New compact construction of eigenstates for supersymmetric spin chains

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ABSTRACT: The problem of separation of variables (SoV) in supersymmetric spin chains is closely related to the calculation of correlation functions in $\mathcal{N} = 4$ SYM theory which is integrable in the planar limit. To address this question we find a compact formula for the spin chain eigenstates, which does not have any sums over auxiliary roots one usually gets in the widely adopted nested Bethe ansatz. Our construction only involves one application of a simple $B^g(u_k)$ operator to the reference state for each of the magnons, in complete analogy with the $\mathfrak{su}(2)$ algebraic Bethe ansatz. This generalizes our SoV based construction for $\mathfrak{su}(n)$ to the supersymmetric $\mathfrak{su}(1|2)$ case.

KEYWORDS: Bethe Ansatz, Lattice Integrable Models

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Contents

1 Introduction

Quantum spin chains serve as a prototypical example of an integrable model combining remarkable algebraic structures with physical relevance. Their supersymmetric versions based on $\mathfrak{su}(m|n)$ superalgebras have also been extensively studied and appear in a wide range of contexts from condensed matter [1, 2] to integrable AdS/CFT [3]. While the spectrum of integrable spin chains is typically governed by a concise system of Bethe equations, it is much more difficult to explicitly construct the eigenstates of the spin chain Hamiltonian. The construction of eigenstates is relevant in particular for the calculation of correlators in $\mathcal{N} = 4$ SYM which is actively being explored (see e.g. [4–9]).

For the simplest spin chains with a rank-1 symmetry algebra such as $\mathfrak{su}(2)$ or $\mathfrak{su}(1|1)$, one can efficiently build the states via algebraic Bethe ansatz, by repeatedly acting on the vacuum with a single 'creation' operator $B(u)$,

$$|\Psi\rangle = B(u_1)B(u_2)\ldots B(u_K)|0\rangle \tag{1.1}$$

where u_i are the Bethe roots which define the excitations' momenta in the spin chain. However, for higher rank spin chains, the standard construction of eigenstates is much more involved. In particular, in the standard nested Bethe ansatz approach the problem is solved recursively, by reducing it to the solution of simpler spin chains with lower rank symmetry [10–12]. The resulting expression for the eigenstate is a complicated sum in which the number of terms grows exponentially with the number of excitations.[1]

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[1] Other remarkable constructions are known, but they are also rather complicated and typically suffer from exponential complexity as well, see e.g. [13–21] and the review [22]. The eigenstates problem has also been discussed in a pure mathematics context, see e.g. the recent works [23].

Surprisingly, it was recently realized in [24] that for rational $\mathfrak{su}(n)$ spin chains it is possible to completely bypass this standard recursive procedure. In fact for any $\mathfrak{su}(n)$ one can build an operator $B^g(u)$ which[2] generates the states just as in the simplest $\mathfrak{su}(2)$ case, by repeated action on the vacuum state:

$$|\Psi\rangle = B^g(u_1)B^g(u_2)\ldots B^g(u_K)|0\rangle\,. \tag{1.2}$$

This operator B^g is an explicit polynomial in the monodromy matrix entries, and it is the same for any spin chain length and number of excitations. The parameters u_i in (1.2) are the momentum-carrying Bethe roots fixed by standard nested Bethe equations or by the Baxter equation. Thus, instead of a complicated nested sum the eigenstate is simply given by one term (1.2). For spin chains in the fundamental representation of $\mathfrak{su}(n)$ the construction was extensively checked numerically and proven in several special cases [24]. For $\mathfrak{su}(2)$ spin chains it already has nontrivial aspects which were explored further in [26] (see also [27] and [28, 29]). Very recently, and with remarkable effort, it was proven rigorously for $\mathfrak{su}(3)$ in [25], and was also shown there to work for any symmetric representation of $\mathfrak{su}(3)$ on the spin chain sites.

In this paper we show how to extend this highly compact construction of eigenstates to the supersymmetric case. We focus on the first nontrivial example of a higher rank super spin chain, which corresponds to the $\mathfrak{su}(1|2)$ superalgebra [30–33]. The $\mathfrak{su}(1|2)$ spin chains are important from a physical point of view as they describe the supersymmetric t-J model widely studied in the context of superconductivity [34, 35]. At the same time, they are interesting conceptually due to their intermediate place in complexity between $\mathfrak{su}(2)$ and $\mathfrak{su}(3)$ models. In particular, the challenging problem of finding compact expressions for scalar products of $\mathfrak{su}(3)$ Bethe states [36–42] (e.g. finding an analog of the remarkable Slavnov determinant [43]) seems to be much more tractable in the $\mathfrak{su}(1|2)$ case [44–49].[3]

We propose an explicit expression for the B^g operator which allows one to build the states simply by repeatedly acting on the vacuum, as in (1.2). In contrast to the $\mathfrak{su}(n)$ case studied in [24], here B^g is not even a polynomial in the monodromy matrix entries. This makes the difference between our construction and the usual nested Bethe ansatz even more striking. While in the bosonic $\mathfrak{su}(n)$ case B^g is written in terms of certain determinants (quantum minors) built from the monodromy matrix, here we find they should be replaced by Berezinians which are known to be non-polynomial. At the same time, by a simple redefinition of the monodromy matrix (mutiplication by an explicit scalar function) we can still render the B^g operator a polynomial in u for the spin chain we consider, and its degree[4] is $2L-1$.

The construction of [24] is directly related to Sklyanin's separation of variables (SoV) program [54, 55], which consists of finding special variables in which the dynamics of a many-particle integrable system decouples into a set of non-interacting one dimensional

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[2]In [24] this operator was denoted by B^{good}. Here we use the shorter notation B^g, also utilized in [25].

[3]Related results for general $\mathfrak{su}(m|n)$ spin chains were obtained in [49–53].

[4]With a more general twist the degree could likely become $2L$, which is the same as for the $\mathfrak{su}(3)$ operator B^g once we extract a trivial factor from it [24]. See the discussion in section 3.

models. In fact eigenstates of the same operator B^g provide the basis of separated co-ordinates for the $\mathfrak{su}(n)$ spin chain (see also [54]). Factorization of the wavefunctions in this basis follows immediately from the construction (1.2) of eigenstates [24]. However, in the supersymmetric case it is not known how to obtain the separated variables even for the simplest $\mathfrak{su}(1|1)$ models. Thus it is all the more nontrivial that a direct analog of the formula for eigenstates (1.2) exists for higher rank super spin chains.

Implementation of the SoV for supersymmetric models remains an important future goal, especially in view of its relevance for $\mathcal{N} = 4$ SYM where the symmetry algebra is $\mathfrak{psu}(2,2|4)$. In the simpler $\mathfrak{su}(1|1)$ case we managed to overcome some of the obstacles towards SoV and we present these results in appendix A. Namely, we propose a B^g operator for $\mathfrak{su}(1|1)$ which, although it does not give separated variables, is diagonalizable unlike the standard B, while having several other curious properties.

This paper is organized as follows. In section 2 we present our notation and overview of supersymmetric spin chains, and also discuss briefly the $\mathfrak{su}(1|1)$ case. Section 3 contains our main results, namely the construction of eigenstates for $\mathfrak{su}(1|2)$ spin chains using only a single operator B^g. We also present the construction for the spin chain with the $(2|1)$ choice of grading, which, although similar to the $(1|2)$ case, is technically different. We conclude in section 4. In appendix A we discuss some observations on the SoV for $\mathfrak{su}(1|1)$ spin chains, in particular presenting the improved B operator.

Note added. When finishing the draft we learned about a work in progress [56] where related results were obtained independently.

2 Supersymmetric spin chains overview

In this section we review the standard algebraic Bethe ansatz description of super spin chains (see e.g. [12, 22] for a review). In the process we introduce notation used in the rest of the paper.

We will work with graded vector spaces, however we will use only standard complex numbers rather than Grassmann variables. A graded vector space $\mathbb{C}^{m|n}$ consists of vectors v with components $v_i \in \mathbb{C}$ with $i = 1, \ldots, m+n$. We assign a parity $[i]$ to the indices so that

$$[i] = 0 \quad \text{for} \quad i = 1, \ldots, m, \qquad [i] = 1 \quad \text{for} \quad i = m+1, \ldots, n. \tag{2.1}$$

This space $\mathbb{C}^{m|n}$ realizes the fundamental representation of $\mathfrak{su}(m|n)$. The Hilbert space $\mathcal{H}$ of the spin chain is a tensor product of L copies of the space $\mathbb{C}^{m|n}$,

$$\mathcal{H} = \mathbb{C}^{m|n} \otimes \mathbb{C}^{m|n} \otimes \cdots \otimes \mathbb{C}^{m|n}. \tag{2.2}$$

The algebraic construction of an integrable quantum spin chain is based on an R-matrix. The standard rational R-matrix in the supersymmetric case acts on $\mathbb{C}^{m|n} \otimes \mathbb{C}^{m|n}$ and is given by[5]

$$R^{ij}_{kl}(u) = \delta^i_k \delta^j_l + \frac{i}{u} \delta^i_l \delta^j_k (-1)^{[i][j]}, \tag{2.3}$$

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[5]The R-matrix is $\mathfrak{gl}(m|n)$ invariant, but we will speak about $\mathfrak{su}(m|n)$ as the symmetry algebra to emphasize that we consider a finite-dimensional representation at each site of the chain.

where the extra signs correspond to using a graded permutation operator. Let us also mention that we will often use the notation

$$f^{\pm} \equiv f(u \pm i/2), \quad f^{[+a]} \equiv f(u + ia/2) \tag{2.4}$$

for shifts of the spectral parameter.

Multiplying several R-matrices together we obtain the monodromy matrix $T(u)$ defining the spin chain, which acts in the tensor product of the Hilbert space $\mathcal{H}$ and an auxiliary space $\mathbb{C}^{m|n}$,

$$T(u) = R_{01}(u - \theta_1) \otimes R_{02}(u - \theta_2) \otimes \cdots \otimes R_{0L}(u - \theta_L) \otimes g. \tag{2.5}$$

We have introduced an extra twist matrix g which acts in the auxiliary space only and corresponds to twisted boundary conditions. We take it to be diagonal,

$$g = \text{diag}\,(\lambda_1, \lambda_2, \ldots, \lambda_{m+n})\,, \tag{2.6}$$

and assume that the twists λ_i are all distinct and in generic position. They serve as regulators in the construction, and also ensure there is a 1-to-1 correspondence between spin chain states and solutions of the nested Bethe equations. We also introduced the parameters θ_L which correspond to inhomogeneities of the spin chain.[6]

The definition of the tensor product of operators in (2.5) is nontrivial in the super-symmetric case and involves extra signs reflecting the graded nature of the vector spaces $\mathbb{C}^{m|n}$, see e.g. [30]. Explicitly, the matrix elements of $T(u)$ read

$$T^{a_1 a_2 \ldots a_L\ j}_{a'_1 a'_2 \ldots a'_L\ j'}(u) = R^{a_1 j}_{a'_1 j''}(u - \theta_1) R^{a_2 j}_{a'_2 j''}(u - \theta_2) \ldots R^{a_L j}_{a'_L j''}(u - \theta_L)\, g^{j''}_{j'} \tag{2.7}$$
$$\times (-1)^{\sum_{\alpha=2}^{L} \sum_{\beta=1}^{\alpha-1} [a'_\beta]([a_\alpha]+[a'_\alpha])}$$

where the indices $a_1, a_2, \ldots$ and $a'_1, a'_2, \ldots$ correspond to individual $\mathbb{C}^{m|n}$ factors of the Hilbert space (2.2) while j, j' label the auxiliary space, and we assume summation over repeated indices.

Although one could consider spin chains with arbitrary representations of $\mathfrak{su}(m|n)$ in the physical and the auxiliary spaces, we only discuss the case when both representations are fundamental. Let us note, however, that other representations on the sites of the chain can be obtained by fusion from the fundamental one, corresponding to a special choice of θ's.

It is very useful to view $T(u)$ as a matrix of size $(m+n) \times (m+n)$ whose elements T^i_j act on the physical Hilbert space $\mathcal{H}$. This matrix satisfies a graded version of the celebrated RTT relation which in components reads[7]

$$R^{ji}_{i'j'}(u - v) T^{i'}_{i''}(u) T^{j'}_{j''}(v)\, (-1)^{[j''][j']} = T^i_{i'}(v) T^j_{j'}(u) R^{j'i'}_{i''j''}(u - v)\, (-1)^{[i'][j]}\,. \tag{2.8}$$

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[6]Although most of the checks we present later in this paper have been done assuming that all θ_k are in generic position, we expect our results to be valid for any choice of θ's.

[7]The notation we use is slightly different compared to [49, 51], so that we have $T^{\text{there}}_{ij} = (-1)^{[j]([i]+1)} T^i_j$. Our notation ensures in particular that the Berezinian has the standard form given below in (3.4), while the notation of [49, 51] would lead to an extra sign in that expression.

Below we will not distinguish between T_j^i and T_{ij}, in order to write some expressions more concisely.

Another key object is the transfer matrix, defined as the supertrace of the monodromy matrix,

$$\mathcal{T}(u) \equiv \operatorname{str} T(u) = \sum_{i=1}^{m+n} (-1)^{[i]}\, T_i^i(u)\,. \tag{2.9}$$

One can show that as a consequence of the RTT relation, the transfer matrices form a commutative family,

$$[\mathcal{T}(u),\ \mathcal{T}(v)] = 0\,, \tag{2.10}$$

which in particular includes the Hamiltonian of the spin chain. Expanding $\mathcal{T}(u)$ as a series in u one therefore obtains a large set of conserved charges commuting wth the Hamiltonian. The main problem we study in this paper is constructing the common eigenbasis of these operators. A particularly simple eigenvector is given by

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \otimes \cdots \otimes \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}\,, \tag{2.11}$$

and plays the role of the 'vacuum' reference state in the algebraic Bethe ansatz.

As an example, let us briefly discuss the $\mathfrak{su}(1|1)$ case which is explored in more detail in appendix A. For $\mathfrak{su}(1|1)$ we may write T as a 2×2 matrix whose entries act on the Hilbert space,

$$T(u) = \begin{pmatrix} A(u) & B(u) \\ C(u) & D(u) \end{pmatrix}\,. \tag{2.12}$$

Then the transfer matrix is given by

$$\mathcal{T}(u) = A(u) - D(u)\,. \tag{2.13}$$

The B operator serves as a creation operator generating the eigenstates $|\Psi\rangle$ of the transfer matrix,

$$|\Psi\rangle = B(u_1)B(u_2)\dots B(u_K)|0\rangle\,, \tag{2.14}$$

provided u_i are fixed by the Bethe equations which read

$$\frac{\lambda_1}{\lambda_2} \prod_{k=1}^{L} \frac{u_j - \theta_k + i/2}{u_j - \theta_k - i/2} = 1,\ \ j = 1,\dots,K\ \ . \tag{2.15}$$

In the next section we extend this highly compact construction of eigenstates to the higher rank $\mathfrak{su}(1|2)$ spin chains.

3 Eigenstates for $\mathfrak{su}(1|2)$ spin chains

In this section we present our main result — the new compact construction of eigenstates for $\mathfrak{su}(1|2)$ spin chains. We first discuss the spin chain with the $(1|2)$ grading in detail and then present the generalization to the $(2|1)$ grading which is similar but technically different. For a pedagogical discussion of the $\mathfrak{su}(1|2)$ algebra and associated spin chains see e.g. [57, 58].[8]

The construction of states we propose is inspired by [24] where it was shown that one can build an operator B^g which generates states for $\mathfrak{su}(n)$ spin chains simply by repeated action on the vacuum as in (1.2), like in the $\mathfrak{su}(2)$ case. This operator is constructed from quantum minors of the monodromy matrix, which are defined as determinants of submatrices of $T(u)$ with extra shifts of the spectral parameter u. These quantum minors are generalizations of the quantum determinant [61]. Explicitly, an $n \times n$ quantum minor is given by[9]

$$T_{j_1,\ldots,j_n|\,k_1,\ldots,k_n}(u) = \sum_{\sigma \in S_n} (-1)^{\text{sign}(\sigma)} T_{j_{\sigma(1)}k_1}(u) T_{j_{\sigma(2)}k_2}(u+i) \ldots T_{j_{\sigma(n)}k_n}(u+ni). \quad (3.1)$$

With this notation the B^g operator for the $\mathfrak{su}(3)$ case takes the simple form

$$B^g(u) = T_{1|3}(u)T_{12|13}(u-i) + T_{2|3}(u)T_{12|23}(u-i), \quad \text{with} \quad T_{ij} \to T_{ij}^g, \quad (3.2)$$

where we indicated that T_{ij} are substituted by the elements of the improved monodromy matrix which is defined by

$$T^g(u) = K^{-1}T(u)K \quad (3.3)$$

with K a generic 3×3 constant matrix. The extra similarity transformation given by K renders the construction non-degenerate while preserving all commutation relations between entries of $T(u)$. It also leaves unchanged the trace of $T(u)$ and consequently the spin chain Hamiltonian.

Our main observation is that in the $\mathfrak{su}(1|2)$ case one should replace the quantum minors appearing in (3.2) by Berezinians which play the role of determinants for supermatrices [62, 63]. For a 2×2 matrix split into four blocks A, B, C, D we define the Berezinian as

$$\text{Ber}\begin{pmatrix} A(u) & B(u) \\ C(u) & D(u) \end{pmatrix} = \left(A(u) - B(u)D^{-1}(u)C(u)\right)D^{-1}(u). \quad (3.4)$$

Applying this formula to the monodromy matrix of an $\mathfrak{su}(1|1)$ spin chain (see (A.1)) gives the operator $\text{Ber}\,T(u)$ which is a central element of the Yangian $Y(\mathfrak{gl}(1|1))$, i.e. it commutes with $A(v), B(v), C(v)$ and $D(v)$ for all values of u and v. This shows that the Berezinian plays the same role for $\mathfrak{su}(1|1)$ as the quantum determinant does for the $\mathfrak{su}(2)$ case. We recall that the quantum determinant for an $\mathfrak{su}(2)$ spin chain monodromy matrix reads

$$\text{qdet}\begin{pmatrix} A(u) & B(u) \\ C(u) & D(u) \end{pmatrix} = A(u)D(u+i) - C(u)B(u+i) \quad (3.5)$$

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[8]Mathematical aspects of representations of the corresponding Yangians were discussed in [59, 60].

[9]We note that the vertical slash appearing in the l.h.s. of (3.1) is unrelated to the graded vector space notation such as $\mathfrak{su}(m|n)$.

and coincides with the 2×2 quantum minor defined in (3.1).

Notice that there are no shifts of the spectral parameter in the 2×2 Berezinian (3.4), in contrast[10] to the quantum determinant (3.5). Due to this we will not make a distinction between the quantum Berezinian and the usual Berezinian of a 2×2 block matrix (both are given by (3.4)).

We will denote the Berezinians similarly to the quantum minors in (3.1), namely

$$\text{Ber}_{i_1 i_2 | j_1 j_2}(u) \equiv \text{Ber} \begin{pmatrix} T_{i_1 j_1}(u) & T_{i_1 j_2}(u) \\ T_{i_2 j_1}(u) & T_{i_2 j_2}(u) \end{pmatrix} . \tag{3.6}$$

With this notation the B^g operator for $\mathfrak{su}(1|2)$ is obtained by simply replacing the quantum minors in the $\mathfrak{su}(3)$ result (3.2) by the Berezinians,

$$B^g(u) = T_{1|3}(u)\text{Ber}_{12|13}(u) + T_{2|3}(u)\text{Ber}_{12|23}(u) \ , \quad \text{with} \ \ T_{ij} \to T_{ij}^g , \tag{3.7}$$

where again one should use elements of the improved monodromy matrix T^g. It is defined by (3.3) like in the $\mathfrak{su}(3)$ case, with the sole difference being that K should only have nonzero entries in its 'even' diagonal blocks,

$$K = \begin{pmatrix} K_{11} & 0 & 0 \\ \hline 0 & K_{22} & K_{23} \\ 0 & K_{32} & K_{33} \end{pmatrix} , \tag{3.8}$$

where the lines emphasize the splitting of K into odd and even elements as an operator on the graded vector space $\mathbb{C}^{1|2}$. The construction works as long as K is a generic matrix of this type. Let us also note that in contrast to the $\mathfrak{su}(3)$ case, there are no shifts of u at all in (3.7).

The main property of this operator is that it allows one to build the transfer matrix eigenstates just as for $\mathfrak{su}(1|1)$ or $\mathfrak{su}(2)$, by repeated action on the vacuum! Namely,

$$|\Psi\rangle = B^g(u_1)B^g(u_2)\dots B^g(u_K)|0\rangle . \tag{3.9}$$

The state $|0\rangle$ here is the standard reference state (2.11),

$$|0\rangle = \begin{pmatrix} 1 \\ \hline 0 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ \hline 0 \\ 0 \end{pmatrix} \otimes \dots \otimes \begin{pmatrix} 1 \\ \hline 0 \\ 0 \end{pmatrix} \tag{3.10}$$

where the horizontal lines again highlight the (1|2) grading of the $\mathbb{C}^{1|2}$ space at each site. Like for $\mathfrak{su}(3)$, the u_i in (3.9) are the momentum-carrying Bethe roots. One way to fix them is to solve the set of usual nested Bethe ansatz equations, which also include auxiliary roots v_j,

$$\prod_{k=1}^{L} \frac{u_i - \theta_k + i/2}{u_i - \theta_k - i/2} = \frac{\lambda_2}{\lambda_1} \prod_{j=1}^{N} \frac{u_i - v_j + i/2}{u_i - v_j - i/2}, \qquad i = 1, \dots, K \tag{3.11}$$

$$\prod_{j=1}^{K} \frac{v_i - u_j + i/2}{v_i - u_j - i/2} = -\frac{\lambda_2}{\lambda_3} \prod_{j=1}^{N} \frac{v_i - v_j + i}{v_i - v_j - i}, \qquad i = 1, \dots, N. \tag{3.12}$$

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[10]Shifts of u do appear in quantum Berezinians of higher size T-matrices [64–68] which generate the center of the Yangian for higher rank spin chains.

The only role of the auxiliary roots is that they indirectly affect the values of the main roots u_i through the Bethe equations.[11] Let us also note that in terms of the Bethe roots the eigenvalues of the transfer matrix read (as one can deduce via standard methods)

$$\mathcal{T}(u) = Q_\theta^- \left[\lambda_1 \frac{Q_\theta^+ Q_u^{--}}{Q_\theta^- Q_u} - \lambda_2 \frac{Q_u^{--} Q_v^+}{Q_u Q_v^-} - \lambda_3 \frac{Q_v^{---}}{Q_v^-} \right] , \qquad (3.13)$$

where

$$Q_\theta(u) = \prod_{k=1}^{L} (u - \theta_k), \quad Q_u = \prod_{j=1}^{K} (u - u_j), \quad Q_v = \prod_{j=1}^{N} (u - v_j) \qquad (3.14)$$

and we also used the compact notation (2.4).

Our construction of the states is clearly free from the recursion inherent in the standard nested Bethe ansatz, where the eigenstates are built in terms of the wavefunctions of an auxiliary lower rank spin chain. Our approach involves only a single operator B^g acting repeatedly on the vacuum. Curiously, while in the nested Bethe ansatz states are built by polynomial combinations of monodromy matrix elements acting on the vacuum (with complicated state-dependent coefficients), the B^g operator is not even a polynomial in the T_{ij} operators.[12] It is also not a polynomial of u in the representation we consider, but we observed that one can make the dependence on u polynomial by multiplying the T_{ij} matrix by a scalar function, namely by replacing $T_{jk}(u) \to Q_\theta(u - \frac{i}{2})T_{jk}(u)$. After that $B^g(u)$ becomes a polynomial of degree $2L - 1$. With a more general twist K (e.g. one involving off-diagonal Grassmann entries) it might be possible to make the degree $2L$, i.e. the same as it is for the B^g operator in $\mathfrak{su}(3)$ once we remove from it a trivial overall factor [24].

Let us highlight a peculiar structural feature of the B^g operator for $\mathfrak{su}(1|2)$. Although all the monodromy matrix entries $T_{ij}(u)$ are just operators acting on the Hilbert space, it is useful to label them as either bosonic/even ($\mathcal{B}$) or fermionic/odd ($\mathcal{F}$) depending on their position inside T viewed as a (1|2) supermatrix, so that schematically

$$T(u) = \begin{pmatrix} \mathcal{B} & \mathcal{F} & \mathcal{F} \\ \hline \mathcal{F} & \mathcal{B} & \mathcal{B} \\ \mathcal{F} & \mathcal{B} & \mathcal{B} \end{pmatrix} . \qquad (3.15)$$

The Berezinian is naturally defined for 2×2 matrices with the standard grading $\begin{pmatrix} \mathcal{B} & \mathcal{F} \\ \mathcal{F} & \mathcal{B} \end{pmatrix}$. While the first Berezinian in the expression (3.7) for B^g is indeed applied to a matrix with this grading, the second one is evaluated for a matrix of the type $\begin{pmatrix} \mathcal{F} & \mathcal{F} \\ \mathcal{B} & \mathcal{B} \end{pmatrix}$. When computing B^g we simply evaluate this Berezinian formally using the definition (3.4). It would be highly interesting to understand the algebraic meaning of such non-conventional super determinants.

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[11]We consider spin chains with generic twists λ_i, which lift degeneracies and ensure that the states are in 1-to-1 correspondence with solutions of the Bethe equations.

[12]Let us note that for $\mathfrak{su}(3)$ one can also write B^g as a non-polynomial combination of T_{ij}, using that the 2×2 quantum minors entering (3.2) and given by (3.5) can be equivalently written as $A(u)(D(u+i) - C(u+i)A^{-1}(u+i)B(u+i))$. For $\mathfrak{su}(1|2)$, however, our B^g operator cannot be recast as a polynomial of T_{ij}, as it is not polynomial in u.

While the proposed construction of eigenstates (3.9) should be regarded as a conjecture, we have extensively checked it numerically. We verified that it produces all the states for the spin chain with $L = 1, 2, 3$ or 4 sites. We also tested it for several states with up to four excitations for $L = 5$, where we already have large 243×243 matrices. In addition, we have proven it analytically for the case with one excitation for any spin chain length L, with a particular simple choice of K

$$ K = \begin{pmatrix} 1 & 0 & 0 \\ \hline 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} . \tag{3.16} $$

The proof is essentially by brute force and follows the one for $\mathfrak{su}(3)$ in [24]. It is based on the fact that the vacuum is an eigenstate for all elements of T_{ij}^g except T_{12}^g and T_{13}^g which serve as creation operators. Namely, we have

$$ T_{ij}^g(u)|0\rangle = 0, \ i > j \tag{3.17} $$

and also

$$ T_{11}^g(u)|0\rangle = \lambda_1 Q_\theta^+ |0\rangle, \qquad T_{22}^g(u)|0\rangle = \lambda_2 Q_\theta^- |0\rangle, \qquad T_{33}^g(u)|0\rangle = \lambda_3 Q_\theta^- |0\rangle, \tag{3.18} $$
$$ T_{23}^g(u)|0\rangle = (\lambda_2 - \lambda_3) Q_\theta^- |0\rangle, \tag{3.19} $$

as one can verify similarly to the $\mathfrak{su}(3)$ case [24].[13] Then using the RTT relations to commute all T_{ij}^g to the right of T_{12}^g and T_{13}^g until they hit the vacuum state $|0\rangle$, we get

$$ B^g(u)|0\rangle = \lambda_2 Q_\theta^- \left(T_{12}^g(u) + \frac{\lambda_2 Q_\theta^- - \lambda_1 Q_\theta^+}{(\lambda_2 - \lambda_3) Q_\theta^-} T_{13}^g(u) \right) |0\rangle . \tag{3.20} $$

Acting on this expression with the transfer matrix one can similarly show that it is an eigenstate on the solutions of Bethe equations.[14] We leave for the future a full general proof for any number of excitations, and hope it can be done using the recent techniques of [25].

Let us note that the $B^g(u)$ operators for $\mathfrak{su}(1|2)$ do not commute at different values of u, and thus naively are not suitable for definition of separated variables (in contrast to the $\mathfrak{su}(n)$ case [24]). Perhaps one may still be able to implement the SoV in some modified way, e.g. making the separated coordinates noncommutative or Grassmannian. We leave this as an important open question for the future. Curiously, we observed[15] that in the standard basis the matrix elements of $B^g(u)B^g(v)$ and $B^g(v)B^g(u)$ are either equal or are related via multiplication by $-\frac{u-v-i}{u-v+i}$. This factor is furthermore precisely the one appearing in the commutation relation of the standard $B(u)$ operators in the $\mathfrak{su}(1|1)$ case (see (A.9)). The same observation is true for the $\mathfrak{su}(1|1)$ B^g operator we present in appendix A. It would be highly interesting to understand the algebraic implications of these commutation relations.

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[13]For the original monodromy matrix elements $T_{ij}(u)$ we have the same action on the vacuum (3.17), (3.18) but instead of (3.19) we find that T_{23} annihilates the vacuum.

[14]Extending this proof to 2 magnons is already nontrivial due to the need to commute $(T_{23}^g)^{-1}$ appearing in the Berezinians through other elements of T^g.

[15]For the first few values of L.

In the $\mathfrak{su}(n)$ case one could use the same operator B^g to build the states starting from a different reference state, using solutions of the appropriate dual Bethe equations corresponding to a particle-hole transformation in the Bethe ansatz. The B^g operator we constructed here for $\mathfrak{su}(1|2)$ does not have the same property as it annihilates the states which could serve as alternative pseudovacua, namely

$$|0'\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \otimes \cdots \otimes \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \tag{3.21}$$

and

$$|0''\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \otimes \cdots \otimes \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} . \tag{3.22}$$

This property is also related to the fact that the B^g operator is nilpotent for a general K of the form (3.8), and thus cannot be diagonalized. This serves as another obstacle to implementing the SoV, as for $\mathfrak{su}(n)$ the eigenvectors of B^g play a key role since they define the basis of separated coordinates in which the wavefunction factorizes (see the discussion in section A.1). However, in the $\mathfrak{su}(1|1)$ case we managed to circumvent this problem by considering a more general K matrix which gives a diagonalizable B^g operator as discussed in appendix A. We hope that this approach may be adapted to the higher rank case, and in addition one could try to use a K matrix with Grassmann elements in the off-diagonal blocks (see section A.2), though the interpretation of the resulting operator remains to be clarified. In any case, we believe that the very existence of the construction of the states (3.9) for $\mathfrak{su}(1|2)$ is encouraging for the prospect of developing the SoV program in the future.

3.1 Extension to the (2|1) grading

While above we discussed the $\mathfrak{su}(1|2)$ spin chains based on the R-matrix with (1|2) grading, one can alternatively consider a spin chain built from the R-matrix with grading chosen as (2|1) (corresponding to $m = 2$ and $n = 1$ in the notation of section 2). This spin chain still realizes the $\mathfrak{su}(1|2)$ symmetry but differs technically from the case we considered. Here we present our construction of eigenstates for this choice of grading, which should also provide important guidance towards its generalization to any $\mathfrak{su}(m|n)$ model.

At the level of transfer matrix eigenvalues, the difference between two choices of the grading can be stated explicitly using the expression for eigenvalues in terms of the Bethe roots, which for the (2|1) case reads[16]

$$\mathcal{T}(u) = -Q_\theta^- \left[\lambda_3 \frac{Q_\theta^{--} Q_u}{Q_\theta^- Q_u^-} - \lambda_2 \frac{Q_u Q_v^{---}}{Q_u^- Q_v^-} - \lambda_1 \frac{Q_v^+}{Q_v} \right] , \tag{3.23}$$

where as before we define

$$Q_u = \prod_{j=1}^{K} (u - u_j) , \quad Q_v = \prod_{j=1}^{N} (u - v_j) , \tag{3.24}$$

---

[16]We have extensively checked this result numerically and it can also be proven using the standard nested Bethe ansatz.

and the Bethe roots u_i, v_j are fixed by standard nested Bethe equations

$$\prod_{k=1}^{L} \frac{u_i - \theta_k + i/2}{u_i - \theta_k - i/2} = \frac{\lambda_3}{\lambda_2} \prod_{j=1}^{N} \frac{u_i - v_j + i/2}{u_i - v_j - i/2}, \qquad i = 1, \dots, K \qquad (3.25)$$

$$\prod_{j=1}^{K} \frac{v_i - u_j + i/2}{v_i - u_j - i/2} = -\frac{\lambda_1}{\lambda_2} \prod_{j=1}^{N} \frac{v_i - v_j + i}{v_i - v_j - i}, \qquad i = 1, \dots, N. \qquad (3.26)$$

Notice that the only difference with the $(1|2)$ Bethe equations (3.11), (3.12) is a reshuffling of the twists λ_i. Comparing (3.23) with the eigenvalues of $\mathcal{T}$ for the $(1|2)$ grading given in (3.13), we see that the eigenvalues for the two gradings are mapped to each other if we apply complex conjugation supplemented by a shift of u, permutation of the twists and overall change of sign,

$$\mathcal{T}^{(1|2)}(\lambda_1, \lambda_2, \lambda_3, \theta_i, u) = -\left[\mathcal{T}^{(2|1)}(\lambda_3^*, \lambda_2^*, \lambda_1^*, \theta_i^*, u^* + i) \right]^*. \qquad (3.27)$$

This equality holds at the level of eigenvalues, with a suitable one-to-one identification between eigenvectors in the two models.

Despite the simplicity of this map, there seems to be no simple relation between $\mathcal{T}(u)$ (or other entries $T_{ij}(u)$) as operators in the standard basis for the two choices of grading, making the realization of our construction in the $(2|1)$ case nontrivial. However, we found that there still exists a B^g operator which allows one to generate the eigenstates, and it reads

$$B^g = T_{32}(u+i)\mathrm{Ber}_{31|32}(u+i) + T_{12}(u+i)\mathrm{Ber}_{31|12}(u+i) \ , \quad \text{with} \ \ T_{ij} \to T_{ij}^g, \qquad (3.28)$$

where now to define $T^g = K^{-1}TK$ we should use a matrix K that is generic but has nonzero entries only in the diagonal blocks corresponding to the $(2|1)$ grading,

$$K = \left(\begin{array}{cc|c} K_{11} & K_{12} & 0 \\ K_{21} & K_{22} & 0 \\ \hline 0 & 0 & K_{33} \end{array} \right). \qquad (3.29)$$

This expression should be compared with the B^g operator for the $(1|2)$ grading given in (3.7). We see there is an extra shift of u in the result for the $(2|1)$ case. Up to this shift, if we replace the Berezinians in both results by the usual quantum minors, we find that both expressions are instances of the $\mathfrak{su}(3)$ B^g operators, simply corresponding to different choices of K for $\mathfrak{su}(3)$. However, for super spin chains only block-diagonal matrices K (like (3.8) or (3.29)) are allowed, so the B^g operators cannot be mapped to each other by adjusting K. Moreover, the B^g operator in the $(2|1)$ case has to act on the dual vacuum of the form[17]

$$|0''\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \otimes \cdots \otimes \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}. \qquad (3.30)$$

---

[17]For completeness we note that the T_{ij} operators in the $(2|1)$ case act on the standard reference state $|0\rangle$ defined in (2.11) as $T_{11}(u)|0\rangle = \lambda_1 Q_\theta^+ |0\rangle$, $T_{22}(u)|0\rangle = \lambda_2 Q_\theta^- |0\rangle$, $T_{33}(u)|0\rangle = \lambda_3 Q_\theta^- |0\rangle$, $T_{23}(u)|0\rangle = 0$, and $T_{ij}(u)|0\rangle = 0$ for $i > j$. For the dual vacuum $|0''\rangle$ we have $T_{11}(u)|0''\rangle = \lambda_1 Q_\theta^- |0''\rangle$, $T_{22}(u)|0''\rangle = \lambda_2 Q_\theta^- |0''\rangle$, $T_{33}(u)|0''\rangle = \lambda_3 Q_\theta^{---} |0''\rangle$, $T_{21}(u)|0''\rangle = 0$, and $T_{ij}(u)|0''\rangle = 0$ for $i < j$.

That is rather natural as this is the image of the original $|0\rangle$ vacuum under the map from $\mathbb{C}^{1|2}$ to $\mathbb{C}^{2|1}$. The states are built as

$$|\Psi\rangle = B^g(u_1)\dots B^g(u_K)|0''\rangle \tag{3.31}$$

where as usual u_i are the momentum-carrying roots fixed by the nested Bethe equations given above in (3.25), (3.26).

As for the $(1|2)$ grading, we have checked numerically that this operator generates the full basis of states for spin chain length $L = 1, 2, 3$ and 4, as well as several states for $L = 5$ with up to four magnons. Its other properties also directly parallel the $(1|2)$ case, in particular it is nilpotent, is not suitable for generating states starting from a different vacuum, and the entries of $B^g(u)B^g(v)$ are related with those of $B^g(v)B^g(u)$ via multiplication by $-\frac{u-v-i}{u-v+i}$ in the cases when they are not equal. Lastly, let us note that the extra shift by i in the result (3.28) is rather intriguing and is similar to the shift needed for $\mathfrak{su}(1|1)$ when using the C operators to build the states from the dual vacuum as discussed in appendix A.

4   Conclusions

In this paper we presented a new and highly compact construction for the eigenstates of higher-rank supersymmetric rational spin chain, for the first nontrivial example which is $\mathfrak{su}(1|2)$. It is inspired by the analogous proposal in the $\mathfrak{su}(n)$ case which in turn has its roots in the separation of variables approach. We find it rather nontrivial that an analogous construction of states exists for $\mathfrak{su}(1|2)$ despite the fact that there is no known implementation of the SoV in the supersymmetric case.

While we have checked the proposal extensively, it would be interesting to prove it rigorously, which is likely to be possible in view of the recent proof in the $\mathfrak{su}(3)$ case [25]. It would be also highly important, though challenging, to uncover the algebraic origins of the operator B^g which generates the states, and to understand its interpretation within the Yangian. In particular, it would be interesting to understand the algebraic meaning of non-conventional super quantum minors entering our $\mathfrak{su}(1|2)$ construction (see discussion after (3.15)). A better algebraic understanding would be important for extending our construction to any $\mathfrak{su}(m|n)$, which is one of the key future directions.

While we have focused on the spin chains with a fundamental representation at each site, we hope the construction should work directly for many other representations, as already proven in the $\mathfrak{su}(3)$ case [25]. It would be important to generalize both the bosonic and supersymmetric constructions to arbitrary representations, in particular to the anti-symmetric representation of $\mathfrak{su}(4)$ relevant for 1-point functions in $\mathcal{N} = 4$ SYM with a defect [69–71]), as well as to noncompact spin chains. Another curious direction is to look for relations with the construction of spin chain Q-operators [72–74]. It is also interesting to explore deformations of our construction corresponding to the trigonometric XXZ case and to the Gaudin models (either bosonic or supersymmetric [75]). In the latter case one may expect an interplay with the remarkable Knizhnik-Zamolodchikov equations [76, 77].

We hope that our results should help to shed light on the yet to be developed SoV program for the supersymmetric case. We present some first steps towards the SoV for $\mathfrak{su}(1|1)$

spin chains in appendix A. Since in the bosonic case the SoV leads to remarkable results for correlators (see e.g. [78–80]), one may hope for similar simplifications in supersymmetric models. For $\mathcal{N} = 4$ SYM drastic simplification of certain correlators in separated variables was observed very recently in [8] (see also [9] and [81–86]), making further development of the SoV program all the more important.

Acknowledgments

A Comments on $\mathfrak{su}(1|1)$ spin chains

In this appendix we discuss the simplest supersymmetric spin chains with $\mathfrak{su}(1|1)$ symmetry. We will see that despite their simplicity it is not clear how to explicitly construct the basis of Sklyanin's separated variables, and we will make some first steps in this direction.

In the $\mathfrak{su}(1|1)$ case, the monodromy matrix $T(u)$ is a 2×2 matrix whose entries A, B, C, D act on the physical Hilbert space,

$$T(u) = \begin{pmatrix} A(u) & B(u) \\ C(u) & D(u) \end{pmatrix} . \tag{A.1}$$

The transfer matrix is given by its supertrace,

$$\mathcal{T}(u) = A(u) - D(u), \tag{A.2}$$

and defines a commutative family of operators,

$$[\mathcal{T}(u), \mathcal{T}(v)] = 0. \tag{A.3}$$

The standard way to construct its eigenstates is by using the $B(u)$ operator as a creation operator on top of the reference state $|0\rangle$ defined by (2.11). The eigenstates are then given by

$$|\Psi\rangle = B(u_1)B(u_2)\ldots B(u_K)|0\rangle \tag{A.4}$$

where u_j are the Bethe roots satisfying the $\mathfrak{su}(1|1)$ Bethe equations,

$$\frac{\lambda_1}{\lambda_2} \prod_{k=1}^{L} \frac{u_j - \theta_k + i/2}{u_j - \theta_k - i/2} = 1, \;\; j = 1, \ldots, K \;\;. \tag{A.5}$$

Let us note that the l.h.s. of (A.5) is the ratio of eigenvalues of $A(u)$ and $D(u)$ on the vector $|0\rangle$,

$$A(u)|0\rangle = \prod_{k=1}^{L}(u_j - \theta_k + i/2), \quad D(u)|0\rangle = \prod_{k=1}^{L}(u_j - \theta_k - i/2). \quad \text{(A.6)}$$

In order to prove that (A.4) gives an eigenstate of $\mathcal{T}$ one uses commutation relations between entries of $T(u)$ following from the RTT relation (2.8). In particular, we have

$$A(u)B(v) = \frac{u - v - i}{u - v}B(v)A(u) + \frac{i}{u - v}B(u)A(v). \quad \text{(A.7)}$$

Moreover $D(u)$ satisfies exactly the same commutation relation (in contrast to the $\mathfrak{su}(2)$ case). This means that we have a commutation relation between B and the full $\mathcal{T} = A - D$,

$$\mathcal{T}(u)B(v) = \frac{u - v - i}{u - v}B(v)\mathcal{T}(u) + \frac{i}{u - v}B(u)\mathcal{T}(v) \quad \text{(A.8)}$$

Using also that as a consequence of the RTT relation

$$B(u)B(v) = -\frac{u - v - i}{u - v + i}B(v)B(u), \quad \text{(A.9)}$$

one can now easily commute $\mathcal{T}(u)$ through all the B operators in (A.4) until it hits $|0\rangle$ which is its eigenstate due to (A.6). It is not hard to show that all unwanted terms generated in the process will cancel due to Bethe equations (A.5), ensuring that $|\Psi\rangle$ is indeed an eigenstate.

Note that the r.h.s. of (A.5) does not include any interaction between the Bethe roots, which are in this sense independent from each other, and moreover they should all be pairwise distinct. This makes the spin chain somewhat similar to a model of free fermions. It is clear that for a given L the complete set of possible Bethe roots is fixed from (A.5) with $K = L$, and any particular state is specified by choosing a subset of these roots.

For completeness let us also discuss how to write the analog of the Baxter $T - Q$ equations for $\mathfrak{su}(1|1)$. Introducing the Q-functions

$$Q_1 = \prod_{j=1}^{K}(u - u_j), \quad Q_\theta = \prod_{k=1}^{L}(u - \theta_k), \quad \text{(A.10)}$$

we can rewrite the Bethe equations (A.5) as

$$\frac{Q_\theta^+(u_j)}{Q_\theta^-(u_j)} = \frac{\lambda_2}{\lambda_1}. \quad \text{(A.11)}$$

Equivalently, we can write the QQ relation

$$\lambda_1 Q_\theta^+ - \lambda_2 Q_\theta^- = (\lambda_1 - \lambda_2)Q_1 Q_2, \quad \text{(A.12)}$$

where Q_2 is also a polynomial. We see that the l.h.s. of this equation does not depend on the state, and the state is specified simply by selecting K Bethe roots out of the zeros of the l.h.s. These will be the zeros of Q_1, while the other zeros of the l.h.s. will be attributed

to Q_2. One can say that for any particular state Q_2 contains those Bethe roots which are not activated for this state.

In terms of the Q-functions the eigenvalue of $\mathcal{T}(u)$ has the simple form

$$\mathcal{T} = (\lambda_1 - \lambda_2)Q_1^{--}Q_2\,, \tag{A.13}$$

and combining this with the QQ relation (A.12) we get the analog of the Baxter equation,

$$\mathcal{T}Q = (\lambda_1 Q_\theta^+ - \lambda_2 Q_\theta^-)Q^{--}\,. \tag{A.14}$$

A.1 Separation of variables overview

While the construction of eigenstates (A.4) for $\mathfrak{su}(1|1)$ directly parallels the $\mathfrak{su}(2)$ case, a crucial difference is that the B operators no longer commute with each other and instead satisfy a Zamolodchikov-Faddeev type relation (A.9). This prevents immediate realization of Sklyanin's separation of variables program for $\mathfrak{su}(1|1)$ and makes it a nontrivial open question. Let us recall briefly how the SoV works for $\mathfrak{su}(2)$ spin chains. The B operators in that case commute,

$$[B^{\mathfrak{su}(2)}(u), B^{\mathfrak{su}(2)}(v)] = 0\,, \tag{A.15}$$

and therefore one can define the commuting operator roots of B denoted as operators x_k,

$$B^{\mathfrak{su}(2)}(u) = B_0 \prod_{k=1}^{L}(u - x_k)\,, \tag{A.16}$$

where B_0 is a constant. The x_k play the role of separated coordinates, and in their common eigenbasis labelled by their eigenvalues x_k we have

$$\langle \mathrm{x}_1, \ldots, \mathrm{x}_L | B^{\mathfrak{su}(2)}(u) = C \prod_{k=1}^{L}(u - \mathrm{x}_k)\langle \mathrm{x}_1, \ldots, \mathrm{x}_L| \tag{A.17}$$

The eigenstates of the transfer matrix can be again built as in (A.4) with the only difference being in the explicit form of Bethe equations satisfied by u_j. Then we see that in the common eigenbasis of x_k the wavefunction factorizes,

$$\langle \mathrm{x}_1, \ldots, \mathrm{x}_L | \Psi \rangle = \prod_{k=1}^{L}(-1)^K Q_1(\mathrm{x}_k)\,, \tag{A.18}$$

where the Q-function $Q_1(u)$ is defined by (A.10) and encodes the Bethe roots. The factorization of the wavefunction into Q-functions in (A.18) shows that the separation of variables has been achieved in the $\mathfrak{su}(2)$ case.

The main problem for $\mathfrak{su}(1|1)$ models is that the B operators do not commute, so one cannot diagonalize their roots x_k simultaneously, making it unclear how to construct the basis $\langle \mathrm{x}_1, \ldots, \mathrm{x}_L|$ of separated coordinates. For $\mathfrak{su}(1|1)$ spin chains there is also another obstacle — namely, the standard B operator is nilpotent and cannot be diagonalized at all.[18] In the next section we will show how to resolve at least this problem, serving as a first step towards the construction of the SoV.

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[18]Informally speaking, the reason why B is nilpotent is that acting with it many times on the reference state $|0\rangle$ we will eventually reach the state where all spins have been flipped, and this state is annihilated by B.

A.2 Improving the B operator

As we discussed above, one problem preventing the SoV implementation for $\mathfrak{su}(1|1)$ spin chains is the fact that $B(u)$ is a nilpotent operator and cannot be diagonalized. In fact the same problem is present also in the $\mathfrak{su}(2)$ case where B is nilpotent as well. There it can be circumvented by redefining the monodromy matrix via an extra similarity transformation with a generic 2×2 constant matrix K acting in the auxiliary space [24, 85],

$$T(u) \to T^g(u) = K^{-1}T(u)K \,. \tag{A.19}$$

This transformation removes degeneracy and makes the new B operator diagonalizable, moreover it is a symmetry of the R-matrix and thus preserves all commutation relations (as well as the trace of T).

For supersymmetric $\mathfrak{su}(m|n)$ spin chains the transformation (A.19) would only preserve the commutation relations if elements of K in its off-diagonal $m \times n$ and $n \times m$ blocks are treated as Grassmann variables anticommuting also with elements of $T(u)$ in the same blocks. However, the resulting B operator for $\mathfrak{su}(1|1)$ would contain Grassmann variables, making unclear the interpretation of its eigenstates and eigenvectors. Nevertheless we can use this approach idea as an inspiration and formally consider for $\mathfrak{su}(1|1)$ the new monodromy matrix

$$T^g = \begin{pmatrix} 1 & \beta \\ 0 & 1 \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} 1 & \alpha \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} A + \beta C & B + A\alpha + \beta D + \beta C\alpha \\ C & D + C\alpha \end{pmatrix} \tag{A.20}$$

where α and β are Grassmann variables commuting with A, D but anticommuting with B, C as well as with each other. Requiring the supertrace of T to be preserved we find $\beta = -\alpha$. Then we can read off the new B-operator $B^g \equiv T^g_{12}$, so explicitly

$$B^g = B(u) + \alpha(A(u) - D(u)) \,. \tag{A.21}$$

The key observation is that one can take α in this equation (A.21) to be a generic complex number rather than a formal Grassmann parameter. The resulting operator will be diagonalizable, and moreover it will still generate the eigenstates! That is, we can again build the eigenstates as

$$|\Psi\rangle = B^g(u_1) \dots B^g(u_K)|0\rangle \tag{A.22}$$

The reason for this is that B^g satisfies exactly the same commutation relation (A.8) with $\mathcal{T}$ as B did,

$$\mathcal{T}(u)B^g(v) = \frac{u - v - i}{u - v}B^g(v)\mathcal{T}(u) + \frac{i}{u - v}B^g(u)\mathcal{T}(v) \,, \tag{A.23}$$

which one can check explicitly using (A.8). This immediately means that the B^g operator is suitable for building eigenstates.

The advantage of using the $B^g(u)$ operator is that unlike $B(u)$ it can be diagonalized. Its eigenvalues read[19]

$$(\lambda_1 - \lambda_2)\alpha \prod_{k=1}^{L}(u - u_k - is_k), \quad s_k = \{0, 1\} \,. \tag{A.24}$$

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[19]We checked this explicitly for the first few values of L.

Curiously, they coincide with eigenvalues of $\mathcal{T}(u)$ (given in (A.13)) up to a simple and u-independent factor. However, the meaning of this fact is not completely clear yet, especially since the $B^g(u)$ operators do not commute for different values of u so their eigenvectors depend on u.

We did not find any simple commutation relation such as (A.9) for two B^g operators. However, curiously, we observed that in the standard basis all matrix elements of $B^g(u)B^g(v)$ and $B^g(v)B^g(u)$ are either equal or are related via multiplication by the same factor as in the B commutation relation (A.9).[20]

Let us also note that the states created by B^g do not depend on α at all. The reason for this is that while $\mathcal{T}$ appears inside the expression for B^g, on Bethe roots we have

$$\mathcal{T}(u_i)|0\rangle = 0 \,, \tag{A.25}$$

which can be easily checked. Thus when we repeatedly act on $|0\rangle$ with B^g we can commute all $\mathcal{T}$'s to the right of the B's using (A.8) until the $\mathcal{T}$'s act on the reference state $|0\rangle$ which is annihilated by them, leaving no dependence on α.

A.3 B^g and dual roots

One can alternatively try to construct the states starting from a different pseudovacuum state where all spins have been flipped,

$$|0'\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \cdots \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \,. \tag{A.26}$$

Then in the standard approach one can build the states using the C operator instead of B,

$$|\Psi\rangle = C(v_1 + i) \ldots C(v_{K'} + i)|0'\rangle \,, \tag{A.27}$$

where the dual Bethe roots v_i satisfy the same Bethe equations as before,

$$\frac{Q_\theta^+(v_j)}{Q_\theta^-(v_j)} = \frac{\lambda_2}{\lambda_1} \,, \quad j = 1, \ldots, K' \quad. \tag{A.28}$$

We shifted the arguments of C operators by i in (A.27) so as to have the Bethe equations take the conventional form (A.28). It is not hard to prove that this gives eigenstates, by using the RTT relations as well as

$$A(u)|0'\rangle = Q_\theta^-|0'\rangle, \quad D(u)|0'\rangle = Q_\theta^{---}|0'\rangle \,. \tag{A.29}$$

In the $\mathfrak{su}(n)$ case one could build the states with B^g starting from any of the n dual pseudovacuum states and using the Bethe roots that solve the corresponding dual Bethe equations. However, we found that the operator B^g we constructed for $\mathfrak{su}(1|1)$ can build states only starting from the usual vacuum $|0\rangle$. The reason for this is that B^g is a linear combination which does not include the C operator, so the dual vacuum $|0'\rangle$ is just an eigenstate of $B^g(u)$ for all u. It would be interesting to see if one may improve the B^g operator even further, and we leave this question for the future.

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[20]We were also able to construct other operators that generate states and satisfy commutation relations of the type (A.9) with different nontrivial factors in the r.h.s. (in one case we get operators that simply anticommute).

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