

## Einstein branes

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**ABSTRACT:** We generalise the standard, flat  $p$ -brane solutions sourced by a dilaton and a form field, by taking the worldvolume to be a curved Einstein space, such as (anti-)de Sitter space. Our method is based on reducing the  $p$ -branes to domain walls and then allowing these domain walls to be curved. For de Sitter worldvolumes this extends some recently constructed warped de Sitter non-compactifications. We restrict our analysis to solutions that possess scaling behavior and demonstrate that these scaling solutions are near-horizon limits of a more general solution. Finally, our framework can equally be used for spacelike branes and the uplift of the domain wall/cosmology correspondence becomes in this context a more general timelike/spacelike brane correspondence.

**KEYWORDS:** Flux compactifications,  $p$ -branes, dS vacua in string theory, Intersecting branes models

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**1 Introduction**

The rôle of  $p$ -branes in string theory and supergravity can hardly be overestimated. They appear as fundamental objects in string theory and as (solitonic) solutions of the equations of motion of supergravity and are crucial for the understanding of many aspects of string theory, such as gauge/gravity duals, string phenomenology, string cosmology and non-perturbative effects, to mention a few.

$p$ -branes typically arise as solutions of the equations of motion of supergravity-like actions that describe  $D$ -dimensional Einstein gravity coupled to a dilaton and form-fields,

$$S_D = \int d^D x \sqrt{|\hat{g}|} \left[ \hat{R} - \frac{1}{2}(\partial\hat{\phi})^2 - \frac{1}{2(p+2)!} e^{b\hat{\phi}} \hat{F}_{\hat{\mu}_1 \dots \hat{\mu}_{p+2}} \hat{F}^{\hat{\mu}_1 \dots \hat{\mu}_{p+2}} \right]. \quad (1.1)$$

Most of the fundamental  $p$ -brane solutions in ten and eleven dimensions are known since the 1990's [1]–[6] and the general solution of the type

$$\begin{aligned} d\hat{s}^2 &= H^{2A}(r) \eta_{ij} dx^i dx^j + H^{2B}(r) (dr^2 + r^2 d\Omega_n^2), \\ e^{-2\hat{\phi}} &= H^C(r), \quad \hat{F}_{i_1 \dots i_{p+1} r} = \partial_r H^E(r) \varepsilon_{i_1 \dots i_{p+1}}, \end{aligned} \quad (1.2)$$

can be found in, for example, [7, 8], with the constants  $A$ ,  $B$ ,  $C$  and  $E$  and the amount of preserved supersymmetry depending on the parameters  $D$ ,  $p$  and  $b$  in the action (1.1).

There are several known generalisations of the above (flat)  $p$ -brane Ansatz. In [9]–[11] it was shown that the Ansatz (1.2) can be easily generalised to include a curved worldvolume metric  $\tilde{g}_{ij} = \tilde{g}_{ij}(x)$ , without changing the constants  $A$ ,  $B$ ,  $C$  and  $E$ , as long as the worldvolume geometry is Ricci-flat,  $\tilde{R}_{ij} = 0$ . In that case, the amount of supersymmetry of the Ricci-flat solution reduces to the number of Killing spinors of  $\tilde{g}_{ij}$ . On the other hand, a vast list of domain wall solutions is known (either exactly or numerically), whose worldvolume has constant curvature, or at least has an Einstein geometry,  $\tilde{R}_{ij} = \tilde{\Lambda} \tilde{g}_{ij}$  [12]–[22]. Recall that domain walls are special kind of  $p$ -branes, defined by having co-dimension one only and that they are magnetically charged with respect to a 0-form “field strength”. This means we can find such solutions from in theories with gravity coupled to a scalar with some scalar potential:

$$S_D = \int d^D x \sqrt{|\hat{g}|} \left[ \hat{R} - \frac{1}{2}(\partial\hat{\phi})^2 - V(\hat{\phi}) \right]. \tag{1.3}$$

Curiously enough, to our knowledge, there are very few known  $p$ -brane solutions for general  $p$  (i.e. not domain walls) that have an Einstein geometry in their worldvolume, and even so, they are sometimes not interpreted as such. Yet the construction of these solutions would be very interesting:  $p$ -branes with a positive worldvolume curvature could yield an alternative way to obtain de Sitter-like solutions in supergravity in the sense of Randall-Sundrum [23] (see also [20]). But also in the context of compact extra dimensions curved brane solutions are relevant. A generic flux compactification down to (anti-)de Sitter space involves brane sources that fill the lower-dimensional non-compact space and wrap some submanifold in the internal space (or are simply pointlike in the internal space). This implies that the brane’s worldvolume is necessarily curved. In most cases these solutions are only understood in the limit that the branes are smeared over the internal space. The full backreacted solutions turn out subtle and might not always exist [24]–[26]. Finally,  $p$ -branes with an AdS geometry in their worldvolume might give rise to a new class of supersymmetric solutions, as domain walls with AdS curvature have been observed to be supersymmetric in specific cases.

It is well known that there is a one to one correspondence between  $D$ -dimensional  $p$ -branes and domain walls in  $(p + 2)$  dimensions, by reducing the solutions (1.2) over the angular part  $d\Omega_n^2$  of the transverse space, where after reduction the radial coordinate  $r$  corresponds to the direction transverse to the domain wall. This suggests a obvious way to find  $p$ -branes with Einstein geometry, by lifting up curved domain walls to  $D = p + n + 2$  dimensions.

Concretely, our strategy to construct  $p$ -brane solutions with Einstein worldvolume will be that of mapping the most general two-block Ansatz for  $D$ -dimensional curved branes

$$d\hat{s}^2 = e^{2A(r)} \tilde{g}_{ij}(x) dx^i dx^j + e^{2B(r)} (dr^2 + r^2 d\Sigma_n^2), \tag{1.4}$$

onto a  $(p + 2)$ -dimensional domain wall Ansatz

$$ds^2 = a^2(z) \tilde{g}_{ij}(x) dx^i dx^j + f^2(z) dz^2, \tag{1.5}$$

by reducing over the angular part  $d\Sigma_n^2 = \bar{h}_{ab}(\theta)d\theta^a d\theta^b$  of the transverse space and then look for solutions whose worldvolume metric  $\tilde{g}_{ij}$  is Einstein. The angular metric  $\bar{h}_{ab}(\theta)$  is allowed to have a positive, negative or zero Einstein curvature,

$$\bar{R}_{ab} = (n-1)\bar{K}\bar{h}_{ab}, \quad \bar{K} = 0, \pm 1. \quad (1.6)$$

For convenience we prefer to work with the dual formulation of the gauge field, where the  $p$ -brane is *magnetically* charged under a  $n$ -form fields strength, with  $n = D - p - 2$ .

## 2 Reduction to a domain wall problem

As explained in the introduction, our strategy to obtain curved  $p$ -brane solutions of the form (1.4) will consist of reducing the problem to that of finding domain wall solutions with curved worldvolume. We therefore consider the following Ansatz for the metric, dilaton and gauge field,<sup>1</sup>

$$\begin{aligned} d\hat{s}^2 &= e^{2\alpha\chi} g_{\mu\nu} dx^\mu dx^\nu + e^{2\beta\chi} \bar{h}_{ab} d\theta^a d\theta^b, \\ \hat{\phi} &= \phi(x), \quad \hat{F}_{a_1\dots a_n} = \frac{1}{\sqrt{|\bar{h}|}} Q \bar{\varepsilon}_{a_1\dots a_n}, \end{aligned} \quad (2.1)$$

where the Greek indices  $\mu, \nu$  run from 0 to  $p+2$ , while the Latin indices  $a, b$  run from 1 to  $n$ . The functions  $g_{\mu\nu}(x)$ ,  $\chi(x)$  and  $\phi(x)$  depend on the external coordinates  $x^\mu$  and the angular metric  $\bar{h}_{ab}(\theta)$  satisfies the property (1.6). The  $(p+2)$ -dimensional metric  $g_{\mu\nu}$  contains both the  $(p+1)$ -dimensional worldvolume and the radial coordinate  $r$  of the transverse space and should be identified with the domain wall spacetime (1.5), while  $\chi$  is a breathing mode. The metric in the form (2.1) can be obtained from (1.4) through the identifications

$$e^{\beta\chi(z)} = e^{B(r)} r, \quad e^{\alpha\chi(z)} a(z) = e^{A(r)}, \quad (2.2)$$

together with the coordinate transformation

$$e^{\alpha\chi(z)} f(z) dz = e^{B(r)} dr. \quad (2.3)$$

Substituting the Ansatz (2.1) in the action (1.1), we find that the latter reduces to the action of  $(p+2)$ -dimensional gravity coupled to two scalars  $\phi$  and  $\chi$  in a double-exponential potential,

$$S_{p+2} = \frac{1}{\kappa} \int d^{p+2}x \sqrt{|g|} \left[ R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}(\partial\chi)^2 - \frac{1}{2}e^{b\phi+c\chi} Q^2 + n(n-1)e^{d\chi} \bar{K} \right], \quad (2.4)$$

where we have imposed the conditions

$$p\alpha = -n\beta, \quad \alpha = \sqrt{\frac{n}{2p(p+n)}}, \quad (2.5)$$

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<sup>1</sup>Our conventions are such that  $\varepsilon^{1\dots n} = 1$  and  $\bar{\varepsilon}_{a_1\dots a_n} = \bar{h}_{a_1 b_1} \dots \bar{h}_{a_n b_n} \varepsilon^{b_1\dots b_n}$ .

in order to be able to write the action in Einstein frame and to canonically normalise  $\chi$ , respectively. Hence  $\beta$  and the parameters  $c$  and  $d$  in the potential are given by

$$\beta = -\sqrt{\frac{p}{2n(p+n)}}, \quad c = (p+1)\sqrt{\frac{2n}{p(p+n)}}, \quad d = \sqrt{\frac{2(p+n)}{pn}}. \quad (2.6)$$

The curved domain wall solutions we will be interested in are of the type

$$\begin{aligned} ds^2 &= a^2(z)\tilde{g}_{ij}(x) dx^i dx^j + f^2(z)dz^2, \\ \phi &= \phi(z), \quad \chi = \chi(z), \end{aligned} \quad (2.7)$$

where the worldvolume metric  $\tilde{g}_{ij}$  satisfies the Einstein condition

$$\tilde{R}_{ij} = p\tilde{\Lambda}\tilde{g}_{ij}. \quad (2.8)$$

The equations of motion, for this Ansatz, can be written as

$$\begin{aligned} f^2 a^{-2} \tilde{\Lambda} + \frac{a''}{a} - \frac{f'a'}{fa} + \frac{1}{2p}(\phi')^2 + \frac{1}{2p}(\chi')^2 &= 0, \\ p(p+1)f^2 a^{-2} \tilde{\Lambda} - p(p+1)\left(\frac{a'}{a}\right)^2 + \frac{1}{2}(\phi')^2 + \frac{1}{2}(\chi')^2 \\ - \frac{1}{2}f^2 e^{b\phi+cx} Q^2 + n(n-1)f^2 e^{d\chi} \bar{K} &= 0, \\ \phi'' + (p+1)\frac{a'}{a}\phi' - \frac{f'}{f}\phi' - \frac{1}{2}b e^{b\phi+cx} f^2 Q^2 &= 0, \\ \chi'' + (p+1)\frac{a'}{a}\chi' - \frac{f'}{f}\chi' - \frac{1}{2}c e^{b\phi+cx} f^2 Q^2 + d n(n-1)e^{d\chi} f^2 \bar{K} &= 0, \end{aligned} \quad (2.9)$$

where a prime denotes differentiation with respect to  $z$ . A few comments are in order. The Einbein  $f(z)$  is not a dynamical degree of freedom, but a gauge choice, as different choices of  $f(z)$  correspond to different parametrisations of the transverse direction. In the forthcoming sections we will usually take  $f(z) = 1$ . This coordinate reparametrisation freedom also implies a degeneracy in the above equations of motion. One can check that only three out of four differential equations are independent, as, for example, conservation of energy together with the last three yield the first equation. The last three equations are therefore sufficient to determine the dynamical degrees of freedom  $a(z)$ ,  $\phi(z)$  and  $\chi(z)$ .

Obviously these equations should be able not only to describe curved  $p$ -brane solutions, but also to recover the known standard (flat)  $p$ -branes (1.2). Indeed, it is easy to show that for example the M2-brane of 11-dimensional supergravity ( $p = 2$ ,  $n = 7$ ,  $\tilde{\Lambda} = 0$ ,  $\bar{K} = 1$ ) corresponds to the following solution:

$$\begin{aligned} a(r) &= (r^6 + R_0^6)^{1/4} r^2, & \chi(r) &= \frac{1}{18\sqrt{7}} \ln(r^6 + R_0^6), \\ f(r) &= (r^6 + R_0^6)^{3/4} r^{-2}, & Q &= \pm 6 R_0^6. \end{aligned} \quad (2.10)$$

### 3 Curved branes without flux and the AJS domain wall

To our knowledge, the first domain wall solution with an Einstein geometry in the world-volume was given in [14], which we will refer to as the AJS domain wall. Though the original derivation was done in five dimensions ( $p = 3$ ), the generalisation to arbitrary  $p$  is straightforward. The domain wall appears as a solution of the Einstein-Hilbert action coupled to a single scalar in an exponential potential,

$$S_{p+2} = \int d^{p+2}x \sqrt{|g|} \left[ R - \frac{1}{2}(\partial\chi)^2 - e^{d\chi}\Lambda \right], \quad (3.1)$$

for general scalar coupling  $d$  and  $(p + 2)$ -dimensional cosmological constant  $\Lambda$ . The AJS domain wall is then given by<sup>2</sup>

$$ds^2 = \left[ 1 + d\sqrt{\frac{-\Lambda}{2p}} z \right]^2 \tilde{g}_{ij} dx^i dx^j + dz^2, \quad e^\chi = \left[ 1 + d\sqrt{\frac{-\Lambda}{2p}} z \right]^{-\frac{2}{d}}, \quad (3.2)$$

where the worldvolume metric  $\tilde{g}_{ij}$  satisfies the Einstein condition (2.8) and where the worldvolume curvature  $\tilde{\Lambda}$  is determined by the scalar coupling  $d$  and the bulk cosmological constant  $\Lambda$ ,

$$\tilde{\Lambda} = \frac{2 - p d^2}{2p^2} \Lambda. \quad (3.3)$$

The minus sign under the square root in (3.2) allows only negative values for  $\Lambda$  and hence we see that we can have de Sitter domain walls for  $d^2 > 2/p$  and AdS ones for  $d^2 < 2/p$ .

In our case,  $d$  and  $\Lambda$  are determined by the reduction Ansatz (2.1),

$$d = \sqrt{\frac{2(p+n)}{pn}}, \quad \Lambda = -n(n-1)\bar{K}. \quad (3.4)$$

In order for  $\Lambda$  to be negative, we are forced to take  $\bar{K} = +1$ , i.e. the angular part in (1.4) and (2.1) is the  $n$ -sphere  $\mathbb{S}^n$ . In this notation the domain wall solution takes the form

$$ds^2 = [1 + \lambda z]^2 \tilde{g}_{ij} dx^i dx^j + dz^2, \quad e^\chi = [1 + \lambda z]^{-\sqrt{\frac{2pn}{p+n}}}, \quad (3.5)$$

with  $\lambda = p^{-1}\sqrt{(n-1)(n+p)}$ . Note that the solutions exist for all values of  $p$  between 1 and  $D - 4$  and that they all require *de Sitter* sliced domain walls<sup>3</sup> with worldvolume curvature

$$\tilde{\Lambda} = \frac{1}{3}(n-1). \quad (3.6)$$

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<sup>2</sup>Using a simple shift of the coordinate  $z$ , one can write the scale factor as  $a(z) = \lambda z$ . This we will occasionally do in this paper without mentioning. The reason to keep the constant term is to make a comparison with [14].

<sup>3</sup>When we consider the coordinate transformation  $y = 1 + \lambda z$  the solution reads  $ds^2 = dy^2 + y^2 d\tilde{s}_{p+1}^2$  with  $d\tilde{s}_{p+1}^2$  being the metric on  $(p + 1)$ -dimensional de Sitter space. This can be confusing, since it is (almost) identical to flat space in Milne coordinates. However the reason that these domain walls do not describe flat space is because the metric  $ds^2 = dy^2 + y^2 d\tilde{s}_{p+1}^2$  only corresponds to flat space for a specific normalisation of the curvature of the de Sitter slice, namely  $\tilde{R}_{p+1} = (p + 1)p$ . In all our solutions the de Sitter curvature is in fact different.

One can understand the range for the values of  $p$  as follows:  $p$ -branes with  $p < 1$  are pointlike and cannot have a curved worldvolume, while  $p$ -branes with  $p > D - 4$  have codimension 2 or smaller, such that the angular part of the transverse space cannot be curved either.

Using the reduction relations (2.2), we can write the solution (3.5) as a purely gravitational solution in  $D = p + n + 2$  dimensions,

$$d\hat{s}^2 = (1 + \lambda z)^{\frac{2p}{p+n}} \tilde{g}_{ij} dx^i dx^j + (1 + \lambda z)^{-\frac{2n}{p+n}} dz^2 + (1 + \lambda z)^{\frac{2p}{p+n}} d\Omega_n^2. \quad (3.7)$$

Applying the coordinate transformation

$$1 + \lambda z = r^\lambda, \quad (3.8)$$

the solution can be written in the more pleasing form

$$d\hat{s}^2 = r^{2\sqrt{\frac{n-1}{p+n}}} \tilde{g}_{ij} dx^i dx^j + r^{2\sqrt{\frac{n-1}{p+n}}-2} [dr^2 + r^2 d\Omega_n^2], \quad (3.9)$$

where  $\tilde{g}_{ij}$  satisfies the Einstein condition (2.8).

The solution (3.5) is a so-called scaling solution, due to the polynomial dependence in the scale factor  $a(y)$ . These solutions are often found in FLRW cosmologies coupled to scalars in an exponential potential. It is therefore no surprise that we find a similar behaviour here, as the domain wall/cosmology correspondence relates these two types of solutions [27, 28]. The fact that we are dealing with scaling solutions will make it easy to generalise them to the case with non-zero flux ( $Q \neq 0$ ).

Recently an extension of this model has been considered in [20] where the exponential potential contains a linear combination of two scalar fields. A suitable rotation in field space maps this model to the model with a single scalar field in the exponential plus one free decoupled scalar field.

## 4 Curved branes with flux

### 4.1 The scaling solutions for general parameters

A more interesting case is that of domain walls with two scalars in a double exponential potential, as these can be interpreted as dilatonic  $p$ -branes with non-zero charge. Inspired by the scaling solution of the previous section, we insist on both  $\chi$  and  $\phi$  depend logarithmically on  $a(z)$ ,

$$\phi = N_1 \log a(z), \quad \chi = N_2 \log a(z). \quad (4.1)$$

Again the requirement that the potential scales like  $a^{-2}$  fixes the constants  $N_1$  and  $N_2$  as

$$N_1 = \frac{2(c-d)}{bd}, \quad N_2 = -\frac{2}{d}, \quad (4.2)$$

and the domain wall solution is then given by

$$ds^2 = [1 + \lambda z]^2 \tilde{g}_{ij} dx^i dx^j + dz^2, \\ e^\phi = [1 + \lambda z]^{\frac{2(c-d)}{bd}}, \quad e^\chi = [1 + \lambda z]^{-\frac{2}{d}}, \quad (4.3)$$

provided that

$$\begin{aligned}\lambda &= bd \sqrt{\frac{-\Lambda}{2p(b^2 + c^2 - cd)}}, \\ Q^2 &= \frac{-2(c-d)d}{b^2 + c^2 - cd} \Lambda, \\ \tilde{\Lambda} &= \frac{b^2 + (c-d)^2 - \frac{1}{2}pb^2d^2}{p^2(b^2 + c^2 - cd)} \Lambda.\end{aligned}\tag{4.4}$$

In the reduction context, where  $c$ ,  $d$  and  $\Lambda$  are given by (2.6) and (3.4), this solution takes the form

$$\begin{aligned}ds^2 &= [1 + \lambda z]^2 \tilde{g}_{ij} dx^i dx^j + dz^2, \\ e^\phi &= [1 + \lambda z]^{\frac{2p(n-1)}{(p+n)b}}, \quad e^\chi = [1 + \lambda z]^{-\sqrt{\frac{2pn}{(p+n)}}},\end{aligned}\tag{4.5}$$

where now

$$\begin{aligned}\lambda &= \frac{b(p+n)}{p} \sqrt{\frac{(n-1)\bar{K}}{(p+n)b^2 + 2(p+1)(n-1)}}, \\ Q^2 &= \frac{4(n-1)^2(p+n)}{(p+n)b^2 + 2(p+1)(n-1)}, \\ \tilde{\Lambda} &= \frac{n-1}{p} \frac{(p+n)b^2 - 2(n-1)^2}{(p+n)b^2 + 2(p+1)(n-1)} \bar{K}.\end{aligned}\tag{4.6}$$

Again we find that necessarily the square root in the expression for  $\lambda$  forces us to take  $\bar{K} = +1$ . We see therefore that in the case with flux, we find both positively and negatively curved  $p$ -branes, depending on the value of the dilaton coupling  $b$ . In particular we can have de Sitter geometry if the dilaton coupling  $b$  is big enough,  $b > (n-1)\sqrt{2/(p+n)}$ .

This solution can be lifted up to  $D = p + n + 2$  dimensions as

$$\begin{aligned}d\hat{s}^2 &= r^A \tilde{g}_{ij} dx^i dx^j + r^{A-2} [dr^2 + r^2 d\Omega_n^2], \\ e^{\hat{\phi}} &= r^B, \quad \hat{F}_{a_1 \dots a_n} = \frac{Q}{\sqrt{|h|}} \bar{\varepsilon}_{a_1 \dots a_n},\end{aligned}\tag{4.7}$$

where  $\tilde{\Lambda}$  and  $Q$  are given in (4.6),  $A$  and  $B$  by

$$A = \sqrt{\frac{4b^2(n-1)}{(p+n)b^2 + 2(p+1)(n-1)}}, \quad B = \sqrt{\frac{4(n-1)^3}{(p+n)b^2 + 2(p+1)(n-1)}},\tag{4.8}$$

and the worldvolume metric  $\tilde{g}_{ij}$  satisfies the Einstein relation

$$\begin{aligned}\tilde{R}_{ij} &= -p \tilde{\Lambda} \tilde{g}_{ij} && \text{for } b^2 > \frac{2(n-1)^2}{(p+n)}, \\ \tilde{R}_{ij} &= +p \tilde{\Lambda} \tilde{g}_{ij} && \text{for } b^2 < \frac{2(n-1)^2}{(p+n)}.\end{aligned}\tag{4.9}$$



## 4.2 The scaling solutions in 10-dimensional supergravity

The general solutions found in the previous subsection simplify remarkably for the case of ten-dimensional Type IIA/B supergravity. For the case of the RR fields, we have that

$$b = \frac{p-3}{2}, \tag{4.10}$$

such that the conditions (4.4) become

$$\lambda = \frac{p-3}{p} \sqrt{\frac{7-p}{2}}, \quad Q^2 = (7-p)^2, \quad \tilde{\Lambda} = -\frac{(7-p)(5-p)}{2p}. \tag{4.11}$$

These Einstein D-branes are negatively curved (AdS) for  $p < 5$  and positive (dS) for  $p > 5$ . The case  $p = 5$  is special, as it is (Ricci) flat. The solution is given by

$$\begin{aligned} d\hat{s}^2 &= r^{\frac{1}{2}} \tilde{g}_{ij} dx^i dx^j + r^{-\frac{3}{2}} [dr^2 + r^2 d\Omega_3^2], \\ e^{\hat{\phi}} &= r, \quad \hat{F}_{a_1 \dots a_3} = \frac{2}{\sqrt{|h|}} \bar{\varepsilon}_{a_1 \dots a_3}, \end{aligned} \tag{4.12}$$

with  $\tilde{R}_{ij} = 0$ . It can easily be shown that this corresponds to the near-horizon geometry of the standard (Ricci-flat) D5-brane. In section 6 we will show that this results is in fact quite general, in the sense that the flat scaling solutions of the equations (2.9) yield the near-horizon geometries of the standard  $p$ -branes.

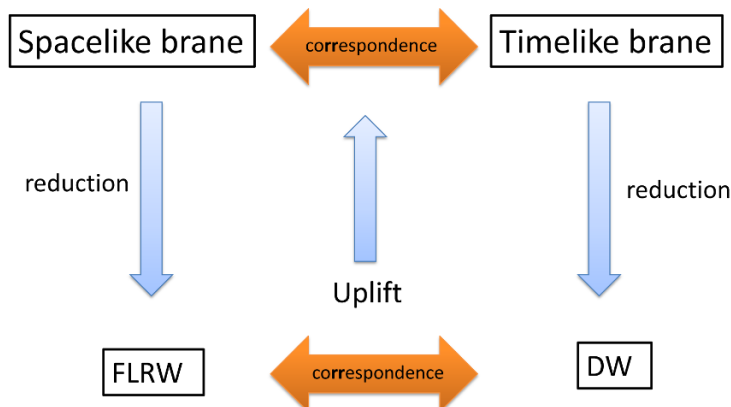
The D7 and D8 are also special, as the angular part of the transverse space is either one- or zero-dimensional and can therefore not be curved. Nevertheless, curved scaling solutions can still be constructed in these cases, by considering solutions with a single exponential scalar potential, coming from the flux form field in higher dimensions (i.e. turning off the exponential proportional to  $\bar{K}$ ). We will not give the exact expressions here, as they can be easily derived from the general results of section 3. Yet it is useful to remark that the flat D8-brane solution is in fact a scaling solution as well. This is not true for the other flat BPS branes, as we explain in section 6.

Note that the metric becomes ill-defined for  $p = 3$ . It is well known that the flat D3-brane does not have a scaling solutions as its near-horizon limit due to the absence of the dilaton. It is easy to see that the same argument extends to the curved case, for which no scaling solutions will exist. The case  $p = 3$  should therefore be excluded from our solutions, as it falls beyond the Ansatz used here.

Finally, the remaining F1 and the NS5-brane are easy to discuss: as they couple (magnetically) to the NSNS 7- and 3-form respectively, we have that

$$\begin{aligned} \text{F1} : \quad & b = -1, \quad p = 1, \\ \text{NS5} : \quad & b = +1, \quad p = 5, \end{aligned} \tag{4.13}$$

and their solutions are the same as the D1 and D5-brane, up to a minus sign in the dilaton exponential (as can be expected from S-duality). We therefore find a negatively curved (AdS) F1 and a Ricci-flat NS5-brane.



**Figure 1.** The general timelike/spacelike brane correspondence from the DW/FLRW correspondence.

### 5 Timelike/spacelike $p$ -brane correspondence

So far we have only considered timelike  $p$ -branes, whose worldvolume is Lorentzian and which have static geometries. Similarly there exist spacelike brane solutions [29], which have Euclidean worldvolume and a time-dependent geometry (i.e. the warp factors in the two-block Ansatz depends solely on time)

$$d\hat{s}^2 = e^{2A(t)} \tilde{g}_{ij}(x) dx^i dx^j + e^{2B(t)} (-dt^2 + t^2 d\Sigma_n^2), \tag{5.1}$$

It should be clear that the same procedure applied to timelike branes can equally well be performed on the spacelike ones: we can dimensionally reduce over the slicing  $d\Sigma_n^2$  of the transverse space, which will give rise to a  $(p + 2)$ -dimensional FLRW solution after reduction,

$$ds^2 = -f^2(t) dt^2 + a^2(t) \tilde{g}_{ij}(x) dx^i dx^j. \tag{5.2}$$

These metrics are solutions to a set of differential equations analogous to (2.9), which differ from that latter in the sign of  $\tilde{\Lambda}$  and the potential. Obviously, the time coordinate  $t$  is the analogue of the transverse direction  $z$  in the domain wall case, as it is transverse to the spatial sections of the FLRW metric. Note that now  $\tilde{\Lambda}$  has the usual interpretation of the FLRW spatial curvature, usually denoted  $k$ .

This analogy is part of what has been named the domain wall/cosmology correspondence [27, 28]. Interestingly, this correspondence here gets uplifted (and hence generalised) to a more general *timelike/spacelike  $p$ -brane* correspondence,<sup>4</sup> as is schematically depicted in figure 1.

<sup>4</sup>This has been discussed briefly before in [30].

Let us quickly discuss this for the simplest case, with  $Q^2 = 0$ . The DW/FLRW correspondence tells us that domain wall scaling solutions get mapped to FLRW solutions with the opposite sign of the potential and opposite sign of the worldvolume curvature (i.e. wall curvature for domain walls and spatial curvature of FLRW metrics). This implies that for  $p = 1, \dots, 6$  we have a set of S-brane scaling solutions with opposite spatial curvature  $\tilde{\Lambda}$  (negative in the  $Q^2 = 0$  case) and a *negatively curved slicing* in the space transverse to the brane,  $\bar{K} < 0$ . The scale factor takes the form

$$a(t) = 1 + \lambda t, \tag{5.3}$$

with the appropriate sign flips for  $\bar{K}$  and  $\tilde{\Lambda}$  in the expression for  $\lambda$ .

In the case with  $Q^2 \neq 0$ , we cannot reverse the sign of the flux contribution. However if we would nonetheless insist on doing so, this would bring us to supergravity theories with ghostlike kinetic terms. In this context the DW/COSM correspondence has been discussed as well [31, 32].

Having hyperbolically sliced transverse directions,  $\bar{K} < 0$ , is often -though not always- part of the definition of an S-brane, as flat S-branes asymptote to flat space in Milne coordinates at late times. This is similar in the way that timelike  $p$ -branes with  $\bar{K} > 0$  approach flat space in radial coordinates at spatial infinity.

## 6 Scaling solutions as near-horizon regions

The above timelike/spacelike brane correspondence allows us to understand better the scaling solutions constructed in this paper, using knowledge from the cosmology side. For the case of FLRW cosmologies with exponential potentials, it is known that scaling solutions do not describe all solutions but they are critical points of an autonomous system of equations of motion, such that they correspond to the late-time or early time behavior of the most general solution. In other words, they can describe attractors or repellers. We refer to [33, 34] for a treatment of this in a general setting that include the scalar potentials discussed here.

In the appendix we have written down the autonomous system corresponding to the equations of motion (2.9) and derived its critical points. In the curved case we reproduce exactly the scaling solutions  $a(z) \sim z$ , presented in the previous sections. In the flat case ( $\tilde{\Lambda} = 0$ ) we can also find scaling solutions, that now go like

$$a(z) \sim z^\ell \quad \text{with} \quad \ell = \frac{(9-p)}{(p-3)^2}. \tag{6.1}$$

Notice that for the 5-brane we have that  $\ell = 1$ , which agrees nicely with the fact that the same solutions appears also as the Ricci-flat case of the Einstein branes (which all have  $\ell = 1$ ). In a certain sense the 5-branes are the intersection of the two classes of solutions.

In order to interpret these flat scaling solutions, let us uplift them to 10 dimensions. Plugging the expressions for  $a(z)$  and  $\chi(z)$  in the full 10-dimensional metric and performing the following coordinate transformation

$$z = r^{\frac{(p-3)^2}{2p}}, \tag{6.2}$$

we find the near-horizon solutions for the standard extremal Dp-branes:

$$ds^2 = H^{\frac{p-7}{8}} ds_{p+1}^2 + H^{\frac{p+1}{8}} (dr^2 + r^2 d\Omega^2) \tag{6.3}$$

with

$$H \sim r^{p-7}. \tag{6.4}$$

The full extremal brane solutions are given by  $H = \alpha + \beta r^{p-7}$ , but only the near-horizon case  $\alpha = 0$  corresponds to a scaling solution (or equivalently, a critical point of the autonomous system).

There are two important lessons that we can draw from this:

1. The scaling solutions (both flat as curved ones) are not the general solutions, but describe the near-horizon of these. We just showed this explicitly for the flat case, but there is no reason to assume that it would be different for the curved case. In fact the autonomous system formalism tells us so: the critical points of the flow equations (the scaling solutions) are particular limits of the full solutions, which in turn are described by the flow lines of the autonomous system. In other words, there must exist extensions of our curved Einstein branes that interpolate between different scaling solutions for small and large  $r$ .
2. In those cases we could not obtain a curved  $p$ -brane scaling solution (e.g. AdS curved branes without flux, or the  $p = 3$  case with flux), there still exist curved brane solutions: they just do not have a scaling regimes. To understand the full space of solutions for either sign of the curvature we refer to [22] for a treatment of the single exponential case.

At the moment we have not been able to find the full analytic solution interpolating between scaling solutions but hope to report on it in the future.

## 7 Discussion

In this paper we have constructed many new solutions that have the interpretation of  $p$ -brane whose worldvolume is a curved Einstein space. The exact solutions we were able to find correspond to scaling solutions. This means that, when the brane is reduced to a (curved) domain wall, the warp factor  $a(z)$  is a simple power-law  $z^\ell$ ,

$$ds^2 = dz^2 + z^\ell \tilde{g}_{ij} dx^i dx^j, \tag{7.1}$$

where the  $(p + 1)$ -dimensional wall geometry is Einstein

$$\tilde{R}_{ij} = p \tilde{\Lambda} \tilde{g}_{ij}. \tag{7.2}$$

When the curvature is non-zero the power-law necessarily has  $\ell = 1$ .

Scaling solutions uplift to  $p$ -brane near-horizons. This is explicitly confirmed in section 6 for the known case of the standard flat branes, but it is natural to extrapolate this

to the case of curved branes. A formal proof of this was achieved by rewriting the equations as an autonomous system for which the scaling solutions are the attractor critical points. Therefore there exist more general solutions that interpolate between a proper scaling solution (the near-horizon) and a non-proper scaling solution, with infinite fields (spatial infinity).

Our approach can be generalised trivially to S-branes. Whereas timelike branes reduce to domain walls, spacelike branes reduce to FLRW cosmologies. The domain wall/cosmology correspondence in this framework gets generalised to a general time-like/spacelike brane correspondence. In this context curved branes are perhaps more natural since they correspond to the curvature of the spatial part of the FLRW metric.

Amongst our explicit curved timelike brane solutions we have many that have de Sitter worldvolumes. This was the case for all fluxless brane solutions and for the solutions with flux when  $p = 6$ . However, when we consider an internal slicing that is not spherical,  $\bar{K} \neq 1$ , we can get de Sitter curved worldvolumes. De Sitter branes have appeared earlier in [14] and [35]–[37] however the solutions in [35, 36] are written in more complicated coordinates. Our approach makes clear that these solutions are not the most general, but should be seen as near-horizons. These de Sitter solutions can not be regarded as warped de Sitter compactifications since the space transverse to the brane worldvolume is necessarily non-compact. However, as warped non-compactifications, such a solution might be relevant if gravity is localised sufficiently and there is some understanding of the presence of gauge forces living on these branes.

Finally we want to compare our method with another well known method that employs dimensional reduction. This method is based on reducing flat  $p$ -branes over their worldvolume [38] (see also [30]), a technique inspired from the special case of black holes [39]. Since the worldvolume is flat, this does not generate a scalar potential, rather one just obtains a sigma model that is solvable and whose integrability can be understood in a formal way using group theory and the Hamilton-Jacobi formalism [40]. This works for a very large class of generalisations with much less worldvolume symmetries [38]. This is in contrast with the technique used in this paper, where we reduce over the curved slice in the transverse space instead of the worldvolume (see for a discussion of the two approaches [41]). However we could equally reduce over the curved worldvolume here and it would generate a scalar potential as well. It would be interesting to see whether in this case the equations of motion could be fully solvable as well.

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## A Autonomous equations of motion

The autonomous variables are given by

$$\begin{aligned} X_1 &= \frac{\phi'}{H\sqrt{2p(p+1)}}, & X_2 &= \frac{\chi'}{H\sqrt{2p(p+1)}}, \\ Y_1 &= \frac{1}{2} \frac{Q^2 e^{b\phi+c\chi}}{p(p+1)H^2}, & Y_2 &= \frac{-\bar{K}n(n-1)e^{d\chi}}{p(p+1)H^2}, \end{aligned} \quad (\text{A.1})$$

where  $H = a'/a$ . One can check that the equations become first-order equations

$$\begin{aligned} \dot{X}_1 &= X_1(p+1) \left( X_1^2 + X_2^2 - 1 + \frac{\epsilon\tilde{\Lambda}}{(p+1)\dot{a}^2} \right) + b\epsilon\sqrt{\frac{p(p+1)}{2}}Y_1, \\ \dot{X}_2 &= X_2(p+1) \left( X_1^2 + X_2^2 - 1 + \frac{\epsilon\tilde{\Lambda}}{(p+1)\dot{a}^2} \right) + c\epsilon\sqrt{\frac{p(p+1)}{2}}Y_1 + d\epsilon\sqrt{\frac{p(p+1)}{2}}Y_2, \\ \dot{Y}_1 &= \sqrt{2p(p+1)}Y_1(bX_1 + cX_2) + 2(p+1)Y_1 \left( X_1^2 + X_2^2 + \frac{\epsilon\tilde{\Lambda}}{(p+1)\dot{a}^2} \right), \\ \dot{Y}_2 &= d\sqrt{2p(p+1)}Y_2X_2 + 2(p+1)Y_2 \left( X_1^2 + X_2^2 + \frac{\epsilon\tilde{\Lambda}}{(p+1)\dot{a}^2} \right), \end{aligned} \quad (\text{A.2})$$

together with the constraint

$$X_1^2 + X_2^2 - \epsilon Y_1 - \epsilon Y_2 + \epsilon \frac{\tilde{\Lambda}}{\dot{a}^2} = 1. \quad (\text{A.3})$$

We used a dot to denote differentiation with respect to  $\ln(a)$  whereas a prime we have been using to denote differentiation with respect to  $z$ . We furthermore introduced  $\epsilon$ , which takes value  $\epsilon = +1$  for domain walls and  $\epsilon = -1$  for FLRW cosmologies.

The constraint equation (A.3) is not really an independent equation. One can show that, when the initial conditions obey the constraint, so will the evolution. This can be proven by taking the derivative of the constraint equation and showing that it is automatically satisfied using the equations (A.2).

When  $\tilde{\Lambda} = 0$  we have a true autonomous system whose dynamics can be understood partially from the critical points, defined as solutions with constant  $X, Y$ . These are the simplest solutions and general solutions interpolate between these critical points. These critical points come in two kinds: those with finite values of the scalars and those with infinite valued scalars [33]. The latter class can describe solutions at spacelike or timelike infinity. Let us here discuss the first class of critical points, with finite scalars. Solving the algebraic equations that one gets when  $\dot{X}_1 = \dot{X}_2 = \dot{Y}_1 = \dot{Y}_2 = 0$  gives the following

solution

$$\begin{aligned}
Y_1 &= \epsilon \frac{2 - 2(p+1)\ell}{p(p+1)\ell^2} (b^{-2} - cb^{-2}d^{-1}), \\
Y_2 &= \epsilon \frac{2 - 2(p+1)\ell}{p(p+1)\ell^2} (-b^{-2}cd^{-1} + c^2b^{-2}d^{-2} + d^{-2}), \\
X_1 &= -\frac{1}{\ell} \sqrt{\frac{2}{p(p+1)}} (b^{-1} - cb^{-1}d^{-1}), \\
X_2 &= -\frac{1}{\ell} \sqrt{\frac{2}{p(p+1)}} d^{-1}, \\
\ell &= \frac{2}{p} (b^{-2} + d^{-2} + b^{-2}c^2d^{-2} - 2b^{-2}cd^{-1}).
\end{aligned} \tag{A.4}$$

When we consider the following equation

$$\frac{H'}{H^2} = -\epsilon \frac{\tilde{\Lambda}}{a^2} - (p+1)X_1^2 - (p+1)X_2^2. \tag{A.5}$$

for the case  $\tilde{\Lambda} = 0$  we find that a critical point must obey

$$\frac{H'}{H^2} \equiv -\ell^{-1} = \text{constant} \implies a(z) \sim z^\ell. \tag{A.6}$$

So the scale factor  $a$  is given by a simple power-law. These solutions are called scaling solutions since every term in the action, or equations of motion, scales in the same way. The scaling solutions, in terms of the fields, read

$$\phi(z) = X_1 \ell \sqrt{p(p+1)} \ln(z) + \phi(1), \quad \chi(z) = X_2 \ell \sqrt{p(p+1)} \ln(z) + \chi(1). \tag{A.7}$$

Let us now consider the case of standard flat timelike  $p$ -branes  $\tilde{\Lambda} = 0, \bar{K} = 1$ . Then we find that the values for the  $Y$ 's are such that a solution exists for  $p = 0, \dots, 6$  excluding  $p = 3$ . If we plug in the specific values for  $b, c$  and  $d$  we find that

$$\ell = \frac{(9-p)}{(p-3)^2}. \tag{A.8}$$

The same strategy still applies when  $\tilde{\Lambda} \neq 0$ . For the latter case we necessarily have that (see e.g. (A.3)) that  $\tilde{\Lambda} \dot{a}^{-2}$  is constant. Therefore

$$a(z) \sim z. \tag{A.9}$$

One can easily check, using the techniques of [34] that we obtain exactly the scaling solutions given in the previous sections. All expressions for  $X$  and  $Y$  (A.4) are still valid if we use  $\ell = 1$ .

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