

A foray on SCFT₃ via super spinor-helicity and Grassmann twistor variables

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ABSTRACT: In this paper, we develop a momentum super space formalism for $\mathcal{N} = 1, 2$ superconformal field theories in three dimensions. First, we solve for super-correlators in the usual momentum superspace variables. However, we found that expressing quantities in super space spinor helicity variables greatly simplifies the analysis. Further, by performing a “half” Fourier transform of the Grassmann coordinates which is analogous to the Twistor transform, an even more remarkable simplification occurs. Using this formalism, we first compute all three point correlation functions involving conserved super-currents with arbitrary spins in $\mathcal{N} = 1, 2$ theories. We discover interesting double copy relations in $\mathcal{N} = 1$ super-correlators. Further, we discovered super double copy relations that take us from $\mathcal{N} = 1$ to $\mathcal{N} = 2$ super-correlators. We also extend our results to correlators involving scalars as well as to higher points. Further, we comment on the connection of our results with the flat space super amplitudes in one higher dimension.

KEYWORDS: Scale and Conformal Symmetries, Superspaces, Extended Supersymmetry

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1 Introduction

Conformal field theory (CFT) forms a cornerstone of theoretical physics. It's applications range from string theory to condensed matter physics. A special class of CFTs are those that also possess supersymmetry, i.e super conformal field theories (SCFTs), see for instance [1, 2]. These theories are of utmost interest as they provide us a window into non-perturbative physics. It is quite advantageous to formulate such theories in momentum superspace. In non supersymmetric CFTs, momentum space has led to various interesting results such as making

a connection to flat space scattering amplitudes in one higher dimension [3, 4], discovering double copy structures at the level of correlation functions [5–7] and even making a connection with early universe cosmology [3, 8–12] and AdS amplitude [13–18]. One natural direction is to explore supersymmetric extension, their connection to supersymmetric scattering amplitude, AdS superamplitude [19], etc. While there has been a lot of development in the Fourier space approach to non-supersymmetric CFTs over the past decade or so [7, 20–34], its supersymmetric counterparts have mostly been left untouched. The development of such a formalism would greatly aid for example the study of connection to flat space S-matrix. This development also have a application in Supersymmetric Chern-Simons matter such as [35–37] and $\mathcal{N} = 6$ ABJM theory [38]. However, such pursuits first require the development of momentum superspace. Most of the literature, however has been in the arena of position superspace [39–46]. Thus, we begin the programme of the development of momentum superspace, starting with three-dimensional SCFTs which possess two or four supercharges.

Outline. In section 2, we introduce the momentum superspace formalism for $\mathcal{N} = 1$ SCFTs. After working out a three-point super-correlator as an example, we show that rather than working in ordinary momentum superspace variables, working with super spinor helicity variables is quite advantageous. We then show that a drastic simplification occurs if we rather work in *Grassmann twistor space*, forming the nexus of our formalism. In section 3, we present our results for all two and three-point correlators with arbitrary (half) integer spin insertions in $\mathcal{N} = 1$ SCFTs. We also discuss double copy relations as well as Ward-Takahashi identities. In section 4, we extend our formalism to theories with four supercharges as well as present a way to obtain $\mathcal{N} = 2$ super-correlators via a super double copy construction of their $\mathcal{N} = 1$ counterparts. In section 5, we discuss four point functions in our new formalism. Finally, in section 6, we summarize our results and discuss several exciting future directions.

We also have several appendices that complement the main text. In appendix A, we present our notations, conventions and some useful spinor-helicity identities. In appendix B, we present the $\mathcal{N} = 1$ and $\mathcal{N} = 2$ Lie superalgebras and the action of their generators on primary superfields in various variables. In appendix C, we prove that conformal invariance of the individual component correlators that appear in a super-correlator implies the conformal invariance of the super-correlator itself. We then present a worked out example of a $\mathcal{N}=1$ three-point function in appendix D.

2 Setting the stage: $\mathcal{N} = 1$ SCFT

The coordinates of $\mathcal{N} = 1$ superspace consist of a pair (x^μ, θ^a) where $x^\mu, \mu = 1, 2, 3$ are the usual position space coordinates and $\theta^a, a = 1, 2$ are the Grassmann coordinates of the superspace [39, 47]. The generators of the $\mathcal{N} = 1$ superconformal algebra that act on the superspace consist of the usual conformal generators $P_\mu, M_{\mu\nu}, D, K_\mu$ along with the supersymmetry generator Q_a and the special superconformal generator S_a . The Lie superalgebra that these generators obey as well as their action on primary super fields are provided in appendix B. We provide our notation in appendix A.

By performing a Fourier transform with respect to the x^μ , we end up in momentum superspace which is described by the coordinates (p_μ, θ^a) , where p_μ is the three-momentum and θ^a are the same Grassmann coordinates as in the position superspace.

Let us now study SCFT in this superspace formalism. The quantities that we are interested in obtaining are correlation functions involving primary superfields. In particular the superfields that we are interested in are symmetric traceless conserved super-currents, which we shall henceforth just refer to as a supercurrent for brevity. A spin s supercurrent has the following component expansion in the superspace [39]:

$$\mathbf{J}_s^{a_1 \cdots a_{2s}}(\theta, \mathbf{x}) = J_s^{a_1 \cdots a_{2s}}(\mathbf{x}) + \theta_m J_{s+\frac{1}{2}}^{(a_1 \cdots a_{2s_1} m)}(\mathbf{x}) - i \frac{\theta^2}{4} (\not{\theta})_m^{a_1} J_s^{a_2 \cdots a_{2s} m}(\mathbf{x}). \quad (2.1)$$

The indices $a_1 \cdots a_{2s}$ are symmetrized. J_s and $J_{s+\frac{1}{2}}$ are component currents that are conserved. The supercurrent (2.1) satisfies the following conservation equation:

$$D_{a_1} \mathbf{J}_s^{a_1 \cdots a_{2s}}(\theta, \mathbf{x}) = 0, \quad (2.2)$$

where $D_{a_1} = \frac{\partial}{\partial \theta^{a_1}} - \frac{i}{2} \theta_b (\sigma^\mu)_{a_1}^b \partial_\mu$, is the supercovariant derivative.

Performing a fourier transform with respect \mathbf{x} , we obtain the momentum superspace counterpart to (2.1):

$$\mathbf{J}_s^{a_1 \cdots a_{2s}}(\theta, \mathbf{p}) = J_s^{a_1 \cdots a_{2s}}(\mathbf{p}) + \theta_m J_{s+\frac{1}{2}}^{(a_1 \cdots a_{2s_1} m)}(\mathbf{p}) + \frac{\theta^2}{4} (\not{\theta})_m^{a_1} J_s^{a_2 \cdots a_{2s} m}(\mathbf{p}). \quad (2.3)$$

We can now construct correlation functions of these operators using the definition (2.3). The super-correlators obey the $\mathcal{N} = 1$ super ward identities. For instance, consider the supersymmetry generator Q_a . It has the following action on superfields:

$$Q_{ia} = \left(\frac{\partial}{\partial \theta_i^a} - \frac{(\not{\theta}_i)_a^b}{2} \theta_{ib} \right). \quad (2.4)$$

For an n point correlation function, invariance under the Q supersymmetry demands that,

$$\sum_{i=1}^n Q_{ia} \langle \mathbf{J}_{s_1}^{a_1 \cdots a_{2s_1}}(\theta_1, \mathbf{p}_1) \cdots \mathbf{J}_{s_n}^{c_1 \cdots c_{2s_n}}(\theta_n, \mathbf{p}_n) \rangle = 0. \quad (2.5)$$

This implies that n point correlators take the following form [35, 37]:

$$\langle \mathbf{J}_{s_1}^{a_1 \cdots a_{2s_1}}(\theta_1, \mathbf{p}_1) \cdots \mathbf{J}_{s_n}^{c_1 \cdots c_{2s_n}}(\theta_n, \mathbf{p}_n) \rangle = e^{-\frac{1}{2n}(\theta_1^a + \cdots + \theta_n^a)(\theta_{1b}(\not{\theta}_1)_{ab} + \cdots + \theta_{nb}(\not{\theta}_n)_{ab})} F^{a_1 \cdots c_{2s_n}}(\{\theta_i - \theta_j\}, \{\mathbf{p}_i\}), \quad (2.6)$$

where,

$$F^{a_1 \cdots c_{2s_n}}(\{\theta_i - \theta_j\}, \{\mathbf{p}_i\}) = F_1^{a_1 \cdots c_{2s_n}}(\{\mathbf{p}_i\}) + (\theta_{1m} - \theta_{2m}) F_2^{a_1 \cdots c_{2s_n} m}(\{\mathbf{p}_i\}) + \cdots, \quad (2.7)$$

contains undetermined functions $F_i(\{\mathbf{p}_i\})$ packaged together via a Grassmann spinor expansion. One can now impose the Ward identities due to the other superconformal generators and constrain the form of these functions. These constraints, however, take the form of coupled partial differential equations involving the $F_i(\{\mathbf{p}_i\})$ and hence quite difficult to solve. There is, however, an alternative method to proceed. We expand the superfields in the correlator using (2.3). The result is a sum of component correlators arranged in a Grassmann spinor expansion. The invariance of the individual component correlators guarantees invariance under the conformal transformations. Please see appendix C for proof. We then substitute this component correlator expansion into the l.h.s. of (2.6) and equate it to the r.h.s. order by order in the Grassmann spinor expansion. Once the resulting *algebraic* equations involving the component correlators and the $F_i(\{\mathbf{p}_i\})$ are solved, the resulting quantity is invariant under the action of the entire superconformal algebra.

2.1 Constraining super correlators in momentum super space

Let us now illustrate the methodology that we just outlined via a three-point example. Consider the spins $s_1 = \frac{3}{2}, s_2 = s_3 = \frac{1}{2}$. Using the superfield expansions provided in (2.3) we obtain,

$$\begin{aligned}
 \langle \mathbf{J}_{\frac{3}{2}}^{(efg)} \mathbf{J}_{\frac{1}{2}}^a \mathbf{J}_{\frac{1}{2}}^b \rangle = & \\
 & \theta_{1h} \langle T^{(efgh)} O_{\frac{1}{2}}^a O_{\frac{1}{2}}^b \rangle - \theta_{2c} \langle J_{\frac{3}{2}}^{(efg)} J^{(ac)} O_{\frac{1}{2}}^b \rangle + \theta_{3d} \langle J_{\frac{3}{2}}^{(efg)} O_{\frac{1}{2}}^a J^{(bd)} \rangle \\
 & + \frac{1}{4} \theta_{1h} \theta_2^2 (\not{p}_2)_c^a \langle T^{(efgh)} O_{\frac{1}{2}}^c O_{\frac{1}{2}}^b \rangle + \frac{1}{4} \theta_{1h} \theta_3^2 (\not{p}_3)_d^b \langle T^{(efgh)} O_{\frac{1}{2}}^a O_{\frac{1}{2}}^d \rangle - \frac{1}{4} \theta_{2c} \theta_1^2 \langle J_{\frac{3}{2}}^{(hfg)} J^{(ac)} O_{\frac{1}{2}}^b \rangle \\
 & - \frac{1}{4} \theta_{2c} \theta_3^2 (\not{p}_3)_d^b \langle J_{\frac{3}{2}}^{(efg)} J^{(ac)} O_{\frac{1}{2}}^d \rangle + \frac{1}{4} \theta_{3d} \theta_1^2 (\not{p}_1)_h^e \langle J_{\frac{3}{2}}^{(hfg)} O_{\frac{1}{2}}^a J^{(bd)} \rangle + \frac{1}{4} \theta_{3d} \theta_2^2 (\not{p}_2)_c^a \langle J_{\frac{3}{2}}^{(efg)} O_{\frac{1}{2}}^c J^{(bd)} \rangle \\
 & + \theta_{1h} \theta_{2c} \theta_{3d} \langle T^{(efgh)} J^{(ac)} J^{(bd)} \rangle + \frac{1}{16} \theta_1^2 \theta_2^2 \theta_{3d} (\not{p}_1)_h^e (\not{p}_2)_c^a \langle J_{\frac{3}{2}}^{(hfg)} O_{\frac{1}{2}}^c J^{(bd)} \rangle \\
 & + \frac{1}{16} \theta_1^2 \theta_3^2 \theta_{2c} (\not{p}_1)_h^e (\not{p}_3)_d^b \langle J_{\frac{3}{2}}^{(hfg)} J^{(ac)} O_{\frac{1}{2}}^d \rangle + \frac{1}{16} \theta_2^2 \theta_3^2 \theta_{1h} (\not{p}_2)_c^a (\not{p}_3)_d^b \langle T^{(efgh)} O_{\frac{1}{2}}^c O_{\frac{1}{2}}^d \rangle, \tag{2.8}
 \end{aligned}$$

while the analogue of (2.6) is given by,

$$\begin{aligned}
 \langle \mathbf{J}_{\frac{3}{2}}^{(efg)} \mathbf{J}_{\frac{1}{2}}^a \mathbf{J}_{\frac{1}{2}}^b \rangle = & e^{\frac{1}{6}(\theta_1^m + \theta_2^m + \theta_3^m)} (\theta_1 - \theta_2)^n (\not{p}_2)_{mn} + (\theta_1 - \theta_3)^n (\not{p}_3)_{mn} \left(A_1^{(efg)abl} (\theta_1 - \theta_2)_l \right. \\
 & \left. + A_2^{(efg)abl} (\theta_1 - \theta_3)_l + B_1^{(efg)abl} (\theta_1 - \theta_2)_l (\theta_1 - \theta_3)^2 + B_2^{(efg)abl} (\theta_1 - \theta_3)_l (\theta_1 - \theta_2)^2 \right). \tag{2.9}
 \end{aligned}$$

We now equate equations (2.8) and (2.9) order by order in the Grassmann spinor expansion. For instance, we obtain at the lowest order,

$$\begin{aligned}
 A_1^{(efg)abl} &= \langle J_{3/2}^{(efg)} J^{(al)} O_{1/2}^b \rangle, \\
 A_2^{(efg)abl} &= -\langle J_{3/2}^{(efg)} O_{1/2}^a J^{(bl)} \rangle, \\
 A_1^{(efg)abl} + A_2^{(efg)abl} &= \langle T^{(efgl)} O_{1/2}^a O_{1/2}^b \rangle, \tag{2.10}
 \end{aligned}$$

which forces the relation,

$$\langle T^{(efgl)} O_{1/2}^a O_{1/2}^b \rangle = \langle J_{3/2}^{(efg)} J^{(al)} O_{1/2}^b \rangle - \langle J_{3/2}^{(efg)} O_{1/2}^a J^{(bl)} \rangle. \tag{2.11}$$

One can independently check that this holds true by computing the l.h.s. and r.h.s. separately. Going to higher orders in the Grassmann expansion yields similar but albeit more complicated and not very insightful relations also involving the component $\langle TJJ \rangle$ correlator.

In order to proceed further, we then use the fact that in a generic 3d CFT, a three point (component) correlator can have at most three independent structures [48]:

$$\langle J_{s_1} J_{s_2} J_{s_3} \rangle = n_b \langle J_{s_1} J_{s_2} J_{s_3} \rangle_B + n_f \langle J_{s_1} J_{s_2} J_{s_3} \rangle_F + n_{\text{odd}} \langle J_{s_1} J_{s_2} J_{s_3} \rangle_{\text{odd}}, \tag{2.12}$$

where the first and the second term are the free bosonic and free fermionic correlators whilst the third term is a parity odd piece. Using (2.12) for each of the component correlators appearing in the superfield expansion and demanding that relations such as (2.11) are satisfied, reduces the number of independent coefficients in (2.12) for every component correlator. In fact, for the correlator in (2.8), we find $n_f = n_b$ and $n_{\text{odd}} = 0$ for all the component correlators thereby obtaining a single parity even solution:

$$\begin{aligned}
 \langle TO_{1/2} O_{1/2} \rangle &= n_b (\langle TO_{1/2} O_{1/2} \rangle_{FB} + \langle TO_{1/2} O_{1/2} \rangle_{FF}), \\
 \langle J_{3/2} J O_{1/2} \rangle &= n_b (\langle J_{3/2} J O_{1/2} \rangle_{FB} + \langle J_{3/2} J O_{1/2} \rangle_{FF}), \\
 \langle TJJ \rangle &= n_b (\langle TJJ \rangle_{FB} + \langle TJJ \rangle_{FF}), \tag{2.13}
 \end{aligned}$$

a result that is consistent with [45], that is, the existence of a single parity even solution. The final expression for the correlator is obtained by substituting (2.13) back into (2.8) or equivalently (2.9). By construction this correlator is invariant under the action of all the generators of the superconformal algebra.

While we were systematically able to solve the equations at every order in the Grassmann spinor expansion, the procedure is quite complicated. Other than dealing with various degeneracy identities,¹ there is no simple generalization to arbitrary spin correlators, no obvious connection to the flat space scattering amplitudes in four dimensions, or the existence of double copy relations between various super-correlators. Hence, we define our new variables in the next section to surpass this tedious process. We give a worked-out example of the same correlator and get the same constraints on the coefficients using our new formalism in appendix D, circumventing all the complexities of momentum super-space variables in one go.

2.2 En route to spinor helicity and Grassmann twistor variables

Rather than attempt a tour de force and try to obtain correlators the above way, we discovered a different set of variables that simplify the analysis significantly. As a first step, let us use spinor helicity variable defined as

$$p_\mu = \frac{1}{2}(\sigma^\mu)_b^a \lambda_a \bar{\lambda}^b, \tag{2.14}$$

where λ and $\bar{\lambda}$ are two component commuting spinors. The momentum p_μ is invariant under the little group transformation, $\lambda \rightarrow r\lambda$, $\bar{\lambda} \rightarrow r^{-1}\bar{\lambda}$, $r \in \mathbb{C}$. As in the case of non-supersymmetric correlators, the introduction of spinor helicity variables greatly helps us in dealing with degeneracies which leads to simplification of algebra as well as expressions. However, to truly exploit the power of spinor helicity variables, we also need to express the Grassmann spinors θ^a in the basis of λ and $\bar{\lambda}$. We define,

$$\theta^a = \frac{\bar{\eta}\lambda^a + \eta\bar{\lambda}^a}{2p}, \tag{2.15}$$

where η is a complex Grassmann variable and $\bar{\eta}$ is its complex conjugate. This definition is consistent with the fact that the dimensionality of θ is $-\frac{1}{2}$. Further, η and $\bar{\eta}$ must transform as $\eta \rightarrow r\eta$, $\bar{\eta} \rightarrow r^{-1}\bar{\eta}$ under little group scalings so that θ remains unaffected.

We then contract the superfield (2.3) with the polarization spinors which are given by,

$$\zeta_a^- = \frac{\lambda_a}{\sqrt{p}}, \quad \zeta_a^+ = \frac{\bar{\lambda}_a}{\sqrt{p}}, \tag{2.16}$$

and express the grassmann spinor θ^a in terms of the η , $\bar{\eta}$ variables defined in (2.15). We obtain the super-current in the $\pm s$ helicities,²

$$\begin{aligned} \mathbf{J}_s^- &= e^{-\frac{\eta\bar{\eta}}{4}} J_s^- + \frac{\bar{\eta}}{2\sqrt{p}} J_{s+\frac{1}{2}}^-, \\ \mathbf{J}_s^+ &= e^{\frac{\eta\bar{\eta}}{4}} J_s^+ + \frac{\eta}{2\sqrt{p}} J_{s+\frac{1}{2}}^+, \end{aligned} \tag{2.17}$$

¹An example of such an identity is $(\not{p}_1)_f^l (\not{p}_2)^{mb} = (\not{p}_1)^{bl} (\not{p}_2)_f^m - (\not{p}_1)^{kl} (\not{p}_2)_k^m \delta_f^b$.

²We also use the fact that the supercurrent (2.3) is symmetric, traceless and conserved and hence it has only two independent components which are the $h = \pm s$ ones.

where, $\mathbf{J}_s^\pm := \zeta_{a_1}^\pm \dots \zeta_{a_{2s}}^\pm \mathbf{J}_s^{a_1 \dots a_{2s}}$. The Super-correlator (2.8) in these variables becomes extremely simple. For instance in the $(- - +)$ helicity configuration we obtain,³

$$\langle \mathbf{J}_{\frac{3}{2}}^- \mathbf{J}_{\frac{1}{2}}^- \mathbf{J}_{\frac{1}{2}}^+ \rangle = \frac{1}{2\sqrt{p_3}} \langle J_{3/2}^- O_{1/2}^- T^+ \rangle \left(e^{-\frac{\eta_1 \bar{\eta}_1}{4}} e^{-\frac{\eta_2 \bar{\eta}_2}{4}} \eta_3 - \frac{1}{E} \left(e^{-\frac{\eta_1 \bar{\eta}_1}{4}} e^{\frac{\eta_3 \bar{\eta}_3}{4}} \bar{\eta}_2 \langle 23 \rangle - e^{-\frac{\eta_2 \bar{\eta}_2}{4}} e^{\frac{\eta_3 \bar{\eta}_3}{4}} \bar{\eta}_1 \langle 31 \rangle - \frac{\bar{\eta}_1 \bar{\eta}_2 \eta_3}{2} \langle 12 \rangle \right) \right). \quad (2.18)$$

Further, the exponential of the Grassmann bilinears that appear in this expression suggests us to perform a Grassmann “half” Fourier transform analogous to the twistor transform [49, 50]. Given a function $F(\eta, \bar{\eta})$ we define the Grassmann Twistor transform as follows:

$$\tilde{F}(\eta, \chi) := \int d\bar{\eta} e^{-\frac{\chi \bar{\eta}}{4}} F(\eta, \bar{\eta}). \quad (2.19)$$

We then make a variable change from (η, χ) to (ξ_+, ξ_-) which are defined as,

$$\xi_\pm = \chi \pm \eta. \quad (2.20)$$

In these new Grassmann “Twistor” Variables, the correlator takes the following beautiful form:⁴

$$\langle \tilde{\mathbf{J}}_{\frac{3}{2}}^- \tilde{\mathbf{J}}_{\frac{1}{2}}^- \tilde{\mathbf{J}}_{\frac{1}{2}}^+ \rangle = -\frac{\xi_{3-}}{256\sqrt{p_3}} \langle J_{\frac{3}{2}}^- J_{\frac{1}{2}}^- J_1^+ \rangle \left[\xi_{1+} \xi_{2+} \xi_{3+} - \frac{8}{E} (\xi_{1+} \langle 23 \rangle + \xi_{2+} \langle 31 \rangle + \xi_{3+} \langle 12 \rangle) \right]. \quad (2.21)$$

Contrasted with (2.8), the new representation (2.21) is not only simpler but also appears in a language that is homologous to the four-dimensional flat space scattering amplitudes [50]. Further, we shall see in the next section that the structure appearing inside the brackets in (2.21) is one of two universal structures that appear in three-point super-correlators with arbitrary (half) integer spin insertions! Motivated by all this simplicity, let us, from first principles, develop our formalism in these new variables.

Grassmann twistor variables

The superfield expansion in the grassmann twistor variable can be obtained by performing the transformation (2.19) on (2.17). The result is,⁵

$$\begin{aligned} \tilde{\mathbf{J}}_s^- &= \frac{1}{4} \left(\xi_+ J_s^- + \frac{2}{\sqrt{p}} J_{s+\frac{1}{2}}^- \right), \\ \tilde{\mathbf{J}}_s^+ &= \frac{\xi_-}{4} \left(J_s^+ + \frac{\xi_+}{4\sqrt{p}} J_{s+\frac{1}{2}}^+ \right). \end{aligned} \quad (2.22)$$

We then construct correlation functions of the super-currents in these new variables using the superfield expansion (2.22). The only Ward identity that we need to impose is the one due to the Q supersymmetry. In these new variables we have,

$$Q_{ia} = 2\lambda_{ia} \frac{\partial}{\partial \xi_{i+}} + \frac{\bar{\lambda}_{ia}}{4} \xi_{i+}. \quad (2.23)$$

³Similar expressions can be obtained for all other helicities.

⁴We can then proceed to obtain similar expressions in all the other helicity configurations as well.

⁵We note the similarity between (2.22) and the $\mathcal{N} = 1$ superfields in the four dimensional flat space literature, see for instance [51].

The associated Ward identity reads,

$$\sum_{i=1}^n Q_{ia} \langle \tilde{\mathbf{J}}_{s_1}^\pm \cdots \tilde{\mathbf{J}}_{s_n}^\pm \rangle = 0. \quad (2.24)$$

As we can see from (2.23), $Q_a = \sum_{i=1}^n Q_{ia}$ is a two component spinor operator. Therefore, it can be written in the following way:⁶

$$Q_a = \lambda_{1a} q + \bar{\lambda}_{1a} \bar{q}, \quad (2.25)$$

for some q, \bar{q} . It is easy to show (using the Schouten identity, see appendix A) that we have,

$$\begin{aligned} q &= 2 \frac{\partial}{\partial \xi_{1+}} + \frac{1}{p_1} \sum_{i=2}^n \left(\langle \bar{1} i \rangle \frac{\partial}{\partial \xi_{i+}} + \frac{\langle \bar{1} \bar{i} \rangle}{8} \xi_{i+} \right), \\ \bar{q} &= \frac{\xi_{1+}}{4} - \frac{1}{p_1} \sum_{i=2}^n \left(\langle 1 i \rangle \frac{\partial}{\partial \xi_{i+}} + \frac{\langle 1 \bar{i} \rangle}{8} \xi_{i+} \right). \end{aligned} \quad (2.26)$$

Thus, the Q Ward identity (2.24), splits into two simpler Ward identities viz,

$$\begin{aligned} q \langle \tilde{\mathbf{J}}_{s_1}^\pm \cdots \tilde{\mathbf{J}}_{s_n}^\pm \rangle &= 0, \\ \bar{q} \langle \tilde{\mathbf{J}}_{s_1}^\pm \cdots \tilde{\mathbf{J}}_{s_n}^\pm \rangle &= 0. \end{aligned} \quad (2.27)$$

We need not impose the Ward identities at the level of the Super-correlator for the same reason that we mentioned just below (2.7), i.e., imposing conformal invariance at the component level suffices to have conformal invariance at the level of the Super-correlator. Further, the different coefficients that appear in the component correlators (2.12) get related to each other due to the Q_a ward identities (2.27).

3 Correlation functions in $\mathcal{N} = 1$ SCFTs

In this section, we shall present our final results for two and three point correlators in the ξ_\pm variables (2.20).

3.1 Two point functions

3.1.1 $\langle \tilde{\mathbf{J}}_s \tilde{\mathbf{J}}_s \rangle, s \in \mathbb{Z}_{>0}$

We obtain the following expressions for the correlator in the $(--)$ and $(++)$ helicity configurations:

$$\begin{aligned} \langle \tilde{\mathbf{J}}_s^- \tilde{\mathbf{J}}_s^- \rangle &= (c_e + ic_o) \frac{\langle 12 \rangle^{2s}}{16p_1} \left(\xi_{1+} \xi_{2+} - \frac{4 \langle 12 \rangle}{p_1} \right), \\ \langle \tilde{\mathbf{J}}_s^+ \tilde{\mathbf{J}}_s^+ \rangle &= -(c_e - ic_o) \xi_{1-} \xi_{2-} - \frac{\langle \bar{1} \bar{2} \rangle^{2s+1}}{256p_1^2} \left(\xi_{1+} \xi_{2+} - \frac{4 \langle 12 \rangle}{p_1} \right), \end{aligned} \quad (3.1)$$

where c_e and c_o are the coefficients of the parity even and parity odd contributions respectively.

⁶We choose to work in the $(\lambda_{1a}, \bar{\lambda}_{1a})$ basis. One can also choose to work in other basis but the results that we obtain will be basis independent.

3.1.2 $\langle \tilde{\mathbf{J}}_s \tilde{\mathbf{J}}_s \rangle$, $s = k + \frac{1}{2}$, $k \in \mathbb{Z}_{\geq 0}$

The expressions of the correlator in the $(--)$ and $(++)$ helicity configurations are given by,

$$\begin{aligned}\langle \tilde{\mathbf{J}}_s^- \tilde{\mathbf{J}}_s^- \rangle &= -(c_e + ic_o) \frac{\langle 12 \rangle^{2s}}{16p_1} \left(\xi_{1+\xi_{2+}} - \frac{4\langle 12 \rangle}{p_1} \right), \\ \langle \tilde{\mathbf{J}}_s^+ \tilde{\mathbf{J}}_s^+ \rangle &= (c_e - ic_o) \xi_{1-\xi_{2-}} \frac{\langle \bar{1} \bar{2} \rangle^{2s+1}}{256p_1^2} \left(\xi_{1+\xi_{2+}} - \frac{4\langle 12 \rangle}{p_1} \right),\end{aligned}\quad (3.2)$$

where again, c_e and c_o are the coefficients of the parity even and parity odd two point function respectively.

3.1.3 Summary

Based on our results, (3.1) and (3.2), we see that general two point functions of any (half) integer spin s conserved super-currents take the following form:

$$\langle \tilde{\mathbf{J}}_s^- \tilde{\mathbf{J}}_s^- \rangle = (c_e + ic_o) (-1)^{2s} \frac{\langle 12 \rangle^{2s}}{16p_1} \Xi_2, \quad \langle \tilde{\mathbf{J}}_s^+ \tilde{\mathbf{J}}_s^+ \rangle = (c_e - ic_o) \xi_{1-\xi_{2-}} (-1)^{2s+1} \frac{\langle \bar{1} \bar{2} \rangle^{2s+1}}{256p_1^2} \Xi_2, \quad (3.3)$$

where we have defined the two point building block,

$$\Xi_2 = \left(\xi_{1+\xi_{2+}} - \frac{4\langle 12 \rangle}{p_1} \right). \quad (3.4)$$

Thus we see that for any spin s , the two point correlators are given by a simple kinematic factor times a universal factor Ξ_2 . We will find a similar structure at the level of three points as well.

3.2 Three point correlation functions

In this subsection, we present our results for three point correlators with arbitrary (half) integer spin insertions. In contrast to the previous analysis in position space [39, 45] etc. . . , we find a universal form for any super-correlator independent of the spins of the insertions.

3.2.1 $\langle \tilde{\mathbf{J}}_{s_1} \tilde{\mathbf{J}}_{s_2} \tilde{\mathbf{J}}_{s_3} \rangle$, $s_1, s_2, s_3 \in \mathbb{Z}_{>0}$

The correlator of three integer spin conserved super-currents is given by the following expressions in the various helicity configurations:

$$\begin{aligned}\langle \tilde{\mathbf{J}}_{s_1}^- \tilde{\mathbf{J}}_{s_2}^- \tilde{\mathbf{J}}_{s_3}^- \rangle &= \frac{1}{64} \langle J_{s_1}^- J_{s_2}^- J_{s_3}^- \rangle \Gamma_3, & \langle \tilde{\mathbf{J}}_{s_1}^+ \tilde{\mathbf{J}}_{s_2}^+ \tilde{\mathbf{J}}_{s_3}^+ \rangle &= \frac{\xi_{1-\xi_{2-}\xi_{3-}}}{512} \langle J_{s_1}^+ J_{s_2}^+ J_{s_3}^+ \rangle \Xi_3, \\ \langle \tilde{\mathbf{J}}_{s_1}^+ \tilde{\mathbf{J}}_{s_2}^+ \tilde{\mathbf{J}}_{s_3}^- \rangle &= -\frac{\xi_{1-\xi_{2-}}}{512} \frac{\langle J_{s_1}^+ J_{s_2}^+ J_{s_3}^- \rangle E}{\langle 12 \rangle} \Gamma_3, & \langle \tilde{\mathbf{J}}_{s_1}^- \tilde{\mathbf{J}}_{s_2}^- \tilde{\mathbf{J}}_{s_3}^+ \rangle &= -\frac{\xi_{3-}}{64} \frac{\langle J_{s_1}^- J_{s_2}^- J_{s_3}^+ \rangle E}{\langle \bar{1} \bar{2} \rangle} \Xi_3, \\ \langle \tilde{\mathbf{J}}_{s_1}^+ \tilde{\mathbf{J}}_{s_2}^- \tilde{\mathbf{J}}_{s_3}^+ \rangle &= -\frac{\xi_{1-\xi_{3-}}}{512} \frac{\langle J_{s_1}^+ J_{s_2}^- J_{s_3}^+ \rangle E}{\langle 13 \rangle} \Gamma_3, & \langle \tilde{\mathbf{J}}_{s_1}^- \tilde{\mathbf{J}}_{s_2}^+ \tilde{\mathbf{J}}_{s_3}^- \rangle &= -\frac{\xi_{2-}}{64} \frac{\langle J_{s_1}^- J_{s_2}^+ J_{s_3}^- \rangle E}{\langle \bar{3} \bar{1} \rangle} \Xi_3, \\ \langle \tilde{\mathbf{J}}_{s_1}^- \tilde{\mathbf{J}}_{s_2}^+ \tilde{\mathbf{J}}_{s_3}^+ \rangle &= -\frac{\xi_{2-\xi_{3-}}}{512} \frac{\langle J_{s_1}^- J_{s_2}^+ J_{s_3}^+ \rangle E}{\langle 23 \rangle} \Gamma_3, & \langle \tilde{\mathbf{J}}_{s_1}^+ \tilde{\mathbf{J}}_{s_2}^- \tilde{\mathbf{J}}_{s_3}^- \rangle &= -\frac{\xi_{1-}}{64} \frac{\langle J_{s_1}^+ J_{s_2}^- J_{s_3}^- \rangle E}{\langle \bar{2} \bar{3} \rangle} \Xi_3,\end{aligned}\quad (3.5)$$

where Γ_3 and Ξ_3 are given by,

$$\begin{aligned}\Gamma_3 &= \left[\xi_{1+}\xi_{2+}\xi_{3+} - \frac{8}{E}(\xi_{1+}\langle 23 \rangle + \xi_{2+}\langle 31 \rangle + \xi_{3+}\langle 12 \rangle) \right], \\ \Xi_3 &= \left[8 - \frac{1}{E}(\xi_{1+}\xi_{2+}\langle \bar{1} \bar{2} \rangle + \xi_{2+}\xi_{3+}\langle \bar{2} \bar{3} \rangle + \xi_{3+}\xi_{1+}\langle \bar{3} \bar{1} \rangle) \right],\end{aligned}\quad (3.6)$$

which we note is similar to the two point case (3.4) where super-correlators are constructed by multiplying a component correlator and a universal spin independent factor, except that in the three point case, we have two such universal factors.⁷ Further, we find that for super-correlators that obey the triangle inequality, the only non zero components are the $(- - -)$ and $(+ + +)$ helicity configurations, i.e, the Super-correlator is homogeneous and has both a parity even and a parity odd solution. In fact we have the universal formulae [31].

$$\begin{aligned}\langle J_{s_1}^- J_{s_2}^- J_{s_3}^- \rangle &= (c_{\text{even}} + ic_{\text{odd}}) \frac{\langle 12 \rangle^{s_1+s_2-s_3} \langle 23 \rangle^{s_2+s_3-s_1} \langle 31 \rangle^{s_1+s_3-s_2}}{E^{s_1+s_2+s_3}} p_1^{s_1-1} p_2^{s_2-1} p_3^{s_3-1}, \\ \langle J_{s_1}^+ J_{s_2}^+ J_{s_3}^+ \rangle &= (c_{\text{even}} - ic_{\text{odd}}) \frac{\langle \bar{1} \bar{2} \rangle^{s_1+s_2-s_3} \langle \bar{2} \bar{3} \rangle^{s_2+s_3-s_1} \langle \bar{3} \bar{1} \rangle^{s_1+s_3-s_2}}{E^{s_1+s_2+s_3}} p_1^{s_1-1} p_2^{s_2-1} p_3^{s_3-1},\end{aligned}\quad (3.7)$$

where c_{even} and c_{odd} are the OPE coefficients corresponding to the parity even and parity odd structures respectively. The relations between the various component correlators as in (3.5) relate the various component correlators to each other, thereby fixing their OPE coefficients just in terms of c_{even} and c_{odd} .

For super-correlators that violate the triangle inequality, there is no parity odd solution as the component correlator on the r.h.s. of (3.5) has no parity odd piece in this case. There exists however, an even piece. In terms of the free bosonic and free fermionic correlators it is given by,

$$\langle J_{s_1}^\pm J_{s_2}^\pm J_{s_3}^\pm \rangle = n_b \left(\langle J_{s_1}^\pm J_{s_2}^\pm J_{s_3}^\pm \rangle_{FB} - \langle J_{s_1}^\pm J_{s_2}^\pm J_{s_3}^\pm \rangle_{FF} \right).\quad (3.8)$$

Substituting (3.8) in (3.5) and reading off the component correlators using (2.22), the other component correlators can be obtained.

3.2.2 $\langle \tilde{J}_{s_1} \tilde{J}_{s_2} \tilde{J}_{s_3} \rangle$, $s_1, s_2 \in \mathbb{Z}_{>0}$, $s_3 = k_3 + \frac{1}{2}$, $k_3 \in \mathbb{Z}_{\geq 0}$

We obtained the following expressions for the various helicity configurations for this half integer spin correlator:

$$\begin{aligned}\langle \tilde{J}_{s_1}^- \tilde{J}_{s_2}^- \tilde{J}_{s_3}^- \rangle &= -\frac{1}{32} \frac{\langle J_{s_1}^- J_{s_2}^- J_{k_3+1}^- \rangle E}{\langle \bar{1} \bar{2} \rangle \sqrt{p_3}} \Xi_3, & \langle \tilde{J}_{s_1}^+ \tilde{J}_{s_2}^+ \tilde{J}_{s_3}^+ \rangle &= -\frac{\xi_{1-}\xi_{2-}\xi_{3-}}{2048} \frac{\langle J_{s_1}^+ J_{s_2}^+ J_{k_3+1}^+ \rangle E}{\langle 12 \rangle \sqrt{p_3}} \Gamma_3, \\ \langle \tilde{J}_{s_1}^+ \tilde{J}_{s_2}^+ \tilde{J}_{s_3}^- \rangle &= \frac{\xi_{1-}\xi_{2-}}{256} \frac{\langle J_{s_1}^+ J_{s_2}^+ J_{k_3+1}^- \rangle}{\sqrt{p_3}} \Xi_3, & \langle \tilde{J}_{s_1}^- \tilde{J}_{s_2}^- \tilde{J}_{s_3}^+ \rangle &= \frac{\xi_{3-}}{256} \frac{\langle J_{s_1}^- J_{s_2}^- J_{k_3+1}^+ \rangle}{\sqrt{p_3}} \Gamma_3, \\ \langle \tilde{J}_{s_1}^+ \tilde{J}_{s_2}^- \tilde{J}_{s_3}^+ \rangle &= \frac{\xi_{1-}\xi_{3-}}{256} \frac{\langle J_{s_1}^+ J_{s_2}^- J_{k_3+1}^+ \rangle E}{\langle \bar{2} \bar{3} \rangle \sqrt{p_3}} \Xi_3, & \langle \tilde{J}_{s_1}^- \tilde{J}_{s_2}^+ \tilde{J}_{s_3}^- \rangle &= \frac{\xi_{2-}}{256} \frac{\langle J_{s_1}^- J_{s_2}^+ J_{k_3+1}^- \rangle E}{\langle 23 \rangle \sqrt{p_3}} \Gamma_3, \\ \langle \tilde{J}_{s_1}^- \tilde{J}_{s_2}^+ \tilde{J}_{s_3}^+ \rangle &= \frac{\xi_{2-}\xi_{3-}}{256} \frac{\langle J_{s_1}^- J_{s_2}^+ J_{k_3+1}^+ \rangle E}{\langle \bar{3} \bar{1} \rangle \sqrt{p_3}} \Xi_3, & \langle \tilde{J}_{s_1}^+ \tilde{J}_{s_2}^- \tilde{J}_{s_3}^- \rangle &= -\frac{\xi_{1-}}{256} \frac{\langle J_{s_1}^+ J_{s_2}^- J_{k_3+1}^- \rangle E}{\langle 31 \rangle \sqrt{p_3}} \Gamma_3.\end{aligned}\quad (3.9)$$

In contrast to the previous case with all integer spins, this class of correlators is non-homogeneous, that is, the component correlators appear as a sum of the bosonic and fermionic correlators $\langle \rangle_B + \langle \rangle_F$. The parity odd structure does not exist for these correlators.

⁷We note that Γ_3 is antisymmetric under any exchange of $(1, 2, 3)$ whilst Ξ_3 is symmetric under any such exchange.

3.2.3 $\langle \tilde{\mathbf{J}}_{s_1} \tilde{\mathbf{J}}_{s_2} \tilde{\mathbf{J}}_{s_3} \rangle$, $s_1 \in \mathbb{Z}_{>0}$, $s_i = k_i + \frac{1}{2}$, $i = 2, 3$ and $k_i \in \mathbb{Z}_{\geq 0}$

This integer spin super-correlator has the following components:

$$\begin{aligned}
 \langle \tilde{\mathbf{J}}_{s_1}^- \tilde{\mathbf{J}}_{s_2}^- \tilde{\mathbf{J}}_{s_3}^- \rangle &= \frac{1}{64} \langle J_{s_1}^- J_{k_2+\frac{1}{2}}^- J_{k_3+\frac{1}{2}}^- \rangle \Gamma_3, & \langle \tilde{\mathbf{J}}_{s_1}^+ \tilde{\mathbf{J}}_{s_2}^+ \tilde{\mathbf{J}}_{s_3}^+ \rangle &= -\frac{\xi_{1-}\xi_{2-}\xi_{3-}}{512} \langle J_{s_1}^+ J_{k_2+\frac{1}{2}}^+ J_{k_3+\frac{1}{2}}^+ \rangle \Xi_3, \\
 \langle \tilde{\mathbf{J}}_{s_1}^+ \tilde{\mathbf{J}}_{s_2}^+ \tilde{\mathbf{J}}_{s_3}^- \rangle &= \frac{\xi_{1-}\xi_{2-}}{512} \frac{\langle J_{s_1}^+ J_{k_2+\frac{1}{2}}^+ J_{k_3+\frac{1}{2}}^- \rangle E}{\langle 12 \rangle} \Gamma_3, & \langle \tilde{\mathbf{J}}_{s_1}^- \tilde{\mathbf{J}}_{s_2}^- \tilde{\mathbf{J}}_{s_3}^+ \rangle &= \frac{\xi_{3-}}{64} \frac{\langle J_{s_1}^- J_{k_2+\frac{1}{2}}^- J_{k_3+\frac{1}{2}}^+ \rangle E}{\langle \bar{1}\bar{2} \rangle} \Xi_3, \\
 \langle \tilde{\mathbf{J}}_{s_1}^+ \tilde{\mathbf{J}}_{s_2}^- \tilde{\mathbf{J}}_{s_3}^+ \rangle &= \frac{\xi_{1-}\xi_{3-}}{512} \frac{\langle J_{s_1}^+ J_{k_2+\frac{1}{2}}^- J_{k_3+\frac{1}{2}}^+ \rangle E}{\langle 13 \rangle} \Gamma_3, & \langle \tilde{\mathbf{J}}_{s_1}^- \tilde{\mathbf{J}}_{s_2}^+ \tilde{\mathbf{J}}_{s_3}^- \rangle &= \frac{\xi_{2-}}{64} \frac{\langle J_{s_1}^- J_{k_2+\frac{1}{2}}^+ J_{k_3+\frac{1}{2}}^- \rangle E}{\langle \bar{3}\bar{1} \rangle} \Xi_3, \\
 \langle \tilde{\mathbf{J}}_{s_1}^- \tilde{\mathbf{J}}_{s_2}^+ \tilde{\mathbf{J}}_{s_3}^+ \rangle &= \frac{\xi_{2-}\xi_{3-}}{512} \frac{\langle J_{s_1}^- J_{k_2+\frac{1}{2}}^+ J_{k_3+\frac{1}{2}}^+ \rangle E}{\langle 23 \rangle} \Gamma_3, & \langle \tilde{\mathbf{J}}_{s_1}^+ \tilde{\mathbf{J}}_{s_2}^- \tilde{\mathbf{J}}_{s_3}^- \rangle &= \frac{\xi_{1-}}{64} \frac{\langle J_{s_1}^+ J_{k_2+\frac{1}{2}}^- J_{k_3+\frac{1}{2}}^- \rangle E}{\langle \bar{2}\bar{3} \rangle} \Xi_3.
 \end{aligned} \tag{3.10}$$

Similar to the all integer spin case (3.5), this correlator also has components that appear as a difference between the bosonic and fermionic structures. In addition, there also exists a parity odd structure when the triangle inequality is satisfied.

3.2.4 $\langle \tilde{\mathbf{J}}_{s_1} \tilde{\mathbf{J}}_{s_2} \tilde{\mathbf{J}}_{s_3} \rangle$, $s_i = k_i + \frac{1}{2}$, $i = 1, 2, 3$ and $k_i \in \mathbb{Z}_{\geq 0}$

This correlator involving three half integer insertions has the following expressions in the various helicities:

$$\begin{aligned}
 \langle \tilde{\mathbf{J}}_{s_1}^- \tilde{\mathbf{J}}_{s_2}^- \tilde{\mathbf{J}}_{s_3}^- \rangle &= \frac{1}{32} \frac{\langle J_{k_1+\frac{1}{2}}^- J_{k_2+\frac{1}{2}}^- J_{k_3+1}^- \rangle E}{\langle \bar{1}\bar{2} \rangle \sqrt{p_3}} \Xi_3, & \langle \tilde{\mathbf{J}}_{s_1}^+ \tilde{\mathbf{J}}_{s_2}^+ \tilde{\mathbf{J}}_{s_3}^+ \rangle &= \frac{\xi_{1-}\xi_{2-}\xi_{3-}}{2048} \frac{\langle J_{k_1+\frac{1}{2}}^+ J_{k_2+\frac{1}{2}}^+ J_{k_3+1}^+ \rangle E}{\langle 12 \rangle \sqrt{p_3}} \Gamma_3, \\
 \langle \tilde{\mathbf{J}}_{s_1}^+ \tilde{\mathbf{J}}_{s_2}^+ \tilde{\mathbf{J}}_{s_3}^- \rangle &= -\frac{\xi_{1-}\xi_{2-}}{256} \frac{\langle J_{k_1+\frac{1}{2}}^+ J_{k_2+\frac{1}{2}}^+ J_{k_3+1}^- \rangle E}{\sqrt{p_3}} \Xi_3, & \langle \tilde{\mathbf{J}}_{s_1}^- \tilde{\mathbf{J}}_{s_2}^- \tilde{\mathbf{J}}_{s_3}^+ \rangle &= -\frac{\xi_{3-}}{256} \frac{\langle J_{k_1+\frac{1}{2}}^- J_{k_2+\frac{1}{2}}^- J_{k_3+1}^+ \rangle E}{\sqrt{p_3}} \Gamma_3, \\
 \langle \tilde{\mathbf{J}}_{s_1}^+ \tilde{\mathbf{J}}_{s_2}^- \tilde{\mathbf{J}}_{s_3}^+ \rangle &= -\frac{\xi_{1-}\xi_{3-}}{256} \frac{\langle J_{k_1+\frac{1}{2}}^+ J_{k_2+\frac{1}{2}}^- J_{k_3+1}^+ \rangle E}{\langle \bar{2}\bar{3} \rangle \sqrt{p_3}} \Xi_3, & \langle \tilde{\mathbf{J}}_{s_1}^- \tilde{\mathbf{J}}_{s_2}^+ \tilde{\mathbf{J}}_{s_3}^- \rangle &= -\frac{\xi_{2-}}{256} \frac{\langle J_{k_1+\frac{1}{2}}^- J_{k_2+\frac{1}{2}}^+ J_{k_3+1}^- \rangle E}{\langle 23 \rangle \sqrt{p_3}} \Gamma_3, \\
 \langle \tilde{\mathbf{J}}_{s_1}^- \tilde{\mathbf{J}}_{s_2}^+ \tilde{\mathbf{J}}_{s_3}^+ \rangle &= -\frac{\xi_{2-}\xi_{3-}}{256} \frac{\langle J_{k_1+\frac{1}{2}}^- J_{k_2+\frac{1}{2}}^+ J_{k_3+1}^+ \rangle E}{\langle \bar{3}\bar{1} \rangle \sqrt{p_3}} \Xi_3, & \langle \tilde{\mathbf{J}}_{s_1}^+ \tilde{\mathbf{J}}_{s_2}^- \tilde{\mathbf{J}}_{s_3}^- \rangle &= \frac{\xi_{1-}}{256} \frac{\langle J_{k_1+\frac{1}{2}}^+ J_{k_2+\frac{1}{2}}^- J_{k_3+1}^- \rangle E}{\langle 31 \rangle \sqrt{p_3}} \Gamma_3.
 \end{aligned} \tag{3.11}$$

This class of correlators are all non homogeneous, that is, their components appear as a sum of bosonic and fermionic structures. We provide a detailed example of one such correlator in appendix D.

3.3 Double-copy relations

Double-copy relations have played an important role in (supersymmetric) scattering amplitudes in the recent decades [52]. Such structures in the non-supersymmetric case were later discovered in conformal field theory [5–7]. It is thus natural to attempt to extend such results to the supersymmetric case as well. Indeed, in the ξ_{\pm} variables that we have been using in this paper, such a structure becomes apparent. Let us illustrate this with an example: consider a three point super-correlator with $s_1 = s_2 = s_3 = 2$, that is, the $\langle \tilde{\mathbf{T}}\tilde{\mathbf{T}}\tilde{\mathbf{T}} \rangle$ correlator. As we discussed below equation (3.5), this correlator is purely homogeneous as it

satisfies the triangle inequality. Therefore, it suffices to consider its $(- - -)$ and $(+ + +)$ components. The $(- - -)$ component correlator is,

$$\langle \tilde{\mathbf{T}}^- \tilde{\mathbf{T}}^- \tilde{\mathbf{T}}^- \rangle = \frac{1}{64} \langle T^- T^- T^- \rangle \Gamma_3. \quad (3.12)$$

Consider now,

$$\langle \tilde{\mathbf{J}}_4^- \tilde{\mathbf{J}}_4^- \tilde{\mathbf{J}}_4^- \rangle = \frac{1}{64} \langle J_4^- J_4^- J_4^- \rangle \Gamma_3. \quad (3.13)$$

If we naively square (3.12) expecting to obtain (3.13), we will obtain zero as $\Gamma_3^2 = 0$ (evident from the definition (3.6)). We note, however, that the relation at the level of *component* correlators,

$$\langle J_4^- J_4^- J_4^- \rangle \propto p_1 p_2 p_3 \langle T^- T^- T^- \rangle^2, \quad (3.14)$$

implies at the superfield level (3.13) that,

$$\langle \tilde{\mathbf{J}}_4^- \tilde{\mathbf{J}}_4^- \tilde{\mathbf{J}}_4^- \rangle \propto p_1 p_2 p_3 \langle T^- T^- T^- \rangle^2 \Gamma_3. \quad (3.15)$$

Equations (3.13) and (3.15) implies a double copy relation between $\langle \tilde{\mathbf{T}}^- \tilde{\mathbf{T}}^- \tilde{\mathbf{T}}^- \rangle$ and $\langle \tilde{\mathbf{J}}_4^- \tilde{\mathbf{J}}_4^- \tilde{\mathbf{J}}_4^- \rangle$. This double copy relation can be extended for arbitrary spin as follows

$$\langle \tilde{\mathbf{J}}_{s_1}^- \tilde{\mathbf{J}}_{s_2}^- \tilde{\mathbf{J}}_{s_3}^- \rangle \propto p_1 p_2 p_3 \langle J_{s_1}^- J_{s_2}^- J_{s_3}^- \rangle \langle J_{s_1}^- J_{s_2}^- J_{s_3}^- \rangle \Gamma_3, \quad (3.16)$$

$$\langle \tilde{\mathbf{J}}_{s_1}^+ \tilde{\mathbf{J}}_{s_2}^+ \tilde{\mathbf{J}}_{s_3}^+ \rangle \propto p_1 p_2 p_3 \langle J_{s_1}^+ J_{s_2}^+ J_{s_3}^+ \rangle \langle J_{s_1}^+ J_{s_2}^+ J_{s_3}^+ \rangle \Xi_3, \quad (3.17)$$

where $s'_1 + s''_1 = s_1$, $s'_2 + s''_2 = s_2$, and $s'_3 + s''_3 = s_3$. Let us also note the spins $s_1, s_2, s_3 \in \mathbb{Z}_{>0}$ and obeys triangle inequality. We shall later see in subsection 4.5, an interesting double copy relation between $\mathcal{N} = 1$ and $\mathcal{N} = 2$ super correlators.

3.4 The super Ward-Takahashi identity

In this subsection we present the super Ward-Takahashi identity for our non homogeneous correlators. We also choose to work with θ variables as they suffice for our purposes. The main aim of this subsection is to show that their is a stark distinction between the Grassmann even and Grassmann odd cases.

Consider a Grassmann odd correlator $\langle \mathbf{J}_{\frac{3}{2}} \mathbf{J}_0 \mathbf{J}_0 \rangle$. The corresponding super Ward-Takahashi identity that we obtain is given by:

$$D_{1a} \langle \mathbf{J}_{\frac{3}{2}}^{abc} \mathbf{J}_0 \mathbf{J}_0 \rangle = a_1 \theta_1^2 \left(\not{p}_2^{bc} \langle \mathbf{J}_0(p_2) \mathbf{J}_0(-p_2) \rangle + \not{p}_3^{bc} \langle \mathbf{J}_0(-p_3) \mathbf{J}_0(p_3) \rangle \right). \quad (3.18)$$

If we consider a Grassmann even correlator such as $\langle \mathbf{T} \mathbf{J}_0 \mathbf{J}_0 \rangle$, we find the following super Ward-Takahashi identity:

$$D_{1a} \langle \mathbf{T}^{abcd} \mathbf{J}_0 \mathbf{J}_0 \rangle = a_2 \theta_1^2 \left(\not{p}_2^{bc} D_2^d \langle \mathbf{J}_0(p_2) \mathbf{J}_0(-p_2) \rangle + \not{p}_3^{bc} D_3^d \langle \mathbf{J}_0(-p_3) \mathbf{J}_0(p_3) \rangle \right), \quad (3.19)$$

where a_1 and a_2 are proportional to the normalization of the $\langle \mathbf{J}_0 \mathbf{J}_0 \rangle$ two point super-correlator and the action of covariant derivative is given below equation (2.2).

Total spin	Homogeneous Even	Homogeneous Odd	Non Homogeneous
$s_1 + s_2 + s_3 \in \mathbb{Z}_{>0}$	✓	✓	-
$s_1 + s_2 + s_3 \in \mathbb{Z}_{>0} + \frac{1}{2}$	-	-	✓

Table 1. Correlation functions obeying the triangle inequality $s_i + s_j \geq s_k \forall i, j, k$.

Total spin	Boson-Fermion	Boson+Fermion	Parity Odd
$s_1 + s_2 + s_3 \in \mathbb{Z}_{>0}$	✓	-	-
$s_1 + s_2 + s_3 \in \mathbb{Z}_{>0} + \frac{1}{2}$	-	✓	-

Table 2. Correlation functions that violate the triangle inequality $s_i + s_j \leq s_k$ for some i, j, k .

3.5 Summary of super correlators: homogeneous and non homogeneous sectors

In this subsection, we summarize our findings of the structures of the super correlators in the language of homogeneity. For correlators that obey the triangle inequality, super correlators with total integer spin are homogeneous⁸ while those with total half-integer spin are non homogeneous.⁹

For correlators that are not inside the triangle, we have the following results: for integer spin correlators, the individual component correlators appear as differences between the bosonic and fermionic results: $\langle \rangle_B - \langle \rangle_F$. In contrast, for half-integer correlators, they appear as a sum $\langle \rangle_B + \langle \rangle_F$. Further, there is no parity odd structure compatible with the supercurrent conservation in either case.

These results are also consistent with the free theory result. The superfield (2.1) for the free theory can be written as follows: for integer spins s , and half integer spins $k + \frac{1}{2}$ we have respectively¹⁰ (B stands for the free bosonic theory result and F stands for the free fermionic theory currents),

$$\begin{aligned} \mathbf{J}_s &= ((J_s)_B - (J_s)_F) + \theta \cdot J_{s+\frac{1}{2}} + \partial((J_s)_B - (J_s)_F), \\ \mathbf{J}_{k+\frac{1}{2}} &= J_{k+\frac{1}{2}} + \theta \cdot ((J_k)_B + (J_k)_F) + \partial J_{k+\frac{1}{2}}. \end{aligned} \tag{3.20}$$

For instance a correlator of three integer spin super currents at the zeroth order (denoted by an $|$) is given by,

$$\langle \mathbf{J}_{s_1} \mathbf{J}_{s_2} \mathbf{J}_{s_3} \rangle | = \langle J_{s_1} J_{s_2} J_{s_3} \rangle_B - \langle J_{s_1} J_{s_2} J_{s_3} \rangle_F, \tag{3.21}$$

which is the definition of the homogeneous piece. similarly, at all orders in the Grassmann spinor expansion we obtain homogeneous component correlators and hence this is consistent with table 1.

Similarly, if we consider three half integer spin supercurrents that obey the triangle inequality, we obtain at say $\mathcal{O}(\theta_1 \theta_2 \theta_3)$ the following expression:

$$\langle \mathbf{J}_{k_1+\frac{1}{2}} \mathbf{J}_{k_2+\frac{1}{2}} \mathbf{J}_{k_3+\frac{1}{2}} \rangle |_{\theta_1 \theta_2 \theta_3} = \theta_1 \theta_2 \theta_3 (\langle J_{k_1+1} J_{k_2+1} J_{k_3+1} \rangle_B + \langle J_{k_1+1} J_{k_2+1} J_{k_3+1} \rangle_F), \tag{3.22}$$

⁸Note that the homogeneous correlator contains both a parity even and parity odd contribution like in (3.7).

⁹In the component language, super correlators that are (non) homogeneous also have component correlators that are also (non) homogeneous.

¹⁰This can easily be seen from the expressions given for the free theory super currents in [39].

which is the definition of the non homogeneous piece and is consistent with¹¹ table 2. Repeating the same analysis for the other cases (two integer, one half integer or one integer, two half integer insertions inside and outside the triangle) we find results consistent with tables 1 and 2. As for the parity odd solution, which is necessarily homogeneous [31], one expects to find one whenever there exists a parity even homogeneous solution as there is little distinction between the two in spinor helicity variables. So far, we have focussed our attention on superfields with non-zero spin. We shall now switch gears and proceed to investigate correlators that also involve scalar superfields.

3.6 Correlators involving scalar superfields

In this subsection, we shall present our results for correlation functions involving scalar superfields. Since the procedure to obtain them is identical to the spinning case, we present just the final results obtained after solving the supersymmetric Ward identities (2.27) and imposing the conformal invariance of each component correlator. The component expansion for the scalar superfield is given by [39]:

$$\mathbf{J}_0 = O_1 + \theta_a O_{\frac{1}{2}}^a + \theta^2 O_2. \tag{3.23}$$

Using (2.15) and performing the Grassmann twistor transform (2.19), we see that it can be written as follows in the η, χ variables:

$$\mathbf{J}_0 = \frac{1}{4} \left(\chi O_1 + \frac{2}{\sqrt{p}} O_{\frac{1}{2}}^- + \frac{1}{2\sqrt{p}} \chi \eta O_{\frac{1}{2}}^+ - 4 \frac{\eta}{p} O_2 \right). \tag{3.24}$$

Changing variables as in (2.20) results in the following expression:

$$\tilde{\mathbf{J}}_0 = \frac{\xi_+ + \xi_-}{8} O_1 + \frac{1}{2\sqrt{p}} O_{\frac{1}{2}}^- + \frac{\xi_- \xi_+}{16\sqrt{p}} O_{\frac{1}{2}}^+ - \frac{\xi_+ - \xi_-}{2p} O_2. \tag{3.25}$$

Let us now begin with the two point function of the scalar superfield.

Two point functions

After solving for the two point function, we see that it can be expressed using the two point building block (3.4) we obtained earlier.

$$\langle \tilde{\mathbf{J}}_0 \tilde{\mathbf{J}}_0 \rangle = \frac{1}{64p_1} \left(2(\alpha_{11} - 4\alpha_{21}) - (\alpha_{11} + 4\alpha_{21}) \xi_{1-} \xi_{2-} - \frac{\langle \bar{1}\bar{2} \rangle}{8p_1} \right) \Xi_2. \tag{3.26}$$

Where $\langle O_1 O_1 \rangle = \frac{\alpha_{11}}{p_1}$ and $\langle O_2 O_1 \rangle = \alpha_{21}$. Let us now proceed to the case of three points.

Three point functions

$$\langle \tilde{\mathbf{J}}_0 \tilde{\mathbf{J}}_0 \tilde{\mathbf{J}}_0 \rangle$$

The three point function of identical scalar superfields after solving the supersymmetric Ward identities (2.27) and imposing conformal invariance of the component correlators can be expressed using the usual three point building blocks (3.6):

$$\langle \tilde{\mathbf{J}}_0 \tilde{\mathbf{J}}_0 \tilde{\mathbf{J}}_0 \rangle = \frac{1}{128p_1 p_2 p_3} \left((f_{111} - 4f_{112}) \Gamma_3 + \frac{1}{8} (f_{111} + 4f_{112}) \xi_{1-} \xi_{2-} \xi_{3-} \Xi_3 \right), \tag{3.27}$$

¹¹Similarly, all component correlators in the grassmann spinor expansion are non-homogeneous.

where,

$$\langle O_1 O_1 O_1 \rangle = \frac{f_{111}}{p_1 p_2 p_3}, \quad \langle O_1 O_1 O_2 \rangle = \frac{f_{112}}{p_1 p_2}. \quad (3.28)$$

In the free theory case $f_{112} = 0$ and hence we have,¹²

$$\langle \tilde{\mathbf{J}}_0 \tilde{\mathbf{J}}_0 \tilde{\mathbf{J}}_0 \rangle_{\text{free theory}} = \frac{f_{111}}{128 p_1 p_2 p_3} \left(\Gamma_3 + \frac{1}{8} \xi_{1-} \xi_{2-} \xi_{3-} \Xi_3 \right). \quad (3.29)$$

$$\langle \tilde{\mathbf{J}}_s \tilde{\mathbf{J}}_0 \tilde{\mathbf{J}}_0 \rangle, \quad s = 2\mathbf{k}, \mathbf{k} \in \mathbb{Z}_{>0}$$

We now consider a three point function with one conserved integer spin super-current (2.22) and two scalar super-fields (3.25). We obtained the following expressions for the two helicity configurations:

$$\begin{aligned} \langle \tilde{\mathbf{J}}_s^- \tilde{\mathbf{J}}_0 \tilde{\mathbf{J}}_0 \rangle &= \frac{\langle J_s^- O_1 O_1 \rangle}{1024} \left(8 - \frac{\langle \bar{2}3 \rangle}{E - 2p_1} \xi_{2-} \xi_{3-} \right) \Gamma_3, \\ \langle \tilde{\mathbf{J}}_s^+ \tilde{\mathbf{J}}_0 \tilde{\mathbf{J}}_0 \rangle &= -\frac{\langle J_s^+ O_1 O_1 \rangle}{1024} \left(\frac{8 \langle 23 \rangle}{E - 2p_1} - \xi_{2-} \xi_{3-} \right) \xi_{1-} \Xi_3. \end{aligned} \quad (3.30)$$

$$\langle \tilde{\mathbf{J}}_s \tilde{\mathbf{J}}_s \tilde{\mathbf{J}}_0 \rangle, \quad s \in \mathbb{Z}_{>0}$$

Consider a three point function with two conserved superfields with integer spins (2.22) and one scalar superfield (3.25). Our results for the various helicity configurations are listed below.

$$\langle \tilde{\mathbf{J}}_s^- \tilde{\mathbf{J}}_s^- \tilde{\mathbf{J}}_0 \rangle = \frac{\langle J_s^- J_s^- O_1 \rangle}{128} \Gamma_3, \quad \langle \tilde{\mathbf{J}}_s^+ \tilde{\mathbf{J}}_s^+ \tilde{\mathbf{J}}_0 \rangle = \frac{\langle J_s^+ J_s^+ O_1 \rangle}{32} \xi_{1-} \xi_{2-} \xi_{3-} \Xi_3, \quad (3.31)$$

while the mixed-helicity cases $\langle \tilde{\mathbf{J}}_s^\mp \tilde{\mathbf{J}}_s^\pm \tilde{\mathbf{J}}_0 \rangle$ vanish for conserved super-currents. This can be understood from the fact that the component correlators of the form $\langle J_s^- J_s^- O_\Delta \rangle$ in mixed helicity configurations are zero [29].

Summary

In this sub-section, we have investigated two and several types of three point super-correlators involving scalar superfields. We have shown that all these correlators can be expressed in terms of the building blocks we obtained earlier viz (3.4) for two points and (3.6) for three points.

Motivated by our success, we now proceed to develop an analogous formalism for CFTs with $\mathcal{N} = 2$ supersymmetry.

4 Extended supersymmetry: the $\mathcal{N} = 2$ case

4.1 The superspace formulation

The $\mathcal{N} = 2$ superspace can be described by the coordinates $(x^\mu, \theta^a, \bar{\theta}^a)$ where x^μ are the usual position space coordinates and θ^a and $\bar{\theta}^a$ are two component Grassmann spinors [44,

¹²An important point to note here is that (3.29) demands the existence of the semi-local term correlator $\langle O_2 O_2 O_1 \rangle$ even in the free theory. Such a contribution can arise from adding the conformally invariant term $\alpha \int d^3x O_1(x) J_2^2(x)$ in the action, where J_2 is the source of the O_2 operator. The a priori arbitrary constant α is fixed by supersymmetry to be equal to $\frac{f_{111}}{16}$.

47]. The generators of the $\mathcal{N} = 2$ superconformal algebra include the usual conformal generators $P_\mu, M_{\mu\nu}, D, K_\mu$ along with the supersymmetry generators Q_a, \bar{Q}_a and the special superconformal generators S_a and \bar{S}_a . More information on this formalism as well as the action of the generators on primary superfields can be found for instance in [39, 44].

We now, parallel our analysis of the $\mathcal{N} = 1$ case in section 2, first convert x^μ to p_μ by a Fourier transform. We then express p_μ in spinor helicity variables as in (2.14). The key change that arises in the $\mathcal{N} = 2$ is the fact the θ^a is no longer Hermitian. Therefore, it has to be described by two complex Grassmann variables in contrast to the single complex Grassmann variables η (2.15). The $\mathcal{N} = 2$ Grassmann spinors can be expressed as follows: for complex Grassmann variables η, μ , we define,

$$\begin{aligned}\theta^a &= \frac{\bar{\eta}\lambda^a + \mu\bar{\lambda}^a}{2p}, \\ \bar{\theta}^a &= \frac{\eta\bar{\lambda}^a + \bar{\mu}\lambda^a}{2p}.\end{aligned}\tag{4.1}$$

Further, just as we performed a Grassmann twistor transform in the $\mathcal{N} = 1$ case (2.19), we do so in the $\mathcal{N} = 2$ case as follows. We define,

$$\tilde{F}(\eta, \chi, \mu, \nu) = \int d\bar{\eta} d\bar{\mu} e^{-\frac{\chi\bar{\eta}}{4} - \frac{\nu\bar{\mu}}{4}} F(\eta, \bar{\eta}, \mu, \bar{\mu}).\tag{4.2}$$

We then define the coordinates:

$$\xi_\pm = \chi \pm \eta, \quad \omega_\pm = \nu \pm \mu.\tag{4.3}$$

In terms of these coordinates, the supersymmetry generators of the $\mathcal{N} = 2$ superconformal algebra take the following simple forms:

$$Q_a = 2\lambda_a \frac{\partial}{\partial \omega_+} + \frac{\bar{\lambda}_a}{4} \xi_+, \quad \bar{Q}_a = 2\lambda_a \frac{\partial}{\partial \xi_+} + \frac{\bar{\lambda}_a}{4} \omega_+.\tag{4.4}$$

The remaining generators are identical to their $\mathcal{N} = 1$ counterparts given in (B.4). However, there is one very important distinction between the $\mathcal{N} = 1$ and $\mathcal{N} = 2$ cases, that is, the presence of a U(1) R-symmetry. The Q and \bar{Q} operators defined in (4.4) have R charges -1 and $+1$ respectively. This implies the following commutation relations

$$[R, Q_a] = -Q_a, \quad [R, \bar{Q}_a] = +\bar{Q}_a.\tag{4.5}$$

The representation of R in the ξ_\pm, ω_\pm , acting on primary super fields is the following:

$$R = \omega_+ \frac{\partial}{\partial \omega_+} - \xi_+ \frac{\partial}{\partial \xi_+},\tag{4.6}$$

where R is the symmetry generator. It can easily be checked using (4.4) and (4.6) that the commutators (4.5) hold.

4.2 Super-currents and Ward identities

We are interested in constraining correlators involving symmetric traceless conserved super-currents. The general form of such currents in $\mathcal{N} = 2$ SCFTs can be found for instance in [39]. Here, we present its momentum superspace avatar:¹³

$$\begin{aligned} \mathbf{J}_s^{a_1 a_2 \dots a_{2s}} &= J_s^{a_1 a_2 \dots a_{2s}} + \theta_m J_{s+\frac{1}{2}}^{a_1 a_2 \dots a_{2s} m} + \bar{\theta}_m \bar{J}_{s+\frac{1}{2}}^{a_1 a_2 \dots a_{2s} m} + \theta_m \bar{\theta}_n J_{s+1}^{a_1 a_2 \dots a_{2s} m n} + \frac{\theta_m \bar{\theta}^m}{2} \not{p}_n^{a_1} J_s^{a_2 \dots a_{2s}} \\ &\quad - \frac{\theta^2 \bar{\theta}_m}{4} \not{p}_n^m J_{s+\frac{1}{2}}^{a_1 a_2 \dots a_{2s} n} - \frac{\bar{\theta}^2 \theta_m}{4} \not{p}_n^m \bar{J}_{s+\frac{1}{2}}^{a_1 a_2 \dots a_{2s} n} - \frac{\theta^2 \bar{\theta}^2}{16} p^2 J_s^{a_1 a_2 \dots a_{2s}}. \end{aligned} \quad (4.7)$$

We then use the definition (4.1) and perform the Grassmann twistor transform (4.2). Their is a drastic simplification compared to (4.7) when the result is expressed in the $(\xi_{\pm}, \omega_{\pm})$ variables (4.3). We have,

$$\begin{aligned} \tilde{\mathbf{J}}_s^- &= \frac{1}{4} \left[\frac{\xi_+ \omega_+}{4} J_s^- - \frac{1}{2\sqrt{p}} \left(\omega_+ J_{s+\frac{1}{2}}^- - \xi_+ \bar{J}_{s+\frac{1}{2}}^- \right) - \frac{J_{s+1}^-}{p} \right], \\ \tilde{\mathbf{J}}_s^+ &= \frac{\xi_- \omega_-}{16} \left[J_s^+ + \frac{1}{4\sqrt{p}} \left(\omega_+ J_{s+\frac{1}{2}}^+ + \xi_+ \bar{J}_{s+\frac{1}{2}}^+ \right) - \frac{\xi_+ \omega_+}{16p} J_{s+1}^+ \right]. \end{aligned} \quad (4.8)$$

We can then construct super-correlators using (4.8), as we did for the $\mathcal{N} = 1$ case. The component correlators can then be constrained using the action of Q_a and \bar{Q}_a , as given in (4.4). The R symmetry is going to play an important role here as correlators have to be R symmetric. Before proceeding further, we note that, ξ_{i+} has R charge -1 , while ω_{i+} has R charge $+1$. Then, correlators are forced to be made of elements of form $\xi_{i+} \omega_{i+}$, where i is the operator label. This is exactly what we got and we present our findings below for two and three point functions.

4.3 Two point functions

The $\mathcal{N} = 2$ two point functions for any integer spin s take the following forms in the two independent helicity configurations:

$$\begin{aligned} \langle \tilde{\mathbf{J}}_s^- \tilde{\mathbf{J}}_s^- \rangle &= \frac{\langle 12 \rangle^{2s}}{16 p_1} \left(\frac{\xi_{1+} \omega_{1+} \xi_{2+} \omega_{2+}}{16} - \frac{\langle 12 \rangle}{4 p_1} (\omega_{1+} \xi_{2+} + \xi_{1+} \omega_{2+}) + \frac{\langle 12 \rangle^2}{p_1^2} \right), \\ \langle \tilde{\mathbf{J}}_s^+ \tilde{\mathbf{J}}_s^+ \rangle &= \xi_{1-} \omega_{1-} \xi_{2-} \omega_{2-} \frac{\langle \bar{1} \bar{2} \rangle^{2s+3}}{65536 p_1^4} \left(\frac{\xi_{1+} \omega_{1+} \xi_{2+} \omega_{2+}}{16} - \frac{\langle 12 \rangle}{4 p_1} (\omega_{1+} \xi_{2+} + \xi_{1+} \omega_{2+}) + \frac{\langle 12 \rangle^2}{p_1^2} \right). \end{aligned} \quad (4.9)$$

Further, the building block of $\mathcal{N} = 2$ two-point functions can be written as a product of two individual building blocks. Then, the correlators can be written as,

$$\begin{aligned} \langle \tilde{\mathbf{J}}_s^- \tilde{\mathbf{J}}_s^- \rangle &= \frac{\langle 12 \rangle^{2s}}{16 p_1} \left(\frac{\xi_{1+} \omega_{2+}}{4} - \frac{\langle 12 \rangle}{p_1} \right) \left(\frac{\omega_{1+} \xi_{2+}}{4} - \frac{\langle 12 \rangle}{p_1} \right), \\ \langle \tilde{\mathbf{J}}_s^+ \tilde{\mathbf{J}}_s^+ \rangle &= \xi_{1-} \omega_{1-} \xi_{2-} \omega_{2-} \frac{\langle \bar{1} \bar{2} \rangle^{2s+3}}{65536 p_1^4} \left(\frac{\xi_{1+} \omega_{2+}}{4} - \frac{\langle 12 \rangle}{p_1} \right) \left(\frac{\omega_{1+} \xi_{2+}}{4} - \frac{\langle 12 \rangle}{p_1} \right). \end{aligned} \quad (4.10)$$

It can be clearly seen after the discussion in the above section, that the correlators are formed by the building blocks which are inherently R symmetric as they are made of elements like $\xi_{i+} \omega_{i+}$.

¹³In an explicit realization of (4.7), the zeroth component current is of the form $J_{\text{boson}} - J_{\text{fermion}}$ whereas the spin $s+1$ component current is of the form $J_{\text{boson}} + J_{\text{fermion}}$ [36].

4.4 Three point function

The structure of the three point function in the $\mathcal{N} = 2$ theories are much more complicated than their $\mathcal{N} = 1$ counterpart. However, by a careful analysis, we were able still able to obtain the three point correlators in the $\mathcal{N} = 2$ theories for any arbitrary integer spin in terms of the following building blocks which is reminiscent of the $\mathcal{N} = 1$ case (3.6).

$$\begin{aligned}\Omega_1 &= \left(\langle 23 \rangle^2 \xi_{1+\omega_{1+}} + \langle 23 \rangle \langle 31 \rangle \xi_{1+\omega_{2+}} + \langle 12 \rangle \langle 23 \rangle \xi_{1+\omega_{3+}} + \langle 23 \rangle \langle 31 \rangle \xi_{2+\omega_{1+}} + \langle 31 \rangle^2 \xi_{2+\omega_{2+}} \right. \\ &\quad \left. + \langle 12 \rangle \langle 31 \rangle \xi_{2+\omega_{3+}} + \langle 12 \rangle \langle 23 \rangle \xi_{3+\omega_{1+}} + \langle 12 \rangle \langle 31 \rangle \xi_{3+\omega_{2+}} + \langle 12 \rangle^2 \xi_{3+\omega_{3+}} \right), \\ \Omega_2 &= \left(- \langle 23 \rangle \xi_{1+\omega_{1+}} (\xi_{2+\omega_{3+}} - \xi_{3+\omega_{2+}}) + \langle 31 \rangle \xi_{2+\omega_{2+}} (\xi_{1+\omega_{3+}} - \xi_{3+\omega_{1+}}) \right. \\ &\quad \left. - \langle 12 \rangle \xi_{3+\omega_{3+}} (\xi_{1+\omega_{2+}} - \xi_{2+\omega_{1+}}) \right), \\ \Omega_3 &= \xi_{1+} \xi_{2+} \xi_{3+\omega_{1+}} \omega_{2+} \omega_{3+}.\end{aligned}\tag{4.11}$$

For example, in the $(- - -)$ helicity we obtain,¹⁴

$$\langle \tilde{\mathbf{J}}_{s_1}^- \tilde{\mathbf{J}}_{s_2}^- \tilde{\mathbf{J}}_{s_3}^- \rangle = \langle J_{s_1}^- J_{s_2}^- J_{s_3}^- \rangle \left(\frac{\Omega_1}{E^2} + \frac{\Omega_2}{8E} - \frac{\Omega_3}{64} \right).\tag{4.12}$$

It is straightforward to obtain analogous formulae in the remaining helicity configurations as well. For spins that satisfy the triangle inequality, (4.12) is homogeneous similar to its $\mathcal{N} = 1$ counterpart (3.5). In $\mathcal{N} = 1$, the non-homogeneous contributions for component correlators that are inside the triangle arose from super-correlators with net half integer spin (see table 1). However, in the $\mathcal{N} = 2$ case, there are no half integer spin super-currents and hence non-homogeneous component correlators that are inside the triangle are not realized. Let us give a short proof of the same. Consider a super-correlator $\langle \tilde{\mathbf{J}}_{s_1} \tilde{\mathbf{J}}_{s_2} \tilde{\mathbf{J}}_{s_3} \rangle$. For it to be non-zero we require $s_1 + s_2 + s_3 = 2n, n \in \mathbb{Z}_{>0}$. In an explicit realization, it has the following component expansion:¹⁵

$$\langle \tilde{\mathbf{J}}_{s_1} \tilde{\mathbf{J}}_{s_2} \tilde{\mathbf{J}}_{s_3} \rangle = \langle J_{s_1} J_{s_2} J_{s_3} \rangle_{FB-FF} + \dots + \text{Grassmann structure} \times \langle J_{s_1+1} J_{s_2+1} J_{s_3+1} \rangle_{FB+FF} + \dots.\tag{4.13}$$

For the FB+FF correlator to be non-zero we require $s_1 + s_2 + s_3 + 3 = 2m, m \in \mathbb{Z}_{>0}$. However, we already required $s_1 + s_2 + s_3 = 2n, n \in \mathbb{Z}_{>0}$ for the super-correlator to be non-vanishing. Reconciling these two conditions yields $2n = 2m - 3, n, m \in \mathbb{Z}_{>0}$ which is clearly a contradiction. Thus, the conclusion is that the non-homogeneous correlator do not occur in the $\mathcal{N} = 2$ spinning three point super correlators.

4.5 Double copy: $\mathcal{N} = 1 \otimes \mathcal{N} = 1 \rightarrow \mathcal{N} = 2$

Two-point function

Consider the following integer two point functions in the $\mathcal{N} = 1$ theory in the $(- -)$ helicity configuration (3.1),

$$\langle \tilde{\mathbf{J}}_s^-(\xi_{1+}) \tilde{\mathbf{J}}_s^-(\omega_{2+}) \rangle_{\mathcal{N}=1} = \frac{\langle 12 \rangle^{2s}}{16p_1} \left(\xi_{1+\omega_{2+}} - \frac{4\langle 12 \rangle}{p_1} \right),$$

¹⁴We actually obtain a two parameter family of solutions. However, in this case, one of them is inconsistent with the operator product expansion in the sense of [31] and hence we set its coefficient to zero. However, this other solution will make its return when we investigate the three point scalar super correlator in subsection 4.6.

¹⁵See footnote 13.

$$\langle \tilde{\mathbf{J}}_s^-(\omega_{1+}) \tilde{\mathbf{J}}_s^-(\xi_{2+}) \rangle_{\mathcal{N}=1} = \frac{\langle 12 \rangle^{2s}}{16p_1} \left(\omega_{1+} \xi_{2+} - \frac{4\langle 12 \rangle}{p_1} \right), \quad (4.14)$$

where we have made the dependence on the Grassmann twistor variables explicit. The reason for the strange choice of the Grassmann twistor variable dependence of the super currents in both lines of (4.14) will become clear shortly. We now take a product between the first and second line of (4.14). The result is,

$$\begin{aligned} & \langle \tilde{\mathbf{J}}_s^-(\xi_{1+}) \tilde{\mathbf{J}}_s^-(\omega_{2+}) \rangle_{\mathcal{N}=1} \langle \tilde{\mathbf{J}}_s^-(\omega_{1+}) \tilde{\mathbf{J}}_s^-(\xi_{2+}) \rangle_{\mathcal{N}=1} = \\ & \frac{\langle 12 \rangle^{4s}}{16p_1^2} \left(\frac{\xi_{1+} \omega_{1+} \xi_{2+} \omega_{2+}}{16} - \frac{\langle 12 \rangle}{4p_1} (\omega_{1+} \xi_{2+} + \xi_{1+} \omega_{2+}) + \frac{\langle 12 \rangle^2}{p_1} \right). \end{aligned} \quad (4.15)$$

By comparing the r.h.s. of the above equation with the $\mathcal{N} = 2$ two point function (4.9), we obtain a super double copy relation!

$$\langle \tilde{\mathbf{J}}_s^-(\xi_{1+}) \tilde{\mathbf{J}}_s^-(\omega_{2+}) \rangle_{\mathcal{N}=1} \langle \tilde{\mathbf{J}}_s^-(\omega_{1+}) \tilde{\mathbf{J}}_s^-(\xi_{2+}) \rangle_{\mathcal{N}=1} = \frac{1}{p_1} \langle \tilde{\mathbf{J}}_{2s}^-(\xi_{1+}, \omega_{1+}) \tilde{\mathbf{J}}_{2s}^-(\xi_{2+}, \omega_{2+}) \rangle_{\mathcal{N}=2}. \quad (4.16)$$

The reason for the weird choices of Grassmann twistor variable dependence in (4.14) is now clear. This and only this particular choice will ensure that the r.h.s. of (4.16), which is a $\mathcal{N} = 2$ two-point function will be invariant under the action of the R symmetry generator (4.6). Similar results in the other helicity configuration as well as double copy relations involving half integer $\mathcal{N} = 1$ supercorrelators can also easily be obtained, all thanks to the simplicity of the Grassmann twistor variables.

Three-point functions that satisfy the spin triangle inequality

Let us now proceed to the three point case. For simplicity, we consider the cases that satisfy the triangle inequality $s_i + s_j \geq s_k \forall i, j, k \in 1, 2, 3$. Our prescription for obtaining $\mathcal{N} = 2$ super correlators from products of $\mathcal{N} = 1$ super correlators is constructing the product such that the resulting object possesses the required R symmetry property. Our goal is to take products of $\mathcal{N} = 1$ three-point functions (3.5) to reproduce the $\mathcal{N} = 2$ answer (4.12). From the required $U(1)$ R symmetry of the result, we know that the total number of ξ_{i+} and ω_{i+} variables appearing in each term must be equal. This then restricts our ansatz for the $\mathcal{N} = 1$ product. Let us concentrate on the $(- - -)$ helicity configuration.¹⁶ We take,

$$\begin{aligned} \text{Ansatz} = & \left[a_1 \langle \tilde{\mathbf{J}}_{s_1}^-(\xi_{1+}) \tilde{\mathbf{J}}_{s_2}^-(\xi_{2+}) \tilde{\mathbf{J}}_{s_3}^-(\xi_{3+}) \rangle_{\mathcal{N}=1} \langle \tilde{\mathbf{J}}_{s_1}^-(\omega_{1+}) \tilde{\mathbf{J}}_{s_2}^-(\omega_{2+}) \tilde{\mathbf{J}}_{s_3}^-(\omega_{3+}) \rangle_{\mathcal{N}=1} \right. \\ & + a_2 \langle \tilde{\mathbf{J}}_{s_1}^-(\xi_{1+}) \tilde{\mathbf{J}}_{s_2}^-(\xi_{2+}) \tilde{\mathbf{J}}_{s_3}^-(\omega_{3+}) \rangle_{\mathcal{N}=1} \langle \tilde{\mathbf{J}}_{s_1}^-(\omega_{1+}) \tilde{\mathbf{J}}_{s_2}^-(\omega_{2+}) \tilde{\mathbf{J}}_{s_3}^-(\xi_{3+}) \rangle_{\mathcal{N}=1} \\ & + a_3 \langle \tilde{\mathbf{J}}_{s_1}^-(\xi_{1+}) \tilde{\mathbf{J}}_{s_2}^-(\omega_{2+}) \tilde{\mathbf{J}}_{s_3}^-(\xi_{3+}) \rangle_{\mathcal{N}=1} \langle \tilde{\mathbf{J}}_{s_1}^-(\omega_{1+}) \tilde{\mathbf{J}}_{s_2}^-(\xi_{2+}) \tilde{\mathbf{J}}_{s_3}^-(\omega_{3+}) \rangle_{\mathcal{N}=1} \\ & \left. + a_4 \langle \tilde{\mathbf{J}}_{s_1}^-(\omega_{1+}) \tilde{\mathbf{J}}_{s_2}^-(\xi_{2+}) \tilde{\mathbf{J}}_{s_3}^-(\xi_{3+}) \rangle_{\mathcal{N}=1} \langle \tilde{\mathbf{J}}_{s_1}^-(\xi_{1+}) \tilde{\mathbf{J}}_{s_2}^-(\omega_{2+}) \tilde{\mathbf{J}}_{s_3}^-(\omega_{3+}) \rangle_{\mathcal{N}=1} \right]. \end{aligned} \quad (4.17)$$

We then demand that the ansatz (4.17) is invariant under the simultaneous action of the R symmetry generator (4.6) on all the insertions. This yields the constraint $a_1 = a_2 = a_3 = a_4$. In fact (4.17) becomes,

$$\langle J_{s_1}^- J_{s_2}^- J_{s_3}^- \rangle^2 \left(\frac{\Omega_1}{E^2} + \frac{\Omega_2}{8E} - \frac{\Omega_3}{64} \right), \quad (4.18)$$

¹⁶One can carry out an analogous analysis in the other helicities.

where the Ω_i are the $\mathcal{N} = 2$ building blocks (4.11). Therefore, we obtain a super double copy,

$$\begin{aligned}
& \langle \tilde{\mathbf{J}}_{2s_1}^-(\xi_{1+}, \omega_{1+}) \tilde{\mathbf{J}}_{2s_2}^-(\xi_{2+}, \omega_{2+}) \tilde{\mathbf{J}}_{2s_3}^-(\xi_{3+}, \omega_{3+}) \rangle_{\mathcal{N}=2} \\
&= p_1 p_2 p_3 \left[\langle \tilde{\mathbf{J}}_{s_1}^-(\xi_{1+}) \tilde{\mathbf{J}}_{s_2}^-(\xi_{2+}) \tilde{\mathbf{J}}_{s_3}^-(\xi_{3+}) \rangle_{\mathcal{N}=1} \langle \tilde{\mathbf{J}}_{s_1}^-(\omega_{1+}) \tilde{\mathbf{J}}_{s_2}^-(\omega_{2+}) \tilde{\mathbf{J}}_{s_3}^-(\omega_{3+}) \rangle_{\mathcal{N}=1} \right. \\
&\quad + \langle \tilde{\mathbf{J}}_{s_1}^-(\xi_{1+}) \tilde{\mathbf{J}}_{s_2}^-(\xi_{2+}) \tilde{\mathbf{J}}_{s_3}^-(\omega_{3+}) \rangle_{\mathcal{N}=1} \langle \tilde{\mathbf{J}}_{s_1}^-(\omega_{1+}) \tilde{\mathbf{J}}_{s_2}^-(\omega_{2+}) \tilde{\mathbf{J}}_{s_3}^-(\xi_{3+}) \rangle_{\mathcal{N}=1} \\
&\quad + \langle \tilde{\mathbf{J}}_{s_1}^-(\xi_{1+}) \tilde{\mathbf{J}}_{s_2}^-(\omega_{2+}) \tilde{\mathbf{J}}_{s_3}^-(\xi_{3+}) \rangle_{\mathcal{N}=1} \langle \tilde{\mathbf{J}}_{s_1}^-(\omega_{1+}) \tilde{\mathbf{J}}_{s_2}^-(\xi_{2+}) \tilde{\mathbf{J}}_{s_3}^-(\omega_{3+}) \rangle_{\mathcal{N}=1} \\
&\quad \left. + \langle \tilde{\mathbf{J}}_{s_1}^-(\omega_{1+}) \tilde{\mathbf{J}}_{s_2}^-(\xi_{2+}) \tilde{\mathbf{J}}_{s_3}^-(\xi_{3+}) \rangle_{\mathcal{N}=1} \langle \tilde{\mathbf{J}}_{s_1}^-(\xi_{1+}) \tilde{\mathbf{J}}_{s_2}^-(\omega_{2+}) \tilde{\mathbf{J}}_{s_3}^-(\omega_{3+}) \rangle_{\mathcal{N}=1} \right]. \quad (4.19)
\end{aligned}$$

The double copy at the level of homogeneous component correlators was essential for this result. For non homogeneous correlators, there exist some double copy relations albeit more complicated ones. Our prescription to obtain $\mathcal{N} = 2$ super-correlators from R symmetry preserving products of $\mathcal{N} = 1$ super-correlators could potentially generalize to higher points.

4.6 Correlators involving scalar superfields

The $\mathcal{N} = 2$ scalar superfield has the following component expansion in momentum superspace:¹⁷

$$\tilde{\mathbf{J}}_0 = O_1 + \theta_a O_{\frac{1}{2}}^a - \bar{\theta}_a \bar{O}_{\frac{1}{2}}^a + \theta_a \bar{\theta}_b J^{ab} + \frac{1}{2} \theta_a \bar{\theta}^a O_2 - \frac{1}{4} \theta^2 \bar{\theta}_b \not{p}^b O_{\frac{1}{2}}^a + \frac{1}{4} \bar{\theta}^2 \theta_b \not{p}^b \bar{O}_{\frac{1}{2}}^a - \frac{1}{16} \theta^2 \bar{\theta}^2 p^2 O_1. \quad (4.20)$$

Expressing the Grassmann spinors as in (2.15), one obtains the following expression:

$$\begin{aligned}
\tilde{\mathbf{J}}_0 &= \left(1 + \frac{\mu \bar{\mu} \eta \bar{\eta}}{16} \right) O_1 + \frac{1}{4p} (\eta \bar{\eta} + \mu \bar{\mu}) O_2 + \frac{1}{2\sqrt{p}} \bar{\eta} \left(1 - \frac{\mu \bar{\mu}}{4} \right) O_{\frac{1}{2}}^- + \frac{1}{2\sqrt{p}} \mu \left(1 + \frac{\eta \bar{\eta}}{4} \right) O_{\frac{1}{2}}^+ \\
&\quad - \frac{1}{2\sqrt{p}} \bar{\mu} \left(1 - \frac{\eta \bar{\eta}}{4} \right) \bar{O}_{\frac{1}{2}}^- - \frac{1}{2\sqrt{p}} \eta \left(1 + \frac{\mu \bar{\mu}}{4} \right) \bar{O}_{\frac{1}{2}}^+ - \frac{1}{2p} (\eta \bar{\mu} J^- - \eta \mu J^+). \quad (4.21)
\end{aligned}$$

Performing Grassmann twistor transforms (2.19) with respect to the variables $\bar{\eta}$ and $\bar{\mu}$, and expressing them in the nicer Grassmann twistor variables given in (2.20), we obtain,

$$\begin{aligned}
\tilde{\mathbf{J}}_0 &= \frac{1}{32} (\xi_+ \omega_+ + \xi_- \omega_-) O_1 - \frac{1}{32p} (\xi_+ \omega_+ - \xi_- \omega_-) O_2 - \frac{1}{8\sqrt{p}} \omega_+ O_{\frac{1}{2}}^- - \frac{1}{8\sqrt{p}} \xi_+ \bar{O}_{\frac{1}{2}}^- \\
&\quad + \frac{1}{64\sqrt{p}} \xi_- \omega_- \omega_+ O_{\frac{1}{2}}^+ - \frac{1}{64\sqrt{p}} \xi_- \omega_- \xi_- \bar{O}_{\frac{1}{2}}^+ + \frac{1}{2p} J^- + \frac{1}{128p} \xi_- \omega_- \xi_+ \omega_+ J^+. \quad (4.22)
\end{aligned}$$

We shall now present explicit results for two and three point functions involving scalar superfields.

Two point function

Much like the $\mathcal{N} = 1$ scenario in subsection 3.6, the two point function of scalar super-fields obtained via supersymmetric Ward identities can be expressed in terms of the same building blocks of spinning super-correlators (4.10).

$$\langle \tilde{\mathbf{J}}_0 \tilde{\mathbf{J}}_0 \rangle = \frac{1}{32p_1} \left((\alpha_{11} + \alpha_{12}) - (\alpha_{11} - \alpha_{12}) \xi_{1-} \xi_{2-} \omega_{1-} \omega_{2-} - \frac{\langle \bar{1}\bar{2} \rangle^2}{256p_1^2} \right) \left(\frac{\xi_{1+} \omega_{2+}}{4} - \frac{\langle 12 \rangle}{p_1} \right) \left(\frac{\omega_{1+} \xi_{2+}}{4} - \frac{\langle 12 \rangle}{p_1} \right). \quad (4.23)$$

¹⁷This can be obtained by a simple Fourier transform of the position space expression provided in say [39] after converting to our conventions.

Three point functions

The three point function of scalar super-fields obtained via solving the supersymmetric Ward identities can be expressed as follows:

$$\begin{aligned} \langle \tilde{\mathbf{J}}_0 \tilde{\mathbf{J}}_0 \tilde{\mathbf{J}}_0 \rangle &= \frac{\langle O_1 O_1 O_1 \rangle}{128} \left(\frac{\xi_1 - \xi_2 - \xi_3 - \omega_1 - \omega_2 - \omega_3}{4096} \left(64 - \frac{8\tilde{\Omega}_2}{E} + \frac{\tilde{\Omega}_1}{E^2} \right) - \left(\frac{\Omega_1}{E^2} + \frac{\Omega_2}{8E} - \frac{\Omega_3}{64} \right) \right) \\ &\quad - \frac{\langle O_2 O_2 O_2 \rangle}{128 p_1 p_2 p_3} \left(\frac{\xi_1 - \xi_2 - \xi_3 - \omega_1 - \omega_2 - \omega_3}{4096} \left(64 - \frac{8\tilde{\Omega}_2}{E} + \frac{\tilde{\Omega}_1}{E^2} \right) + \left(\frac{\Omega_1}{E^2} + \frac{\Omega_2}{8E} - \frac{\Omega_3}{64} \right) \right). \end{aligned} \quad (4.24)$$

Note that in addition to the building block (made out of the $\Omega_i, i = 1, 2, 3$) for the spinning correlators given in (4.11), we also require the additional one that was thrown away for the spinning case due to OPE inconsistency (see footnote 14). Here, the other building block is essential and does not lead to any OPE inconsistency. The components of the second building block, the $\tilde{\Omega}_i, i = 1, 2, 3$, are given by,

$$\begin{aligned} \tilde{\Omega}_1 &= \left(\langle \bar{1}\bar{2} \rangle^2 \xi_{1+\omega_1+\xi_{2+\omega_2+}} + \langle \bar{2}\bar{3} \rangle^2 \xi_{2+\omega_2+\xi_{3+\omega_3+}} + \langle \bar{3}\bar{1} \rangle^2 \xi_{3+\omega_3+\xi_{1+\omega_1+}} \right. \\ &\quad \left. - \langle \bar{1}\bar{2} \rangle \langle \bar{3}\bar{1} \rangle \xi_{1+\omega_1+} (\xi_{2+\omega_3+} + \xi_{3+\omega_2+}) - \langle \bar{2}\bar{3} \rangle \langle \bar{1}\bar{2} \rangle \xi_{2+\omega_2+} (\xi_{3+\omega_1+} + \xi_{1+\omega_3+}) \right. \\ &\quad \left. - \langle \bar{3}\bar{1} \rangle \langle \bar{2}\bar{3} \rangle \xi_{3+\omega_3+} (\xi_{1+\omega_2+} + \xi_{2+\omega_1+}) \right) \\ \tilde{\Omega}_2 &= \left(\langle \bar{1}\bar{2} \rangle (\xi_{1+\omega_2+} - \xi_{2+\omega_1+}) + \langle \bar{2}\bar{3} \rangle (\xi_{2+\omega_3+} - \xi_{3+\omega_2+}) + \langle \bar{3}\bar{1} \rangle (\xi_{3+\omega_1+} - \xi_{1+\omega_3+}) \right). \end{aligned} \quad (4.25)$$

It is now a straightforward exercise with these building blocks to obtain mixed correlators (those involving scalars and spinning operators). We shall however, not pursue this right now but rather, switch gears once more and investigate higher point functions.

5 Four point functions in $\mathcal{N} = 1$ SCFT

In this section, we investigate the consequences of $\mathcal{N} = 1$ super conformal invariance for four point functions. As usual, we begin with the case where all operators have non-zero spin.

$$\langle \tilde{\mathbf{J}}_{s_1} \tilde{\mathbf{J}}_{s_2} \tilde{\mathbf{J}}_{s_3} \tilde{\mathbf{J}}_{s_4} \rangle, \quad s_i \in \mathbb{Z}_{>0}$$

Consider the four-point functions of conserved super-currents with integer spins. Upon solving for the q and \bar{q} Ward identities (2.27), we find that the super-correlator can be expressed using two seed component correlators. Here, we present some prototypical examples of the various helicity configurations, with the rest of the permutations easily obtainable given the building blocks in (5.2). We have,

$$\begin{aligned} \langle \tilde{\mathbf{J}}_{s_1}^- \tilde{\mathbf{J}}_{s_2}^- \tilde{\mathbf{J}}_{s_3}^- \tilde{\mathbf{J}}_{s_4}^- \rangle &= \frac{\langle J_{s_1}^- J_{s_2}^- J_{s_3}^- J_{s_4}^- \rangle}{256} \Gamma_{4E} + \frac{\langle J_{s_1+\frac{1}{2}}^- J_{s_2+\frac{1}{2}}^- J_{s_3+\frac{1}{2}}^- J_{s_4+\frac{1}{2}}^- \rangle}{128\sqrt{p_1 p_2 p_3 p_4}} \Xi_{4E}, \\ \langle \tilde{\mathbf{J}}_{s_1}^+ \tilde{\mathbf{J}}_{s_2}^+ \tilde{\mathbf{J}}_{s_3}^+ \tilde{\mathbf{J}}_{s_4}^+ \rangle &= \xi_1 - \xi_2 - \xi_3 - \xi_4 - \frac{\langle J_{s_1}^+ J_{s_2}^+ J_{s_3}^+ J_{s_4}^+ \rangle}{2048} \Xi_{4E} + \xi_1 - \xi_2 - \xi_3 - \xi_4 - \frac{\langle J_{s_1+\frac{1}{2}}^+ J_{s_2+\frac{1}{2}}^+ J_{s_3+\frac{1}{2}}^+ J_{s_4+\frac{1}{2}}^+ \rangle}{65536\sqrt{p_1 p_2 p_3 p_4}} \Gamma_{4E}, \end{aligned}$$

$$\begin{aligned}
 \langle \tilde{\mathbf{J}}_{s_1}^- \tilde{\mathbf{J}}_{s_2}^- \tilde{\mathbf{J}}_{s_3}^+ \tilde{\mathbf{J}}_{s_4}^+ \rangle &= \frac{\xi_3 - \xi_{4-}}{256(2p_3 + 2p_4 - E)} \langle J_{s_1}^- J_{s_2}^- J_{s_3}^+ J_{s_4}^+ \rangle \left(\langle 12 \rangle \Xi_{4E} - \frac{\langle \bar{3}\bar{4} \rangle}{8} \Gamma_{4E} \right) \\
 &\quad + \frac{\xi_3 - \xi_{4-}}{1024(2p_3 + 2p_4 - E) \sqrt{p_1 p_2 p_3 p_4}} \langle J_{s_1+\frac{1}{2}}^- J_{s_2+\frac{1}{2}}^- J_{s_3+\frac{1}{2}}^+ J_{s_4+\frac{1}{2}}^+ \rangle \left(\langle 34 \rangle \Xi_{4E} - \frac{\langle \bar{1}\bar{2} \rangle}{8} \Gamma_{4E} \right), \\
 \langle \tilde{\mathbf{J}}_{s_1}^- \tilde{\mathbf{J}}_{s_2}^- \tilde{\mathbf{J}}_{s_3}^- \tilde{\mathbf{J}}_{s_4}^+ \rangle &= \xi_{4-} \frac{\langle J_{s_1}^- J_{s_2}^- J_{s_3}^- J_{s_4}^+ \rangle}{256} \Gamma_{4O} - \xi_{4-} \frac{\langle J_{s_1+\frac{1}{2}}^- J_{s_2+\frac{1}{2}}^- J_{s_3+\frac{1}{2}}^- J_{s_4+\frac{1}{2}}^+ \rangle}{1024 \sqrt{p_1 p_2 p_3 p_4}} \Xi_{4O}, \\
 \langle \tilde{\mathbf{J}}_{s_1}^+ \tilde{\mathbf{J}}_{s_2}^+ \tilde{\mathbf{J}}_{s_3}^- \tilde{\mathbf{J}}_{s_4}^- \rangle &= \xi_{1-} \xi_{2-} \xi_{3-} \frac{\langle J_{s_1}^+ J_{s_2}^+ J_{s_3}^- J_{s_4}^- \rangle}{2048} \Xi_{4O} - \xi_{1-} \xi_{2-} \xi_{3-} \frac{\langle J_{s_1+\frac{1}{2}}^+ J_{s_2+\frac{1}{2}}^+ J_{s_3+\frac{1}{2}}^- J_{s_4+\frac{1}{2}}^- \rangle}{8192 \sqrt{p_1 p_2 p_3 p_4}} \Gamma_{4O},
 \end{aligned} \tag{5.1}$$

where,

$$\begin{aligned}
 \Gamma_{4E} &= \left[\xi_{1+} \xi_{2+} \xi_{3+} \xi_{4+} - \frac{8}{E} (\xi_{1+} \xi_{2+} \langle 34 \rangle - \xi_{1+} \xi_{3+} \langle 24 \rangle + \xi_{1+} \xi_{4+} \langle 23 \rangle + \xi_{2+} \xi_{3+} \langle 14 \rangle - \xi_{2+} \xi_{4+} \langle 13 \rangle \right. \\
 &\quad \left. + \xi_{3+} \xi_{4+} \langle 12 \rangle \right], \\
 \Xi_{4E} &= \left[8 - \frac{1}{E} (\xi_{1+} \xi_{2+} \langle \bar{1}\bar{2} \rangle + \xi_{1+} \xi_{3+} \langle \bar{1}\bar{3} \rangle + \xi_{1+} \xi_{4+} \langle \bar{1}\bar{4} \rangle + \xi_{2+} \xi_{3+} \langle \bar{2}\bar{3} \rangle + \xi_{2+} \xi_{4+} \langle \bar{2}\bar{4} \rangle + \xi_{3+} \xi_{4+} \langle \bar{3}\bar{4} \rangle) \right], \\
 \Gamma_{4O} &= \left[\xi_{1+} \xi_{2+} \xi_{3+} - \frac{1}{E - 2p_4} (\xi_{1+} \xi_{2+} \xi_{4+} \langle 3\bar{4} \rangle - \xi_{1+} \xi_{3+} \xi_{4+} \langle 2\bar{4} \rangle + \xi_{2+} \xi_{3+} \xi_{4+} \langle 1\bar{4} \rangle + 8\xi_{1+} \langle 23 \rangle \right. \\
 &\quad \left. - 8\xi_{2+} \langle 13 \rangle + 8\xi_{3+} \langle 12 \rangle \right], \\
 \Xi_{4O} &= \left[8\xi_{4+} - \frac{1}{E - 2p_4} (\xi_{1+} \xi_{2+} \xi_{4+} \langle \bar{1}\bar{2} \rangle + \xi_{1+} \xi_{3+} \xi_{4+} \langle \bar{1}\bar{3} \rangle + \xi_{2+} \xi_{3+} \xi_{4+} \langle \bar{2}\bar{3} \rangle + 8\xi_{1+} \langle \bar{1}\bar{4} \rangle + 8\xi_{2+} \langle \bar{2}\bar{4} \rangle \right. \\
 &\quad \left. + 8\xi_{3+} \langle \bar{3}\bar{4} \rangle \right],
 \end{aligned} \tag{5.2}$$

where, the subscripts ‘‘O’’ and ‘‘E’’ are to denote the building blocks being Grassmann odd and even respectively. These can be viewed as generalizations of the three point building blocks Γ_3 and Ξ_3 (3.6).

Our main interest in investigating four point functions is to see if supersymmetry can be used to obtain spinning correlators from scalar ones. Let us therefore investigate the latter.

5.1 Scalar superfield four point function

In the literature, general scalar four point functions are known in the form of simplex integrals [53–55]. Since supersymmetry connects scalar correlators to spinning ones, one would hope to obtain simplex representations for the latter. Let us thus analyze the scalar superfield four point function. Recall that the expansion for the scalar superfield in the η, χ variables is as follows (3.24):

$$\mathbf{J}_0 = \frac{1}{4} \left(\chi O_1 + \frac{2}{\sqrt{p}} O_{1/2}^- + \frac{1}{2\sqrt{p}} \chi \eta O_{1/2}^+ - 4 \frac{\eta}{p} O_2 \right). \tag{5.3}$$

Using this expansion, we see that the four point function consists of 256 terms. However, half of these terms are correlators with an odd number of the spin half operators and hence vanish by Lorentz invariance, thus leaving us with 128 of them. Demanding q and \bar{q} invariance

of the super correlator brings this number down to 32. As there are a total of only 16 scalar component correlators, the spinning correlators are not entirely determined by the scalar ones. For instance,

$$\frac{\langle O_2 O_2 O_{1/2}^- O_{1/2}^- \rangle}{\sqrt{p_3 p_4}} = -\frac{\langle 34 \rangle}{32 p_1 p_2} \left(\langle O_1 O_2 O_1 O_1 \rangle - 4 \frac{\langle O_1 O_2 O_1 O_2 \rangle}{p_4} - 4 \frac{\langle O_1 O_2 O_2 O_1 \rangle}{p_3} + 16 \frac{\langle O_1 O_2 O_2 O_2 \rangle}{p_3 p_4} \right) + \dots, \quad (5.4)$$

where the dots include correlators with non-zero spin. Although we can use a simplex representations for the scalar correlators in the r.h.s. of the above equation using the expressions obtained in [53–55], one only obtains a partial answer for the spinning correlator.

We find that the four-point function of the spin half operator (in a particular helicity) is as follows:

$$\begin{aligned} \frac{1}{\sqrt{p_1 p_2 p_3 p_4}} \langle O_{\frac{1}{2}}^+ O_{\frac{1}{2}}^+ O_{\frac{1}{2}}^- O_{\frac{1}{2}}^- \rangle &= \frac{\langle \bar{1}3 \rangle}{2 p_1 \sqrt{p_2 p_4}} \langle O_1 O_{\frac{1}{2}}^+ O_1 O_{\frac{1}{2}}^- \rangle - \frac{2 \langle \bar{1}3 \rangle}{p_1 p_3 \sqrt{p_2 p_4}} \langle O_1 O_{\frac{1}{2}}^+ O_2 O_{\frac{1}{2}}^- \rangle \\ &\quad - \frac{\langle \bar{1}4 \rangle}{2 p_1 \sqrt{p_2 p_3}} \langle O_1 O_{\frac{1}{2}}^+ O_{\frac{1}{2}}^- O_1 \rangle + \frac{2 \langle \bar{1}4 \rangle}{p_1 p_4 \sqrt{p_2 p_3}} \langle O_1 O_{\frac{1}{2}}^+ O_{\frac{1}{2}}^- O_2 \rangle \\ &\quad + \frac{2 \langle \bar{1}2 \rangle}{p_1 p_2 \sqrt{p_3 p_4}} \langle O_1 O_2 O_{\frac{1}{2}}^- O_{\frac{1}{2}}^- \rangle + \frac{\langle \bar{1}2 \rangle}{2 p_1 \sqrt{p_3 p_4}} \langle O_1 O_1 O_{\frac{1}{2}}^- O_{\frac{1}{2}}^- \rangle. \end{aligned} \quad (5.5)$$

Notice that it cannot be expressed purely in terms of the scalar component correlators since the correlators $\langle O_{1/2}^\pm O_{1/2}^\pm O_{\Delta_1} O_{\Delta_2} \rangle$ themselves cannot be as we saw in (5.4). However, a more thorough analysis may enable us to make progress and we hope to return to this problem in the future.

6 Discussion and future directions

Summary

In this paper, we have developed a formalism to study three dimensional super conformal field theories. Rather than working in momentum space with the usual grassmann spinors θ^a , we found it advantageous to express both the momenta and the grassmann spinors in spinor helicity variables, i.e, super spinor helicity variables. Looking at the structures that one obtains in these variables motivated us to perform a grassmann twistor transform where we noticed a remarkable simplification at the level of symmetry generators and super-correlation functions. We then went on to constrain and solve for all (half) integer two and three point functions in $\mathcal{N} = 1$ SCFTs in these new variables. In contrast to the case by case approach like in the earlier position space analysis, we obtain a universal structure for these super-correlators. This immediately gave us a double copy relation at the level of the super-correlators as well as give us a structure reminiscent of the four dimensional flat space scattering amplitudes. We then investigated super-correlators that also involve scalars. We also developed the formalism for the $\mathcal{N} = 2$ case where we discovered similar universal structures for the correlators independent of spin. Further, we presented a prescription to obtain $\mathcal{N} = 2$ super correlators from their $\mathcal{N} = 1$ counterparts through a super double copy relation. Finally, we obtained some preliminary results at the four point level.

Future directions

Extended supersymmetry

The most natural generalization of our work is to extend our formalism to theories with higher supersymmetry. The spinor helicity variable representation of θ_a^A where A is the R-symmetry index is a simple generalization of the $\mathcal{N} = 1$ definition (2.15).

$$\theta_a^A = \frac{\bar{\eta}^A \lambda_a + \eta^A \bar{\lambda}_a}{2p}, \quad (6.1)$$

which is just obtained from (2.15) by promoting the complex grassmann variable η to η^A to accommodate the R-symmetry. Likewise, one can obtain the generators and the super-currents and perform an analysis parallel to the one presented in this paper to obtain correlation functions in these theories as well.

(Slightly broken) higher spin supersymmetry

Yet another important extension of our work is the study of higher spin super conformal field theories. It would be interesting to see the interplay between higher spin symmetry and super symmetry as both these symmetries connect correlators with different amounts of spin. Further, the case when the higher spin symmetry is slightly broken is yet another avenue that is worth exploring [56–66]. Supersymmetric Chern Simons+matter theories provide one such quintessential example. We plan to pursue the development of the supersymmetric slightly broken higher spin algebra to analyze and solve for correlation functions in the near future. This will allow us to obtain the supersymmetric analogue of results such as those obtained in [67] where all n point functions involving single trace primary operators in CS+fermionic matter theory were obtained in terms of the free theory correlators. The maximally supersymmetric version of the CS+matter theory is the $\mathcal{N} = 6$ ABJM theory. We hope that our analysis will prove also prove useful in the pursuit of its study.

A simplex representation for spinning correlators

A simplex representation for four (and higher) point scalar correlators in CFT has been obtained [53–55]. However, an extension to the spinning cases has not yet been achieved. Supersymmetry, as it connects correlators with different spins could provide a way to obtain analogous simplex representations for spinning correlators starting with the known expression for scalars. We attempted this briefly in section 5 and obtained some preliminary results but it would be good to perform a more detailed and systematic analysis. Extending the technology of weight shifting and spin raising operators to supersymmetric theories also is also another venue of interest.

Making a precise connection to the four dimensional S matrices

The fascinating connection between non supersymmetric correlators in three dimensions and flat space amplitudes in four dimensions has been analyzed thoroughly and well understood. A similar statement for supersymmetric theories has not yet been obtained. Four dimensional super symmetric scattering amplitudes take extremely simple forms and whilst our building

blocks for three point super-correlators (3.6) are similar in form to the four dimensional three point amplitude building blocks [50, 51], we have not attempted to establish a precise connection between the two. Such a connection could facilitate a number of interesting directions such as importing recursion relation techniques such as the BCFW recursion relation to three dimensional SCFTs, color kinematics duality for CFT correlators and so on.

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A Notation, conventions and some useful formulae

In this appendix, we summarize our conventions and notations as well as provide some formulae that we found useful.

Notations

Momentum conservation in all our (super) correlation functions is implicit, i.e we do not explicitly write the momentum conserving Dirac-delta functions. The component fields are presented in italics, whereas superfields are always presented in bold typeface. For instance, J_s is a component field whereas \mathbf{J}_s is a super field. Superfields when expressed in the grassmann twistor variables are denoted in bold and with a tilde such as $\tilde{\mathbf{J}}_s$. The subscripts b or B and f or F for component correlators refer to the free bosonic and free fermionic theory correlators respectively.

Conventions

We work with the usual flat Euclidean metric,

$$\delta_{\mu\nu} = \mathbf{diag}(1, 1, 1), \tag{A.1}$$

with which vector indices are raised and lowered. Since upper and lower indices are identical in this case, we do not need to distinguish between them. Our convention for the three dimensional Levi-Civita symbol is,

$$\begin{aligned} \epsilon_{123} &= \epsilon^{123} = 1, \\ \epsilon_{\mu\nu\rho}\epsilon_{\alpha\beta\gamma} &= \delta_{\mu\alpha}\delta_{\nu\beta}\delta_{\rho\gamma} - \delta_{\mu\beta}\delta_{\nu\alpha}\delta_{\rho\gamma}. \end{aligned} \tag{A.2}$$

Spinor indices on the other hand are raised and lowered using the two dimensional Levi-Civita symbol ϵ_{ab} which is given by,

$$\begin{aligned} \epsilon_{12} &= \epsilon^{12} = 1, \\ \epsilon_{ab}\epsilon^{ac} &= \delta_b^c. \end{aligned} \tag{A.3}$$

Our convention for raising and lowering indices is as follows: given a spinor A_a we have,

$$A^a = \epsilon^{ab} A_b \iff A_a = \epsilon_{ba} A^b. \quad (\text{A.4})$$

Further, spinorial derivatives in our conventions are as follows:

$$\frac{\partial A^a}{\partial A^b} = \delta_b^a. \quad (\text{A.5})$$

However, in contrast to the index raising and lowering for spinors (A.4), we have the following conventions for the derivatives:

$$\epsilon^{ab} \frac{\partial}{\partial A^b} = -\frac{\partial}{\partial A_a}. \quad (\text{A.6})$$

We choose the following representations of the Pauli matrices

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (\text{A.7})$$

which satisfy,

$$(\sigma^\mu)_b^a (\sigma^\nu)_a^c = \delta^{\mu\nu} \delta_b^c + i \epsilon^{\mu\nu\rho} (\sigma^\rho)_b^c. \quad (\text{A.8})$$

Three dimensional spinor-helicity variables

Given a three-vector p_μ , we can trade it for a matrix $(\not{p})_b^a$ in the following way:

$$(\not{p})_b^a = p_\mu (\sigma^\mu)_b^a = \lambda_b \bar{\lambda}^a + p \delta_b^a. \quad (\text{A.9})$$

We can extract the magnitude of the momentum (energy) through the following bracket:

$$p = -\frac{1}{2} \langle \lambda \bar{\lambda} \rangle. \quad (\text{A.10})$$

We can also contract form spinor dot products belonging to different momenta in the following way:

$$\langle ij \rangle = \lambda_{ia} \lambda_j^a. \quad (\text{A.11})$$

Since we work with spinning operators, we require the use of polarization vectors. Our conventions for the same are,

$$z_\mu^- (\sigma^\mu)_b^a = (\not{z}^-)_b^a = \frac{\lambda_b \lambda^a}{p}, \quad (\text{A.12})$$

$$z_\mu^+ (\sigma^\mu)_b^a = (\not{z}^+)_b^a = \frac{\bar{\lambda}_b \bar{\lambda}^a}{p}. \quad (\text{A.13})$$

For a spin half operator we define the polarization spinor to be,

$$\zeta_a^- = \frac{\lambda_a}{\sqrt{p}}, \quad \zeta_a^+ = \frac{\bar{\lambda}_a}{\sqrt{p}}. \quad (\text{A.14})$$

In a correlation function involving n operator insertions, momentum conservation reads,

$$\sum_{i=1}^n p_i^\mu = 0. \quad (\text{A.15})$$

Contracting this equation with $(\sigma_\mu)_b^a$ yields momentum conservation in terms of the spinor variables.

$$\sum_{i=1}^n \lambda_{ib} \bar{\lambda}_i^a = -E \delta_b^a. \quad (\text{A.16})$$

The three dimensional dot product of two three vectors x and y can be written in spinor notation using,

$$x \cdot y = \frac{1}{2} (\not{x})_b^a (\not{y})_a^b. \quad (\text{A.17})$$

Since we work with parity odd correlation functions as well, we will require the following formula:

$$\epsilon^{\mu\nu\rho} = \frac{1}{2i} (\sigma^\mu)_b^a (\sigma^\nu)_a^c (\sigma^\rho)_c^b. \quad (\text{A.18})$$

For any three vectors v_1, v_2 and v_3 we define for convenience,

$$\epsilon^{v_1 v_2 v_3} = v_{1\mu} v_{2\nu} v_{3\rho} \epsilon^{\mu\nu\rho}. \quad (\text{A.19})$$

Some useful spinor-helicity variables identities

Contracting the momentum conservation equation (A.16) with different combinations of spinors, we get

$$\langle ji \rangle \langle \bar{i} \bar{k} \rangle = E \langle j \bar{k} \rangle, \quad (\text{A.20})$$

$$\langle ji \rangle \langle \bar{i} k \rangle = (E - 2p_k) \langle j k \rangle, \quad (\text{A.21})$$

$$\langle \bar{j} i \rangle \langle \bar{i} k \rangle = (E - 2p_j) \langle \bar{j} \bar{k} \rangle, \quad (\text{A.22})$$

$$\langle \bar{j} i \rangle \langle \bar{i} k \rangle = (E - 2p_j - 2p_k) \langle \bar{j} k \rangle, \quad (\text{A.23})$$

$$\langle ji \rangle \langle \bar{i} j \rangle + \langle jk \rangle \langle \bar{k} j \rangle = 0, \quad (\text{A.24})$$

$$\langle ij \rangle \langle \bar{i} \bar{j} \rangle = E(E - 2p_k), \quad (\text{A.25})$$

where i, j and k are all distinct labels.

The dot products of the polarizations with the momenta are given by,

$$\begin{aligned} p_i \cdot z_j^- &= -\frac{\langle ij \rangle \langle \bar{i} \bar{j} \rangle}{2p_j}, & z_i^- \cdot z_j^- &= -\frac{\langle ij \rangle^2}{2p_i p_j}, \\ p_i \cdot z_j^+ &= -\frac{\langle \bar{i} \bar{j} \rangle \langle i j \rangle}{2p_j}, & z_i^- \cdot z_j^+ &= -\frac{\langle \bar{i} \bar{j} \rangle^2}{2p_i p_j}, \\ & & z_i^+ \cdot z_j^+ &= -\frac{\langle i j \rangle^2}{2p_i p_j}. \end{aligned} \quad (\text{A.26})$$

For the contractions of momentum and polarization vectors with the three dimensional Levi-Civita symbol we have,

$$\begin{aligned}
 \epsilon^{z_i^- z_j^- p_k} &= i \frac{\langle ij \rangle \left(\langle i\bar{k} \rangle \langle k\bar{j} \rangle + \langle ij \rangle p_k \right)}{2p_i p_j}, & \epsilon^{z_i^- p_j p_k} &= i \frac{\langle i\bar{k} \rangle \left(\langle ij \rangle \langle k\bar{j} \rangle - \langle ki \rangle p_j \right) + \langle i\bar{j} \rangle \langle ij \rangle p_k}{2p_i}, \\
 \epsilon^{z_i^- z_j^+ p_k} &= i \frac{\langle i\bar{j} \rangle \left(\langle i\bar{k} \rangle \langle k\bar{j} \rangle + \langle i\bar{j} \rangle p_k \right)}{2p_i p_j}, & \epsilon^{z_i^+ p_j p_k} &= i \frac{\langle i\bar{k} \rangle \left(\langle i\bar{j} \rangle \langle k\bar{j} \rangle - \langle k\bar{i} \rangle p_j \right) + \langle i\bar{j} \rangle \langle i\bar{j} \rangle p_k}{2p_i}, \\
 \epsilon^{z_i^+ z_j^+ p_k} &= i \frac{\langle i\bar{j} \rangle \left(\langle i\bar{k} \rangle \langle k\bar{j} \rangle + \langle i\bar{j} \rangle p_k \right)}{2p_i p_j}, & &
 \end{aligned}
 \tag{A.27}$$

We also used the two dimensional Schouten identity on many occasions which reads

$$\delta_f^a \epsilon^{bc} + \delta_f^b \epsilon^{ac} + \delta_f^c \epsilon^{ba} = 0.
 \tag{A.28}$$

Another very important manifestation of the Schouten identity is the following. For any two-component spinor λ_{ia} we decompose it as a linear combination of λ_{1a} and $\bar{\lambda}_{1a}$.

$$\lambda_i = -\frac{\langle i\bar{j} \rangle}{2p_j} \lambda_j + \frac{\langle ij \rangle}{2p_j} \bar{\lambda}_j,
 \tag{A.29}$$

$$\bar{\lambda}_i = -\frac{\langle i\bar{j} \rangle}{2p_j} \lambda_j + \frac{\langle ij \rangle}{2p_j} \bar{\lambda}_j.
 \tag{A.30}$$

B $\mathcal{N} = 1$ and $\mathcal{N} = 2$ superconformal algebra

$\mathcal{N} = 1$ superconformal algebra

The $\mathcal{N} = 1$ superconformal algebra in three dimensions consists of the following generators: the generator of translations, P_μ , the generator of rotations, $M_{\mu\nu}$, the generator of dilatations and the generator of special conformal transformations which are respectively denoted as D and K_μ .

$$\begin{aligned}
 [M_{\mu\nu}, M_{\rho\lambda}] &= i (\delta_{\mu\rho} M_{\nu\lambda} - \delta_{\nu\rho} M_{\mu\lambda} - \delta_{\mu\lambda} M_{\nu\rho} + \delta_{\nu\lambda} M_{\mu\rho}), \\
 [M_{\mu\nu}, P_\alpha] &= i (\delta_{\mu\alpha} P_\nu - \delta_{\nu\alpha} P_\mu), & [M_{\mu\nu}, K_\alpha] &= i (\delta_{\mu\alpha} K_\nu - \delta_{\nu\alpha} K_\mu), \\
 [D, P_\mu] &= iP_\mu, & [P_\mu, K_\nu] &= 2i (\delta_{\mu\nu} D - M_{\mu\nu}), \\
 \{Q_a, Q_b\} &= i (\sigma^\mu)_{ab} P^\mu, & \{S_a, S_b\} &= i (\sigma^\mu)_{ab} K^\mu, \\
 [D, Q_a] &= \frac{i}{2} Q_a, & [D, K_\mu] &= -i K_\mu, \\
 [K_\mu, Q_a] &= i (\sigma_\mu)_a^b S_b, & [D, S_a] &= -\frac{i}{2} S_a, \\
 [M_{\mu\nu}, Q_a] &= \frac{i}{2} \epsilon_{\mu\nu\rho} (\sigma^\rho)_a^b Q_b, & [P_\mu, S_a] &= i (\sigma_\mu)_b^a Q_b, \\
 [M_{\mu\nu}, S_a] &= \frac{i}{2} \epsilon_{\mu\nu\rho} (\sigma^\rho)_a^b S_b, & \{Q_a, S_b\} &= \epsilon_{ab} D - \frac{i}{2} \epsilon_{\mu\nu\rho} (\sigma^\rho)_{ab} M^{\mu\nu}.
 \end{aligned}
 \tag{B.1}$$

The (anti) commutators not listed above are zero. The representation of these generators acting on primary superfields are as follows:

We have,

$$\begin{aligned}
 P_\mu &= -i\partial_\mu, \\
 M_{\mu\nu} &= -i\left(x_\mu\partial_\nu - x_\nu\partial_\mu + \frac{i}{2}\epsilon_{\mu\nu\rho}(\sigma^\rho)_b^a\theta^b\frac{\partial}{\partial\theta^a}\right) + \mathcal{M}_{\mu\nu}, \\
 D &= -i\left(x^\nu\partial_\nu + \frac{1}{2}\theta^a\frac{\partial}{\partial\theta^a} + \Delta\right), \\
 K_\mu &= i\left(2x_\mu x^\nu\partial_\nu - x^2\partial_\mu + 2\Delta x_\mu\right) - 2x^\nu\mathcal{M}_{\mu\nu} + i(\sigma^\mu)_b^c(\sigma^\nu)_c^a x_\nu\theta^b\frac{\partial}{\partial\theta^a}, \\
 Q_a &= \frac{\partial}{\partial\theta^a} + \frac{i}{2}\theta_b(\sigma^\mu)_a^b\partial_\mu,
 \end{aligned} \tag{B.2}$$

where Δ is the scaling dimension of the primary and $\mathcal{M}_{\mu\nu}$ encodes the non trivial transformation of operators with spin. The expression for S_a is obtained by computing $[K_\mu, Q_a]$ as in (B.1). Given the position superspace generators in (B.2), it is straightforward to obtain the momentum superspace generators by performing a Fourier transform to go from the x^μ to the p_μ variables.

$$\begin{aligned}
 P_\mu &= p_\mu, \\
 M_{\mu\nu} &= -i\left(p_\mu\frac{\partial}{\partial p_\nu} - p_\nu\frac{\partial}{\partial p_\mu} + \frac{i}{2}\epsilon_{\mu\nu\rho}(\sigma^\rho)_b^a\theta^b\frac{\partial}{\partial\theta^a}\right) + \mathcal{M}_{\mu\nu}, \\
 D &= i\left(p_\nu\frac{\partial}{\partial p_\nu} + (3 - \Delta) - \frac{1}{2}\theta^a\frac{\partial}{\partial\theta^a}\right), \\
 K_\mu &= -\left(p_\mu\frac{\partial^2}{\partial p_\nu\partial p^\nu} + 2(\Delta - 3)\frac{\partial}{\partial p_\mu} - 2p_\nu\frac{\partial^2}{\partial p_\nu\partial p_\mu}\right) - 2i\frac{\partial}{\partial p^\nu}\mathcal{M}_{\mu\nu} - (\sigma^\mu)_b^c(\sigma^\nu)_c^a\theta^b\frac{\partial}{\partial\theta^a}\frac{\partial}{\partial p^\nu}, \\
 Q_a &= \frac{\partial}{\partial\theta^a} - \frac{1}{2}\theta_b(\sigma^\mu)_a^b p_\mu,
 \end{aligned} \tag{B.3}$$

with S_a obtained via (B.1).

The generators in the super spinor helicity grassmann twistor variables (2.20) are given by,

$$\begin{aligned}
 P_\mu &= \frac{1}{2}(\sigma^\mu)_b^a\lambda_a\bar{\lambda}^b, \\
 M_{\mu\nu} &= \frac{1}{2}\epsilon_{\mu\nu\rho}(\sigma^\rho)_b^a\left(\bar{\lambda}^b\frac{\partial}{\partial\bar{\lambda}^a} + \lambda^b\frac{\partial}{\partial\lambda^a}\right), \\
 D &= \frac{i}{2}\left(\bar{\lambda}^a\frac{\partial}{\partial\bar{\lambda}^a} + \lambda^a\frac{\partial}{\partial\lambda^a} + 2\right), \\
 K^\mu &= 2(\sigma^\mu)^{ab}\frac{\partial^2}{\partial\lambda^a\bar{\lambda}^b}, \\
 Q_a &= \left(2\lambda_a\frac{\partial}{\partial\xi_+} + \frac{\bar{\lambda}_a}{4}\xi_+\right), \\
 S_a &= -2i\left(2\frac{\partial}{\partial\xi_+}\frac{\partial}{\partial\bar{\lambda}^a} + \frac{\xi_+}{4}\frac{\partial}{\partial\lambda^a}\right).
 \end{aligned} \tag{B.4}$$

$\mathcal{N} = 2$ superconformal algebra

Similarly we get the $\mathcal{N} = 2$ algebra, all the generators will remain the same except the Q_a and S_a and we will have two more generators \bar{Q}_a and \bar{S}_a .

Q_a and \bar{Q}_a were given in the Main text (4.4), but we give them here again with S_a and \bar{S}_a .

$$Q_a = 2\lambda_a \frac{\partial}{\partial \omega_+} + \frac{\bar{\lambda}_a}{4} \xi_+, \quad \bar{Q}_a = 2\lambda_a \frac{\partial}{\partial \xi_+} + \frac{\bar{\lambda}_a}{4} \omega_+, \quad (\text{B.5})$$

$$S_a = -2i \left(2 \frac{\partial}{\partial \omega_+} \frac{\partial}{\partial \bar{\lambda}^a} + \frac{\xi_+}{4} \frac{\partial}{\partial \lambda^a} \right), \quad \bar{S}_a = -2i \left(2 \frac{\partial}{\partial \xi_+} \frac{\partial}{\partial \bar{\lambda}^a} + \frac{\omega_+}{4} \frac{\partial}{\partial \lambda^a} \right). \quad (\text{B.6})$$

For the sake of completeness we again write the R-symmetry generator (4.6)

$$R = \omega_+ \frac{\partial}{\partial \omega_+} - \xi_+ \frac{\partial}{\partial \xi_+}. \quad (\text{B.7})$$

C $K_{\text{SCFT}} \implies K_{\text{CFT}}$ for the components

In this section we will prove that the invariance under special conformal transformations of the Super-correlators translates to the conformal invariance of the individual component correlators. Let us first consider an example viz $\langle \mathbf{J}^{(ab)} \mathbf{J}^{(cd)} \mathbf{T}^{(ijkl)} \rangle$. Using the superfield expansion (2.3), we obtain,

$$\langle \mathbf{J}^{(ab)} \mathbf{J}^{(cd)} \mathbf{T}^{(ijkl)} \rangle = \langle J^{ab} J^{cd} T^{ijkl} \rangle - \theta_{1\epsilon} \theta_{2f} \langle J_{3/2}^{abe} J_{3/2}^{cdf} T^{ijkl} \rangle + \dots \quad (\text{C.1})$$

The action of K_μ on a three point function is given by,¹⁸

$$\begin{aligned} K_\mu &= K_{1\mu} + K_{2\mu} \\ &= - \sum_{i=1}^2 \left(p_{i\mu} \frac{\partial^2}{\partial p_{i\nu} \partial p_i^\nu} + 2(\Delta_i - 3) \frac{\partial}{\partial p_{i\mu}} - 2p_{i\nu} \frac{\partial^2}{\partial p_{i\nu} \partial p_{i\mu}} \right) - \sum_{i=1}^2 2i \frac{\partial}{\partial p_i^\nu} \mathcal{M}_{i\mu\nu} \\ &\quad - \sum_{i=1}^2 (\sigma^\mu)_b^c (\sigma^\nu)_c^a \theta_i^b \frac{\partial}{\partial \theta_i^a} \frac{\partial}{\partial p_i^\nu}. \end{aligned} \quad (\text{C.2})$$

Acting with K_μ on (C.1), we get,¹⁹

$$K_\mu \langle \mathbf{J}^{(ab)} \mathbf{J}^{(cd)} \mathbf{T}^{(ijkl)} \rangle = K_\mu \langle J^{ab} J^{cd} T^{ijkl} \rangle - K_\mu (\theta_{1\epsilon} \theta_{2f} \langle J_{3/2}^{abe} J_{3/2}^{cdf} T^{ijkl} \rangle) + \dots \quad (\text{C.3})$$

At the zeroth order of this Grassmann expansion, K_μ^θ (which we define below) doesn't play any role, but it will play a crucial role at higher orders of expansion in getting the correct special conformal transformation. We can write the action abstractly as,

$$\begin{aligned} K_\mu \langle J^{ab} J^{cd} T \rangle &= K_\mu^{\langle JJT \rangle} \langle J^{ab} J^{cd} T \rangle + K_\mu^\theta \langle J^{ab} J^{cd} T \rangle, \\ &= K_\mu^{\langle JJT \rangle} \langle J^{ab} J^{cd} T \rangle + 0, \\ \implies K_\mu \langle J^{ab} J^{cd} T \rangle &= K_\mu^{\langle JJT \rangle} \langle J^{ab} J^{cd} T \rangle, \end{aligned}$$

i.e, the usual conformal ward identity for the component correlator is reproduced.

¹⁸The momentum space generators are provided in (B.3).

¹⁹Note that K_μ does not mix correlators appearing at different orders in the Grassmann spinor expansion.

where we write the first part of K_μ as $K^{\langle JJT \rangle}$, as its form is fixed by the zeroth component,

$$K_\mu^{\langle JJT \rangle} = - \sum_{i=1}^2 \left(p_{i\mu} \frac{\partial^2}{\partial p_{i\nu} \partial p_i^\nu} + 2(\Delta_i - 3) \frac{\partial}{\partial p_{i\mu}} - 2p_{i\nu} \frac{\partial^2}{\partial p_{i\nu} \partial p_{i\mu}} \right) - \sum_{i=1}^2 2i \frac{\partial}{\partial p_i^\nu} \mathcal{M}_{i\mu\nu}, \quad (\text{C.4})$$

$$K_\mu^\theta = - \sum_{i=1}^2 (\sigma^\mu)_b^c (\sigma^\nu)_c^a \theta_i^b \frac{\partial}{\partial \theta_i^a} \frac{\partial}{\partial p_i^\nu}. \quad (\text{C.5})$$

Now, we give the actual form of $K_\mu^{\langle JJT \rangle}$. First, the action of $\mathcal{M}_{\mu\nu}$ on a spinor is given as

$$\mathcal{M}_{\mu\nu} F^a = -\frac{1}{2} \epsilon_{\mu\nu\rho} (\sigma^\rho)_b^a F^b. \quad (\text{C.6})$$

Now, let's evaluate the action of $\mathcal{M}_{\mu\nu}$ part on the zeroth order correlator.

$$\begin{aligned} - \sum_{i=1}^2 2i \frac{\partial}{\partial p_i^\nu} \mathcal{M}_{i\mu\nu} \langle J^{ab} J^{cd} T \rangle &= -2i \left[\frac{\partial}{\partial p_1^\nu} \mathcal{M}_{1\mu\nu} \langle J^{ab} J^{cd} T \rangle + \frac{\partial}{\partial p_2^\nu} \mathcal{M}_{2\mu\nu} \langle J^{ab} J^{cd} T \rangle \right] \\ &= -2i \left[-\frac{1}{2} \epsilon_{\mu\nu\rho} (\sigma^\rho)_k^a \frac{\partial}{\partial p_1^\nu} \langle J^{kb} J^{cd} T \rangle - \frac{1}{2} \epsilon_{\mu\nu\rho} (\sigma^\rho)_k^b \frac{\partial}{\partial p_1^\nu} \langle J^{ak} J^{cd} T \rangle \right. \\ &\quad \left. - \frac{1}{2} \epsilon_{\mu\nu\rho} (\sigma^\rho)_k^c \frac{\partial}{\partial p_2^\nu} \langle J^{ab} J^{kd} T \rangle - \frac{1}{2} \epsilon_{\mu\nu\rho} (\sigma^\rho)_k^d \frac{\partial}{\partial p_2^\nu} \langle J^{ab} J^{ck} T \rangle \right] \end{aligned} \quad (\text{C.7})$$

Hence, the representation of $\mathcal{M}_{\mu\nu}$ part is fixed, and it acts on the a, b index of first insertion and on the c, d index of second insertion. Thus, the whole expression can be given as

$$\begin{aligned} K_\mu^{\langle JJT \rangle} \langle J^{ab} J^{cd} T \rangle &= \\ \left[- \sum_{i=1}^2 \left(p_{i\mu} \frac{\partial^2}{\partial p_{i\nu} \partial p_i^\nu} + 2(\Delta_i - 3) \frac{\partial}{\partial p_{i\mu}} - 2p_{i\nu} \frac{\partial^2}{\partial p_{i\nu} \partial p_{i\mu}} \right) - \sum_{i=1}^2 2i \frac{\partial}{\partial p_i^\nu} \mathcal{M}_{i\mu\nu} \right] \langle J^{ab} J^{cd} T \rangle, \end{aligned} \quad (\text{C.8})$$

where $\Delta_i = 2$ i.e. the scaling dimension of spin-1 conserved currents, and the $\mathcal{M}_{\mu\nu}$ action is given by (C.7). Hence, the action of $K_\mu^{\langle JJT \rangle}$ is fixed by the zeroth order, given in (C.8).

At the second order i.e. $O(\theta_1 \theta_2)$, we obtain,

$$\begin{aligned} K_\mu(\theta_{1e} \theta_{2f} \langle J_{3/2}^{abe} J_{3/2}^{cdf} T \rangle) &= \\ &= \theta_{1e} \theta_{2f} K_\mu^{\langle JJT \rangle} \langle J_{3/2}^{abe} J_{3/2}^{cdf} T \rangle + K_\theta^\mu \left(\theta_{1e} \theta_{2f} \langle J_{3/2}^{abe} J_{3/2}^{cdf} T \rangle \right), \\ &= \theta_{1e} \theta_{2f} \left[- \sum_{i=1}^2 \left(p_{i\mu} \frac{\partial^2}{\partial p_{i\nu} \partial p_i^\nu} + 2(\Delta_i - 3) \frac{\partial}{\partial p_{i\mu}} - 2p_{i\nu} \frac{\partial^2}{\partial p_{i\nu} \partial p_{i\mu}} \right) - \sum_{i=1}^2 2i \frac{\partial}{\partial p_i^\nu} \mathcal{M}_{i\mu\nu} \right] \langle J_{3/2}^{abe} J_{3/2}^{cdf} T \rangle \\ &\quad - \left[\sum_{i=1}^2 (\sigma^\mu)_b^c (\sigma^\nu)_c^a \theta_i^b \frac{\partial}{\partial \theta_i^a} \frac{\partial}{\partial p_i^\nu} \right] \left(\theta_{1e} \theta_{2f} \langle J_{3/2}^{abe} J_{3/2}^{cdf} T \rangle \right), \end{aligned} \quad (\text{C.9})$$

where $\Delta_i = 2$ are scaling dimensions of the first two insertions in $\langle JJT \rangle$.

Consider now the third of line of (C.9). We have,

$$\begin{aligned}
 K_\mu^\theta \left(\theta_{1e} \theta_{2f} \langle J_{3/2}^{abe} J_{3/2}^{cdf} T \rangle \right) &= \\
 &= - \left[\sum_{i=1}^2 (\sigma^\mu)_b^c (\sigma^\nu)_c^a \theta_i^b \frac{\partial}{\partial \theta_i^a} \frac{\partial}{\partial p_i^\nu} \right] (\theta_{1e} \theta_{2f}) \langle J_{3/2}^{abe} J_{3/2}^{cdf} T \rangle, \\
 &= - \left(\delta^{\mu\nu} \delta_k^l + i \epsilon^{\mu\nu\rho} (\sigma^\rho)_k^l \right) \left(\theta_1^k \theta_{2f} \epsilon_{le} \frac{\partial}{\partial p_1^\nu} \langle J_{3/2}^{abe} J_{3/2}^{cdf} T \rangle + \theta_{1e} \theta_2^k \epsilon_{lf} \frac{\partial}{\partial p_2^\nu} \langle J_{3/2}^{abe} J_{3/2}^{cdf} T \rangle \right), \\
 &= - \theta_{1e} \theta_{2f} \left[\frac{\partial}{\partial p_1^\mu} \langle J_{3/2}^{abe} J_{3/2}^{cdf} T \rangle + \frac{\partial}{\partial p_2^\mu} \langle J_{3/2}^{abe} J_{3/2}^{cdf} T \rangle \right] \\
 &\quad + \theta_{1e} \theta_{2f} \left[i \epsilon^{\mu\nu\rho} (\sigma_\rho)_k^e \frac{\partial}{\partial p_1^\nu} \langle J_{3/2}^{abk} J_{3/2}^{cdf} T \rangle + i \epsilon^{\mu\nu\rho} (\sigma_\rho)_k^f \frac{\partial}{\partial p_2^\nu} \langle J_{3/2}^{abe} J_{3/2}^{cdk} T \rangle \right], \\
 &= - 2 \theta_{1e} \theta_{2f} \left[\frac{1}{2} \frac{\partial}{\partial p_1^\mu} \langle J_{3/2}^{abe} J_{3/2}^{cdf} T \rangle + \frac{1}{2} \frac{\partial}{\partial p_2^\mu} \langle J_{3/2}^{abe} J_{3/2}^{cdf} T \rangle \right] \\
 &\quad - 2 \theta_{1e} \theta_{2f} \left[- \frac{1}{2} \epsilon^{\mu\nu\rho} (\sigma_\rho)_k^e \frac{\partial}{\partial p_1^\nu} \langle J_{3/2}^{abk} J_{3/2}^{cdf} T \rangle - \frac{1}{2} \epsilon^{\mu\nu\rho} (\sigma_\rho)_k^f \frac{\partial}{\partial p_2^\nu} \langle J_{3/2}^{abe} J_{3/2}^{cdk} T \rangle \right]. \tag{C.10}
 \end{aligned}$$

Hence,

$$\begin{aligned}
 K_\mu^\theta \left(\theta_{1e} \theta_{2f} \langle J_{3/2}^{abe} J_{3/2}^{cdf} T \rangle \right) &= - 2 \theta_{1e} \theta_{2f} \left[\frac{1}{2} \frac{\partial}{\partial p_1^\mu} \langle J_{3/2}^{abe} J_{3/2}^{cdf} T \rangle + \frac{1}{2} \frac{\partial}{\partial p_2^\mu} \langle J_{3/2}^{abe} J_{3/2}^{cdf} T \rangle \right] \\
 &\quad - 2 \theta_{1e} \theta_{2f} \left[- \frac{1}{2} \epsilon^{\mu\nu\rho} (\sigma_\rho)_k^e \frac{\partial}{\partial p_1^\nu} \langle J_{3/2}^{abk} J_{3/2}^{cdf} T \rangle - \frac{1}{2} \epsilon^{\mu\nu\rho} (\sigma_\rho)_k^f \frac{\partial}{\partial p_2^\nu} \langle J_{3/2}^{abe} J_{3/2}^{cdk} T \rangle \right]. \tag{C.11}
 \end{aligned}$$

Using (C.11) in (C.9), it becomes,

$$\begin{aligned}
 K_\mu \left(\theta_{1e} \theta_{2f} \langle J_{3/2}^{abe} J_{3/2}^{cdf} T \rangle \right) &= \\
 &= \theta_{1e} \theta_{2f} K_\mu^{\langle JJT \rangle} \langle J_{3/2}^{abe} J_{3/2}^{cdf} T \rangle + K_\mu^\theta \left(\theta_{1e} \theta_{2f} \langle J_{3/2}^{abe} J_{3/2}^{cdf} T \rangle \right), \\
 &= \theta_{1e} \theta_{2f} \left[- \sum_{i=1}^2 \left(p_{i\mu} \frac{\partial^2}{\partial p_{i\nu} \partial p_i^\nu} + 2 \left(\Delta_i + \frac{1}{2} - 3 \right) \frac{\partial}{\partial p_{i\mu}} - 2 p_{i\nu} \frac{\partial^2}{\partial p_{i\nu} \partial p_{i\mu}} \right) - \sum_{i=1}^2 2i \frac{\partial}{\partial p_i^\nu} \mathcal{M}_{i\mu\nu} \right] \langle J_{3/2}^{abe} J_{3/2}^{cdf} T \rangle, \\
 &= \theta_{1e} \theta_{2f} \left(K_\mu^{\langle J_{3/2} J_{3/2} T \rangle} \langle J_{3/2}^{abe} J_{3/2}^{cdf} T \rangle \right). \tag{C.12}
 \end{aligned}$$

Hence, after the addition of K_μ^θ action to $K_\mu^{\langle JJT \rangle}$, we get the correct action on $\langle J_{3/2} J_{3/2} T \rangle$. This can be seen as the dimensions have shifted, $\Delta_i \rightarrow \Delta_i + \frac{1}{2}$, which are the correct scaling dimension of spin-3/2 conserved current. Also, the correct action of the index e and f for both first and second insertion is added to the $\mathcal{M}_{\mu\nu}$ part. Thus making it the special conformal transformation for the component correlator. This method can be applied to any component. Hence, we have shown that the invariance under special conformal transformations of the Super-correlators translates to the conformal invariance of the individual component correlators

D A worked out example of a $\mathcal{N} = 1$ three point function

In this appendix, we outline the steps for solving for $\mathcal{N} = 1$ three-point correlators in the Grassmann twistor variables, demonstrating the significant reduction of work in getting

the constraints between different component correlator. Consider the Super-correlator $\langle \tilde{\mathbf{J}}_{3/2} \tilde{\mathbf{J}}_{1/2} \tilde{\mathbf{J}}_{1/2} \rangle$. We have, (see equation (3.11))

$$\langle \tilde{\mathbf{J}}_{3/2}^- \tilde{\mathbf{J}}_{1/2}^- \tilde{\mathbf{J}}_{1/2}^- \rangle = \frac{E}{32 \langle \bar{1} \bar{2} \rangle \sqrt{p_3}} \langle J_{3/2}^- O_{1/2}^- J^- \rangle \Xi_3, \quad (\text{D.1})$$

$$\langle \tilde{\mathbf{J}}_{3/2}^- \tilde{\mathbf{J}}_{1/2}^- \tilde{\mathbf{J}}_{1/2}^+ \rangle = -\frac{\xi_{3-}}{256 \sqrt{p_3}} \langle J_{3/2}^- O_{1/2}^- J^+ \rangle \Gamma_3. \quad (\text{D.2})$$

In equation (D.1), we can extract the component correlators on the l.h.s. using the superfield expansion (2.22) and compare with the corresponding expression in the r.h.s. Let's first consider (D.1). By matching the l.h.s. and r.h.s. at the zeroth order, $\xi_{2+} \xi_{3+}$, and $\xi_{1+} \xi_{3+}$, we get the following relations

$$\langle T^- J^- J^- \rangle = \frac{2E \sqrt{p_1 p_2}}{\langle \bar{1} \bar{2} \rangle} \langle J_{3/2}^- O_{1/2}^- J^- \rangle, \quad (\text{D.3})$$

$$\langle T^- O_{1/2}^- O_{1/2}^- \rangle = \frac{\langle \bar{2} \bar{3} \rangle \sqrt{p_1}}{\langle \bar{1} \bar{2} \rangle \sqrt{p_3}} \langle J_{3/2}^- O_{1/2}^- J^- \rangle, \quad (\text{D.4})$$

$$\langle J_{3/2}^- J^- O_{1/2}^- \rangle = \frac{\langle \bar{1} \bar{3} \rangle \sqrt{p_2}}{\langle \bar{1} \bar{2} \rangle \sqrt{p_3}} \langle J_{3/2}^- O_{1/2}^- J^- \rangle. \quad (\text{D.5})$$

This implies that,

$$\langle T^- J^- J^- \rangle = \frac{2E \sqrt{p_2 p_3}}{\langle \bar{2} \bar{3} \rangle} \langle T^- O_{1/2}^- O_{1/2}^- \rangle, \quad \langle T^- J^- J^- \rangle = \frac{2E \sqrt{p_1 p_3}}{\langle \bar{1} \bar{3} \rangle} \langle J_{3/2}^- J^- O_{1/2}^- \rangle. \quad (\text{D.6})$$

Similarly, in the for $(- - +)$ helicity (D.2) we get the following relations between the component correlators:

$$\langle T^- J^- J^+ \rangle = 2\sqrt{p_2 p_3} \frac{\langle 12 \rangle}{\langle 31 \rangle} \langle T^- O_{1/2}^- O_{1/2}^+ \rangle, \quad \langle T^- J^- J^+ \rangle = 2\sqrt{p_1 p_3} \frac{\langle 12 \rangle}{\langle 23 \rangle} \langle J_{3/2}^- J^- O_{1/2}^+ \rangle. \quad (\text{D.7})$$

Using the decomposition of a three dimensional CFT correlator as in (2.12), we can write explicit forms for these correlators. Substituting these forms in equations (D.6) and (D.7), we can constrain their individual coefficients and reduce the independent coefficients from six to one. Then, correlators can be written as

$$\langle TJJ \rangle = n_b (\langle TJJ \rangle_B + \langle TJJ \rangle_F), \quad (\text{D.8})$$

$$\langle TO_{1/2} O_{1/2} \rangle = n_b (\langle TO_{1/2} O_{1/2} \rangle_B + \langle TO_{1/2} O_{1/2} \rangle_F), \quad (\text{D.9})$$

$$\langle J_{3/2} JO_{1/2} \rangle = n_b (\langle J_{3/2} JO_{1/2} \rangle_B + \langle J_{3/2} JO_{1/2} \rangle_F). \quad (\text{D.10})$$

Thus, we obtain exactly one parity even solution, which is exactly in line with the results of [45].

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