3 algebra with central charges.

We now turn to generalizing the boundary conditions of the section (2.1) to the 3-dimensional higher spin theory based on two copies of $sl(3,\mathbb{R})$ or $su(1,2)$ algebra [16, 20]. Motivated by the CIG boundary conditions of the previous section we write the connections again as deformations of AdS_3 solution. We work in the principal embedding basis for the gauge algebra. Our conventions very closely follow those of [20] and may be found in appendix B. We start with the following ansatz for the gauge connections:

$$A = b^{-1}\partial_r b\, dr + b^{-1}\left[(L_1 - \kappa\, L_{-1} - \omega\, W_{-2})\, dx^+\right] b \tag{3.1}$$

$$\tilde{A} = b\,\partial_r b^{-1}\, dr + b\left[(-L_{-1} + \tilde{\kappa}\, L_1 + \tilde{\omega}\, W_2)\, dx^- + \left(\sum_{a=-1}^{1} f^{(a)} L_a + \sum_{i=-2}^{2} g^{(i)} W_i\right) dx^+\right] b^{-1}$$

where b is again $e^{L_0 \ln \frac{r}{l}}$. Note that, as in $sl(2,\mathbb{R})$ case, our ansatz is the Dirichlet one of [20] for the left sector. Similarly, the right sector includes the right sector ansatz of previous section as a special case.[4] All the coefficients of the algebra generators above are understood to be functions of x^+ and x^-.

Imposing flatness conditions on A and $\tilde{A}$ leads to a set of first order partial differential equations which may be found in (C.1) in appendix C. The flatness condition of A simply implies that the functions κ and ω depend only on x^+. The equations coming from the flatness condition on $\tilde{A}$ enable one to solve for $\{f^{(0)},\, f^{(1)},\, g^{(-1)},\, g^{(0)},\, g^{(1)},\, g^{(2)}\}$ in terms of $\{\tilde{\kappa},\, \tilde{\omega},\, f^{(-1)},\, g^{(-2)}\}$ and their derivatives, provided the functions $\{\tilde{\kappa},\, \tilde{\omega},\, f^{(-1)},\, g^{(-2)}\}$ satisfy the constraints coming from the 5th and the 10th equations in (C.1):

$$(\partial_+ + 2\,\partial_- f^{(-1)} + f^{(-1)}\,\partial_-)\,\tilde{\kappa} - \alpha^2\,(12\,\partial_- g^{(-2)} + 8\, g^{(-2)}\partial_-)\,\tilde{\omega} = \frac{1}{2}\partial_-^3 f^{(-1)}, \tag{3.2}$$

$$\begin{aligned} &12\,(\partial_+ + 3\,\partial_- f^{(-1)} + f^{(-1)}\partial_-)\,\tilde{\omega} \\ &\quad +(10\,\partial_-^3 g^{(-2)} + 15\,\partial_-^2 g^{(-2)}\,\partial_- + 9\,\partial_- g^{(-2)}\partial_-^2 + 2\,g^{(-2)}\partial_-^3)\,\tilde{\kappa} \\ &\qquad -16\,(2\,\partial_- g^{(-2)} + g^{(-2)}\,\partial_-)\,\tilde{\kappa}^2 = \frac{1}{2}\partial_-^5 g^{(-2)}. \end{aligned} \tag{3.3}$$

We point out that these are the Ward identities that the induced W_3 gravity theory is expected to satisfy. See Ooguri et al. [14] for a comparison. These have also appeared recently in [22] in a related context.

[4] A similar ansatz has been considered recently in [22].

Next, we seek the residual gauge transformations of our solutions. Defining the gauge parameter to be $\lambda = \lambda^{(a)} L_a + \eta^{(i)} W_i$ and imposing the condition that the gauge field configuration $a = (L_1 - \kappa L_{-1} - \omega W_{-2})\, dx^+$ is left form-invariant under the corresponding gauge transformation leads to equations (C.2) [20]. Under these (relabeling $\lambda^{(1)} \to \lambda$ and $\eta^{(2)} \to \eta$) we have

$$\delta\kappa = \lambda\,\kappa' + 2\,\lambda'\,\kappa - \frac{1}{2}\lambda''' - 8\,\alpha^2\,\eta\,\omega' - 12\,\alpha^2\,\omega\,\eta' \tag{3.4}$$

$$\delta\omega = \lambda\,\omega' + 3\,\omega\,\lambda' - \frac{8}{3}\,\kappa\,(\kappa\,\eta' + \eta\,\kappa') + \frac{1}{4}(5\,\kappa'\,\eta'' + 3\,\eta'\,\kappa'') + \frac{1}{6}\,(5\,\kappa\,\eta''' + \eta\,\kappa''') - \frac{1}{24}\eta''''' \tag{3.5}$$

We parametrize the residual gauge transformations of $\tilde{a}$ by the gauge parameters $\tilde{\lambda} = \tilde{\lambda}^{(a)} L_a + \tilde{\eta}^{(i)} W_i$. The constraints on this parameter are given in (C.3) of appendix C. These induce the following variations:

$$\delta\tilde{\kappa} = -2\,\tilde{\kappa}\,\partial_-\tilde{\lambda}^{(-1)} - \tilde{\lambda}^{(-1)}\partial_-\tilde{\kappa} + 8\,\alpha^2\,\tilde{\eta}^{(-2)}\,\partial_-\tilde{\omega} + 12\,\alpha^2\,\tilde{\omega}\,\partial_-\tilde{\eta}^{(-2)} + \frac{1}{2}\partial_-^3\tilde{\lambda}^{(-1)} \tag{3.6}$$

$$\begin{aligned}\delta\tilde{\omega} =& -\tilde{\lambda}^{(-1)}\,\partial_-\tilde{\omega} - 3\,\tilde{\omega}\,\partial_-\lambda^{(-1)} + \frac{8}{3}\tilde{\kappa}\,(\tilde{\kappa}\,\partial_-\tilde{\eta}^{(-2)} + \tilde{\eta}^{(-2)}\,\partial_-\tilde{\kappa})\\ &-\frac{1}{4}(5\,\partial_-\tilde{\kappa}\,\partial_-^2\tilde{\eta}^{(-2)} + 3\,\partial_-\tilde{\eta}^{(-2)}\,\partial_-^2\tilde{\kappa})\\ &-\frac{1}{6}(5\,\tilde{\kappa}\,\partial_-^3\tilde{\eta}^{(-2)} + \tilde{\eta}^{(-2)}\,\partial_-^3\tilde{\kappa}) + \frac{1}{24}\partial_-^5\tilde{\eta}^{(-2)}\end{aligned}$$

$$\begin{aligned}\delta f^{(-1)} =& \partial_+\tilde{\lambda}^{(-1)} + f^{(-1)}\,\partial_-\tilde{\lambda}^{(-1)} - \tilde{\lambda}^{(-1)}\,\partial_- f^{(-1)} + \frac{32}{3}\,\alpha^2\,\tilde{\kappa}\,(g^{(-2)}\,\partial_-\tilde{\eta}^{(-2)} - \tilde{\eta}^{(-2)}\,\partial_- g^{(-2)})\\ &+\alpha^2(\partial_- g^{(-2)}\,\partial_-^2\tilde{\eta}^{(-2)} - \partial_-\tilde{\eta}^{(-2)}\,\partial_-^2 g^{(-2)}) - \frac{2}{3}\,\alpha^2\,(g^{(-2)}\,\partial_-^3\tilde{\eta}^{(-2)} - \tilde{\eta}^{(-2)}\,\partial_-^3 g^{(-2)})\end{aligned} \tag{3.7}$$

$$\delta g^{(-2)} = \partial_+\tilde{\eta}^{(-2)} + f^{(-1)}\,\partial_-\tilde{\eta}^{(-2)} - \tilde{\lambda}^{(-1)}\,\partial_- g^{(-2)} + 2\,(g^{(-2)}\,\partial_-\tilde{\lambda}^{(-1)} - \tilde{\eta}^{(-2)}\,\partial_- f^{(-1)}) \tag{3.8}$$

For the residual gauge transformations to be global symmetries of the boundary theory one needs to impose the variational principle $\delta S = 0$ as well. We add the following boundary action:

$$S_{\text{bdy.}} = \frac{k}{4\pi}\int d^2x\, tr\Bigg(-L_0[\tilde{a}_+, \tilde{a}_-] + 2\,\tilde{\kappa}_0\,L_1\,\tilde{a}_+ \\ + \frac{1}{2\,\alpha}\,W_0\{\tilde{a}_+, \tilde{a}_-\} + \frac{1}{3}\tilde{a}_+\,\tilde{a}_- + 2\,\tilde{\omega}_0\,W_2\,\tilde{a}_+\Bigg)$$

With this the variation of the total action becomes:

$$\delta S_{\text{total}} = -\frac{k}{2\pi}\int d^2x\ [(\tilde{\kappa} - \tilde{\kappa}_0)\,\delta f^{(-1)} + 4\,\alpha^2\,(\tilde{\omega} - \tilde{\omega}_0)\,\delta g^{(-2)}] \tag{3.9}$$

where $\tilde{\kappa}_0$ and $\tilde{\omega}_0$ are some real numbers. Again we have several ways to satisfy $\delta S = 0$:

1. $\delta f^{(-1)} = 0$ and $\delta g^{(-2)} = 0$.

   This is the Dirichlet condition again and leads to W_3 as the asymptotic symmetry algebra [21].

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2. $\tilde{\kappa} = \tilde{\kappa}_0$ ($\tilde{\omega} = \tilde{\omega}_0$) and $\delta g^{(-2)} = 0$ ($\delta f^{(-1)} = 0$).

   These are mixed boundary conditions. Since we are interested in freeing up both $f^{(-1)}$ and $g^{(-2)}$ we will not consider them further here.

3. $\tilde{\kappa} = \tilde{\kappa}_0$ and $\tilde{\omega} = \tilde{\omega}_0$.

   These conditions do allow both $f^{(-1)}$ and $g^{(-2)}$ to fluctuate freely as we require. Therefore we will consider this case in detail below.

3.1 Solutions of W_3 Ward identities

The W_3 Ward identities (3.2), (3.3) are also expected to be integrable (just as the Virasoro one in section 2 was) and general solutions can be written down by an appropriate reparametrization of $f^{(-1)}$ and $g^{(-2)}$.

As stated before we restrict to the case of $\tilde{\kappa} = \tilde{\kappa}_0$ and $\tilde{\omega} = \tilde{\omega}_0$ for constant $\tilde{\kappa}_0$ and $\tilde{\omega}_0$ as this allows for classical solutions with fluctuating $f^{(-1)}$ and $g^{(-2)}$. Let us solve the W_3 Ward identities in this case. These read

$$\begin{aligned} \partial_-^3 f^{(-1)} + 24\,\alpha^2\,\tilde{\omega}_0\,\partial_- g^{(-2)} - 4\,\tilde{\kappa}_0\,\partial_- f^{(-1)} &= 0 \\ \partial_-^5 g^{(-2)} - 20\,\tilde{\kappa}_0\,\partial_-^3 g^{(-2)} + 64\,\tilde{\kappa}_0^2\,\partial_- g^{(-2)} - 72\,\tilde{\omega}_0\,\partial_- f^{(-1)} &= 0 \end{aligned} \tag{3.10}$$

There are two distinct cases: $\tilde{\omega}_0 = 0$ and $\tilde{\omega}_0 \neq 0$. When $\tilde{\omega}_0 = 0$ there are further two distinct cases:

1. $\tilde{\omega}_0 = 0$ and $\tilde{\kappa}_0 = 0$ gives:

$$\begin{aligned} f^{(-1)} &= f_{-1}(x^+) + x^-\,f_0(x^+) + (x^-)^2 f_1(x^+), \\ g^{(-2)} &= g_{-2}(x^+) + x^-\,g_{-1}(x^+) + (x^-)^2\,g_0(x^+) + (x^-)^3\,g_1(x^+) + (x^-)^4\,g_2(x^+) \end{aligned} \tag{3.11}$$

   This solution is suitable for non-compact x^+ and x^- (such as the boundary of AdS_3 in Poincare coordinates).

2. $\tilde{\omega}_0 = 0$ and $\tilde{\kappa}_0 \neq 0$:

$$\begin{aligned} f^{(-1)} &= f_\kappa(x^+) + g_\kappa(x^+)\,e^{2\sqrt{\tilde{\kappa}_0}\,x^-} + \bar{g}_\kappa(x^+)\,e^{-2\sqrt{\tilde{\kappa}_0}x^-}, \\ g^{(-2)} &= f_\omega(x^+) + g_\omega(x^+)\,e^{2\sqrt{\tilde{\kappa}_0}\,x^-} + \bar{g}_\omega(x^+)\,e^{-2\sqrt{\tilde{\kappa}_0}\,x^-} \\ &\quad + h_\omega(x^+)\,e^{4\sqrt{\tilde{\kappa}_0}\,x^-} + \bar{h}_\omega(x^+)\,e^{-4\sqrt{\tilde{\kappa}_0}\,x^-} \end{aligned} \tag{3.12}$$

   Again any positive value for $\tilde{\kappa}_0$ is suitable for non-compact boundary coordinates. Among the negative values $\tilde{\kappa}_0 = -\frac{1}{4}$ (times square of any integer) is suitable for compact boundary coordinates (such as the boundary of global AdS_3).

3. When $\tilde{\kappa}_0 \neq 0$ and $\tilde{\omega}_0 \neq 0$, again the Ward identities can be solved. The general solutions involve eight arbitrary functions of x^+ (just as in the cases with $\tilde{\omega}_0 = 0$). Here we will only consider the special case where the solutions do not depend on x^-:

$$f^{(-1)} = f(x^+), \quad g^{(-2)} = g(x^+). \tag{3.13}$$

   This case is the analogue of [7] in the higher spin context.

Next we analyze these cases one by one and find the asymptotic symmetries.

4 Asymptotic symmetries, charges and Poisson brackets

To find the asymptotic symmetries to which we can associate charges one needs to look for the residual gauge transformations of the solutions of interest. Just as in the $sl(2,\mathbb{R})$ case we can look at the residual gauge transformations of a and $\tilde{a}$ and translate the corresponding gauge parameters λ and $\tilde{\lambda}$ using $\Lambda = b^{-1}\lambda\, b$ and $\tilde{\Lambda} = b\,\tilde{\lambda}\, b^{-1}$. After finding these one can compute the corresponding charges.

A method for computing the charges corresponding to residual gauge transformations is provided by the Barnich et al. [23, 24]. Using their method one can show that the change in the charge $\not\delta Q$ along the space of solutions of one copy of the Chern-Simons theory is:

$$\not\delta Q_\Lambda = -\frac{k}{2\pi}\int_0^{2\pi} d\phi\; tr[\Lambda\,\delta A_\phi]. \tag{4.1}$$

where Λ is the gauge transformation parameter. We will see that these charges are integrable for all the residual gauge transformations considered below.

Now, demanding that the charge (corresponding to a given residual gauge transformation) generates the right variations of the functions parametrizing the solutions via

$$\delta_\Lambda f(x) = \{Q_\Lambda, f(x)\}, \tag{4.2}$$

allows one to read out the Poisson brackets between those functions.

We are now ready to carry out this exercise for the left sector and each of the cases (3.11)–(3.13) in the right sector one by one.

4.1 The left sector symmetry algebra

The left sector is common for all of the cases we consider in this paper. The corresponding $\not\delta Q$ is

$$\not\delta Q_\Lambda = -\frac{k}{2\pi}\int_0^{2\pi} d\phi\,(\lambda\,\delta\kappa - 4\,\alpha^2\,\eta\,\delta\omega) \tag{4.3}$$

This when integrated between ($\kappa = 0$, $\omega = 0$) and generic (κ, ω) gives

$$Q_{(\lambda,\eta)} = -\frac{k}{2\pi}\int_0^{2\pi} d\phi\;[\lambda\,\kappa - 4\,\alpha^2\,\eta\,\omega] \tag{4.4}$$

This charge generates the variations (3.4), (3.5) provided we take the Poisson brackets amongst κ and ω to be:

$$\begin{aligned}
-\frac{k}{2\pi}\,\{\kappa(x^+),\kappa(\tilde{x}^+)\} &= -\kappa'(x^+)\,\delta(x^+-\tilde{x}^+) - 2\,\kappa\,(x^+)\,\delta'(x^+-\tilde{x}^+) + \frac{1}{2}\,\delta'''(x^+-\tilde{x}^+),\\
-\frac{k}{2\pi}\,\{\kappa(x^+),\omega(\tilde{x}^+)\} &= -2\,\omega'(x^+)\,\delta(x^+-\tilde{x}^+) - 3\,\omega(x^+)\,\delta'(x^+-\tilde{x}^+),\\
-\frac{2k\alpha^2}{\pi}\,\{\omega(x^+),\omega(\tilde{x}^+)\} &= \frac{8}{3}[\kappa^2(x^+)\,\delta'(x^+-\tilde{x}^+) + \kappa(x^+)\,\kappa'(x^+)\delta(x^+-\tilde{x}^+)]\\
&\quad -\frac{1}{6}[5\kappa(x^+)\delta'''(x^+-\tilde{x}^+) + \kappa'''(x^+)\delta(x^+-\tilde{x}^+)]\\
&\quad -\frac{1}{4}[3\kappa''(x^+)\delta'(x^+-\tilde{x}^+) + 5\kappa'(x^+)\delta''(x^+-\tilde{x}^+)]\\
&\quad +\frac{1}{24}\delta^{(5)}(x^+-\tilde{x}^+)
\end{aligned} \tag{4.5}$$

These brackets were first computed by Campoleoni et al. [21]. To compare with their answers one has to take $\kappa \to -\frac{2\pi}{k}\kappa$, $\omega \to \frac{\pi}{2k\alpha^2}\omega$, $\alpha^2 \to -\sigma$ in the expressions here. The result, therefore, is that the left sector gives a classical W_3 algebra as its asymptotic algebra.

Next we turn to computing the charges and Poisson brackets on the right sector for all the cases of interest.

4.2 $\tilde{\kappa}_0 = 0$ and $\tilde{\omega}_0 = 0$

In this case the residual gauge transformation parameters are

$$\begin{aligned}\tilde{\lambda}^{(-1)} &= \lambda_{-1}(x^+) + x^- \,\lambda_0(x^+) + (x^-)^2\,\lambda_1(x^+)\\ \tilde{\eta}^{(-2)} &= \eta_{-2}(x^+) + x^- \,\eta_{-1}(x^+) + (x^-)^2\,\eta_0(x^+) + (x^-)^3\,\eta_1(x^+) + (x^-)^4\,\eta_2(x^+)\end{aligned} \tag{4.6}$$

The corresponding action on the fields can be found in (C.4) of appendix C. Defining

$$\begin{aligned}\{J^a, a = 1, \cdots, 8\} &= \{f_{-1}, f_0, f_1, g_{-2}, g_{-1}, g_0, g_1, g_2\}\\ \{\lambda^a, a = 1, \cdots, 8\} &= \{\lambda_{-1}, \lambda_0, \lambda_1, \eta_{-2}, \eta_{-1}, \eta_0, \eta_1, \eta_2\}\end{aligned} \tag{4.7}$$

these expressions can also be written in a compact form:

$$\delta J^a = \partial_+\lambda^a - f^a{}_{bc}J^b\lambda^c \tag{4.8}$$

where $f^a{}_{bc}$ are structure constants of our gauge algebra given in appendix B. The charge in this case is integrable and has the expression:

$$Q[\tilde{\lambda}] = \frac{k}{4\pi}\int dx^+\,\eta_{ab}J^a\lambda^b \tag{4.9}$$

where η_{ab} is the Killing form of the gauge algebra. The Poisson brackets can be read out and we find:

$$\{J^a(x^+), J^b(\tilde{x}^+)\} = f^{ab}{}_c\,J^c(x^+)\,\delta(x^+ - \tilde{x}^+) - \frac{k}{4\pi}\,\eta^{ab}\,\delta'(x^+ - \tilde{x}^+) \tag{4.10}$$

where we have redefined: $J^a \to -\frac{4\pi}{k}J^a$. This may be recognized as level-k Kac-Moody extension of the algebra used in defining the higher spin theory.

4.3 $\tilde{\kappa}_0 = -\frac{1}{4}$ and $\tilde{\omega}_0 = 0$

In this case the residual gauge transformation parameters are

$$\begin{aligned}\tilde{\lambda}^{(-1)} &= \lambda_f(x^+) + \lambda_g(x^+)\,e^{i\,x^-} + \bar{\lambda}_{\bar{g}}(x^+)\,e^{-i\,x^-}\\ \tilde{\eta}^{(-2)} &= \eta_f(x^+) + \eta_g(x^+)\,e^{i\,x^-} + \bar{\eta}_{\bar{g}}(x^+)\,e^{-i\,x^-} + \eta_h(x^+)\,e^{2i\,x^-} + \bar{\eta}_{\bar{h}}(x^+)\,e^{-2i\,x^-}\end{aligned} \tag{4.11}$$

The symmetry transformations are given in (C.5) in appendix C. Defining the currents J^a and parameters λ^a as

$$\begin{aligned}\{J^a, a = 1, \cdots, 8\} &= \{\bar{g}_\kappa,\, f_\kappa,\, g_\kappa,\, \bar{h}_\omega,\, \bar{g}_\omega,\, f_\omega,\, g_\omega,\, h_\omega\}\\ \{\lambda^a, a = 1, \cdots, 8\} &= \{\bar{\lambda}_{\bar{g}},\, \lambda_f,\, \lambda_g,\, \bar{\eta}_{\bar{h}},\, \bar{\eta}_{\bar{g}},\, \eta_f,\, \eta_g,\, \eta_h\}\end{aligned} \tag{4.12}$$

these expressions can also be written in a compact form:

$$\delta J^a = \partial_+ \lambda^a - i\,\hat{f}^a{}_{bc} J^b \lambda^c \tag{4.13}$$

where (somewhat surprisingly) $\hat{f}^a{}_{bc}$ are obtained from the structure constants $f^a{}_{bc}$ of the gauge algebra by replacing $\alpha^2 \to -\alpha^2$. In this case the charge is:

$$Q[\lambda^a] = -\frac{k}{4\pi} \int_0^{2\pi} d\phi\, \hat{\eta}_{ab}\, \lambda^a J^b \tag{4.14}$$

where $\hat{\eta}_{ab}$ is the one obtained from the Killing form η_{ab} of the gauge algebra by replacing α^2 by $-\alpha^2$. The corresponding Poisson brackets are

$$\{J^a(x^+), J^b(\tilde{x}^+)\} = i\hat{f}^{ab}{}_c\, J^c(x^+)\, \delta(x^+ - \tilde{x}^+) + \frac{k}{4\pi} \hat{h}^{ab} \delta'(x^+ - \tilde{x}^+). \tag{4.15}$$

where again we have redefined: $J^a \to \frac{4\pi}{k} J^a$. This again is a level-$k$ Kac-Moody algebra, but for the difference that it is obtained from the gauge algebra by $\alpha^2 \to -\alpha^2$ replacement.

Note that if we start with the Lie algebra of the matrices given in appendix B with $\alpha^2 = 1$ corresponding to the $sl(3,\mathbb{R})$ algebra ($\alpha^2 = -1$ corresponding to the $su(1,2)$ algebra) and make the replacement $\alpha^2 \to -\alpha^2$ we end up the $su(1,2)$ algebra ($sl(3,\mathbb{R})$ algebra). Therefore, the result in this case is that the asymptotic symmetry algebra is the $su(1,2)$ current algebra at level-k when the gauge algebra used in the definition of the higher spin theory is $sl(3,\mathbb{R})$.

4.4 $\tilde{\kappa}_0 \neq 0$, $\tilde{\omega}_0 \neq 0$, $\partial_- f^{(-1)} = \partial_- g^{(-2)} = 0$

In this case the residual gauge transformation parameters are

$$\tilde{\lambda}^{(-1)} = \tilde{\lambda}(x^+), \quad \tilde{\eta}^{(-2)} = \tilde{\eta}(x^+). \tag{4.16}$$

Under these gauge transformations the fields transform as

$$\delta f^{(-1)} = \partial_+ \tilde{\lambda}, \quad \delta g^{(-2)} = \partial_+ \tilde{\eta}. \tag{4.17}$$

Thus the residual gauge symmetries generate two commuting copies of U(1) classically. Restricted to the $sl(2,\mathbb{R})$ sub-sector this case corresponds to [7]. The charge is

$$Q_{\tilde{a}} = \frac{k}{2\pi} \int_0^{2\pi} d\phi\, 2 \left[\tilde{\lambda}\, (\tilde{\kappa}_0\, f - 6\,\alpha^2\, \tilde{\omega}_0\, g) + \tilde{\eta}\, 2\, \alpha^2 \left(\frac{8}{3} \tilde{\kappa}_0^2\, g - 3\, \tilde{\omega}_0\, f \right) \right] \tag{4.18}$$

This leads to Poisson brackets:

$$\begin{aligned}
\{\tilde{\kappa}_0\, f(x^+) - 6\,\alpha^2\, \tilde{\omega}_0\, g(x^+),\, f(\tilde{x}^+)\} &= -\frac{\pi}{k} \delta'(x^+ - x^{+\prime}), \\
\left\{ \frac{8}{3} \tilde{\kappa}_0^2\, g(x^+) - 3\, \tilde{\omega}_0\, f(x^+),\, f(\tilde{x}^+) \right\} &= 0 \\
\{\tilde{\kappa}_0\, f(x^+) - 6\,\alpha^2\, \tilde{\omega}_0\, g(x^+),\, g(\tilde{x}^+)\} &= 0, \\
\left\{ \frac{8}{3} \tilde{\kappa}_0^2\, g(x^+) - 3\, \tilde{\omega}_0\, f(x^+),\, g(\tilde{x}^+) \right\} &= -\frac{\pi}{2k\alpha^2} \delta'(x^+ - \tilde{x}^+)
\end{aligned} \tag{4.19}$$

These four relations are solved by the following three equations:

$$\begin{aligned}
\{f(x^+),\, f(\tilde{x}^+)\} &= -\frac{\pi}{k}\frac{\tilde{\kappa}_0^2}{\Delta}\delta'(x^+ - \tilde{x}^+), \\
\{g(x^+),\, g(\tilde{x}^+)\} &= -\frac{\pi}{k}\frac{3\,\tilde{\kappa}_0}{16\,\Delta\alpha^2}\delta'(x^+ - \tilde{x}^+), \\
\{f(x^+),\, g(\tilde{x}^+)\} &= -\frac{\pi}{k}\frac{9\,\tilde{\omega}_0}{8\,\Delta}\delta'(x^+ - \tilde{x}^+)
\end{aligned} \tag{4.20}$$

where $\Delta = \tilde{\kappa}_0^3 - \frac{27}{4}\,\alpha^2\,\tilde{\omega}_0^2$ which we have to assume not to vanish.[5]

5 Discussion

In this paper, generalizing the results of [4, 5] we proposed boundary conditions for higher spin gauge theories in 3d in their first order formalism that are different from the usual Dirichlet boundary conditions.[6] The left sector is treated with the usual Dirichlet boundary conditions whereas in the right sector we chose free boundary conditions. We restricted our attention to the spin-3 case for calculational convenience. The Dirichlet boundary conditions for general higher spin theory based on $sl(n,\mathbb{R})$ Chern-Simons was discussed in [20] and for $hs[\lambda]$ case in [25]. One should be able to generalize our considerations to these other higher spin theories as well.

The boundary conditions considered here give one copy of W_3 and a copy of $sl(3,\mathbb{R})$ (or $su(1,2)$ or $u(1)\oplus u(1)$) Kac-Moody algebra. This matches with the symmetry algebra expected of the 2d chiral induced W-gravity with an appropriate field content.

Let us emphasize that there appears to be a surprising difference between the asymptotic symmetry algebras of section 4.2 and section 4.3: namely the maximal finite subalgebra of (4.10) is isomorphic to the gauge algebra of the higher spin theory where as that in (4.15) differs from the gauge algbra by $\alpha^2 \to -\alpha^2$ (this interchanges $sl(3,\mathbb{R})$ and $su(1,2)$). It will be interesting to understand the source of this possibility of getting a different real-form of the complexified gauge algebra out of our boundary conditions.

The Poisson brackets between κ or ω of the left sector and any of the right sector currents vanish. Recall that in the $sl(2,\mathbb{R})$ case, motivated by how the asymptotic vector fields in the second order formalism [4] acted on the fields, we made (current dependent) redefinitions of the residual gauge parameters. Here too one can do such a redefinition. For instance, if we change variables

$$\lambda^a \to \lambda^a + \alpha_1 J^a\,\lambda + \alpha_2\, d^a{}_{bc} J^b\, J^c\,\eta + \cdots \tag{5.1}$$

where λ^a are the parameters defined in (4.7) and λ and η are the gauge parameters of the left sector, $d_{abc} \sim \mathrm{Tr}(T_a\{T_b, T_c\})$, then one finds that

$$\kappa \to \kappa + \#\,\eta_{ab}J^a J^b + \cdots\,, \quad \omega \to \omega + \#\,d_{abc}J^a J^b J^c + \cdots \tag{5.2}$$

[5] Taking linear combinations $f + \chi\, g$ and $f - \chi\, g$ (for some constant χ) as the currents one can decouple these two $u(1)$ Kac-Moody algebras.

[6] It should be noted that the ansatz for the right sector gauge field (3.1) studied here also appeared recently in [22, 26] where the authors were only interested in generalizations of Dirichlet type boundary conditions.

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where the dots in these redefinitions represent possible terms higher order in J^as or terms involving derivatives of the currents. The additional terms here may be recognized as the (classical analogues) of Sugawara constructions of spin-2 and spin-3 currents out of the Kac-Moody currents. After such redefinitions the generators of the asymptotic symmetry algebras of the left and the right sectors will not commute any longer.

It is of interest to understand the holographic duals of the higher spin theories with our boundary conditions better. For instance, how does one construct the action of the CIWG theories given the bulk theory and its boundary conditions. This question was addressed in the $n = 2$ case by Banados et al. [29]. For the case of $n = 3$ we point out that the boundary conditions considered here can be seen to be consistent with the constraints imposed on the gauge connection in [14] in their definition of CIWG as an $sl(3, \mathbb{R})$ gauged WZW model. It will be important to understand this connection better. In fact, Verlinde [32] anticipated that the CIWG theories defined via the constrained gauged WZW model could be defined through 3d gravity theories and our proposal can be considered as a realization of that anticipation.

The case considered in section 4.4 is the generalization of [7] where the motivation to allow for only x^+ dependence of the boundary metric component $F(x^+, x^-)$ is to ensure that the scalar curvature of the class of metrics $ds^2 = -dx^+dx^- + F(x^+, x^-)(dx^+)^2$ vanishes. Recall that the vanishing scalar curvature implies that the Weyl anomaly for a CFT on this background also vanishes. It will be interesting to see if the case of section 4.4 also satisfies a generalization of such a condition to the higher spin context.

The CIG boundary conditions of section 2 do not allow the general BTZ [27] black hole solution (say when $F = 0$) as a bulk configuration consistent with the variational principle [4, 5]. To see this note that the variational principle is satisfied by taking the constant $\tilde{\kappa}$ to be $-1/4$ (or zero if we do not have a compact direction on the boundary) whereas the general BTZ black hole would require it to be any arbitrary (positive) constant given by a linear combination of its energy and the angular momentum. Therefore setting $\tilde{\kappa} = -1/4$ or $\tilde{\kappa} = 0$ (for $F = 0$) imposes a linear relation between the energy and angular momentum of the corresponding black hole (an extremal condition). This continues to be true even for the CIWG boundary conditions considered here as the CIG boundary conditions can be embedded into these more general ones (by simply restricting to an $sl(2, \mathbb{R})$ subalgebra of $sl(3, \mathbb{R})$). The solutions carrying all possible higher spin charges (see [28]) are not necessarily allowed classical solutions of our boundary conditions. It will be interesting to study the consequences of this to the properties of black holes in the CIWG boundary conditions.

We have used the first order formalism to do our computations. Our boundary conditions can be translated to the metric and the spin-3 fields in the second order language. There is a second order formalism of the 3d higher spin theories [30, 31]. It will be interesting to work out the details in that formalism too.

Finally it will be interesting to see how to generalize these chiral boundary conditions to other contexts, such as other embeddings of gravity sector into the higher spin theory, supersymmetric theories etc.

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A AdS_3 gravity in first order formulation

The AdS_3 gravity in the Hilbert-Palatini formulation can be recast as a gauge theory with action

$$S[A,\tilde{A}] = \frac{k}{4\pi}\int \mathrm{tr}\Big(A\wedge A + \frac{2}{3}A\wedge A\wedge A\Big) - \frac{k}{4\pi}\int \mathrm{tr}(\tilde{A}\wedge\tilde{A} + \frac{2}{3}\tilde{A}\wedge\tilde{A}\wedge\tilde{A}) \quad \text{(A.1)}$$

up to boundary terms, where the gauge group is $\mathrm{SL}(2,\mathbb{R})$. The equations of motion are $F = dA + A\wedge A = 0$ and $\tilde{F} := d\tilde{A} + \tilde{A}\wedge\tilde{A} = 0$. We work with the following defining representation of the $sl(2,\mathbb{R})$ algebra.

$$L_{-1} = \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix}, \quad L_0 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad L_1 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad , \quad \text{(A.2)}$$

Satisfying $[L_m, L_n] = (m-n)L_{m+n}$. The metric defined by $\mathrm{Tr}(T_aT_b) = \frac{1}{2}h_{ab}$ is

$$h_{ab} = \begin{pmatrix} 0 & 0 & -2 \\ 0 & 1 & 0 \\ -2 & 0 & 0 \end{pmatrix} \quad \text{(A.3)}$$

It is known that the connections

$$\begin{aligned} A &= b^{-1}\partial_r b\, dr + b^{-1}(L_1 - \kappa(x^+)\, L_{-1})\, b\, dx^+ \\ \tilde{A} &= b\,\partial_r b^{-1}\, dr + b\,(\tilde{\kappa}(x^-)\, L_1 - L_{-1})\, b^{-1}\, dx^- \end{aligned} \quad \text{(A.4)}$$

represent all the solutions of AdS_3 gravity satisfying Brown-Henneaux (Dirichlet) boundary conditions (in FG coordinates) where $b = e^{L_0 \ln\frac{r}{l}}$. In fact, any solution of the Chern-Simons theory (locally) can be written as

$$A = b^{-1}\partial_r b\, dr + b^{-1}\, a\, b, \quad \tilde{A} = b\,\partial_r b^{-1}\, dr + b\,\tilde{a}\, b^{-1} \quad \text{(A.5)}$$

where a and $\tilde{a}$ are flat connections in two dimensions with coordinates (x^+, x^-). The general solution can be written as $a = g^{-1}\, dg$ and $\tilde{a} = \tilde{g}\, d\tilde{g}^{-1}$ where g and $\tilde{g}$ are $\mathrm{SL}(2,\mathbb{R})$ group elements that depend on (x^+, x^-). We now present general solution to this flatness condition in a different parametrization that will be useful to us. Consider the most general $sl(2,\mathbb{R})$ 1-form on the boundary

$$a = (a_+^{(+)}\, L_1 + a_+^{(-)}\, L_{-1} + a_+^{(0)}\, L_0)\, dx^+ + (a_-^{(+)}\, L_1 + a_-^{(-)}\, L_{-1} + a_-^{(0)}\, L_0)\, dx^- \quad \text{(A.6)}$$

Assuming that $a_+^{(+)}$ does not vanish, the flatness conditions imply:

$$\begin{aligned} a_-^{(0)} &= \frac{1}{a_+^{(+)}}\left(a_-^{(+)}\, a_+^{(0)} + \partial_- a_+^{(+)} - \partial_+ a_-^{(+)}\right), \\ a_-^{(-)} &= \frac{1}{2a_+^{(+)}}\left(2\, a_-^{(+)}\, a_+^{(-)} + \partial_- a_+^{(0)} - \partial_+ a_-^{(0)}\right) \\ \frac{1}{2}\partial_+^3 f &= \partial_-\kappa - 2\,\kappa\,\partial_+ f - f\,\partial_+\kappa \end{aligned} \quad \text{(A.7)}$$

where $\kappa = a_+^{(+)}\, a_+^{(-)} - \frac{1}{4}(a_+^{(0)})^2 - \frac{1}{2}\partial_+ a_+^{(0)} + \frac{1}{2} a_+^{(0)}\, \partial_+ \ln a_+^{(+)} + \frac{1}{2}\partial_+^2 \ln a_+^{(+)} - \frac{1}{4}(\partial_+ \ln a_+^{(+)})^2$ and $f = \frac{a_-^{(+)}}{a_+^{(+)}}$. Similarly if we consider the 1-form

$$\tilde a = (\tilde a_+^{(+)}\, L_1 + \tilde a_+^{(-)}\, L_{-1} + \tilde a_+^{(0)}\, L_0)\, dx^+ + (\tilde a_-^{(+)}\, L_1 + \tilde a_-^{(-)}\, L_{-1} + \tilde a_-^{(0)}\, L_0)\, dx^- \tag{A.8}$$

Then, assuming now that $\tilde a_-^{(-)}$ does not vanish, the flatness conditions read

$$\begin{aligned} \tilde a_+^{(+)} &= \frac{1}{2\,\tilde a_-^{(-)}}(2\,\tilde a_-^{(+)}\,\tilde a_+^{(-)} + \partial_- \tilde a_+^{(0)} - \partial_+ \tilde a_-^{(0)}), \\ \tilde a_+^{(0)} &= \frac{1}{\tilde a_-^{(-)}}\,(\tilde a_-^{(0)}\,\tilde a_+^{(-)} + \partial_- \tilde a_+^{(-)} - \partial_+ \tilde a_-^{(-)}) \\ \frac{1}{2}\partial_-^3 \tilde f &= \partial_+ \tilde\kappa - 2\,\tilde\kappa\,\partial_- \tilde f - \tilde f\,\partial_- \tilde\kappa \end{aligned} \tag{A.9}$$

where $\tilde f = \frac{\tilde a_+^{(-)}}{\tilde a_-^{(-)}}$ and $\tilde\kappa = \tilde a_-^{(-)}\,\tilde a_-^{(+)} - \frac{1}{4}(\tilde a_-^{(0)})^2 + \frac{1}{2}\partial_- \tilde a_-^{(0)} - \frac{1}{2}\tilde a_-^{(0)}\,\partial_- \ln \tilde a_-^{(-)} + \frac{1}{2}\partial_-^2 \ln \tilde a_-^{(-)} - \frac{1}{4}(\partial_- \ln \tilde a_-^{(-)})^2$. The last equation is again the famous Virasoro Ward identity that can be solved explicitly as in section 2. Some special cases of the above formulae have appeared before, for instance, in [33].

B $sl(3,\mathbb{R})$ conventions

We work with the following basis of 3×3 matrices (see [21]) for the fundamental representation of the gauge group used in the definition of the higher spin theory:

$$\begin{aligned} L_{-1} &= \begin{pmatrix} 0 & -2 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{pmatrix}, & L_0 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \\ L_1 &= \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, & W_{-2} &= \alpha \begin{pmatrix} 0 & 0 & 8 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ W_{-1} &= \alpha \begin{pmatrix} 0 & -2 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}, & W_0 &= \alpha\,\frac{2}{3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\ W_1 &= \alpha \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}, & W_2 &= \alpha \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 2 & 0 & 0 \end{pmatrix}. \end{aligned} \tag{B.1}$$

The algebra satisfied by these matrices is

$$\begin{aligned} [L_m, L_n] &= (m-n)\, L_{m+n}, \qquad [L_m, W_n] = (2m-n)\, w_{m+n}, \\ [W_m, W_n] &= -\frac{\alpha^2}{3}(m-n)(2m^2 + 2n^2 - mn - 8)\, L_{m+n} \end{aligned} \tag{B.2}$$

For $\alpha^2 = -1$ this is the $su(1,2)$ algebra and for $\alpha^2 = 1$ this is the $sl(3,\mathbb{R})$ algebra. We take the Killing metric as $\eta_{ab} = \frac{1}{2}\,\mathrm{Tr}(T_a T_b)$ where T_a are the above matrices. The structure constants are $f_{abc} = \frac{1}{2}\mathrm{Tr}(T_a\,[T_b, T_c])$.

C Equations referred to in the text

It is easily seen that imposing the flatness condition on A and $\tilde{A}$ of (3.1) in section 3 leads to the following coupled partial first order differential equations:

$$\begin{aligned}
\partial_-\kappa &= 0\\
\partial_-\omega &= 0,\\
\partial_- f^{(-1)} + f^{(0)} &= 0,\\
\partial_- f^{(0)} + 2\, f^{(1)} + 2\,\tilde{\kappa}\, f^{(-1)} - 16\,\alpha^2\,\tilde{\omega}\, g^{(-2)} &= 0,\\
-\partial_+\tilde{\kappa} + \partial_- f^{(1)} + \tilde{\kappa}\, f^{(0)} - 4\,\alpha^2\,\tilde{\omega}\, g^{(-1)} &= 0\\
\\
\partial_- g^{(-2)} + g^{(-1)} &= 0,\\
\partial_- g^{(-1)} + 2\, g^{(0)} + 4\,\tilde{\kappa}\, g^{(-2)} &= 0,\\
\partial_- g^{(0)} + 3\, g^{(1)} + 3\,\tilde{\kappa}\, g^{(-1)} &= 0,\\
\partial_- g^{(1)} + 4\, g^{(2)} + 2\,\tilde{\kappa}\, g^{(0)} + 4\,\tilde{\omega}\, f^{(-1)} &= 0,\\
-\partial_+\tilde{\omega} + \partial_- g^{(2)} + \tilde{\kappa}\, g^{(1)} + 2\,\tilde{\omega}\, f^{(0)} &= 0.
\end{aligned} \tag{C.1}$$

The condition that the connection A in (3.1) is left form-invariant under a gauge transformation using the gauge parameter $\lambda = \lambda^{(a)} L_a + \eta^{(i)}\, W_i$ gives rise to the following equations:

$$\begin{aligned}
\partial_-\lambda^{(a)} = \partial_-\eta^{(i)} &= 0,\\
\partial_+\lambda^{(0)} + 2\,\lambda^{(-1)} + 2\,\kappa\,\lambda^{(1)} - 16\,\alpha^2\,\omega\,\eta^{(2)} &= 0,\\
\partial_+\lambda^{(1)} + \lambda^{(0)} &= 0,\\
\partial_+\eta^{(-1)} + 4\,\eta^{(-2)} + 2\,\kappa\,\eta^{(0)} + 4\,\omega\,\lambda^{(1)} &= 0,\\
\partial_+\eta^{(0)} + 3\,\eta^{(-1)} + 3\,\kappa\,\eta^{(1)} &= 0,\\
\partial_+\eta^{(1)} + 2\,\eta^{(0)} + 4\,\kappa\,\eta^{(2)} &= 0,\\
\partial_+\eta^{(2)} + \eta^{(1)} &= 0.
\end{aligned} \tag{C.2}$$

Similarly the corresponding condition on $\tilde{A}$ in (3.1) under the gauge transformation with the parameter $\tilde{\lambda} = \tilde{\lambda}^{(a)} L_a + \tilde{\eta}^{(i)}\, W_i$ leads to:

$$\begin{aligned}
\tilde{\lambda}_0 + \partial_-\tilde{\lambda}^{(-1)} &= 0,\\
\partial_-\tilde{\lambda}_0 + 2\,\tilde{\lambda}^{(1)} + 2\,\tilde{\kappa}\,\tilde{\lambda}^{(-1)} - 16\,\alpha^2\,\tilde{\omega}\,\tilde{\eta}^{(-2)} &= 0,\\
\partial_-\tilde{\eta}^{(-2)} + \tilde{\eta}^{(-1)} &= 0,\\
\partial_-\tilde{\eta}^{(-1)} + 2\,\tilde{\eta}^{(0)} + 4\,\tilde{\eta}^{(-2)}\,\tilde{\kappa} &= 0,\\
\partial_-\tilde{\eta}^{(0)} + 3\,\tilde{\eta}^{(1)} + 3\,\tilde{\eta}^{(-1)}\,\tilde{\kappa} &= 0,\\
\partial_-\tilde{\eta}^{(1)} + 4\,\tilde{\eta}^{(2)} + 2\,\tilde{\eta}^{(0)}\,\tilde{\kappa} + 4\,\tilde{\lambda}^{(-1)}\,\tilde{\omega} &= 0.
\end{aligned} \tag{C.3}$$

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In the case of $\tilde{\kappa}_0 = 0$, $\tilde{\omega}_0 = 0$ considered in section 4.2 we find that the the fields in A transform as:

$$\begin{aligned}
\delta f_0 &= \lambda_0' + 2\,(f_{-1}\,\lambda_1 - \lambda_{-1}\,f_1) - 2\,\alpha^2\,(\eta_{-1}\,g_1 - \eta_1\,g_{-1}) - 16\,\alpha^2\,(\eta_2\,g_{-2} - \eta_{-2}\,g_2)\\
\delta f_1 &= \lambda_1' + (\lambda_1\,f_0 - \lambda_0\,f_1) - 2\,\alpha^2\,(\eta_0\,g_1 - \eta_1\,g_0) - 4\,\alpha^2\,(\eta_2\,g_{-1} - \eta_{-1}\,g_2)\\
\delta f_{-1} &= \lambda_{-1}' + (\lambda_0\,f_{-1} - \lambda_{-1}\,f_0) - 2\,\alpha^2\,(\eta_{-1}\,g_0 - \eta_0\,g_{-1}) - 4\,\alpha^2\,(\eta_1\,g_{-2} - \eta_{-2}\,g_1)\\
\delta g_0 &= \eta_0' + 3\,(\eta_1\,f_{-1} - \eta_{-1}\,f_1) + 3\,(\lambda_1\,g_{-1} - \lambda_{-1} g_1)\\
\delta g_1 &= \eta_1' + (\eta_1\,f_0 - \lambda_0\,g_1) + 2\,(\lambda_1\,g_0 - \eta_0\,f_1) + 4\,(\eta_2\,f_{-1} - \lambda_{-1}\,g_2)\\
\delta g_{-1} &= \eta_{-1}' + (\lambda_0\,g_{-1} - \eta_{-1}\,f_0) + 2\,(\eta_0\,f_{-1} - \lambda_{-1}\,g_0) + 4\,(\lambda_1\,g_{-2} - \eta_{-2}\,f_1)\\
\delta g_2 &= \eta_2' + (\lambda_1\,g_1 - \eta_1\,f_1) + 2\,(\eta_2\,f_0 - \lambda_0\,g_2)\\
\delta g_{-2} &= \eta_{-2}' + (\eta_{-1}\,f_{-1} - \lambda_{-1}\,g_{-1}) + 2\,(\lambda_0\,g_{-2} - \eta_{-2}\,f_0)
\end{aligned} \tag{C.4}$$

In the case of $\tilde{\kappa}_0 = -\frac{1}{4}$ and $\tilde{\omega}_0 = 0$ considered in section 4.3 the variations of the functions in the gauge connection $\tilde{A}$ under the gauge transformation with the parameter in (4.11) are

$$\begin{aligned}
\delta f_\kappa &= \lambda_f' + 2\,i\,(\bar{g}_\kappa\,\lambda_g - \bar{\lambda}_{\bar{g}}\,g_\kappa) + 2\,i\,\alpha^2\,(\bar{\eta}_{\bar{g}}\,g_\omega - \eta_g\,\bar{g}_\omega) + 16\,i\,\alpha^2\,(\eta_h\,\bar{h}_\omega - \bar{\eta}_{\bar{h}}\,h_\omega)\\
\delta g_\kappa &= \lambda_g' + i\,(\lambda_g\,f_\kappa - \lambda_f\,g_\kappa) + 2\,i\,\alpha^2\,(\eta_f\,g_\omega - \eta_g\,f_\omega) + 4\,i\,\alpha^2\,(\eta_h\,\bar{g}_\omega - \bar{\eta}_{\bar{g}}\,h_\omega)\\
\delta \bar{g}_\kappa &= \bar{\lambda}_{\bar{g}}' + i\,(\lambda_f\,\bar{g}_\kappa - \bar{\lambda}_{\bar{g}}\,f_\kappa) + 2\,i\,\alpha^2\,(\bar{\eta}_{\bar{g}}\,f_\omega - \eta_f\,\bar{g}_\omega) + 4\,i\,\alpha^2\,(\eta_g\,\bar{h}_\omega - \bar{\eta}_{\bar{h}}\,g_\omega)\\
\delta f_\omega &= \eta_f' + 3i\,(\eta_g\,\bar{g}_\kappa - \bar{\eta}_{\bar{g}}\,g_\kappa) + 3i\,(\lambda_g\,\bar{g}_\omega - \bar{\lambda}_{\bar{g}} g_\omega)\\
\delta g_\omega &= \eta_g' + i\,(\eta_g\,f_\kappa - \lambda_f\,g_\omega) + 2i\,(\lambda_g\,f_\omega - \eta_f\,g_\kappa) + 4i\,(\eta_h\,\bar{g}_\kappa - \bar{\lambda}_{\bar{g}}\,h_\omega)\\
\delta \bar{g}_\omega &= \bar{\eta}_{\bar{g}}' + i\,(\lambda_f\,\bar{g}_\omega - \bar{\eta}_{\bar{g}}\,f_\kappa) + 2i\,(\eta_f\,\bar{g}_\kappa - \bar{\lambda}_{\bar{g}}\,f_\omega) + 4i\,(\lambda_g\,\bar{h}_\omega - \bar{\eta}_{\bar{h}}\,g_\kappa)\\
\delta h_\omega &= \eta_h' + i(\lambda_g\,g_\omega - \eta_g\,g_\kappa) + 2i\,(\eta_h\,f_\kappa - \lambda_f\,h_\omega)\\
\delta \bar{h}_\omega &= \bar{\eta}_{\bar{h}}' + i\,(\bar{\eta}_{\bar{g}}\,\bar{g}_\kappa - \bar{\lambda}_{\bar{g}}\,\bar{g}_\omega) + 2i\,(\lambda_f\,\bar{h}_\omega - \bar{\eta}_{\bar{h}}\,f_\kappa)
\end{aligned} \tag{C.5}$$

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