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Type IIB supergravity solution for the T-dual of the η-deformed $AdS_5 \times S^5$ superstring

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Abstract: We find an exact type IIB supergravity solution that represents a one-parameter deformation of the T-dual of the $AdS_5 \times S^5$ background (with T-duality applied in all 6 abelian bosonic isometric directions). The non-trivial fields are the metric, dilaton and RR 5-form only. The latter has remarkably simple "undeformed" form when written in terms of a "deformation-rotated" vielbein basis. An unusual feature of this solution is that the dilaton contains a linear dependence on the isometric coordinates of the metric precluding a straightforward reversal of T-duality. If we still formally dualize back, we find exactly the metric, B-field and product of dilaton with RR field strengths as recently extracted from the η-deformed $AdS_5 \times S^5$ superstring action in arXiv:1507.04239. We also discuss similar solutions for deformed $AdS_n \times S^n$ backgrounds with $n = 2, 3$. In the $\eta \to i$ limit we demonstrate that all these backgrounds can be interpreted as special limits of gauged WZW models and are also related to (a limit of) the Pohlmeyer-reduced models of the $AdS_n \times S^n$ superstrings.

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Contents

1 Introduction

Finding the supergravity background corresponding to the η-deformation of $AdS_5 \times S^5$ superstring sigma model [1–4] ("η-model") has turned out to be a non-trivial problem. The corresponding metric and B-field were read off the superstring action in [5]. It was then found that for low-dimensional analogs [6] of the deformed metric, corresponding to deformations of the $AdS_2 \times S^2$ and $AdS_3 \times S^3$ sigma models, it is possible [7] to find special combinations of RR fluxes and the dilaton that complete the metrics to full type IIB supergravity solutions. However, it was noticed that there may be many dilaton/flux backgrounds supporting the same metric and it was not checked that the solutions that were found (with particular non-factorized dilatons) correspond to the η-deformed $AdS_2 \times S^2$ and $AdS_3 \times S^3$ superstring sigma models. Very recently, the RR background that follows directly from quadratic fermionic term in the η-deformed $AdS_5 \times S^5$ sigma model was finally found in [8] but surprisingly it was found that the resulting metric, B-field and RR fluxes cannot be supported by a dilaton to promote them to a consistent type IIB supergravity solution.

In a parallel development, a one-parameter deformation of the non-abelian dual of the $AdS_5 \times S^5$ superstring sigma model was constructed [9, 10] (generalizing the bosonic models of [11]). This "λ-model" is closely connected (via an analytic continuation) to the η-model at the classical phase space level (the associated Poisson bracket algebras are effectively isomorphic [12]). Furthermore, it was found in [13] that the metric of the η-model can be obtained from the metric of the λ-model by a certain singular limit (involving infinite shifts of coordinates corresponding to Cartan directions of the original symmetry group) and an analytic continuation relating the two deformation parameters ($\eta = i\frac{1-\lambda}{1+\lambda}$).

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More precisely, the metric that originated from this singular limit of the λ-model metric was not the η-deformed $AdS_5 \times S^5$ metric itself but its T-dual with respect to all 6 isometric directions associated to Cartan generators of $\mathrm{SO}(2,4) \times \mathrm{SO}(6)$. The reason for this can be traced to the fact that the λ-deformation was applied to the non-abelian T-dual of the $AdS_5 \times S^5$ coset model and performing the non-abelian duality implies dualizing with respect to the whole symmetry group. Applying the singular limit gives preference to the Cartan directions, such that it should produce a deformation of the abelian T-dual of the $AdS_5 \times S^5$ model [13]. This observation [13] of the special role of the T-dual of the deformed $AdS_5 \times S^5$ model turns out to be crucial in what follows.

Guided by the existence of a supergravity solution that supports the metric of the $AdS_5 \times S^5$ λ-model by a particularly simple (factorized) dilaton and just the RR 5-form flux [14, 15], in [13] we applied the above singular limit to its $AdS_2 \times S^2$ counterpart and found a new supergravity solution (different to the T-dual of the solution found in [7]) that supports the T-dual of the η-deformed $AdS_2 \times S^2$ metric by a RR 2-form flux and factorized dilaton. A peculiar feature of this solution was that the dilaton contained a term linear in the two isometric coordinates of the metric. This precluded us from applying the standard rules to reverse the T-duality and find the supergravity background supporting the original η-deformation of $AdS_2 \times S^2$ metric.

Motivated by the observations of [13], here we directly construct similar type IIB supergravity solutions supporting the T-duals of the η-deformations of the $AdS_3 \times S^3$ and $AdS_5 \times S^5$ metrics. Again, the resulting dilatons contain terms linear in (some of) the isometric coordinates, which precludes us from undoing the T-duality and thus finding similar solutions supporting the η-deformed $AdS_3 \times S^3$ and $AdS_5 \times S^5$ metrics themselves.

The solution we find for the T-dual of the η-deformed $AdS_5 \times S^5$ metric contains only the dilaton Φ and the RR 5-form flux F_5. Surprisingly, if we formally apply the standard T-duality rules [16–21] to this background, we obtain the metric, B-field *and* precisely the product of the mixed RR fluxes with the dilaton, $e^{\Phi} F_n$, as found directly from the η-deformed $AdS_5 \times S^5$ sigma model action in [5, 8]. Since this T-dualization can be done explicitly at the level of the classical string action (ignoring the issue of the quantum dilaton shift) this supports the idea that the solution we find is the one associated with the η-model.

This also explains the conclusion of [8] that the background extracted from the η-model action cannot be promoted to a supergravity solution for any choice of the dilaton. Indeed, the usual expectation that T-duality should map from one supergravity solution to another does not apply in cases in which the dilaton explicitly depends on the isometries of the metric — the new T-dual dilaton will depend on the original isometric coordinates, while the dual metric and other fields will describe their dual analogs. One might attempt to interpret the resulting background as a solution of some "doubled" version of type IIB string theory where both the original and dual coordinates are treated on an equal footing [22, 23], or, possibly, of "doubled" type IIB supergravity [24–27] but with the strong constraint relaxed.[1] An alternative is to search for a T-dual solution where the

[1] The usual discussions of "doubled" field theory assume the weak $\partial_i \tilde{\partial}^i X = 0$ as well as the strong constraint $\partial_i X \tilde{\partial}^i Y = 0$ for any two fields X, Y (cf., however, [28]). The former is satisfied in our case while the latter is not as we have $\tilde{\partial}^i \Phi \neq 0$ for the dilaton or RR flux while $\partial_i g \neq 0$ for the metric.

dilaton depends linearly on the same dual coordinates that appear in the metric, i.e. to map the "momentum" mode of the dilaton into the "winding" one. We leave an investigation of this idea for the future.

To sum up, here we will show that, while it is presently still unclear how to directly interpret the background found [5, 8] from the $AdS_5 \times S^5$ η-model as a type IIB supergravity solution, the background formally related to it by T-duality in all 6 isometric directions can indeed be promoted to an exact supergravity solution by properly adjusting the dilaton (in particular, adding terms linear in some isometric coordinates). We shall also provide another interpretation of these linear dilaton terms in the special $\eta \to i$ limit by showing that they appear from the dilaton of the standard gauged WZW model upon taking a special limit required to obtain the η-deformed metric as in [13].

We shall start in section 2.1 with a review of the solution of type IIB supergravity for the T-dual of the η-deformed $AdS_2 \times S^2$ metric supported by a factorized, non-isometric dilaton and just a single imaginary RR 2-form flux (originating from the 5-form in 10d supergravity upon compactification on T^6) [13]. Then in sections 2.2 and 2.3 we shall construct the analogous backgrounds in the $AdS_3 \times S^3$ and $AdS_5 \times S^5$ cases. These solutions will possess the same features, i.e. they will be supporting the T-dual of the η-deformed metric with a factorized, non-isometric dilaton and just a single imaginary RR 3-form flux for $AdS_3 \times S^3$ and self-dual 5-form flux for $AdS_5 \times S^5$. In the $AdS_5 \times S^5$ case we shall explicitly check (in appendix A) that after formally T-dualizing along the isometric directions of the metric we recover the background fields (metric, B-field and $e^\Phi F$) of the supercoset η-model of [1, 2], which were found in [8].

Furthermore, these backgrounds should appear as limits of the λ-model backgrounds constructed in [14] and [15]. In section 3.1 we provide evidence for this in the direct $\eta \to i$ or, equivalently, $\varkappa \equiv \frac{2\eta}{1-\eta^2} \to i$ limit, in which the RR fluxes vanish. In section 3.2 we shall consider a refined $\varkappa \to i$ limit in which one also rescales the "longitudinal" coordinates, resulting in a pp-wave background [6, 13], which is related to the Pohlmeyer-reduced model. Some concluding remarks will be made in section 4.

2 Supergravity backgrounds for T-duals to η-deformed $AdS_n \times S^n$ models

We shall consider the deformed models for $AdS_n \times S^n$ with $n = 2, 3, 5$ in parallel. The undeformed $AdS_5 \times S^5$ metric is a solution of type IIB supergravity with constant dilaton and homogeneous F_5 flux. Applying T-duality in all 3+3 isometric directions we formally arrive at another supergravity solution with non-constant dilaton and (since T-duality is applied in the time direction [29]) an *imaginary* 5-form flux. Similarly, starting with the $AdS_2 \times S^2$ ($AdS_3 \times S^3$) solution of type II supergravity compactified on T^6 (T^4) we again find a solution supported by a non-trivial dilaton and imaginary 2-form (3-form) flux in the effective 4d (6d) supergravity.

One may then look for solutions which represent deformations of these T-dual $AdS_n \times S^n$ backgrounds, i.e. such that their metrics are the same as T-duals of the η-deformed

$AdS_n \times S^n$ metrics in [5, 6]. As in [5, 6, 13] we shall use

$$\varkappa \equiv \frac{2\eta}{1-\eta^2} \tag{2.1}$$

as the deformation parameter in the supergravity fields. The minimal assumption is that such solutions should be again supported just by a dilaton and a single (imaginary) RR n-form. Indeed, we shall find such solutions but, as discussed in the Introduction, we will be unable to dualize back to get real backgrounds due to linear terms in the dilaton present for $\varkappa \neq 0$, which break the isometries of the undeformed background.

2.1 $AdS_2 \times S^2$

Let us start by reviewing the supergravity solutions for the η-model in the $AdS_2 \times S^2$ case found in [13], which were constructed by taking limits of the λ-model backgrounds presented in [14].

Here we compactify 10d type IIB supergravity on T^6 to four dimensions retaining the metric, dilaton and a single RR 2-form flux. The field content of the corresponding truncation of the 10d supergravity is given by the metric, dilaton and self-dual RR 5-form, which is built from the 2-form flux F and the holomorphic 3-form on T^6, see, for example, [7, 30]. The resulting bosonic 4d action is then given by

$$\mathcal{S}_2 = \int d^4x\,\sqrt{-g}\Big[e^{-2\Phi}\left[R + 4(\nabla\Phi)^2\right] - \frac{1}{4}F_{mn}F^{mn}\Big]\,. \tag{2.2}$$

The corresponding equations of motion and Bianchi identities are $(m, n, \ldots = 0, 1, 2, 3)$

$$\begin{aligned} R + 4\nabla^2\Phi - 4(\nabla\Phi)^2 = 0\,, \qquad & R_{mn} + 2\nabla_m\nabla_n\Phi = \frac{e^{2\Phi}}{2}\left(F_{mp}F_n{}^p - \frac{1}{4}g_{mn}F^2\right) \\ \partial_n\left(\sqrt{-g}F^{mn}\right) = 0\,, \qquad & \partial_{[p}F_{mn]} = 0\,. \end{aligned} \tag{2.3}$$

The first two equations imply that the dilaton should satisfy $\nabla^2 e^{-2\Phi} = 0$.

The metric of the η-model in this case is given by [5, 6]

$$d\hat{s}_2^2 = -\frac{1+\rho^2}{1-\varkappa^2\rho^2}d\hat{t}^2 + \frac{d\rho^2}{(1-\varkappa^2\rho^2)(1+\rho^2)} + \frac{1-r^2}{1+\varkappa^2 r^2}d\hat{\varphi}^2 + \frac{dr^2}{(1+\varkappa^2 r^2)(1-r^2)}\,, \tag{2.4}$$

which has a $\mathrm{U}(1)^2$ isometry,[2] corresponding to the rank of $P\mathrm{SU}(1,1|2)$, given by shifts in $\hat{t}$ and $\hat{\varphi}$. We use hats to denote isometries that we will T-dualize; the corresponding dual coordinates will have no hats.

In [13] two solutions of the equations (2.3) that support the metric (2.4), up to T-dualities, were given. Here we will mainly consider the solution corresponding to the metric (2.4) T-dualized in both isometries (the target-space indices $m, n, \ldots = 0, 1, 2, 3$

[2] Here and below we shall formally refer to abelian isometries corresponding to translations in some direction as "U(1) isometries", i.e. we will not distinguish between compact and non-compact isometries.

correspond to t, ρ, φ, r)[3]

$$ds_2^2 = \eta_{ab} e^a e^b \,, \qquad e^0 = \frac{\sqrt{1-\varkappa^2\rho^2}}{\sqrt{1+\rho^2}} dt \,, \qquad e^1 = \frac{d\rho}{\sqrt{1-\varkappa^2\rho^2}\sqrt{1+\rho^2}} \,,$$
$$e^2 = \frac{\sqrt{1+\varkappa^2 r^2}}{\sqrt{1-r^2}} d\varphi \,, \qquad e^3 = \frac{dr}{\sqrt{1+\varkappa^2 r^2}\sqrt{1-r^2}} \,,$$
$$e^{\Phi_2} F_2 = \frac{\sqrt{2} i \sqrt{1+\varkappa^2}}{\sqrt{1-\varkappa^2\rho^2}\sqrt{1+\varkappa^2 r^2}} \big[(e^0 \wedge e^3 + e^1 \wedge e^2) - \varkappa^2 \rho r (e^0 \wedge e^3 - e^1 \wedge e^2)$$
$$+ \varkappa\rho (e^0 \wedge e^2 + e^1 \wedge e^3) + \varkappa r (e^0 \wedge e^2 - e^1 \wedge e^3) \big] \,,$$
$$e^{\Phi_2} = e^{\Phi_0 - \varkappa(t+\varphi)} \frac{\sqrt{1-\varkappa^2\rho^2}\sqrt{1+\varkappa^2 r^2}}{\sqrt{1+\rho^2}\sqrt{1-r^2}} \,. \tag{2.5}$$

The RR flux actually has a remarkably simple form that can be made explicit by introducing the boosted/rotated zweibein bases

$$\mathrm{e}^t \equiv \frac{1}{\sqrt{1-\varkappa^2\rho^2}} \left(e^0 + \varkappa\rho\, e^1 \right) = \frac{1}{\sqrt{1+\rho^2}} \left(dt + \frac{\varkappa\rho}{1-\varkappa^2\rho^2} d\rho \right) \,,$$
$$\mathrm{e}^\rho \equiv \frac{1}{\sqrt{1-\varkappa^2\rho^2}} \left(e^1 + \varkappa\rho\, e^0 \right) = \frac{1}{\sqrt{1+\rho^2}} \left(\frac{1}{1-\varkappa^2\rho^2} d\rho + \varkappa\rho\, dt \right) \,,$$
$$\mathrm{e}^\varphi \equiv \frac{1}{\sqrt{1+\varkappa^2 r^2}} \left(e^2 - \varkappa r\, e^3 \right) = \frac{1}{\sqrt{1-r^2}} \left(d\varphi - \frac{\varkappa r}{1+\varkappa^2 r^2} dr \right) \,,$$
$$\mathrm{e}^r \equiv \frac{1}{\sqrt{1+\varkappa^2 r^2}} \left(e^3 + \varkappa r\, e^2 \right) = \frac{1}{\sqrt{1-r^2}} \left(\frac{1}{1+\varkappa^2 r^2} dr + \varkappa r\, d\varphi \right) \,, \tag{2.6}$$

such that

$$ds_2^2 = \eta_{ab} e'^a e'^b \,, \qquad e^{\Phi_2} F_2 = \sqrt{2} i \sqrt{1+\varkappa^2} \left(e'^0 \wedge e'^3 + e'^1 \wedge e'^2 \right) \,,$$
$$e'^0 = \mathrm{e}^t \,, \qquad e'^1 = \mathrm{e}^\rho \,, \qquad e'^2 = \mathrm{e}^\varphi \,, \qquad e'^3 = \mathrm{e}^r \,. \tag{2.7}$$

Thus the $\varkappa$-deformation is a "rotation" that preserves the structure of the undeformed background; it only affects the definition of the tangent basis (and dilaton). In particular, in this basis $e^{\Phi_2} F_2$ remains constant and is just rescaled by a factor of $\sqrt{1+\varkappa^2}$.[4] Furthermore, the RR potential C_1 for $F_2 = dC_1$ also takes a simple form in this basis[5]

$$e^{\Phi_2} C_1 = \sqrt{2} i \sqrt{1+\varkappa^{-2}} \left(\frac{e'^3}{\sqrt{1+\rho^2}} - \frac{e'^1}{\sqrt{1-r^2}} \right) \,. \tag{2.8}$$

Some other important features of this solution that are worth noting are:

[3]Let us emphasize that the solution of (2.3) supporting the T-dual to the metric (2.4) is not unique. For example, the solution of the same dilaton equation $\nabla^2 e^{-2\Phi} = 0$ corresponding to the T-dual of the solution found in [7] is different: $e^{\Phi_2'} = \frac{1-\varkappa\rho\, r}{\sqrt{1+\rho^2}\sqrt{1-r^2}}$, i.e. this dilaton is isometric but not factorizable.

[4]As in the undeformed limit, there is actually a one-parameter family of fluxes that solve the supergravity equations (2.3) given by

$$e^{\Phi_2} F_2 = \sqrt{2} i \sqrt{1+\varkappa^2} \left(\mathrm{c}_1\, e'^0 \wedge e'^3 + \mathrm{c}_2\, e'^1 \wedge e'^2 \right) \,, \qquad \mathrm{c}_1^2 + \mathrm{c}_2^2 = 2 \,.$$

[5]Note that the singular term in C_1 appearing in the limit $\varkappa \to 0$ is pure gauge, i.e. $C_1 \to \sqrt{2} i \varkappa^{-1} (dr - d\rho) + \ldots$. One can of course choose an alternative gauge in which C_1 is manifestly regular for $\varkappa \to 0$.

- The background fields entering the classical Green-Schwarz action (the metric and $e^\Phi F_2$) are invariant under the $\mathrm{U}(1)^2$ isometry given by shifts in t and φ. This isometry is broken to (a "null") U(1) in the dilaton by the linear $t+\varphi$ term

- For $\varkappa \in \mathbb{R}$ the metric and dilaton are real, while the RR flux is imaginary.

- For $\varkappa \to 0$ the $\mathrm{U}(1)^2$ isometry is restored in the full background, and we can T-dualize back recovering the standard Bertotti-Robinson solution for $AdS_2 \times S^2$ (i.e. the $\varkappa \to 0$ limit of (2.4)) with constant dilaton and *real* homogeneous 2-form flux which has a factorized "2+2" form.

- For $\varkappa \to \infty$, rescaling the fields and the string tension, we find a non-standard background, i.e. the dilaton still has a linear dependence on the isometric coordinates.

- For $\varkappa \to i$ the flux vanishes, while the t and φ directions become free. We will discuss this limit in more detail in section 3.

The dependence of the dilaton on $t+\varphi$ in (2.5) prohibits one directly T-dualizing in these directions to recover the metric (2.4).[6] Still, it is interesting to note that if we formally T-dualize the metric using the standard rules we will get an additional shift of the dilaton that will cancel the square root factors in e^{Φ_2} in (2.5). One may thus attribute the origin of these factors to T-dualizing from (2.4) to (2.5). Then we get simply

$$\hat{\Phi}_2 = \Phi_0 - \varkappa(t+\varphi) \tag{2.9}$$

as the dilaton associated to (2.4). Note, however, that this dilaton depends on the dual counterparts t and φ of the original coordinates $\hat{t}$ and $\hat{\varphi}$ in (2.4), i.e. the resulting background will not have an immediate interpretation as a standard type IIB supergravity solution.

The other background considered in [13] is for the metric (2.4), i.e. with no T-dualities. This solution is related to (2.5) by the formal map[7]

$$t \to \frac{i\hat{t}}{\varkappa}\,, \qquad \rho \to \frac{i}{\varkappa\rho}\,, \qquad \varphi \to \frac{i\hat{\varphi}}{\varkappa}\,, \qquad r \to \frac{i}{\varkappa r}\,. \tag{2.10}$$

The corresponding dilaton and RR flux are then complex so the interpretation of this solution is unclear. Indeed, the $\varkappa \to 0$ limit of the resulting background represents a non-standard solution — the undeformed $AdS_2 \times S^2$ metric supported by a complex dilaton with a linear dependence on the isometric directions and complex RR flux. It does, however, have a natural $\varkappa \to \infty$ limit if we first use the rescaling

$$\begin{aligned} &\hat{t} \to \varkappa^{-1}\hat{t}\,, && \rho \to \varkappa^{-1}\rho\,, && \hat{\varphi} \to \varkappa^{-1}\hat{\varphi}\,, && r \to \varkappa^{-1} r\,, \\ &ds^2 \to \varkappa^2 ds^2\,, && e^{\Phi_2}F_2 \to \varkappa e^{\Phi_2}F_2\,, && e^{\Phi_2} \to \varkappa^{-2}e^{\Phi_2}\,. && \end{aligned} \tag{2.11}$$

[6] One could still perform T-duality in the orthogonal "null" direction $t-\varphi$, but that appears to give a complicated background with an extra B-field, i.e. it does not bring us back to (2.4).

[7] For $\varkappa^2 \in (0,-1]$, i.e. including the point $\varkappa = i$, the map (2.10) is a real diffeomorphism.

The resulting real background is the "mirror" model one constructed in [31, 32] and is related to a $dS_2 \times H^2$ solution by T-dualities in $\hat{t}$ and $\hat{\varphi}$, with a constant dilaton and imaginary homogeneous RR flux. Thus the parameter region around $\varkappa = \infty$ corresponds to considering the η-model for $dS_2 \times H^2$ with deformation parameter $\widetilde{\varkappa} = \varkappa^{-1}$.

2.2 $AdS_3 \times S^3$

Next, let us consider the background corresponding to the η-model for $AdS_3 \times S^3$. The solution we find can be embedded into 10d type IIB supergravity by compactifying on T^4 to six dimensions retaining only the metric, dilaton and a single RR 3-form flux. The field content of the corresponding 10d supergravity solution will also be given by the metric, dilaton and a RR 3-form flux. The resulting truncated bosonic 6d action is then given by

$$\mathcal{S}_6 = \int d^6x\, \sqrt{-g}\Big[e^{-2\Phi}\big[R + 4(\nabla\Phi)^2\big] - \frac{1}{12}F_{mnp}F^{mnp}\Big] \,. \tag{2.12}$$

The corresponding equations of motion and Bianchi identities are $(m, n, \ldots = 0, 1, 2, 3, 4, 5)$

$$\begin{aligned} R + 4\nabla^2\Phi - 4(\nabla\Phi)^2 = 0\,, \qquad & R_{mn} + 2\nabla_m\nabla_n\Phi = \frac{e^{2\Phi}}{4}\Big(F_{mpq}F_n{}^{pq} - \frac{1}{6}g_{mn}F^2\Big) \\ \partial_p(\sqrt{-g}F^{mnp}) = 0\,, \qquad & \partial_{[q}F_{mnp]} = 0\,. \end{aligned} \tag{2.13}$$

The first two equations imply again that the dilaton should satisfy $\nabla^2 e^{-2\Phi} = 0$.

The metric of the η-model in this case is given by [5, 6]

$$d\hat{s}_3^2 = -\frac{1+\rho^2}{1-\varkappa^2\rho^2}d\hat{t}^2 + \frac{d\rho^2}{(1-\varkappa^2\rho^2)(1+\rho^2)} + \rho^2 d\hat{\psi}_1^2 + \frac{1-r^2}{1+\varkappa^2 r^2}d\hat{\varphi}^2 + \frac{dr^2}{(1+\varkappa^2 r^2)(1-r^2)} + r^2 d\hat{\phi}_1^2\,. \tag{2.14}$$

It has a $U(1)^4$ isometry represented by shifts in $\hat{t}$, $\hat{\psi}_1$, $\hat{\varphi}$ and $\hat{\phi}_1$ (we again use hats to denote isometric directions that we will T-dualize).

As in the $AdS_2 \times S^2$ case, the solution of (2.13) we find is for the metric (2.14) T-dualized in all four isometric directions. The resulting background is (the target-space indices $m, n, \ldots = 0, 1, 2, 3, 4, 5$ correspond to $t, \psi_1, \rho, \varphi, \phi_1, r$)

$$\begin{aligned} ds_3^2 = \eta_{ab}e^a e^b\,, \qquad & e^0 = \frac{\sqrt{1-\varkappa^2\rho^2}}{\sqrt{1+\rho^2}}dt\,, \qquad e^1 = \frac{d\psi_1}{\rho}\,, \qquad e^2 = \frac{d\rho}{\sqrt{1-\varkappa^2\rho^2}\sqrt{1+\rho^2}}\,, \\ & e^3 = \frac{\sqrt{1+\varkappa^2 r^2}}{\sqrt{1-r^2}}d\varphi\,, \qquad e^4 = \frac{d\phi_1}{r}\,, \qquad e^5 = \frac{dr}{\sqrt{1+\varkappa^2 r^2}\sqrt{1-r^2}}\,, \\ e^{\Phi_3}F_3 = {} & \frac{2i\sqrt{1+\varkappa^2}}{\sqrt{1-\varkappa^2\rho^2}\sqrt{1+\varkappa^2 r^2}}\big[(e^0\wedge e^1\wedge e^5 + e^2\wedge e^3\wedge e^4) + \varkappa^2\rho r(e^0\wedge e^4\wedge e^5 - e^1\wedge e^2\wedge e^3) \\ & + \varkappa\rho(e^0\wedge e^3\wedge e^4 - e^1\wedge e^2\wedge e^5) + \varkappa r(e^0\wedge e^1\wedge e^3 + e^2\wedge e^4\wedge e^5)\big]\,, \\ e^{\Phi_3} = {} & e^{\Phi_0 - 2\varkappa(t+\varphi)}\frac{(1-\varkappa^2\rho^2)(1+\varkappa^2 r^2)}{\rho r\sqrt{1+\rho^2}\sqrt{1-r^2}}\,. \end{aligned} \tag{2.15}$$

As in the $AdS_2 \times S^2$ case in (2.5), (2.7) the RR flux takes very simple form when written in terms of the "deformed" basis introduced in (2.6)

$$\begin{aligned} &ds_3^2 = \eta_{ab} e'^a e'^b \,, \qquad\qquad e^{\Phi_3} F_3 = 2i\sqrt{1+\varkappa^2}\,\left(e'^0\wedge e'^1\wedge e'^5 + e'^2\wedge e'^3\wedge e'^4\right), \\ &e'^0 = \mathrm{e}^t\,, \quad e'^1 = \mathrm{e}^{\psi_1}\equiv e^1\,, \quad e'^2 = \mathrm{e}^{\rho}\,, \quad e'^3 = \mathrm{e}^{\varphi}\,, \quad e'^4 = \mathrm{e}^{\phi_1}\equiv e^4\,, \quad e'^5 = \mathrm{e}^r\,. \end{aligned} \tag{2.16}$$

In this basis we again see that the $\varkappa$-deformation preserves the structure of the undeformed background; $e^{\Phi_3}F_3$ remains constant and is just rescaled by a factor of $\sqrt{1+\varkappa^2}$.[8] Furthermore, the RR potential C_2 for $F_3 = dC_2$ again takes a simple form in this basis (cf. (2.8))

$$e^{\Phi_3} C_2 = i\sqrt{1+\varkappa^{-2}}\left(\frac{e'^1\wedge e'^5}{\sqrt{1+\rho^2}} - \frac{e'^2\wedge e'^4}{\sqrt{1-r^2}}\right). \tag{2.17}$$

Some other important features of this solution are:

- The background fields entering the classical Green-Schwarz action (the metric and $e^{\Phi}F$) are invariant under the $\mathrm{U}(1)^4$ isometry given by shifts in t, ψ_1, φ and ϕ_1. This isometry is broken to $\mathrm{U}(1)^3$ in the dilaton by the linear $t+\varphi$ term.
- For $\varkappa \in \mathbb{R}$ the metric and dilaton are real, while the RR flux is imaginary.
- For $\varkappa \to 0$ the $\mathrm{U}(1)^4$ isometry is restored in the full background, and T-dualizing in all four isometries we recover the standard solution for undeformed $AdS_3 \times S^3$ with constant dilaton and real homogeneous RR flux.
- For $\varkappa \to \infty$, rescaling the fields and the string tension, we find a non-standard background, i.e. the dilaton still has a linear dependence on the isometric coordinates.
- For $\varkappa \to i$ the 3-form flux vanishes, while the t and φ directions become free in the metric. We will discuss this limit in more detail in section 3.

As in the $AdS_2 \times S^2$ case we may formally T-dualize the background (2.15) in all four isometries of the metric to recover the metric in (2.14). The resulting dilaton will then be (cf. (2.9))

$$e^{\hat{\Phi}_3} = e^{\Phi_0 - 2\varkappa(t+\varphi)}\sqrt{1-\varkappa^2\rho^2}\sqrt{1+\varkappa^2 r^2}\,, \tag{2.18}$$

i.e. linear in the dual coordinates and constant in the $\varkappa \to 0$ limit.

[8] As written, the 3-form flux satisfies the self-duality equation $F_{3mnp} = \frac{1}{3!}\sqrt{-g}\,\epsilon_{mnpqrs}F_3^{qrs}$, $\epsilon_{012345} = -1$. As in the $AdS_2 \times S^2$ case, there is actually a one-parameter family of fluxes that solve the supergravity equations (2.13) given by

$$e^{\Phi_3}F_3 = 2i\sqrt{1+\varkappa^2}\,\left(\mathrm{c}_1\, e'^0\wedge e'^1\wedge e'^5 + \mathrm{c}_2\, e'^2\wedge e'^3\wedge e'^4\right), \qquad \mathrm{c}_1^2 + \mathrm{c}_2^2 = 2\,.$$

Therefore, with an appropriate choice of $\mathrm{c}_{1,2}$, the sign of the self-duality equation can be reversed, for example, taking $\mathrm{c}_1 = -1$, $\mathrm{c}_2 = 1$ ($\mathrm{c}_1 = 1$, $\mathrm{c}_2 = -1$). This can also be understood as reversing the sign of the isometric coordinate ψ_1 (ϕ_1). Indeed, reversing the sign of an odd number of the isometries of the metric will reverse the sign of the self-duality equation. However, reversing the sign of t or φ will modify the dilaton as well as the 3-form flux.

Again, there is a second solution we can find — directly for the metric (2.14) with no T-dualities applied. This solution is related to (2.15) by the formal map[9]

$$t \to \frac{i\hat{t}}{\varkappa}\,, \qquad \rho \to \frac{i}{\varkappa\rho}\,, \qquad \psi_1 \to \frac{i\hat{\psi}_1}{\varkappa}\,, \qquad \varphi \to \frac{i\hat{\varphi}}{\varkappa}\,, \qquad r \to \frac{i}{\varkappa r}\,, \qquad \phi_1 \to \frac{i\hat{\phi}_1}{\varkappa}\,. \tag{2.19}$$

However, the $\varkappa \to 0$ limit of the resulting background gives a non-standard solution — the undeformed $AdS_3 \times S^3$ metric supported by a complex dilaton (with a linear dependence on the isometric directions) and complex RR flux. But it does have a natural $\varkappa \to \infty$ limit if we first use the rescaling

$$\begin{aligned} \hat{t} &\to \varkappa^{-1}\hat{t}\,, \qquad & \rho &\to \varkappa^{-1}\rho\,, \qquad & \hat{\varphi} &\to \varkappa^{-1}\hat{\varphi}\,, \qquad r \to \varkappa^{-1} r\,, \\ ds^2 &\to \varkappa^2 ds^2\,, \qquad & e^{\Phi_3}F_3 &\to \varkappa^2 e^{\Phi_3}F_3\,, \qquad & e^{\Phi_3} &\to \varkappa^{-4}e^{\Phi_3}\,, \end{aligned} \tag{2.20}$$

along with a shift of the constant part of the dilaton Φ_0 by $\frac{i\pi}{2}$. The resulting real background is the "mirror" model one [31, 32] and is related to a $dS_3 \times H^3$ background by T-dualities in $\hat{t}$ and $\hat{\varphi}$, with a constant dilaton and imaginary homogeneous RR flux (as could be expected). In this sense expanding around $\varkappa = \infty$ corresponds to considering the η-model for $dS_3 \times H^3$ with deformation parameter $\widetilde{\varkappa} = \varkappa^{-1}$.

2.3 $AdS_5 \times S^5$

Let us now turn to the case of the η-deformed $AdS_5 \times S^5$. The background that we find can be interpreted as a deformation of the T-dual of $AdS_5 \times S^5$ and is a solution of 10d type IIB supergravity with just the metric, dilaton and RR 5-form flux switched on.[10]

The relevant part of the type II 10d supergravity action is then given by

$$\mathcal{S}_{10} = \int d^{10}x\,\sqrt{-g}\Big[e^{-2\Phi}\big[R + 4(\nabla\Phi)^2\big] - \frac{1}{4\cdot 5!}F_{mnpqr}F^{mnpqr}\Big]\,. \tag{2.21}$$

The corresponding equations of motion and Bianchi identities are $(m, n, \ldots = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9)$

$$\begin{aligned} &R + 4\nabla^2\Phi - 4(\nabla\Phi)^2 = 0\,, \qquad && R_{mn} + 2\nabla_m\nabla_n\Phi = \frac{e^{2\Phi}}{4\cdot 4!}F_{mpqrs}F_n{}^{pqrs} \\ &\partial_r(\sqrt{-g}F^{mnpqr}) = 0\,, \quad \partial_{[s}F_{mnpqr]} = 0\,, \qquad && F_{mnpqr} = -\frac{1}{5!}\sqrt{-g}\,\epsilon_{mnpqrstuvw}F^{stuvw}\,, \end{aligned} \tag{2.22}$$

where we have also included the self-duality equation ($\epsilon_{0123456789} = -1$) for the RR 5-form flux, which needs to be imposed separately. Again, the first two equations imply that the dilaton should satisfy $\nabla^2 e^{-2\Phi} = 0$.

[9] If we also analytically continue $\varkappa$ to the region $\varkappa^2 \in (0, -1]$ including the point $\varkappa = i$ then the map (2.19) is a real diffeomorphism.

[10] Applying T-duality in all 6 isometric directions of undeformed $AdS_5 \times S^5$ metric one gets a formal type IIB solution with T-dual metric supported by non-constant dilaton and (imaginary) 5-form flux. The reason for finding only non-zero RR 5-form flux can be understood heuristically by observing that T-duality is applied to the time and 2 longitudinal directions (angles of S^3) of the D3-branes as well as 3 transverse directions (3 angles of the transverse S^5) with the longitudinal and transverse directions interchanging their roles.

The metric and B-field corresponding to the η-model in this case are [5] (as before, we use hats to denote isometries that we will T-dualize)

$$\begin{aligned} d\hat{s}_5^2 = & -\frac{1+\rho^2}{1-\varkappa^2\rho^2}d\hat{t}^2 + \frac{d\rho^2}{(1-\varkappa^2\rho^2)(1+\rho^2)} + \frac{\rho^2\cos^2\zeta}{1+\varkappa^2\rho^4\sin^2\zeta}d\hat{\psi}_1^2 + \frac{d\zeta^2}{1+\varkappa^2\rho^4\sin^2\zeta} + \rho^2\sin^2\zeta d\hat{\psi}_2^2 \\ & + \frac{1-r^2}{1+\varkappa^2r^2}d\hat{\varphi}^2 + \frac{dr^2}{(1+\varkappa^2r^2)(1-r^2)} + \frac{r^2\cos^2\xi}{1+\varkappa^2r^4\sin^2\xi}d\hat{\phi}_1^2 + \frac{d\xi^2}{1+\varkappa^2r^4\sin^2\xi} + r^2\sin^2\xi d\hat{\phi}_2^2, \\ \hat{B} = & \frac{\varkappa\rho^4\sin\zeta\cos\zeta}{1+\varkappa^2\rho^4\sin^2\zeta}d\hat{\psi}_1\wedge d\zeta - \frac{\varkappa r^4\sin\xi\cos\xi}{1+\varkappa^2r^4\sin^2\xi}d\hat{\phi}_1\wedge d\xi \ . \end{aligned} \tag{2.23}$$

Both have a $U(1)^6$ isometry (corresponding to the Cartan directions of the bosonic subgroup of the undeformed $PSU(2,2|4)$ symmetry) represented by shifts in $\hat{t}$, $\hat{\psi}_1$, $\hat{\psi}_2$, $\hat{\varphi}$, $\hat{\phi}_1$ and $\hat{\phi}_2$.

The type IIB solution supporting *that* metric is not known (cf. [8]) but as in the lower-dimensional examples above we will find a consistent solution that supports the fully T-dual metric and B-field, i.e. the background (2.23) T-dualized in all six isometries

$$\begin{aligned} ds_5^2 = & -\frac{1-\varkappa^2\rho^2}{1+\rho^2}dt^2 + \frac{d\rho^2}{(1+\rho^2)(1-\varkappa^2\rho^2)} \\ & + \frac{1+\varkappa^2\rho^4\sin^2\zeta}{\rho^2\cos^2\zeta}\left(d\psi_1 + \frac{\varkappa\rho^4\sin\zeta\cos\zeta}{1+\varkappa^2\rho^4\sin^2\zeta}d\zeta\right)^2 + \frac{\rho^2 d\zeta^2}{1+\varkappa^2\rho^4\sin^2\zeta} + \frac{d\psi_2^2}{\rho^2\sin^2\zeta} \\ & + \frac{1+\varkappa^2r^2}{1-r^2}d\varphi^2 + \frac{dr^2}{(1-r^2)(1+\varkappa^2r^2)} \\ & + \frac{1+\varkappa^2r^4\sin^2\xi}{r^2\cos^2\xi}\left(d\phi_1 - \frac{\varkappa r^4\sin\xi\cos\xi}{1+\varkappa^2r^4\sin^2\xi}d\xi\right)^2 + \frac{r^2d\xi^2}{1+\varkappa^2r^4\sin^2\xi} + \frac{d\phi_2^2}{r^2\sin^2\xi} \\ = & -\frac{1-\varkappa^2\rho^2}{1+\rho^2}dt^2 + \frac{d\rho^2}{(1+\rho^2)(1-\varkappa^2\rho^2)} + \frac{d\psi_1^2}{\rho^2\cos^2\zeta} + (\rho\, d\zeta + \varkappa\rho\tan\zeta\, d\psi_1)^2 + \frac{d\psi_2^2}{\rho^2\sin^2\zeta} \\ & + \frac{1+\varkappa^2r^2}{1-r^2}d\varphi^2 + \frac{dr^2}{(1-r^2)(1+\varkappa^2r^2)} + \frac{d\phi_1^2}{r^2\cos^2\xi} + (r\, d\xi - \varkappa r\tan\xi\, d\phi_1)^2 + \frac{d\phi_2^2}{r^2\sin^2\xi}\,, \end{aligned} \tag{2.24}$$

$$B = 0\ . \tag{2.25}$$

Here the T-dualities in $\hat{\psi}_1$ and $\hat{\phi}_1$ removed the B-field at the expense of introducing off-diagonal terms in the metric as in [7]. The T-duality conventions we use are given in appendix A.

In (2.24) we have presented two forms of the dualized metric; the first is the one that arises naturally from the T-duality procedure [7], while the second has a particularly simple form, which will be useful later.

Introducing the shorthand notation $E^{abcde} \equiv e^a\wedge e^b\wedge e^c\wedge e^d\wedge e^e$, and taking the target-space indices $m, n, \ldots = 0,1,2,3,4,5,6,7,8,9$ to correspond to $t, \psi_2, \psi_1, \zeta, \rho, \varphi, \phi_2, \phi_1, \xi, r$, the solution of (2.22) we found is

$$\begin{aligned} & ds_5^2 = \eta_{ab}e^ae^b\,, \qquad e^0 = \frac{\sqrt{1-\varkappa^2\rho^2}}{\sqrt{1+\rho^2}}dt\,, \qquad e^1 = \frac{d\psi_2}{\rho\sin\zeta}\,, \\ & e^2 = \frac{\sqrt{1+\varkappa^2\rho^4\sin^2\zeta}}{\rho\cos\zeta}d\psi_1 + \frac{\varkappa\rho^3\sin\zeta}{\sqrt{1+\varkappa^2\rho^4\sin^2\zeta}}d\zeta\,, \\ & e^3 = \frac{\rho\, d\zeta}{\sqrt{1+\varkappa^2\rho^4\sin^2\zeta}}\,, \qquad e^4 = \frac{d\rho}{\sqrt{1-\varkappa^2\rho^2}\sqrt{1+\rho^2}}\,, \end{aligned}$$

$$e^5 = \frac{\sqrt{1+\varkappa^2 r^2}}{\sqrt{1-r^2}} d\varphi\,, \qquad e^6 = \frac{d\phi_2}{r\sin\xi}\,, \qquad e^7 = \frac{\sqrt{1+\varkappa^2 r^4 \sin^2\xi}}{r\cos\xi} d\phi_1 - \frac{\varkappa r^3 \sin\xi}{\sqrt{1+\varkappa^2 r^4\sin^2\xi}} d\xi\,,$$
$$e^8 = \frac{r\,d\xi}{\sqrt{1+\varkappa^2 r^4 \sin^2\xi}}\,, \qquad e^9 = \frac{dr}{\sqrt{1+\varkappa^2 r^2}\sqrt{1-r^2}}\,,$$

$$e^{\Phi_5} F_5 = \frac{4i\sqrt{1+\varkappa^2}}{\sqrt{1-\varkappa^2\rho^2}\sqrt{1+\varkappa^2\rho^4\sin^2\zeta}\sqrt{1+\varkappa^2 r^2}\sqrt{1+\varkappa^2\rho^4\sin^2\xi}}$$
$$\Big[\big[E^{01289} + E^{34567} - \varkappa\rho(E^{03567} - E^{12489}) - \varkappa r(E^{01258} + E^{34679}) + \varkappa^2\rho r(E^{03679} - E^{12458})\big]$$
$$-\varkappa\rho^2\sin\zeta\big[E^{01389} - E^{24567} + \varkappa\rho(E^{02567} + E^{13489}) - \varkappa r(E^{01358} - E^{24679}) - \varkappa^2\rho r(E^{02679} + E^{13458})\big]$$
$$-\varkappa r^2\sin\xi\big[E^{01279} - E^{34568} + \varkappa\rho(E^{03568} + E^{12479}) - \varkappa r(E^{01257} - E^{34689}) - \varkappa^2\rho r(E^{03689} + E^{12457})\big]$$
$$+\varkappa^2\rho^2 r^2\sin\zeta\sin\xi\big[E^{01379} + E^{24568} - \varkappa\rho(E^{02568} - E^{13479}) - \varkappa r(E^{01357} + E^{24689})$$
$$+\varkappa^2\rho r(E^{02689} - E^{13457})\big]\Big]$$
$$e^{\Phi_5} = e^{\Phi_0 - 4\varkappa(t+\varphi) - 2\varkappa(\psi_1-\phi_1)} \frac{(1-\varkappa^2\rho^2)^2(1+\varkappa^2 r^2)^2}{\rho^2 r^2\sqrt{1+\rho^2}\sqrt{1-r^2}\sin 2\zeta\sin 2\xi}\,. \tag{2.26}$$

Remarkably, as in the lower-dimensional cases in (2.7), (2.16), this complicated-looking RR flux once again takes a very simple form when written in terms of the "deformed" basis in (2.6) and the angular part of the vielbein associated with the second form of the metric in (2.24)

$$ds_5^2 = \eta_{ab} e'^a e'^b\,, \quad e^{\Phi_5} F_5 = 4i\sqrt{1+\varkappa^2}\big(e'^0\wedge e'^1\wedge e'^2\wedge e'^8\wedge e'^9 + e'^3\wedge e'^4\wedge e'^5\wedge e'^6\wedge e'^7\big)\,, \tag{2.27}$$

where

$$e'^0 = \mathrm{e}^t\,, \quad e'^1 = \mathrm{e}^{\psi_2} \equiv e^1\,, \quad e'^2 = \mathrm{e}^{\psi_1}\,, \quad e'^3 = \mathrm{e}^{\zeta}\,, \quad e'^4 = \mathrm{e}^{\rho}\,, \tag{2.28}$$
$$e'^5 = \mathrm{e}^{\varphi}\,, \quad e'^6 = \mathrm{e}^{\phi_2} \equiv e^6\,, \quad e'^7 = \mathrm{e}^{\phi_1}\,, \quad e'^8 = \mathrm{e}^{\xi}\,, \quad e'^9 = \mathrm{e}^{r}\,,$$
$$\mathrm{e}^{\psi_1} \equiv \frac{e^2 - \varkappa\rho^2\sin\zeta\, e^3}{\sqrt{1+\varkappa^2\rho^4\sin^2\zeta}} = \frac{d\psi_1}{\rho\cos\zeta}\,, \qquad \mathrm{e}^{\zeta} \equiv \frac{e^3 + \varkappa\rho^2\sin\zeta\, e^2}{\sqrt{1+\varkappa^2\rho^4\sin^2\zeta}} = \rho d\zeta + \varkappa\rho\tan\zeta\, d\psi_1\,,$$
$$\mathrm{e}^{\phi_1} \equiv \frac{e^7 + \varkappa r^2\sin\xi\, e^8}{\sqrt{1+\varkappa^2 r^4\sin^2\xi}} = \frac{d\phi_1}{r\cos\xi}\,, \qquad \mathrm{e}^{\xi} \equiv \frac{e^8 - \varkappa r^2\sin\xi\, e^7}{\sqrt{1+\varkappa^2 r^4\sin^2\xi}} = r d\xi - \varkappa r\tan\xi\, d\phi_1\,.$$

In this basis we again see that the $\varkappa$-deformation preserves the structure of the undeformed background; $e^{\Phi_5}F_5$ remains constant and is just rescaled by a factor of $\sqrt{1+\varkappa^2}$.[11] Further-

[11] By construction, the 5-form flux satisfies the self-duality equation given in (2.22). If we drop the requirement of self-duality, there are a discrete set of four fluxes that solve the supergravity equations (2.22) given by

$$e^{\Phi_5} F_5 = 4i\sqrt{1+\varkappa^2}\big(\mathrm{c}_1\, e'^0\wedge e'^1\wedge e'^2\wedge e'^8\wedge e'^9 + \mathrm{c}_2\, e'^3\wedge e'^4\wedge e'^5\wedge e'^6\wedge e'^7\big)\,, \qquad \mathrm{c}_1^2 = \mathrm{c}_2^2 = 1\,.$$

This is in contrast to the lower-dimensional cases for which there was a one-parameter family of fluxes. However, as in those cases, with an appropriate choice of $\mathrm{c}_{1,2}$ the sign of the self-duality equation can still be reversed, for example, taking $\mathrm{c}_1 = -1$, $\mathrm{c}_2 = 1$ ($\mathrm{c}_1 = 1$, $\mathrm{c}_2 = -1$). This can also be understood as reversing the sign of the isometric coordinate ψ_2 (ϕ_2). Indeed, reversing the sign of an odd number of the isometries of the metric will reverse the sign of the self-duality equation. However, reversing the sign of t, ψ_1, φ or ϕ_1 will modify the metric and dilaton as well as the 5-form flux.

more, as in the lower-dimensional cases (2.8) and (2.17), the RR potential C_4 for $F_5 = dC_4$ takes a simple form in this basis

$$e^{\Phi_5} C_4 = i\sqrt{1+\varkappa^{-2}}\Big(\frac{e'^1 \wedge e'^2 \wedge e'^8 \wedge e'^9}{\sqrt{1+\rho^2}} + \frac{e'^3 \wedge e'^4 \wedge e'^6 \wedge e'^7}{\sqrt{1-r^2}}\Big) . \tag{2.29}$$

The singular part of C_4 in the $\varkappa \to 0$ limit is again pure gauge.

Some important features of this solution are:

- The background fields entering the classical Green-Schwarz action (the metric and $e^{\Phi} F_5$) are invariant under the $\mathrm{U}(1)^6$ isometry given by shifts in t, ψ_1, ψ_2, φ, ϕ_1 and ϕ_2. This isometry is broken to $\mathrm{U}(1)^5$ by the presence of one linear combination of four of these directions in the dilaton Φ_5, and hence the isometry is also broken in F_5.
- For $\varkappa \in \mathbb{R}$ the metric and dilaton are real, while the RR flux is imaginary (which may be attributed to T-duality in time direction being secretly involved).
- For $\varkappa \to 0$ the $\mathrm{U}(1)^6$ isometry is restored in the full background, and we can T-dualize back to the frame of (2.23). T-dualizing in all six isometries we recover the standard solution for $AdS_5 \times S^5$ (i.e. the $\varkappa \to 0$ limit of (2.23)) with constant dilaton and real homogeneous RR flux.
- For $\varkappa \to \infty$, rescaling the fields and the string tension, we find a non-standard background, i.e. the dilaton still has a linear dependence on the isometric coordinates.
- For $\varkappa \to i$ the RR flux vanishes, while the t and φ directions become free. We will discuss this limit in more detail in section 3.

If we were allowed to T-dualize the background (2.26) in all six isometries of the metric using the standard rules and ignoring the linear terms in the dilaton, we would end up with the metric and B-field in (2.23) and a RR background $e^{\hat{\Phi}_5}\hat{F}_n$, which is precisely the one extracted from the quadratic fermionic term of the deformed supercoset action in [8] (see appendix A). The dilaton formally corrected by the standard factor originating from T-duality will then be (cf. (2.9))

$$e^{\hat{\Phi}_5} = e^{\Phi_0 - 4\varkappa(t+\varphi) - 2\varkappa(\psi_1 - \phi_1)} \Big[\frac{(1-\varkappa^2\rho^2)^3(1+\varkappa^2 r^2)^3}{(1+\varkappa^2\rho^4\sin^2\zeta)(1+\varkappa^2 r^4 \sin^2\xi)}\Big]^{1/2} , \tag{2.30}$$

i.e. will be constant in the $\varkappa \to 0$ limit. Once again, the resulting background will not have an interpretation as a standard type IIB solution as the dilaton (and thus the RR flux) will depend on the original isometric coordinates while the metric will describe their dual counterparts.

As in the lower-dimensional cases there is formally a second solution we can find, which is for the metric and B-field in (2.23) T-dualized *only* in $\hat{\psi}_1$ and $\hat{\phi}_1$ (i.e. obtained by doing only the T-dualities that remove the B-field at the expense of introducing off-diagonal terms in the metric). This T-duality turns out to be necessary — in contrast to the lower-dimensional $AdS_2 \times S^2$ and $AdS_3 \times S^3$ cases, here we still are unable to find (even a formal, complex) type IIB solution supporting directly the original metric and B-field in (2.23). This solution is related to (2.26) by the map

$$t \to \frac{i\hat{t}}{\varkappa}, \quad \rho \to \frac{i}{\varkappa\rho}, \quad \psi_1 \to \psi_1 + \frac{1}{\varkappa}\log\sin\zeta, \quad \zeta \to i\log\tan\frac{\zeta}{2} + \frac{\pi}{2}, \quad \psi_2 \to \frac{i\hat{\psi}_2}{\varkappa},$$
$$\varphi \to \frac{i\hat{\varphi}}{\varkappa}, \quad r \to \frac{i}{\varkappa r}, \quad \phi_1 \to \phi_1 - \frac{1}{\varkappa}\log\sin\xi, \quad \xi \to i\log\tan\frac{\xi}{2} + \frac{\pi}{2}, \quad \phi_2 \to \frac{i\hat{\phi}_2}{\varkappa}. \quad (2.31)$$

The $\varkappa \to 0$ limit of the resulting background is an unfamiliar solution representing the undeformed $AdS_5 \times S^5$ metric supported by a complex dilaton with a linear dependence on the isometric directions and complex RR flux. It does, however, have a natural $\varkappa \to \infty$ limit if we first do the rescaling

$$\begin{aligned} &\hat{t} \to \varkappa^{-1}\hat{t}, && \rho \to \varkappa^{-1}\rho, && \psi_1 \to \varkappa^{-2}\psi_1, \\ &\hat{\varphi} \to \varkappa^{-1}\hat{\varphi}, && r \to \varkappa^{-1}r, && \phi_1 \to \varkappa^{-2}\phi_1, \\ &ds^2 \to \varkappa^2 ds^2, && e^{\Phi_5}F_5 \to \varkappa^4 e^{\Phi_5}F_5, && e^{\Phi_5} \to \varkappa^{-8}e^{\Phi_5}, \end{aligned} \quad (2.32)$$

along with a shift of the constant part of the dilaton Φ_0 by $\frac{i\pi}{2}$. T-dualizing the resulting real background in ψ_1 and ϕ_1 we find the "mirror" model solution constructed in [31, 32], which is furthermore related to a $dS_5 \times H^5$ background (by T-dualities in $\hat{t}$ and $\hat{\varphi}$) with constant dilaton and imaginary homogeneous RR 5-form flux. Thus considering $\varkappa$ in the region around $\varkappa = \infty$ corresponds to the η-model for $dS_5 \times H^5$ with the deformation parameter $\widetilde{\varkappa} = \varkappa^{-1}$.

3 $\varkappa \to i$ limit

In this section we will consider the $\eta \to i$, or, equivalently, $\varkappa \to i$ (cf. (2.1)) limit of the above backgrounds.[12]

This limit can be taken in two different ways. Setting $\varkappa = i$ directly (with all coordinates fixed) one finds that the RR flux vanishes, the two "longitudinal" coordinates t and φ decouple, and the resulting "transverse" metric-dilaton background can be interpreted as corresponding to a special limit of a gauged WZW model.

Alternatively, the limit can be taken by combining $\varkappa \to i$ with a special rescaling of the coordinates $x^\pm = t \mp \varphi$ which leads to a pp-wave type background [6]. As was found in [6], for the $AdS_n \times S^n$ cases with $n = 2, 3$, fixing the light-cone gauge in the string action for the pp-wave background one arrives at the Pohlmeyer-reduced (PR) model [33, 34] for the corresponding undeformed $AdS_n \times S^n$ model.[13] In the $n = 5$ case we shall show that a similar procedure leads to a special limit [13] of the PR model for the $AdS_5 \times S^5$ superstring.

[12] This limit will be considered formally, i.e. the resulting background may be complex and one may need further analytic continuation to get a real solution.

[13] In [6] the corresponding pp-wave RR background for the cases $n = 2, 3$ was reconstructed directly in the pp-wave limit, while here we will derive it from the full deformed solution.

3.1 Direct $\varkappa \to i$ limit and connection with the gauged WZW model

Setting $\varkappa = i$ in the backgrounds (2.5), (2.15) and (2.26) we find the following common structure of the corresponding metric, dilaton and RR fluxes ($n = 2, 3, 5$)

$$ds_n^2 = -dt^2 + d\varphi^2 + ds_{n\perp}^2\,, \quad \Phi_n = \Phi_{n\|} + \Phi_{n\perp}\,, \quad \Phi_{n\|} = -i(n-1)(t+\varphi)\,, \quad F_n = 0\,. \tag{3.1}$$

First, as the RR fluxes in (2.5), (2.15), (2.26) all have an overall factor of $\sqrt{1+\varkappa^2}$ they vanish for $\varkappa = i$. As a result, we get a purely NS-NS metric-dilaton background that must be a solution of the supergravity equations. Second, the "longitudinal" t and φ directions effectively decouple from the remaining transverse directions (they form an $\mathbb{R}^{1,1}$ subspace in the metric). The "null" linear dilaton term $\Phi_{n\|}$ then does not affect the value of the central charge, i.e. one also gets a solution when taking this term with an arbitrary coefficient.[14]

Thus the transverse metric and dilaton should represent a conformal sigma model on their own. Since there is no RR flux and the metric and dilaton have a direct-product $M_A^{n-1} \times M_S^{n-1}$ structure, we should end up with the direct sum of the two transverse conformal models corresponding to the AdS_n and S^n parts of the deformed background, i.e. having $(n-1)$-dimensional target spaces ($n = 2, 3, 5$).[15]

Explicitly, setting $\varkappa = i$ in (2.5) and (2.15) and redefining the "radial" coordinates as

$$\rho \equiv \tan\alpha\,, \qquad r \equiv \tanh\beta\,, \tag{3.2}$$

we find that for $n = 2$ and $n = 3$

$$ds_{2\perp}^2 = d\alpha^2 + d\beta^2\,, \qquad \Phi_{2\perp} = \Phi_0\,, \tag{3.3}$$

$$ds_{3\perp}^2 = d\alpha^2 + \cot^2\alpha\, d\psi_1^2 + d\beta^2 + \coth^2\beta\, d\phi_1^2\,, \qquad e^{\Phi_{3\perp}} = \frac{e^{\Phi_0}}{\sin\alpha\ \sinh\beta}\,. \tag{3.4}$$

Thus in the $AdS_2 \times S^2$ case we find a free transverse theory, while in the $AdS_3 \times S^3$ case it is represented by direct sum of the (vector) gauged WZW models for $G/H = \mathrm{SO}(3)/\mathrm{SO}(2)$ and $\mathrm{SO}(1,2)/\mathrm{SO}(2)$ [35, 36]. In the $n = 5$ case we get from (2.26)

$$ds_{5\perp}^2 = ds_{5A\perp}^2 + ds_{5S\perp}^2\,, \qquad \Phi_{5\perp} = \Phi_0 + \Phi_{5A\perp} + \Phi_{5S\perp}\,, \tag{3.5}$$

$$\begin{aligned} ds_{5A\perp}^2 &= d\alpha^2 + \cot^2\alpha \left(\frac{d\psi_1^2}{\cos^2\zeta} + \frac{d\psi_2^2}{\sin^2\zeta}\right) + \tan^2\alpha\,(d\zeta + i\tan\zeta\, d\psi_1)^2\,, \\ e^{\Phi_{5A\perp}} &= \frac{e^{-2i\psi_1}}{\cos\alpha\ \sin^2\alpha\ \sin 2\zeta}\,, \end{aligned} \tag{3.6}$$

$$\begin{aligned} ds_{5S\perp}^2 &= d\beta^2 + \coth^2\beta \left(\frac{d\phi_1^2}{\cos^2\xi} + \frac{d\phi_2^2}{\sin^2\xi}\right) + \tanh^2\beta\,(d\xi - i\tan\xi\, d\phi_1)^2\,, \\ e^{\Phi_{5S\perp}} &= \frac{e^{2i\phi_1}}{\cosh\beta\ \sinh^2\beta\ \sin 2\xi}\,. \end{aligned} \tag{3.7}$$

[14] This dilaton can be made real by an analytic continuation interchanging the roles of t and φ, i.e. $t = i\varphi'$, $\varphi = it'$.

[15] One can also formally consider the 3d model corresponding to the $n = 4$ case (i.e. $AdS_4 \times S^4$), which can be viewed as a truncation of the $n = 5$ case.

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As for the [SO(3)/SO(2)] × [SO(1, 2)/SO(2)] gauged WZW model in the $n = 3$ case (3.4) above, one can check directly that each of these two 4d metric-dilaton backgrounds satisfies the corresponding equations in (2.22) with the constant shifts of the central charge from the free-theory value, 4, cancelling between the two factors. Note that the metrics and dilatons here can be made real by the analytic continuation $\psi_1 \to i\psi_1$, $\phi_1 \to i\phi_1$.

The gauged WZW interpretation of the two 4d backgrounds in (3.6), (3.7) may seem doubtful at first as here each factor has 2 isometries, while the metrics corresponding to G/H gauged WZW models with non-abelian H should have no isometries. However, the isometries can be effectively generated by taking special singular limits as pointed out in [13]. Indeed, as we shall now explain, these two 4d backgrounds may be viewed as (an analytic continuation of) a singular limit of those associated to the SO(5)/SO(4) and SO(1, 4)/SO(4) gauged WZW models.

Let us start with the metric-dilaton background corresponding to the SO(5)/SO(4) gauged WZW model as given in [37–39][16]

$$\begin{aligned} ds^2_{\text{gwzw}} &= d\alpha^2 + \cot^2\alpha \left(\frac{d\tilde{\psi}_1^2}{\cos^2\zeta} + \frac{d\psi_2^2}{\sin^2\zeta} \right) \\ &\quad + \tan^2\alpha \left(d\zeta - \frac{\tan\zeta \, \sin 2\tilde{\psi}_1 \, d\tilde{\psi}_1 - \cot\zeta \, \sin 2\psi_2 \, d\psi_2}{\cos 2\tilde{\psi}_1 + \cos 2\psi_2} \right)^2 , \end{aligned} \tag{3.8}$$

$$e^{\Phi_{\text{gwzw}}} = \frac{1}{\cos\alpha \, \sin^2\alpha \, \sin 2\zeta \, (\cos 2\tilde{\psi}_1 + \cos 2\psi_2)} \, . \tag{3.9}$$

We now consider the following singular limit (a special case of the limit in [13]). We first analytically continue $\tilde{\psi}_1 \to i\vartheta$ and shift ϑ by an infinite constant L, i.e. set

$$\tilde{\psi}_1 = i(\vartheta + L) \, , \qquad L \to \infty \, . \tag{3.10}$$

As a result, ϑ becomes an isometry of the metric (3.8) while the dilaton (with $\Phi_0 = \Phi'_0 + 2L - \log 2$) now has a linear dependence on ϑ

$$ds'^2_{\text{gwzw}} = d\alpha^2 + \cot^2\alpha \left(-\frac{d\vartheta^2}{\cos^2\zeta} + \frac{d\psi_2^2}{\sin^2\zeta} \right) + \tan^2\alpha \, \big(d\zeta + \tan\zeta \, d\vartheta\big)^2 \, , \tag{3.11}$$

$$e^{\Phi'_{\text{gwzw}}} = \frac{e^{-2\vartheta}}{\cos\alpha \, \sin^2\alpha \, \sin 2\zeta} \, . \tag{3.12}$$

After taking the limit the coordinate ψ_2 also becomes an isometry of the full background (this is a special feature of the limits discussed in [13]). Comparing (3.11), (3.12) to the metric and dilaton in (3.6) we see that analytically continuing back, $\vartheta = i\psi_1$, we recover the AdS_5 part of the background (3.5) found from the $\varkappa \to i$ limit of (2.26).

Furthermore, we can start from the SO(1, 4)/SO(4) gauged WZW model, the metric and dilaton of which can be found by analytically continuing (3.8), (3.9) as

$$\alpha \to i\beta \, , \qquad \tilde{\psi}_1 \to \tilde{\phi}_1 \, , \qquad \psi_2 \to \phi_2 \, , \qquad \zeta \to \xi \, , \qquad ds^2 \to -ds^2 \, . \tag{3.13}$$

[16] This solution can be found by analytically continuing the result of [37, 38] in the $\epsilon = \epsilon' = 1$ coordinate patch, which is different to the patch used to obtain the metric given in [39]. The notation we use for the four angular coordinates is chosen for comparison with (3.6).

Then taking a similar limit we recover the S^5 part of the background (3.7) found from the $\varkappa \to i$ limit of (2.26) (up to a constant shift of the dilaton).

Let us note that these metrics can be put into simple diagonal forms where the shift isometry becomes a rescaling symmetry. For example, introducing $w = -\cos 2\psi_2$, $v = \cos 2\tilde{\psi}_1$, $u = \sin^2 \zeta \cos 2\tilde{\psi}_1 - \cos^2 \zeta \cos 2\psi_2$, one may write (3.8) as [37, 38]

$$ds^2_{\rm gwzw} = d\alpha^2 + \cot^2\alpha \left[\frac{(v-w)\,dw^2}{4(u-w)(1-w^2)} + \frac{(v-w)\,dv^2}{4(v-u)(1-v^2)}\right] + \tan^2\alpha\,\frac{du^2}{4(u-w)(v-u)} \,. \tag{3.14}$$

The special limit described above then translates into an infinite rescaling of the two coordinates $v = e^{2L}v'$, $u = e^{2L}u'$. This produces the diagonal form of the background (3.11), (3.12)

$$ds'^2_{\rm gwzw} = d\alpha^2 + \frac{\cot^2\alpha\,dx^2 - \tan^2\alpha\,dy^2}{y^2 - x^2} + \cot^2\alpha\,\frac{x^2}{y^2}\,d\psi_2^2 \,, \tag{3.15}$$

$$e^{\Phi'_{\rm gwzw}} = \frac{1}{2\cos\alpha\,\sin^2\alpha\,y\sqrt{x^2-y^2}} \,, \qquad x = \sqrt{v'} = e^{\vartheta} \,, \qquad y = \sqrt{u'} = \sin\zeta\,e^{\vartheta} \,. \tag{3.16}$$

The two isometries of the metric (3.15) are (i) the simultaneous rescaling of x and y and (ii) the shift of ψ_2.[17]

We have thus demonstrated that the $\varkappa \to i$ limit of the $\varkappa$-deformed solution we have found has an interpretation as a limiting background for the standard bosonic gauged WZW model. This connection "explains" the origin of the terms linear in ψ_1 and ϕ_1 in the dilaton in (2.26), relating them to particular terms in the dilaton of the gauged WZW model before the limit. Note that to establish this connection we needed to start with the deformation of the T-dual $AdS_n \times S^n$ background. For $n = 5$ the "reversal" of the T-duality is still not possible even for $\varkappa = i$ because of the linear terms in ψ_1 and ϕ_1 in the dilaton.

3.2 pp-wave $\varkappa \to i$ limit and connection with Pohlmeyer reduction

It was shown in [6] that the $\varkappa \to i$ limit can be made more non-trivial by combining it with a particular rescaling of the directions $x^\pm = t \mp \varphi$ as follows

$$t = \epsilon x^- + \frac{x^+}{\epsilon} \,, \qquad \varphi = \epsilon x^- - \frac{x^+}{\epsilon} \,, \qquad \varkappa = i\sqrt{1+\epsilon^2} \,, \qquad \epsilon \to 0 \,. \tag{3.17}$$

Then taking the $\epsilon \to 0$ limit in the solutions (2.5), (2.15), (2.26) one finds the following pp-wave backgrounds ($n = 2, 3, 5$)[18]

$$ds^2_n = -4dx^-dx^+ + \tfrac{1}{2}(\cos 2\alpha - \cosh 2\beta)(dx^+)^2 + ds^2_{n\perp} \,, \tag{3.18}$$

$$F_n = dC_{n-1} = d\mathcal{C}_{n-2} \wedge dx^+ \,, \qquad\qquad \Phi_n = \Phi_{n\perp} \,. \tag{3.19}$$

[17] Note that this metric is a direct generalization of the metric that appears upon taking a scaling limit ($p \to e^L p$, $q \to e^L q$, $L \to \infty$) in the SO(4)/SO(3) gauged WZW metric [40] $ds^2 = d\alpha^2 + \frac{\cot^2\alpha\ dp^2 + \tan^2\alpha\ dq^2}{1-p^2-q^2}$ with $p = x$, $q = iy$.

[18] If we send $\epsilon \to i\epsilon$ and $x^\pm \to \pm i x^\pm$ we end up with the same solution with the opposite sign for the coefficient of $(dx^+)^2$ and an imaginary flux. As we will discuss, the light-cone gauge-fixing of (3.18) is related to the Pohlmeyer-reduced theories for $AdS_n \times S^n$. In an analogous way, the light-cone gauge-fixing of these backgrounds after the analytic continuation, is related to the Pohlmeyer-reduced theories for $dS_n \times H^n$.

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Here the transverse metric $ds^2_{n\perp}$ and the dilaton $\Phi_{n\perp}$ are the same as in (3.1) (i.e. given by (3.3)–(3.7)). Note that since in the limit (3.17) one has $t+\varphi = 2\epsilon x^- \to 0$ the longitudinal part of the dilaton in (3.1) is absent, i.e. the dilaton here depends only on the transverse coordinates. The RR potential C_{n-1} also depends only on the transverse coordinates $x_\perp = (\alpha, \beta, \ldots)$, where again we use (3.2).

Let us present the explicit form of the "null" RR backgrounds F_n in (3.19). In the $AdS_2 \times S^2$ case, which was discussed in [13], from (2.5) one finds[19]

$$C_1 = \sqrt{2} e^{-\Phi_0} (\cos\alpha \,\sinh\beta + \sin\alpha \,\cosh\beta) dx^+ \ . \tag{3.20}$$

In this case it was shown in [6] that the light-cone gauge-fixing ($x^+ = \mu\tau$) of the string in the background (3.18), (3.3), (3.19), (3.20) yields the corresponding PR model, which is equivalent [33, 34] to the $\mathcal{N} = 2$ supersymmetric sine-Gordon model.

In the $AdS_3 \times S^3$ case, the transverse metric and dilaton were given in (3.4). From (2.15) we find

$$C_2 = e^{-\Phi_0} (\cos^2\alpha \,\sinh^2\beta \, d\psi_1 - \sin^2\alpha \,\cosh^2\beta \, d\phi_1) \wedge dx^+ \ . \tag{3.21}$$

T-dualizing in ψ_1 and ϕ_1, or, alternatively, considering the analytic continuation $x^\pm \to \pm i x^\pm$, $\alpha \to \alpha + \frac{\pi}{2}$, $\beta \to \beta + \frac{i\pi}{2}$, $\Phi_0 \to \Phi_0 + \frac{i\pi}{2}$, we recover the "null" 3-form background supporting the corresponding pp-wave metric (3.18), (3.4) that was found directly in [6] (see also [41] for a similar 10d solution supported by F_5). The light-cone gauge-fixed form of the resulting string sigma model is equivalent [6] to the axial-gauged version of the PR model for $AdS_3 \times S^3$ [33, 34]. Therefore, the light-cone gauge-fixing of this model before T-dualizing or analytically continuing is equivalent to the vector-gauged version of the PR model.

As was explained above, in the $AdS_5 \times S^5$ case the transverse metric and dilaton in (3.18), (3.19) are the same as the special limit of the metric-dilaton background for the $[SO(5)/SO(4)] \times [SO(1,4)/SO(4)]$ gauged WZW model. The latter defines the bosonic kinetic part of the PR model associated to the $AdS_5 \times S^5$ superstring [33, 34, 42].[20] Thus (in contrast to the $n = 2$ and 3 cases) in the $n = 5$ case the light-cone gauge-fixed action of the superstring model corresponding to the background (3.18), (3.19) should be only a *limit* of the full $AdS_5 \times S^5$ PR model, in agreement with the picture suggested in [13].

In the $n = 5$ case we find from (2.26) that $F_5 = dC_4$ with[21]

$$\begin{aligned} C_4 = &- i e^{-\Phi_0} \Big[\cos^4\alpha \,\sinh^4\beta \, d\psi_2 \wedge d(e^{2i\psi_1}) \wedge d(\sin^2\xi \, e^{-2i\phi_1}) \\ &+ \sin^4\alpha \,\cosh^4\beta \, d(\sin^2\zeta \, e^{2i\psi_1}) \wedge d\phi_2 \wedge d(e^{-2i\phi_1}) \Big] \wedge dx^+ \ . \end{aligned} \tag{3.22}$$

[19] This matches the expression in [13] with a symmetric choice of the free constants there.

[20] Note that, as for the $n = 2, 3$ cases [6, 13], the $\varkappa \to i$ limit of the deformed AdS part of the background is associated to the PR model for the sphere and vice versa.

[21] Note that C_4 takes a simple form when written in terms of the diagonal-metric coordinates used in (3.15), i.e. $y^2 = \sin^2\zeta \, e^{2i\psi_1}$, $x^2 = e^{2i\psi_1}$ and $\mathrm{y}^2 = \sin^2\xi \, e^{-2i\phi_1}$, $\mathrm{x}^2 = e^{-2i\phi_1}$.

As for the transverse metric and dilaton (3.18), (3.19) the corresponding 5-form can be made real by analytically continuing $\psi_1 \to i\psi_1$, $\phi_1 \to i\phi_1$, along with shifting Φ_0 by $\frac{i\pi}{2}$.

To conclude, we have seen that in each of the $AdS_2 \times S^2$ and $AdS_3 \times S^3$ cases there is a background for which the corresponding superstring theory taken in light-cone gauge gives precisely the PR model for the original $AdS_n \times S^n$ superstring. The PR model was found by solving the conformal-gauge constraints of the $AdS_n \times S^n$ superstring at the classical level using a non-local change of variables and then reconstructing a new local action.[22]

At the same time, in the $n = 5$ case the situation is different; we first need to take a certain limit of the $AdS_5 \times S^5$ PR model (generating 2+2 isometries in the kinetic terms, cf. (3.6), (3.7)) in order to relate it to the light-cone gauge-fixed superstring on the pp-wave background (3.18), (3.19), (3.22). Originating directly from light-cone gauge-fixed pp-wave superstring, this "limiting" PR model should have some special features and deserves further investigation.

4 Summary and concluding remarks

In this paper we have found a type IIB supergravity solution (with only the metric, dilaton and 5-form being non-trivial) that can be interpreted as one-parameter ($\varkappa$ or η (2.1)) deformation of the background obtained from the maximally symmetric $AdS_5 \times S^5$ background by applying T-duality in all 6 isometric directions. The latter ($\varkappa = 0$) solution has imaginary RR flux, which is a consequence of the formal T-duality being applied in time and this feature persists for $\varkappa \neq 0$. Another unusual property of the solution (2.26) is that for $\varkappa \neq 0$ the dilaton has a linear term $\Phi_{5\,\text{lin}} = -4\varkappa(t + \varphi) - 2\varkappa(\psi_1 - \phi_1)$ depending on a linear combination of 4 out of 6 isometric directions of the metric.

Still, the metric and $e^{\Phi_5} F_5$, which enter the corresponding classical Green-Schwarz superstring action, are invariant under shifts in the isometric directions (i.e. these coordinates enter the Green-Schwarz action only through their derivatives) and so one can formally T-dualize in them, as, e.g., in [17–19] — assuming one can first ignore the non-invariant linear piece in the "quantum" dilaton term of the action. Remarkably, the resulting T-dual sigma model is equivalent (at least to quadratic order in fermions) to the η-deformation [1, 2] of the $AdS_5 \times S^5$ superstring, i.e. the T-dual background has exactly the same metric, B-field and combination of RR fluxes with the dilaton $e^{\hat{\Phi}_5} \hat{F}_n$ $(n = 1, 3, 5)$ as found in [5, 8] directly from the action of the η-model of [1, 2]. However, the presence of $\Phi_{5\,\text{lin}}$ in the dilaton, which depends on the what are now "dual" coordinates means that, in contrast to the solution (2.26) found here, the T-dual background of [5, 8] cannot be directly interpreted as (the non-dilaton part of) a standard type IIB supergravity solution — the full quantum T-dual sigma model including the dilaton term appears to be defined on a

[22]This connection gives a new perspective on the properties of the PR model like its UV finiteness and the possibility of world-sheet supersymmetry. pp-waves with curved transverse space in general do not preserve space-time supersymmetry and thus a priori one should not expect explicit world-sheet supersymmetry for the $n > 2$ cases, although there may be a hidden one (see [6, 41] for related discussions).

"doubled" space.[23] The precise meaning of the relation between our solution (2.26), the background of [5, 8] and the η-model of [1, 2] thus remains to be clarified.[24]

The RR flux vanishes at the special value of the deformation parameter $\varkappa = i$, at which point our background factorizes into the product of the three factors: a flat 2d "longitudinal" space (t, φ) with linear imaginary dilaton $-4i(t + \varphi)$ in (3.1) and two 4d "transverse" metric-dilaton backgrounds (3.6) and (3.7). Each of these factors represents a conformal bosonic sigma model and they also solve the 10d supergravity equations, with central charge shifts cancelling between the two 4d models. We have shown that the two 4d factors can be interpreted as special limits [13] of the backgrounds corresponding to the SO(5)/SO(4) and SO(1, 4)/SO(4) gauged WZW models respectively. This relates the linear terms in the two dilatons to "blowing up" certain angular factors in the standard gauged WZW dilatons.

We have also observed that these "transverse" backgrounds may be viewed as defining the kinetic term of a limit [13] of the Pohlmeyer-reduced model associated with the $AdS_5 \times S^5$ superstring. We still cannot (due to the linear dilaton term) T-dualize back to relate this to the $\varkappa = i$ limit of the η-model directly, however, we can formally do so at the level of the Green-Schwarz action. As shown in [5] the η-model is associated to the vertex (particle-like) q-deformed S-matrix of [43, 44], in particular in the $\varkappa \to i$ limit it should be associated to the limit of that S-matrix investigated in [45, 46]. The full PR model should, however, be associated with the soliton S-matrix. Therefore, this limit may be implementing the "soliton-like picture to particle-like picture" transformation [47]. It would be interesting to clarify if this is indeed the case.

Finally, let us note that according to the discussion in [13] we should expect a relation between the solution found here and the one constructed in [15] that supports the metric of the λ-model, i.e. a one-parameter deformation of the non-abelian dual of $AdS_5 \times S^5$ superstring. Such a relation is known to hold in the $AdS_2 \times S^2$ case [13]. The background of [15] also contains only the metric, dilaton and 5-form flux and therefore, may indeed

[23]The usual T-duality transformation at the level of type II supergravity maps solutions to solutions but it applies only in the presence of an (abelian) isometry; upon compactifying on an isometric direction the T-duality becomes equivalent to a field redefinition. This logic does not apply in the case when the dilaton depends on isometric directions of the metric, suggesting one should start with a "doubled" string theory extension of type IIB supergravity, cf. [22–27]. Another possible idea for bypassing the complication of the linear non-isometric dilaton is to replace it in the action with the term $e^{-2\Phi_{5\,\rm lin}}\partial_+ u \partial_- v$, where u and v are two extra coordinates; integrating out u and v produces the dilaton shift equal to $\Phi_{5\,\rm lin}$. Redefining u and v by $e^{-\Phi_{5\,\rm lin}}$ the total action will depend only on derivatives of the isometric coordinates so one will be able to T-dualize in them. However, one will not be able to easily integrate out u, v after doing the T-duality, i.e. the resulting background will be a 12-dimensional one (with signature $-, -, +, \dots, +$) and hence its interpretation is unclear.

[24]This (partial) T-duality relation between our solution and the background of [5, 8] appears to imply that the η-model of [1, 2] should be one-loop UV finite at least in the bosonic sector. Indeed, the Green-Schwarz superstring action corresponding to a type II supergravity background should be Weyl invariant, and, in particular, UV finite. This should be true for the Green-Schwarz model built on our solution. Since the formal T-dual of the latter corresponds to the metric, B-field and $e^{\Phi}F_k$ background of [5, 8], i.e. leads to the bosonic part and quadratic fermionic terms of the η-model action, the conditions of scale invariance of the η-model in the metric part, i.e. the generalized Einstein equations modulo reparametrizations ($R_{\mu\nu} + \sum(e^{\Phi}F_k)^2_{\mu\nu} + \ldots = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$), should be satisfied regardless of the dilaton issue.

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reduce to (2.26) after taking a limit and doing an analytic continuation. The two metrics are related by such a procedure [13] but the precise correspondence between the dilatons and 5-forms remains to be checked.

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A 6-fold T-dual of the $AdS_5 \times S^5$ η-model background

In this appendix we give the details of the formal T-duality transformations of the η-model metric, B-field and RR fluxes of [8] that can be used to establish a relation to our supergravity solution (2.26). The "background" fields found in [8] by assuming that the η-model action of [1, 2] can be interpreted as a classical Green-Schwarz sigma model in a particular type IIB supergravity background are given by[25]

$$
\begin{aligned}
d\hat{s}_5^2 &= -\frac{1+\rho^2}{1-\varkappa^2\rho^2}d\hat{t}^2 + \frac{d\rho^2}{(1-\varkappa^2\rho^2)(1+\rho^2)} + \frac{\rho^2\cos^2\zeta}{1+\varkappa^2\rho^4\sin^2\zeta}d\hat{\psi}_1^2 + \frac{d\zeta^2}{1+\varkappa^2\rho^4\sin^2\zeta} + \rho^2\sin^2\zeta\, d\hat{\psi}_2^2 \\
&\quad + \frac{1-r^2}{1+\varkappa^2 r^2}d\hat{\varphi}^2 + \frac{dr^2}{(1+\varkappa^2 r^2)(1-r^2)} + \frac{r^2\cos^2\xi}{1+\varkappa^2 r^4\sin^2\xi}d\hat{\phi}_1^2 + \frac{d\xi^2}{1+\varkappa^2 r^4\sin^2\xi} + r^2\sin^2\xi\, d\hat{\phi}_2^2\,, \\
\hat{B} &= \frac{\varkappa\rho^4\sin\zeta\cos\zeta}{1+\varkappa^2\rho^4\sin^2\zeta}d\hat{\psi}_1\wedge d\zeta - \frac{\varkappa r^4\sin\xi\cos\xi}{1+\varkappa^2 r^4\sin^2\xi}d\hat{\phi}_1\wedge d\xi\,, \\
e^{\hat{\Phi}}\hat{F}^{(1)} &= \varkappa^2 \mathrm{F}\left[\rho^4\sin^2\zeta\, d\hat{\psi}_2 - r^4\sin^2\xi\, d\hat{\phi}_2\right], \\
e^{\hat{\Phi}}\hat{F}^{(3)} &= \varkappa\, \mathrm{F}\left[\frac{\rho^3\sin^2\zeta}{1-\varkappa^2\rho^2}d\hat{t}\wedge d\hat{\psi}_2\wedge d\rho + \frac{r^3\sin^2\xi}{1+\varkappa^2 r^2}d\hat{\varphi}\wedge d\hat{\phi}_2\wedge dr \right. \\
&\quad + \frac{\rho^4\sin\zeta\cos\zeta}{1+\varkappa^2\rho^4\sin^2\zeta}d\hat{\psi}_2\wedge d\hat{\psi}_1\wedge d\zeta + \frac{r^4\sin\xi\cos\xi}{1+\varkappa^2 r^4\sin^2\xi}d\hat{\phi}_2\wedge d\hat{\phi}_1\wedge d\xi \\
&\quad + \frac{\varkappa^2\rho r^4\sin^2\xi}{1-\varkappa^2\rho^2}d\hat{t}\wedge d\rho\wedge d\hat{\phi}_2 - \frac{\varkappa^2\rho^4 r\sin^2\zeta}{1+\varkappa^2 r^2}d\hat{\psi}_2\wedge d\hat{\varphi}\wedge dr \\
&\quad \left. + \frac{\varkappa^2\rho^4 r^4\sin\zeta\cos\zeta\sin^2\xi}{1+\varkappa^2\rho^4\sin^2\zeta}d\hat{\psi}_1\wedge d\zeta\wedge d\hat{\phi}_2 + \frac{\varkappa^2\rho^4 r^4\sin^2\zeta\sin\xi\cos\xi}{1+\varkappa^2 r^4\sin^2\xi}d\hat{\psi}_2\wedge d\hat{\phi}_1\wedge d\xi\right],
\end{aligned}
$$

[25] It is interesting to note that reversing the signs of $\hat{\psi}_2$ and $\hat{\phi}_2$ reverses the sign of all the RR k-form strengths, while reversing the signs of all the isometric directions reverses the signs of the B-field along with the RR 1- and 5-forms. Combining the two transformations reverses the signs of the B-field and the RR 3-form. These all correspond to well-known $\mathbb{Z}_2$ symmetries of Type IIB theory.

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$$
\begin{aligned}
e^{\hat\Phi}\hat F^{(5)} = \mathrm{F}\Bigg[& \frac{\rho^3\sin\zeta\cos\zeta}{(1-\varkappa^2\rho^2)(1+\varkappa^2\rho^4\sin^2\zeta)} d\hat t\wedge d\hat\psi_2\wedge d\hat\psi_1\wedge d\zeta\wedge d\rho \\
& - \frac{r^3\sin\xi\cos\xi}{(1+\varkappa^2 r^2)(1+\varkappa^2 r^4\sin^2\xi)} d\hat\varphi\wedge d\hat\phi_2\wedge d\hat\phi_1\wedge d\xi\wedge dr \\
& - \frac{\varkappa^2\rho r}{(1-\varkappa^2\rho^2)(1+\varkappa^2 r^2)}(\rho^2\sin^2\zeta\, d\hat t\wedge d\hat\psi_2\wedge d\rho\wedge d\hat\varphi\wedge dr + r^2\sin^2\xi\, d\hat t\wedge d\rho\wedge d\hat\varphi\wedge d\hat\phi_2\wedge dr) \\
& + \frac{\varkappa^2\rho^4 r^4\sin\zeta\cos\zeta\sin\xi\cos\xi}{(1+\varkappa^2\rho^4\sin^2\zeta)(1+\varkappa^2 r^4\sin^2\xi)}(d\hat\psi_2\wedge d\hat\psi_1\wedge d\zeta\wedge d\hat\phi_1\wedge d\xi - d\hat\psi_1\wedge d\zeta\wedge d\hat\phi_2\wedge d\hat\phi_1\wedge d\xi) \\
& + \frac{\varkappa^2\rho r^4\sin\xi\cos\xi}{(1-\varkappa^2\rho^2)(1+\varkappa^2 r^4\sin^2\xi)}(\rho^2\sin^2\zeta\, d\hat t\wedge d\hat\psi_2\wedge d\rho\wedge d\hat\phi_1\wedge d\xi - d\hat t\wedge d\rho\wedge d\hat\phi_2\wedge d\hat\phi_1\wedge d\xi) \\
& - \frac{\varkappa^2\rho^4 r\sin\zeta\cos\zeta}{(1+\varkappa^2 r^2)(1+\varkappa^2\rho^4\sin^2\zeta)}(r^2\sin^2\xi\, d\hat\psi_1\wedge d\zeta\wedge d\hat\varphi\wedge d\hat\phi_2\wedge dr + d\hat\psi_2\wedge d\hat\psi_1\wedge d\zeta\wedge d\hat\varphi\wedge dr) \\
& - \frac{\varkappa^4\rho^5 r^4\sin\zeta\cos\zeta\sin^2\xi}{(1-\varkappa^2\rho^2)(1+\varkappa^2\rho^4\sin^2\zeta)} d\hat t\wedge d\hat\psi_1\wedge d\zeta\wedge d\rho\wedge d\hat\phi_2 \\
& - \frac{\varkappa^4\rho^4 r^5\sin^2\zeta\sin\xi\cos\xi}{(1+\varkappa^2 r^2)(1+\varkappa^2 r^4\sin^2\xi)} d\hat\psi_2\wedge d\hat\varphi\wedge d\hat\phi_1\wedge d\xi\wedge dr\Bigg], \\
\mathrm{F} \equiv & \frac{4\sqrt{1+\varkappa^2}}{\sqrt{1-\varkappa^2\rho^2}\sqrt{1+\varkappa^2\rho^4\sin^2\zeta}\sqrt{1+\varkappa^2 r^2}\sqrt{1+\varkappa^2 r^4\sin^2\xi}}\,. \qquad \text{(A.1)}
\end{aligned}
$$

Here $\hat\Phi$ is some a priori unknown dilaton and $\hat F^{(k)}$ are RR k-form strengths of type IIB theory. The self-duality equation for the RR 5-form used in [8] has the opposite sign to that used in (2.22) in section 2.3, that is[26]

$$
\hat F^{(5)}_{mnpqr} = \frac{1}{5!}\sqrt{-\hat g}\,\epsilon_{mnpqrstuvw}\hat F^{(5)stuvw}\,. \qquad \text{(A.2)}
$$

We also introduce the usual 7- and 9-forms (defined in terms of the dual RR 3- and 1-forms)

$$
\hat F^{(7)}_{mnpqrst} = -\frac{1}{3!}\sqrt{-\hat g}\,\epsilon_{mnpqrstuvw}\hat F^{(3)uvw}\,, \qquad \hat F^{(9)}_{mnpqrstuv} = \sqrt{-\hat g}\,\epsilon_{mnpqrstuvw}\hat F^{(1)w}\,. \quad \text{(A.3)}
$$

For the conventions of type IIB theory used in [8], the T-duality transformation rules can be presented as follows [16, 20, 21] (here $\hat y$ stands for an isometric direction along which we dualize, while y is the dual coordinate)

$$
\begin{aligned}
g_{yy} &= \hat g^{-1}_{\hat y\hat y}\,, \qquad g_{ym} = \hat g^{-1}_{\hat y\hat y}\hat B_{\hat y m}\,, \qquad B_{ym} = \hat g^{-1}_{\hat y\hat y}\hat g_{\hat y m}\,, \qquad e^{\Phi} = \hat g^{-1/2}_{\hat y\hat y}e^{\hat\Phi}\,, \\
g_{mn} &= \hat g_{mn} - \hat g^{-1}_{\hat y\hat y}(\hat g_{\hat y m}\hat g_{\hat y n} - \hat B_{\hat y m}\hat B_{\hat y n})\,, \qquad B_{mn} = \hat B_{mn} - \hat g^{-1}_{yy}(\hat g_{\hat y m}\hat B_{\hat y n} - \hat B_{\hat y m}\hat g_{\hat y n})\,, \\
F^{(k)}_{ym_2\dots m_k} &= -\hat F^{(k-1)}_{m_2\dots m_k} + (k-1)\hat g^{-1}_{\hat y\hat y}\hat g_{\hat y[m_2}\hat F^{(k-1)}_{\hat y m_3\dots m_k]}\,, \\
F^{(k)}_{m_1 m_2\dots m_k} &= -\hat F^{(k+1)}_{\hat y m_1\dots m_k} + k\hat B_{\hat y[m_1}F^{(k)}_{y m_2\dots m_k]}\,, \qquad \text{(A.4)}
\end{aligned}
$$

while the transformation rules for the RR potentials are similar to those of the RR fluxes. Note that these relations imply that the vielbein components of $e^\Phi F$ get mapped into vielbein components of $e^{\hat\Phi}\hat F$, in agreement with how T-duality acts on the quadratic fermionic term in the Green-Schwarz action [17].

[26] Recall that $\epsilon_{0123456789} = -1$ and that $m, n, \ldots = 0,1,2,3,4,5,6,7,8,9$ correspond to $t, \psi_2, \psi_1, \zeta, \rho, \varphi, \phi_2, \phi_1, \xi, r$.

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We can formally apply these rules to the background fields in (A.1) with the transformation of the combination of RR flux and dilaton, $e^{\hat{\Phi}}\hat{F}^{(n)}$, found by combining the transformations of the individual factors/components. To compensate for the different choice of self-duality equation we first reverse the sign of $\hat{\psi}_2$ in (A.1), after which, applying the T-duality rules (A.4), we recover our solution (2.26). To summarize, even though the background fields in (A.1) cannot be extended to a solution of type IIB supergravity and the fluxes are inconsistent with the Bianchi identities [8], those found after applying the T-duality rules (A.4) can be and are. This is not in contradiction with the standard logic that T-duality maps from one background solving the supergravity equations of motion and Bianchi identities to another, as this assumes that the directions in which one dualizes are isometries of the full background, and not just the combinations of fields appearing in the classical Green-Schwarz action.

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