

Anomalous dimensions of non-chiral operators from AdS/CFT

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ABSTRACT: Non-chiral operators with positive anomalous dimensions can have interesting applications to supersymmetric model building. Motivated by this, we develop a new method for obtaining the anomalous dimensions of non-chiral double-trace operators in $\mathcal{N} = 1$ superconformal field theories (SCFTs) with weakly-coupled AdS duals. Via the Hamiltonian formulation of AdS/CFT, we show how to directly compute the anomalous dimension as a bound state energy in the gravity dual. This simplifies previous approaches based on the four-point function and the OPE. We apply our method to a class of effective AdS₅ supergravity models, and we find that the binding energy can have either sign. If such models can be UV completed, they will provide the first calculable examples of SCFTs with positive anomalous dimensions.

KEYWORDS: AdS-CFT Correspondence, Conformal and W Symmetry, Supergravity Models

ARXIV EPRINT: [1104.5013](https://arxiv.org/abs/1104.5013)

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1 Introduction

Recently, there has been significant progress in characterizing the operator product expansion (OPE) in 4d CFTs using general principles such as crossing symmetry and unitarity. For real scalar primary operators ϕ , the authors of [1, 2] were able to place an upper bound on the anomalous dimension of ϕ^2 , the first scalar primary appearing in the $\phi \times \phi$ OPE. Here, following the literature, we define the anomalous dimension of ϕ^2 to be:

$$\gamma_{\phi^2} \equiv \Delta_{\phi^2} - 2\Delta_{\phi}. \quad (1.1)$$

In [3], these bounds were extended to the case of a complex scalar field transforming under a global symmetry group.

Upper bounds on anomalous dimensions of composite operators in CFTs are interesting from a phenomenological point of view. In the context of non-supersymmetric CFTs, such bounds can have implications for conformal technicolor theories where the Higgs field H is subject to strong conformal dynamics [4]. In general, one wants $H^\dagger H$ to have dimension ~ 4 for the gauge hierarchy problem, but H to have dimension ~ 1 for flavor. Thus one wants a large positive anomalous dimension for $H^\dagger H$. Upper bounds on anomalous dimensions can constrain or rule out such technicolor models.

In this paper, we will be interested in analogous issues in 4d superconformal field theories (SCFTs), where now we take ϕ to be a chiral primary operator. Here, interesting applications of positive anomalous dimensions arise when ϕ participates in SUSY-breaking. Then the soft masses in the MSSM come from operators of the form

$$\int d^2\theta \frac{\phi \Phi_i \Phi_j}{M}, \quad \int d^4\theta \frac{\phi^\dagger \phi \Phi_i \Phi_j}{M^2}, \quad (1.2)$$

where Φ_i are MSSM fields and M is a UV scale where these operators are generated. In general, the latter operators are unconstrained by supersymmetry, and in various contexts they can easily lead to too-large SUSY-breaking effects. One way to suppress these effects is to imagine that $\gamma_{\phi^\dagger \phi} > 0$ due to strong SCFT dynamics. Then the coefficients of (1.2) run strongly as one flows into the IR, and at a scale $\mu \ll M$ one finds

$$\int d^2\theta \left(\frac{\mu}{M}\right)^{\Delta_\phi - 1} \frac{\phi \Phi_i \Phi_j}{M}, \quad \int d^4\theta \left(\frac{\mu}{M}\right)^{2\Delta_{\phi^\dagger \phi} - 2} \frac{\phi^\dagger \phi \Phi_i \Phi_j}{M^2}. \quad (1.3)$$

This mechanism is used, for example, in solutions to the μ/B_μ problem in gauge mediation [5–8]; solutions to the problem of flavor-violation in gravity mediation [9–11]; and gaugino mediation in the context of “general messenger gauge mediation” [12].

Despite the many potential applications of positive anomalous dimensions, so far there do not actually exist any examples of SCFTs with $\gamma_{\phi^\dagger \phi} > 0$. Thus it is interesting to speculate on whether $\gamma_{\phi^\dagger \phi} \leq 0$ in all SCFTs. The authors of [13] were able to prove a general upper bound on $\gamma_{\phi^\dagger \phi}$, by extending the crossing-symmetry-based methods of [1] to the supersymmetric case. Their bound allows for positive anomalous dimensions, but it can almost certainly be significantly improved with further numerical work, given the stronger results of [3] on complex scalars, which did not assume supersymmetry. Thus it is still conceivable that general SCFT constraints could imply the strongest possible bound $\gamma_{\phi^\dagger \phi} \leq 0$.

In this paper, we will approach the question of positive anomalous dimensions in SCFTs in a complementary way. Rather than attempting to refine and improve the general bounds, we will instead use the tools of AdS/CFT to study explicit examples [14–16]. As is well known, local supergravity theories in AdS₅ (supposing they have a stable UV completion) provide constructive examples of a certain class of 4d SCFTs, namely theories which have a large N ’t Hooft limit. SCFTs realized in this way have conformally invariant correlation functions that satisfy the constraints of crossing symmetry and unitarity, order by order in the $1/N$ expansion. Thus they are well-suited to exploring the space of possibilities consistent with general bounds.

As was recently emphasized in [17–19], an especially useful simplifying limit is where the AdS theory has only a handful of light states (compared parametrically to the Planck scale), so that one may decouple all massive string states and focus only on a minimal light sector. On the SCFT side, this corresponds to decoupling all but a small number of single-trace operators, and focusing on the multi-trace operators built out of these.

In the following sections, we will develop the necessary tools for calculating $\gamma_{\phi^\dagger\phi}$ in such SCFTs. Since we are interested primarily in the sign of $\gamma_{\phi^\dagger\phi}$, we are free to focus on the leading-order effect in the $1/N$ expansion. This corresponds to doing semi-classical supergravity in an AdS₅ background. In section 2, we will describe the setup in more detail. For simplicity, we will focus on a single charged, complex scalar field ϕ minimally coupled to gravity and the graviphoton:

$$S = \frac{1}{\kappa^2} \int d^5x \sqrt{-g} \left[-(D^\mu \phi)^\dagger (D_\mu \phi) - m^2 \phi^\dagger \phi - \frac{a}{R^2} (\phi^\dagger \phi)^2 - b (\phi^\dagger \phi) (\partial_\mu \phi^\dagger \partial^\mu \phi) \right]. \quad (1.4)$$

Here a and b are dimensionless coefficients that are a priori free parameters of the model, and R is the AdS radius.

Even after restricting to the leading-order effect in the $1/N$ expansion and to the minimal model (1.4), calculations in AdS₅ can be dauntingly complex. The conventional method [20–23] for calculating the dimension of $\phi^\dagger\phi$ in AdS/CFT has been to first obtain the four-point function $\langle \phi^\dagger(x_1) \phi(x_2) \phi^\dagger(x_3) \phi(x_4) \rangle$ using the standard techniques, go to a short-distance limit dominated by the OPE, and read off the anomalous dimensions from the expansion in conformal blocks. The correlation functions cannot be written in closed form, but rather must be expressed in terms of special integral functions, and the anomalous dimensions are related to the coefficients of logarithmically-singular terms in these special functions.

In section 3, we will present a much simpler and more direct method for computing anomalous dimensions. Our method is based on the Hamiltonian formulation of AdS/CFT, rather than the more commonly used Lagrangian formulation which leads to correlation functions. It makes use of the fact that the anomalous dimension of $\phi^\dagger\phi$ in the SCFT is dual to the binding energy of two-particle state in AdS. We show how to calculate the binding energies directly, thereby bypassing the four point function altogether.

Our method also simplifies and extends previous approaches based on the Hamiltonian formalism, in particular the recent work of [19]. We generalize the results of [19] to include complex scalar fields with arbitrary gauge boson and graviton exchange. Furthermore, we show how all these different contributions to the binding energy can be understood in a uniform, semiclassical framework. Integrating out the gauge boson and graviton using their classical equations of motion, we will derive a non-local, quartic, effective interaction Hamiltonian δH_{eff} for the dual bulk field. This will allow us to obtain the binding energy of $\phi^\dagger\phi$ and $\phi\phi$ using *first-order* perturbation theory:

$$\begin{aligned} \gamma_{\phi^\dagger\phi} &= \int d^4x \sqrt{-g} \langle \phi^\dagger\phi | \delta H_{\text{eff}}[\phi, \phi^\dagger] | \phi^\dagger\phi \rangle \\ \gamma_{\phi\phi} &= \int d^4x \sqrt{-g} \langle \phi\phi | \delta H_{\text{eff}}[\phi, \phi^\dagger] | \phi\phi \rangle. \end{aligned} \quad (1.5)$$

By contrast, in [19], it was necessary to go to *second-order* perturbation theory to deal with particle exchange. This in turn necessitated understanding the wavefunctions for an infinite tower of excited intermediate states, which was only carried out for *s*-channel exchange of scalar fields.

In general, the dimension of chiral operators is protected by supersymmetry, so that $\gamma_{\phi\phi}$ necessarily vanishes. Consequently, our calculation of $\gamma_{\phi\phi}$ imposes a relation on the coefficients a and b in (1.4):

$$a = \Delta(\Delta - 2)b - \frac{2\Delta^2}{3}. \tag{1.6}$$

Our final result for $\gamma_{\phi^\dagger\phi}$ turns out to be

$$\gamma_{\phi^\dagger\phi} \propto \left(b - \frac{2\Delta(2\Delta + 3)}{3(2\Delta + 1)} \right), \tag{1.7}$$

where the proportionality constant is strictly positive for $\Delta > 1$. As a highly non-trivial check of our method, we rederive the $\phi\phi$ and $\phi^\dagger\phi$ anomalous dimensions using the conventional four-point function approach in section 4.

Interestingly, although (1.7) is a binding energy between oppositely charged particles, it is not always negative. At large Δ , one can take a flat-space limit and $\gamma_{\phi^\dagger\phi} < 0$ as expected. However, for Δ small, the AdS curvature modifies the gravitational force and the anomalous dimension can have either sign.

In section 5, we apply our general tools to the study of a specific, minimal supergravity theory for a single hypermultiplet, based on the coset $SU(2, 1)/U(2) \times U(1)$. Starting from the explicit $\mathcal{N} = 2$ supergravity Lagrangian for the hypermultiplet sigma model, we derive the Lagrangian (1.4) with specific values of a and b . We find that for a range of choices of the parameters of this model, one has $\gamma_{\phi^\dagger\phi} > 0$. Finally, in section 6, we conclude with a summary of our results, and a discussion of potential model-building applications and future directions.

2 General scalar fields coupled to $\mathcal{N} = 2$ supergravity

2.1 Brief overview of $\mathcal{N} = 2$ $d = 5$ supergravity

In this section, we will briefly describe the structure of AdS₅ theories which are dual to 4d $\mathcal{N} = 1$ SCFTs in the large N limit at strong 't Hooft coupling. At low energies and weak coupling, the effective AdS₅ theory falls into the framework of $\mathcal{N} = 2$, $d = 5$ supergravity. Useful references include [24–28].

Chiral primaries in the SCFT are dual to hypermultiplets in the 5d bulk. A hypermultiplet has four real degrees of freedom; these are dual to the chiral primary and its F -component.

Besides the hypermultiplet, the other basic BPS multiplets in $\mathcal{N} = 2$, $d = 5$ supergravity are the vector multiplet and the gravity multiplet. The gravity multiplet is dual to the multiplet containing the stress tensor and the $U(1)_R$ -current of $\mathcal{N} = 1$ SCFT. Its bosonic degrees of freedom consist of the funfbein and the graviphoton. Meanwhile, vector

multiplets are dual to the current supermultiplets of global symmetries in the SCFT. Each vector multiplet’s bosonic degrees of freedom consist of a real scalar and a gauge field.

Supersymmetry dictates that the vector multiplet scalars take values on a “special Kähler manifold” and the hypermultiplet scalars take values on a “quaternion Kähler manifold.” Interactions arise from gauging isometries in these manifolds.

The gravity and vector multiplet are examples of “massless multiplets”, and the hypermultiplets are examples of “chiral multiplets”. These multiplets are protected by supersymmetry and are dual to shortened representations of the $\mathcal{N} = 1$ superconformal algebra in $d = 4$. In addition, there are many other possible multiplets in $\mathcal{N} = 2, d = 5$ supergravity. These include higher-spin shortened multiplets. Others, known as “massive multiplets,” are less constrained by supersymmetry, and are dual to “semi-short” and “long” representations of the $\mathcal{N} = 1$ superconformal algebra in $d = 4$. These are described in detail in [29, 30]. (See also the appendix of [31] for a concise summary of the unitary representations of the $\mathcal{N} = 1$ superconformal algebra.) Massive multiplets generally correspond to KK modes, so they will usually be present in any compactification down to 5d.

We note that while the representations of the $\mathcal{N} = 1$ superconformal algebra include multiplets with arbitrarily high spin, fields with spin > 2 are not believed to arise in any local, weakly-coupled supergravity theory. So we will assume these are absent in the effective AdS theory, and focus our attention on spin ≤ 2 .

In addition, we will restrict ourselves to supergravity theories without massless vector multiplets, for the following reason. As discussed above, massless vector multiplets in AdS are dual to conserved non-R global symmetries in the SCFT. As emphasized by many authors (see e.g. [5, 7, 32, 33]), theories where ϕ is charged under a (non-R) global symmetry necessarily have $\gamma_{\phi^\dagger\phi} \leq 0$.¹ Given that we are interested in the possibility of $\gamma_{\phi^\dagger\phi} > 0$, it makes sense to exclude massless vector multiplets from our setup.

So, to summarize, for the problem we are interested in, we can consider hypermultiplets coupled to themselves, to the gravity multiplet, and to other massive multiplets with spin ≤ 2 . Now let us describe the setup in more detail.

2.2 Our setup

We are interested in obtaining the anomalous dimension of $\phi^\dagger\phi$ to leading order in the $1/N$ expansion. According to the AdS/CFT dictionary, $1/N$ corresponds to the 5d gravitational coupling $\kappa = \sqrt{8\pi G_N}$ in the bulk dual

$$\frac{1}{N} \sim \frac{1}{(m_5 R)^{3/2}} \sim \frac{\kappa}{R^{3/2}}, \tag{2.1}$$

where m_5 is the five-dimensional Planck scale and R is the AdS radius. In what follows, we will work in units of $R = 1$. Then κ controls the strength of the interactions in the 5d supergravity theory. To compute the leading anomalous dimension, we must specify the interactions of ϕ with itself and with all the other fields in the 5d bulk dual, to lowest order

¹In such theories, the conserved current of the global symmetry lives in a protected multiplet whose lowest component is a dimension-2 scalar J , and J must appear in the OPE of $\phi^\dagger \times \phi$ with a nonzero coefficient fixed by the global symmetry. Since unitarity restricts $\Delta_\phi \geq 1$, one necessarily has $\gamma_{\phi^\dagger\phi} \leq \Delta_J - 2\Delta_\phi \leq 0$.

in κ . As we will see below, the leading anomalous dimensions arise at $\mathcal{O}(\kappa^2)$ and come from tree-level diagrams in the bulk theory.

As discussed above, ϕ is dual to a complex scalar field (which we will denote with the same symbol) inside a hypermultiplet. Clearly, ϕ must couple canonically to gravity and to the graviphoton. It can also couple to itself via quartic interactions. (Cubic self-interactions are forbidden by charge conservation.) So far we have described the following setup:

$$S = \frac{1}{\kappa^2} \int d^5x \sqrt{-g} \left(\frac{1}{2} (\mathcal{R} + 12) - \frac{1}{4g^2} F^2 - (D^\mu \phi)^\dagger (D_\mu \phi) - m^2 \phi^\dagger \phi - V[\phi, \phi^\dagger] \right), \quad (2.2)$$

where $D_\mu = \partial_\mu - iqA_\mu$ and

$$V[\phi, \phi^\dagger] = a(\phi^\dagger \phi)^2 + b(\phi^\dagger \phi)(\partial_\mu \phi^\dagger \partial^\mu \phi) \quad (2.3)$$

for some coefficients a and b . Note that in (2.2), all the fields are dimensionless and the only dependence on κ is out front. Thus all the dimensions in (2.2) are made up with powers of R .

Demanding the canonical relation between the central charge of the energy-momentum tensor and the central charge of the R-current fixes²

$$g^2 = \frac{3}{2}. \quad (2.5)$$

According to the usual AdS/CFT dictionary, the scalar mass-squared is related to the dimension in the SCFT via:

$$m^2 = \Delta(\Delta - 4). \quad (2.6)$$

The U(1) charge here is not independent of the mass, but is given by

$$q = \frac{2}{3}\Delta. \quad (2.7)$$

This relation is the bulk counterpart of the usual dimension/R-charge relation for chiral primaries in $\mathcal{N} = 1$ SCFT. Finally, a and b are not independent in this class of models. Since ϕ is a chiral primary in the SCFT, the dimension of ϕ^2 is protected. Therefore, the energy of the $\phi\phi$ two-particle state must be exactly 2Δ . As we will see in section 3.1, this imposes a relation between a and b :

$$a = \Delta(\Delta - 2)b - \frac{2\Delta^2}{3}. \quad (2.8)$$

²Conserved currents J^μ and the energy-momentum tensor $T^{\mu\nu}$ generate symmetry transformations in the CFT, which provides them a canonical normalization based on their three-point functions with other operators. The central charges c_V and c_T of J^μ and $T^{\mu\nu}$ respectively are then defined through their two-point functions:

$$\langle J^\mu(x) J^\nu(0) \rangle = \frac{12c_V}{(2\pi)^4} \frac{I^{\mu\nu}}{x^6}, \quad \langle T^{\mu\nu}(x) T^{\rho\sigma}(0) \rangle = \frac{40c_T}{\pi^4} \left(\frac{1}{2} (I^{\mu\rho} I^{\nu\sigma} + I^{\mu\sigma} I^{\nu\rho}) - \frac{1}{4} \delta^{\mu\nu} \delta^{\rho\sigma} \right), \quad (2.4)$$

where $I^{\mu\nu} = \delta^{\mu\nu} - 2\frac{x^\mu x^\nu}{x^2}$. From the action (2.2) for A_μ and $h_{\mu\nu}$, one may calculate the two-point functions of J^μ and $T^{\mu\nu}$ and compare to (2.4), with the result $c_V = \frac{8\pi^2}{g^2\kappa^2}$, $c_T = \frac{\pi^2}{\kappa^2}$. However, since J^μ here is the R-current, it falls in the same multiplet as $T^{\mu\nu}$. Consequently, the central charges are related by $c_V = \frac{16}{3}c_T$ [34], which enforces the value for g stated in the text.

We believe this is a new relation between the quartic hypermultiplet couplings of $\mathcal{N} = 2$, $d = 5$ supergravity theories. Below in section 5, we will see that it is respected in the specific example of the “universal hypermultiplet.” It would be interesting to test this relation further in more general examples and in actual string compactifications.

The action (2.2), subject to the relations above, is the most general setup we will consider in this paper. According to the discussion in the previous subsection, we are ignoring couplings to additional massive multiplets. This is purely for simplicity; our methods should be easily extendable to include these modes as well, and it would be very interesting to do so.

Aside from the possibility of massive multiplets, we claim that (2.2) can be used to calculate the binding energies of $\phi\phi$ and $\phi^\dagger\phi$ in any $\mathcal{N} = 2$, $d = 5$ supergravity theory, to leading order in the gravitational coupling κ . To prove this claim, it is useful to keep in mind some diagrammatic intuition. As we will describe in more detail in section 4, the binding energies arise from diagrams with four ϕ (or ϕ^\dagger) external legs. Then the leading order binding energies arise at $\mathcal{O}(\kappa^2)$ from single insertions of the quartic potential (2.3), as well as from tree-level graviton and gravi-photon exchange. All other supergravity interactions (higher order self-interactions of ϕ , interactions with fermions and other charged scalars) can contribute only at loop order. Thus they involve more internal lines, and hence more powers of κ . We can also consider higher derivative corrections due to massive string states. These are suppressed by powers of α' inside the parentheses in (2.2). Since we are assuming $\alpha' \ll R^2$, these are also subleading effects. We conclude that (2.2) captures the leading-order binding energies of $\phi\phi$ and $\phi^\dagger\phi$, up to possible couplings to additional massive multiplets.

Finally, we should emphasize that the effective theory (2.2) is not UV complete. Thus any results derived from it are subject to the usual caveats of whether a given effective field theory can be UV-completed. In practice this is usually accomplished by finding a string theory embedding. This extremely interesting line of investigation is beyond the scope of this paper; we look forward to returning to it in a future publication.

The form of the action (2.2) is ideally suited for power-counting of κ . However, for computations, it is more convenient to canonically normalize the fields

$$(\phi, A_\mu, h_{\mu\nu}) \rightarrow \kappa(\phi, gA_\mu, h_{\mu\nu}) \tag{2.9}$$

which we shall do in the following sections.

3 A new approach to anomalous dimensions in AdS/CFT

3.1 Hamiltonian formulation of AdS/CFT – the free theory

In this section, we will derive general formulas for the leading order binding energies of $\phi\phi$ and $\phi^\dagger\phi$ in the Hamiltonian formulation of AdS/CFT. By directly focusing from the outset on the Hamiltonian and the spectrum of operators, we can bypass the correlation functions of the theory and all the extra complications they bring.

Our strategy will be to integrate out the photon and graviton in (2.2), leaving behind an effective action for the scalar alone. Passing from the effective action to the Hamiltonian,

the leading-order binding energy can be read off using first-order perturbation theory, simply by looking at matrix elements of the Hamiltonian. As we will see, most of the effort of calculating the binding energies in the present method will go into integrating out the photon and graviton. Even this step will be performed in a very physically transparent way, essentially by treating them as semi-classical fields sourced by the two-particle states, $\phi^\dagger\phi$ or $\phi\phi$, whose binding energies we wish to calculate. The high degree of symmetry of these sources drastically simplifies the response of the photon and graviton fields, which can be solved for in simple, closed form for any value of Δ .

To begin, let us describe the Hamiltonian formulation of the free theory ($\kappa \rightarrow 0$). We work with the Hamiltonian which generates time evolution in AdS₅ global coordinates:

$$ds^2 = \frac{1}{\cos^2 \rho} (-dt^2 + d\rho^2 + \sin^2 \rho d\Omega_3^2) . \tag{3.1}$$

(So $\rho \in [0, \frac{\pi}{2})$ with the boundary of AdS occurring at $\rho \rightarrow \pi/2$.) As is well known, this Hamiltonian corresponds to that of the dual CFT in radial quantization, i.e. the dilatation operator. Thus its spectrum should correspond to the dimension of operators in the dual CFT. Indeed, for a free scalar field in AdS, the spacetime curvature acts like a potential well and the energy spectrum of single particle ϕ states is discrete:

$$E_{n,\ell}^{(0)} = \Delta + 2n + \ell \tag{3.2}$$

where ℓ is the total spin. This is in direct correspondence with the dimensions of a single-trace scalar primary operator ϕ and its descendants in the dual CFT [15, 35]. More generally, all of the single-particle states of AdS are in one-to-one correspondence with the single-trace states in the CFT in radial quantization. In AdS, as in flat space, the creation and annihilation operators for these states and their anti-particles are used to construct the second-quantized scalar field $\phi(x)$:

$$\phi(x) = \sum_{n,l,J} (\psi_{n,l,J}^*(x) a_{n,l,J} + \psi_{n,l,J}(x) b_{n,l,J}^\dagger) . \tag{3.3}$$

The sum is over the discrete eigenmodes of AdS, with J labelling additional spin quantum numbers (e.g. azimuthal spin). $\psi_{n,l,J}$ are the appropriate Klein-Gordon wavefunctions, normalized so that the Hamiltonian constructed from the conventional free scalar Lagrangian is simply

$$H_{\text{free}} = \sum_{n,l,J} E_{n,l}^{(0)} \left(a_{n,l,J}^\dagger a_{n,l,J} + b_{n,l,J}^\dagger b_{n,l,J} \right) . \tag{3.4}$$

The only wavefunction that will turn out to be relevant to our calculation is that of the lowest-energy mode:

$$\psi_0(x) \equiv \psi_{0,0,0}(x) = N_\Delta (e^{it} \cos \rho)^\Delta, \quad N_\Delta = \sqrt{\frac{\Delta - 1}{2\pi^2}} . \tag{3.5}$$

Here the $2\pi^2$ in the denominator is the volume of S^3 . The relevant two-particle states in the free theory are just

$$|\phi\phi\rangle = \frac{1}{\sqrt{2}} b_0^\dagger b_0^\dagger |0\rangle, \quad |\phi^\dagger\phi\rangle = a_0^\dagger b_0^\dagger |0\rangle, \tag{3.6}$$

($a_0 \equiv a_{0,0,0}, b_0 \equiv b_{0,0,0}$). In the free theory, these states clearly have energy that is just 2Δ , as one can verify by acting on them with H_{free} .

3.2 Adding interactions

Now consider turning on κ -suppressed interactions in (2.2) (remembering that we have canonicalized the fields via (2.9)). These deform the Hamiltonian away from (3.4),

$$H_{\text{free}} \rightarrow H_{\text{free}} + \kappa \delta H_{\text{exchange}} + \kappa^2 \delta H_{\text{contact}}. \quad (3.7)$$

Here we have separated out the interactions due to photon and graviton exchange, which start at $\mathcal{O}(\kappa)$, and the quartic scalar contact interactions, which start at $\mathcal{O}(\kappa^2)$. We would like to find the perturbed spectrum of two particle states. One idea would be to directly apply time-independent perturbation theory to (3.7). Then the energy of $\phi\phi$ would be given by [19]:

$$E_{\phi\phi} = E_{\phi\phi}^{(0)} + \kappa^2 \left(\langle \phi\phi | \delta H_{\text{contact}} | \phi\phi \rangle + \sum_{|\alpha\rangle \neq |\phi\phi\rangle} \frac{|\langle \phi\phi | \delta H_{\text{exchange}} | \alpha \rangle|^2}{E_{\phi\phi}^{(0)} - E_{\alpha}^{(0)}} \right) + \mathcal{O}(\kappa^4), \quad (3.8)$$

with an analogous formula for $E_{\phi^\dagger\phi}$. So to extract the effect of photon and graviton exchange using (3.8), we must perform second order perturbation theory. This is a formidable task — the sum in (3.8) is over all possible intermediate states in the Hilbert space, including descendants and states with arbitrary spin. We will now instead develop a better approach that allows us to treat all contributions using first-order perturbation theory.

As described at the beginning of this section, our approach is to integrate out the photon and graviton classically, leaving behind a non-local effective potential for the scalar field alone. Formally, we can write the effective theory for ϕ as:

$$S_{\text{eff}} = \int d^5x \sqrt{-g} \left(-(|\partial\phi|^2 + m^2|\phi|^2) - V_{\text{eff}}[\phi, \phi^\dagger] + \mathcal{O}(\phi^6) \right), \quad (3.9)$$

$$V_{\text{eff}}[\phi, \phi^\dagger] = V[\phi, \phi^\dagger] - \frac{1}{2}\kappa A_\mu[\phi, \phi^\dagger] J^\mu[\phi, \phi^\dagger] - \frac{1}{4}\kappa h_{\mu\nu}[\phi, \phi^\dagger] T^{\mu\nu}[\phi, \phi^\dagger].$$

In this formula, $A_\mu[\phi, \phi^\dagger]$ and $h^{\mu\nu}[\phi, \phi^\dagger]$ are understood to be non-local functionals of ϕ given by the solutions to the usual (linearized) field equations for electromagnetism and gravity in curved space. We can write these

$$\Delta_V^{\mu\nu} A_\nu[\phi, \phi^\dagger] = -\kappa J^\mu[\phi, \phi^\dagger]$$

$$\Delta_T^{\mu\nu, \rho\sigma} h_{\rho\sigma}[\phi, \phi^\dagger] = -\frac{1}{2}\kappa T^{\mu\nu}[\phi, \phi^\dagger]. \quad (3.10)$$

where Δ_V and Δ_T are second-order differential operators in the AdS coordinates, and J_μ and $T_{\mu\nu}$ are the usual U(1) current and the stress tensor operators, respectively:

$$J_\mu = igq(\phi\partial_\mu\phi^\dagger - \phi^\dagger\partial_\mu\phi), \quad T_{\mu\nu} = (\partial_\mu\phi\partial_\nu\phi^\dagger + (\mu \leftrightarrow \nu)) - g_{\mu\nu}(\partial_\rho\phi\partial^\rho\phi^\dagger + m^2\phi\phi^\dagger). \quad (3.11)$$

Of course, to even define $A_\mu[\phi, \phi^\dagger]$ and $h^{\mu\nu}[\phi, \phi^\dagger]$ via (3.10), it is necessary to choose a gauge. We will do this below; the final result for the anomalous dimensions is independent of this choice.

According to (3.10), V_{eff} is formally quartic in ϕ , ϕ^\dagger and is $\mathcal{O}(\kappa^2)$. All other terms in the Lagrangian (e.g. $A_\mu A^\mu \phi^\dagger \phi$) contribute to the effective potential only at higher order in ϕ , and hence higher order in κ . Performing canonical quantization, we find that the leading order interaction Hamiltonian density δH_{eff} is also just V_{eff} .³ Therefore, we can use first order perturbation theory to obtain the leading $\mathcal{O}(\kappa^2)$ binding energies of the states $|\phi\phi\rangle$ and $|\phi^\dagger\phi\rangle$:⁴

$$\begin{aligned}\gamma_{\phi\phi} &= \int d^4x \sqrt{-g} \langle \phi\phi | V_{\text{eff}}[\phi, \phi^\dagger] | \phi\phi \rangle = 2 \int d^4x \sqrt{-g} V_{\text{eff}}[\phi, \phi^\dagger] \Big|_{b_0^\dagger b_0^\dagger b_0 b_0} \\ \gamma_{\phi^\dagger\phi} &= \int d^4x \sqrt{-g} \langle \phi^\dagger\phi | V_{\text{eff}}[\phi, \phi^\dagger] | \phi^\dagger\phi \rangle = \int d^4x \sqrt{-g} V_{\text{eff}}[\phi, \phi^\dagger] \Big|_{a_0^\dagger b_0^\dagger a_0 b_0}.\end{aligned}\tag{3.14}$$

This is the main result of this paper. As we shall see, it represents a significantly simpler and more direct calculation of the anomalous dimensions in AdS/CFT compared with previous approaches based on the four point function. In essence, the simplification gained here is due to the fact that the four point function requires full knowledge of the photon and graviton propagators in AdS₅, while this approach only requires certain matrix elements (3.14) of the propagators.

The only nontrivial step at this point is solving for the matrix elements of (3.10), and even here we will find astonishing simplifications due to the high degree of symmetry of the source wavefunctions (3.5). In the following subsections, we will flesh out the rest of this calculation. Some miscellaneous technical details are relegated to appendix A. Since V_{eff} is proportional to κ^2 , we will set $\kappa = 1$ below to avoid cluttering the equations.

3.3 Calculation of $\phi\phi$ anomalous dimension

Let us first evaluate $\gamma_{\phi\phi}$, which is simpler. According to (3.14), we need to extract $b_0^\dagger b_0^\dagger b_0 b_0$ from $V_{\text{eff}}[\phi, \phi^\dagger]$. Since V_{eff} is quartic in ϕ , ϕ^\dagger , there is only one way to do this given the expansion (3.3), namely to pull $\psi_0(x) b_0^\dagger$ from ϕ and $\psi_0^*(x) b_0$ from ϕ^\dagger . So (3.14) becomes

$$\gamma_{\phi\phi} = 2 \int d^4x \sqrt{-g} V_{\text{eff}}[\phi(x) = \psi_0(x), \phi^\dagger(x) = \psi_0^*(x)].\tag{3.15}$$

Using (3.9), the individual quartic, photon, and graviton contributions to (3.15) are as follows:

³From (3.9), we compute the conjugate momenta

$$\Pi_\phi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \dot{\phi}^\dagger - \frac{\partial V_{\text{eff}}}{\partial \dot{\phi}}, \quad \Pi_{\phi^\dagger} = \frac{\partial \mathcal{L}}{\partial \dot{\phi}^\dagger} = \dot{\phi} - \frac{\partial V_{\text{eff}}}{\partial \dot{\phi}^\dagger},\tag{3.12}$$

and then the Hamiltonian density:

$$\mathcal{H} = \Pi_\phi \dot{\phi} + \Pi_{\phi^\dagger} \dot{\phi}^\dagger - \mathcal{L} = |\Pi_\phi|^2 + |\nabla\phi|^2 + m^2|\phi|^2 + V_{\text{eff}} + \dots = \mathcal{H}_{\text{free}} + V_{\text{eff}} + \dots\tag{3.13}$$

where \dots is higher order in ϕ or κ . Note that the linear terms $\propto \frac{\partial V_{\text{eff}}}{\partial \dot{\phi}}$ cancel out when passing to the fields and their conjugate momenta.

⁴Here we ignore self-contractions inside V_{eff} , i.e. we treat it as normal ordered. Such self-contractions correct the mass of ϕ itself, and cancel out of the leading-order binding energy.

1. The easiest term in V_{eff} to evaluate is the contribution from the original scalar potential, V , since this simply involves substituting $\psi_0(x)$, $\psi_0^*(x)$ from (3.5) in the appropriate places. Performing this, we obtain

$$\gamma_{\phi\phi}^{(\text{quartic})} = \frac{\pi^2 N_{\Delta}^4 (a + b\Delta(2 - \Delta))}{(\Delta - 1)(2\Delta - 1)}. \quad (3.16)$$

2. Next, consider the photon contribution. According to (3.9) and (3.15), this takes the form

$$\gamma_{\phi\phi}^{(\text{photon})} = - \int d^4x \sqrt{-g} A_{\mu}[\psi_0(x), \psi_0^*(x)] J^{\mu}[\psi_0(x), \psi_0^*(x)] \quad (3.17)$$

where now A_{μ} is now a function of x (rather than an operator) which satisfies (3.10) with $\phi(x) \rightarrow \psi_0(x)$, $\phi^{\dagger}(x) \rightarrow \psi_0^*(x)$. We note that $J^{\mu}[\psi_0(x), \psi_0^*(x)]$ is time-independent, and has a very simple form:

$$J^0[\psi_0(x), \psi_0^*(x)] = -2\Delta N_{\Delta}^2 g q y^{2\Delta+2}, \quad J^i[\psi_0(x), \psi_0^*(x)] = 0. \quad (3.18)$$

where $y \equiv \cos \rho$. The lack of any current component or time-dependence means that we may choose a gauge where the only non-vanishing component of A_{μ} is the potential A_0 . Solving (3.10) with the source (3.18), we find a simple formula for A_0 (for details, see appendix A):

$$A_0 = - \frac{N_{\Delta}^2 g q (y^2 - y^{2\Delta})}{2(\Delta - 1)(1 - y^2)}. \quad (3.19)$$

Substituting back into (3.17), we obtain

$$\gamma_{\phi\phi}^{(\text{photon})} = \frac{\pi^2 N_{\Delta}^4 g^2 q^2}{2\Delta - 1}. \quad (3.20)$$

3. Finally, let us evaluate the graviton contribution. According to (3.9) and (3.15), this takes the form

$$\gamma_{\phi\phi}^{(\text{graviton})} = - \frac{1}{2} \int d^4x \sqrt{-g} h^{\mu\nu}[\psi_0(x), \psi_0^*(x)] T_{\mu\nu}[\psi_0(x), \psi_0^*(x)]. \quad (3.21)$$

The energy-momentum tensor $T_{\mu\nu}[\psi_0, \psi_0^*]$ is again time-independent, with a simple form:

$$T_{\nu}^{\mu}[\psi_0(x), \psi_0^*(x)] = 2\Delta N_{\Delta}^2 y^{2\Delta} \cdot \text{diag}(2 - \Delta, 2, 2 - \Delta + \Delta y^2, \dots, 2 - \Delta + \Delta y^2). \quad (3.22)$$

Solving (3.10) with the source (3.22), we again find a simple result:

$$h_{tt} = \frac{2\Delta N_{\Delta}^2 (y^2 - y^{2\Delta})}{3(\Delta - 1)(1 - y^2)}, \quad h_{\rho\rho} = h_{tt} - \frac{2\Delta N_{\Delta}^2}{3} y^{2\Delta-2}, \quad (3.23)$$

with all other metric components vanishing. The energy shift for $\phi\phi$ may now be calculated by substituting (3.22) and (3.23) into (3.21). We obtain

$$\gamma_{\phi\phi}^{(\text{graviton})} = - \frac{2\pi^2 N_{\Delta}^4 \Delta^2 (\Delta - 2)}{3(\Delta - 1)(2\Delta - 1)}. \quad (3.24)$$

Finally, let us put together all the different contributions. The $\phi\phi$ anomalous dimension is the sum of all contributions (3.16), (3.20), and (3.24). Because $\phi\phi$ is a chiral operator, its dimension is protected by supersymmetry, so $\gamma_{\phi\phi}$ must vanish. Using (2.5) and (2.7), the result is

$$\gamma_{\phi\phi} = \gamma_{\phi\phi}^{(\text{quartic})} + \gamma_{\phi\phi}^{(\text{photon})} + \gamma_{\phi\phi}^{(\text{graviton})} = \frac{\pi^2 N_{\Delta}^4 (2\Delta^2 + 3a - 3b\Delta(\Delta - 2))}{3(\Delta - 1)(2\Delta - 1)} = 0, \quad (3.25)$$

where the fact that $\gamma_{\phi\phi}$ must vanish enforces the relation (2.8) between a and b .

3.4 Calculation of $\phi^\dagger\phi$ anomalous dimension

The calculation of $\gamma_{\phi^\dagger\phi}$ in (3.14) is more difficult, because there are multiple ways to get $a_0^\dagger b_0^\dagger a_0 b_0$ from $V_{\text{eff}}[\phi, \phi^\dagger]$. Essentially this is because now both t -channel and s -channel terms contribute. To aid in handling all possible contractions systematically, let us label each of the four ϕ 's in V_{eff} separately:

$$V_{\text{eff}}[\phi_1, \phi_2; \phi_1^\dagger, \phi_2^\dagger] = a(\phi_1\phi_1^\dagger)(\phi_2\phi_2^\dagger) + b(\phi_1\phi_1^\dagger)(\partial_\mu\phi_2\partial^\mu\phi_2^\dagger) - \frac{1}{2}A_\mu[\phi_1, \phi_1^\dagger]J^\mu[\phi_2, \phi_2^\dagger] - \frac{1}{4}h^{\mu\nu}[\phi_1, \phi_1^\dagger]T_{\mu\nu}[\phi_2, \phi_2^\dagger]. \quad (3.26)$$

Thus we have $V_{\text{eff}}[\phi, \phi^\dagger] = V_{\text{eff}}[\phi_1 = \phi, \phi_2 = \phi; \phi_1^\dagger = \phi^\dagger, \phi_2^\dagger = \phi^\dagger]$.

Note that there is a symmetry of V_{eff} under interchange of the ϕ_i 's that will be useful in reducing the number of terms to be evaluated. If we switch $\phi_1 \leftrightarrow \phi_2$ and $\phi_1^\dagger \leftrightarrow \phi_2^\dagger$, then via integration by parts V_{eff} remains unchanged:

$$V_{\text{eff}}[\phi_1, \phi_2; \phi_1^\dagger, \phi_2^\dagger] = V_{\text{eff}}[\phi_2, \phi_1; \phi_2^\dagger, \phi_1^\dagger]. \quad (3.27)$$

This identity is true term by term in (3.26).

With this new notation, the contribution from all contractions is very simply stated. Extracting $a_0^\dagger b_0^\dagger a_0 b_0$ from (3.26), and using (3.27), we obtain

$$\begin{aligned} \gamma_{\phi^\dagger\phi} &= 2 \int d^4x \sqrt{-g} V_{\text{eff}}[\phi_1 = \psi_0(x), \phi_2 = \psi_0^*(x); \phi_1^\dagger = \psi_0(x), \phi_2^\dagger = \psi_0^*(x)] \\ &\quad + 2 \int d^4x \sqrt{-g} V_{\text{eff}}[\phi_1 = \psi_0(x), \phi_2 = \psi_0^*(x); \phi_1^\dagger = \psi_0^*(x), \phi_2^\dagger = \psi_0(x)]. \end{aligned} \quad (3.28)$$

The individual quartic, photon and graviton contributions are now (keep in mind we are setting $\kappa = 1$ for convenience):

1. The easiest term in V_{eff} to evaluate is again the contribution from the original scalar potential, V , since this simply involves substituting $\psi_0(x)$, $\psi_0^*(x)$ in the appropriate places. Performing this, we obtain

$$\gamma_{\phi^\dagger\phi}^{(\text{quartic})} = \frac{2\pi^2 N_{\Delta}^4 (a + b\Delta)}{(\Delta - 1)(2\Delta - 1)}. \quad (3.29)$$

2. Next, consider the photon contribution from (3.27). This is, explicitly,

$$\gamma_{\phi^\dagger\phi}^{(\text{photon})} = - \int d^4x \sqrt{-g} \left(A_\mu[\psi_0, \psi_0] J^\mu[\psi_0^*, \psi_0^*] + A_\mu[\psi_0, \psi_0^*] J^\mu[\psi_0^*, \psi_0] \right). \quad (3.30)$$

Since $J^\mu[\psi_1, \psi_2]$ is antisymmetric in its arguments, the first term clearly vanishes, and the second gives the opposite of the $\phi\phi$ anomalous dimension (3.17). Therefore we conclude that

$$\gamma_{\phi^\dagger\phi}^{(\text{photon})} = -\gamma_{\phi\phi}^{(\text{photon})} = -\frac{\pi^2 N_\Delta^4 g^2 q^2}{2\Delta - 1}. \quad (3.31)$$

3. Now we come to the graviton contribution. This is

$$\gamma_{\phi^\dagger\phi}^{(\text{graviton})} = -\frac{1}{2} \int d^4x \sqrt{-g} \left(h_{\mu\nu}[\psi_0, \psi_0] T^{\mu\nu}[\psi_0^*, \psi_0^*] + h_{\mu\nu}[\psi_0, \psi_0^*] T^{\mu\nu}[\psi_0^*, \psi_0] \right). \quad (3.32)$$

In contrast to the photon case, here $T^{\mu\nu}$ is symmetric in its arguments, so the first term no longer vanishes, and the second term is equal to the $\phi\phi$ anomalous dimension (3.21). The first term corresponds to s-channel graviton exchange. To evaluate it, we need to take into account the fact that $T_{\mu\nu}[\psi_0^*, \psi_0^*]$ is time-dependent. So the metric response is as well:

$$h_{\rho\rho} = \frac{2}{3} N_\Delta^2 \Delta (y^2 - 1) y^{2\Delta-2} e^{2i\Delta t}, \quad h_{tt} = -\frac{2N_\Delta^2 \Delta (\Delta - 1)}{3(\Delta + 1)} y^{2\Delta} e^{2i\Delta t}. \quad (3.33)$$

Substituting this into (3.32), we find

$$-\frac{1}{2} \int d^4x \sqrt{-g} h_{\mu\nu}[\psi_0, \psi_0] T^{\mu\nu}[\psi_0^*, \psi_0^*] = -\frac{2\pi^2 N_\Delta^4 \Delta^2 (2\Delta - 5)}{3(\Delta - 1)(2\Delta - 1)(2\Delta + 1)}. \quad (3.34)$$

Adding this to (3.21), we obtain

$$\gamma_{\phi^\dagger\phi}^{(\text{graviton})} = -\frac{2\pi^2 N_\Delta^4 \Delta^2 (2\Delta^2 - \Delta - 7)}{3(\Delta - 1)(2\Delta - 1)(2\Delta + 1)}. \quad (3.35)$$

Finally, let us put together all the different contributions (3.29), (3.31), and (3.35). The total $\phi^\dagger\phi$ anomalous dimension is:

$$\gamma_{\phi^\dagger\phi} = \gamma_{\phi^\dagger\phi}^{(\text{quartic})} + \gamma_{\phi^\dagger\phi}^{(\text{photon})} + \gamma_{\phi^\dagger\phi}^{(\text{graviton})} = \frac{2\pi^2 N_\Delta^4 \Delta}{2\Delta - 1} \left(b - \frac{2\Delta(2\Delta + 3)}{3(2\Delta + 1)} \right). \quad (3.36)$$

In the second equation, we have again substituted (2.8) for a . Thus $\gamma_{\phi^\dagger\phi}$ depends on two parameters — Δ and b . A contour plot of $\gamma_{\phi^\dagger\phi}$ is shown in figure 1. At large Δ , the anomalous dimension is always negative. This fact can be understood physically by noting that at large Δ , the wavefunctions ψ_0 are very narrowly concentrated at small ρ , and thus the binding energies are controlled by the flat-space limit of AdS. In this limit, the gravitational and electromagnetic binding energies dominate over the contact term. Since both of these forces between two particles of opposite charge are attractive, the binding energy is always negative at large Δ .

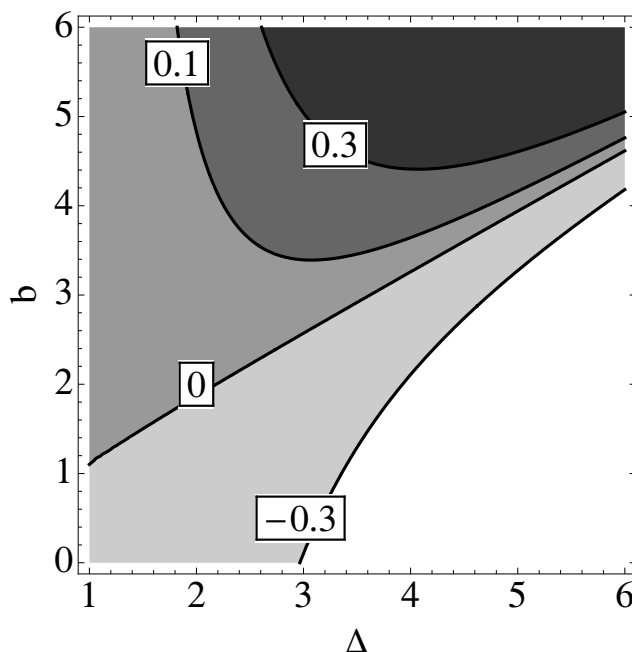


Figure 1. Contours of $\gamma_{\phi^\dagger\phi}$ as a function of the parameters b and Δ . Positive $\gamma_{\phi^\dagger\phi}$ occurs only for $b > \frac{10}{9}$, and then for Δ below a critical value that grows with increasing b .

More generally, however, the anomalous dimension can take either sign. For $b > \frac{10}{9}$, it becomes positive in the range

$$1 \leq \Delta < \frac{3}{4} \left(\sqrt{b^2 - \frac{2}{3}b + 1} + (b - 1) \right). \quad (3.37)$$

For $b < \frac{10}{9}$, $\gamma_{\phi^\dagger\phi}$ is negative for all $\Delta \geq 1$.

4 Anomalous dimensions via the four point function in AdS/CFT

4.1 Anomalous dimensions via the four point function in general

In this section, we will obtain the leading-order anomalous dimensions a different way, by calculating the four-point function and then taking different short distance limits where the lowest dimension operators in the $\phi\phi$ and $\phi^\dagger\phi$ OPE's dominate the expansion in conformal blocks. This will serve as a check of our new and improved method presented in the previous section. The more casual reader should feel free to skip over this technical section and head directly for section 5.

We start by summarizing how one calculates operator dimensions from the four point function in general CFTs. Consider the four point function $\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_1^\dagger(x_3)\mathcal{O}_2^\dagger(x_4) \rangle$ involving two scalar primaries \mathcal{O}_1 and \mathcal{O}_2 . We will restrict our attention to the special case where both \mathcal{O}_1 and \mathcal{O}_2 have the same dimension Δ . This four-point function is constrained

by conformal symmetry to take the form:

$$\mathcal{C}_{\mathcal{O}_1\mathcal{O}_2}(x_1, x_2, x_3, x_4) \equiv \langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_1^\dagger(x_3)\mathcal{O}_2^\dagger(x_4) \rangle = \frac{\mathcal{F}_{\mathcal{O}_1\mathcal{O}_2}(u, v)}{x_{12}^{2\Delta} x_{34}^{2\Delta}}, \quad (4.1)$$

where u and v are the conformal cross ratios:

$$u \equiv \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, \quad v \equiv \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}. \quad (4.2)$$

It is often convenient to change variables from u and v to x and z , defined by $u = xz$, $v = (1-x)(1-z)$.

In general, information about $\mathcal{O}_1 \times \mathcal{O}_2$, including the dimension and OPE coefficients of all operators appearing in the OPE, can be extracted from (4.1) in the $x_1 \rightarrow x_2$ limit. In this limit, one has $u \rightarrow 0$ and $v \rightarrow 1$ (equivalently, $x, z \rightarrow 0$), and $\mathcal{F}_{\mathcal{O}_1\mathcal{O}_2}(u, v)$ can be expanded in conformal blocks:

$$\mathcal{F}_{\mathcal{O}_1\mathcal{O}_2}(u, v) = \sum_{\mathcal{O}} |C_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}}|^2 G(\Delta_{\mathcal{O}}, \ell_{\mathcal{O}}; x, z). \quad (4.3)$$

Here the sum is over conformal primaries \mathcal{O} (with dimension $\Delta_{\mathcal{O}}$ and spin $\ell_{\mathcal{O}}$) appearing in the OPE of $\mathcal{O}_1\mathcal{O}_2$, and $C_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}}$ is the OPE coefficient.⁵ In [36, 37], Dolan & Osborn discovered a simple closed form expression for the conformal blocks. If we take $x = \epsilon a$ and $z = \epsilon b$ with $\epsilon \rightarrow 0$, we find from their general result that

$$G(\Delta_{\mathcal{O}}, \ell_{\mathcal{O}}; x = \epsilon a, z = \epsilon b) = \epsilon^{\Delta_{\mathcal{O}}} (ab)^{\frac{\Delta_{\mathcal{O}} - \ell_{\mathcal{O}}}{2}} \left(\frac{a^{\ell_{\mathcal{O}}+1} - b^{\ell_{\mathcal{O}}+1}}{a - b} \right) + \dots \quad (4.4)$$

So the expansion in conformal blocks (4.3) can be thought of as an expansion in operator dimensions.

Now let us specialize to the problem at hand: determining the leading-order anomalous dimension of the double-trace scalar operator $\mathcal{O}_{\min} = \mathcal{O}_1\mathcal{O}_2$ in an SCFT with a local, weakly-coupled gravity dual. According to (4.4), perturbing $\Delta_{\mathcal{O}} = \Delta_{\mathcal{O}}^{(0)} + \kappa^2 \gamma_{\mathcal{O}}$ gives rise to log u -singular terms at leading order in κ^2 :

$$\mathcal{F}_{\mathcal{O}_1\mathcal{O}_2}(u, v) = \frac{1}{2} \kappa^2 \log u \sum_{\mathcal{O}} \gamma_{\mathcal{O}} |C_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}}^{(0)}|^2 \epsilon^{\Delta_{\mathcal{O}}^{(0)}} (ab)^{\frac{\Delta_{\mathcal{O}}^{(0)} - \ell_{\mathcal{O}}}{2}} \left(\frac{a^{\ell_{\mathcal{O}}+1} - b^{\ell_{\mathcal{O}}+1}}{a - b} \right) + \dots \quad (4.5)$$

Here $C_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}}^{(0)}$ are the OPE coefficients in the free theory. But the $\mathcal{O}_1 \times \mathcal{O}_2$ free OPE is equivalent to Taylor expansion: only double-trace operators of the schematic form

$$\mathcal{O} = \mathcal{O}_1 \overleftrightarrow{\partial}_{\mu_1} \dots \overleftrightarrow{\partial}_{\mu_\ell} (\overleftrightarrow{\partial})^{2n} \mathcal{O}_2 \quad (4.6)$$

appear. These have dimension and spin given by

$$\Delta_{\mathcal{O}}^{(0)} = 2\Delta + 2n + \ell, \quad \ell_{\mathcal{O}} = \ell \quad (n, \ell \geq 0) \quad (4.7)$$

⁵The overall normalizations of the OPE coefficients are fixed by requiring all the primary operators to have canonically normalized two-point functions. For example, for scalar primaries, one has $\langle \mathcal{O}^\dagger(x)\mathcal{O}(0) \rangle = |x|^{-2\Delta_{\mathcal{O}}}$.

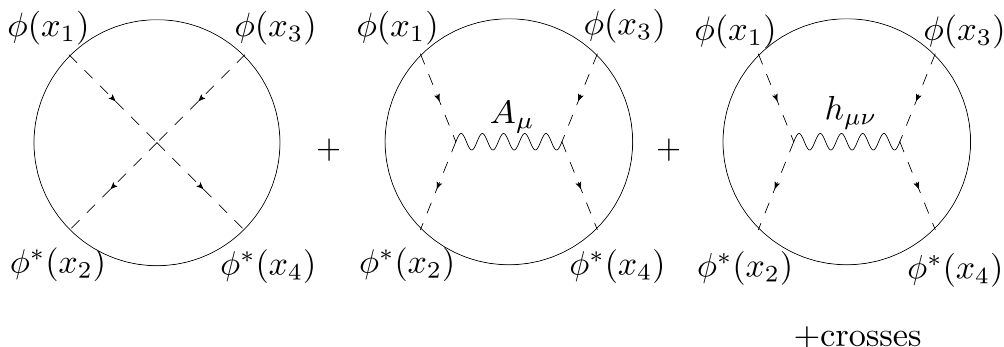


Figure 2. Witten Diagrams for the quartic contact interaction (left) and photon, graviton exchange (middle, right).

in the free theory. So their dimension is bounded from below by 2Δ , saturated only by $\mathcal{O}_{\min} = \mathcal{O}_1\mathcal{O}_2$ with $n = \ell = 0$. Therefore, in the expansion (4.5), the leading term has u^Δ and corresponds to precisely the operator we are interested in. We conclude that

$$\mathcal{F}_{\mathcal{O}_1\mathcal{O}_2}(u, v) = \frac{1}{2}\kappa^2\gamma_{\mathcal{O}_{\min}}|C_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_{\min}}|^2 u^\Delta \log u + \dots \tag{4.8}$$

That is, by selecting out the leading $\log u$ singularity in the $\mathcal{O}(\kappa^2)$ correction to the four-point function, we can infer the leading-order anomalous dimension of $\mathcal{O}_1\mathcal{O}_2$.

Below, we will derive the leading-order anomalous dimensions for $(\mathcal{O}_1, \mathcal{O}_2) = (\phi, \phi)$ and (ϕ^\dagger, ϕ) , using (4.8). For our purposes, we will need

$$|C_{\phi, \phi, \phi, \phi}^{(0)}|^2 = 2, \quad |C_{\phi^\dagger, \phi, \phi^\dagger, \phi}^{(0)}|^2 = 1 \tag{4.9}$$

which follows from requiring $\phi\phi$ and $\phi^\dagger\phi$ to have canonically normalized two-point functions.

4.2 AdS/CFT calculation of four-point functions

The $\mathcal{O}(\kappa^2)$ correction to the four-point function is calculated using the standard techniques of AdS/CFT. For the model (2.2), the relevant Witten diagrams are shown in figure 2. We see that there are three types of contributions to the four-point function in general — those from scalar quartic interactions, photon exchange, and graviton exchange. We will separate out these contributions and write

$$\begin{aligned} \mathcal{C}_{\mathcal{O}_1\mathcal{O}_2}(x_1, x_2, x_3, x_4) &= \mathcal{C}_{\mathcal{O}_1\mathcal{O}_2}^{(\text{quartic})}(x_1, x_2, x_3, x_4) + \mathcal{C}_{\mathcal{O}_1\mathcal{O}_2}^{(\text{photon})}(x_1, x_2, x_3, x_4) \\ &\quad + \mathcal{C}_{\mathcal{O}_1\mathcal{O}_2}^{(\text{graviton})}(x_1, x_2, x_3, x_4). \end{aligned} \tag{4.10}$$

Witten diagram calculations are generally more tractable in Euclidean AdS Poincaré patch coordinates

$$ds^2 = \frac{d\vec{w}^2 + (dw^0)^2}{(w^0)^2}. \tag{4.11}$$

In what follows, we will need the formula for the bulk-to-boundary propagator:

$$K_{\Delta}(w, \vec{x}) = N_{\Delta} \left(\frac{w^0}{(w^0)^2 + (\vec{w} - \vec{x})^2} \right)^{\Delta}. \tag{4.12}$$

The normalization factor is fixed by imposing the canonically normalized two-point function in the CFT; it is exactly equal to the normalization of the ground-state single-particle wavefunction $\psi_0(x)$ derived in section 3.⁶ Note that for indices, we will be following the conventions of [40]. So in general, indices will be raised, lowered, and contracted using the flat Euclidean metric $\delta_{\mu\nu}$. For instance, $w^{\mu} = w_{\mu}$. This includes the squares of coordinates, e.g. $w^2 = w^{\mu}w^{\mu}$, etc.. When the AdS metric is called for, it will be exhibited explicitly.

Now we are finally ready to derive the leading $1/N$ correction to the anomalous dimensions, using (4.8), (4.9), and (4.10), starting with the $\phi\phi$ case.

4.3 $\phi\phi$ anomalous dimension

As discussed in the previous subsection, the four-point function has three contributions:

$$\begin{aligned} \mathcal{C}_{\phi\phi}(x_1, x_2, x_3, x_4) &= \mathcal{C}_{\phi\phi}^{(\text{quartic})}(x_1, x_2, x_3, x_4) + \mathcal{C}_{\phi\phi}^{(\text{photon})}(x_1, x_2, x_3, x_4) \\ &\quad + \mathcal{C}_{\phi\phi}^{(\text{graviton})}(x_1, x_2, x_3, x_4). \end{aligned} \tag{4.13}$$

Using the Lagrangian (2.2), we obtain for the scalar contribution:

$$\begin{aligned} \mathcal{C}_{\phi\phi}^{(\text{quartic})} &= -a \int d^5w \sqrt{g} K_{\Delta}(w, \vec{x}_1) K_{\Delta}(w, \vec{x}_2) K_{\Delta}(w, \vec{x}_3) K_{\Delta}(w, \vec{x}_4) \\ &\quad - b \int d^5w \sqrt{g} g^{\mu\nu}(w) \partial_{\mu} K_{\Delta}(w, \vec{x}_1) K_{\Delta}(w, \vec{x}_2) \partial_{\nu} K_{\Delta}(w, \vec{x}_3) K_{\Delta}(w, \vec{x}_4) \\ &\quad + (\vec{x}_1 \leftrightarrow \vec{x}_2, \vec{x}_3 \leftrightarrow \vec{x}_4). \end{aligned} \tag{4.14}$$

It is straightforward to extract the log u divergence in (4.14) in the limit $x_1 \rightarrow x_2, x_4 \rightarrow \infty$. (We can always take $x_4 \rightarrow \infty$ via a conformal transformation, without loss of generality.) After a shift in the integration variable $\vec{w} \rightarrow \vec{w} + \vec{x}_1$, the divergence comes from the region of integration where $w^0 \sim |\vec{w}| \sim |x_{12}| \ll |x_{13}|$. Then a quick way to extract the coefficient of the log divergence is to set $x_1 = x_2$ in the integrand, and cutoff the divergent integral at $|x_{12}|$. The result is

$$\lim_{x_1 \rightarrow x_2} \mathcal{F}_{\phi\phi}^{(\text{quartic})} = \frac{\pi^2 \beta_{\Delta}^2 (a + b\Delta(2 - \Delta))}{(2\Delta - 1)(\Delta - 1)} u^{\Delta} \log u + \dots \tag{4.15}$$

⁶For $1 < \Delta < 2$, there are two available branches Δ_+, Δ_- of the mass-dimension relation $m^2 = \Delta(\Delta - 4)$, and one must modify the AdS/CFT prescription for obtaining correlation functions. There are several equivalent procedures: the earliest method was to use (4.12) and Legendre transform the result [38]; alternatively, one may impose modified boundary conditions for the bulk field in order to select the smaller branch Δ_- , in which case the bulk-to-boundary propagator is modified [39]. In either approach, the answers at the end of the day seem to be trivially obtainable by ignoring all these subtleties and just analytically continuing from $\Delta > 2$ to $\Delta < 2$ [39]. This is what we will do here. We note that an advantage of the method in section 3 is that such subtleties never arise. There, the wavefunctions ψ_0 are valid for any Δ above the unitarity bound.

where ... is higher order in u . Using (4.8) and (4.9), we obtain the anomalous dimension quoted in (3.16).⁷

For the photon contribution:

$$\mathcal{C}_{\phi\phi}^{(\text{photon})} = \frac{q^2}{2!} \int d^5w \sqrt{g} g^{\mu\nu}(w) J_\mu(w; \vec{x}_2, \vec{x}_4) I_\nu(w; \vec{x}_1, \vec{x}_3) + (x_1 \leftrightarrow x_2, x_3 \leftrightarrow x_4). \quad (4.16)$$

Here

$$J_\mu(w; x, y) = i \left(K_\Delta(w, \vec{x}) \partial_\mu K_\Delta(w, \vec{y}) - K_\Delta(w, \vec{y}) \partial_\mu K_\Delta(w, \vec{x}) \right), \quad (4.17)$$

is the U(1) current (3.11) evaluated with bulk-to-boundary propagators, and

$$I_\mu(w; \vec{x}_1, \vec{x}_3) = \int d^5z \sqrt{g} g^{\nu\rho}(z) J_\nu(z; \vec{x}_1, \vec{x}_3) G_{\rho\mu}(z, w) \quad (4.18)$$

with $G_{\rho\mu}(z, w)$ being the massless photon propagator (explicit formulas are given in [41, 42]). We obtain a closed-form expression for $I_\mu(w; \vec{x}_1, \vec{x}_3)$ using the ingenious method of [43], whereby the problem of integrating over z is converted into the much easier problem of solving a differential equation in w . The differential equation follows from the fact that the photon propagator is a Green's function in AdS₅.⁸ Solving this differential equation with appropriate boundary conditions, one finds:

$$I_\mu(w; \vec{x}_1, \vec{x}_3) = -\frac{iN_\Delta^2}{2(\Delta-1)} |\vec{x}_{13}|^{-2\Delta} \left(\frac{(w-x_3)^\mu}{(w-x_3)^2} - \frac{(w-x_1)^\mu}{(w-x_1)^2} \right) f(t) \quad (4.19)$$

where

$$\begin{aligned} f(t) &\equiv \frac{t(1-t^{\Delta-1})}{1-t} \\ t &\equiv \frac{(w^0)^2 \vec{x}_{13}^2}{(w-x_1)^2 (w-x_3)^2}. \end{aligned} \quad (4.20)$$

This is explained in more detail in appendix B.

To extract the $\log u$ divergence from (4.16)(4.19)(4.20), we must first expand $f(t)$ in powers of t :

$$f(t) = \sum_{k \geq 1} t^k - \sum_{k \geq 1} t^{k+\Delta-1}. \quad (4.21)$$

Using the fact that $t^\alpha \sim |\vec{x}_{13}|^{2\alpha} K_\alpha(w, \vec{x}_1) K_\alpha(w, \vec{x}_3)$ (up to normalization), the $\log u$ divergence from each term in the series representation of $f(t)$ can be extracted as in the scalar case above. The answer for a given power of t is:

$$\lim_{x_1 \rightarrow x_2} \mathcal{F}_{\phi\phi}^{(\text{photon})} \Big|_{t^\alpha} = -\frac{2\pi^2 N_\Delta^4 q^2 \Delta}{(\alpha + \Delta - 1)(\alpha + \Delta)} u^\Delta \log u + \dots \quad (4.22)$$

Substituting this in (4.21), we obtain the photon anomalous dimension quoted in (3.20).

⁷One can also extract the $\log u$ divergence more properly (and painfully) using the identities for D and H functions listed in [36], see especially (C.8) of that paper. We have checked all our results here using this method.

⁸In [43], arbitrary spacetime dimension d was considered, but attention was restricted to specific values of the operator dimension Δ . Here we focus on $d = 4$, and we find that I_μ can be obtained for any Δ .

Finally, the graviton contribution is:

$$\mathcal{C}_{\phi\phi}^{(\text{graviton})} = \frac{1}{2! \cdot 4} \int d^5 w \sqrt{g} g^{\mu\rho}(w) g^{\nu\lambda}(w) T_{\mu\nu}(w; \vec{x}_2, \vec{x}_4) I_{\rho\lambda}(w; \vec{x}_1, \vec{x}_3) + (x_1 \leftrightarrow x_2, x_3 \leftrightarrow x_4). \quad (4.23)$$

Here

$$T_{\mu\nu}(w; x, y) = \left(\partial_\mu K_\Delta(w, \vec{x}) \partial_\nu K_\Delta(w, \vec{y}) + (\mu \leftrightarrow \nu) \right) - g_{\mu\nu} \left(g^{\rho\lambda}(w) \partial_\rho K_\Delta(w, \vec{x}) \partial_\lambda K_\Delta(w, \vec{y}) + m^2 K_\Delta(w, \vec{x}) K_\Delta(w, \vec{y}) \right) \quad (4.24)$$

is the stress-energy tensor (3.11) evaluated on the bulk-to-boundary propagators, and

$$I_{\mu\nu}(w; \vec{x}_1, \vec{x}_3) = \int d^5 z \sqrt{g} g^{\mu'\rho'}(z) g^{\nu'\lambda'}(z) T_{\mu'\nu'}(z; \vec{x}_1, \vec{x}_3) G_{\rho'\lambda'\mu\nu}(z, w) \quad (4.25)$$

with $G_{\rho'\lambda'\mu\nu}(z, w)$ being the graviton propagator (explicit formulas in [42]). Using again the methods of [43], we obtain for $I_{\mu\nu}$:

$$I_{\mu\nu}(w; \vec{x}_1, \vec{x}_3) = \frac{N_\Delta^2 \Delta}{(\Delta - 1)} |\vec{x}_{31}|^{-2\Delta} \frac{1}{(w^0)^2} \left(\frac{1}{3} \delta_{\mu\nu} - J_{\mu 0}(w - x_1) J_{\nu 0}(w - x_1) \right) f(t) + (\text{gauge - dependent}) \quad (4.26)$$

where $J_{\mu\nu}(w) = \delta_{\mu\nu} - 2w^\mu w^\nu / w^2$ is the inversion tensor. Note that (4.26) is not symmetric in $x_1 \leftrightarrow x_3$. This is because we have neglected gauge-dependent terms in evaluating the z integral. These must drop out after doing the w integral.

Repeating the same manipulations as for the scalar and photon cases, we obtain

$$\lim_{x_1 \rightarrow x_2} \mathcal{F}_{\phi\phi}^{(\text{graviton})} \Big|_{t^\alpha} = \frac{4\pi^2 N_\Delta^4 \Delta^2 (\Delta(\Delta - 4) + \alpha(\Delta + 2))}{3(\alpha + \Delta)(\alpha + \Delta - 1)(\alpha + \Delta - 2)} u^\Delta \log u + \dots \quad (4.27)$$

Substituting this in (4.21), we obtain the graviton anomalous dimension quoted in (3.24).

4.4 $\phi^\dagger\phi$ anomalous dimension

Next we consider the $\phi^\dagger\phi$ anomalous dimensions. These are more complicated, because generally they have contributions from s -channel diagrams as well as t, u channel diagrams. Here it makes sense to separate out the s -channel contribution, because the t and u channel pieces have already been computed above.

The four-point function relevant to $\phi^\dagger\phi$ is easily obtained from that for $\phi\phi$:

$$\mathcal{C}_{\phi^\dagger\phi}(x_1, x_2, x_3, x_4) = \mathcal{C}_{\phi\phi}(x_1, x_3, x_2, x_4) \quad (4.28)$$

Applying this to (4.14), and extracting the $\log u$ divergence as above, we find

$$\lim_{x_1 \rightarrow x_2} \mathcal{F}_{\phi^\dagger\phi}^{(\text{quartic})} = \left(\frac{(a + b\Delta^2)}{2(2\Delta - 1)(\Delta - 1)} + \frac{(a + b\Delta(2 - \Delta))}{2(2\Delta - 1)(\Delta - 1)} \right) \pi^2 N_\Delta^4 u^\Delta \log u + \dots \quad (4.29)$$

The first term is the s -channel contribution; the second term is the contribution of the $t+u$ channels, which we have computed above.

For the photon interaction, we obtain the opposite of the $\phi\phi$ case (4.22), as expected (the factor of 2 difference is due to (4.9)):

$$\lim_{x_1 \rightarrow x_2} F_{\phi^\dagger \phi}^{(\text{photon})} \Big|_{t^\alpha} = \frac{q^2 \Delta}{(\alpha + \Delta - 1)(\alpha + \Delta)} \pi^2 N_\Delta^4 u^\Delta \log u + \dots \quad (4.30)$$

Here the s -channel contribution is found to vanish — essentially there is an extra suppression as $x_1 \rightarrow x_2$ from $J_\mu(w; \vec{x}_1, \vec{x}_2) \rightarrow 0$ in (4.16). Meanwhile, the $t + u$ channels differ by a sign from (4.22), which is a consequence of $J_\mu(w; x, y) = -J_\mu(w; y, x)$. This all agrees perfectly with (3.31).

Finally, for the graviton interaction, we find

$$\lim_{x_1 \rightarrow x_2} \mathcal{F}_{\phi^\dagger \phi}^{(\text{graviton})} \Big|_{t^\alpha} = \left(-\delta_\alpha \frac{\Delta^2(2\Delta - 5)}{3(2\Delta - 1)(2\Delta + 1)} + \frac{\Delta^2(\Delta(\Delta - 4) + \alpha(\Delta + 2))}{3(\alpha + \Delta)(\alpha + \Delta - 1)(\alpha + \Delta - 2)} \right) \times \pi^2 N_\Delta^4 u^\Delta \log u + \dots \quad (4.31)$$

Here the s -channel only contributes for a certain power of t ; the other channels are again the same as above. The anomalous dimension obtained from (4.31) is in perfect agreement with (3.35).

5 Example: $SU(2, 1)/U(2) \times U(1)$ coset

Here we will study an explicit example of $\mathcal{N} = 2$, $d = 5$ gauged supergravity with one hypermultiplet and no vector multiplets. This is the sigma model whose target space is the quaternionic Kähler manifold

$$\mathcal{M} = SU(2, 1)/(SU(2) \times U(1)). \quad (5.1)$$

In $d = 4$, this sigma model corresponds to the “universal hypermultiplet” (containing the axion-dilaton) of type II compactification on a general Calabi-Yau threefold [44]. In $d = 5$, it similarly describes the axion-dilaton sector of general type II compactifications on $AdS_5 \times SE_5$ [45–48]. This space is studied in fairly explicit detail in the literature; we have found [26, 27, 49] to be especially useful. Generalizations to n hypermultiplets exist, and it would be interesting to study these as well.

A convenient representation of the coset is:

$$L(z) = \frac{1}{1 - z^\dagger z} \begin{pmatrix} \mathbf{1}_2(1 - z^\dagger z) + 2zz^\dagger & 2z \\ 2z^\dagger & 1 + z^\dagger z \end{pmatrix} \in SU(2, 1) \quad (5.2)$$

where $z = (z^1, z^2)$ are complex coordinates on \mathcal{M} .⁹ The unique left- and right-invariant metric on \mathcal{M} is inherited from

$$ds^2 = \frac{1}{4} \text{Tr} L^{-1} dL L^{-1} dL = -\frac{1}{4} \text{Tr} \eta dL^\dagger \eta dL \quad (5.4)$$

⁹The complex coordinates used here are related to the real coordinates $(V, \sigma, \theta, \tau)$ in [26] via

$$V = \frac{1 - z^\dagger z}{(1 + z^1)(1 + \bar{z}^1)}, \quad \sigma = -2\text{Im} \left(\frac{z^1}{1 + z^1} \right), \quad \theta = \text{Re} \left(\frac{z^2}{1 + z^1} \right), \quad \tau = -\text{Im} \left(\frac{z^2}{1 + z^1} \right). \quad (5.3)$$

up to multiplication by a constant. (Here the constant is fixed to agree with the conventions in the literature, e.g. [26].) Explicitly we have:

$$ds^2 = 2g_{i\bar{j}}dz^i d\bar{z}^{\bar{j}}, \quad g_{i\bar{j}} = \frac{\bar{z}^{\bar{i}}z^j}{(1-z^\dagger z)^2} + \frac{\delta_{ij}}{(1-z^\dagger z)}, \quad (5.5)$$

which comes from the following Kähler potential:

$$\mathcal{K} = -\log(1 - z^\dagger z). \quad (5.6)$$

The manifold \mathcal{M} has an $SU(2,1)$ isometry group. The Killing vectors $\delta_a z^i = K_a^i$ ($a = 1, \dots, 8$) are:

$$\begin{aligned} K_1 &= -i \begin{pmatrix} z^2 \\ z^1 \end{pmatrix}, & K_2 &= \begin{pmatrix} -z^2 \\ z^1 \end{pmatrix}, & K_3 &= i \begin{pmatrix} -z^1 \\ z^2 \end{pmatrix}, & K_4 &= i \begin{pmatrix} z^1 \\ z^2 \end{pmatrix} \\ K_5 &= \begin{pmatrix} 1 - (z^1)^2 \\ -z^1 z^2 \end{pmatrix}, & K_6 &= i \begin{pmatrix} 1 + (z^1)^2 \\ z^1 z^2 \end{pmatrix}, & K_7 &= \begin{pmatrix} -z^1 z^2 \\ 1 - (z^2)^2 \end{pmatrix}, & K_8 &= i \begin{pmatrix} z^1 z^2 1 + (z^2)^2 \end{pmatrix}. \end{aligned} \quad (5.7)$$

Note that $K_{1,2,3}$ generates an $SU(2)$ subgroup of $SU(2,1)$, and K_4 generates a trivial $U(1)$ subgroup.

As discussed above, the hypermultiplet interactions arise from gauging the isometries (5.7). That is, we choose an isometry generated by some linear combination of (5.7),

$$K = c_a K_a. \quad (5.8)$$

This determines the hypermultiplet Lagrangian

$$\mathcal{L} = -g_{i\bar{j}} D_\mu z^i D^\mu \bar{z}^{\bar{j}} - V(z, \bar{z}) \quad (5.9)$$

via

$$D_\mu z^i = \partial_\mu z^i + A_\mu K^i \quad (5.10)$$

and

$$V(z, \bar{z}) = \frac{3}{2} K^i \bar{K}_i - \frac{1}{4} (D^i \bar{K}^{\bar{j}} - D^{\bar{j}} K^i) (g_{i\bar{j}} g_{k\bar{\ell}} - 2g_{i\bar{\ell}} g_{k\bar{j}}) (D^k \bar{K}^{\bar{\ell}} - D^{\bar{\ell}} K^k). \quad (5.11)$$

Here indices are being raised and lowered with the Kähler metric (5.5). This formula for the scalar potential is derived in appendix C.

According to the hyperino SUSY transformation, the SUSY vacua are located at the solutions to $K^i = 0$ [25]. Since the target space is a symmetric space, we lose no generality by assuming that the vacuum is located at $z^1 = z^2 = 0$. Then the isometries that can be gauged are $K_{1,2,3,4}$ from (5.7). Via an $SU(2)$ rotation, we also lose no generality if we only take K_3 out of $K_{1,2,3}$ to be gauged. Thus the most general isometry that can be gauged is:

$$K = \sqrt{\frac{3}{2}} (K^3 + cK^4) \quad (5.12)$$

for an arbitrary real constant c , whose physical significance will become apparent shortly. The overall normalization in (5.12) is chosen so that the AdS radius $R = 1$. Substituting (5.12) into (5.9), (5.10), and (5.11), and expanding around $z^i = 0$, we obtain the hypermultiplet Lagrangian to quartic order:

$$\begin{aligned} \mathcal{L} = & -D_\mu z^1 D^\mu \bar{z}^1 - D_\mu z^2 D^\mu \bar{z}^2 + 6 - m_+^2 |z^1|^2 - m_-^2 |z^2|^2 \\ & - \left((2|z^1|^2 + |z^2|^2) \partial_\mu z^1 \partial^\mu \bar{z}^1 + (|z^1|^2 + 2|z^2|^2) \partial_\mu z^2 \partial^\mu \bar{z}^2 + z^2 \bar{z}^1 \partial_\mu z^1 \partial^\mu \bar{z}^2 + z^1 \bar{z}^2 \partial_\mu z^2 \partial^\mu \bar{z}^1 \right) \\ & + 3 \left((1 - c^2) |z^1|^4 + (3 - 2c^2) |z^1|^2 |z^2|^2 + (1 - c^2) |z^2|^4 \right) + \dots \end{aligned} \quad (5.13)$$

Here:

$$\begin{pmatrix} D_\mu z^1 \\ D_\mu z^2 \end{pmatrix} = \begin{pmatrix} \partial_\mu z^1 + i\sqrt{\frac{3}{2}} A_\mu (c - 1) z^1 \\ \partial_\mu z^2 + i\sqrt{\frac{3}{2}} A_\mu (c + 1) z^2 \end{pmatrix} \quad (5.14)$$

the masses are given by:

$$m_\pm^2 = \left(\frac{3c}{2} \mp \frac{3}{2} \right) \left(\frac{3c}{2} \pm \frac{5}{2} \right) \quad (5.15)$$

the second line of (5.13) comes from expanding out the target space metric, and the third line comes from expanding out the scalar potential. From the AdS/CFT dictionary (2.5), (2.6), and (2.7), we conclude that z^2 is dual to a chiral primary ϕ with dimension and R-charge given by

$$\Delta_\phi = \frac{3c}{2} + \frac{3}{2}, \quad R_\phi = \frac{2}{3} \Delta_\phi \quad (5.16)$$

while z^1 is dual to its F -component with dimension and R-charge given by

$$\Delta_{F_\phi} = \Delta_\phi + 1, \quad R_{F_\phi} = \frac{2}{3} \Delta_\phi - 2. \quad (5.17)$$

Other branches of the mass-dimension relation do not obey $R = \frac{2}{3} \Delta$ for the chiral primary, and therefore are not consistent with supersymmetry.

The quartic interactions for the chiral primary z^2 are:

$$\mathcal{L}_{quartic} = -2|z^2|^2 \partial_\mu z^2 \partial^\mu \bar{z}^2 + 3(1 - c^2) |z^2|^4. \quad (5.18)$$

In terms of our earlier parameterization (2.3), this sets the potential coefficients to be

$$b = 2, \quad a = -3(1 - c^2) = \frac{4}{3} \Delta (\Delta - 3). \quad (5.19)$$

This does indeed satisfy the relation (2.8), as required by supersymmetry. Correspondingly, the $\phi\phi$ anomalous dimension vanishes to this order. Meanwhile, the $\phi^\dagger\phi$ anomalous dimension is easily read off from (3.36):

$$\gamma_{\phi^\dagger\phi} = -\frac{4\pi^2 N_\Delta^4 \Delta (2\Delta^2 - 3\Delta - 3)}{3(2\Delta - 1)(2\Delta + 1)}. \quad (5.20)$$

This is negative for large Δ , but crosses over to positive for $\Delta \lesssim 2.2$.

6 Summary and future directions

In this paper, we developed a new method for computing anomalous dimensions of double-trace operators in 4d $\mathcal{N} = 1$ SCFTs with local, weakly-coupled AdS₅ supergravity duals, at leading order in the $1/N$ approximation. Anomalous dimensions are dual to binding energies of two-particle states in the bulk. By directly computing these binding energies in the Hamiltonian formulation of AdS/CFT, we have considerably simplified previous indirect approaches based on the four-point function.

For the sake of concreteness and simplicity, we have focused here on a minimal effective model consisting of a single complex scalar ϕ coupled supersymmetrically to gravity and the graviphoton in AdS₅. We calculated the anomalous dimensions of $\phi\phi$ and $\phi^\dagger\phi$, using both our new method and the four-point function method. We found complete agreement between the two methods, which provides a strong check of our results. Using the fact that $\gamma_{\phi\phi}$ must be zero in any $\mathcal{N} = 1$ SCFT, we derived a new constraint (2.8) on the parameters of the effective theory. Although we have focused on the AdS duals of supersymmetric CFTs in this paper, our general techniques and results clearly apply equally well, with minor modifications, to the AdS duals of non-supersymmetric CFTs.

Our result (3.36) for the anomalous dimension of $\phi^\dagger\phi$ in our minimal model is illuminating. We found that depending on the parameters of the model, the anomalous dimension can be positive or negative. That is, we can have either $\Delta_{\phi^\dagger\phi} > 2\Delta_\phi$ or $\Delta_{\phi^\dagger\phi} < 2\Delta_\phi$. In section 5, we considered a specific supergravity model (the “universal hypermultiplet”) which is a special case of our minimal toy model, and which confirms that both signs of the anomalous dimensions are possible.

This has interesting consequences for what can be proved about $\gamma_{\phi^\dagger\phi}$ using general CFT principles such as crossing symmetry and unitarity. In the framework of effective AdS/CFT, correlation functions obtained at any fixed order in the $1/N$ expansion automatically satisfy crossing symmetry and unitarity. Thus even without a UV completion, any results derived in an effective AdS/CFT setup are guaranteed to be compatible with any and all bounds extracted using crossing symmetry and unitarity alone. The results in this paper, for example, robustly demonstrate that $\gamma_{\phi^\dagger\phi} > 0$ is consistent with crossing symmetry and unitarity (and supersymmetry) of the four-point function.

The present paper is just the first step in a much broader research programme. Clearly, a more exhaustive study going beyond the minimal model (2.2) is needed. For instance, one could consider models with more hypermultiplets, and also models with massive supergravity modes, which can arise in realistic string compactifications (see e.g. [29, 30] and [45–48]). It would be also interesting to take our methods beyond leading order, to understand the effects of loops in the effective theory, as well as α' -suppressed higher derivative corrections. Including such modes and effects will lead to many more contributions to binding energies, and it will be fascinating to compare the full range of such generalizations with the bounds on various SCFT quantities in the literature. Bounds on quantities other than anomalous dimensions should be explored as well; although we have focused here on anomalous dimensions, SCFTs also face bounds on central charges and OPE coefficients.

While the positive anomalous dimensions we have obtained here are certainly tantalizing and suggestive, there is no guarantee that our setup (2.2) (with general b and Δ) can

always be UV-completed. Thus we cannot claim to have constructed the first existence proof of positive anomalous dimensions in SCFT. At best we have an almost-existence proof. Clearly, UV completions in string theory are sorely needed. A promising direction here would be to study consistent truncations of type IIB string theory compactified on Sasaki-Einstein manifolds. The papers of [45–48] should prove useful for this purpose.

Alternatively, it is also conceivable that while positive anomalous dimensions are possible in effective theories such as the one studied here, such effective theories are never realized in string theory. Along these lines, it would be interesting to see if one could deduce nontrivial constraints on the parameters of the effective theory using general principles of QFT, as was done in [50]. Such constraints usually derive from analyzing the theory in a background of non-zero ϕ field configurations. Most of the region of parameter space with positive anomalous dimension has b positive and large, which naively seems to be good for stability, since it behaves like a correct-sign kinetic term at $\langle\phi\rangle \neq 0$.

Ultimately, one would also like to take advantage of these AdS constructions and apply them to supersymmetric model-building. As discussed in the introduction, SCFTs with positive anomalous dimensions have several interesting applications to the hidden sector of SUSY-breaking theories, where they are used to suppress undesired Kähler potential terms under RG evolution. For instance, scalar mass terms in the MSSM generated at the Planck scale generically lead to large flavor violation at low scales. Positive anomalous dimensions for the SUSY-breaking field can solve this problem by causing such scalar masses to flow to zero, relative to gaugino mass terms, restoring the Standard Model flavor-breaking structure. This is essentially the proposal of gaugino mediation [9–11]. (In fact, the Planck-scale is inessential for the *spectrum* of gaugino mediation; positive anomalous dimensions can lead to the same type of spectrum also in low-scale SUSY breaking models [12].) One can similarly imagine using positive anomalous dimensions to suppress B_μ relative to μ in gauge mediation, thereby solving the μ/B_μ problem [5–8]. In all such applications, an actual calculable example of an SCFT with the desired properties is currently lacking. Perhaps the AdS/CFT approach employed here could one day lead to such an example. Of course, a complete model along these lines would require incorporating dynamical SUSY breaking into the hidden sector in the AdS description, and coupling it to the MSSM. How to do this in general is interesting to contemplate.

Acknowledgments

We would like to thank A. Dymarsky, D. Green, T. Hartman, J. Kaplan, J. Liu, G. Moore, J. Penedones, A. Royston, M. Schmaltz, P. Szepietowski, Y. Tachikawa, and especially J. Maldacena for useful discussions. The research of ALF is supported in part by DOE grant DE-FG02-01ER-40676 and NSF CAREER grant PHY-0645456. The research of DS is supported in part by a DOE Early Career Award.

A Details of the binding energy calculations

In this appendix, we will provide the detailed solution of the equations of motion (3.10) in the $\phi\phi$ and $\phi^\dagger\phi$ binding energy calculations of section 3.

First, we begin with the photon equation of motion (3.10) in the $\phi\phi$ case. The starting point is the source (3.18), which we repeat here for convenience:

$$J^0[\psi_0(x), \psi_0^*(x)] = -2\Delta q N_\Delta^2 \cos^{2\Delta+2} \rho, \quad J^i[\psi_0(x), \psi_0^*(x)] = 0. \quad (\text{A.1})$$

Due to the high degree of symmetry of the wavefunctions, we may take as an ansatz that A_0 depends only on the radial coordinate ρ , so the equation of motion $D_\mu F^{\mu 0} = J^0$ simplifies dramatically:

$$\left(\frac{y^5}{(1-y^2)} \right) \partial_y \left(\frac{(1-y^2)^2}{y} \partial_y A_0 \right) = 2\Delta q N_\Delta^2 \Delta y^{2\Delta+2}, \quad (\text{A.2})$$

where $y = \cos \rho$. This can be trivially solved for A_0 , yielding the result (3.19) quoted in the body of the paper. Here the two integration constants are fixed by the conditions that the potential die off at $\rho \rightarrow \frac{\pi}{2}$ and that it be smooth at the origin $\rho \rightarrow 0$.

Next we consider the graviton equation of motion in the $\phi\phi$ case. Here the starting point is the stress tensor source (3.22), which we again repeat for convenience:

$$T_{\nu}^{\mu}[\psi_0(x), \psi_0^*(x)] = 2\Delta N_\Delta^2 \cos^{2\Delta} \rho \cdot \text{diag} (2 - \Delta, 2, 2 - \Delta \sin^2 \rho, \dots, 2 - \Delta \sin^2 \rho). \quad (\text{A.3})$$

The symmetry of the source again suggests a very symmetric ansatz for the metric:

$$g_{\mu\nu} = g_{\mu\nu}^{(\text{AdS})} + h_{\mu\nu}(y) \quad (\text{A.4})$$

with only h_{tt} and $h_{\rho\rho}$ nonzero. Linearizing in $h_{\mu\nu}$, (3.10) has only two independent equations of motion: $G^t_t = T^t_t, G^\rho_\rho = T^\rho_\rho$ (the remaining components of Einstein's equation are linear combinations of these and their derivatives). These two are respectively,

$$\begin{aligned} \frac{3}{2} y h'_{\rho\rho}(y) + \frac{3}{y^2 - 1} h_{\rho\rho}(y) &= -2\Delta(\Delta - 2) N_\Delta^2 y^{2\Delta-2}, \\ \frac{3}{2} y h'_{tt}(y) - \frac{3(y^2 - 2)}{y^2 - 1} h_{\rho\rho}(y) + 3h_{tt}(y) &= 4\Delta N_\Delta^2 y^{2\Delta-2}. \end{aligned} \quad (\text{A.5})$$

These are again easily solved, and yield the formulas (3.23) quoted in the text. Again, the integration constants have been fixed by demanding that the solution be regular at $y = 1$ and die off at $y = 0$.

Finally, we come to the graviton equation of motion in the $\phi^\dagger\phi$ case, with the time-dependent source $T_{\mu\nu}[\psi_0, \psi_0]$. Now we modify our ansatz for the metric:

$$g_{\mu\nu} = g_{\mu\nu}^{(\text{AdS})} + e^{2i\Delta t} h_{\mu\nu}(y), \quad (\text{A.6})$$

with again only h_{tt} and $h_{\rho\rho}$ nonzero. The components of Einstein's equation corresponding to G^t_t and G^ρ_ρ are now

$$\begin{aligned} 3y^3 h'_{\rho\rho}(y)(y^2 - 1) + 6y^2 h_{\rho\rho}(y) - N_\Delta^2 (y^2 - 1)(4y^{2\Delta} \Delta(2 + \Delta(y^2 - 1))) &= 0, \\ 3y^3 h'_{tt}(y) + 6y^2 (h_{tt}(y) - h_{\rho\rho}(y) \frac{y^2 - 2}{y^2 - 1}) + 4N_\Delta^2 y^{2\Delta} \Delta(\Delta y^2 - 2) &= 0. \end{aligned} \quad (\text{A.7})$$

These are solved by (3.33).

B Details of the four-point function calculations

Here we will review the clever technique of [43] for evaluating the z integrals in the calculation of AdS four-point functions (4.18) and (4.25). Keep in mind our conventions for the indices, discussed below (4.12).

After a shift to $y = w - x_1$, (4.18) becomes

$$I_\mu(y; 0, \vec{x}_{31}) = \int d^5 z \sqrt{g} g^{\nu\rho}(z) J_\nu(z; 0, \vec{x}_{31}) G_{\rho\mu}(z, y). \quad (\text{B.1})$$

Next, we perform an inversion on the r.h.s. of $I^\mu(y; 0, \vec{x}_{31})$ above. This yields

$$I_\mu(y; 0, \vec{x}_{31}) = |x_{31}|^{-2\Delta} \frac{1}{y^2} J_{\mu\nu}(y) I_\nu(y' - x'_{31}) \quad (\text{B.2})$$

where $J_{\mu\nu}(y) = \delta_{\mu\nu} - 2y^\mu y^\nu / y^2$ is the inversion tensor, $y'^\mu = \frac{y^\mu}{y^2}$, and

$$I_\mu(x) = iN_\Delta^2 \int \frac{d^5 z}{(z^0)^5} g^{\nu\rho}(z) \left((z^0)^\Delta \partial_\nu \left(\frac{z^0}{z^2} \right)^\Delta - \left(\frac{z^0}{z^2} \right)^\Delta \partial_\nu (z^0)^\Delta \right) G_{\rho\mu}(z, x). \quad (\text{B.3})$$

The point of these manipulations is that we have reduced the z integral in (B.1) to a function of a single variable $I_\mu(x)$. Lorentz invariance, dimensional analysis, and current conservation imply:

$$I_\mu(x) = iN_\Delta^2 \frac{x^\mu}{x^2} g(t), \quad t \equiv \frac{(x^0)^2}{x^2}. \quad (\text{B.4})$$

One can check that substituting $x = y' - x'_{31}$, one obtains t as in (4.20). Finally, $g(t)$ satisfies the following differential equation, derived from the fact that the photon propagator is a AdS Green's function:

$$2t^2(t-1)g'' + 4t^2g' = -\Delta t^\Delta. \quad (\text{B.5})$$

After imposing appropriate boundary conditions (namely that $g(t)$ is zero at $t = 0$ and smooth at $t = 1$), we obtain the solution $g(t) = -\frac{1}{2(\Delta-1)}f(t)$ with $f(t)$ given in (4.20). Substituting this into (B.4), and then into (B.2), we obtain the result (4.19) quoted in the text.

Next, consider the same shift and inversion applied to the graviton z integral (4.25). This yields:

$$\begin{aligned} I_{\mu\nu}(y; 0, \vec{x}_{31}) &= \int d^5 z \sqrt{g} g^{\mu'\rho'}(z) g^{\nu'\lambda'}(z) T_{\mu'\nu'}(z; 0, \vec{x}_{31}) G_{\rho'\lambda'\mu\nu}(z, y) \\ &= |x_{31}|^{-2\Delta} \frac{1}{y^4} J_{\mu\rho}(y) J_{\nu\lambda}(y) I_{\rho\lambda}(y' - x'_{31}) \end{aligned} \quad (\text{B.6})$$

where

$$\begin{aligned} I_{\mu\nu}(x) &= N_\Delta^2 \int \frac{d^5 z}{(z^0)^5} g^{\mu'\rho'}(z) g^{\nu'\lambda'}(z) \left[2\partial_{\mu'}(z^0)^\Delta \partial_{\nu'} \left(\frac{z^0}{z^2} \right)^\Delta \right. \\ &\quad \left. - g_{\mu'\nu'} \left(\partial_{\kappa'}(z^0)^\Delta \partial^{\kappa'} \left(\frac{z^0}{z^2} \right)^\Delta + m^2 (z^0)^\Delta \left(\frac{z^0}{z^2} \right)^\Delta \right) \right] G_{\rho'\lambda'\mu\nu}(z, x). \end{aligned} \quad (\text{B.7})$$

According to D'Hoker et al, $I_{\mu\nu}(x)$ takes the form

$$I_{\mu\nu}(x) = 2N_{\Delta}^2 \left[g_{\mu\nu} h(t) + \frac{\delta_{0\mu}\delta_{0\nu}}{(x^0)^2} \phi(t) + D_{\mu} D_{\nu} X(t) + D_{\{\mu} \left(\frac{\delta_{\nu\}0}{x^0} Y(t) \right) \right]. \quad (\text{B.8})$$

Here X and Y are gauge artifacts that drop out of the final integral over w . Differential equations analogous to (B.5) can be derived for h and ϕ , leading to:

$$h(t) = -\frac{1}{3}\phi(t) = -\frac{\Delta}{3}g(t). \quad (\text{B.9})$$

Using (B.9) in (B.6) and (B.8), we arrive at the result (4.26) quoted in the body of the paper.

C Calculation of the hypermultiplet potential

In this appendix, we will derive the formula (5.11) for the hypermultiplet scalar potential. Our starting point is the general formula for the hypermultiplet potential valid for any quaternionic Kähler (QK) manifold. To write down this formula, we must first review some basic facts about QK manifolds.

Let \mathcal{M}_{4n_H} denote a QK manifold of real dimension $4n_H$, with real coordinates q^x , $x = 1, \dots, 4n_H$. Since \mathcal{M}_{4n_H} has holonomy contained within $\text{Sp}(n_H) \times \text{SU}(2)$, it is convenient to pass from the curved coordinates q^x to flat coordinates $e^{\alpha} \in \text{Sp}(n_H) \times \text{SU}(2)$ via the vielbein $f_{A\alpha}^x$, with $A = 1, \dots, 2n_H$ and $\alpha = 1, 2$. Let $\omega_{\alpha}^{\beta} = \omega_{x\alpha}^{\beta} dq^x$ denote the $\text{SU}(2)$ spin connection. Explicitly, we have¹⁰

$$\omega_{x\alpha}^{\beta} = -\frac{1}{2}(\Gamma_{xz}^y f_{yA}^{\beta} f_{\alpha}^{zA} + f_{A\alpha}^y \partial_x f_y^{A\beta}). \quad (\text{C.1})$$

We can change to an explicit adjoint-valued basis with $\omega^{r=1,2,3} = -\frac{i}{2}\omega_{\alpha}^{\beta}(\sigma^r)_{\beta}^{\alpha}$. The $\text{SU}(2)$ curvature is given by

$$\mathcal{R}^r = d\omega^r - \epsilon^{rst}\omega^s \wedge \omega^t. \quad (\text{C.2})$$

From this, we construct the *quaternionic prepotentials* $P^{r=1,2,3}$. They are given by the formula

$$P^r = \frac{1}{2}D^x K^y \mathcal{R}_{xy}^r. \quad (\text{C.3})$$

Finally, we have all the ingredients we need to write down the general formula for the hypermultiplet potential. It is:

$$V = \frac{3}{4}K^x K_x - 4P^r P^r = \frac{3}{4}K^x K_x - D^x K^y D^w K^z \mathcal{R}_{xy}^r \mathcal{R}_{wz}^r. \quad (\text{C.4})$$

Now, we would like to specialize this formula to the case of interest, $\mathcal{M}_{4(n_H=1)} = \text{SU}(2, 1)/\text{SU}(2) \times \text{U}(1)$, which is also a Kähler manifold. Passing from the real coordinates q^x to the complex coordinates $z^i, \bar{z}^{\bar{i}}$, the first term in (C.4) can be written $\frac{3}{2}K^i \bar{K}_{\bar{i}}$. To simplify

¹⁰Indices x, y, \dots are raised and lowered with the target space metric g_{xy} ; indices α, β, \dots are raised and lowered with $\epsilon_{\alpha\beta}$; and indices A, B, \dots are raised and lowered with the invariant two-tensor C_{AB} on $\text{Sp}(n_H)$.

the second term in (C.4), we first notice that in this case, the only nonzero components of \mathcal{R}_{xy}^r are $\mathcal{R}_{i\bar{j}}^r = -\mathcal{R}_{\bar{j}i}^r$. Moreover, we find that

$$\mathcal{R}_{i\bar{j}}^r \mathcal{R}_{k\bar{l}}^r = \frac{1}{4}(g_{i\bar{j}} g_{k\bar{l}} - 2g_{i\bar{l}} g_{k\bar{j}}). \quad (\text{C.5})$$

Substituting this into (C.4), we obtain (5.11).

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