Published for SISSA by Springer

Received: *June 28, 2011*
Revised: *September 5, 2011*
Accepted: *October 3, 2011*
Published: *October 14, 2011*

The general gaugings of maximal $d = 9$ supergravity

J.J. Fernández-Melgarejo,[a] **T. Ortín**[b] **and E. Torrente-Luján**[a]

[a] *Grupo de Física Teórica y Cosmología, Dept. de Física, Universidad de Murcia, Campus de Espinardo, E-30100-Murcia, Spain*
[b] *Instituto de Física Teórica UAM/CSIC, C/ Nicolás Cabrera, 13-15, C.U. Cantoblanco, E-28049-Madrid, Spain*

E-mail: jj.fernandezmelgarejo@um.es, Tomas.Ortin@csic.es, torrente@cern.ch

Abstract: We use the embedding tensor method to construct the most general maximal gauged/massive supergravity in $d = 9$ dimensions and to determine its extended field content. Only the 8 independent deformation parameters (embedding tensor components, mass parameters etc.) identified by Bergshoeff et al. (an $SL(2,\mathbb{R})$ triplet, two doublets and a singlet) can be consistently introduced in the theory, but their simultaneous use is subject to a number of quadratic constraints. These constraints have to be kept and enforced because they cannot be used to solve some deformation parameters in terms of the rest. The deformation parameters are associated to the possible 8-forms of the theory, and the constraints are associated to the 9-forms, all of them transforming in the conjugate representations. We also give the field strengths and the gauge and supersymmetry transformations for the electric fields in the most general case. We compare these results with the predictions of the E_{11} approach, finding that the latter predicts one additional doublet of 9-forms, analogously to what happens in $N = 2$ $d = 4, 5, 6$ theories.

Keywords: Extended Supersymmetry, Supergravity Models, p-branes

ArXiv ePrint: 1106.1760

doi:10.1007/JHEP10(2011)068

JHEP10(2011)068

Contents

JHEP10(2011)068

1 Introduction

The discovery of the relation between RR $(p+1)$-form potentials in 10-dimensional type II supergravity theories and D-branes [1] made it possible to associate most of the fields of the string low-energy effective field theories (supergravity theories in general) to extended objects (*branes*) of diverse kinds: fundamental, Dirichlet, solitonic, Kaluza-Klein etc. This

association has been fruitfully used in two directions: to infer the existence of new supergravity fields from the known existence in the String Theory of a given brane or string state and *vice versa*. Thus, the knowledge of the existence of Dp-branes with large values of p made it necessary to learn how to deal consistently with the magnetic duals of the RR fields that were present in the standard formulations of the supergravity theories constructed decades before, because in general it is impossible to dualize and rewrite the theory in terms of the dual magnetic fields. The existence of NS-NS $(p+1)$-forms in the supergravity theories that could also be dualized made it necessary to include solitonic branes dual to the fundamental ones (strings, basically). It was necessary to include all the objects and fields that could be reached from those already known by U-duality transformations and this effort led to the discovery of new branes and the introduction of the *democratic* formulations of the type II supergravities [2] dealing simultaneously with all the relevant electric and magnetic supergravity fields in a consistent way.

The search for all the extended states of String Theory has motivated the search for all the fields that can be consistently introduced in the corresponding Supergravity Theories, a problem that has no simple answer for the d-, $(d-1)$ and $(d-2)$-form fields, which are not the duals of electric fields already present in the standard formulation, at least in any obvious way. The branes that would couple to them can play important rôles in String Theory models, which makes this search more interesting.

As mentioned before, U-duality arguments have been used to find new supergravity fields but U-duality can only reach new fields belonging to the same orbits as the known fields. To find other possible fields, a systematic study of the possible consistent supersymmetry transformation rules for p-forms has been carried out in the 10-dimensional maximal supergravities in refs. [2–7] but this procedure is long and not systematic. The conjectured E_{11} symmetry [8–10] can be used to determine the bosonic extended field content of maximal supergravity in different dimensions.[1] Thee results have been recently used to construct the U-duality-covariant Wess-Zumino terms of all possible branes in all dimensions [12, 13]. In this approach supersymmetry is not explicitly taken into account, only through the U-duality group.

Another possible systematic approach to this problem (that does not take supersymmetry into account explicitly either) is provided by the embedding-tensor formalism.[2] This formalism, introduced in refs. [17–21] allows the study of the most general deformations of field theories and, in particular, of supergravity theories [22–30]. One of the main features of this formalism is that it requires the systematic introduction of new higher-rank potentials which are related by Stückelberg gauge transformations. This structure is known as the *tensor hierarchy* of the theory [20, 21, 27, 31–33] and can be taken as the (bosonic) extended field content of the theory. In Supergravity Theories one may need to take into account additional constraints on the possible gaugings, but, if the gauging is allowed by supersymmetry, then gauge invariance will require the introduction of all the fields in the associated tensor hierarchy and, since gauge invariance is a *sine qua non* condition for su-

[1] Smaller Kač-Moody algebras can be used in supergravities with smaller number of supercharges such as $N = 2$ theories in $d = 4, 5, 6$ dimensions [11].

[2] For recent reviews see refs. [14–16].

JHEP10(2011)068

persymmetry, the tensor hierarchy will be automatically compatible with supersymmetry. Furthermore, if we set to zero all the deformation parameters (gauge coupling constants, Romans-like mass parameters [34] etc.) the fields that we have introduced will remain in the undeformed theory.

This formalism, therefore, provides another systematic way of finding the extended field content of Supergravity Theories. However, it cannot be used in the most interesting cases, $N = 1, d = 11$ and $N = 2A, B, d = 10$ Supergravity, because these theories cannot be gauged because they do not have 1-forms ($N = 1, d = 11$ and $N = 2B, d = 10$) or the 1-form transforms under the only (Abelian) global symmetry ($N = 2A, d = 10$). Only $N = 2A, d = 10$ can be deformed through the introduction of Romans' mass parameter, but the consistency of this deformation does not seem to require the introduction of any higher-rank potentials. The dimensional reduction to $d = 9$ of these theories, though, has 3 vector fields, and their embedding-tensor formalism can be used to study all its possible gaugings and find its extended field content.

Some gaugings of the maximal $d = 9$ supergravity have been obtained in the past by generalized dimensional reduction [35] of the 10-dimensional theories with respect to the $\mathrm{SL}(2, \mathbb{R})$ global symmetry of the $N = 2B$ theory [36–38] or other rescaling symmetries [39].[3] All these possibilities were systematically and separately studied in ref. [41], taking into account the dualities that relate the possible deformation parameters introduced with the generalized dimensional reductions. However, the possible combinations of deformations were not studied, and, as we will explain, some of the higher-rank fields are associated to the constraints on the combinations of deformations. Furthermore, we do not know if other deformations, with no higher-dimensional origin (such as Romans' massive deformation of the $N = 2A, d = 10$ supergravity) are possible.

Our goal in this paper will be to make a systematic study of all these possibilities using the embedding-tensor formalism plus supersymmetry to identify the extended-field content of the theory, finding the rôle played by the possible 7-, 8- and 9-form potentials, and compare the results with the prediction of the E_{11} approach. We expect to get at least compatible results, as in the $N = 2, d = 4, 5, 6$ cases studied in [30] and [11].

This paper is organized as follows: in section 2 we review the undeformed maximal 9-dimensional supergravity and its global symmetries. In section 3 we study the possible deformations of the theory using the embedding-tensor formalism and checking the closure of the local supersymmetry algebra for each electric p-form of the theory. In section 4 we summarize the results of the previous section describing the possible deformations and the constraints they must satisfy. We discuss the relations between those results and the possible 7- 8- and 9-form potentials of the theory and how these results compare with those obtained in the literature using the E_{11} approach. Section 5 contains our conclusions. Our conventions are briefly discussed in appendix A. The Noether currents of the undeformed theory are given in appendix B. A summary of our results for the deformed theory (deformed field strengths, gauge transformations and covariant derivatives, supersymmetry transformations etc.) is contained in appendix C.

[3] An SO(2)-gauged version of the theory was directly constructed in ref. [40].

JHEP10(2011)068

2 Maximal $d = 9$ supergravity: the undeformed theory

There is only one undeformed (i.e. ungauged, massless) maximal (i.e. $N = 2$, containing no dimensionful parameters in their action, apart from the overall Newton constant) 9-dimensional supergravity [42]. Both the dimensional reduction of the massless $N = 2A, d = 10$ theory and that of the $N = 2B, d = 10$ theory on a circle give the same undeformed $N = 2, d = 9$ theory, a property related to the T duality between type IIA and IIB string theories compactified on circles [43, 44] and from which the type II Buscher rules can be derived [45].

The fundamental (*electric*) fields of this theory are,

$$\left\{ e_\mu{}^a, \varphi, \tau \equiv \chi + ie^{-\phi}, A^I{}_\mu, B^i{}_{\mu\nu}, C_{\mu\nu\rho}, \psi_\mu, \tilde{\lambda}, \lambda, \right\} . \tag{2.1}$$

where $I = 0, \mathbf{i}$, with $\mathbf{i}, \mathbf{j}, \mathbf{k} = 1, 2$ and $i, j, k = 1, 2$.[4] The complex scalar τ parametrizes an $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ coset that can also be described through the symmetric $\mathrm{SL}(2,\mathbb{R})$ matrix

$$\mathcal{M} \equiv e^\phi \begin{pmatrix} |\tau|^2 & \chi \\ \chi & 1 \end{pmatrix} , \qquad \mathcal{M}^{-1} \equiv e^\phi \begin{pmatrix} 1 & -\chi \\ -\chi & |\tau|^2 \end{pmatrix} . \tag{2.2}$$

The undeformed field strengths of the electric p-forms are, in our conventions[5,6]

$$F^I = dA^I , \tag{2.3}$$

$$H^i = dB^i + \tfrac{1}{2}\delta^i{}_{\mathbf{i}}(A^0 \wedge F^{\mathbf{i}} + A^{\mathbf{i}} \wedge F^0) , \tag{2.4}$$

$$G = d[C - \tfrac{1}{6}\varepsilon_{\mathbf{ij}} A^{0\mathbf{ij}}] - \varepsilon_{\mathbf{i}j} F^{\mathbf{i}} \wedge \left(B^j + \tfrac{1}{2}\delta^j{}_{\mathbf{j}} A^{0\mathbf{j}} \right) , \tag{2.5}$$

and are invariant under the undeformed gauge transformations

$$\delta_\Lambda A^I = -d\Lambda^I , \tag{2.6}$$

$$\delta_\Lambda B^i = -d\Lambda^i + \delta^i{}_{\mathbf{i}} \left[\Lambda^{\mathbf{i}} F^0 + \Lambda^0 F^{\mathbf{i}} + \tfrac{1}{2} \left(A^0 \wedge \delta_\Lambda A^{\mathbf{i}} + A^{\mathbf{i}} \wedge \delta_\Lambda A^0 \right) \right] , \tag{2.7}$$

$$\begin{aligned} \delta_\Lambda [C - \tfrac{1}{6}\varepsilon_{\mathbf{ij}} A^{0\mathbf{ij}}] = & -d\Lambda - \varepsilon_{\mathbf{i}j} \left(F^{\mathbf{i}} \wedge \Lambda^j + \Lambda^{\mathbf{i}} \wedge H^j - \delta_\Lambda A^{\mathbf{i}} \wedge B^j \right. \\ & \left. + \tfrac{1}{2}\delta^j{}_{\mathbf{j}} A^{0\mathbf{i}} \wedge \delta_\Lambda A^{\mathbf{j}} \right) . \end{aligned} \tag{2.8}$$

The bosonic action is, in these conventions, given by

$$\begin{aligned} S = \int \Big\{ & - \star R + \tfrac{1}{2} d\varphi \wedge \star d\varphi + \tfrac{1}{2} \left[d\phi \wedge \star d\phi + e^{2\phi} d\chi \wedge \star d\chi \right] + \tfrac{1}{2} e^{\frac{4}{\sqrt{7}}\varphi} F^0 \wedge \star F^0 \\ & + \tfrac{1}{2} e^{\frac{3}{\sqrt{7}}\varphi} (\mathcal{M}^{-1})_{\mathbf{ij}} F^{\mathbf{i}} \wedge \star F^{\mathbf{j}} + \tfrac{1}{2} e^{-\frac{1}{\sqrt{7}}\varphi} (\mathcal{M}^{-1})_{ij} H^i \wedge \star H^j + \tfrac{1}{2} e^{\frac{2}{\sqrt{7}}\varphi} G \wedge \star G \\ & - \tfrac{1}{2} \left[G + \varepsilon_{\mathbf{i}j} A^{\mathbf{i}} \wedge \left(H^j - \tfrac{1}{2}\delta^j{}_{\mathbf{j}} A^{\mathbf{j}} \wedge F^0 \right) \right] \wedge \left\{ \left[G + \varepsilon_{\mathbf{i}j} A^{\mathbf{i}} \wedge \left(H^j - \tfrac{1}{2}\delta^j{}_{\mathbf{j}} A^{\mathbf{j}} \wedge F^0 \right) \right] \wedge A^0 \right. \\ & \left. - \varepsilon_{ij} \left(H^i - \delta^i{}_{\mathbf{i}} A^{\mathbf{i}} \wedge F^0 \right) \wedge \left(B^j - \tfrac{1}{2}\delta^j{}_{\mathbf{j}} A^{0\mathbf{j}} \right) \right\} \Big\} . \end{aligned} \tag{2.9}$$

[4] Sometimes we need to distinguish the indices $1, 2$ of the 1-forms (and their dual 6-forms) from those of the 2-forms (and their dual 5-forms). We will use boldface indices for the former and their associated gauge parameters.

[5] We use the shorthand notation $A^{IJ} \equiv A^I \wedge A^J$, $B^{ijk} \equiv B^i \wedge B^j \wedge B^k$ etc.

[6] The relation between these fields and those of refs. [37] and [41] are given in appendix A.2.

JHEP10(2011)068

The kinetic term for the $\mathrm{SL}(2,\mathbb{R})$ scalars ϕ and χ can be written in the alternative forms

$$\tfrac{1}{2}\left[d\phi\wedge\star d\phi+e^{2\phi}d\chi\wedge\star d\chi\right]=\frac{d\tau\wedge\star d\bar\tau}{2(\Im\mathrm{m}\tau)^2}=\tfrac{1}{4}\mathrm{Tr}\left[d\mathcal{M}\mathcal{M}^{-1}\wedge\star d\mathcal{M}\mathcal{M}^{-1}\right]\,, \tag{2.10}$$

the last of which is manifestly $\mathrm{SL}(2,\mathbb{R})$-invariant. The Chern-Simons term of the action (the last two lines of eq. (2.9)) can also be written in the alternative form

$$\begin{aligned}&-\tfrac{1}{2}d\left[C-\tfrac{1}{6}\varepsilon_{\mathbf{ij}}A^{\mathbf{0ij}}-\varepsilon_{\mathbf{i}j}A^{\mathbf{i}}\wedge B^j\right]\wedge\left\{d\left[C-\tfrac{1}{6}\varepsilon_{\mathbf{ij}}A^{\mathbf{0ij}}-\varepsilon_{\mathbf{i}j}A^{\mathbf{i}}\wedge B^j\right]\wedge A^0\right.\\&\left.-\varepsilon_{ij}d\left(B^i-\tfrac{1}{2}\delta^i{}_{\mathbf{i}}A^{\mathbf{0i}}\right)\wedge\left(B^j-\tfrac{1}{2}\delta^j{}_{\mathbf{j}}A^{\mathbf{0j}}\right)\right\}\,,\end{aligned} \tag{2.11}$$

that has an evident 11-dimensional origin.

The equations of motion of the scalars, derived from the action above, are

$$\begin{aligned}&d\star d\varphi-\tfrac{2}{\sqrt{7}}e^{\frac{4}{\sqrt{7}}\varphi}F^0\wedge\star F^0-\tfrac{3}{2\sqrt{7}}e^{\frac{3}{\sqrt{7}}\varphi}(\mathcal{M}^{-1})_{\mathbf{ij}}F^{\mathbf{i}}\wedge\star F^{\mathbf{j}}\\&\qquad+\tfrac{1}{2\sqrt{7}}e^{-\frac{1}{\sqrt{7}}\varphi}(\mathcal{M}^{-1})_{ij}H^i\wedge\star H^j-\tfrac{1}{\sqrt{7}}e^{\frac{2}{\sqrt{7}}\varphi}G\wedge\star G=0\,,\end{aligned} \tag{2.12}$$

$$d\left[\star\frac{d\bar\tau}{(\Im\mathrm{m}\tau)^2}\right]-i\frac{d\tau\wedge\star d\bar\tau}{(\Im\mathrm{m}\tau)^3}-\partial_\tau(\mathcal{M}^{-1})_{\mathbf{ij}}\left[F^{\mathbf{i}}\wedge\star F^{\mathbf{j}}+H^i\wedge\star H^j\right]=0\,, \tag{2.13}$$

and those of the fundamental p-forms ($p\geq 1$), after some algebraic manipulations, take the form

$$d\left(e^{\frac{4}{\sqrt{7}}\varphi}\star F^0\right)=-e^{-\frac{1}{\sqrt{7}}\varphi}\mathcal{M}^{-1}_{\mathbf{ij}}F^{\mathbf{i}}\wedge\star H^j+\tfrac{1}{2}G\wedge G\,, \tag{2.14}$$

$$d\left(e^{\frac{3}{\sqrt{7}}\varphi}\mathcal{M}^{-1}_{\mathbf{ij}}\star F^{\mathbf{j}}\right)=-e^{\frac{3}{\sqrt{7}}\varphi}\mathcal{M}^{-1}_{\mathbf{ij}}F^0\wedge\star H^j+\varepsilon_{\mathbf{i}j}e^{\frac{2}{\sqrt{7}}\varphi}H^j\wedge\star G\,, \tag{2.15}$$

$$d\left(e^{-\frac{1}{\sqrt{7}}\varphi}\mathcal{M}^{-1}_{ij}\star H^j\right)=\varepsilon_{\mathbf{i}j}e^{\frac{2}{\sqrt{7}}\varphi}F^{\mathbf{j}}\wedge\star G-\varepsilon_{ij}H^j\wedge G\,, \tag{2.16}$$

$$d\left(e^{\frac{2}{\sqrt{7}}\varphi}\star G\right)=F^0\wedge G+\tfrac{1}{2}\varepsilon_{ij}H^i\wedge H^j\,. \tag{2.17}$$

2.1 Global symmetries

The undeformed theory has as (classical) global symmetry group $\mathrm{SL}(2,\mathbb{R})\times(\mathbb{R}^+)^2$. The $(\mathbb{R}^+)^2$ symmetries correspond to scalings of the fields, the first of which, that we will denote by α,[7] acts on the metric and only leaves the equations of motion invariant while the second of them, which we will denote by β, leaves invariant both the metric and the action. The β rescaling corresponds to the so-called *trombone symmetry* which may not survive higher-derivative string corrections.

One can also discuss two more scaling symmetries γ and δ, but γ is just a subgroup of $\mathrm{SL}(2,\mathbb{R})$ and δ is related to the other scaling symmetries by

$$\tfrac{4}{9}\alpha-\tfrac{8}{3}\beta-\gamma-\tfrac{1}{2}\delta=0\,. \tag{2.18}$$

[7] This discussion follows closely that of ref. [41] in which the higher-dimensional origin of each symmetry is also studied. In particular, we use the same names and definitions for the scaling symmetries and we reproduce the table of scaling weights for the electric fields.

JHEP10(2011)068

$\mathbb{R}^+$	${e_\mu}^a$	e^φ	e^ϕ	χ	A^0	A^1	A^2	B^1	B^2	C	ψ_μ	λ	$\tilde{\lambda}$	ϵ	$\mathcal{L}$
α	9/7	$6/\sqrt{7}$	0	0	3	0	0	3	3	3	9/14	−9/14	−9/14	9/14	9
β	0	$\sqrt{7}/4$	3/4	−3/4	1/2	−3/4	0	−1/4	1/2	−1/4	0	0	0	0	0
γ	0	0	−2	2	0	1	−1	1	−1	0	0	0	0	0	0
δ	8/7	$-4/\sqrt{7}$	0	0	0	2	2	2	2	4	4/7	−4/7	−4/7	4/7	8

Table 1. The scaling weights of the electric fields of maximal $d = 9$ supergravity.

We will take α and β as the independent symmetries. The weights of the electric fields under all the scaling symmetries are given in table 1. We can see that each of the three gauge fields ${A^I}_\mu$ has zero weight under *two* (linear combinations) of these three symmetries: one is a symmetry of the action, the other is a symmetry of the equations of motion only. The 1-form that has zero weight under a given rescaling is precisely the one that can be used to gauge that rescaling, but this kind of conditions are automatically taken into account by the embedding-tensor formalism and we will not have to discuss them in detail.

The action of the element of $\mathrm{SL}(2,\mathbb{R})$ given by the matrix

$$\left(\Omega^i_j\right) = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \qquad ad - bc = 1\,, \tag{2.19}$$

on the fields of the theory is

$$\begin{aligned} \tau' &= \frac{a\tau + b}{c\tau + d}\,, & \mathcal{M}'_{ij} &= \Omega_i{}^k \mathcal{M}_{kl} \Omega_j{}^l\,, \\ A^{\mathbf{i}\prime} &= \Omega_{\mathbf{j}}{}^{\mathbf{i}} A^{\mathbf{j}}\,, & B^{i\prime} &= \Omega_j{}^i B^j\,, \\ \psi'_\mu &= e^{\frac{i}{2}l}\psi_\mu\,, & \lambda &= e^{\frac{3i}{2}l}\lambda\,, \\ \tilde{\lambda}' &= e^{-\frac{i}{2}l}\tilde{\lambda}\,, & \epsilon' &= e^{\frac{i}{2}l}\epsilon\,. \end{aligned} \tag{2.20}$$

where

$$e^{2il} \equiv \frac{c\,\tau^* + d}{c\,\tau + d}\,. \tag{2.21}$$

The rest of the fields $({e^a}_\mu, \varphi, {A^0}_\mu, C_{\mu\nu\rho})$, are invariant under $\mathrm{SL}(2,\mathbb{R})$.

We are going to label the 5 generators of these global symmetries by T_A, $A = 1, \cdots, 5$. $\{T_1, T_2, T_3\}$ will be the 3 generators of $\mathrm{SL}(2,\mathbb{R})$ (collectively denoted by $\{T_m\}$, $m = 1, 2, 3$), and T_4 and T_5 will be, respectively, the generators of the rescalings α and β. Our choice for the generators of $\mathrm{SL}(2,\mathbb{R})$ acting on the doublets of 1-forms $A^{\mathbf{i}}$ and 2-forms B^i is

$$T_1 = \tfrac{1}{2}\sigma^3\,, \qquad T_2 = \tfrac{1}{2}\sigma^1\,, \qquad T_3 = \tfrac{i}{2}\sigma^2\,, \tag{2.22}$$

where the σ^m are the standard Pauli matrices, so

$$[T_1, T_2] = T_3\,, \qquad [T_2, T_3] = -T_1\,, \qquad [T_3, T_1] = -T_2\,. \tag{2.23}$$

Then, the 3×3 matrices corresponding to generators acting (contravariantly) on the 3 1-forms A^I (and covariantly on their dual 6-forms $\tilde{A}_I$ to be introduced later) are

$$\begin{aligned} \left((T_1)_J{}^I\right) &= \tfrac{1}{2}\left(\begin{array}{c|c} 0 & 0 \\ \hline 0 & \sigma^3 \end{array}\right), \quad \left((T_2)_J{}^I\right) = \tfrac{1}{2}\left(\begin{array}{c|c} 0 & 0 \\ \hline 0 & \sigma^1 \end{array}\right), \quad \left((T_3)_J{}^I\right) = \tfrac{1}{2}\left(\begin{array}{c|c} 0 & 0 \\ \hline 0 & i\sigma^2 \end{array}\right), \\ \left((T_4)_J{}^I\right) &= \mathrm{diag}(3,0,0)\,, \quad \left((T_5)_J{}^I\right) = \mathrm{diag}(1/2, -3/4, 0)\,. \end{aligned} \tag{2.24}$$

We will sometimes denote this representation by $T_A^{(3)}$. The 2×2 matrices corresponding to generators acting (contravariantly) on the doublet of 2-forms B^i (and covariantly on their dual 5-forms $\tilde{B}_i$ to be introduced later) are

$$\begin{aligned} \big((T_1)_j{}^i\big) &= \tfrac{1}{2}\sigma^3\,, & \big((T_2)_j{}^i\big) &= \tfrac{1}{2}\sigma^1\,, \quad \big((T_3)_j{}^i\big) = \tfrac{i}{2}\sigma^2\,, \\ \big((T_4)_j{}^i\big) &= \mathrm{diag}(3,3)\,, & \big((T_5)_j{}^i\big) &= \mathrm{diag}(-1/4,1/2)\,. \end{aligned} \tag{2.25}$$

We will denote this representation by $T_A^{(2)}$. The generators that act on the 3-form C (sometimes denoted by $T_A^{(1)}$) are

$$T_1 = T_2 = T_3 = 0\,, \quad T_4 = 3\,, \quad T_5 = -1/4\,. \tag{2.26}$$

We will also need the generators that act on the magnetic 4-form $\tilde{C}$ (see next section), also denoted by $T_A^{(\tilde{1})}$

$$\tilde{T}_1 = \tilde{T}_2 = \tilde{T}_3 = 0\,, \quad \tilde{T}_4 = 6\,, \quad \tilde{T}_5 = 1/4\,. \tag{2.27}$$

We define the structure constants $f_{AB}{}^C$ by

$$[T_A, T_B] = f_{AB}{}^C T_C\,. \tag{2.28}$$

The symmetries of the theory are isometries of the scalar manifold $(\mathbb{R}\times \mathrm{SL}(2,\mathbb{R}/\mathrm{U}(1))$. The Killing vector associated with the generator T_A will be denoted by k_A and will be normalized so that their Lie brackets are given by

$$[k_A, k_B] = -f_{AB}{}^C k_C\,. \tag{2.29}$$

The $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$ factor of the scalar manifold is a Kähler space with Kähler potential, Kähler metric and Kähler 1-form, respectively given by

$$\mathcal{K} = -\log \Im\mathrm{m}\tau = \phi\,, \quad \mathcal{G}_{\tau\tau^*} = \partial_\tau\partial_{\tau^*}\mathcal{K} = \tfrac{1}{4}e^{2\phi}\,, \quad \mathcal{Q} = \tfrac{1}{2i}\left(\partial_\tau \mathcal{K} d\tau - \text{c.c.}\right) = \tfrac{1}{2}e^{\phi} d\chi\,. \tag{2.30}$$

In general, the isometries of the Kähler metric only leave invariant the Kähler potential up to Kähler transformations:

$$\pounds_{k_m}\mathcal{K} = k_m{}^\tau \partial_\tau \mathcal{K} + \text{c.c.} = \lambda_m(\tau) + \text{c.c.}\,, \qquad \pounds_{k_m}\mathcal{Q} = -\tfrac{i}{2} d\lambda_m\,, \tag{2.31}$$

where the λ_m are holomorphic functions of the coordinates that satisfy the equivariance property

$$\pounds_{k_m}\lambda_n - \pounds_{k_n}\lambda_m = -f_{mn}{}^p\lambda_p\,. \tag{2.32}$$

Then, for each of the $\mathrm{SL}(2,\mathbb{R})$ Killing vectors k_m, $m = 1,2,3$, it is possible to find a real *Killing prepotential* or *momentum map* $\mathcal{P}_m$ such that

$$\begin{aligned} k_{m\,\tau^*} &= \mathcal{G}_{\tau^*\tau} k_m{}^\tau = i\partial_{\tau^*}\mathcal{P}_m\,, \\ k_m{}^\tau \partial_\tau \mathcal{K} &= i\mathcal{P}_m + \lambda_m\,, \\ \pounds_{k_m}\mathcal{P}_n &= -f_{mn}{}^p\mathcal{P}_p\,. \end{aligned} \tag{2.33}$$

JHEP10(2011)068

The non-vanishing components of all the Killing vectors are[8]

$$k_1{}^\tau = \tau\,, \qquad k_2{}^\tau = \tfrac{1}{2}(1-\tau^2)\,, \qquad k_3{}^\tau = \tfrac{1}{2}(1+\tau^2)\,, \qquad k_4{}^\tau = 0\,, \qquad k_5{}^\tau = -\tfrac{3}{4}\tau\,. \tag{2.34}$$

and

$$k_4{}^\varphi = 6/\sqrt{7}\,, \qquad\qquad k_5{}^\varphi = \sqrt{7}/4\,. \tag{2.35}$$

The holomorphic functions $\lambda_m(\tau)$ take the values

$$\lambda_1 = -\tfrac{1}{2}\,, \qquad \lambda_2 = \tfrac{1}{2}\tau\,, \qquad \lambda_3 = -\tfrac{1}{2}\tau\,, \tag{2.36}$$

and the momentum maps are given by:

$$\mathcal{P}_1 = \tfrac{1}{2}e^{\phi}\chi\,, \qquad \mathcal{P}_2 = \tfrac{1}{4}e^{\phi}(1-|\tau|^2)\,, \qquad \mathcal{P}_3 = \tfrac{1}{4}e^{\phi}(1+|\tau|^2)\,. \tag{2.37}$$

These objects will be used in the construction of $\mathrm{SL}(2,\mathbb{R})$-covariant derivatives for the fermions.

2.2 Magnetic fields

As it is well known, for each p-form potential with $p>0$ one can define a *magnetic* dual which in $d-9$ dimensions will be a $(7-p)$-form potential. Then, we will have magnetic 4-, 5- and 6-form potentials in the theory.

A possible way to define those potentials and identify their $(8-p)$-form field strengths consists in writing the equations of motion of the p-forms as total derivatives. Let us take, for instance, the equation of motion of the 3-form C eq. (2.17). It can be written as

$$\begin{aligned} d\frac{\partial\mathcal{L}}{\partial G} = d\Big\{ & e^{\frac{2}{\sqrt{7}}\varphi}\star G - \Big[G + \varepsilon_{\mathbf{i}j}A^{\mathbf{i}}\wedge\Big(H^j - \tfrac{1}{2}\delta^j{}_{\mathbf{j}}A^{\mathbf{j}}\wedge F^0\Big)\Big]\wedge A^0 \\ & + \tfrac{1}{2}\varepsilon_{ij}\Big(H^i - \delta^i{}_{\mathbf{i}}A^{\mathbf{i}}\wedge F^0\Big)\wedge\Big(B^j - \tfrac{1}{2}\delta^j{}_{\mathbf{j}}A^{0\mathbf{j}}\Big)\Big\} = 0\,. \end{aligned} \tag{2.38}$$

We can transform this equation of motion into a Bianchi identity by replacing the combination of fields on which the total derivative acts by the total derivative of a 4-form which we choose for the sake of convenience[9]

$$\begin{aligned} d\Big[\tilde{C} - C\wedge A^0 - \tfrac{3}{4}\varepsilon_{\mathbf{i}j}A^{0\mathbf{i}}\wedge B^j\Big] \equiv\ & e^{\frac{2}{\sqrt{7}}\varphi}\star G - \Big[G + \varepsilon_{\mathbf{i}j}A^{\mathbf{i}}\wedge\Big(H^j - \tfrac{1}{2}\delta^j{}_{\mathbf{j}}A^{\mathbf{j}}\wedge F^0\Big)\Big]\wedge A^0 \\ & + \tfrac{1}{2}\varepsilon_{ij}\Big(H^i - \delta^i{}_{\mathbf{i}}A^{\mathbf{i}}\wedge F^0\Big)\wedge\Big(B^j - \tfrac{1}{2}\delta^j{}_{\mathbf{j}}A^{0\mathbf{j}}\Big)\,, \end{aligned} \tag{2.39}$$

where $\tilde{C}$ will be the magnetic 4-form. This relation can be put in the form of a duality relation

$$e^{\frac{2}{\sqrt{7}}\varphi}\star G = \tilde{G}\,, \tag{2.40}$$

where we have defined the magnetic 5-form field strength

$$\tilde{G} \equiv d\tilde{C} + C\wedge F^0 - \tfrac{1}{24}\varepsilon_{\mathbf{ij}}A^{0\mathbf{ij}}\wedge F^0 - \varepsilon_{ij}\big(H^i - \tfrac{1}{2}dB^i\big)\wedge B^j\,. \tag{2.41}$$

The equation of motion for $\tilde{C}$ is just the Bianchi identity of G rewritten in terms of $\tilde{G}$.

[8] The holomorphic and anti-holomorphic components are defined by $k = k^\tau\partial_\tau + \text{c.c.} = k^\chi\partial_\chi + k^\phi\partial_\phi$.

[9] With this definition $\tilde{G}$ will have exactly the same form that we will obtain from the embedding tensor formalism.

JHEP10(2011)068

0	1	2	3	4	5	6	7	8	9
j_A	A^I	B^i	C	$\tilde{C}$	$\tilde{B}_i$	$\tilde{A}_I$	$\tilde{A}^A_{(7)}$	$\tilde{A}_{(8)}$	$\tilde{A}_{(9)}$
	F^I	H^i	G	$\tilde{G}$	$\tilde{H}_i$	$\tilde{F}_I$	$\tilde{F}^A_{(8)}$	$\tilde{F}_{(9)}$	

Table 2. Electric and magnetic forms and their field strengths.

$\mathbb{R}^+$	$\tilde{C}$	$\tilde{B}_2$	$\tilde{B}_1$	$\tilde{A}_2$	$\tilde{A}_1$	$\tilde{A}_0$
α	6	6	6	9	9	6
β	1/4	$-1/2$	$+1/4$	0	$+3/4$	$-1/2$
γ	0	1	-1	1	-1	0
δ	4	6	6	6	6	8

Table 3. The scaling weights of the magnetic fields of maximal $d = 9$ supergravity can be determined by requiring that the sum of the weights of the electric and magnetic potentials equals that of the Lagrangian. The scaling weights of the 7-, 8- and 9-forms can be determined in the same way after we find the entities they are dual to (Noether currents, embedding-tensor components and constraints, see section 4).

In a similar fashion we can define a doublet of 5-forms $\tilde{B}_i$ with field strengths denoted by $\tilde{H}_i$, and a singlet and a doublet of 6-forms $\tilde{A}_0, \tilde{A}_{\mathbf{i}}$ with field strengths denoted, respectively, by $\tilde{F}_0$ and $\tilde{F}_{\mathbf{i}}$. The field strengths can be chosen to have the form

$$\begin{aligned}
\tilde{H}_i =& d\tilde{B}_i - \delta_{ij}B^j \wedge G + \delta_{i\mathbf{j}}\tilde{C} \wedge F^{\mathbf{j}} + \tfrac{1}{2}\delta_{i\mathbf{j}}\left(A^0 \wedge F^{\mathbf{j}} + A^{\mathbf{j}} \wedge F^0\right) \wedge C \\
&+ \tfrac{1}{2}\delta_{ij}\varepsilon_{k\mathbf{l}}B^{jk} \wedge F^{\mathbf{l}}\,, \qquad (2.42)\\
\tilde{F}_0 =& d\tilde{A}_0 + \tfrac{1}{2}C \wedge G - \varepsilon_{\mathbf{i}j}F^{\mathbf{i}} \wedge \left(\delta^{jk}\tilde{B}_k - \tfrac{2}{3}B^j \wedge C\right) \\
&- \tfrac{1}{18}\varepsilon_{\mathbf{ij}}A^{\mathbf{ij}} \wedge \left(\tilde{G} - F^0 \wedge C - \tfrac{1}{2}\varepsilon_{kl}B^k \wedge H^l\right) \\
&- \tfrac{1}{6}\varepsilon_{\mathbf{i}j}A^{\mathbf{i}} \wedge \left(B^j \wedge G - C \wedge H^j - \tfrac{2}{3}\delta^j{}_{\mathbf{j}}\tilde{C} \wedge F^{\mathbf{j}} - \varepsilon_{k\mathbf{l}}B^{jk} \wedge F^{\mathbf{l}}\right)\,, \qquad (2.43)\\
\tilde{F}_{\mathbf{i}} =& d\tilde{A}_{\mathbf{i}} + \delta_{\mathbf{i}j}\left(B^j + \tfrac{7}{18}\delta^j{}_{\mathbf{k}}A^{0\mathbf{k}}\right) \wedge \tilde{G} - \delta_{\mathbf{i}}{}^j F^0 \wedge \tilde{B}_j - \tfrac{1}{9}\delta_{\mathbf{ij}}\left(8A^0 \wedge F^{\mathbf{j}} + A^{\mathbf{j}} \wedge F^0\right) \wedge \tilde{C} \\
&- \tfrac{1}{3}\delta_{\mathbf{i}j}\varepsilon_{lm}\left(B^j + \tfrac{1}{3}\delta^j{}_{\mathbf{k}}A^{0\mathbf{k}}\right) \wedge B^l \wedge H^m - \tfrac{1}{6}\delta_{\mathbf{i}j}\varepsilon_{\mathbf{k}l}\left(A^0 \wedge H^j - B^j \wedge F^0\right) \wedge A^{\mathbf{k}} \wedge B^l \\
&- \tfrac{1}{9}A^0 \wedge F^0 \wedge \delta_{\mathbf{ij}}\left(\tfrac{7}{2}A^{\mathbf{j}} \wedge C + \delta^{\mathbf{j}}{}_k\varepsilon_{\mathbf{lm}}A^{\mathbf{lm}} \wedge B^k\right)\,, \qquad (2.44)
\end{aligned}$$

and the duality relations are

$$\tilde{H}_i = e^{-\frac{1}{\sqrt{7}}\varphi}\mathcal{M}^{-1}_{ij} \star H^j\,, \qquad (2.45)$$

$$\tilde{F}_0 = e^{\frac{4}{\sqrt{7}}\varphi} \star F^0\,, \qquad (2.46)$$

$$\tilde{F}_{\mathbf{i}} = e^{\frac{3}{\sqrt{7}}\varphi}\mathcal{M}^{-1}_{\mathbf{ij}} \star F^{\mathbf{j}}\,. \qquad (2.47)$$

The situation is summarized in table 2. The scaling weights of the magnetic fields are given in table 3.

JHEP10(2011)068

This dualization procedure is made possible by the gauge symmetries associated with all the p-form potentials for $p > 0$ (actually, by the existence of gauge transformations with constant parameters) and, therefore, it always works for massless p-forms with $p > 0$ and generically fails for 0-form fields. However, in maximal supergravity theories at least, there is a global symmetry group that acts on the scalar manifold and whose dimension is larger than that of the scalar manifold. Therefore, there is one Noether 1-form current j_A associated with each of the generators of the global symmetries of the theory T_A. These currents are conserved on-shell, *i. e.* they satisfy

$$d \star j_A = 0 \,,$$

on-shell, and we can define a $(d-2)$-form potential $\tilde{A}^A_{(d-2)}$ by

$$d\tilde{A}^A_{(d-2)} = G^{AB} \star j_B \,,$$

where G^{AB} is the inverse Killing metric of the global symmetry group, so that the conservation law (dynamical) becomes a Bianchi identity.

Thus, while the dualization procedure indicates that for each electric p-form with $p > 0$ there is a dual magnetic $(7-p)$-form transforming in the conjugate representation, it tells us that there are as many magnetic $(d-2)$-form duals of the scalars as the dimension of the global group (and not of as the dimension of the scalar manifold) and that they transform in the co-adjoint representation. Actually, since there is no need to have scalar fields in order to have global symmetries, it is possible to define magnetic $(d-2)$-form potentials even in the total absence of scalars.[10]

According to these general arguments, which are in agreement with the general results of the embedding-tensor formalism [29–31, 33], we expect a triplet of 7-form potentials $\tilde{A}^m_{(7)}$ associated with the $\mathrm{SL}(2,\mathbb{R})$ factor of the global symmetry group [37] and two singlets $\tilde{A}^4_{(7)}, \tilde{A}^5_{(7)}$ associated with the rescalings α, β (see table 2).

Finding or just determining the possible magnetic $(d-1)$- and d-form potentials in a given theory is more complicated. In the embedding-tensor formalism it is natural to expect as many $(d-1)$-form potentials as deformation parameters (embedding-tensor components, mass parameters etc.) can be introduced in the theory since the rôle of the $(d-1)$-forms in the action is that of being Lagrange multipliers enforcing their constancy.[11] The number of deformation parameters that can be introduced in this theory is, as we are going to see, very large, but there are many constraints that they have to satisfy to preserve gauge and supersymmetry invariance. Furthermore, there are many Stückelberg shift symmetries acting on the possible $(d-1)$-form potentials. Solving the constraints leaves us with the independent deformation parameters that we can denote by $m_\sharp$ and, correspondingly,

[10] See refs. [29, 30] for examples.

[11] The embedding-tensor formalism gives us a reason to introduce the $(d-1)$-form potentials based on the deformation parameters but the $(d-1)$-form potentials do not disappear when the deformation parameters are set equal to zero.

JHEP10(2011)068

with a reduced number of $(d-1)$-form potentials $\tilde{A}^{\sharp}_{(d-1)}$ on which only a few Stückelberg symmetries (or none at all) act.[12]

The d-form field strengths $\tilde{F}^{\sharp}_{(d)}$ are related to the scalar potential of the theory through the expression [29–31, 33]

$$\tilde{F}^{\sharp}_{(d)} = \tfrac{1}{2} \star \frac{\partial V}{\partial m_{\sharp}} \,. \tag{2.48}$$

Thus, in order to find the possible 8-form potentials of this theory we need to study its independent consistent deformations $m_{\sharp}$. We will consider this problem in the next section.

In the embedding-tensor formalism, the d-form potentials are associated with constraints of the deformation parameters since they would be the Lagrange multipliers enforcing them in the action [26]. If we do not solve any of the constraints there will be many d-form potentials but there will be many Stückelberg symmetries acting on them as well. Thus, only a small number of *irreducible* constraints that cannot be solved[13] and of associated d-forms may be expected in the end, but we have to go through the whole procedure to identify them. This identification will be one of the main results of the following section.

However, this is not the end of the story for the possible 9-forms. As it was shown in ref. [30] in 4- 5- and 6-dimensional cases, in the ungauged case one can find more d-forms with consistent supersymmetric transformation rules than predicted by the embedding-tensor formalism. Those additional fields are predicted by the Kač-Moody approach [11]. However, after gauging, the new fields do not have consistent, independent, supersymmetry transformation rules to all orders in fermions,[14] and have to be combined with other d-forms, so that, in the end, only the number of d-forms predicted by the embedding-tensor formalism survive.

This means that the results obtained via the embedding-tensor formalism for the 9-forms have to be interpreted with special care and have to be compared with the results obtained with other approaches.

The closure of the local supersymmetry algebra needs to be checked on all the fields in the tensor hierarchy predicted by the embedding-tensor formalism and, in particular, on the 9-forms to all orders in fermions. However, given that gauge invariance is requirement for local supersymmetry invariance, we expect consistency in essentially all cases with the possible exception of the 9-forms, according to the above discussion. In the next section we will do this for the electric fields of the theory.

3 Deforming the maximal $d = 9$ supergravity

In this section we are going to study the possible deformations of $d = 9$ supergravity, starting from its possible gaugings using the embedding-tensor formalism and constructing the corresponding tensor hierarchy [17–21, 31, 33] up to the 4-form potentials.

[12] The $(d-1)$-form potentials that "disappear" when we solve the constraints are evidently associated with the gauge-fixing of the missing Stückelberg symmetries.

[13] In general, the quadratic constraints cannot be used to solve some deformation parameters in terms of the rest. For instance, in this sense, if a and b are two of them, a constraint of the form $ab = 0$ cannot be solved and we can call it *irreducible*.

[14] The insufficience of first-order in fermions checks was first noticed in ref. [6].

JHEP10(2011)068

If we denote by $\Lambda^I(x)$ the scalar parameters of the gauge transformations of the 1-forms A^I and by α^A the constant parameters of the global symmetries, we want to promote

$$\alpha^A \longrightarrow \Lambda^I(x)\vartheta_I{}^A\,, \tag{3.1}$$

where $\vartheta_I{}^A$ is the *embedding tensor*, in the transformation rules of all the fields, and we are going to require the theory to be covariant under the new local transformations using the 1-forms as gauge fields.

To achieve this goal, starting with the transformations of the scalars, the successive introduction of higher-rank p-form potentials is required, which results in the construction of a tensor hierarchy. Most of these fields are already present in the supergravity theory or can be identified with their magnetic duals but this procedure allows us to introduce consistently the highest-rank fields (the d-, $(d-1)$- and $(d-2)$-form potentials), which are not dual to any of the original electric fields. Actually, as explained in section 2.2, the highest-rank potentials are related to the symmetries (Noether currents), the independent deformation parameters and the constraints that they satisfy, but we need to determine these, which requires going through this procedure checking the consistency with gauge and supersymmetry invariance at each step.

Thus, we are going to require invariance under the new gauge transformations for the scalar fields and we are going to find that we need new couplings to the gauge 1-form fields (as usual). Then we will study the modifications of the supersymmetry transformation rules of the scalars and fermion fields which are needed to ensure the closure of the local supersymmetry algebra on the scalars. Usually we do not expect modifications in the bosons' supersymmetry transformations, but the fermions' transformations need to be modified by replacing derivatives and field strengths by covariant derivatives and covariant field strengths and, furthermore, by adding *fermion shifts*. The local supersymmetry algebra will close provided that we impose certain constraints on the embedding tensor components and on the fermion shifts.

Repeating this procedure on the 1-forms (which requires the coupling to the 2-forms) etc. we will find a set of constraints that we can solve, determining the independent components of the deformation tensors[15] and the fermions shifts. Some constraints (typically quadratic in deformation parameters) have to be left unsolved and we will have to take them into account towards the end of this procedure.

As a result we will identify the independent deformations of the theory and the constraints that they satisfy. From this we will be able to extract information about the highest-rank potentials in the tensor hierarchy.

3.1 The 0-forms φ, τ

Under the global symmetry group, the scalars transform according to

$$\delta_\alpha\varphi = \alpha^A k_A{}^\varphi\,, \qquad \delta_\alpha\tau = \alpha^A k_A{}^\tau\,, \tag{3.2}$$

[15] As we are going to see, besides the embedding tensor, one can introduce many other deformation tensors.

JHEP10(2011)068

where the α^A are the constant parameters of the transformations, labeled by $A = 1, \cdots, 5$, and where $k_A{}^\varphi$ and $k_A{}^\tau$ are the corresponding components of the Killing vectors of the scalar manifold, given in eq. (2.35) (eq. (2.34)).

According to the general prescription eq. (3.1), we want to gauge these symmetries making the theory invariant under the local transformations

$$\delta_\Lambda \varphi = \Lambda^I \vartheta_I{}^A k_A{}^\varphi \,, \qquad \delta_\Lambda \tau = \Lambda^I \vartheta_I{}^A k_A{}^\tau \,, \tag{3.3}$$

where $\Lambda^I(x)$, $I = 0, \mathbf{1}, \mathbf{2}$, are the 0-form gauge parameters of the 1-form gauge fields A^I and $\vartheta_I{}^A$ is the embedding tensor.

To construct gauge-covariant field strengths for the scalars it is enough to replace their derivatives by covariant derivatives.

3.1.1 Covariant derivatives

The covariant derivatives of the scalars have the standard form

$$\mathfrak{D}\varphi = d\varphi + A^I \vartheta_I^A k_A{}^\varphi \,, \qquad \mathfrak{D}\tau = d\tau + A^I \vartheta_I^A k_A{}^\tau \,, \tag{3.4}$$

and they transform covariantly provided that the 1-form gauge fields transform as

$$\delta_\Lambda A^I = -\mathfrak{D}\Lambda^I + Z^I{}_i \Lambda^i \,, \tag{3.5}$$

where the Λ^i, $i = 1, 2$, are two possible 1-form gauge parameters and $Z^I{}_i$ is a possible new deformation parameter that must satisfy the orthogonality constraint

$$\vartheta_I{}^A Z^I{}_i = 0 \,. \tag{3.6}$$

Furthermore, it is necessary that the embedding tensor satisfies the standard quadratic constraint

$$\vartheta_I{}^A T_{A\,J}{}^K \vartheta_K{}^C - \vartheta_I{}^A \vartheta_J{}^B f_{AB}{}^C = 0 \,, \tag{3.7}$$

that expresses the gauge-invariance of the embedding tensor.

As a general rule, all the deformation tensors have to be gauge-invariant and we can anticipate that we will have to impose the constraint that expresses the gauge-invariance of $Z^I{}_i$, namely

$$X_{J\,K}{}^I Z^K{}_i - X_{J\,i}{}^j Z^I{}_j = 0 \,, \tag{3.8}$$

where

$$X_{I\,J}{}^K \equiv \vartheta_I{}^A T_{A\,J}{}^K \,, \qquad X_{J\,i}{}^j \equiv \vartheta_J{}^A T_{A\,i}{}^j \,. \tag{3.9}$$

3.1.2 Supersymmetry transformations of the fermion fields

We will assume for simplicity that the supersymmetry transformations of the fermion fields in the deformed theory have essentially the same form as in the undeformed theory but

JHEP10(2011)068

covariantized (derivatives and field strengths) and, possibly, with the addition of fermion shifts which we add in the most general form:

$$\begin{aligned}
\delta_\epsilon \psi_\mu &= \mathfrak{D}_\mu \epsilon + f\gamma_\mu \epsilon + k\gamma_\mu \epsilon^* + \tfrac{i}{8\cdot 2!} e^{-\frac{2}{\sqrt{7}}\varphi} \left(\tfrac{5}{7}\gamma_\mu \gamma^{(2)} - \gamma^{(2)}\gamma_\mu\right) F^0 \epsilon \\
&\quad - \tfrac{1}{8\cdot 2!} e^{\frac{3}{2\sqrt{7}}\varphi + \frac{1}{2}\phi} \left(\tfrac{5}{7}\gamma_\mu \gamma^{(2)} - \gamma^{(2)}\gamma_\mu\right) (F^1 - \tau F^2)\epsilon^* \\
&\quad - \tfrac{i}{8\cdot 3!} e^{-\frac{1}{2\sqrt{7}}\varphi} \left(\tfrac{3}{7}\gamma_\mu \gamma^{(3)} + \gamma^{(3)}\gamma_\mu\right) (H^1 - \tau H^2)\epsilon^* \\
&\quad - \tfrac{1}{8\cdot 4!} e^{\frac{1}{\sqrt{7}}\varphi} \left(\tfrac{1}{7}\gamma_\mu \gamma^{(4)} - \gamma^{(4)}\gamma_\mu\right) G\epsilon \,, \qquad (3.10) \\
\delta_\epsilon \tilde{\lambda} &= i\, \not{\mathfrak{D}}\varphi \epsilon^* + \tilde{g}\epsilon + \tilde{h}\epsilon^* - \tfrac{1}{\sqrt{7}} e^{-\frac{2}{\sqrt{7}}\varphi} \not{F}^0 \epsilon^* - \tfrac{3i}{2\cdot 2!\sqrt{7}} e^{\frac{3}{2\sqrt{7}}\varphi + \frac{1}{2}\phi} (\not{F}^1 - \tau^* \not{F}^2)\epsilon \\
&\quad - \tfrac{1}{2\cdot 3!\sqrt{7}} e^{-\frac{1}{2\sqrt{7}}\varphi + \frac{1}{2}\phi} (\not{H}^1 - \tau^* \not{H}^2)\epsilon - \tfrac{i}{4!\sqrt{7}} e^{\frac{1}{\sqrt{7}}\varphi} \not{G}\epsilon^* \,, \qquad (3.11) \\
\delta_\epsilon \lambda &= -e^{\phi} \not{\mathfrak{D}}\tau \epsilon^* + g\epsilon + h\epsilon^* - \tfrac{i}{2\cdot 2!} e^{\frac{3}{2\sqrt{7}}\varphi + \frac{1}{2}\phi} (\not{F}^1 - \tau \not{F}^2)\epsilon \\
&\quad + \tfrac{1}{2\cdot 3!} e^{-\frac{1}{2\sqrt{7}}\varphi + \frac{1}{2}\phi} (\not{H}^1 - \tau \not{H}^2)\epsilon \,. \qquad (3.12)
\end{aligned}$$

In these expressions, $f, k, g, h, \tilde{g}, \tilde{h}$ are six functions of the scalars and deformation parameters to be determined, the covariant field strengths have the general form predicted by the tensor hierarchy (to be determined) and the covariant derivatives of the scalars have the forms given above. Furthermore, in $\delta_\epsilon \psi_\mu$, $\mathfrak{D}_\mu \epsilon$ stands for the Lorentz- and gauge-covariant derivative of the supersymmetry parameter, which turns out to be given by

$$\mathfrak{D}_\mu \epsilon \equiv \left\{ \nabla_\mu + \tfrac{i}{2} \left[\tfrac{1}{2} e^{\phi} \mathfrak{D}^5_\mu \chi + A^I{}_\mu \vartheta_I{}^m \mathcal{P}_m \right] + \tfrac{9}{14} \gamma_\mu \not{A}^I \vartheta_I{}^4 \right\} \epsilon \qquad (3.13)$$

where $\mathcal{P}_m$ $1, 2, 3$ are the momentum maps of the holomorphic Killing vectors of $\mathrm{SL}(2, \mathbb{R})$, defined in eq. (2.33) and given in eq. (2.37), ∇_μ is the Lorentz-covariant derivative and

$$\mathfrak{D}^5_\mu \chi \equiv \partial_\mu \chi - \tfrac{3}{4} A^I{}_\mu \vartheta_I{}^5 \chi \qquad (3.14)$$

is the derivative of χ covariant only with respect to the β rescalings. It can be checked that $\mathfrak{D}_\mu \epsilon$ transforms covariantly under gauge transformations if and only if the embedding tensor satisfies the standard quadratic constraint eq. (3.7).

An equivalent expression for it is

$$\mathfrak{D}_\mu \epsilon = \left\{ \nabla_\mu + \tfrac{i}{2} \left[\tfrac{1}{2} e^{\phi} \mathfrak{D}_\mu \chi - A^I{}_\mu \vartheta_I{}^m \Im\mathrm{m} \lambda_m \right] + \tfrac{9}{14} \gamma_\mu \not{A}^I \vartheta_I{}^4 \right\} \epsilon \,, \qquad (3.15)$$

where the λ_m, $m = 1, 2, 3$, of $\mathrm{SL}(2, \mathbb{R})$ and defined in eq. (2.33) and given in eq. (2.36) and where now

$$\mathfrak{D}_\mu \chi \equiv \partial_\mu \chi + A^I{}_\mu \vartheta_I{}^A k_A{}^\chi \,, \qquad (3.16)$$

is the total covariant derivative of χ (which is invariant under both the α and β scaling symmetries as well as under $\mathrm{SL}(2, \mathbb{R})$).

The actual form of the $(p+1)$-form field strengths will not be needed until the moment in which study the closure of the supersymmetry algebra on the corresponding p-form potential.

JHEP10(2011)068

3.1.3 Closure of the supersymmetry algebra on the 0-forms φ, τ

We assume that the supersymmetry transformations of the scalars are the same as in the undeformed theory

$$\delta_\epsilon \varphi = -\tfrac{i}{4}\bar{\epsilon}\tilde{\lambda}^* + \text{h.c.}\,, \tag{3.17}$$

$$\delta_\epsilon \tau = -\tfrac{1}{2}e^{-\phi}\bar{\epsilon}^*\lambda\,. \tag{3.18}$$

To lowest order in fermions, the commutator of two supersymmetry transformations gives

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}]\,\varphi = \xi^\mu \mathfrak{D}_\mu \varphi + \Re\mathrm{e}(\tilde{h})b - \Im\mathrm{m}(\tilde{g})c + \Re\mathrm{e}(\tilde{g})d\,, \tag{3.19}$$

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}]\,\tau = \xi^\mu \mathfrak{D}_\mu \tau + e^{-\phi}\left[g(c - id) - ihb\right]\,, \tag{3.20}$$

where ξ^μ is one of the spinor bilinears defined in appendix A.1 that clearly plays the rôle of parameter of the general coordinate transformations and a, b, c, d are the scalar bilinears defined in the same appendix.

In the right hand side of these commutators, to lowest order in fermions, we expect a general coordinate transformation (the Lie derivative $\pounds_\xi$ of the scalars with respect to ξ^μ) and a gauge transformation which has the form of eq. (3.3) for the scalars. Therefore, the above expressions should be compared with

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}]\,\varphi = \pounds_\xi \varphi + \Lambda^I \vartheta_I{}^A k_A{}^\varphi\,, \tag{3.21}$$

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}]\,\tau = \pounds_\xi \tau + \Lambda^I \vartheta_I{}^A k_A{}^\tau\,, \tag{3.22}$$

from which we get the relations

$$\Re\mathrm{e}(\tilde{h})b - \Im\mathrm{m}(\tilde{g})c + \Re\mathrm{e}(\tilde{g})d = (\Lambda^I - a^I)\vartheta_I{}^A k_A{}^\varphi\,, \tag{3.23}$$

$$g(c - id) - ihb = e^{\phi}(\Lambda^I - a^I)\vartheta_I{}^A k_A{}^\tau\,, \tag{3.24}$$

which would allow us to determine the fermion shift functions if we knew the gauge parameters Λ^I. In order to determine the Λ^Is we have to close the supersymmetry algebra on the 1-forms. In these expressions and in those that will follow, we use the shorthand notation

$$a^I \equiv \xi^\mu A^I{}_\mu\,, \qquad b^i{}_\mu \equiv \xi^\nu B^i{}_{\nu\mu}\,, \qquad c_{\mu\nu} \equiv \xi^\rho C_{\rho\mu\nu}\,, \qquad \text{etc.} \tag{3.25}$$

3.2 The 1-forms A^I

The next step in this procedure is to consider the 1-forms that we just introduced to construct covariant derivatives for the scalars.

3.2.1 The 2-form field strengths F^I

The gauge transformations of the 1-forms are given in eq. (3.5) and we first need to determine their covariant field strengths. A general result of the embedding-tensor formalism tells us that we need to introduce 2-form potentials in the covariant field strengths. In

JHEP10(2011)068

this case only have the $\mathrm{SL}(2,\mathbb{R})$ doublet B^i at our disposal and, therefore, the 2-form field strengths have the form

$$F^I = dA^I + \tfrac{1}{2} X_{JK}{}^I A^J \wedge A^K + Z^I{}_i B^i \,, \tag{3.26}$$

where $X_{JK}{}^I$ has been defined in eq. (3.9) and $Z^I{}_i$ is precisely the deformation tensor we introduced in eq. (3.5). F^I will transform covariantly under eq. (3.5) if simultaneously the 2-forms B^i transform according to

$$\delta_\Lambda B^i = -\mathfrak{D}\Lambda^i - 2 h_{IJ}{}^i \left[\Lambda^I F^J + \tfrac{1}{2} A^I \wedge \delta_\Lambda A^J\right] + Z^i \Lambda \,, \tag{3.27}$$

where $h_{IJ}{}^i$ and Z^i are two possible new deformation tensors the first of which must satisfy the constraint

$$X_{(JK)}{}^I + Z^I{}_i h_{JK}{}^i = 0 \,, \tag{3.28}$$

while Z^i must satisfy the orthogonality constraint

$$Z^I{}_i Z^i = 0 \,. \tag{3.29}$$

Both of them must satisfy the constraints that express their gauge invariance:

$$X_{I\,j}{}^i h_{JK}{}^j - 2 X_{I(J}{}^L h_{K)L}{}^i = 0 \,, \tag{3.30}$$

$$X_I Z^i - X_{I\,j}{}^i Z^j = 0 \,, \tag{3.31}$$

where

$$X_I \equiv \vartheta_I{}^A T^{(1)}_A \,. \tag{3.32}$$

3.2.2 Closure of the supersymmetry algebra on the 1-forms A^I

We assume, as we are doing with all the bosons, that the supersymmetry transformations of the 1-forms of the theory are not deformed by the gauging, so they take the form

$$\delta_\epsilon A^0{}_\mu = \tfrac{i}{2} e^{\frac{2}{\sqrt{7}}\varphi} \bar\epsilon \left(\psi_\mu - \tfrac{i}{\sqrt{7}} \gamma_\mu \tilde\lambda^*\right) + \text{h.c.} \,, \tag{3.33}$$

$$\delta_\epsilon A^{\mathbf{1}}{}_\mu = \tfrac{i}{2} \tau^* e^{-\frac{3}{2\sqrt{7}}\varphi + \frac{1}{2}\phi} \left(\bar\epsilon^* \psi_\mu - \tfrac{i}{4} \bar\epsilon \gamma_\mu \lambda + \tfrac{3i}{4\sqrt{7}} \bar\epsilon^* \gamma_\mu \tilde\lambda^*\right) + \text{h.c.} \,, \tag{3.34}$$

$$\delta_\epsilon A^{\mathbf{2}}{}_\mu = \tfrac{i}{2} e^{-\frac{3}{2\sqrt{7}}\varphi + \frac{1}{2}\phi} \left(\bar\epsilon^* \psi_\mu - \tfrac{i}{4} \bar\epsilon \gamma_\mu \lambda + \tfrac{3i}{4\sqrt{7}} \bar\epsilon^* \gamma_\mu \tilde\lambda^*\right) + \text{h.c.} \tag{3.35}$$

The commutator of two of them gives, to lowest order in fermions,

$$\begin{aligned}
[\delta_{\epsilon_1}, \delta_{\epsilon_2}]\, A^0{}_\mu &= \xi^\nu F^0{}_{\nu\mu} - \mathfrak{D}_\mu \left(e^{\frac{2}{\sqrt{7}}\varphi} b\right) + \tfrac{2}{\sqrt{7}} e^{\frac{2}{\sqrt{7}}\varphi} \Big\{ \left[\Re\mathrm{e}(\tilde h) - \sqrt{7}\, \Im\mathrm{m}(f)\right] \xi_\mu \\
&\quad + \left[\Re\mathrm{e}(\tilde g) - \sqrt{7}\, \Im\mathrm{m}(k)\right] \sigma_\mu + \left[\Im\mathrm{m}(\tilde g) - \sqrt{7}\, \Re\mathrm{e}(k)\right] \rho_\mu \Big\} \,, \qquad (3.36) \\
[\delta_{\epsilon_1}, \delta_{\epsilon_2}]\, A^{\mathbf{1}}{}_\mu &= \xi^\nu F^{\mathbf{1}}{}_{\nu\mu} - \partial_\mu \left[e^{-\frac{3}{2\sqrt{7}}\varphi + \frac{1}{2}\phi} (\chi d + e^{-\phi} c)\right] \\
&\quad - A^I{}_\mu \left[(\tfrac{1}{2}\vartheta_I{}^1 - \tfrac{3}{4}\vartheta_I{}^5) e^{-\frac{3}{2\sqrt{7}}\varphi + \frac{1}{2}\phi} (\chi d + e^{-\phi} c) + \tfrac{1}{2}(\vartheta_I{}^2 + \vartheta_I{}^3) e^{-\frac{3}{2\sqrt{7}}\varphi \frac{1}{2}\phi} d\right] \qquad (3.37) \\
&\quad - 2 e^{-\frac{3}{2\sqrt{7}}\varphi + \frac{1}{2}\phi} \Big\{ \chi \left[\Im\mathrm{m}(k) + \tfrac{3}{4\sqrt{7}} \Re\mathrm{e}(\tilde g) - \tfrac{1}{4} \Re\mathrm{e}(g)\right] + e^{-\phi} \left[-\Re\mathrm{e}(k) - \tfrac{3}{4\sqrt{7}} \Im\mathrm{m}(\tilde g) - \tfrac{1}{4} \Im\mathrm{m}(g)\right] \Big\} \xi_\mu \\
&\quad - 2 e^{-\frac{3}{2\sqrt{7}}\varphi + \frac{1}{2}\phi} \Big\{ \chi \left[-\Re\mathrm{e}(f) - \tfrac{3}{4\sqrt{7}} \Im\mathrm{m}(\tilde h) + \tfrac{1}{4} \Im\mathrm{m}(h)\right] + e^{-\phi} \left[-\Im\mathrm{m}(f) - \tfrac{3}{4\sqrt{7}} \Re\mathrm{e}(\tilde h) - \tfrac{1}{4} \Re\mathrm{e}(h)\right] \Big\} \rho_\mu \\
&\quad - 2 e^{-\frac{3}{2\sqrt{7}}\varphi + \frac{1}{2}\phi} \Big\{ \chi \left[\Im\mathrm{m}(f) + \tfrac{3}{4\sqrt{7}} \Re\mathrm{e}(\tilde h) - \tfrac{1}{4} \Re\mathrm{e}(h)\right] + e^{-\phi} \left[-\Re\mathrm{e}(f) - \tfrac{3}{4\sqrt{7}} \Im\mathrm{m}(\tilde h) - \tfrac{1}{4} \Im\mathrm{m}(h)\right] \Big\} \sigma_\mu ,
\end{aligned}$$

JHEP10(2011)068

and

$$
\begin{aligned}
[\delta_{\epsilon_1},\delta_{\epsilon_2}]\,A^{\mathbf{2}}{}_{\mu} = {}& \xi^{\nu}F^{\mathbf{2}}{}_{\nu\mu} - \partial_{\mu}\left(e^{-\frac{3}{2\sqrt{7}}\varphi+\frac{1}{2}\phi}d\right)\\
& -A^{I}{}_{\mu}\left[\tfrac{1}{2}(\vartheta_I{}^2-\vartheta_I{}^3)e^{-\frac{3}{2\sqrt{7}}\varphi+\frac{1}{2}\phi}(\chi d+e^{-\phi}c)-\tfrac{1}{2}\vartheta_I{}^1 e^{-\frac{3}{2\sqrt{7}}\varphi+\frac{1}{2}\phi}d\right]\\
& -2e^{-\frac{3}{2\sqrt{7}}\varphi+\frac{1}{2}\phi}\left[\Im\mathrm{m}(k)+\tfrac{3}{4\sqrt{7}}\Re\mathrm{e}(\tilde{g})-\tfrac{1}{4}\Re\mathrm{e}(g)\right]\xi_{\mu}\\
& -2e^{-\frac{3}{2\sqrt{7}}\varphi+\frac{1}{2}\phi}\left[-\Re\mathrm{e}(f)-\tfrac{3}{4\sqrt{7}}\Im\mathrm{m}(\tilde{h})+\tfrac{1}{4}\Im\mathrm{m}(h)\right]\rho_{\mu}\\
& -2e^{-\frac{3}{2\sqrt{7}}\varphi+\frac{1}{2}\phi}\left[\Im\mathrm{m}(f)+\tfrac{3}{4\sqrt{7}}\Re\mathrm{e}(\tilde{h})-\tfrac{1}{4}\Re\mathrm{e}(h)\right]\sigma_{\mu}\,,
\end{aligned}
\tag{3.38}
$$

where σ_μ and ρ_μ are spinor bilinears defined in appendix A.1.

The closure of the local supersymmetry algebra requires the commutators to take the form

$$
[\delta_{\epsilon_1},\delta_{\epsilon_2}]\,A^{I}{}_{\mu} = \pounds_{\xi}A^{I}{}_{\mu} - \mathfrak{D}_{\mu}\Lambda^{I} + Z^{I}{}_{i}\Lambda^{i}{}_{\mu}\,,
\tag{3.39}
$$

which will only happen if gauge parameters Λ^I are given by

$$
\begin{aligned}
\Lambda^{0} &= a^{0} + e^{\frac{2}{\sqrt{7}}\varphi}b\,,\\
\Lambda^{\mathbf{1}} &= a^{\mathbf{1}} + e^{-\frac{3}{2\sqrt{7}}\varphi+\frac{1}{2}\phi}(\chi d+e^{-\phi}c)\,,\\
\Lambda^{\mathbf{2}} &= a^{\mathbf{2}} + e^{-\frac{3}{2\sqrt{7}}\varphi+\frac{1}{2}\phi}d\,,
\end{aligned}
\tag{3.40}
$$

and the 1-form gauge parameters $\Lambda^i{}_\mu$ satisfy the relations

$$
\begin{aligned}
&\left[\Re\mathrm{e}(\tilde{h})-\sqrt{7}\,\Im\mathrm{m}(f)\right]\xi_{\mu}+\left[\Re\mathrm{e}(\tilde{g})-\sqrt{7}\,\Im\mathrm{m}(k)\right]\sigma_{\mu}+\left[\Im\mathrm{m}(\tilde{g})-\sqrt{7}\,\Re\mathrm{e}(k)\right]\rho_{\mu}\\
&= \tfrac{\sqrt{7}}{2}e^{-\frac{2}{\sqrt{7}}\varphi}Z^{0}{}_{i}\left[\Lambda^{i}{}_{\mu}-(b^{i}{}_{\mu}-h_{IJ}{}^{i}a^{I}A^{J}{}_{\mu})\right]\,, \qquad (3.41)\\
&\quad\left\{\chi\left[\Im\mathrm{m}(k)+\tfrac{3}{4\sqrt{7}}\Re\mathrm{e}(\tilde{g})-\tfrac{1}{4}\Re\mathrm{e}(g)\right]+e^{-\phi}\left[-\Re\mathrm{e}(k)-\tfrac{3}{4\sqrt{7}}\Im\mathrm{m}(\tilde{g})-\tfrac{1}{4}\Im\mathrm{m}(g)\right]\right\}\xi_{\mu}\\
&\quad+\left\{\chi\left[-\Re\mathrm{e}(f)-\tfrac{3}{4\sqrt{7}}\Im\mathrm{m}(\tilde{h})+\tfrac{1}{4}\Im\mathrm{m}(h)\right]+e^{-\phi}\left[-\Im\mathrm{m}(f)-\tfrac{3}{4\sqrt{7}}\Re\mathrm{e}(\tilde{h})-\tfrac{1}{4}\Re\mathrm{e}(h)\right]\right\}\rho_{\mu}\\
&\quad+\left\{\chi\left[\Im\mathrm{m}(f)+\tfrac{3}{4\sqrt{7}}\Re\mathrm{e}(\tilde{h})-\tfrac{1}{4}\Re\mathrm{e}(h)\right]+e^{-\phi}\left[-\Re\mathrm{e}(f)-\tfrac{3}{4\sqrt{7}}\Im\mathrm{m}(\tilde{h})-\tfrac{1}{4}\Im\mathrm{m}(h)\right]\right\}\sigma_{\mu}\,,\\
&= -\tfrac{1}{2}e^{+\frac{3}{2\sqrt{7}}\varphi-\frac{1}{2}\phi}Z^{\mathbf{1}}{}_{i}\left[\Lambda^{i}{}_{\mu}-(b^{i}{}_{\mu}-h_{IJ}{}^{i}a^{I}A^{J}{}_{\mu})\right]\,, \qquad (3.42)\\
&\quad\left[\Im\mathrm{m}(k)+\tfrac{3}{4\sqrt{7}}\Re\mathrm{e}(\tilde{g})-\tfrac{1}{4}\Re\mathrm{e}(g)\right]\xi_{\mu}+\left[-\Re\mathrm{e}(f)-\tfrac{3}{4\sqrt{7}}\Im\mathrm{m}(\tilde{h})+\tfrac{1}{4}\Im\mathrm{m}(h)\right]\rho_{\mu}\\
&\quad+\left[\Im\mathrm{m}(f)+\tfrac{3}{4\sqrt{7}}\Re\mathrm{e}(\tilde{h})-\tfrac{1}{4}\Re\mathrm{e}(h)\right]\sigma_{\mu}\,,\\
&= -\tfrac{1}{2}e^{+\frac{3}{2\sqrt{7}}\varphi-\frac{1}{2}\phi}Z^{\mathbf{2}}{}_{i}\left[\Lambda^{i}{}_{\mu}-(b^{i}{}_{\mu}-h_{IJ}{}^{i}a^{I}A^{J}{}_{\mu})\right]\,. \qquad (3.43)
\end{aligned}
$$

Using the values of the parameters Λ^I that we just have determined in the relations eqs. (3.23) and (3.24) we can determine some of the fermions shifts:

$$
\Re\mathrm{e}(\tilde{h}) = \vartheta_0{}^{A}k_{A}{}^{\varphi}e^{\frac{2}{\sqrt{7}}\varphi}\,, \tag{3.44}
$$

$$
\tilde{g} = (\vartheta_{\mathbf{1}}{}^{A}\tau^{*}+\vartheta_{\mathbf{2}}{}^{A})k_{A}{}^{\varphi}e^{-\frac{3}{2\sqrt{7}}\varphi+\frac{1}{2}\phi}\,, \tag{3.45}
$$

$$
h = i\vartheta_0{}^{A}k_{A}{}^{\tau}e^{\frac{2}{\sqrt{7}}\varphi+\phi}\,, \tag{3.46}
$$

$$
g = \vartheta_{\mathbf{1}}{}^{A}k_{A}{}^{\tau}e^{-\frac{3}{2\sqrt{7}}\varphi+\frac{1}{2}\phi}\,. \tag{3.47}
$$

JHEP10(2011)068

As a matter of fact, g is overdetermined: we get two different expression for it that give the same value if and only if

$$(\vartheta_{\mathbf{1}}{}^{A}\tau + \vartheta_{\mathbf{2}}{}^{A})k_{A}{}^{\tau} = 0\,, \tag{3.48}$$

which, upon use of the explicit expressions of the holomorphic Killing vectors $k_A{}^{\tau}$ in section 2.1, leads to the following linear constraints on the components of the embedding tensor:

$$\begin{aligned} \vartheta_{\mathbf{2}}{}^{2} + \vartheta_{\mathbf{2}}{}^{3} &= 0\,, \\ \vartheta_{\mathbf{1}}{}^{2} + \vartheta_{\mathbf{1}}{}^{3} + 2\vartheta_{\mathbf{2}}{}^{1} - \tfrac{3}{2}\vartheta_{\mathbf{2}}{}^{5} &= 0\,, \\ \vartheta_{\mathbf{2}}{}^{2} - \vartheta_{\mathbf{2}}{}^{3} - 2\vartheta_{\mathbf{1}}{}^{1} + \tfrac{3}{2}\vartheta_{\mathbf{1}}{}^{5} &= 0\,, \\ \vartheta_{\mathbf{1}}{}^{2} - \vartheta_{\mathbf{1}}{}^{3} &= 0\,. \end{aligned} \tag{3.49}$$

These constraints allow us to express 4 of the 15 components of the embedding tensor in terms of the remaining 11, but we are only going to do this after we take into account the constraints that we are going to find in the closure of the local supersymmetry algebra on the doublet of 2-forms B^i.

The values of $g, h.\tilde{g}, \tilde{h}$ and the above constraints are compatible with those of the primary deformations found in ref. [41].

3.3 The 2-forms B^i

In the previous subsection we have introduced a doublet of 2-forms B^i with given gauge transformations to construct the 2-form field strengths F^I. We now have to construct their covariant field strengths and check the closure of the local supersymmetry algebra on them.

3.3.1 The 3-form field strengths H^i

In general we need to introduce 3-form potentials to construct the covariant 3-form field strengths and, since in maximal 9-dimensional supergravity, we only have C at our disposal, the 3-form field strengths will be given by

$$H^i = \mathfrak{D}B^i - h_{IJ}{}^{i}A^I \wedge dA^J - \tfrac{1}{3}X_{[IJ}{}^{L}h_{K]L}{}^{i}A^{IJK} + Z^iC\,, \tag{3.50}$$

and they transform covariantly under the gauge transformations of the 1- and 2-forms that we have previously determined provided if the 3-form C transforms as

$$\delta_\Lambda C = -\mathfrak{D}\Lambda + g_{Ii}\left[-\Lambda^I H^i - F^I \wedge \Lambda^i + \delta_\Lambda A^I \wedge B^i - \tfrac{1}{3}h_{JK}{}^{i}A^{IJ} \wedge \delta_\Lambda A^K\right] + Z\tilde{\Lambda}\,. \tag{3.51}$$

where g_{Ii} and Z are two possible new deformation parameters. g_{Ii} must satisfy the constraint

$$2h_{IJ}{}^{i}Z^{J}{}_{j} + X_{I\,j}{}^{i} + Z^{i}g_{Ij} = 0\,, \tag{3.52}$$

while Z must satisfy the orthogonality constraint

$$Z^iZ = 0\,. \tag{3.53}$$

JHEP10(2011)068

Both must by gauge-invariant, which implies the constraints

$$X_{IJ}{}^{L} g_{Li} + X_{I\,i}{}^{j} g_{Jj} - X_I g_{Ji} = 0\,, \tag{3.54}$$

$$(X_I - \tilde{X}_I) Z = 0\,, \tag{3.55}$$

where

$$\tilde{X}_I \equiv \vartheta_I{}^{A} T_A^{(\tilde{1})}\,. \tag{3.56}$$

Using the constraints obeyed by the deformation parameters and the explicit form of the 2-form field strengths F^I we can rewrite the 3-form field strengths in the useful form

$$H^i = \mathfrak{D} B^i - h_{IJ}{}^{i} A^I \wedge F^J + \tfrac{1}{6} X_{[IJ}{}^{L} h_{K]L}{}^{i} A^{IJK} - \tfrac{1}{2} X_{Ij}{}^{i} A^I \wedge B^j + Z^i \big(C - \tfrac{1}{2} g_{Ij} A^I \wedge B^j\big)\,. \tag{3.57}$$

3.3.2 Closure of the supersymmetry algebra on the 2-forms B^i

In the undeformed theory, the supersymmetry transformation rules for the 2-forms are

$$\begin{aligned} \delta_\epsilon B^1 \;=\; & \tau^* e^{\frac{1}{2\sqrt{7}}\varphi + \frac{1}{2}\phi} \left[\bar{\epsilon}^* \gamma_{[\mu} \psi_{\nu]} - \tfrac{i}{8} \bar{\epsilon} \gamma_{\mu\nu} \lambda - \tfrac{i}{8\sqrt{7}} \bar{\epsilon}^* \gamma_{\mu\nu} \tilde{\lambda}^*\right] \\ & - \delta^1{}_{\mathbf{i}} \left(A^0{}_{[\mu|} \delta_\epsilon A^{\mathbf{i}}{}_{|\nu]} + A^{\mathbf{i}}{}_{[\mu|} \delta_\epsilon A^0{}_{|\nu]}\right) + \text{h.c.}\,, \end{aligned} \tag{3.58}$$

$$\begin{aligned} \delta_\epsilon B^2 \;=\; & e^{\frac{1}{2\sqrt{7}}\varphi + \frac{1}{2}\phi} \left[\bar{\epsilon}^* \gamma_{[\mu} \psi_{\nu]} - \tfrac{i}{8} \bar{\epsilon} \gamma_{\mu\nu} \lambda - \tfrac{i}{8\sqrt{7}} \bar{\epsilon}^* \gamma_{\mu\nu} \tilde{\lambda}^*\right] \\ & - \delta^2{}_{\mathbf{i}} \left(A^0{}_{[\mu|} \delta_\epsilon A^{\mathbf{i}}{}_{|\nu]} + A^{\mathbf{i}}{}_{[\mu|} \delta_\epsilon A^0{}_{|\nu]}\right) + \text{h.c.}\,. \end{aligned} \tag{3.59}$$

The last terms in both transformations are associated with the presence of derivatives of $A^{\mathbf{1}}$ and $A^{\mathbf{2}}$ in the field strengths of B^1 and B^2 in the undeformed theory (see eq. (2.4)). In the deformed theory, the terms $-(A^0 \wedge dA^{\mathbf{i}} + A^{\mathbf{i}} \wedge dA^0)$ are replaced by more general couplings $-h_{IJ}{}^{i} A^I \wedge dA^J$ and, therefore, it would be natural to replace the last terms in $\delta_\epsilon B^i{}_{\mu\nu}$ by

$$-\,2 h_{IJ}{}^{i} A^I{}_{[\mu|} \delta_\epsilon A^J{}_{|\nu]}\,. \tag{3.60}$$

In the commutator of two supersymmetry transformations on the 2-forms, these terms give the right contributions to the terms $-2h_{IJ}{}^{i} \Lambda^I F^J$ of the gauge transformations (see eq. (3.27)). However, these terms must receive other contributions in order to be complete and it turns out that the only terms of the form $-2h_{IJ}{}^{i} \Lambda^I F^J$ that can be completed are precisely those of the undeformed theory, which correspond to

$$h_{\mathbf{i}0}{}^{j} = -\tfrac{1}{2} \delta_{\mathbf{i}}{}^{j}\,. \tag{3.61}$$

In order to get more general $h_{IJ}{}^{i}$s it would be necessary to deform the fermions' supersymmetry rules, something we will not do here. Furthermore, the structure of the Chern-Simons terms of the field strengths is usually determined by the closure of the supersymmetry algebra at higher orders in fermions and it is highly unlikely that a more general structure of the Chern-Simons terms will be allowed by supersymmetry. Therefore, from now on, we will set $h_{IJ}{}^{i}$ to the above value and we will set the values of the deformation tensors in the Chern-Simons terms of the higher-rank field strengths, to the values of the undeformed theory. Using the above value of $h_{IJ}{}^{i}$ in the constraints in which it occurs will

JHEP10(2011)068

help us to solve them, sometimes completely, as we will see. Nevertheless, we will keep using the notation $h_{IJ}{}^{i}$ for convenience.

Using the identity

$$\begin{aligned}\xi^{\rho}H^{i}{}_{\rho\mu\nu} - 2h_{IJ}{}^{i}A^{I}{}_{\mu}\pounds_{\xi}A^{J}{}_{\nu} = {} & \pounds_{\xi}B^{i}{}_{\mu\nu} - 2\mathfrak{D}_{[\mu|}(b^{i}{}_{|\nu]} - h_{IJ}{}^{i}a^{I}A^{J}{}_{|\nu]})] \\ & -2h_{IJ}{}^{i}a^{I}F^{J}{}_{\mu\nu} \\ & +Z^{i}\left(c_{\mu\nu} - g_{Ij}a^{I}B^{j}{}_{\mu\nu} + \tfrac{2}{3}g_{Jj}h_{IK}{}^{j}a^{I}A^{JK}{}_{\mu\nu]}\right) ,\end{aligned} \tag{3.62}$$

we find that the local supersymmetry algebra closes on the B^{i}s in the expected form (to lowest order in fermions)

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}]\, B^{i}{}_{\mu\nu} = \pounds_{\xi}B^{i}{}_{\mu\nu} + \delta_{\Lambda}B^{i}{}_{\mu\nu} , \tag{3.63}$$

where $\delta_{\Lambda}B^{i}{}_{\mu\nu}$ is the gauge transformation given in eq. (3.27) in which the 0-form gauge parameters Λ^{I} are as in eqs. (3.40), the 1-form gauge parameters $\Lambda^{i}{}_{\mu}$ are given by

$$\Lambda^{i}{}_{\mu} = \lambda^{i}{}_{\mu} + b^{i}{}_{\mu} - h_{IJ}{}^{i}a^{I}A^{J}{}_{\mu} , \tag{3.64}$$

where

$$\begin{aligned}\lambda^{1}{}_{\mu} &\equiv e^{\frac{1}{2\sqrt{7}}\varphi+\frac{1}{2}\phi}(\chi\sigma_{\mu} - e^{-\phi}\rho_{\mu}) , \\ \lambda^{2}{}_{\mu} &\equiv e^{\frac{1}{2\sqrt{7}}\varphi}\sigma_{\mu} ,\end{aligned} \tag{3.65}$$

and the shift term is given by

$$\begin{aligned}& Z^{1}\left[\Lambda_{\mu\nu} - \left(c_{\mu\nu} - g_{Ij}a^{I}B^{j}{}_{\mu\nu} + \tfrac{2}{3}g_{Jj}h_{IK}{}^{j}a^{I}A^{JK}{}_{\mu\nu}\right)\right] \\ & \quad = e^{\frac{1}{2\sqrt{7}}\varphi+\frac{1}{2}\phi}\left[\left(\tfrac{1}{2}\Im\mathrm{m}(g) - 4\Re\mathrm{e}(k) + \tfrac{1}{2\sqrt{7}}\Im\mathrm{m}(\tilde{g})\right)\chi\right. \\ & \qquad \left. - \left(\tfrac{1}{2}\Re\mathrm{e}(g) + 4\Im\mathrm{m}(k) - \tfrac{1}{2\sqrt{7}}\Re\mathrm{e}(\tilde{g})\right)e^{-\phi}\right]\xi_{\mu\nu} ,\end{aligned} \tag{3.66}$$

$$\begin{aligned}& Z^{2}\left[\Lambda_{\mu\nu} - \left(c_{\mu\nu} - g_{Ij}a^{I}B^{j}{}_{\mu\nu} - \tfrac{2}{3}g_{Jj}h_{IK}{}^{j}a^{I}A^{JK}{}_{\mu\nu}\right)\right] \\ & \quad = e^{\frac{1}{2\sqrt{7}}\varphi+\frac{1}{2}\phi}\left(\tfrac{1}{2}\Im\mathrm{m}(g) - 4\Re\mathrm{e}(k) + \tfrac{1}{2\sqrt{7}}\Im\mathrm{m}(\tilde{g})\right)\xi_{\mu\nu} .\end{aligned} \tag{3.67}$$

Now, let us analyze the constraints that involve $h_{IJ}{}^{i}$. From those that only involve the embedding tensor we find seven linear constraints that imply those in eqs. (3.49) and that can be used to eliminate seven components of the embedding tensor:

$$\begin{aligned}&\vartheta_{\mathbf{2}}{}^{1} = 0 , && \vartheta_{\mathbf{1}}{}^{2} = \tfrac{3}{4}\vartheta_{\mathbf{2}}{}^{5} , && \vartheta_{\mathbf{1}}{}^{3} = \tfrac{3}{4}\vartheta_{\mathbf{2}}{}^{5} , \\ &\vartheta_{\mathbf{1}}{}^{1} = \tfrac{3}{2}\vartheta_{\mathbf{1}}{}^{5} , && \vartheta_{\mathbf{2}}{}^{2} = \tfrac{3}{4}\vartheta_{\mathbf{1}}{}^{5} , && \vartheta_{\mathbf{2}}{}^{3} = -\tfrac{3}{4}\vartheta_{\mathbf{1}}{}^{5} , \\ &\vartheta_{0}{}^{4} = -\tfrac{1}{6}\vartheta_{0}{}^{5} ,\end{aligned} \tag{3.68}$$

leaving the eight components (a triplet of $\mathrm{SL}(2,\mathbb{R})$ in the upper component, a singlet and two doublets of $\mathrm{SL}(2,\mathbb{R})$ in the lower components)

$$\vartheta_{0}{}^{m} , \quad m = 1,2,3 , \quad \vartheta_{0}{}^{5} , \quad \vartheta_{\mathbf{i}}{}^{4} , \quad \vartheta_{\mathbf{i}}{}^{5} , \quad \mathbf{i} = \mathbf{1},\mathbf{2} , \tag{3.69}$$

as the only independent ones. These components correspond to the eight deformation parameters of the primary deformations studied in ref. [41]. More precisely, the relation between them are

$$\begin{aligned}&\vartheta_{0}{}^{m} = m_{m} , \quad (m = 1,2,3) && \vartheta_{\mathbf{1}}{}^{4} = -m_{11} , && \vartheta_{\mathbf{1}}{}^{5} = \tilde{m}_{4} , \\ &\vartheta_{0}{}^{5} = -\tfrac{16}{3}m_{\mathrm{IIB}} , && \vartheta_{\mathbf{2}}{}^{4} = m_{\mathrm{IIA}} , && \vartheta_{\mathbf{2}}{}^{5} = m_{4} .\end{aligned} \tag{3.70}$$

JHEP10(2011)068

From the constraints that relate $h_{IJ}{}^{i}$ to $Z^{I}{}_{i}, Z^{i}$ and g_{Ii} we can determine all these tensors, up to a constant ζ, in terms of the independent components of the embedding tensor:

$$\begin{aligned} Z^{\mathbf{i}}{}_{j} &= \vartheta_0{}^{m}(T_m)_{\mathbf{j}}{}^{\mathbf{i}} - \tfrac{3}{4}\vartheta_0{}^{5}\delta_j{}^{1}\delta_1{}^{\mathbf{i}}\,, & Z^{0}{}_{i} &= 3\vartheta_{\mathbf{i}}{}^{4} + \tfrac{1}{2}\vartheta_{\mathbf{i}}{}^{5}\,, \\ g_{0i} &= 0\,, & g_{\mathbf{i}j} &= \varepsilon_{\mathbf{i}j}\,. \end{aligned} \tag{3.71}$$

The constant ζ is the coefficient of a Chern-Simons term in the 4-form field strength and, therefore, will be completely determined by supersymmetry.

Finally, using all these results in eqs. (3.41)–(3.43) we find

$$k = -\tfrac{9i}{14}e^{-\frac{3}{2\sqrt{7}}\varphi+\frac{1}{2}\phi}(\vartheta_{\mathbf{1}}{}^{4}\tau + \vartheta_{\mathbf{2}}{}^{4})\,, \tag{3.72}$$

$$\Im\mathrm{m}(f) = \tfrac{3}{28}\vartheta_0{}^{5}e^{\frac{2}{\sqrt{7}}\varphi}\,, \tag{3.73}$$

$$\begin{aligned} \Re\mathrm{e}(f) + \tfrac{3}{4\sqrt{7}}\Im\mathrm{m}(\tilde{h}) = \tfrac{1}{4}e^{\frac{2}{\sqrt{7}}\varphi+\phi}\big\{&\tfrac{1}{2}(\vartheta_0{}^{2} + \vartheta_0{}^{3}) + \big(\vartheta_0{}^{1} - \tfrac{3}{4}\vartheta_0{}^{5}\big)\chi \\ &-\tfrac{1}{2}(\vartheta_0{}^{2} - \vartheta_0{}^{3})|\tau|^{2}\big\}\,, \end{aligned} \tag{3.74}$$

which determines almost completely all the fermion shifts. We find that, in order to determine completely $\Re\mathrm{e}(f)$ and $\Im\mathrm{m}(\tilde{h})$, separately, one must study the closure of the supersymmetry algebra on the fermions of the theory or on the bosons at higher order in fermions. The result is

$$\Re\mathrm{e}(f) = \tfrac{1}{14}e^{\frac{2}{\sqrt{7}}\varphi}\vartheta_0{}^{m}\mathcal{P}_m\,, \tag{3.75}$$

$$\Im\mathrm{m}(\tilde{h}) = \tfrac{4}{\sqrt{7}}e^{\frac{2}{\sqrt{7}}\varphi}\vartheta_0{}^{m}\mathcal{P}_m\,. \tag{3.76}$$

All these results are collected in appendix C.

3.4 The 3-form C

In the next step we are going to consider the last of the fundamental, electric p-forms of the theory, the 3-form C, whose gauge transformation is given in eq. (3.51).

3.4.1 The 4-form field strength G

The 4-form field strength G is given by

$$G = \mathfrak{D}C - g_{Ii}\left(F^{I} - \tfrac{1}{2}Z^{I}{}_{j}B^{j}\right)\wedge B^{i} - \tfrac{1}{3}h_{IK}{}^{i}g_{Ji}A^{IJ}\wedge dA^{K} + Z\tilde{C}\,, \tag{3.77}$$

and it is covariant under general gauge transformations provided that the 4-form $\tilde{C}$ transforms as

$$\begin{aligned} \delta_\Lambda\tilde{C} = &-\mathfrak{D}\tilde{\Lambda} - \tilde{g}_I\left[\Lambda^{I}G + C\wedge\delta_\Lambda A^{I} + F^{I}\wedge\Lambda + \tfrac{1}{12}g_{Ji}h_{KL}{}^{i}A^{IJK}\wedge\delta_\Lambda A^{L}\right] \\ &-\tilde{g}_{ij}[2H^{i}\wedge\Lambda^{j} - B^{i}\wedge\delta_\Lambda B^{j} + 2h_{IJ}{}^{i}B^{j}\wedge A^{I}\wedge\delta_\Lambda A^{J}] \\ &-\tilde{g}_{IJK}\left[3\Lambda^{I}F^{JK} + 2(F^{I} - Z^{I}{}_{i}B^{i})\wedge A^{J}\wedge\delta_\Lambda A^{K} - \tfrac{1}{4}X_{LM}{}^{J}A^{ILM}\wedge\delta_\Lambda A^{K}\right] \\ &+Z^{i}\tilde{\Lambda}_i\,, \end{aligned} \tag{3.78}$$

JHEP10(2011)068

where the new deformation tensors that we have introduced, $\tilde{g}_I, \tilde{g}_{ij} = -\tilde{g}_{ji}$ and $\tilde{g}_{IJK} = \tilde{g}_{(IJK)}$, are subject to the constraints

$$g_{I[i}Z^I{}_{j]} + Z\tilde{g}_{ij} = 0\,, \tag{3.79}$$

$$X_I + g_{Ii}Z^i + Z\tilde{g}_I = 0\,, \tag{3.80}$$

$$h_{(IJ}{}^i g_{K)i} - Z\tilde{g}_{IJK} = 0\,, \tag{3.81}$$

plus the constraints that express the gauge invariance of the new deformation parameters

$$\tilde{X}_I\tilde{g}_J - X_I{}_J{}^K\tilde{g}_K = 0\,, \tag{3.82}$$

$$\tilde{X}_I\tilde{g}_{ij} - 2X_{I\,[i|}{}^k\tilde{g}_{k|j]} = 0\,, \tag{3.83}$$

$$\tilde{X}_I\tilde{g}_{JKL} - 3X_{I\,(J}{}^M\tilde{g}_{KL)M} = 0\,. \tag{3.84}$$

3.4.2 Closure of the supersymmetry algebra on the 3-form C

Taking into account the form of $\delta_\epsilon C_{\mu\nu\rho}$ in the undeformed case and the form of the field strength G, we arrive at the following Ansatz for the supersymmetry transformation of the 3-form C:

$$\delta_\epsilon C_{\mu\nu\rho} = -\tfrac{3}{2}e^{-\frac{1}{\sqrt{7}}\varphi}\bar{\epsilon}\gamma_{[\mu\nu}\left(\psi_{\rho]} + \tfrac{i}{6\sqrt{7}}\tilde{\lambda}^*\right) + \text{h.c.} + 3\delta_\epsilon A^I{}_{[\mu|}\left(g_{Ii}B^i{}_{|\nu\rho]} + \tfrac{2}{3}h_{IJ}{}^i g_{Ki}A^{JK}{}_{|\nu\rho]}\right). \tag{3.85}$$

The last two terms are written in terms of the tensors g_{Ii} and $h_{IJ}{}^i$. In the undeformed theory these tensors have values which are determined by supersymmetry (at orders in fermions higher than we are considering here) and that cannot be changed in the deformed theory, as we already discussed when we considered the 2-forms for $h_{IJ}{}^i$. Thus, $h_{IJ}{}^i$ is given by eq. (3.61) and g_{Ii} is given by eqs. (3.71) with $\zeta = +1$

Using the identity

$$\begin{aligned}
&\xi^\sigma G_{\sigma\mu\nu\rho} + 3\pounds_\xi A^I{}_{[\mu|}\left[g_{Ii}B^i{}_{|\nu\rho]} + \tfrac{2}{3}h_{IJ}{}^i g_{Ki}A^{JK}{}_{|\nu\rho]}\right] = \\
&\quad = \pounds_\xi C_{\mu\nu\rho} - 3\mathfrak{D}_{[\mu|}\left[\left(c_{|\nu\rho]} - g_{Ij}a^I B^j{}_{|\nu\rho]} + \tfrac{2}{3}g_{Jj}h_{IK}{}^j a^I A^{JK}{}_{|\nu\rho]}\right)\right] \\
&\qquad + g_{Ii}\left[-a^I H^i{}_{\mu\nu\rho} - 3F^I{}_{[\mu\nu|}(b^i{}_{|\rho]} - h_{JK}{}^i a^J A^K{}_{|\rho]})\right] \\
&\qquad + Z\left\{\tilde{c}_{\mu\nu\rho} - \tilde{g}_I a^I C_{\mu\nu\rho} + 3\tilde{g}_{ij}B^i{}_{[\mu\nu|}(b^j{}_{|\rho]} - h_{JK}{}^j a^J A^K{}_{\rho)} - 12\tilde{g}_{IJK}a^I A^J{}_{[\mu}\partial_\nu A^K{}_{\rho]}\right. \\
&\qquad \left. + 3h_{IJ}{}^i\tilde{g}_{ij}a^I A^J{}_{[\mu}B^j{}_{\nu\rho]} - \tfrac{1}{4}\left(h_{IJ}{}^i g_{Ki}\tilde{g}_L + 3X_{JK}{}^M\tilde{g}_{ILM}\right)a^I A^{JKL}{}_{\mu\nu\rho}\right\}\,,
\end{aligned} \tag{3.86}$$

one can see that the local supersymmetry algebra closes into a general coordinate transformation plus a gauge transformation of C of the form eq. (3.51) with

$$\Lambda_{\mu\nu} = e^{\frac{1}{\sqrt{7}}\varphi}\xi_{\mu\nu} + \left(c_{\mu\nu} - g_{Ij}a^I B^j{}_{\mu\nu} - \tfrac{2}{3}g_{Jj}h_{IK}{}^j a^I A^{JK}{}_{\mu\nu}\right)\,, \tag{3.87}$$

and with the identification

$$\begin{aligned}
&Z\left\{\tilde{\Lambda}_{\mu\nu\rho} - \tilde{c}_{\mu\nu\rho} + \tilde{g}_I a^I C_{\mu\nu\rho} + 3\tilde{g}_{ij}B^i{}_{[\mu\nu|}\left(b^j{}_{|\rho]} - h_{JK}{}^j a^J A^K{}_{|\rho]}\right) - 12\tilde{g}_{IJK}a^I A^J{}_{[\mu}\partial_\nu A^K{}_{\rho]}\right. \\
&\quad \left. - 3\tilde{g}_{ij}h_{IJ}{}^i a^I A^J{}_{[\mu}B^j{}_{\nu\rho]} + \tfrac{1}{4}\left(\tilde{g}_L g_{Ki}h_{IJ}{}^i + 3\tilde{g}_{ILN}X_{JK}{}^N\right)a^I A^{JKL}{}_{\mu\nu\rho}\right\} \\
&\quad = 6e^{-\frac{1}{\sqrt{7}}\varphi}\left[\Im\mathrm{m}(f) + \tfrac{1}{6\sqrt{7}}\Re\mathrm{e}(\tilde{h})\right]\zeta_{\mu\nu\rho}\,.
\end{aligned} \tag{3.88}$$

JHEP10(2011)068

Comparing eq. (3.87) with eqs. (3.66) and (3.67) we find that

$$Z^1 = X_{\mathbf{2}} = 3\vartheta_{\mathbf{2}}{}^4 - \tfrac{1}{4}\vartheta_{\mathbf{2}}{}^5\,, \qquad Z^2 = -X_{\mathbf{1}} = -3\vartheta_{\mathbf{1}}{}^4 + \tfrac{1}{4}\vartheta_{\mathbf{1}}{}^5\,. \tag{3.89}$$

To make further progress it is convenient to compute the 5-form $\tilde{G}$ since it will contain the tensors $\tilde{g}_I, \tilde{g}_{ij}, \tilde{g}_{IJK}$ that appear in the above expression. These tensors cannot be deformed (just as it happens with $h_{IJ}{}^i$) and their values can be found by comparing the general form of $\tilde{G}$ with the value found by duality, eq. (2.41).

The generic form of the magnetic 5-form field strength $\tilde{G}$ is

$$\begin{aligned}\tilde{G} = {} & \mathfrak{D}\tilde{C} - \tilde{g}_J\left[(F^J - Z^J{}_j B^j)\wedge C + \tfrac{1}{12} g_{Kj} h_{MN}{}^j A^{JKM}\wedge dA^N\right] \\ & + 2\tilde{g}_{ij}\left(H^i - \tfrac{1}{2}\mathfrak{D}B^i\right)\wedge B^j - \tilde{g}_{JKL}\left(A^J\wedge dA^{KL} + \tfrac{3}{4} X_{MN}{}^L A^{JMN}\wedge dA^K\right) \\ & + Z^i \tilde{B}_i\,,\end{aligned} \tag{3.90}$$

and comparing this generic expression with eq. (2.41) we find that

$$\tilde{g}_I = -\delta_I{}^0\,, \qquad \tilde{g}_{ij} = -\tfrac{1}{2}\varepsilon_{ij}\,, \qquad \tilde{g}_{IJK} = 0\,. \tag{3.91}$$

Plugging these values into the constraints that involve Z eqs. (3.53), (3.55), and (3.79)–(3.81) we find that it must be related to $\vartheta_0{}^5$ by

$$Z = -\tfrac{3}{4}\vartheta_0{}^5\,, \tag{3.92}$$

and that $\vartheta_0{}^5$ must satisfy the two doublets of quadratic constraints

$$\vartheta_{\mathbf{i}}{}^4\vartheta_0{}^5 = 0\,, \tag{3.93}$$

$$\vartheta_{\mathbf{i}}{}^5\vartheta_0{}^5 = 0\,. \tag{3.94}$$

Plugging our results into all the other constraints between deformation tensors, we find that all of them are satisfied provided that the quadratic constraints

$$\varepsilon^{\mathbf{ij}}\vartheta_{\mathbf{i}}{}^4\vartheta_{\mathbf{j}}{}^5 = 0\,, \tag{3.95}$$

$$\vartheta_0{}^m\left(12\vartheta_{\mathbf{i}}{}^4 + 5\vartheta_{\mathbf{i}}{}^5\right) = 0\,, \tag{3.96}$$

$$\vartheta_{\mathbf{j}}{}^4\left(\vartheta_0^m T_m\right)_{\mathbf{i}}{}^{\mathbf{j}} = 0\,, \tag{3.97}$$

are also satisfied. This set of irreducible quadratic constraints that cannot be used to solve some deformation parameters in terms of the rest in an analytic form, and with which the 9-form potentials of the theory may be associated as explained in section 2.2 is one of our main results.

4 Summary of results and discussion

In the previous section we have constructed order by order in the rank of the p-forms the supersymmetric tensor hierarchy of maximal 9-dimensional supergravity, up to $p = 3$, which covers all the fundamental fields of the theory.

JHEP10(2011)068

As it usually happens in all maximal supergravity theories, all the deformation parameters can be expressed in terms of components of the embedding tensor. Furthermore, we have shown that gauge invariance and local supersymmetry allow for one triplet, two doublets and one singlet of independent components of the embedding tensor

$$\vartheta_0{}^m\,,\quad m=1,2,3\,,\quad \vartheta_0{}^5\,,\quad \vartheta_{\mathbf{i}}{}^4\,,\quad \vartheta_{\mathbf{i}}{}^5\,,\quad \mathbf{i}=\mathbf{1},\mathbf{2}\,. \tag{4.1}$$

They can be identified with the deformation parameters studied in ref. [41]:

$$\begin{aligned} &\vartheta_0{}^m = m_m\,,\ (m=1,2,3) && \vartheta_{\mathbf{1}}{}^4 = -m_{11}\,, && \vartheta_{\mathbf{1}}{}^5 = \tilde{m}_4\,,\\ &\vartheta_0{}^5 = -\tfrac{16}{3}m_{\rm IIB}\,, && \vartheta_{\mathbf{2}}{}^4 = m_{\rm IIA}\,, && \vartheta_{\mathbf{2}}{}^5 = m_4\,. \end{aligned} \tag{4.2}$$

This proves, on the one hand, that no more deformations are possible and, on the other hand, that all the deformations of maximal 9-dimensional supergravity have a higher-dimensional origin, as shown in ref. [41].

Furthermore, we have also shown that it is not possible to give non-zero values to all the deformation parameters at the same time, since they must satisfy the quadratic constraints

$$\vartheta_0{}^m\left(12\vartheta_{\mathbf{i}}{}^4+5\vartheta_{\mathbf{i}}{}^5\right)\equiv\mathcal{Q}^m{}_{\mathbf{i}}=0\,, \tag{4.3}$$

$$\vartheta_{\mathbf{i}}{}^4\vartheta_0{}^5\equiv\mathcal{Q}^4{}_{\mathbf{i}}=0\,, \tag{4.4}$$

$$\vartheta_{\mathbf{i}}{}^5\vartheta_0{}^5\equiv\mathcal{Q}^5{}_{\mathbf{i}}=0\,, \tag{4.5}$$

$$\vartheta_{\mathbf{j}}{}^4\left(\vartheta_0^m T_m\right)_{\mathbf{i}}{}^{\mathbf{j}}\equiv\mathcal{Q}_{\mathbf{i}}=0\,, \tag{4.6}$$

$$\varepsilon^{\mathbf{ij}}\vartheta_{\mathbf{i}}{}^4\vartheta_{\mathbf{j}}{}^5\equiv\mathcal{Q}=0\,, \tag{4.7}$$

all of which are related to gauge invariance.

Using these results, we can now apply the arguments developed in section 2.2 to relate the number of symmetries (Noether currents), deformation parameters, and quadratic constraints to the numbers (and symmetry properties) of 7-, 8- and 9-forms of the theory. Our results can be compared with those presented in ref. [12] (table 6) and ref. [13] (table 3) and found from E_{11} level decomposition.

Associated with the symmetry group of the equations of motion of the theory, $\mathrm{SL}(2,\mathbb{R})\times\mathbb{R}^2$ there are 5 Noether currents j_A that fit into one triplet and two singlets of $\mathrm{SL}(2,\mathbb{R})$ and are explicitly given in appendix B. Their weights are given in table 4. They can be dualized as explained in section 2.2 into a triplet and two singlets of 7-forms $\tilde{A}_{(7)}$ whose weights are given in table 7. In refs. [12, 13] the β rescaling has not been considered. As mentioned before, it corresponds to the so-called trombone symmetry which may not survive to higher-derivative string corrections. The associated 7-form singlet $\tilde{A}^5_{(7)}$ does not appear in their analysis. The weights assigned in those references to the fields correspond to one third of the weight of the α rescaling in our conventions.

Associated with each of the $\mathrm{SL}(2,\mathbb{R})$ multiplets of independent embedding-tensor components there is a dual multiplet of 8-forms $\tilde{A}_{(8)}$ (i.e. one triplet, two doublets and one singlet) whose weights are given in table 7. The doublet and singlet associated with the gauging of the trombone symmetry using the doublet and singlet of 1-forms are missing in

JHEP10(2011)068

$\mathbb{R}^+$	j_1	$j_2 - j_3$	$j_2 + j_3$	j_4	j_5
α	0	0	0	0	0
β	0	$+3/4$	$-3/4$	0	0
γ	0	-2	$+2$	0	0
δ	0	0	0	0	0

Table 4. Weights of the Noether currents.

$\mathbb{R}^+$	$\vartheta_0{}^1$	$\vartheta_0{}^2 - \vartheta_0{}^3$	$\vartheta_0{}^2 + \vartheta_0{}^3$	$\vartheta_{\mathbf{1}}{}^4, \vartheta_{\mathbf{1}}{}^5$	$\vartheta_{\mathbf{1}}{}^4, \vartheta_{\mathbf{2}}{}^5$	$\vartheta_0{}^5$
α	-3	-3	-3	0	0	-3
β	$-1/2$	$-5/4$	$1/4$	$3/4$	0	$-1/2$
γ	0	2	-2	-1	1	0
δ	0	0	0	-2	-2	0

Table 5. Weights of the embedding tensor components.

$\mathbb{R}^+$	$\mathcal{Q}_{\mathbf{1}}{}^1$	$\mathcal{Q}_{\mathbf{2}}{}^1$	$\mathcal{Q}_{\mathbf{1}}{}^{2-3}$	$\mathcal{Q}_{\mathbf{2}}{}^{2-3}$	$\mathcal{Q}_{\mathbf{1}}{}^{2+3}$	$\mathcal{Q}_{\mathbf{2}}{}^{2+3}$	$\mathcal{Q}_{\mathbf{1}}{}^4, \mathcal{Q}_{\mathbf{1}}{}^5$	$\mathcal{Q}_{\mathbf{2}}{}^4, \mathcal{Q}_{\mathbf{2}}{}^5$	$\mathcal{Q}_{\mathbf{1}}$	$\mathcal{Q}_{\mathbf{2}}$	$\mathcal{Q}$
α	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	0
β	$1/4$	$-1/2$	$-1/2$	$-5/4$	1	$1/4$	$1/4$	$-1/2$	$1/4$	$-1/2$	$3/4$
γ	-1	1	1	3	-3	-1	-1	1	-1	1	0
δ	-2	-2	-2	-2	-2	-2	-2	-2	-2	-2	-4

Table 6. Weights of quadratic constraints components.

refs. [12, 13], but the rest of the 8-forms and their weights are in perfect agreement with those obtained from E_{11}. Given the amount of work that it takes to determine which are the independent components of the embedding tensor allowed by supersymmetry, this is a quite non-trivial test of the consistency of the E_{11} and the embedding-tensor approaches.

Finally, associated with each of the quadratic constraints that the components of the embedding tensor must satisfy $\mathcal{Q}_{\mathbf{i}}{}^m, \mathcal{Q}_{\mathbf{i}}{}^4, \mathcal{Q}_{\mathbf{i}}{}^5, \mathcal{Q}_{\mathbf{i}}, \mathcal{Q}$ there is a 9-form potential $\tilde{A}_{(9)}$. The weights of these potentials are given in table 7. If we set to zero the embedding-tensor components associated with the trombone symmetry $\vartheta_A{}^5$, the only constraints which are not automatically solved are

$$\mathcal{Q}_{\mathbf{i}}{}^m = 12\vartheta_0{}^m\vartheta_{\mathbf{i}}{}^4 = 0\,, \qquad \mathcal{Q}_{\mathbf{i}} = \vartheta_{\mathbf{j}}{}^4\,(\vartheta_0^m T_m)_{\mathbf{i}}{}^{\mathbf{j}} = 0\,. \tag{4.8}$$

The first of these constraints can be decomposed into a quadruplet and a doublet: rewriting $\mathcal{Q}_{\mathbf{i}}{}^m$ in the equivalent form

$$\mathcal{Q}_{\mathbf{i(jk)}} = \vartheta_{\mathbf{i}}{}^4\,(\vartheta_0^m T_m)_{\mathbf{j}}{}^{\mathbf{l}}\varepsilon_{\mathbf{kl}}\,, \tag{4.9}$$

the quadruplet corresponds to the completely symmetric part $\mathcal{Q}_{\mathbf{(ijk)}}$ and the doublet to

$$\varepsilon^{\mathbf{jk}}\mathcal{Q}_{\mathbf{j(ki)}} = -\mathcal{Q}_{\mathbf{i}}\,, \tag{4.10}$$

which is precisely the other doublet. Therefore, we get the quadruplet and one doublet of 9-forms with weight 4 under $\alpha/3$, while one more doublet is found in refs. [12, 13].

JHEP10(2011)068

$\mathbb{R}^+$	$\tilde{A}^{m}_{(7)}$	$\tilde{A}^{4}_{(7)}$	$\tilde{A}^{5}_{(7)}$	$\tilde{A}^{m}_{(8)}$	$\tilde{A}^{4\,\mathbf{i}}_{(8)}$	$\tilde{A}^{5\,\mathbf{i}}_{(8)}$	$\tilde{A}^{4}_{(8)}$	$\tilde{A}^{\mathbf{i}}_{(9)\,m}$	$\tilde{A}^{\mathbf{i}}_{(9)\,4}$	$\tilde{A}^{\mathbf{i}}_{(9)\,5}$	$\tilde{A}^{\mathbf{i}}_{(9)}$	$\tilde{A}_{(9)}$
α	9	9	9	12	9	9	12	12	12	12	12	9
δ	8	8	8	8	2	2	8	10	10	10	10	12

Table 7. Weights of the 7-, 8- and 9-form fields.

This situation is similar to the one encountered in the $N = 2$ theories in $d = 4, 5, 6$ dimensions [30]. In those cases, the Kač-Moody (here E_{11}) approach predicts one doublet of d-form potentials more than the embedding-tensor formalism [11]. However, it can be seen that taking the undeformed limit of the results obtained in the embedding-tensor formalism, one additional doublet of d-forms arises because some Stückelberg shifts proportional to deformation tensors that could be used to eliminate them, now vanish. Furthermore, the local supersymmetry algebra closes on them as independent fields.

By analogy with what happens in the $N = 2$ theories in $d = 4, 5, 6$ dimensions, the same mechanism can make our results compatible with those of the E_{11} approach (up to the trombone symmetry): we expect the existence of two independent doublets of 9-forms in the undeformed theory but we also expect new Stückelberg transformations in the deformed theory such that one a combination of them is independent and the supersymmetry algebra closes.

This possibility (and the exclusion of any further 9-forms) can only be proven by the direct exploration of all the possible candidates to 9-form supersymmetry transformation rules, to all orders in fermions, something that lies outside the boundaries of this work.

5 Conclusions

In this paper we have applied the embedding-tensor formalism to the study of the most general deformations (i.e. gaugings and massive deformations) of maximal 9-dimensional supergravity. We have used the complete global $\mathrm{SL}(2,\mathbb{R}) \times \mathbb{R}^2$ symmetry of its equations of motion, which includes the so-called *trombone symmetry*. We have found the constraints that the deformation parameters must satisfy in order to preserve both gauge and supersymmetry invariance (the latter imposed through the closure of the local supersymmetry algebra to lowest order in fermions). We have used most of the constraints to express some components of the deformation tensors in terms of a few components of the embedding tensor which we take to be independent and which are given in eq. (4.1). At that point we have started making contact with the results of ref. [41], since those independent components are precisely the 8 possible deformations identified there. All of them have a higher-dimensional origin discussed in detail in ref. [41]. The field strengths, gauge transformations and supersymmetry transformations of the deformed theory, written in terms of the independent deformation tensors, are collected in appendix C.

The 8 independent deformation tensors are still subject to quadratic constraints, given in eq. (4.3), but those constraints cannot be used to express analytically some of them in terms of the rest, and, therefore, we must keep the 8 deformation parameters and we must enforce these irreducible quadratic constraints.

JHEP10(2011)068

In section 4 we have used our knowledge of the global symmetries (and corresponding Noether 1-forms), the independent deformation tensors and the irreducible quadratic constraints of the theory, together with the general arguments of section 2.2 to determine the possible 7-, 8- and 9-forms of the theory (table 7), which are dual to the Noether currents, independent deformation tensors and irreducible quadratic constraints. We have compared this spectrum of higher-rank forms with the results of refs. [12, 13], based on E_{11} level decomposition. We have found that, in the sector unrelated to the trombone symmetry, which was excluded from that analysis, the embedding-tensor formalism predicts one doublet of 9-forms less than the E_{11} approach. However, both predictions are not contradictory: the extra doublet of 9-forms may not survive the deformations on which the embedding-tensor formalism is built: new 9-form Stückelberg shifts proportional to the deformation parameters may occur that can be used to eliminate it so only one combination of the two 9-form doubles survives. This mechanism is present in the $N = 2$ $d = 4, 5, 6$ theories [30], although the physics behind it is a bit mysterious.

We can conclude that we have satisfactorily identified the extended field content (the tensor hierarchy) of maximal 9-dimensional supergravity and, furthermore, that all the higher-rank fields have an interpretation in terms of symmetries and gaugings. This situation is in contrast with our understanding of the extended field content of the maximal 10-dimensional supergravities ($N = 2A, B$) for which the E_{11} approach can be used to get a prediction of the higher-rank forms (which turns out to be correct [4–6]) but thw embedding-tensor approach apparently cannot be used[16] for this end. This seems to preclude an interpretation for the 9- and 10-form fields in terms of symmetries and gaugings,[17] at least if we insist in the standard construction of the tensor hierarchy that starts with the gauging of global symmetries. Perhaps a more general point of view is necessary.

Acknowledgments

TO would like to thank E. Bergshoeff for several useful conversations. This work has been supported in part by the Spanish grants FPA2009-07692, FIS2007-1234 and FPA2008-453, the Comunidad de Madrid grant HEPHACOS S2009ESP-1473 and the Spanish Consolider-Ingenio 2010 program CPAN CSD2007-00042 and Comunidad de Murcia-Fundación Seneca research grants. The work of JJFM has been supported by the Spanish Ministry of Education FPU grant AP2008-00919. JJFM would like to thank CERN and IFT-UAM/CSIC for their hospitality dn Prof. J. Bernabéu for his kind support . ET wishes to thank the hospitality of the CERN-TH division. JJFM would like to thank the hospitality of the CERN-TH Division and the IFT-UAM/CSIC. TO wishes to thank M.M. Fernández for her permanent support.

[16] In the $N = 2B$ case there are no 1-forms to be used as gauge fields and in the $N = 2A$ case the only 1-form available is not invariant under the only rescaling symmetry available.

[17] The 8-form fields are dual to the Noether currents of the global symmetries.

JHEP10(2011)068

n	0	1	2	3	4	5	6	7	8	9
a_n	$-$	$+$	$-$	$+$	$-$	$+$	$-$	$+$	$-$	$+$
b_n	$+$	$-$	$-$	$+$	$+$	$-$	$-$	$+$	$+$	$-$

Table 8. Values of the coefficients a_n and b_n defined in eqs. (A.3).

A Conventions

We follow the conventions of ref. [41]. In particular, we use mostly plus signature $(-,+,\cdots,+)$ and the gamma matrices satisfy

$$\gamma_a^* = -\gamma_a\,, \qquad \gamma_a = \eta_{aa}\gamma_a^\dagger\,. \tag{A.1}$$

The Dirac conjugate of a spinor ϵ is defined by

$$\bar\epsilon \equiv \epsilon^\dagger\gamma_0\,. \tag{A.2}$$

Then, we have

$$\begin{aligned}(\bar\epsilon\gamma^{(n)}\lambda)^* &= a_n\bar\epsilon^*\gamma^{(n)}\lambda^*\,,\\ (\bar\epsilon\gamma^{(n)}\lambda)^* &= b_n\bar\lambda\gamma^{(n)}\epsilon\,,\end{aligned} \tag{A.3}$$

where the signs a_n and b_n are given in table 8

A.1 Spinor bilinears

We define the following real bilinears of the supersymmetry parameters ϵ_1 and ϵ_2:

$$\bar\epsilon_2\epsilon_1 \equiv a + ib\,, \tag{A.4}$$

$$\bar\epsilon_2\epsilon_1^* \equiv c + id\,, \tag{A.5}$$

$$\bar\epsilon_2\gamma_{\mu_1\cdots\mu_n}\epsilon_1 \equiv \xi_{\mu_1\cdots\mu_n} + i\zeta_{\mu_1\cdots\mu_n}\,, \tag{A.6}$$

$$\bar\epsilon_2\gamma_{\mu_1\cdots\mu_n}\epsilon_1^* \equiv \sigma_{\mu_1\cdots\mu_n} + i\rho_{\mu_1\cdots\mu_n}\,, \tag{A.7}$$

A.2 Relation with other conventions

The electric fields used in this paper are related to those used in ref. [37] (which uses a mostly minus signature) as follows:

$$K = e^{\frac{\sqrt{7}}{3}\varphi}\,, \tag{A.8}$$

$$\lambda \equiv C^{(0)} + ie^{-\varphi} = \tau \equiv \chi + ie^{-\phi}\,, \tag{A.9}$$

$$A_{(1)} = A^0\,, \tag{A.10}$$

$$\mathbf{A}_{(1)} = A^{\mathbf{i}}\,, \tag{A.11}$$

$$\mathbf{A}_{(2)} = B^i + \tfrac{1}{2}A^{0\mathbf{i}}\,, \tag{A.12}$$

$$A_{(3)} = -C + \tfrac{1}{2}\varepsilon_{\mathbf{i}j}A^{\mathbf{i}}\wedge B^j - \tfrac{1}{12}\varepsilon_{\mathbf{ij}}A^{0\mathbf{ij}}\,, \tag{A.13}$$

$$A_{(4)} = -\tilde C + C\wedge A^0 - \tfrac{1}{4}\varepsilon_{i\mathbf{j}}B^i\wedge A^{0\mathbf{j}}\,. \tag{A.14}$$

JHEP10(2011)068

The field strengths are related by

$$F_{(2)} = F^0\,, \tag{A.15}$$
$$\mathbf{F}_{(2)} = F^{\mathbf{i}}\,, \tag{A.16}$$
$$\mathbf{F}_{(3)} = H^i\,, \tag{A.17}$$
$$F_{(4)} = -G\,, \tag{A.18}$$
$$F_{(5)} = -\tilde{G}\,. \tag{A.19}$$

The relation with the fields used in ref. [41] (which also uses mostly plus signature) is given by (our fields are in the r.h.s. of these equations)

$$B^i = -\left(B^i + \tfrac{1}{2}A^{0\mathbf{i}}\right)\,, \tag{A.20}$$
$$C = -\left(C - \tfrac{1}{6}\varepsilon_{\mathbf{ij}}A^{0\mathbf{ij}}\right)\,, \tag{A.21}$$

while the field strengths are related by

$$H^i = -H^i\,, \tag{A.22}$$
$$G = -G\,. \tag{A.23}$$

The rest of the fields are identical.

B Noether currents

The Noether 1-form currents of the undeformed theory j_A are given by

$$\begin{aligned}
\star j_m =& \star\, d\mathcal{M}_{ij}\left(\mathcal{M}^{-1}\right)_{jk} T_{mi}{}^{k} + e^{\frac{4}{\sqrt{7}}\varphi}(\mathcal{M}^{-1}_{\mathbf{ij}})T_{m\mathbf{k}}{}^{\mathbf{i}}A^{\mathbf{k}}\wedge\star F^{\mathbf{j}}\\
&+ T_{mk}{}^{i}\left[e^{-\frac{1}{\sqrt{7}}\varphi}\mathcal{M}^{-1}_{ij}\left(B^k - \tfrac{1}{2}A^{0k}\right)\wedge\star H^j + \tfrac{1}{2}\varepsilon_{ij}\left(-2e^{\frac{2}{\sqrt{7}}\varphi}A^{\mathbf{j}}\wedge B^k\wedge\star G\right.\right.\\
&+ \left(B^j - A^{0j}\right)\wedge B^k\wedge G + \varepsilon_{ln}A^l\wedge B^{jk}\wedge\left(H^n - \tfrac{1}{2}A^n\wedge F^0\right)\\
&\left.\left.+\tfrac{1}{4}\varepsilon_{ln}A^{0ln}\wedge B^k\wedge H^j\right)\right]\,,
\end{aligned} \tag{B.1}$$

$$\begin{aligned}
\star j_4 =& \tfrac{6}{\sqrt{7}}\star d\varphi + 3\Big[e^{\frac{4}{\sqrt{7}}\varphi}A^0\wedge\star F^0 + e^{-\frac{1}{\sqrt{7}}\varphi}\mathcal{M}^{-1}_{ij}\left(B^i+\tfrac{1}{2}A^{0i}\right)\wedge\star H^j + e^{\frac{2}{\sqrt{7}}\varphi}\left(C-\tfrac{1}{6}\varepsilon_{\mathbf{ij}}A^{0\mathbf{ij}}\right)\wedge\star G\\
&+A^0\wedge\left(C+\varepsilon_{\mathbf{i}j}A^{\mathbf{i}}\wedge B^j\right)\wedge G\Big] + \tfrac{3}{2}\varepsilon_{ij}\left[\left(-C+\varepsilon_{kl}A^k\wedge B^l - \tfrac{7}{12}\varepsilon_{kl}A^{0kl}\right)\wedge B^i\wedge H^j\right.\\
&\left.-\tfrac{3}{2}A^{0i}\wedge C\wedge H^j + \left(A^i\wedge B^j - \tfrac{1}{2}A^{0ij}\right)\wedge F^0\wedge C\right]\,,
\end{aligned} \tag{B.2}$$

$$\begin{aligned}
\star j_5 =& \tfrac{\sqrt{7}}{4}\star d\varphi - \tfrac{3}{8}\star\frac{\tau d\bar{\tau} + \text{c.c.}}{(\Im\text{m}\tau)^2} + e^{\frac{4}{\sqrt{7}}\varphi}T_{50}{}^{0}A^0\wedge\star F^0 + e^{\frac{3}{\sqrt{7}}\varphi}T_{5\mathbf{k}}{}^{\mathbf{i}}\mathcal{M}^{-1}_{ij}A^{\mathbf{k}}\wedge\star F^{\mathbf{j}}\\
&+ e^{-\frac{1}{\sqrt{7}}\varphi}\mathcal{M}^{-1}_{ij}\left[T_{5k}{}^{i}\left(B^k - \tfrac{1}{2}A^{0k}\right) + \tfrac{1}{4}A^{0i}\right]\wedge\star H^j\\
&+ e^{\frac{2}{\sqrt{7}}\varphi}\left(T_5 C - \tfrac{1}{12}\varepsilon_{ij}A^{0ij} - T_{5k}{}^{i}\varepsilon_{ij}\left(A^k\wedge B^j - \tfrac{1}{6}A^{0kj}\right)\right)\wedge\star G\\
&+ \tfrac{1}{4}\varepsilon_{ij}\left[T_{5k}{}^{i}\left(-2B^{jk} + 3A^{0j}\wedge B^k - 5A^{0k}\wedge B^j\right) - \tfrac{1}{2}A^{0i}\wedge B^j\right]\wedge G\\
&+ \tfrac{1}{4}\varepsilon_{ij}\left[T_{5k}{}^{i}\left(+2\varepsilon_{ln}A^l\wedge B^{nk} - \varepsilon_{ln}A^{0ln}\wedge B^k\right) - T_5\left(6A^{0i}+B^i\right)\wedge C - \tfrac{1}{12}\varepsilon_{kl}A^{0kl}\wedge B^i\right]\wedge H^j\\
&+ \varepsilon_{ij}\varepsilon_{ln}T_{5k}{}^{i}\left[\tfrac{5}{6}A^{0jk}\wedge B^l - A^{0lj}\wedge B^k + \tfrac{1}{2}A^k\wedge B^{jl}\right]\wedge H^n\\
&+ T_5\left[A^0\wedge C\wedge G + \tfrac{1}{2}\varepsilon_{ij}\left(B^j + \tfrac{1}{2}A^{0j}\right)\wedge A^i\wedge F^0\wedge C\right]
\end{aligned} \tag{B.3}$$

C Final results

In this appendix we give the final form of the deformed covariant field strengths, covariant derivatives, gauge and supersymmetry transformations in terms of the independent deformation parameters given in eq. (4.1). We must bear in mind that they are assumed to satisfy the irreducible quadratic constraints given in eq. (4.3) and only then the field strengths etc. have the right transformation properties.

The covariant derivatives of the scalar fields are given by

$$\mathfrak{D}\varphi = d\varphi - \tfrac{137}{24\sqrt{7}}\vartheta_0{}^5 A^0 + \left(-\tfrac{\sqrt{7}}{4}\vartheta_{\mathbf{i}}{}^4 + \tfrac{6}{\sqrt{7}}\vartheta_{\mathbf{i}}{}^5\right) A^{\mathbf{i}}\,, \tag{C.1}$$

$$\mathfrak{D}\tau = d\tau + \vartheta_0{}^m k_m{}^\tau A^0 - \tfrac{3}{4}\vartheta_0{}^5\tau A^0 + \tfrac{3}{4}\left(\vartheta_{\mathbf{1}}{}^5\tau + \vartheta_{\mathbf{2}}{}^5\right)\left(A^{\mathbf{1}} - \tau A^{\mathbf{2}}\right)\,, \tag{C.2}$$

and their gauge transformations are explictly given by

$$\delta_\Lambda\varphi = -\tfrac{137}{24\sqrt{7}}\vartheta_0{}^5\Lambda^0 + \left(-\tfrac{\sqrt{7}}{4}\vartheta_{\mathbf{i}}{}^4 + \tfrac{6}{\sqrt{7}}\vartheta_{\mathbf{i}}{}^5\right)\Lambda^{\mathbf{i}}\,, \tag{C.3}$$

$$\delta_\Lambda\tau = \vartheta_0{}^m k_m{}^\tau\Lambda^0 - \tfrac{3}{4}\vartheta_0{}^5\tau\Lambda^0 + \tfrac{3}{4}\left(\vartheta_{\mathbf{1}}{}^5\tau + \vartheta_{\mathbf{2}}{}^5\right)\left(\Lambda^{\mathbf{1}} - \tau\Lambda^{\mathbf{2}}\right)\,. \tag{C.4}$$

The deformed p-form field strengths are given by

$$F^0 = dA^0 - \tfrac{1}{2}\left(3\vartheta_{\mathbf{i}}{}^4 + \tfrac{1}{2}\vartheta_{\mathbf{i}}{}^5\right)A^{0\mathbf{i}} + \left(3\vartheta_{\mathbf{i}}{}^4 + \tfrac{1}{2}\vartheta_{\mathbf{i}}{}^5\right)B^{\mathbf{i}}\,, \tag{C.5}$$

$$\begin{aligned} F^{\mathbf{i}} = {}& dA^{\mathbf{i}} + \tfrac{1}{2}\left(\vartheta_0{}^m(T_m^{(3)})_{\mathbf{j}}{}^{\mathbf{i}}A^{0\mathbf{j}} - \tfrac{3}{4}\delta_{\mathbf{1}}{}^{\mathbf{i}}\vartheta_0{}^5 A^{0\mathbf{1}} + \tfrac{3}{2}\varepsilon^{\mathbf{ij}}\vartheta_{\mathbf{j}}{}^5 A^{\mathbf{12}}\right) \\ & + \vartheta_0{}^m(T_m^{(3)})_{\mathbf{j}}{}^{\mathbf{i}}B^j - \tfrac{3}{4}\delta_1{}^{\mathbf{i}}\vartheta_0{}^5 B^1\,, \end{aligned} \tag{C.6}$$

$$\begin{aligned} H^i = {}& \mathfrak{D}B^i + \tfrac{1}{2}\left(A^0\wedge dA^{\mathbf{i}} + A^{\mathbf{i}}\wedge dA^0\right) + \tfrac{1}{6}\varepsilon^{i\mathbf{j}}\left(3\vartheta_{\mathbf{j}}{}^4 + \tfrac{1}{2}\vartheta_{\mathbf{j}}{}^5\right)A^{0\mathbf{12}} \\ & + \varepsilon^{i\mathbf{j}}\left(3\vartheta_{\mathbf{j}}{}^4 - \tfrac{1}{4}\vartheta_{\mathbf{j}}{}^5\right)C\,, \end{aligned} \tag{C.7}$$

$$\begin{aligned} G = {}& \mathfrak{D}C - \varepsilon_{\mathbf{i}j}\left[F^{\mathbf{i}}\wedge B^j - \tfrac{1}{2}\delta^j{}_{\mathbf{j}}\left(A^{\mathbf{i}}\wedge dA^{\mathbf{j}} - \tfrac{1}{3}d(A^{0\mathbf{ij}})\right)\right] \\ & + \tfrac{1}{2}\left(\varepsilon_{ij}\vartheta_0{}^m(T_m^{(2)})_k{}^i B^{jk} - \tfrac{3}{4}\vartheta_0{}^5 B^{12}\right) + Z\tilde{C}\,, \end{aligned} \tag{C.8}$$

where the covariant derivatives acting on the different fields are given by

$$\begin{aligned} \mathfrak{D}B^i = {}& dB^i + \vartheta_0{}^m(T_m^{(2)})_j{}^i A^0\wedge B^j - \tfrac{3}{4}\delta_1{}^i\vartheta_0{}^5 A^0\wedge B^1 \\ & + \left(3\vartheta_{\mathbf{k}}{}^4 - \tfrac{1}{4}\vartheta_{\mathbf{k}}{}^5\right)A^{\mathbf{k}}\wedge B^i + \tfrac{3}{4}\delta_{\mathbf{j}}{}^i\vartheta_k{}^5 A^{\mathbf{j}}\wedge B^k\,, \end{aligned} \tag{C.9}$$

$$\mathfrak{D}C = dC - \tfrac{3}{4}\vartheta_0{}^5 A^0\wedge C + \left(3\vartheta_{\mathbf{i}}{}^4 - \tfrac{1}{4}\vartheta_{\mathbf{i}}{}^5\right)A^{\mathbf{i}}\wedge C\,. \tag{C.10}$$

The field strengths transform covariantly under the gauge transformations

$$\delta_\Lambda A^0 = -\mathfrak{D}\Lambda^0 + \left(3\vartheta_{\mathbf{i}}{}^4 + \tfrac{1}{2}\vartheta_{\mathbf{i}}{}^5\right)\Lambda^i\,, \tag{C.11}$$

$$\delta_\Lambda A^{\mathbf{i}} = -\mathfrak{D}\Lambda^{\mathbf{i}} + \vartheta_0{}^m(T_m^{(3)})_{\mathbf{j}}{}^{\mathbf{i}}\Lambda^j - \tfrac{3}{4}\delta_1{}^{\mathbf{i}}\vartheta_0{}^5\Lambda^1\,, \tag{C.12}$$

$$\begin{aligned} \delta_\Lambda B^i = {}& -\mathfrak{D}\Lambda^i + F^0\wedge\Lambda^{\mathbf{i}} + F^{\mathbf{i}}\Lambda^0 + \tfrac{1}{2}\left(A^0\wedge\delta_\Lambda A^{\mathbf{i}} + A^{\mathbf{i}}\wedge\delta_\Lambda A^0\right) \\ & + \varepsilon^{i\mathbf{j}}\left(3\vartheta_{\mathbf{j}}{}^4 - \tfrac{1}{4}\vartheta_{\mathbf{j}}{}^5\right)\Lambda\,, \end{aligned} \tag{C.13}$$

$$\begin{aligned} \delta_\Lambda\left(C - \tfrac{1}{6}\varepsilon_{\mathbf{ij}}A^{0\mathbf{ij}}\right) = {}& -\mathfrak{D}\Lambda - \varepsilon_{\mathbf{i}j}\left(\Lambda^{\mathbf{i}}H^j + F^{\mathbf{i}}\wedge\Lambda^j - \delta_\Lambda A^{\mathbf{i}}\wedge B^j\right) \\ & - \tfrac{1}{2}\varepsilon_{\mathbf{ij}}A^{0\mathbf{i}}\delta_\Lambda A^{\mathbf{j}} + Z\tilde{\Lambda}\,, \end{aligned} \tag{C.14}$$

JHEP10(2011)068

where the covariant derivatives of the different gauge parameters are given by

$$\mathfrak{D}\Lambda^0 = d\Lambda^0 + \left(3\vartheta_{\mathbf{i}}{}^4 + \tfrac{1}{2}\vartheta_{\mathbf{i}}{}^5\right) A^{\mathbf{i}}\Lambda^0\,, \tag{C.15}$$

$$\mathfrak{D}\Lambda^{\mathbf{i}} = d\Lambda^{\mathbf{i}} + \vartheta_0{}^m (T_m^{(3)})_{\mathbf{j}}{}^{\mathbf{i}} A^0\Lambda^{\mathbf{j}} - \tfrac{3}{4}\delta_{\mathbf{1}}{}^{\mathbf{i}}\vartheta_0{}^5 A^0\Lambda^{\mathbf{1}} + \tfrac{3}{4}\varepsilon^{\mathbf{ij}}\varepsilon_{\mathbf{kl}}\vartheta_{\mathbf{j}}{}^5 A^{\mathbf{k}}\Lambda^{\mathbf{l}}\,, \tag{C.16}$$

$$\begin{aligned}\mathfrak{D}\Lambda^i &= d\Lambda^i + \vartheta_0{}^m (T_m^{(2)})_j{}^i A^0\wedge\Lambda^j + \left(3\vartheta_{\mathbf{k}}{}^4 - \tfrac{1}{4}\vartheta_{\mathbf{k}}{}^5\right) A^{\mathbf{k}}\wedge\Lambda^i \\ &\quad + \tfrac{3}{4}\delta_{\mathbf{j}}{}^i\vartheta_{\mathbf{k}}{}^5 A^{\mathbf{j}}\wedge\Lambda^k\,,\end{aligned} \tag{C.17}$$

$$\mathfrak{D}\Lambda = d\Lambda - \tfrac{3}{4}\vartheta_0{}^5 A^0\wedge\Lambda + \left(3\vartheta_{\mathbf{i}}{}^4 - \tfrac{1}{4}\vartheta_{\mathbf{i}}{}^5\right) A^{\mathbf{i}}\wedge\Lambda\,. \tag{C.18}$$

The supersymmetry transformation rules of the fermion fields are given by

$$\begin{aligned}\delta_\epsilon\psi_\mu &= \mathfrak{D}_\mu\epsilon + f\gamma_\mu\epsilon + k\gamma_\mu\epsilon^* + \tfrac{i}{8\cdot 2!}e^{-\frac{2}{\sqrt{7}}\varphi}\left(\tfrac{5}{7}\gamma_\mu\gamma^{(2)} - \gamma^{(2)}\gamma_\mu\right)F^0\epsilon \\ &\quad - \tfrac{1}{8\cdot 2!}e^{\frac{3}{2\sqrt{7}}\varphi+\frac{1}{2}\phi}\left(\tfrac{5}{7}\gamma_\mu\gamma^{(2)} - \gamma^{(2)}\gamma_\mu\right)(F^1 - \tau F^2)\epsilon^* \\ &\quad - \tfrac{i}{8\cdot 3!}e^{-\frac{1}{2\sqrt{7}}\varphi}\left(\tfrac{3}{7}\gamma_\mu\gamma^{(3)} + \gamma^{(3)}\gamma_\mu\right)(H^1 - \tau H^2)\epsilon^* \\ &\quad - \tfrac{1}{8\cdot 4!}e^{\frac{1}{\sqrt{7}}\varphi}\left(\tfrac{1}{7}\gamma_\mu\gamma^{(4)} - \gamma^{(4)}\gamma_\mu\right)G\epsilon\,,\end{aligned} \tag{C.19}$$

$$\begin{aligned}\delta_\epsilon\tilde{\lambda} &= i\,\not{\mathfrak{D}}\varphi\epsilon^* + \tilde{g}\epsilon + \tilde{h}\epsilon^* - \tfrac{1}{\sqrt{7}}e^{-\frac{2}{\sqrt{7}}\varphi}\not{F}^0\epsilon^* - \tfrac{3i}{2\cdot 2!\sqrt{7}}e^{\frac{3}{2\sqrt{7}}\varphi+\frac{1}{2}\phi}(\not{F}^1 - \tau^*\not{F}^2)\epsilon \\ &\quad - \tfrac{1}{2\cdot 3!\sqrt{7}}e^{-\frac{1}{2\sqrt{7}}\varphi+\frac{1}{2}\phi}(\not{H}^1 - \tau^*\not{H}^2)\epsilon - \tfrac{i}{4!\sqrt{7}}e^{\frac{1}{\sqrt{7}}\varphi}\not{G}\epsilon^*\,,\end{aligned} \tag{C.20}$$

$$\begin{aligned}\delta_\epsilon\lambda &= -e^{\phi}\not{\mathfrak{D}}\tau\epsilon^* + g\epsilon + h\epsilon^* - \tfrac{i}{2\cdot 2!}e^{\frac{3}{2\sqrt{7}}\varphi+\frac{1}{2}\phi}(\not{F}^1 - \tau\not{F}^2)\epsilon \\ &\quad + \tfrac{1}{2\cdot 3!}e^{-\frac{1}{2\sqrt{7}}\varphi+\frac{1}{2}\phi}(\not{H}^1 - \tau\not{H}^2)\epsilon\,,\end{aligned} \tag{C.21}$$

where

$$\mathfrak{D}_\mu\epsilon = \left\{\nabla_\mu + \tfrac{i}{2}\left[\tfrac{1}{2}e^{\phi}\mathfrak{D}_\mu^5\chi + A^I{}_\mu\vartheta_I{}^m\mathcal{P}_m\right] + \tfrac{9}{14}\gamma_\mu\not{A}^I\vartheta_I{}^4\right\}\epsilon\,, \tag{C.22}$$

$$\mathfrak{D}_\mu^5\chi = \partial_\mu\chi - \tfrac{3}{4}A^I{}_\mu\vartheta_I{}^5\chi\,, \tag{C.23}$$

and where the fermion shifts are given by

$$f = \tfrac{1}{14}e^{\frac{2}{\sqrt{7}}\varphi}\left(\vartheta_0{}^m\mathcal{P}_m + \tfrac{3i}{2}\vartheta_0{}^5\right)\,, \tag{C.24}$$

$$k = -\tfrac{9i}{14}e^{-\frac{3\varphi}{2\sqrt{7}}+\frac{\phi}{2}}\left(\vartheta_{\mathbf{1}}{}^4\tau + \vartheta_{\mathbf{2}}{}^4\right)\,, \tag{C.25}$$

$$\tilde{g} = e^{-\frac{3\varphi}{2\sqrt{7}}+\frac{\phi}{2}}\left[\tfrac{6}{\sqrt{7}}\left(\vartheta_{\mathbf{1}}{}^4\tau^* + \vartheta_{\mathbf{2}}{}^4\right) + \tfrac{\sqrt{7}}{4}\left(\vartheta_{\mathbf{1}}{}^5\tau^* + \vartheta_{\mathbf{2}}{}^5\right)\right]\,, \tag{C.26}$$

$$\tilde{h} = \tfrac{4}{\sqrt{7}}e^{\frac{2}{\sqrt{7}}\varphi}\left(\tfrac{3}{16}\vartheta_0{}^5 + i\vartheta_0{}^m\mathcal{P}_m\right)\,, \tag{C.27}$$

$$g = \tfrac{3}{4}e^{-\frac{3\varphi}{2\sqrt{7}}+\frac{\phi}{2}}\left(\vartheta_{\mathbf{1}}{}^5\tau + \vartheta_{\mathbf{2}}{}^5\right)\,, \tag{C.28}$$

$$h = ie^{\frac{2\varphi}{\sqrt{7}}+\phi}\left(\vartheta_0{}^m k_m{}^\tau - \tfrac{3}{4}\vartheta_0{}^5\tau\right)\,. \tag{C.29}$$

JHEP10(2011)068

The supersymmetry transformations of the bosonic fields are

$$\delta_\epsilon \varphi = -\tfrac{i}{4}\bar{\epsilon}\tilde{\lambda}^* + \text{h.c.}\,, \tag{C.30}$$

$$\delta_\epsilon \tau = -\tfrac{1}{2}e^{-\phi}\bar{\epsilon}^*\lambda\,, \tag{C.31}$$

$$\delta_\epsilon A^0{}_\mu = \tfrac{i}{2}e^{\frac{2}{\sqrt{7}}\varphi}\bar{\epsilon}\left(\psi_\mu - \tfrac{i}{\sqrt{7}}\gamma_\mu\tilde{\lambda}^*\right) + \text{h.c.}\,, \tag{C.32}$$

$$\delta_\epsilon A^{\mathbf{1}}{}_\mu = \tfrac{i}{2}\tau^* e^{-\frac{3}{2\sqrt{7}}\varphi+\frac{1}{2}\phi}\left(\bar{\epsilon}^*\psi_\mu - \tfrac{i}{4}\bar{\epsilon}\gamma_\mu\lambda + \tfrac{3i}{4\sqrt{7}}\bar{\epsilon}^*\gamma_\mu\tilde{\lambda}^*\right) + \text{h.c.}\,, \tag{C.33}$$

$$\delta_\epsilon A^{\mathbf{2}}{}_\mu = \tfrac{i}{2}e^{-\frac{3}{2\sqrt{7}}\varphi+\frac{1}{2}\phi}\left(\bar{\epsilon}^*\psi_\mu - \tfrac{i}{4}\bar{\epsilon}\gamma_\mu\lambda + \tfrac{3i}{4\sqrt{7}}\bar{\epsilon}^*\gamma_\mu\tilde{\lambda}^*\right) + \text{h.c.} \tag{C.34}$$

$$\begin{aligned}\delta_\epsilon B^1 = {}& \tau^* e^{\frac{1}{2\sqrt{7}}\varphi+\frac{1}{2}\phi}\left[\bar{\epsilon}^*\gamma_{[\mu}\psi_{\nu]} - \tfrac{i}{8}\bar{\epsilon}\gamma_{\mu\nu}\lambda - \tfrac{i}{8\sqrt{7}}\bar{\epsilon}^*\gamma_{\mu\nu}\tilde{\lambda}^*\right] + \text{h.c.} \\ & -\delta^1{}_{\mathbf{i}}\left(A^0{}_{[\mu|}\delta_\epsilon A^{\mathbf{i}}{}_{|\nu]} + A^{\mathbf{i}}{}_{[\mu|}\delta_\epsilon A^0{}_{|\nu]}\right)\,,\end{aligned} \tag{C.35}$$

$$\begin{aligned}\delta_\epsilon B^2 = {}& e^{\frac{1}{2\sqrt{7}}\varphi+\frac{1}{2}\phi}\left[\bar{\epsilon}^*\gamma_{[\mu}\psi_{\nu]} - \tfrac{i}{8}\bar{\epsilon}\gamma_{\mu\nu}\lambda - \tfrac{i}{8\sqrt{7}}\bar{\epsilon}^*\gamma_{\mu\nu}\tilde{\lambda}^*\right] + \text{h.c.} \\ & -\delta^2{}_{\mathbf{i}}\left(A^0{}_{[\mu|}\delta_\epsilon A^{\mathbf{i}}{}_{|\nu]} + A^{\mathbf{i}}{}_{[\mu|}\delta_\epsilon A^0{}_{|\nu]}\right)\,,\end{aligned} \tag{C.36}$$

$$\begin{aligned}\delta_\epsilon C_{\mu\nu\rho} = {}& -\tfrac{3}{2}e^{-\frac{1}{\sqrt{7}}\varphi}\bar{\epsilon}\gamma_{[\mu\nu}\left(\psi_{\rho]} + \tfrac{i}{6\sqrt{7}}\tilde{\lambda}^*\right) + \text{h.c.} \\ & +3\delta_\epsilon A^I{}_{[\mu|}\left(g_{Ii}B^i{}_{|\nu\rho]} + \tfrac{2}{3}h_{IJ}{}^i g_{Ki}A^{JK}{}_{|\nu\rho]}\right)\,.\end{aligned} \tag{C.37}$$

References

[1] J. Polchinski, *Dirichlet-Branes and Ramond-Ramond Charges*, *Phys. Rev. Lett.* **75** (1995) 4724 [hep-th/9510017] [SPIRES].

[2] E. Bergshoeff, R. Kallosh, T. Ortín, D. Roest and A. Van Proeyen, *New Formulations of $D = 10$ Supersymmetry and D8-O8 Domain Walls*, *Class. Quant. Grav.* **18** (2001) 3359 [hep-th/0103233] [SPIRES].

[3] E. Bergshoeff, M. de Roo, B. Janssen and T. Ortín, *The super D9-brane and its truncations*, *Nucl. Phys.* **B 550** (1999) 289 [hep-th/9901055] [SPIRES].

[4] E.A. Bergshoeff, M. de Roo, S.F. Kerstan and F. Riccioni, *IIB Supergravity Revisited*, *JHEP* **08** (2005) 098 [hep-th/0506013] [SPIRES].

[5] E.A. Bergshoeff, M. de Roo, S.F. Kerstan, T. Ortín and F. Riccioni, *IIA ten-forms and the gauge algebras of maximal supergravity theories*, *JHEP* **07** (2006) 018 [hep-th/0602280] [SPIRES].

[6] E.A. Bergshoeff, J. Hartong, P.S. Howe, T. Ortín and F. Riccioni, *IIA/ IIB Supergravity and Ten-forms*, *JHEP* **05** (2010) 061 [arXiv:1004.1348] [SPIRES].

[7] J. Greitz and P.S. Howe, *Maximal supergravity in $D = 10$: forms, Borcherds algebras and superspace cohomology*, *JHEP* **08** (2011) 146 [arXiv:1103.5053] [SPIRES].

[8] B.L. Julia, *Dualities in the classical supergravity limits: Dualizations, dualities and a detour via $(4k + 2)$-dimensions*, hep-th/9805083 [SPIRES].

[9] P.C. West, *$E(11)$ and M theory*, *Class. Quant. Grav.* **18** (2001) 4443 [hep-th/0104081].

[10] F. Riccioni, D. Steele and P. West, *The E_{11} origin of all maximal supergravities — the hierarchy of field-strengths*, *JHEP* **09** (2009) 095 [arXiv:0906.1177] [SPIRES].

JHEP10(2011)068

[11] A. Kleinschmidt and D. Roest, *Extended Symmetries in Supergravity: the Semi-simple Case*, *JHEP* **07** (2008) 035 [arXiv:0805.2573] [SPIRES].

[12] E.A. Bergshoeff and F. Riccioni, *D-Brane Wess-Zumino Terms and U-duality*, *JHEP* **11** (2010) 139 [arXiv:1009.4657] [SPIRES].

[13] E.A. Bergshoeff and F. Riccioni, *String Solitons and T-duality*, *JHEP* **05** (2011) 131 [arXiv:1102.0934] [SPIRES].

[14] M. Trigiante, *Dual Gauged Supergravities*, hep-th/0701218 [SPIRES].

[15] M. Weidner, *Gauged Supergravities in Various Spacetime Dimensions*, *Fortsch. Phys.* **55** (2007) 843 [hep-th/0702084] [SPIRES].

[16] H. Samtleben, *Lectures on Gauged Supergravity and Flux Compactifications*, *Class. Quant. Grav.* **25** (2008) 214002 [arXiv:0808.4076] [SPIRES].

[17] F. Cordaro, P. Fré, L. Gualtieri, P. Termonia and M. Trigiante, $N = 8$ *gaugings revisited: An exhaustive classification*, *Nucl. Phys.* **B 532** (1998) 245 [hep-th/9804056] [SPIRES].

[18] B. de Wit, H. Samtleben and M. Trigiante, *On Lagrangians and gaugings of maximal supergravities*, *Nucl. Phys.* **B 655** (2003) 93 [hep-th/0212239] [SPIRES].

[19] B. de Wit, H. Samtleben and M. Trigiante, *Maximal supergravity from IIB flux compactifications*, *Phys. Lett.* **B 583** (2004) 338 [hep-th/0311224] [SPIRES].

[20] B. de Wit and H. Samtleben, *Gauged maximal supergravities and hierarchies of nonabelian vector-tensor systems*, *Fortsch. Phys.* **53** (2005) 442 [hep-th/0501243] [SPIRES].

[21] B. de Wit, H. Samtleben and M. Trigiante, *Magnetic charges in local field theory*, *JHEP* **09** (2005) 016 [hep-th/0507289] [SPIRES].

[22] B. de Wit, H. Samtleben and M. Trigiante, *The maximal $D = 5$ supergravities*, *Nucl. Phys.* **B 716** (2005) 215 [hep-th/0412173] [SPIRES].

[23] H. Samtleben and M. Weidner, *The maximal $D = 7$ supergravities*, *Nucl. Phys.* **B 725** (2005) 383 [hep-th/0506237] [SPIRES].

[24] J. Schon and M. Weidner, *Gauged $N = 4$ supergravities*, *JHEP* **05** (2006) 034 [hep-th/0602024] [SPIRES].

[25] B. de Wit, H. Samtleben and M. Trigiante, *The maximal $D = 4$ supergravities*, *JHEP* **06** (2007) 049 [arXiv:0705.2101] [SPIRES].

[26] E.A. Bergshoeff, J. Gomis, T.A. Nutma and D. Roest, *Kac-Moody Spectrum of (Half-)Maximal Supergravities*, *JHEP* **02** (2008) 069 [arXiv:0711.2035] [SPIRES].

[27] B. de Wit, H. Nicolai and H. Samtleben, *Gauged Supergravities, Tensor Hierarchies and M-theory*, *JHEP* **02** (2008) 044 [arXiv:0801.1294] [SPIRES].

[28] E.A. Bergshoeff, O. Hohm, D. Roest, H. Samtleben and E. Sezgin, *The Superconformal Gaugings in Three Dimensions*, *JHEP* **09** (2008) 101 [arXiv:0807.2841] [SPIRES].

[29] J. Hartong, M. Hubscher and T. Ortín, *The supersymmetric tensor hierarchy of $N = 1$, $d = 4$ supergravity*, *JHEP* **06** (2009) 090 [arXiv:0903.0509] [SPIRES].

[30] M. Huebscher, T. Ortín and C.S. Shahbazi, *The Tensor Hierarchies of Pure $N = 2$, $d = 4, 5, 6$ Supergravities*, *JHEP* **11** (2010) 130 [arXiv:1006.4457] [SPIRES].

[31] E.A. Bergshoeff, J. Hartong, O. Hohm, M. Huebscher and T. Ortín, *Gauge Theories, Duality Relations and the Tensor Hierarchy*, *JHEP* **04** (2009) 123 [arXiv:0901.2054] [SPIRES].

JHEP10(2011)068

[32] B. de Wit and M. van Zalk, *Supergravity and M-theory*, *Gen. Rel. Grav.* **41** (2009) 757 [arXiv:0901.4519] [SPIRES].

[33] J. Hartong and T. Ortín, *Tensor Hierarchies of 5- and 6-Dimensional Field Theories*, *JHEP* **09** (2009) 039 [arXiv:0906.4043] [SPIRES].

[34] L.J. Romans, *Massive $N = 2a$ Supergravity in Ten-Dimensions*, *Phys. Lett.* **B 169** (1986) 374 [SPIRES].

[35] J. Scherk and J.H. Schwarz, *How to Get Masses from Extra Dimensions*, *Nucl. Phys.* **B 153** (1979) 61 [SPIRES].

[36] I.V. Lavrinenko, H. Lu and C.N. Pope, *Fiber bundles and generalized dimensional reduction*, *Class. Quant. Grav.* **15** (1998) 2239 [hep-th/9710243] [SPIRES].

[37] P. Meessen and T. Ortín, *An* Sl$(2, Z)$ *multiplet of nine-dimensional type-II supergravity theories*, *Nucl. Phys.* **B 541** (1999) 195 [hep-th/9806120] [SPIRES].

[38] J. Gheerardyn and P. Meessen, *Supersymmetry of massive $D = 9$ supergravity*, *Phys. Lett.* **B 525** (2002) 322 [hep-th/0111130] [SPIRES].

[39] P.S. Howe, N.D. Lambert and P.C. West, *A New massive type IIA supergravity from compactification*, *Phys. Lett.* **B 416** (1998) 303 [hep-th/9707139] [SPIRES].

[40] H. Nishino and S. Rajpoot, *Gauged $N = 2$ supergravity in nine-dimensions and domain wall solutions*, *Phys. Lett.* **B 546** (2002) 261 [hep-th/0207246] [SPIRES].

[41] E. Bergshoeff, T. de Wit, U. Gran, R. Linares and D. Roest, *(Non-)Abelian gauged supergravities in nine dimensions*, *JHEP* **10** (2002) 061 [hep-th/0209205] [SPIRES].

[42] S.J. Gates, Jr., H. Nishino and E. Sezgin, *Supergravity in $d = 9$ and its coupling to the non-compact σ-model*, *Class. Quant. Grav.* **3** (1986) 21 [SPIRES].

[43] J. Dai, R.G. Leigh and J. Polchinski, *New Connections Between String Theories*, *Mod. Phys. Lett.* **A 4** (1989) 2073 [SPIRES].

[44] M. Dine, P.Y. Huet and N. Seiberg, *Large and Small Radius in String Theory*, *Nucl. Phys.* **B 322** (1989) 301 [SPIRES].

[45] E. Bergshoeff, C.M. Hull and T. Ortín, *Duality in the type II superstring effective action*, *Nucl. Phys.* **B 451** (1995) 547 [hep-th/9504081] [SPIRES].

JHEP10(2011)068