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The sparticle spectrum in minimal gaugino-gauge mediation

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Abstract: We compute the sparticle mass spectrum in the minimal four-dimensional construction that interpolates between gaugino mediation and ordinary gauge mediation.

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Contents

1 Introduction

In this note, we compute the soft masses in the minimal four-dimensional construction [1, 2] of "gaugino mediation" [3, 4] (see also [5] for a recent discussion). The model is presented in figure 1. The chiral superfields $Q, \tilde{Q}$ are the matter fields of MSSM, $L, \tilde{L}$ is a single pair of "link fields" in the bifundamental of $G_{SM_1} \times G_{SM_2}$, whose VEV breaks this product group to the diagonal Standard-Model (SM) gauge group, G_{SM}, and $T, \tilde{T}$ is a single pair of messengers, which couple to the spurion of SUSY-breaking, S, whose scalar components get VEVs,

$$S = M + \theta^2 F \,, \tag{1.1}$$

as in Minimal Gauge Mediation (MGM). The superpotential of the model takes the form

$$W = ST\tilde{T} + K(L\tilde{L} - v^2) \,, \tag{1.2}$$

where K is a Lagrange multiplier superfield, introduced to set the VEV of the link fields to v.

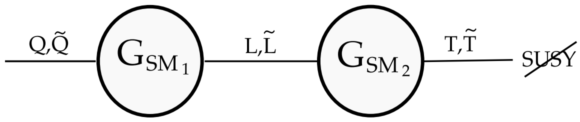

Figure 1. Quiver diagram for our setting.

For simplicity, we first take $G_{SM_1} \times G_{SM_2}$ to be $\mathrm{U}(1)_1 \times \mathrm{U}(1)_2$.[1] Let us introduce the two dimensionless parameters x and y:

$$x \equiv \frac{F}{M^2}\,, \qquad y \equiv \frac{m_v}{M}\,, \qquad m_v \equiv 2v\sqrt{g_1^2 + g_2^2}\,, \tag{1.3}$$

where m_v is the mass of the massive combination of gauge bosons of the broken $\mathrm{U}(1)_1 \times \mathrm{U}(1)_2$, and $g_{1,2}$ are the gauge couplings of $\mathrm{U}(1)_{1,2}$, respectively. The parameter x is a measure of the SUSY-breaking scale, F/M, relative to the messenger scale, M, while y interpolates between MGM (as $y \to \infty$) and minimal gaugino mediation (when $y \ll 1$). We thus refer to this model as "Minimal gaugino-Gauge Mediation" (MgGM).

The main result of this note is the following. The soft scalar masses (at the messenger scale) in this theory, $m_{\tilde{f}}^2$, are obtained by adding to the two-loop integrands in [6] the common factor,

$$f(k^2, m_v^2) \equiv \left(\frac{m_v^2}{k^2 - m_v^2}\right)^2\,, \tag{1.4}$$

namely,

$$m_{\tilde{f}}^2 = \int \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\,([6])\,f\,, \tag{1.5}$$

where ([6]) in the integrand is the same as for MGM in [6], and the momentum k amounts to the one on the massless propagator in each of the two-loop diagrams. The SM coupling, g_e, is given in terms of $g_{1,2}$ by

$$\frac{1}{g_e^2} = \frac{1}{g_1^2} + \frac{1}{g_2^2}\,. \tag{1.6}$$

When v is much smaller than M, eq. (1.5) implies that $m_{\tilde{f}}^2$ has a suppression factor of order v^2/M^2 relative to MGM, while $f \to 1$ if $v \to \infty$, in which case one recovers the results of MGM [6, 7]. On the other hand, the gaugino masses, $m_{\tilde{g}}$, are as in MGM, with the SM coupling given by (1.6), for any v.

This note is organized as follows. In section 2, the theoretical setting is introduced. In section 3, the two-loop graphs contributing to the sfermions mass are discussed, and in section 4 they are evaluated. In section 5, we present the soft masses for MSSM, and in section 6 we discuss our results. Finally, in a couple of appendices, we list some technical details about the evaluation of the gaugino and sfermion graphs.

[1] The generalization to $G_{SM} = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ is simple, and will be presented in section 5.

2 Theoretical setting

We consider the setting in figure 1, discussed in the introduction. The matter fields are taken with charge ± 1, with the following sign choice:

$$\begin{aligned} D_\mu Q &= \partial_\mu Q + i g_1 A^1_\mu Q\,, \qquad D_\mu T = \partial_\mu T + i g_2 A^2_\mu T\,, \\ D_\mu L &= \partial_\mu L + i g_1 A^1_\mu L - i g_2 A^2_\mu L\,. \end{aligned} \tag{2.1}$$

The potential is the sum of D and F terms:

$$\begin{aligned} V_D &= \frac{g_1^2}{2}\left(QQ^\dagger - \tilde{Q}^\dagger\tilde{Q} + LL^\dagger - \tilde{L}^\dagger\tilde{L}\right)^2 + \frac{g_2^2}{2}\left(-LL^\dagger + \tilde{L}^\dagger\tilde{L} + TT^\dagger - \tilde{T}^\dagger\tilde{T}\right)^2\,, \\ V_F &= |L\tilde{L} - v^2|^2 + |KL|^2 + |K\tilde{L}|^2 + |ST|^2 + |S\tilde{T}|^2 + |T\tilde{T} + F|^2\,. \end{aligned} \tag{2.2}$$

The VEVs of the scalars are:

$$L = \tilde{L} = v\,, \qquad K = T = \tilde{T} = Q = \tilde{Q} = 0\,, \qquad S = M\,.$$

2.1 Tree-level masses

After the VEV insertion, the following term gives mass to a combination of the two U(1)'s:

$$2v^2(g_1^2 + g_2^2)\left(\frac{g_1 A^1_\mu - g_2 A^2_\mu}{\sqrt{g_1^2 + g_2^2}}\right)^2\,, \tag{2.3}$$

which gives $m_v = 2v\sqrt{g_1^2 + g_2^2}$ for the combination of the two vectors which gets a mass.

The part of the Lagrangian corresponding to the scalar masses reads:

$$\begin{aligned} &\begin{pmatrix} \delta T^* & \delta\tilde{T} \end{pmatrix} \begin{pmatrix} M^2 & F \\ F & M^2 \end{pmatrix} \begin{pmatrix} \delta T \\ \delta\tilde{T}^* \end{pmatrix} + 2v^2|\delta K|^2 \\ &+ v^2|\delta L + \delta\tilde{L}|^2 + \frac{g_1^2 + g_2^2}{2} v^2 (\delta L + \delta L^* - \delta\tilde{L} - \delta\tilde{L}^*)^2\,. \end{aligned} \tag{2.4}$$

The imaginary part of the scalar $\frac{(\delta L - \delta\tilde{L})}{\sqrt{2}}$ is eaten by Higgs mechanism; the real part of the same scalar takes the same mass m_v as the gauge boson (it is in the same supermultiplet). The scalar messengers $T_\pm = (T \pm \tilde{T}^*)/\sqrt{2}$ get mass squared $m_\pm^2 = M^2 \pm F$.

The piece corresponding to the fermion masses is:

$$i v\sqrt{2}(g_1\lambda_1 - g_2\lambda_2)(\psi_L - \psi_{\tilde{L}}) - (M\psi_T\psi_{\tilde{T}} + v\psi_K\psi_{\tilde{L}} + v\psi_K\psi_L) + \text{c.c.} \tag{2.5}$$

The combination

$$\lambda_A = i\,\frac{g_2\lambda_1 + g_1\lambda_2}{\sqrt{g_1^2 + g_2^2}}\,, \tag{2.6}$$

remains massless at tree level, while

$$\lambda_B = i\,\frac{g_1\lambda_1 - g_2\lambda_2}{\sqrt{g_1^2 + g_2^2}}\,, \qquad \eta = \frac{\psi_{\tilde{L}} - \psi_L}{\sqrt{2}}\,, \tag{2.7}$$

mix to make the following Dirac fermion

$$\kappa = \begin{pmatrix} (\lambda_B)_\alpha \\ (\eta^*)^{\dot{\alpha}} \end{pmatrix}\,, \tag{2.8}$$

whose mass is m_v. Finally, the fermionic messengers $\psi_T, \psi_{\tilde{T}}$ get a mass $m_f = M$.

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2.2 Gaugino couplings

In Weyl spinor notation, the gaugino couplings with the $Q, \tilde{Q}, T, \tilde{T}$ hypermultiplets are:

$$
\begin{aligned}
& -ig_2\sqrt{2}\left(T\psi_T^*\lambda_2^* - T^*\psi_T\lambda_2 - \tilde{T}\psi_{\tilde{T}}^*\lambda_2^* + \tilde{T}^*\psi_{\tilde{T}}\lambda_2\right) \\
& -ig_1\sqrt{2}\left(Q\psi_Q^*\lambda_1^* - Q^*\psi_Q\lambda_1 - \tilde{Q}\psi_{\tilde{Q}}^*\lambda_1^* + \tilde{Q}^*\psi_{\tilde{Q}}\lambda_1\right) .
\end{aligned}
\tag{2.9}
$$

After some manipulations these couplings are:

$$
\begin{aligned}
\frac{1}{\sqrt{g_1^2+g_2^2}} & \left(g_1 g_2 T_+(\psi_T^*\lambda_A^* - \psi_{\tilde{T}}\lambda_A) + g_1 g_2 T_-(\psi_T^*\lambda_A^* + \psi_{\tilde{T}}\lambda_A)\right. \\
& \left. + g_2^2 T_+(\psi_{\tilde{T}}\lambda_B - \psi_T^*\lambda_B^*) - g_2^2 T_-(\psi_T^*\lambda_B^* + \psi_{\tilde{T}}\lambda_B)\right) \\
& + \frac{\sqrt{2}}{\sqrt{g_1^2+g_2^2}} Q\left(g_1 g_2 \psi_Q^*\lambda_A^* + g_1^2\psi_Q^*\lambda_B^*\right) + \text{c.c.} .
\end{aligned}
\tag{2.10}
$$

It is useful to write some of these couplings in Dirac notation; the following spinors are introduced for this purpose:

$$
\omega_T = \begin{pmatrix} (\psi_T)_\alpha \\ (\psi_{\tilde{T}}^*)^{\dot{\alpha}} \end{pmatrix} , \qquad \omega_Q = \begin{pmatrix} (\psi_Q)_\alpha \\ (\psi_{\tilde{Q}}^*)^{\dot{\alpha}} \end{pmatrix} , \qquad \lambda_M = \begin{pmatrix} (\lambda_A)_\alpha \\ (\lambda_A^*)^{\dot{\alpha}} \end{pmatrix} .
\tag{2.11}
$$

The couplings involving λ_M, $\omega_{Q,T}$ and Q, T_+, T_- are:

$$
g_e\left(\sqrt{2}Q\bar{\omega}_Q\frac{1+\gamma^5}{2}\lambda_M + T_+\bar{\omega}_T\gamma_5\lambda_M + T_-\bar{\omega}_T\lambda_M\right) + \text{c.c.} ,
\tag{2.12}
$$

where g_e is defined in (1.6), while the couplings involving the Dirac spinor κ (2.8) are:

$$
\begin{aligned}
\frac{1}{\sqrt{g_1^2+g_2^2}} & \left(\sqrt{2}g_1^2 Q\bar{\omega}_Q\frac{1+\gamma^5}{2}\kappa^c + g_2^2 T_+\left(\bar{\omega}_T\frac{1-\gamma_5}{2}\kappa - \bar{\omega}_T\frac{1+\gamma_5}{2}\kappa^c\right)\right. \\
& \left. - g_2^2 T_-\left(\bar{\omega}_T\frac{1-\gamma_5}{2}\kappa + \bar{\omega}_T\frac{1+\gamma_5}{2}\kappa^c\right)\right) + \text{c.c.} .
\end{aligned}
\tag{2.13}
$$

Here κ^c is the charge conjugate spinor of κ:

$$
\kappa^c = \begin{pmatrix} (\eta)_\alpha \\ (\lambda_B^*)^{\dot{\alpha}} \end{pmatrix} .
\tag{2.14}
$$

3 Calculation of the sfermion masses

The aim is to generalize the two-loops calculations by Martin [6] in minimal gauge mediation. These graphs come in three different classes: there is a graph due to the exchange of scalars, some graphs which are due to the exchange of gauge bosons and a graph which is due to exchange of gauginos. In this section we examine each of these contributions separately.

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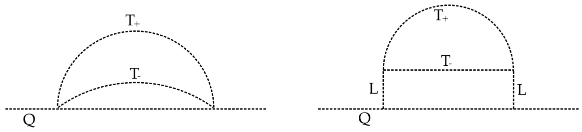

Figure 2. Graph corresponding to the contribution due to D-term (on the left at infinite v, on the right at finite v).

3.1 Scalar graph

The graph corresponding to the contribution due to scalar exchange is shown in figure 2. The two Φ^4 interactions

$$g_e^2 QQ^* (T_+ T_-^* + T_+^* T_-)\,,$$

of the minimal gauge mediation case are replaced by four Φ^3 interactions.

The detailed form of these interactions is:

$$-2v\left(g_1^2 \frac{\delta L_R - \delta\tilde{L}_R}{\sqrt{2}} QQ^* + g_2^2 \frac{\delta L_R - \delta\tilde{L}_R}{\sqrt{2}} (T_+ T_-^* + T_- T_+^*)\right)\,, \tag{3.1}$$

where $L = v + (\delta L_R + i\delta L_I)/\sqrt{2}$ and $\tilde{L} = v + (\delta\tilde{L}_R + i\delta\tilde{L}_I)/\sqrt{2}$. Notice that this cubic vertex couples just with the eigenvector of the mass matrix whose mass is m_v (1.3). So the propagator that must be inserted between each couple of vertical cubic vertices is

$$\frac{i}{k^2 - m_v^2}\,.$$

In the $v \to \infty$ limit the usual Φ^4 interaction is recovered, with the diagonal U(1) effective coupling constant g_e^2. A direct evaluation gives:

$$\begin{aligned}
&-2\int \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\frac{1}{(k-p)^2 - m_+^2}\frac{1}{p^2 - m_-^2}\frac{1}{k^2}\left(\frac{4g_1^2 g_2^2 v^2}{k^2 - m_v^2}\right)^2 \\
&\quad = -2g_e^4 \int \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\frac{1}{(k-p)^2 - m_+^2}\frac{1}{p^2 - m_-^2}\frac{1}{k^2} f(k^2, m_v^2) \\
&\quad = \int \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\,([6])\, f\,,
\end{aligned} \tag{3.2}$$

where f is given in (1.4). This proves the claim in eq. (1.5) for the scalar graph.

3.2 Gauge boson graphs

The graphs which give the contribution due to the exchange of gauge bosons are shown in figure 3. In the case of minimal gauge mediation [6], which corresponds to the $v \to \infty$ limit of our setting, only the contribution of a massless gauge boson must be taken into account.

In our more general setting, we can introduce the following mass eigenstates:

$$A_\mu^A = \frac{g_2 A_\mu^1 + g_1 A_\mu^2}{\sqrt{g_1^2 + g_2^2}}\,, \qquad A_\mu^B = \frac{g_1 A_\mu^1 - g_2 A_\mu^2}{\sqrt{g_1^2 + g_2^2}}\,, \tag{3.3}$$

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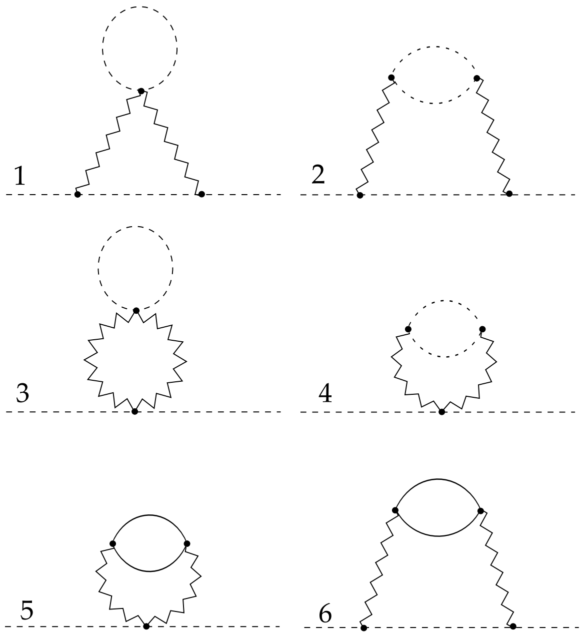

Figure 3. Graphs corresponding to gauge boson exchange. In the minimal gauge mediation case there is just the contribution from a massless gauge boson; in our setting the contribution of both the massless and the massive gauge bosons must be taken into account.

The combination A_μ^A is massless, while A_μ^B get a mass m_v due to Higgs mechanism. The covariant derivatives of Q and T in the new variables are:

$$
\begin{aligned}
D_\mu Q &= \partial_\mu Q + i g_e A_\mu^A Q + \frac{i g_1^2}{\sqrt{g_1^2+g_2^2}} A_\mu^B Q \,, \\
D_\mu T &= \partial_\mu T + i g_e A_\mu^A T - \frac{i g_2^2}{\sqrt{g_1^2+g_2^2}} A_\mu^B T \,,
\end{aligned}
\tag{3.4}
$$

where g_e is defined in (1.6).

Let us denote with k the momentum on the gauge boson propagators. Three kinds of graphs must then be taken into account: the one with two massless A_μ^A propagators, the ones with two massive A_μ^B exchanges and the ones with one massless and one massive propagators. The contribution of the last kind of graphs comes with a relative minus sign with respect to the first two; the result is:

$$
\int \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\,([6])\left(1+\frac{(k^2)^2}{(k^2-m_v^2)^2}-\frac{2k^2}{(k^2-m_v^2)}\right). \tag{3.5}
$$

Here ([6]) is the same as the integrand for MGM in [6], while the expression in the second parentheses gives the common factor $f(k^2, m_v^2)$, where the momentum k corresponds to the one on the massless propagator. This proves the claim in eq. (1.5) for the gauge boson graphs; the detailed evaluation of the graphs is presented in appendix B.

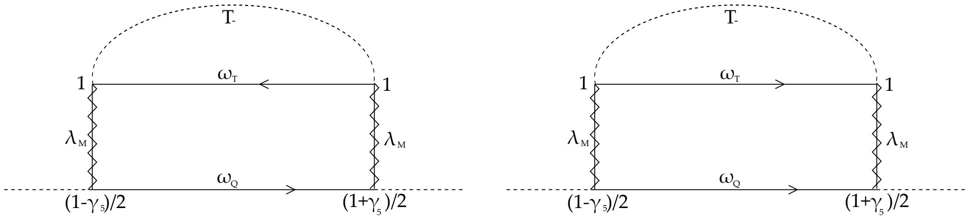

Figure 4. Contribution due to the mediation of the massless gaugino.

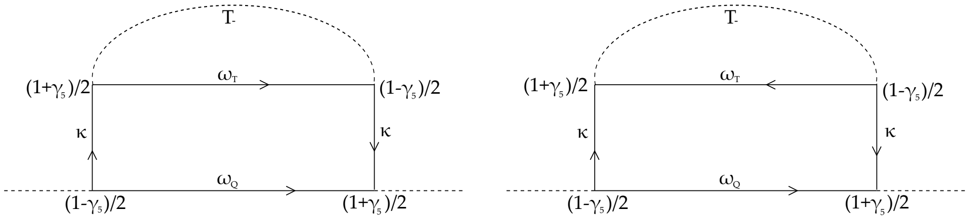

Figure 5. Contribution due to the mediation of the dirac fermion κ.

3.3 Gaugino graphs

The contribution due to gaugino exchange is given by three classes of graphs, one for the combination of the two gauginos that is massless at tree level, one for the combination that gets a tree-level Dirac mass, and a mixed one. It is very useful for the evaluation to use the Feynman rules given in [8] for Majorana fermions and for interactions with explicit charge conjugate spinors.

We first recall the evaluation of the gaugino graph in MGM. The contribution due to T_- is shown in figure 4; the contribution due to T_+ is similar.[2] The evaluation gives:

$$4g^4 \int \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\frac{\mathrm{Tr}(\not{k}\frac{1-\gamma_5}{2}\not{k}\frac{1+\gamma_5}{2}\not{k}(\not{k}-\not{p}+m_f))}{(k^2)^3((k-p)^2-m_f^2)(p^2-m_\pm^2)}$$

$$= 4g^4 \int \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\frac{2(k^2-kp)}{(k^2)^2((k-p)^2-m_f^2)(p^2-m_\pm^2)}\,.$$

In the case of MgGM there is the same diagram, corresponding to the exchange of the massless gaugino λ_A, weighted by $g^4 = g_e^4$.

There is also a diagram corresponding to the exchange of the Dirac fermion κ (see figure 5):

$$4g_e^4 \int \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\frac{2k(k-p)}{(k^2-m_v^2)^2((k-p)^2-m_f^2)(p^2-m_\pm^2)}\,.$$

Finally, there are also mixed diagrams which exchange both λ_M, κ, as shown in figure 6. They have a minus sign with respect to the previous ones, and they all give the same contribution; the total is:

$$-8g_e^4 \int \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\frac{2k(k-p)}{(k^2-m_v^2)(k^2)((k-p)^2-m_f^2)(p^2-m_\pm^2)}\,.$$

[2] There are some extra $\pm\gamma_5$ factors which at the end give rise to the same evaluation, with the replacement $m_- \to m_+$.

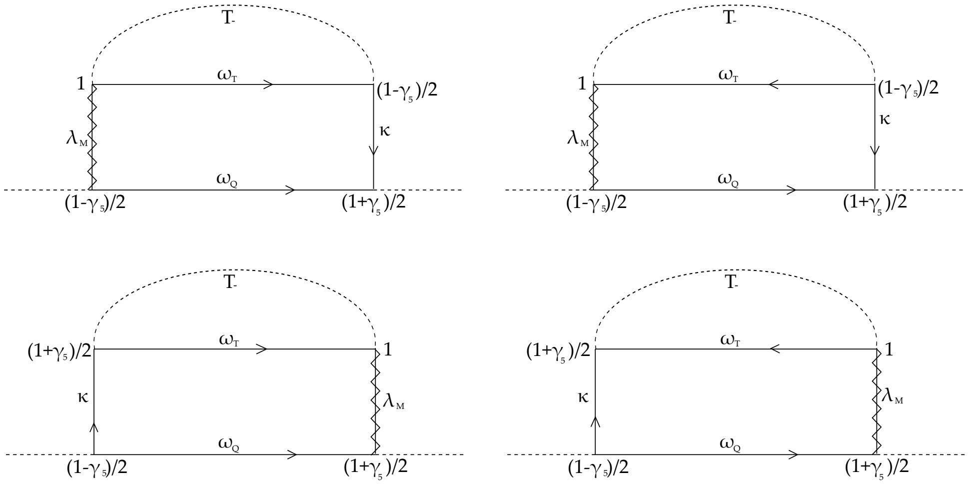

Figure 6. Mixed contribution due to the combined action of λ_M, κ.

All in all, the sum of the three kinds of diagrams is:[3]

$$4g_e^4 \int \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\frac{2(k^2-kp)}{(k^2)^2((k-p)^2-m_f^2)(p^2-m_\pm^2)}\left(1+\frac{(k^2)^2}{(k^2-m_v^2)^2}-\frac{2k^2}{(k^2-m_v^2)}\right),$$

which gives the same factor inside the integral as for the other contributions. This completes the proof of eq. (1.5).

4 Evaluation of the integrals

In this section, we write the integrals for the sfermions mass in a notation similar to the one in [9]; we pass to Euclidean variables, and define

$$\begin{aligned}&\langle m_{11},\ldots,m_{1n_1}|m_{21},\ldots,m_{2n_2}|m_{31},\ldots,m_{3n_3}\rangle \\ &=\int\frac{d^dk}{\pi^{d/2}}\frac{d^dq}{\pi^{d/2}}\prod_{i=1}^{n_1}\prod_{j=1}^{n_2}\prod_{l=1}^{n_3}\frac{1}{k^2+m_{1i}^2}\frac{1}{q^2+m_{2j}^2}\frac{1}{(k-q)^2+m_{3l}^2}\,.\end{aligned} \tag{4.1}$$

In this notation the integral that should be evaluated in order to compute the sfermions mass is:

$$\begin{aligned}&(g_e^4m_v^4/(4\pi)^d)\left(-\langle m_+|m_+|0,m_v,m_v\rangle-\langle m_-|m_-|0,m_v,m_v\rangle\right.\\ &-4\langle m_f|m_f|0,m_v,m_v\rangle-2\langle m_+|m_-|0,m_v,m_v\rangle+4\langle m_+|m_f|0,m_v,m_v\rangle\\ &+4\langle m_-|m_f|0,m_v,m_v\rangle-4m_+^2\langle m_+|m_+|0,0,m_v,m_v\rangle\\ &-4m_-^2\langle m_-|m_-|0,0,m_v,m_v\rangle+8m_f^2\langle m_f|m_f|0,0,m_v,m_v\rangle\\ &\left.+4(m_+^2-m_f^2)\langle m_+|m_f|0,0,m_v,m_v\rangle+4(m_-^2-m_f^2)\langle m_-|m_f|0,0,m_v,m_v\rangle\right)\,.\end{aligned} \tag{4.2}$$

Note that this is obtained from the result in [6] by adding the last two entries in each term: $\langle[6]\rangle \to \langle[6], m_v, m_v\rangle$.

[3]More manipulations with this integral are presented in appendix B.

We will use the following expression taken from [9], with the convention $d = 4 - 2\epsilon$:

$$\langle m_0|m_1|m_2\rangle = \frac{1}{-1+2\epsilon}\left(m_0^2\langle m_0, m_0|m_1|m_2\rangle + m_1^2\langle m_1, m_1|m_0|m_2\rangle + m_2^2\langle m_2, m_2|m_0|m_1\rangle\right) . \tag{4.3}$$

The basic object to compute then is

$$\begin{aligned}\langle m_0, m_0|m_1|m_2\rangle &= \frac{1}{2\epsilon^2} + \frac{1/2 - \gamma - \log m_0^2}{\epsilon} \\ &\quad +\gamma^2 - \gamma + \frac{\pi^2}{12} + (2\gamma - 1)\log m_0^2 + \log^2 m_0^2 - \frac{1}{2} + h(a, b) .\end{aligned} \tag{4.4}$$

The function h is given by the integral [9]:

$$h(a, b) = \int_0^1 dx \left(1 + \mathrm{Li}_2(1 - \mu^2) - \frac{\mu^2}{1 - \mu^2}\log\mu^2\right) , \tag{4.5}$$

where the dilogarithm is defined by $\mathrm{Li}_2(x) = -\int_0^1 \frac{dt}{t}\log(1 - xt)$, $a = m_1^2/m_0^2$, $b = m_2^2/m_0^2$, and

$$\mu^2 = \frac{ax + b(1 - x)}{x(1 - x)} . \tag{4.6}$$

For $a = 0$, the function h simplifies to $h(0, b) = 1 + \mathrm{Li}_2(1 - \mathrm{b})$. It is also possible to write an analytical expression:

$$\begin{aligned}h(a, b) &= 1 - \frac{\log a \log b}{2} - \frac{a + b - 1}{\sqrt{\Delta}}\left(\mathrm{Li}_2\left(-\frac{u_2}{v_1}\right) + \mathrm{Li}_2\left(-\frac{v_2}{u_1}\right)\right. \\ &\quad \left. +\frac{1}{4}\log^2\frac{u_2}{v_1} + \frac{1}{4}\log^2\frac{v_2}{u_1} + \frac{1}{4}\log^2\frac{u_1}{v_1} - \frac{1}{4}\log^2\frac{u_2}{v_2} + \frac{\pi^2}{6}\right) ,\end{aligned} \tag{4.7}$$

where

$$\Delta = 1 - 2(a + b) + (a - b)^2 , \qquad u_{1,2} = \frac{1 + b - a \pm \sqrt{\Delta}}{2} , \qquad v_{1,2} = \frac{1 - b + a \pm \sqrt{\Delta}}{2} . \tag{4.8}$$

The integrals with two massless propagators are infrared divergent and so a mass m_ϵ must be introduced there as an infrared cutoff; this artificial parameter will disappear at the end of the calculation. A useful relation [6] is:

$$\begin{aligned}\langle m_a|m_b|m_\epsilon, m_\epsilon\rangle &= \frac{\Gamma(1 + 2\epsilon)}{2}\left(\frac{1}{\epsilon^2} + \frac{1 - 2\log m_\epsilon^2}{\epsilon} + 1 - \frac{\pi^2}{6}\right. \\ &\quad \left. -F_2(m_a^2, m_b^2) - 2F_3(m_a^2, m_b^2) + (-2 + 2F_1(m_a^2, m_b^2)\log m_\epsilon^2 + \log^2 m_\epsilon^2)\right) ,\end{aligned} \tag{4.9}$$

where

$$\begin{aligned}F_1(a, b) &= \frac{a\log a - b\log b}{a - b} , \qquad F_2(a, b) = \frac{a\log^2 a - b\log^2 b}{a - b} , \\ F_3(a, b) &= \frac{a\,\mathrm{Li}_2(1 - b/a) - b\,\mathrm{Li}_2(1 - a/b)}{a - b} ,\end{aligned} \tag{4.10}$$

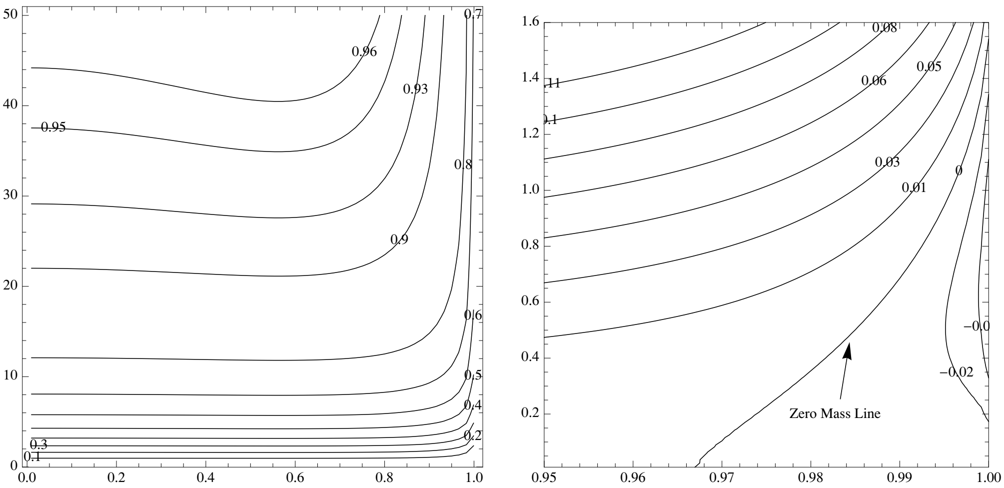

Figure 7. Contour plot for $s(x, y)$. On the right we zoom on the regime near $x = 1$ and small y, and we find that the sfermion is tachyonic below the zero mass line.

for $a \neq b$ and

$$F_1(a, a) = 1 + \log a\,, \qquad F_2(a, a) = 2 \log a + \log^2 a\,, \qquad F_3(a, a) = 2\,. \tag{4.11}$$

We can then use the following expressions [10] to relate the integrals to the known objects $\langle m_0|m_1|m_2\rangle$ or $\langle m_0, m_0|m_1|m_2\rangle$:

$$\begin{aligned} \langle m_a|m_b|0, m_v, m_v\rangle &= \frac{\langle m_a|m_b|0\rangle - \langle m_a|m_b|m_v\rangle}{m_v^4} - \frac{\langle m_a|m_b|m_v, m_v\rangle}{m_v^2}\,, \qquad (4.12) \\ \langle m_a|m_b|m_\epsilon, m_\epsilon, m_v, m_v\rangle &= \frac{\langle m_a|m_b|m_v, m_v\rangle + \langle m_a|m_b|m_\epsilon, m_\epsilon\rangle}{(m_v^2 - m_\epsilon^2)^2} + 2\frac{\langle m_a|m_b|m_v\rangle - \langle m_a|m_b|m_\epsilon\rangle}{(m_v^2 - m_\epsilon^2)^3}\,. \end{aligned}$$

The sfermions mass can be expressed as:[4]

$$m^2_{\tilde{f}} = 4 \left(\frac{F}{M}\right)^2 \left(\frac{\alpha_e}{4\pi}\right)^2 s(x, y)\,, \tag{4.13}$$

where x and y are defined in eq. (1.3); note that $x < 1$ (to avoid unstable messengers).

The analytic expression for $s(x, y)$ is:

$$s(x, y) = \frac{1}{2x^2}\left(s_0 + \frac{s_1 + s_2}{y^2} + s_3 + s_4 + s_5\right) + (x \to -x)\,, \tag{4.14}$$

[4]The 4 factor is due to our choice of U(1) charges.

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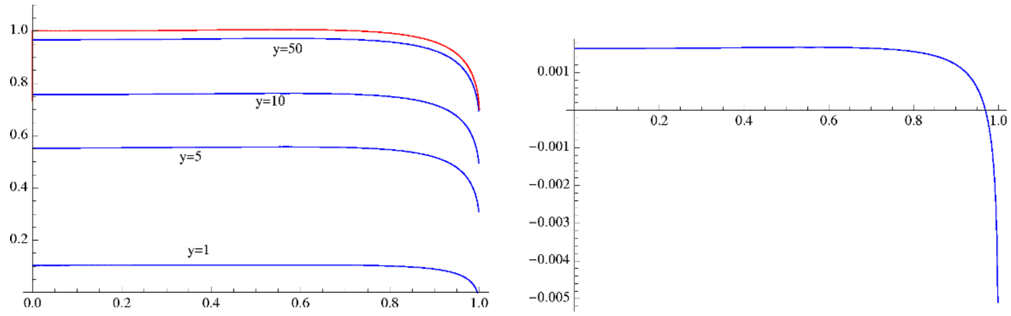

Figure 8. Left: the function $s(x, y)$, plotted along the x axis for $y = 1, 5, 10, 50$. The top line corresponds to the gauge mediation case (formally $y \to \infty$). Right: the same plot for $y = 1/10$. The sfermion becomes tachyonic near $x = 1$.

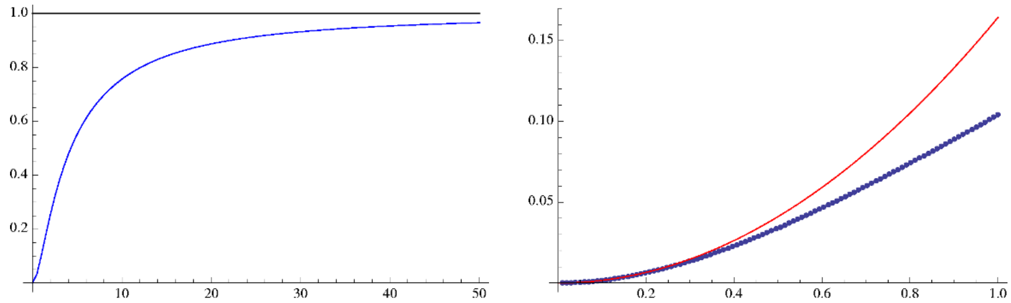

Figure 9. The function $s(x, y)$, plotted along the y axis for $x = 1/100$. On the right we zoom on the small y regime; the upper line corresponds to a quadratic fit on the values with $y \leq 0.1$, which gives $s \approx 0.1643\, y^2$ in this regime; this fit is a good approximation as long as $m_v < M/3$.

where

$$
\begin{aligned}
s_0 &= 2(1+x)\left(\log(1+x) - 2\mathrm{Li}_2\left(\frac{x}{1+x}\right) + \frac{1}{2}\mathrm{Li}_2\left(\frac{2x}{1+x}\right)\right), \qquad (4.15)\\
s_1 &= -4x^2 - 2x(1+x)\log^2(1+x) - x^2\,\mathrm{Li}_2(x^2)\,,\\
s_2 &= 8\,(1+x)^2\, h\left(\frac{y^2}{1+x}, 1\right) - 4x\,(1+x)\, h\left(\frac{y^2}{1+x}, \frac{1}{1+x}\right) - 4xh\left(y^2, 1+x\right) - 8h\left(y^2, 1\right)\,,\\
s_3 &= -2h\left(\frac{1}{y^2}, \frac{1}{y^2}\right) - 2x\, h\left(\frac{1+x}{y^2}, \frac{1}{y^2}\right) + 2(1+x)h\left(\frac{1+x}{y^2}, \frac{1+x}{y^2}\right),\\
s_4 &= (1+x)\left(2h\left(\frac{y^2}{1+x}, \frac{1}{1+x}\right) - h\left(\frac{y^2}{1+x}, 1\right) - h\left(\frac{y^2}{1+x}, \frac{1-x}{1+x}\right)\right),\\
s_5 &= 2h\left(y^2, 1+x\right) - 2h\left(y^2, 1\right)\,.
\end{aligned}
$$

The expressions $s_0(x)$ and $s_1(x)$ were simplified by using standard dilogarithm identities. Note that in the $y \to \infty$ limit only s_0 contributes; the result then reduces to the one in minimal gauge mediation [6, 7].

Some plots of the function $s(x, y)$ are shown in figures 7, 8 and 9. In particular, we see that the sfermion is tachyonic in some regime in parameters space.

5 MSSM sparticle mass spectrum

In the case of the MSSM the result for the sfermions mass is:

$$m_{\tilde{f}}^2 = 2\left(\frac{F}{M}\right)^2 \sum_r \left(\frac{\alpha_r}{4\pi}\right)^2 C_r^{\tilde{f}} n_r s(x, y_r)\,, \tag{5.1}$$

where

$$y_r = \frac{m_{v_r}}{M}\,, \qquad m_{v_r} = 2v\sqrt{\left(g_1^{(r)}\right)^2 + \left(g_2^{(r)}\right)^2}\,, \tag{5.2}$$

with $g_{1,2}^{(r)}$ being the couplings of $G_{SM_{1,2}}$ in figure 1, respectively; $r = 1, 2, 3$ for U(1), SU(2), SU(3), respectively, and

$$\alpha_r \equiv \frac{\left(g_{SM}^{(r)}\right)^2}{4\pi}\,, \qquad \frac{1}{\left(g_{SM}^{(r)}\right)^2} = \frac{1}{\left(g_1^{(r)}\right)^2} + \frac{1}{\left(g_2^{(r)}\right)^2}\,. \tag{5.3}$$

In eq. (5.1), $C_r^{\tilde{f}}$ is the quadratic Casimir invariant of the MSSM scalar field $\tilde{f}$, in a normalization where $C_3 = 4/3$ for color triplets, $C_2 = 3/4$ for SU(2) doublets and $C_1 = \frac{3}{5}Y^2$; n_r is the Dynkin index for the pair of messengers in a normalization where $n_r = 1$ for $N + \bar{N}$ of SU(N), and $n_1 = \frac{6}{5}Y^2$ for a messenger pair with weak hypercharge $Y = Q_{\mathrm{EM}} - T_3$ (we use the GUT normalization for α_1, as in [6]).

In the limit $m_v \to \infty$ the well known result of [6, 7] is recovered, with $s = t(x)$ (see the previous section):

$$t(x) = \frac{1+x}{x^2}\left(\log(1+x) - 2\mathrm{Li}_2\left(\frac{x}{1+x}\right) + \frac{1}{2}\mathrm{Li}_2\left(\frac{2x}{1+x}\right)\right) + (x \to -x)\,. \tag{5.4}$$

The gauginos mass is instead the same as in minimal gauge mediation:

$$m_{\tilde{g}_r} = \frac{\alpha_r}{4\pi}\frac{F}{M} n_r\, q(x)\,, \tag{5.5}$$

where α_r are given in (5.3), and

$$q(x) = \frac{1}{x^2}\left((1+x)\log(1+x) + (1-x)\log(1-x)\right)\,. \tag{5.6}$$

6 Discussion

In this note we computed the sparticle mass spectrum in Minimal gaugino-Gauge Mediation (MgGM) as a function of the parameters x and y in (1.3). We have not studied the Renormalization Group Evolution of the soft masses, and it should be interesting to investigate how it affects the sparticle spectrum at the weak scale.

One peculiar result is that in low-scale gaugino mediation, the sfermions become tachyonic (at the messenger scale M) when the effective SUSY-breaking scale, F/M, approaches M. This occurs in a very small corner of the (x, y) plane, where it is likely that the RGE

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flips the sign of $m^2_{\tilde{f}}$. For small v, there are also important three-loop contributions [5], which we ignored in this note; in particular, these may also cure the instabilities mentioned above.

The models studied here provide a particular class of General Gauge Mediation (GGM) models [11] (although they do not fall into the class of General Messenger Gauge Mediation (GMGM) models [12, 13]). Some possible generalizations of our work are the following. First, one may define General gaugino-Gauge Mediation (GgGM) models and compute their soft masses. In particular, it will be interesting to compute the soft masses in the "Direct Gaugino Mediation" models of [5] and their generalizations, namely, in dynamical realizations of MgGM and its generalizations in (deformed) SQCD. It should also be interesting to find which of the parameters space of GGM is being covered, and to investigate the phenomenological aspects, e.g. constraints on the spectrum, the NLSP and the experimental signatures for the classes of models above.

Note added. The result (1.4), (1.5) was generalized to an arbitrary SUSY-breaking sector in [14]. See also the recent work [15].

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A One-loop gaugino masses

For completeness, the 1-loop gaugino mass is presented; it is given by the MGM one, with the gauge couplings in the unbroken G_{SM} group. This is obtained from the sum of two diagrams, one with the scalar messenger with mass m_- (whose coupling is proportional to $\bar{\lambda}_M \omega_T$) and one with mass m_+ (whose coupling is proportional to $\bar{\lambda}_M(\gamma^5)\omega_T$) running in the loop:

$$\begin{aligned} g^2 \int \frac{d^4k}{(2\pi)^4} \frac{F}{(k^2-M^2+F)(k^2-M^2-F)} \frac{\not{k}+M}{k^2-M^2} \\ = g^2 \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4k}{(2\pi)^4} \frac{2FM}{(k^2-M^2-F(y-x))^3} \,. \end{aligned} \tag{A.1}$$

Going to Euclidean variables, the evaluation gives:

$$\frac{\alpha}{4\pi} \frac{F}{M} \int_0^1 dx \int_0^{1-x} dy \frac{1}{1+(x-y)\frac{F}{M^2}} \,,$$

which after an integration gives the well known result (which is in eqs. (5.5), (5.6) of this note). In the case of MgGM, the same formula applies with $g = g_e$ (1.6), since the gaugino is in the gauge multiplet of the unbroken G_{SM} group.

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B Evaluation of the gauge boson and gaugino graphs

Let us evaluate the graphs in figure 3 explicitly; Feynman gauge is used. The evaluation of graph 1 is:

$$2g_e^4 \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - m_\pm^2} \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2)^2} f(k^2, m_v^2)\,, \tag{B.1}$$

where $f(k)$ is given in (1.4), and there is a symmetry factor $S = 2$. Here and below, a $\sum_{m_+,m_-}$ is understood.

The evaluation of graph 2 gives:

$$\begin{aligned}
&-g_e^4 \int \frac{d^4p}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{((2p+k)k)^2}{(k^2)^3((p+k)^2 - m_\pm^2)(p^2 - m_\pm^2)} f(k^2, m_v^2) \\
&\quad = -g_e^4 \int \frac{d^4p}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \left(\frac{1}{p^2 - m_\pm^2} - \frac{1}{(p+k)^2 - m_\pm^2} \right) \frac{2pk + k^2}{(k^2)^3} f(k^2, m_v^2) \\
&\quad = -g_e^4 \int \frac{d^4p}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{-2pk}{(k^2)^3((p+k)^2 - m_\pm^2)} f(k^2, m_v^2) \\
&\quad = -g_e^4 \int \frac{d^4s}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{-2sk + 2k^2}{(k^2)^3(s^2 - m_\pm^2)} f(k^2, m_v^2) \\
&\quad = -g_e^4 \int \frac{d^4s}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{2}{(k^2)^2(s^2 - m_\pm^2)} f(k^2, m_v^2)\,,
\end{aligned} \tag{B.2}$$

where at last we have done the change of variable $s = p + k$. So we have that the total contribution of graphs 1 and 2 cancels, as in [6].

Graph 3 is very similar to graph 1, up to a numerical constant and negative relative sign: there is a 4 coming for the $g_{\mu\nu}g^{\mu\nu}$, the symmetry factor is 2 and there is a 4 from the two photon-scalar vortices. At the end the evaluation gives -4 times graph 1.

A similar strategy can be employed with graph 4:

$$\begin{aligned}
&g_e^4 \int \frac{d^4p}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} f(k^2, m_v^2) \frac{(2p+k)^2}{(k^2)^2(p^2 - m_\pm^2)((p+k)^2 - m_\pm)^2} \\
&\quad = g_e^4 \int \frac{d^4p}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} f(k^2, m_v^2) \left(\frac{4}{(k^2)^2(p^2 - m_\pm^2)} - \frac{1}{(k^2)(p^2 - m_\pm^2)((p+k)^2 - m_\pm^2)} \right. \\
&\qquad \left. + \frac{4m_\pm^2}{(k^2)^2(p^2 - m_\pm^2)((p+k)^2 - m_\pm^2)} - \frac{4pk + 2k^2}{(k^2)^2(p^2 - m_\pm^2)((p+k)^2 - m_\pm^2)} \right) .
\end{aligned} \tag{B.3}$$

The last term is zero because it is proportional to the integral

$$\int \frac{d^4p}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} f(k^2, m_v^2) \frac{2}{(k^2)^2} \left(\frac{1}{p^2 - m_\pm^2} - \frac{1}{(p+k)^2 - m_\pm^2} \right) = 0\,.$$

The symmetry factor is $S = 2$.

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The evaluation of graph 5 is:

$$
\begin{aligned}
&-g_e^4\int \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\, f(k^2,m_v^2)\,\frac{\mathrm{Tr}(\gamma^\mu(\not{k}+\not{p}+m_f)\gamma^\rho(\not{p}+m_f))g_{\mu\rho}}{(k^2)^2(p^2-m_f^2)((p+k)^2-m_f^2)}\\
&\quad= g_e^4\int \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\, f(k^2,m_v^2)\,\frac{8p(p+k)-16m_f^2}{(k^2)^2(p^2-m_f^2)((p+k)^2-m_f^2)}\\
&\quad= g_e^4\int \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\, f(k^2,m_v^2)\left(\frac{4}{(k^2)^2(p^2-m_f^2)}+\frac{4}{(k^2)^2((p+k)^2-m_f^2)}\right.\\
&\qquad\left.-\frac{8m_f^2}{(k^2)^2((p+k)^2-m_f^2)(p^2-m_f^2)}-\frac{4}{(k^2)((p+k)^2-m_f^2)(p^2-m_f^2)}\right).
\end{aligned}
$$

There is a symmetry factor $S=2$.

Now let us check that graph 6 is zero (this is just in Feynman gauge, which is the one used in the calculation):

$$
\begin{aligned}
&g_e^4\int \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\, f(k^2,m_v^2)\,\frac{k_\mu k_\sigma}{(k^2)^3}\frac{\mathrm{Tr}(\gamma^\mu(\not{k}+\not{p}+m_f))\gamma^\sigma(\not{p}+m_f)}{((k+p)^2-m_f^2)(p^2-m_f^2)}\\
&\quad= g_e^4\int \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\, f(k^2,m_v^2)\, 4\frac{2(pk)^2+(pk)k^2-k^2p^2+k^2m_f^2}{(k^2)^3((k+p)^2-m_f^2)(p^2-m_f^2)}.
\end{aligned}
$$

Let us then subtract from that

$$
g_e^4\int \frac{dp^4}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\, f(k^2,m_v^2)\,\frac{4kp}{(k^2)^3(p^2-m_f^2)},
$$

which is clearly zero by symmetry. What is left is:

$$
-4g_e^4\int \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\, f(k^2,m_v^2)\left(\frac{1}{(k^2)^2((p+k)^2-m_f^2)}+\frac{kp}{(k^2)^3((p+k)^2-m_f^2)}\right),
$$

which vanishes (this can be shown by using the auxiliary variable $s=p+k$).

Finally, the gaugino graphs give:

$$
\begin{aligned}
&4g_e^4\int \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\,\frac{\mathrm{Tr}(\not{k}\frac{1-\gamma_5}{2}\not{k}\frac{1+\gamma_5}{2}\not{k}(\not{k}-\not{p}+m_f))}{(k^2)^3((k-p)^2-m_f^2)(p^2-m_\pm^2)}f(k)\\
&\quad= 4g_e^4\int \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\,\frac{2(k^2-kp)}{(k^2)^2((k-p)^2-m_f^2)(p^2-m_\pm^2)}f(k)\\
&\quad= 4g_e^4\int \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}\left(\frac{1}{(k^2)^2(p^2-m_\pm^2)}+\frac{1}{(k^2)((k-p)^2-m_f^2)(p^2-m_\pm^2)}\right.\\
&\qquad\left.-\frac{1}{(k^2)^2((k-p)^2-m_f^2)}-\frac{(m_\pm^2-m_f^2)}{(k^2)^2((k-p)^2-m_f^2)(p^2-m_\pm^2)}\right)f(k).
\end{aligned}
$$

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