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Higher spin supercurrents in anti-de Sitter space

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Abstract: We propose higher spin supercurrent multiplets for $\mathcal{N} = 1$ supersymmetric field theories in four-dimensional anti-de Sitter space (AdS). Their explicit realisations are derived for various supersymmetric theories, including a model of N massive chiral scalar superfields with an arbitrary mass matrix. We also present new off-shell gauge formulations for the massless $\mathcal{N} = 1$ supersymmetric multiplet of integer superspin s in AdS, where $s = 2, 3, \ldots$, as well as for the massless gravitino multiplet (superspin $s = 1$) which requires special consideration.

Keywords: Supergravity Models, Superspaces, Supersymmetric Effective Theories, Supersymmetry and Duality

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1 Introduction

There exist only five maximally supersymmetric backgrounds in off-shell $\mathcal{N}=1$ supergravity in four dimensions [1].[1] Only two of them support maximally symmetric spacetimes. The latter backgrounds are: Minkowski superspace $\mathbb{M}^{4|4}$ [6, 7] and anti-de Sitter (AdS) superspace $\mathrm{AdS}^{4|4}$ [8–10].

The structure of consistent supercurrent multiplets in $\mathcal{N}=1$ AdS supersymmetry [11, 12] considerably differs from that in the $\mathcal{N}=1$ super-Poincaré case, see e.g. [13, 14]. Specifically, there exist three minimal supercurrents with 12 + 12 degrees of freedom in $\mathbb{M}^{4|4}$ [14], and only one in $\mathrm{AdS}^{4|4}$ [11], the latter being the AdS extension of the Ferrara-Zumino supercurrent [15]. Furthermore, the so-called $\mathcal{S}$-multiplet advocated by Komargodski and Seiberg [16] does not admit a minimal extension to AdS.[2] These differences between the supercurrent multiplets in $\mathbb{M}^{4|4}$ and $\mathrm{AdS}^{4|4}$ have nontrivial dynamical implications. For instance, since every $\mathcal{N}=1$ supersymmetric field theory in AdS should have a well-defined Ferrara-Zumino supercurrent [11, 12], the Kähler target space of every supersymmetric nonlinear σ-model in AdS must be non-compact and possess an exact Kähler two-form, in accordance with the analysis of Komargodski and Seiberg [16].[3] The same conclusion was also obtained by direct studies of the most general $\mathcal{N}=1$ supersymmetric nonlinear σ-models in AdS [1, 20].

It should be pointed out that the consistent AdS supercurrents [11, 12] are closely related to two classes of supersymmetric gauge theories: (i) the known off-shell formulations, minimal (see, e.g., [3, 21] for reviews) and non-minimal [12], for $\mathcal{N}=1$ AdS supergravity; and (ii) the two dually equivalent series of massless higher spin supermultiplets in AdS proposed in [22]. More specifically, as discussed in [12], there are only two irreducible AdS supercurrents, with (12+12) and (20+20) degrees of freedom.[4] The former is naturally associated with the so-called longitudinal action $S^{||}_{(3/2)}$ for a massless superspin-3/2 multiplet in AdS [22], which is formulated in terms of a real vector prepotential $H_{\alpha\dot\alpha}$ and a covariantly chiral superfield σ. The latter is associated with a unique dual formulation $S^{\perp}_{(3/2)}$ where the chiral superfield is replaced by a complex linear superfield Γ. The functional $S^{||}_{(3/2)}$

[1] The classification by Festuccia and Seiberg [1] was given purely at the component level. It was re-derived in [2] using the superspace formalism developed in the mid-1990s [3]. As curved $\mathcal{N}=1$ superspaces, all maximally supersymmetric backgrounds were described in [4] (see also [5] for a new derivation of the results in [2, 4], which works equally well for all known off-shell formulations for $\mathcal{N}=1$ supergravity).

[2] The consistent supergravity extension of the $\mathcal{S}$-multiplet was given in [11].

[3] In the $\mathcal{N}=2$ extended case, AdS supersymmetry imposes nontrivial restrictions on the structure of the hyperkähler target spaces of supersymmetric nonlinear σ-models [17–19].

[4] These supercurrents are related to each other via a well-defined improvement transformation [12].

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proves to be the linearised action for minimal $\mathcal{N}=1$ AdS supergravity. The dual action $S^{\perp}_{(3/2)}$ results from the linearisation around the AdS background of non-minimal $\mathcal{N}=1$ AdS supergravity [12].[5] Both actions represent the lowest superspin limits of two infinite series of dual models, $S^{||}_{(s+\frac{1}{2})}$ and $S^{\perp}_{(s+\frac{1}{2})}$, for off-shell massless gauge supermultiplets in AdS of half-integer superspin $(s+\frac{1}{2})$, where $s=1,2\dots$, constructed in [22]. Off-shell formulations for massless gauge supermultiplets in AdS of integer superspin s, with $s=1,2\dots$, were also constructed in [22]. In the flat-superspace limit, the supersymmetric higher spin theories of [22] reduce to those proposed in [23, 24].

Making use of the gauge off-shell formulations for massless higher spin supermultiplets in AdS [22], one can define consistent higher spin supercurrent multiplets in AdS superspace (i.e. higher spin extensions of the supercurrent) that contain ordinary bosonic and fermionic conserved currents in AdS. One can then look for explicit realisations of such higher spin supercurrents in concrete supersymmetric theories in AdS, for instance models for massless and massive chiral scalar superfields. Such a program is a natural extension of the flat-space results obtained in recent papers [25, 26] in which two of us built on the structure of higher spin supercurrent multiplets in models for superconformal chiral superfields [27]. In accordance with the standard Noether method (see, e.g., [28] for a review), the construction of conserved higher spin supercurrents for various supersymmetric theories in AdS is equivalent to generating consistent cubic vertices of the type $\int HJ$, where H denotes some off-shell higher spin gauge multiplet [22], and $J=\mathcal{D}^p\Phi\mathcal{D}^q\Psi$ is the higher spin current which is constructed in terms of some matter multiplets Φ and Ψ and the AdS covariant derivatives $\mathcal{D}$. This is one of the important applications of the results presented in the present paper. In the flat-superspace case, several cubic vertices involving the off-shell higher spin multiplets of [23, 24] were constructed recently in [29–31], as an extension of the superconformal cubic couplings between a chiral scalar superfield and an infinite tower of gauge massless multiplets of half-integer superspin given in [27].

It should be pointed out that conserved higher spin currents for scalar and spinor fields in Minkowski space have been studied in numerous publications. To the best of our knowledge, the spinor case was first described by Migdal [32] and Makeenko [33], while the conserved higher spin currents for scalar fields were first obtained in [33–35] (see also [36, 37]). The conserved higher spin currents for scalar fields in AdS were studied, e.g., in [38–42]. Since the curvature of AdS space is non-zero, explicit calculations of conserved higher spin currents are much harder than in Minkowski space. This is one of the reasons why refs. [38, 39] studied only the conformal scalar, and only the first order correction to the flat-space expression was given explicitly. The construction presented in [42] is more complete in the sense that all conserved higher spin currents were computed exactly for a free massive scalar field. This was achieved by making use of a somewhat unorthodox formulation in the so-called ambient space. All these works dealt with integer spin currents. The important feature of supersymmetric theories is that they also possess half-integer spin currents. They belong to the higher spin supercurrent multiplets we construct in this

[5] It was believed for almost thirty years that there is no off-shell non-minimal formulation for $\mathcal{N}=1$ AdS supergravity [21]. However, such a formulation was constructed in [12].

work. Another nice feature of the supersymmetric case is that the calculation of higher spin supercurrent multiplets in AdS superspace is considerably simpler than the problem of computing the ordinary conserved higher spin currents in AdS space.

Various aspects of supersymmetric field theories on AdS_4 have been studied in detail over the last forty years, see, e.g., [1, 9, 10, 17, 18, 20, 43–49] and references therein. The energy-momentum tensor of such a theory belongs to the Ferrara-Zumino supercurrent (or, equivalently, to the non-minimal AdS supercurrent which is related to the Ferrara-Zumino supermultiplet by a well-defined improvement transformation [12].). In this paper we present, for the first time, higher spin extensions of the AdS supercurrents and derive their explicit realisations for various supersymmetric theories on AdS, including a model of N massive chiral scalar superfields with an arbitrary mass matrix. Our results have numerous applications. For instance, the conserved higher spin supercurrents computed in section 5 and 6 can readily be reduced to component fields. This will give closed-form expressions for conserved higher spin bosonic and fermionic currents in models with massive scalar and spinor fields, thus leading to more general results than those known in the literature [38–42]. Another applications of the results obtained are consistent cubic coupling between chiral scalar supermultiplets and massless higher spin supermultiplets. Our results also make it possible to develop an effective action approach to massless higher spin supermultiplets along the lines advocated in [50–52] and more recently in [27]. We also refine some statements given recently in the literature, see section 7.

This paper is organised as follows. Section 2 contains a summary of the results concerning supersymmetric field theory in AdS superspace. Section 3 is devoted to a novel formulation for the massless integer superspin multiplets in AdS. This formulation is shown to reduce to that proposed in [22] upon partially fixing the gauge freedom. We also describe off-shell formulations (including a novel one) for the massless gravitino multiplet in AdS. In section 4 we introduce higher spin supercurrent multiplets in AdS and describe improvement transformations for them. Sections 5 and 6 are devoted to the explicit construction of higher spin supercurrents for N massive chiral multiplets. Several nontrivial applications of the results obtained are given in section 7. The main body of the paper is accompanied by three technical appendices. Appendix A reviews the irreducible supercurrent multiplets in AdS following [11, 12]. Appendices B and C review the conserved higher spin currents for N scalars and spinors, respectively, with arbitrary mass matrices. These results are scattered in the literature, including [32–35].

2 Field theory in AdS superspace

In this section we give a summary of the results which are absolutely essential when doing $\mathcal{N} = 1$ supersymmetric field theory in AdS in a manifestly OSp(1|4)-invariant way. We mostly follow the presentation in [22]. Our notation and two-component spinor conventions agree with [3], except for the notation for superspace integration measures.

Let $z^M = (x^m, \theta^\mu, \bar{\theta}_{\dot{\mu}})$ be local coordinates for $\mathcal{N} = 1$ AdS superspace, $\text{AdS}^{4|4}$. The geometry of $\text{AdS}^{4|4}$ may be described in terms of covariant derivatives of the form

$$\mathcal{D}_A = (\mathcal{D}_a, \mathcal{D}_\alpha, \bar{\mathcal{D}}^{\dot{\alpha}}) = E_A + \Omega_A\,, \qquad E_A = E_A{}^M \partial_M\,, \tag{2.1}$$

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where $E_A{}^M$ is the inverse superspace vielbein, and

$$\Omega_A = \frac{1}{2}\,\Omega_A{}^{bc} M_{bc} = \Omega_A{}^{\beta\gamma} M_{\beta\gamma} + \bar{\Omega}_A{}^{\dot\beta\dot\gamma} \bar{M}_{\dot\beta\dot\gamma}\,, \tag{2.2}$$

is the Lorentz connection. The Lorentz generators $M_{bc} \Leftrightarrow (M_{\beta\gamma}, \bar{M}_{\dot\beta\dot\gamma})$ act on two-component spinors as follows:

$$M_{\alpha\beta}\,\psi_\gamma = \frac{1}{2}(\varepsilon_{\gamma\alpha}\psi_\beta + \varepsilon_{\gamma\beta}\psi_\alpha)\,, \qquad M_{\alpha\beta}\,\bar\psi_{\dot\gamma} = 0\,, \tag{2.3a}$$

$$\bar{M}_{\dot\alpha\dot\beta}\,\bar\psi_{\dot\gamma} = \frac{1}{2}(\varepsilon_{\dot\gamma\dot\alpha}\bar\psi_{\dot\beta} + \varepsilon_{\dot\gamma\dot\beta}\bar\psi_{\dot\alpha})\,, \qquad \bar{M}_{\dot\alpha\dot\beta}\,\psi_\gamma = 0\,. \tag{2.3b}$$

The covariant derivatives of $\mathrm{AdS}^{4|4}$ satisfy the following algebra

$$\{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot\alpha}\} = -2\mathrm{i}\mathcal{D}_{\alpha\dot\alpha}\,, \tag{2.4a}$$

$$\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = -4\bar\mu\, M_{\alpha\beta}\,, \qquad \{\bar{\mathcal{D}}_{\dot\alpha}, \bar{\mathcal{D}}_{\dot\beta}\} = 4\mu\,\bar{M}_{\dot\alpha\dot\beta}\,, \tag{2.4b}$$

$$[\mathcal{D}_\alpha, \mathcal{D}_{\beta\dot\beta}] = \mathrm{i}\bar\mu\,\varepsilon_{\alpha\beta}\bar{\mathcal{D}}_{\dot\beta}\,, \qquad [\bar{\mathcal{D}}_{\dot\alpha}, \mathcal{D}_{\beta\dot\beta}] = -\mathrm{i}\mu\,\varepsilon_{\dot\alpha\dot\beta}\mathcal{D}_\beta\,, \tag{2.4c}$$

$$[\mathcal{D}_{\alpha\dot\alpha}, \mathcal{D}_{\beta\dot\beta}] = -2\bar\mu\mu\Big(\varepsilon_{\alpha\beta}\bar{M}_{\dot\alpha\dot\beta} + \varepsilon_{\dot\alpha\dot\beta}M_{\alpha\beta}\Big)\,, \tag{2.4d}$$

with $\mu \neq 0$ being a complex parameter, which is related to the scalar curvature $\mathcal{R}$ of AdS space by the rule $\mathcal{R} = -12|\mu|^2$.

In our calculations, we often make use of the following identities, which can be readily derived from the covariant derivatives algebra (2.4):

$$\mathcal{D}_\alpha\mathcal{D}_\beta = \frac{1}{2}\varepsilon_{\alpha\beta}\mathcal{D}^2 - 2\bar\mu\,M_{\alpha\beta}\,, \qquad \bar{\mathcal{D}}_{\dot\alpha}\bar{\mathcal{D}}_{\dot\beta} = -\frac{1}{2}\varepsilon_{\dot\alpha\dot\beta}\bar{\mathcal{D}}^2 + 2\mu\,\bar{M}_{\dot\alpha\dot\beta}\,, \tag{2.5a}$$

$$\mathcal{D}_\alpha\mathcal{D}^2 = 4\bar\mu\,\mathcal{D}^\beta M_{\alpha\beta} + 4\bar\mu\,\mathcal{D}_\alpha\,, \qquad \mathcal{D}^2\mathcal{D}_\alpha = -4\bar\mu\,\mathcal{D}^\beta M_{\alpha\beta} - 2\bar\mu\,\mathcal{D}_\alpha\,, \tag{2.5b}$$

$$\bar{\mathcal{D}}_{\dot\alpha}\bar{\mathcal{D}}^2 = 4\mu\,\bar{\mathcal{D}}^{\dot\beta}\bar{M}_{\dot\alpha\dot\beta} + 4\mu\,\bar{\mathcal{D}}_{\dot\alpha}\,, \qquad \bar{\mathcal{D}}^2\bar{\mathcal{D}}_{\dot\alpha} = -4\mu\,\bar{\mathcal{D}}^{\dot\beta}\bar{M}_{\dot\alpha\dot\beta} - 2\mu\,\bar{\mathcal{D}}_{\dot\alpha}\,, \tag{2.5c}$$

$$\big[\bar{\mathcal{D}}^2, \mathcal{D}_\alpha\big] = 4\mathrm{i}\mathcal{D}_{\alpha\dot\beta}\bar{\mathcal{D}}^{\dot\beta} + 4\mu\,\mathcal{D}_\alpha = 4\mathrm{i}\bar{\mathcal{D}}^{\dot\beta}\mathcal{D}_{\alpha\dot\beta} - 4\mu\,\mathcal{D}_\alpha\,, \tag{2.5d}$$

$$\big[\mathcal{D}^2, \bar{\mathcal{D}}_{\dot\alpha}\big] = -4\mathrm{i}\mathcal{D}_{\beta\dot\alpha}\mathcal{D}^\beta + 4\bar\mu\,\bar{\mathcal{D}}_{\dot\alpha} = -4\mathrm{i}\mathcal{D}^\beta\mathcal{D}_{\beta\dot\alpha} - 4\bar\mu\,\bar{\mathcal{D}}_{\dot\alpha}\,, \tag{2.5e}$$

where $\mathcal{D}^2 = \mathcal{D}^\alpha\mathcal{D}_\alpha$, and $\bar{\mathcal{D}}^2 = \bar{\mathcal{D}}_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}$. These relations imply the identity

$$\mathcal{D}^\alpha(\bar{\mathcal{D}}^2 - 4\mu)\mathcal{D}_\alpha = \bar{\mathcal{D}}_{\dot\alpha}(\mathcal{D}^2 - 4\bar\mu)\bar{\mathcal{D}}^{\dot\alpha}\,, \tag{2.6}$$

which guarantees the reality of the action functionals considered in the next sections.

Complex tensor superfields $\Gamma_{\alpha(m)\dot\alpha(n)} := \Gamma_{\alpha_1\dots\alpha_m\dot\alpha_1\dots\dot\alpha_n} = \Gamma_{(\alpha_1\dots\alpha_m)(\dot\alpha_1\dots\dot\alpha_n)}$ and $G_{\alpha(m)\dot\alpha(n)}$ are referred to as transverse linear and longitudinal linear, respectively, if the constraints

$$\bar{\mathcal{D}}^{\dot\beta}\Gamma_{\alpha(m)\dot\beta\dot\alpha(n-1)} = 0\,, \qquad n \neq 0\,, \tag{2.7a}$$

$$\bar{\mathcal{D}}_{(\dot\alpha_1}G_{\alpha(m)\dot\alpha_2\dots\dot\alpha_{n+1})} = 0 \tag{2.7b}$$

are satisfied. For $n = 0$ the latter constraint coincides with the condition of covariant chirality, $\bar{\mathcal{D}}_{\dot\beta}G_{\alpha(m)} = 0$. With the aid of (2.5a), the relations (2.7) lead to the linearity conditions

$$(\bar{\mathcal{D}}^2 - 2(n+2)\mu)\,\Gamma_{\alpha(m)\dot\alpha(n)} = 0\,, \tag{2.8a}$$

$$(\bar{\mathcal{D}}^2 + 2n\mu)\,G_{\alpha(m)\dot\alpha(n)} = 0\,. \tag{2.8b}$$

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The transverse condition (2.7a) is not defined for $n = 0$. However its corollary (2.8a) remains consistent for the choice $n = 0$ and corresponds to complex linear superfields $\Gamma_{\alpha(m)}$ constrained by

$$(\bar{\mathcal{D}}^2 - 4\mu)\,\Gamma_{\alpha(m)} = 0\,. \tag{2.9}$$

In the family of constrained superfields $\Gamma_{\alpha(m)}$ introduced, the scalar multiplet, $m = 0$, is used most often in applications. One can define projectors $P_n^{\perp}$ and $P_n^{||}$ on the spaces of transverse linear and longitudinal linear superfields respectively:

$$P_n^{\perp} = \frac{1}{4(n+1)\mu}(\bar{\mathcal{D}}^2 + 2n\mu)\,, \tag{2.10a}$$

$$P_n^{||} = -\frac{1}{4(n+1)\mu}(\bar{\mathcal{D}}^2 - 2(n+2)\mu)\,, \tag{2.10b}$$

with the properties

$$\big(P_n^{\perp}\big)^2 = P_n^{\perp}\,, \quad \big(P_n^{||}\big)^2 = P_n^{||}\,, \quad P_n^{\perp}P_n^{||} = P_n^{||}P_n^{\perp} = 0\,, \quad P_n^{\perp} + P_n^{||} = \mathbb{1}\,. \tag{2.11}$$

Superfields (2.7) were introduced and studied by Ivanov and Sorin [10] in their analysis of the representations of the AdS supersymmetry. A nice review of the results of [10] is given in the book [53].

Given a complex tensor superfield $V_{\alpha(m)\dot\alpha(n)}$ with $n \neq 0$, it can be represented as a sum of transverse linear and longitudinal linear multiplets,

$$V_{\alpha(m)\dot\alpha(n)} = -\frac{1}{2\mu(n+2)}\bar{\mathcal{D}}^{\dot\gamma}\bar{\mathcal{D}}_{(\dot\gamma}V_{\alpha(m)\dot\alpha_1\dots\dot\alpha_n)} - \frac{1}{2\mu(n+1)}\bar{\mathcal{D}}_{(\dot\alpha_1}\bar{\mathcal{D}}^{|\dot\gamma|}V_{\alpha(m)\dot\alpha_2\dots\dot\alpha_n)\dot\gamma}\,. \tag{2.12}$$

Choosing $V_{\alpha(m)\dot\alpha(n)}$ to be transverse linear ($\Gamma_{\alpha(m)\dot\alpha(n)}$) or longitudinal linear ($G_{\alpha(m)\dot\alpha(n)}$), the above relation gives

$$\Gamma_{\alpha(m)\dot\alpha(n)} = \bar{\mathcal{D}}^{\dot\beta}\Phi_{\alpha(m)\,(\dot\beta\dot\alpha_1\cdots\dot\alpha_n)}\,, \tag{2.13a}$$

$$G_{\alpha(m)\dot\alpha(n)} = \bar{\mathcal{D}}_{(\dot\alpha_1}\Psi_{\alpha(m)\,\dot\alpha_2\cdots\dot\alpha_n)}\,, \tag{2.13b}$$

for some prepotentials $\Phi_{\alpha(m)\dot\alpha(n+1)}$ and $\Psi_{\alpha(m)\dot\alpha(n-1)}$. The constraints (2.7) hold for unconstrained $\Phi_{\alpha(m)\dot\alpha(n+1)}$ and $\Psi_{\alpha(m)\dot\alpha(n-1)}$. These prepotentials are defined modulo gauge transformations of the form:

$$\delta_\xi\Phi_{\alpha(m)\,\dot\alpha(n+1)} = \bar{\mathcal{D}}^{\dot\beta}\xi_{\alpha(m)\,(\dot\beta\dot\alpha_1\cdots\dot\alpha_{n+1})}\,, \tag{2.14a}$$

$$\delta_\zeta\Psi_{\alpha(m)\,\dot\alpha(n-1)} = \bar{\mathcal{D}}_{(\dot\alpha_1}\zeta_{\alpha(m)\,\dot\alpha_2\cdots\dot\alpha_{n-1})}\,, \tag{2.14b}$$

with the gauge parameters $\xi_{\alpha(m)\,\dot\alpha(n+2)}$ and $\zeta_{\alpha(m)\,\dot\alpha(n-2)}$ being unconstrained.

The isometry group of $\mathcal{N} = 1$ AdS superspace is OSp(1|4). The isometries transformations of AdS$^{4|4}$ are generated by the Killing vector fields $\Lambda^A E_A$ which are defined to solve the Killing equation

$$\left[\Lambda + \frac{1}{2}\omega^{bc}M_{bc}, \mathcal{D}_A\right] = 0\,, \qquad \Lambda := \Lambda^B\mathcal{D}_B = \lambda^b\mathcal{D}_b + \lambda^\beta\mathcal{D}_\beta + \bar\lambda_{\dot\beta}\bar{\mathcal{D}}^{\dot\beta}\,, \tag{2.15}$$

for some Lorentz superfield parameter $\omega^{bc} = -\omega^{cb}$. As shown in [3], the equations in (2.15) are equivalent to

$$\mathcal{D}_{(\alpha}\lambda_{\beta)\dot\beta} = 0\,, \qquad \bar{\mathcal{D}}^{\dot\beta}\lambda_{\alpha\dot\beta} + 8\mathrm{i}\lambda_\alpha = 0\,, \tag{2.16a}$$

$$\mathcal{D}_\alpha\lambda^\alpha = 0\,, \qquad \bar{\mathcal{D}}_{\dot\alpha}\lambda_\alpha + \frac{\mathrm{i}}{2}\mu\lambda_{\alpha\dot\alpha} = 0\,, \tag{2.16b}$$

$$\omega_{\alpha\beta} = \mathcal{D}_\alpha\lambda_\beta\,. \tag{2.16c}$$

The solution to these equations is given in [3]. If T is a tensor superfield (with suppressed indices), its infinitesimal OSp(1|4) transformation is

$$\delta T = \left(\Lambda + \frac{1}{2}\omega^{bc}M_{bc}\right)T\,. \tag{2.17}$$

In Minkowski space, there are two ways to generate supersymmetric invariants, one of which corresponds to the integration over the full superspace and the other over its chiral subspace. In AdS superspace, every chiral integral can be always recast as a full superspace integral. Associated with a scalar superfield $\mathcal{L}$ is the following OSp(1|4) invariant

$$\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta\, E\,\mathcal{L} = -\frac{1}{4}\int \mathrm{d}^4x\mathrm{d}^2\theta\,\mathcal{E}\,(\bar{\mathcal{D}}^2 - 4\mu)\mathcal{L}\,, \qquad E^{-1} = \mathrm{Ber}\,(E_{\mathrm{A}}{}^M)\,, \tag{2.18}$$

where $\mathcal{E}$ denotes the chiral integration measure.[6] Let $\mathcal{L}_\mathrm{c}$ be a chiral scalar, $\bar{\mathcal{D}}_{\dot\alpha}\mathcal{L}_\mathrm{c} = 0$. It generates the supersymmetric invariant $\int \mathrm{d}^4x\mathrm{d}^2\theta\,\mathcal{E}\,\mathcal{L}_\mathrm{c}$. The specific feature of AdS superspace is that the chiral action can equivalently be written as an integral over the full superspace [54, 55]

$$\int \mathrm{d}^4x\mathrm{d}^2\theta\,\mathcal{E}\,\mathcal{L}_\mathrm{c} = \frac{1}{\mu}\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta\,E\,\mathcal{L}_\mathrm{c}\,. \tag{2.19}$$

Unlike the flat superspace case, the integral on the right does not vanish in AdS.

3 Massless integer superspin multiplets

Let s be a positive integer. The longitudinal formulation for the massless superspin-s multiplet in AdS was realised in [22] in terms of the following dynamical variables

$$v^{\parallel}_{(s)} = \left\{H_{\alpha(s-1)\dot\alpha(s-1)}(z), G_{\alpha(s)\dot\alpha(s)}(z), \bar G_{\alpha(s)\dot\alpha(s)}(z)\right\}\,. \tag{3.1}$$

Here, $H_{\alpha(s-1)\dot\alpha(s-1)}$ is an unconstrained real superfield, and $G_{\alpha(s)\dot\alpha(s)}$ is a longitudinal linear superfield. The latter is a field strength associated with a complex unconstrained prepotential $\Psi_{\alpha(s)\dot\alpha(s-1)}$,

$$G_{\alpha_1\dots\alpha_s\dot\alpha_1\dots\dot\alpha_s} := \bar{\mathcal{D}}_{(\dot\alpha_1}\Psi_{\alpha_1\dots\alpha_s\dot\alpha_2\dots\dot\alpha_s)} \quad\Longrightarrow\quad \bar{\mathcal{D}}_{(\dot\alpha_1}G_{\alpha_1\dots\alpha_s\dot\alpha_2\dots\dot\alpha_{s+1})} = 0\,. \tag{3.2}$$

[6] In the chiral representation [3, 21], the chiral measure is $\mathcal{E} = \varphi^3$, where φ is the chiral compensator of old minimal supergravity [54].

The gauge freedom postulated in [22] is given by

$$\delta H_{\alpha(s-1)\dot\alpha(s-1)} = \mathcal{D}^\beta L_{\beta\alpha(s-1)\dot\alpha(s-1)} - \bar{\mathcal{D}}^{\dot\beta} \bar{L}_{\alpha(s-1)\dot\beta\dot\alpha(s-1)}\,, \tag{3.3a}$$

$$\delta G_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)} = \frac{1}{2}\bar{\mathcal{D}}_{(\dot\beta}\mathcal{D}_{(\beta}\mathcal{D}^{|\gamma|} L_{\alpha(s-1))\gamma\dot\alpha(s-1))}\,, \tag{3.3b}$$

where the gauge parameter is $L_{\alpha(s)\dot\alpha(s-1)}$ is unconstrained.

In this section we propose a reformulation of the longitudinal theory that is obtained by enlarging the gauge freedom (3.3) at the cost of introducing a new purely gauge superfield variables in addition to $H_{\alpha(s-1)\dot\alpha(s-1)}$, $\Psi_{\alpha(s)\dot\alpha(s-1)}$ and $\bar\Psi_{\alpha(s-1)\dot\alpha(s)}$. In such a setting, the gauge freedom of $\Psi_{\alpha(s)\dot\alpha(s-1)}$ coincides with that of a superconformal multiplet of superspin s [27]. The new formulation will be an extension of the one given in [26] in the flat-superspace case.

3.1 New formulation

Given a positive integer $s \geq 2$, a massless superspin-s multiplet can be described in $\text{AdS}^{4|4}$ by using the following superfield variables: (i) an unconstrained prepotential $\Psi_{\alpha(s)\dot\alpha(s-1)}$ and its complex conjugate $\bar\Psi_{\alpha(s-1)\dot\alpha(s)}$; (ii) a real superfield $H_{\alpha(s-1)\dot\alpha(s-1)} = \bar{H}_{\alpha(s-1)\dot\alpha(s-1)}$; and (iii) a complex superfield $\Sigma_{\alpha(s-1)\dot\alpha(s-2)}$ and its conjugate $\bar\Sigma_{\alpha(s-2)\dot\alpha(s-1)}$, where $\Sigma_{\alpha(s-1)\dot\alpha(s-2)}$ is constrained to be transverse linear,

$$\bar{\mathcal{D}}^{\dot\beta}\Sigma_{\alpha(s-1)\dot\beta\dot\alpha(s-3)} = 0\,. \tag{3.4}$$

The constraint (3.4) is solved in terms of a complex unconstrained prepotential $Z_{\alpha(s-1)\dot\alpha(s-1)}$ by the rule

$$\Sigma_{\alpha(s-1)\dot\alpha(s-2)} = \bar{\mathcal{D}}^{\dot\beta} Z_{\alpha(s-1)(\dot\beta\dot\alpha_1\dots\dot\alpha_{s-2})}\,. \tag{3.5}$$

This prepotential is defined modulo gauge transformations

$$\delta_\xi Z_{\alpha(s-1)\dot\alpha(s-1)} = \bar{\mathcal{D}}^{\dot\beta}\xi_{\alpha(s-1)(\dot\beta\dot\alpha_1\dots\dot\alpha_{s-1})}\,, \tag{3.6}$$

with the gauge parameter $\xi_{\alpha(s-1)\dot\alpha(s)}$ being unconstrained.

The gauge freedom of $\Psi_{\alpha_1\dots\alpha_s\dot\alpha_1\dots\dot\alpha_{s-1}}$ is chosen to coincide with that of the superconformal superspin-s multiplet [27], which is

$$\delta_{\mathfrak{V},\zeta}\Psi_{\alpha_1\dots\alpha_s\dot\alpha_1\dots\dot\alpha_{s-1}} = \frac{1}{2}\mathcal{D}_{(\alpha_1}\mathfrak{V}_{\alpha_2\dots\alpha_s)\dot\alpha_1\dots\dot\alpha_{s-1}} + \bar{\mathcal{D}}_{(\dot\alpha_1}\zeta_{\alpha_1\dots\alpha_s\dot\alpha_2\dots\dot\alpha_{s-1})}\,, \tag{3.7a}$$

with unconstrained gauge parameters $\mathfrak{V}_{\alpha(s-1)\dot\alpha(s-1)}$ and $\zeta_{\alpha(s)\dot\alpha(s-2)}$. The $\mathfrak{V}$-transformation is defined to act on the superfields $H_{\alpha(s-1)\dot\alpha(s-1)}$ and $\Sigma_{\alpha(s-1)\dot\alpha(s-2)}$ as follows

$$\delta_{\mathfrak{V}} H_{\alpha(s-1)\dot\alpha(s-1)} = \mathfrak{V}_{\alpha(s-1)\dot\alpha(s-1)} + \bar{\mathfrak{V}}_{\alpha(s-1)\dot\alpha(s-1)}\,, \tag{3.7b}$$

$$\delta_{\mathfrak{V}}\Sigma_{\alpha(s-1)\dot\alpha(s-2)} = \bar{\mathcal{D}}^{\dot\beta}\bar{\mathfrak{V}}_{\alpha(s-1)\dot\beta\dot\alpha(s-2)} \quad\Longrightarrow\quad \delta_{\mathfrak{V}} Z_{\alpha(s-1)\dot\alpha(s-1)} = \bar{\mathfrak{V}}_{\alpha(s-1)\dot\alpha(s-1)}\,. \tag{3.7c}$$

The longitudinal linear superfield defined by (3.2) is invariant under the ζ-transformation (3.7a) and varies under the $\mathfrak{V}$-transformation as

$$\delta_{\mathfrak{V}} G_{\alpha_1\dots\alpha_s\dot\alpha_1\dots\dot\alpha_s} = \frac{1}{2}\bar{\mathcal{D}}_{(\dot\alpha_1}\mathcal{D}_{(\alpha_1}\mathfrak{V}_{\alpha_2\dots\alpha_s)\dot\alpha_2\dots\dot\alpha_s)}\,. \tag{3.8}$$

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Our next task is to derive an AdS extension of the gauge-invariant action in Minkowski superspace (given by eq. (2.8) in [26]). The geometry of AdS superspace is completely determined by the algebra (2.4). We start with the following action functional in AdS superspace, which is a minimal AdS extension of the action constructed in [26].

$$
\begin{aligned}
S^{\|}_{(s)} = \Big(-\frac{1}{2}\Big)^s \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta\, E\, \Big\{ & \frac{1}{8} H^{\alpha(s-1)\dot\alpha(s-1)} \mathcal{D}^\beta \bar{\mathcal{D}}^2 \mathcal{D}_\beta H_{\alpha(s-1)\dot\alpha(s-1)} \\
& + \frac{s}{s+1} H^{\alpha(s-1)\dot\alpha(s-1)} \Big(\mathcal{D}^\beta \bar{\mathcal{D}}^{\dot\beta} G_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)} - \bar{\mathcal{D}}^{\dot\beta}\mathcal{D}^\beta \bar G_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)}\Big) \\
& + 2\bar G^{\alpha(s)\dot\alpha(s)} G_{\alpha(s)\dot\alpha(s)} + \frac{s}{s+1}\Big(G^{\alpha(s)\dot\alpha(s)} G_{\alpha(s)\dot\alpha(s)} + \bar G^{\alpha(s)\dot\alpha(s)} \bar G_{\alpha(s)\dot\alpha(s)}\Big) \\
& + \frac{s-1}{4s} H^{\alpha(s-1)\dot\alpha(s-1)} \Big(\mathcal{D}_{\alpha_1} \bar{\mathcal{D}}^2 \bar\Sigma_{\alpha_2\dots\alpha_{s-1}\dot\alpha(s-1)} - \bar{\mathcal{D}}_{\dot\alpha_1} \mathcal{D}^2 \Sigma_{\alpha(s-1)\dot\alpha_2\dots\dot\alpha_{s-1}}\Big) \\
& + \frac{1}{s}\Psi^{\alpha(s)\dot\alpha(s-1)} \Big(\mathcal{D}_{\alpha_1}\bar{\mathcal{D}}_{\dot\alpha_1} - 2\mathrm{i}(s-1)\mathcal{D}_{\alpha_1\dot\alpha_1}\Big)\Sigma_{\alpha_2\dots\alpha_s\dot\alpha_2\dots\dot\alpha_{s-1}} \\
& + \frac{1}{s}\bar\Psi^{\alpha(s-1)\dot\alpha(s)} \Big(\bar{\mathcal{D}}_{\dot\alpha_1}\mathcal{D}_{\alpha_1} - 2\mathrm{i}(s-1)\mathcal{D}_{\alpha_1\dot\alpha_1}\Big)\bar\Sigma_{\alpha_2\dots\alpha_{s-1}\dot\alpha_2\dots\dot\alpha_s} \\
& + \frac{s-1}{8s}\Big(\Sigma^{\alpha(s-1)\dot\alpha(s-2)}\mathcal{D}^2\Sigma_{\alpha(s-1)\dot\alpha(s-2)} - \bar\Sigma^{\alpha(s-2)\dot\alpha(s-1)}\bar{\mathcal{D}}^2\bar\Sigma_{\alpha(s-2)\dot\alpha(s-1)}\Big) \\
& - \frac{1}{s^2}\bar\Sigma^{\alpha(s-2)\dot\alpha(s-2)\dot\beta}\Big(\frac{1}{2}(s^2+1)\mathcal{D}^\beta\bar{\mathcal{D}}_{\dot\beta} + \mathrm{i}(s-1)^2\mathcal{D}^\beta{}_{\dot\beta}\Big)\Sigma_{\beta\alpha(s-2)\dot\alpha(s-2)}\Big\} + \dots
\end{aligned} \tag{3.9}
$$

The gauge-invariant action in AdS is expected to differ from (3.9) by some μ-dependent terms, which are required to ensure invariance under the gauge transformations (3.7) and, by construction, (3.6). We compute the variation of (3.9) under (3.7) and then add certain μ-dependent terms to achieve an invariant action. The identities (2.5) prove to be useful in carrying out such calculations.

The above procedure leads to the following action in AdS, which is invariant under the gauge transformations (3.7) and (3.6)

$$
\begin{aligned}
S^{\|}_{(s)} = \Big(-\frac{1}{2}\Big)^s \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta\, E\, \Big\{ & \frac{1}{8} H^{\alpha(s-1)\dot\alpha(s-1)} \mathcal{D}^\beta (\bar{\mathcal{D}}^2 - 4\mu) \mathcal{D}_\beta H_{\alpha(s-1)\dot\alpha(s-1)} \\
& + \frac{s}{s+1} H^{\alpha(s-1)\dot\alpha(s-1)} \Big(\mathcal{D}^\beta \bar{\mathcal{D}}^{\dot\beta} G_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)} - \bar{\mathcal{D}}^{\dot\beta}\mathcal{D}^\beta \bar G_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)}\Big) \\
& + \frac{(s+1)^2}{2}\bar\mu\mu H^{\alpha(s-1)\dot\alpha(s-1)} H_{\alpha(s-1)\dot\alpha(s-1)} \\
& + 2\bar G^{\alpha(s)\dot\alpha(s)} G_{\alpha(s)\dot\alpha(s)} + \frac{s}{s+1}\Big(G^{\alpha(s)\dot\alpha(s)} G_{\alpha(s)\dot\alpha(s)} + \bar G^{\alpha(s)\dot\alpha(s)} \bar G_{\alpha(s)\dot\alpha(s)}\Big) \\
& + \frac{s-1}{4s} H^{\alpha(s-1)\dot\alpha(s-1)} \Big(\mathcal{D}_{\alpha_1} \bar{\mathcal{D}}^2 \bar\Sigma_{\alpha_2\dots\alpha_{s-1}\dot\alpha(s-1)} - \bar{\mathcal{D}}_{\dot\alpha_1} \mathcal{D}^2 \Sigma_{\alpha(s-1)\dot\alpha_2\dots\dot\alpha_{s-1}}\Big) \\
& + \frac{1}{s}\Psi^{\alpha(s)\dot\alpha(s-1)} \Big(\mathcal{D}_{\alpha_1}\bar{\mathcal{D}}_{\dot\alpha_1} - 2\mathrm{i}(s-1)\mathcal{D}_{\alpha_1\dot\alpha_1}\Big)\Sigma_{\alpha_2\dots\alpha_s\dot\alpha_2\dots\dot\alpha_{s-1}} \\
& + \frac{1}{s}\bar\Psi^{\alpha(s-1)\dot\alpha(s)} \Big(\bar{\mathcal{D}}_{\dot\alpha_1}\mathcal{D}_{\alpha_1} - 2\mathrm{i}(s-1)\mathcal{D}_{\alpha_1\dot\alpha_1}\Big)\bar\Sigma_{\alpha_2\dots\alpha_{s-1}\dot\alpha_2\dots\dot\alpha_s} \\
& - \mu\frac{s^2+4s-1}{2s} H^{\alpha(s-1)\dot\alpha(s-1)}\mathcal{D}_{\alpha_1}\bar\Sigma_{\alpha_2\dots\alpha_{s-1}\dot\alpha(s-1)} \\
& + \bar\mu\frac{s^2+4s-1}{2s} H^{\alpha(s-1)\dot\alpha(s-1)}\bar{\mathcal{D}}_{\dot\alpha_1}\Sigma_{\alpha(s-1)\dot\alpha_2\dots\dot\alpha_{s-1}}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{s-1}{8s}\Big(\Sigma^{\alpha(s-1)\dot\alpha(s-2)}\mathcal{D}^2\Sigma_{\alpha(s-1)\dot\alpha(s-2)}-\bar\Sigma^{\alpha(s-2)\dot\alpha(s-1)}\bar{\mathcal{D}}^2\bar\Sigma_{\alpha(s-2)\dot\alpha(s-1)}\Big)\\
&-\frac{1}{s^2}\bar\Sigma^{\alpha(s-2)\dot\alpha(s-2)\dot\beta}\Big(\frac{1}{2}(s^2+1)\mathcal{D}^\beta\bar{\mathcal{D}}_{\dot\beta}+\mathrm{i}(s-1)^2\mathcal{D}^\beta{}_{\dot\beta}\Big)\Sigma_{\beta\alpha(s-2)\dot\alpha(s-2)}\\
&+\mu\frac{s^2+4s-1}{4s}\bar\Sigma^{\alpha(s-2)\dot\alpha(s-1)}\bar\Sigma_{\alpha(s-2)\dot\alpha(s-1)}\\
&+\bar\mu\frac{s^2+4s-1}{4s}\Sigma_{\alpha(s-1)\dot\alpha(s-2)}\Sigma^{\alpha(s-1)\dot\alpha(s-2)}\Big\}\,. \qquad (3.10)
\end{aligned}
$$

The above action is real due to the identity (2.6). In the limit of vanishing curvature of the AdS superspace ($\mu\to 0$), (3.10) reduces to the action constructed in [26].

The $\mathfrak{V}$-gauge freedom (3.7) allows us to gauge away $\Sigma_{\alpha(s-1)\dot\alpha(s-2)}$,

$$
\Sigma_{\alpha(s-1)\dot\alpha(s-2)}=0\,. \qquad (3.11)
$$

In this gauge, the action (3.10) reduces to that describing the longitudinal formulation for the massless superspin-s multiplet [22]. The gauge condition (3.11) does not fix completely the $\mathfrak{V}$-gauge freedom. The residual gauge transformations are generated by

$$
\mathfrak{V}_{\alpha(s-1)\dot\alpha(s-1)}=\mathcal{D}^\beta L_{(\beta\alpha_1\dots\alpha_{s-1})\dot\alpha(s-1)}\,, \qquad (3.12)
$$

with $L_{\alpha(s)\dot\alpha(s-1)}$ being an unconstrained superfield. With this expression for $\mathfrak{V}_{\alpha(s-1)\dot\alpha(s-1)}$, the gauge transformations (3.7a) and (3.7b) coincide with (3.3). Thus, the action (3.10) indeed provides an off-shell formulation for the massless superspin-s multiplet in the AdS superspace.

One can impose an alternative gauge fixing

$$
H_{\alpha(s-1)\dot\alpha(s-1)}=0\,. \qquad (3.13)
$$

In accordance with (3.7b), in this gauge the residual gauge freedom is described by

$$
\mathfrak{V}_{\alpha(s-1)\dot\alpha(s-1)}=\mathrm{i}\mathfrak{R}_{\alpha(s-1)\dot\alpha(s-1)}\,,\qquad \bar{\mathfrak{R}}_{\alpha(s-1)\dot\alpha(s-1)}=\mathfrak{R}_{\alpha(s-1)\dot\alpha(s-1)}\,. \qquad (3.14)
$$

The action (3.10) includes a single term which involves the 'naked' gauge field $\Psi_{\alpha(s)\dot\alpha(s-1)}$ and not the field strength $G_{\alpha(s)\dot\alpha(s)}$, the latter being defined by (3.2) and invariant under the ζ-transformation (3.7a). This is actually a BF term, for it can be written in two different forms

$$
\begin{aligned}
&\frac{1}{s}\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta\,E\,\Psi^{\alpha(s)\dot\alpha(s-1)}\Big(\mathcal{D}_{\alpha_1}\bar{\mathcal{D}}_{\dot\alpha_1}-2\mathrm{i}(s-1)\mathcal{D}_{\alpha_1\dot\alpha_1}\Big)\Sigma_{\alpha_2\dots\alpha_s\dot\alpha_2\dots\dot\alpha_{s-1}}\\
&=-\frac{1}{s+1}\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta\,E\,G^{\alpha(s)\dot\alpha(s)}\Big(\bar{\mathcal{D}}_{\dot\alpha_1}\mathcal{D}_{\alpha_1}+2\mathrm{i}(s+1)\mathcal{D}_{\alpha_1\dot\alpha_1}\Big)Z_{\alpha_2\dots\alpha_s\dot\alpha_2\dots\dot\alpha_s}\,. \qquad (3.15)
\end{aligned}
$$

The former makes the gauge symmetry (3.6) manifestly realised, while the latter turns the ζ-transformation (3.7a) into a manifest symmetry.

Making use of (3.15) leads to a different representation for the action (3.10). It is

$$
\begin{aligned}
S^{\|}_{(s)} = \Big(-\frac{1}{2}\Big)^s \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta\, E\, \Big\{ & \frac{1}{8} H^{\alpha(s-1)\dot\alpha(s-1)}\mathcal{D}^\beta(\bar{\mathcal{D}}^2-4\mu)\mathcal{D}_\beta H_{\alpha(s-1)\dot\alpha(s-1)} \\
& + \frac{s}{s+1} H^{\alpha(s-1)\dot\alpha(s-1)}\Big(\mathcal{D}^\beta\bar{\mathcal{D}}^{\dot\beta}G_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)} - \bar{\mathcal{D}}^{\dot\beta}\mathcal{D}^\beta\bar G_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)}\Big) \\
& + \frac{(s+1)^2}{2}\bar\mu\mu H^{\alpha(s-1)\dot\alpha(s-1)}H_{\alpha(s-1)\dot\alpha(s-1)} \\
& + 2\bar G^{\alpha(s)\dot\alpha(s)}G_{\alpha(s)\dot\alpha(s)} + \frac{s}{s+1}\Big(G^{\alpha(s)\dot\alpha(s)}G_{\alpha(s)\dot\alpha(s)} + \bar G^{\alpha(s)\dot\alpha(s)}\bar G_{\alpha(s)\dot\alpha(s)}\Big) \\
& + \frac{s-1}{4s} H^{\alpha(s-1)\dot\alpha(s-1)}\Big(\mathcal{D}_{\alpha_1}\bar{\mathcal{D}}^2\bar\Sigma_{\alpha_2\dots\alpha_{s-1}\dot\alpha(s-1)} - \bar{\mathcal{D}}_{\dot\alpha_1}\mathcal{D}^2\Sigma_{\alpha(s-1)\dot\alpha_2\dots\dot\alpha_{s-1}}\Big) \\
& - \frac{1}{s+1} G^{\alpha(s)\dot\alpha(s-1)}\Big(\bar{\mathcal{D}}_{\dot\alpha_1}\mathcal{D}_{\alpha_1} + 2\mathrm{i}(s+1)\mathcal{D}_{\alpha_1\dot\alpha_1}\Big)Z_{\alpha_2\dots\alpha_s\dot\alpha_2\dots\dot\alpha_s} \\
& + \frac{1}{s+1} G^{\alpha(s)\dot\alpha(s-1)}\Big(\mathcal{D}_{\alpha_1}\bar{\mathcal{D}}_{\dot\alpha_1} + 2\mathrm{i}(s+1)\mathcal{D}_{\alpha_1\dot\alpha_1}\Big)\bar Z_{\alpha_2\dots\alpha_s\dot\alpha_2\dots\dot\alpha_s} \\
& - \mu\frac{s^2+4s-1}{2s} H^{\alpha(s-1)\dot\alpha(s-1)}\mathcal{D}_{\alpha_1}\bar\Sigma_{\alpha_2\dots\alpha_{s-1}\dot\alpha(s-1)} \\
& + \bar\mu\frac{s^2+4s-1}{2s} H^{\alpha(s-1)\dot\alpha(s-1)}\bar{\mathcal{D}}_{\dot\alpha_1}\Sigma_{\alpha(s-1)\dot\alpha_2\dots\dot\alpha_{s-1}} \\
& + \frac{s-1}{8s}\Big(\Sigma^{\alpha(s-1)\dot\alpha(s-2)}\mathcal{D}^2\Sigma_{\alpha(s-1)\dot\alpha(s-2)} - \bar\Sigma^{\alpha(s-2)\dot\alpha(s-1)}\bar{\mathcal{D}}^2\bar\Sigma_{\alpha(s-2)\dot\alpha(s-1)}\Big) \\
& - \frac{1}{s^2}\bar\Sigma^{\alpha(s-2)\dot\alpha(s-2)\dot\beta}\Big(\frac{1}{2}(s^2+1)\mathcal{D}^\beta\bar{\mathcal{D}}_{\dot\beta} + \mathrm{i}(s-1)^2\mathcal{D}^\beta{}_{\dot\beta}\Big)\Sigma_{\beta\alpha(s-2)\dot\alpha(s-2)} \\
& + \mu\frac{s^2+4s-1}{4s}\bar\Sigma^{\alpha(s-2)\dot\alpha(s-1)}\bar\Sigma_{\alpha(s-2)\dot\alpha(s-1)} \\
& + \bar\mu\frac{s^2+4s-1}{4s}\Sigma_{\alpha(s-1)\dot\alpha(s-2)}\Sigma^{\alpha(s-1)\dot\alpha(s-2)}\Big\}\,.
\end{aligned} \tag{3.16}
$$

3.2 Dual formulation

As in the case of the flat superspace [26], the theory with action (3.16) can be reformulated in terms of a transverse linear superfield by applying the duality transformation introduced in [22].

We now associate with our theory (3.16) the following first-order action[7]

$$
\begin{aligned}
S_{\text{first-order}} &= S^{\|}_{(s)}[U,\bar U,H,Z,\bar Z] \\
&\quad + \Big(\frac{-1}{2}\Big)^s\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta\, E\,\Big(\frac{2}{s+1}\Gamma^{\alpha(s)\dot\alpha(s)}U_{\alpha(s)\dot\alpha(s)} + \text{c.c.}\Big)\,,
\end{aligned} \tag{3.17}
$$

where $S^{\|}_{(s)}[U,\bar U,H,Z,\bar Z]$ is obtained from the action (3.16) by replacing $G_{\alpha(s)\dot\alpha(s)}$ with an unconstrained complex superfield $U_{\alpha(s)\dot\alpha(s)}$, and the Lagrange multiplier $\Gamma_{\alpha(s)\dot\alpha(s)}$ is transverse linear,

$$\bar{\mathcal{D}}^{\dot\beta}\Gamma_{\alpha(s)\dot\beta\dot\alpha_1\dots\dot\alpha_{s-1}} = 0\,. \tag{3.18}$$

Varying (3.17) with respect to the Lagrange multiplier and taking into account the constraint (3.18) yields $U_{\alpha(s)\dot\alpha(s)} = G_{\alpha(s)\dot\alpha(s)}$; then, $S_{\text{first-order}}$ turns into the original action (3.16). On the other hand, we can eliminate the auxiliary superfields $U_{\alpha(s)\dot\alpha(s)}$ and

[7] The specific normalisation of the Lagrange multiplier in (3.17) is chosen to match that of [22].

$\bar{U}_{\alpha(s)\dot\alpha(s)}$ from (3.17) using their equations of motion. This leads to the dual action

$$
\begin{aligned}
S^{\perp}_{(s)} = -\Big(-\frac{1}{2}\Big)^s \int d^4x d^2\theta d^2\bar\theta\, E \bigg\{ & -\frac{1}{8} H^{\alpha(s-1)\dot\alpha(s-1)} \mathcal{D}^\beta (\bar{\mathcal{D}}^2 - 4\mu) \mathcal{D}_\beta H_{\alpha(s-1)\dot\alpha(s-1)} \\
& + \frac{1}{8} \frac{s^2}{(s+1)(2s+1)} [\mathcal{D}^\beta, \bar{\mathcal{D}}^{\dot\beta}] H^{\alpha(s-1)\dot\alpha(s-1)} [\mathcal{D}_{(\beta}, \bar{\mathcal{D}}_{(\dot\beta}] H_{\alpha(s-1))\dot\alpha(s-1))} \\
& + \frac{1}{2} \frac{s^2}{s+1} \mathcal{D}^{\beta\dot\beta} H^{\alpha(s-1)\dot\alpha(s-1)} \mathcal{D}_{(\beta(\dot\beta} H_{\alpha(s-1))\dot\alpha(s-1))} \\
& - \frac{(s+1)^2}{2} \bar\mu\mu H^{\alpha(s-1)\dot\alpha(s-1)} H_{\alpha(s-1)\dot\alpha(s-1)} \\
& + \frac{2\mathrm{i}s}{2s+1} H^{\alpha(s-1)\dot\alpha(s-1)} \mathcal{D}^{\beta\dot\beta} \Big(\boldsymbol{\Gamma}_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)} - \bar{\boldsymbol{\Gamma}}_{\beta\alpha(s-1)\dot\beta\dot\alpha(s-1)} \Big) \\
& + \frac{2}{2s+1} \bar{\boldsymbol{\Gamma}}^{\alpha(s)\dot\alpha(s)} \boldsymbol{\Gamma}_{\alpha(s)\dot\alpha(s)} - \frac{s}{(s+1)(2s+1)} \Big(\boldsymbol{\Gamma}^{\alpha(s)\dot\alpha(s)} \boldsymbol{\Gamma}_{\alpha(s)\dot\alpha(s)} + \bar{\boldsymbol{\Gamma}}^{\alpha(s)\dot\alpha(s)} \bar{\boldsymbol{\Gamma}}_{\alpha(s)\dot\alpha(s)} \Big) \\
& - \frac{s-1}{2(2s+1)} H^{\alpha(s-1)\dot\alpha(s-1)} \Big(\mathcal{D}_{\alpha_1} \bar{\mathcal{D}}^2 \bar\Sigma_{\alpha_2\dots\alpha_{s-1}\dot\alpha(s-1)} - \bar{\mathcal{D}}_{\dot\alpha_1} \mathcal{D}^2 \Sigma_{\alpha(s-1)\dot\alpha_2\dots\dot\alpha_{s-1}} \Big) \\
& + \frac{1}{2(2s+1)} H^{\alpha(s-1)\dot\alpha(s-1)} \Big(\mathcal{D}^2 \bar{\mathcal{D}}_{\dot\alpha_1} \Sigma_{\alpha(s-1)\dot\alpha_2\dots\dot\alpha_{s-1}} - \bar{\mathcal{D}}^2 \mathcal{D}_{\alpha_1} \bar\Sigma_{\alpha_2\dots\alpha_{s-1}\dot\alpha(s-1)} \Big) \\
& - \mathrm{i} \frac{(s-1)^2}{s(2s+1)} H^{\alpha(s-1)\dot\alpha(s-1)} \mathcal{D}_{\alpha_1\dot\alpha_1} \Big(\mathcal{D}^\beta \Sigma_{\beta\alpha_2\dots\alpha_{s-1}\dot\alpha_2\dots\dot\alpha_{s-1}} + \bar{\mathcal{D}}^{\dot\beta} \bar\Sigma_{\alpha_2\dots\alpha_{s-1}\dot\beta\dot\alpha_2\dots\dot\alpha_{s-1}} \Big) \\
& + \mu \frac{(s+2)(s+1)}{2s+1} H^{\alpha(s-1)\dot\alpha(s-1)} \mathcal{D}_{\alpha_1} \bar\Sigma_{\alpha_2\dots\alpha_{s-1}\dot\alpha(s-1)} \\
& - \bar\mu \frac{(s+2)(s+1)}{2s+1} H^{\alpha(s-1)\dot\alpha(s-1)} \bar{\mathcal{D}}_{\dot\alpha_1} \Sigma_{\alpha(s-1)\dot\alpha_2\dots\dot\alpha_{s-1}} \\
& - \frac{s-1}{8s} \Big(\Sigma^{\alpha(s-1)\dot\alpha(s-2)} \mathcal{D}^2 \Sigma_{\alpha(s-1)\dot\alpha(s-2)} - \bar\Sigma^{\alpha(s-2)\dot\alpha(s-1)} \bar{\mathcal{D}}^2 \bar\Sigma_{\alpha(s-2)\dot\alpha(s-1)} \Big) \\
& + \frac{1}{s^2} \bar\Sigma^{\alpha(s-2)\dot\alpha(s-2)\dot\beta} \Big(\frac{1}{2}(s^2+1) \mathcal{D}^\beta \bar{\mathcal{D}}_{\dot\beta} + \mathrm{i}(s-1)^2 \mathcal{D}^\beta{}_{\dot\beta} \Big) \Sigma_{\beta\alpha(s-2)\dot\alpha(s-2)} \\
& - \mu \frac{s^2+4s-1}{4s} \bar\Sigma^{\alpha(s-2)\dot\alpha(s-1)} \bar\Sigma_{\alpha(s-2)\dot\alpha(s-1)} \\
& - \bar\mu \frac{s^2+4s-1}{4s} \Sigma_{\alpha(s-1)\dot\alpha(s-2)} \Sigma^{\alpha(s-1)\dot\alpha(s-2)} \bigg\},
\end{aligned} \tag{3.19}
$$

where we have defined

$$
\boldsymbol{\Gamma}_{\alpha(s)\dot\alpha(s)} = \Gamma_{\alpha(s)\dot\alpha(s)} - \frac{1}{2} \bar{\mathcal{D}}_{(\dot\alpha_1} \mathcal{D}_{(\alpha_1} Z_{\alpha_2\dots\alpha_s)\dot\alpha_2\dots\dot\alpha_s)} - \mathrm{i}(s+1) \mathcal{D}_{(\alpha_1(\dot\alpha_1} Z_{\alpha_2\dots\alpha_s)\dot\alpha_2\dots\dot\alpha_s)} \,. \tag{3.20}
$$

The first-order model introduced is equivalent to the original theory (3.16). The action (3.17) is invariant under the gauge ξ-transformation (3.6) which acts on $U_{\alpha(s)\dot\alpha(s)}$ and $\Gamma_{\alpha(s)\dot\alpha(s)}$ by the rule

$$
\delta_\xi U_{\alpha(s)\dot\alpha(s)} = 0 \,, \tag{3.21a}
$$

$$
\delta_\xi \Gamma_{\alpha(s)\dot\alpha(s)} = \bar{\mathcal{D}}^{\dot\beta} \Big\{ \frac{s+1}{2(s+2)} \bar{\mathcal{D}}_{(\dot\beta} \mathcal{D}_{\alpha_1} \xi_{\alpha_2\dots\alpha_s\dot\alpha_1\dots\dot\alpha_s)} + \mathrm{i}(s+1) \mathcal{D}_{\alpha_1(\dot\beta} \xi_{\alpha_2\dots\alpha_s\dot\alpha_1\dots\dot\alpha_s)} \Big\} . \tag{3.21b}
$$

$\boldsymbol{\Gamma}_{\alpha(s)\dot\alpha(s)}$ is invariant under the gauge transformations (3.6) and (3.21b).

The first-order action (3.17) is also invariant under the gauge $\mathfrak{V}$-transformation (3.7b) and (3.7c), which acts on $U_{\alpha(s)\dot\alpha(s)}$ and $\Gamma_{\alpha(s)\dot\alpha(s)}$ as

$$\delta_{\mathfrak{V}} U_{\alpha(s)\dot\alpha(s)} = \frac{1}{2}\bar{\mathcal{D}}_{(\dot\alpha_1}\mathcal{D}_{(\alpha_1}\mathfrak{V}_{\alpha_2\dots\alpha_s)\dot\alpha_2\dots\dot\alpha_s)}\,, \tag{3.22a}$$

$$\delta_{\mathfrak{V}}\Gamma_{\alpha(s)\dot\alpha(s)} = 0\,. \tag{3.22b}$$

In accordance with (3.7c), the $\mathfrak{V}$-gauge freedom may be used to impose the condition

$$Z_{\alpha(s-1)\dot\alpha(s-1)} = 0\,. \tag{3.23}$$

In this gauge the action (3.19) reduces to the one defining the transverse formulation for the massless superspin-s multiplet [22]. The gauge condition (3.23) is preserved by residual local $\mathfrak{V}$- and ξ-transformations of the form

$$\bar{\mathcal{D}}^{\dot\beta}\xi_{\alpha(s-1)\dot\beta\dot\alpha(s-1)} + \bar{\mathfrak{V}}_{\alpha(s-1)\dot\alpha(s-1)} = 0\,. \tag{3.24}$$

Making use of the parametrisation (3.12), the residual gauge freedom is

$$\delta H_{\alpha(s-1)\dot\alpha(s-1)} = \mathcal{D}^{\beta}L_{\beta\alpha(s-1)\dot\alpha(s-1)} - \bar{\mathcal{D}}^{\dot\beta}\bar{L}_{\alpha(s-1)\dot\beta\dot\alpha(s-1)}\,, \tag{3.25a}$$

$$\delta\Gamma_{\alpha(s)\dot\alpha(s)} = \frac{s+1}{2(s+2)}\bar{\mathcal{D}}^{\dot\beta}\Big\{\bar{\mathcal{D}}_{(\dot\beta}\mathcal{D}_{(\alpha_1} + 2\mathrm{i}(s+2)\mathcal{D}_{(\alpha_1(\dot\beta}\Big\}\bar{L}_{\alpha_2\dots\alpha_s)\dot\alpha_1\dots\dot\alpha_s)}\,, \tag{3.25b}$$

which is exactly the gauge symmetry of the transverse formulation for the massless superspin-s multiplet [22].

3.3 Models for the massless gravitino multiplet in AdS

The massless gravitino multiplet (i.e., the massless superspin-1 multiplet) was excluded from the above consideration. Here we will fill the gap.

The (generalised) longitudinal formulation for the gravitino multiplet is described by the action

$$\begin{aligned} S^{\parallel}_{\rm GM} = -\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta\, E\,\Big\{&\frac{1}{16}H\mathcal{D}^{\alpha}(\bar{\mathcal{D}}^2-4\mu)\mathcal{D}_{\alpha}H + \frac{1}{4}H\big(\mathcal{D}^{\alpha}\bar{\mathcal{D}}^{\dot\alpha}G_{\alpha\dot\alpha} - \bar{\mathcal{D}}^{\dot\alpha}\mathcal{D}^{\alpha}\bar{G}_{\alpha\dot\alpha}\big)\\ &+\bar{G}^{\alpha\dot\alpha}G_{\alpha\dot\alpha} + \frac{1}{4}\big(G^{\alpha\dot\alpha}G_{\alpha\dot\alpha} + \bar{G}^{\alpha\dot\alpha}\bar{G}_{\alpha\dot\alpha}\big)\\ &+|\mu|^2\left(H - \frac{\Phi}{\mu} - \frac{\bar\Phi}{\bar\mu}\right)^2 + \left(\frac{\Phi}{\mu} + \frac{\bar\Phi}{\bar\mu}\right)\big(\mu\mathcal{D}^{\alpha}\Psi_{\alpha} + \bar\mu\bar{\mathcal{D}}_{\dot\alpha}\bar\Psi^{\dot\alpha}\big)\Big\}\,, \end{aligned} \tag{3.26a}$$

where Φ is a chiral scalar superfield, $\bar{\mathcal{D}}_{\dot\alpha}\Phi = 0$, and

$$G_{\alpha\dot\alpha} = \bar{\mathcal{D}}_{\dot\alpha}\Psi_{\alpha}\,, \qquad \bar{G}_{\alpha\dot\alpha} = -\mathcal{D}_{\alpha}\bar\Psi_{\dot\alpha}\,. \tag{3.26b}$$

This action is invariant under gauge transformations of the form

$$\delta H = \mathfrak{V} + \bar{\mathfrak{V}}\,, \tag{3.27a}$$

$$\delta\Psi_{\alpha} == \frac{1}{2}\mathcal{D}_{\alpha}\mathfrak{V} + \eta_{\alpha}\,, \qquad \bar{\mathcal{D}}_{\dot\alpha}\eta_{\alpha} = 0\,, \tag{3.27b}$$

$$\delta\Phi = -\frac{1}{4}(\bar{\mathcal{D}}^2 - 4\mu)\bar{\mathfrak{V}}\,. \tag{3.27c}$$

This is one of the two models for the massless gravitino multiplet in AdS introduced in [11]. In a flat-superspace limit, the action reduces to that given in [56]. Imposing the gauge condition $\Phi = 0$ reduces the action (3.26) to the original longitudinal formulation for the massless gravitino multiplet in AdS [22].

The action (3.26) involves the chiral scalar Φ and its conjugate only in the combination $(\varphi + \bar{\varphi})$, where $\varphi = \Phi/\mu$. This means that the model (3.26) possesses a dual formulation realised in terms of a real linear superfield L,

$$\big(\bar{\mathcal{D}}^2 - 4\mu\big)L = 0\,, \qquad \bar{L} = L\,. \tag{3.28}$$

The dual model is described by the action [11]

$$\begin{aligned}S_{\rm GM} = -\int {\rm d}^4x{\rm d}^2\theta{\rm d}^2\bar{\theta}\, E\,\Big\{&\frac{1}{16}H\mathcal{D}^\alpha(\bar{\mathcal{D}}^2-4\mu)\mathcal{D}_\alpha H + \frac{1}{4}H\big(\mathcal{D}^\alpha\bar{\mathcal{D}}^{\dot\alpha}G_{\alpha\dot\alpha} - \bar{\mathcal{D}}^{\dot\alpha}\mathcal{D}^\alpha\bar{G}_{\alpha\dot\alpha}\big)\\ &+\bar{G}^{\alpha\dot\alpha}G_{\alpha\dot\alpha} + \frac{1}{4}\big(G^{\alpha\dot\alpha}G_{\alpha\dot\alpha} + \bar{G}^{\alpha\dot\alpha}\bar{G}_{\alpha\dot\alpha}\big) + |\mu|^2H^2\\ &-\frac{1}{4}\Big(2|\mu|H + L - \frac{\mu}{|\mu|}\mathcal{D}^\alpha\Psi_\alpha - \frac{\bar{\mu}}{|\mu|}\bar{\mathcal{D}}_{\dot\alpha}\bar{\Psi}^{\dot\alpha}\Big)^2\Big\}\,.\end{aligned} \tag{3.29}$$

This action is invariant under the gauge transformations (3.27a), (3.27b) and

$$\delta L = \frac{1}{|\mu|}\big(\mu\mathcal{D}^\alpha\eta_\alpha + \bar{\mu}\bar{\mathcal{D}}_{\dot\alpha}\bar{\eta}^{\dot\alpha}\big)\,. \tag{3.30}$$

In a flat-superspace limit, the action (3.29) reduces to that given in [57].

In Minkowski superspace, there exists one more dual realisation for the massless gravitino multiplet model [26] which is obtained by performing a Legendre transformation converting Φ into a complex linear superfield. This formulation cannot be lifted to the AdS case, the reason being the fact that the action (3.26) involves the chiral scalar Φ and its conjugate only in the combination $(\varphi + \bar{\varphi})$, where $\varphi = \Phi/\mu$.

The dependence on Ψ_α and $\bar{\Psi}_{\dot\alpha}$ in the last term of (3.26) can be expressed in terms of $G_{\alpha\dot\alpha}$ and $\bar{G}_{\alpha\dot\alpha}$ if we introduce a complex unconstrained prepotential U for Φ in the standard way

$$\Phi = -\frac{1}{4}(\bar{\mathcal{D}}^2 - 4\mu)U\,. \tag{3.31}$$

Then making use of (2.5d) gives

$$\int {\rm d}^4x{\rm d}^2\theta{\rm d}^2\bar{\theta}\, E\,\Phi\mathcal{D}^\alpha\Psi_\alpha = -\int {\rm d}^4x{\rm d}^2\theta{\rm d}^2\bar{\theta}\, E\, G^{\alpha\dot\alpha}\Big(\frac{1}{4}\bar{\mathcal{D}}_{\dot\alpha}\mathcal{D}_\alpha + {\rm i}\mathcal{D}_{\alpha\dot\alpha}\Big)U\,. \tag{3.32}$$

Since the resulting action depends on $G_{\alpha\dot\alpha}$ and $\bar{G}_{\alpha\dot\alpha}$, we can introduce a dual formulation for the theory that is obtained turning $G_{\alpha\dot\alpha}$ and $\bar{G}_{\alpha\dot\alpha}$ into a transverse linear superfield

$$\Gamma_{\alpha\dot\alpha} = \bar{\mathcal{D}}^{\dot\beta}\Phi_{\alpha\,\dot\alpha\dot\beta}\,, \qquad \Phi_{\alpha\,\dot\beta\dot\alpha} = \Phi_{\alpha\,\dot\alpha\dot\beta} \tag{3.33}$$

and its conjugate using the scheme described in [22]. The resulting action is

$$
\begin{aligned}
S^{\perp}_{\rm GM} = \int {\rm d}^4x{\rm d}^2\theta{\rm d}^2\bar\theta\, E\,\Big\{ & -\frac{1}{16}H\mathcal{D}^\alpha(\bar{\mathcal{D}}^2-4\mu)\mathcal{D}_\alpha H \\
& +\frac{1}{96}[\mathcal{D}^\alpha,\bar{\mathcal{D}}^{\dot\alpha}]H\,[\mathcal{D}_\alpha,\bar{\mathcal{D}}_{\dot\alpha}]H+\frac{1}{8}\mathcal{D}^{\alpha\dot\alpha}H\,\mathcal{D}_{\alpha\dot\alpha}H \\
& +\frac{1}{3}\bar{\mathbf{\Gamma}}^{\alpha\dot\alpha}\mathbf{\Gamma}_{\alpha\dot\alpha}-\frac{1}{12}\Big(\mathbf{\Gamma}^{\alpha\dot\alpha}\mathbf{\Gamma}_{\alpha\dot\alpha}+\bar{\mathbf{\Gamma}}^{\alpha\dot\alpha}\bar{\mathbf{\Gamma}}_{\alpha\dot\alpha}\Big)+\frac{\rm i}{3}\Big(\bar{\mathbf{\Gamma}}^{\alpha\dot\alpha}-\mathbf{\Gamma}^{\alpha\dot\alpha}\Big)\mathcal{D}_{\alpha\dot\alpha}H \\
& -\frac{1}{6}\Phi\mathcal{D}^2H-\frac{1}{6}\bar\Phi\bar{\mathcal{D}}^2H-|\mu|^2\Big(H-\frac{\Phi}{\mu}-\frac{\bar\Phi}{\bar\mu}\Big)^2\Big\}\,,
\end{aligned}
\tag{3.34}
$$

where we have defined

$$
\mathbf{\Gamma}_{\alpha\dot\alpha}:=\Gamma_{\alpha\dot\alpha}-\frac{1}{2}\bar{\mathcal{D}}_{\dot\alpha}\mathcal{D}_\alpha U-2{\rm i}\,\mathcal{D}_{\alpha\dot\alpha}U\,. \tag{3.35}
$$

The action (3.34) is invariant under the following gauge transformations

$$
\delta_\xi U=\bar{\mathcal{D}}_{\dot\alpha}\bar\xi^{\dot\alpha}\,, \tag{3.36a}
$$

$$
\delta_\xi\Gamma_{\alpha\dot\alpha}=-\frac{1}{3}\bar{\mathcal{D}}^{\dot\beta}\Big\{\bar{\mathcal{D}}_{(\dot\beta}\mathcal{D}_\alpha\bar\xi_{\dot\alpha)}+6{\rm i}\,\mathcal{D}_{\alpha(\dot\beta}\bar\xi_{\dot\alpha)}\Big\}\,. \tag{3.36b}
$$

Both Φ and $\mathbf{\Gamma}_{\alpha\dot\alpha}$ are invariant under ξ-gauge transformations. The action (3.34) is also invariant under the gauge transformations (3.27a), (3.27c) and

$$
\delta_{\mathfrak{V}}U=\bar{\mathfrak{V}}\,, \tag{3.37a}
$$

$$
\delta_{\mathfrak{V}}\Gamma_{\alpha\dot\alpha}=0\,. \tag{3.37b}
$$

Imposing the gauge condition $U=0$ reduces the action (3.34) to the original transverse formulation for the massless gravitino multiplet in AdS [22].

4 Higher spin supercurrents

In this section we introduce higher spin supercurrent multiplets in AdS. First of all, we recall the structure of the gauge superfields in terms of which the massless half-integer superspin multiplets are described [22].

4.1 Massless half-integer superspin multiplets

For a massless multiplet of half-integer superspin $s+1/2$, with $s=2,3,\ldots$, there exist two off-shell formulations [22] which are referred to as transverse and longitudinal. They are described in terms of the following dynamical variables:

$$
\mathcal{V}^{\perp}_{s+1/2}=\Big\{H_{\alpha(s)\dot\alpha(s)}\,,\ \Gamma_{\alpha(s-1)\dot\alpha(s-1)}\,,\ \bar\Gamma_{\alpha(s-1)\dot\alpha(s-1)}\Big\}\,, \tag{4.1a}
$$

$$
\mathcal{V}^{\parallel}_{s+1/2}=\Big\{H_{\alpha(s)\dot\alpha(s)}\,,\ G_{\alpha(s-1)\dot\alpha(s-1)}\,,\ \bar G_{\alpha(s-1)\dot\alpha(s-1)}\Big\}\,. \tag{4.1b}
$$

Here $H_{\alpha(s)\dot\alpha(s)}$ is a real unconstrained superfield. The complex superfields $\Gamma_{\alpha(s-1)\dot\alpha(s-1)}$ and $G_{\alpha(s-1)\dot\alpha(s-1)}$ are transverse linear and longitudinal linear, respectively,

$$
\bar{\mathcal{D}}^{\dot\beta}\,\Gamma_{\alpha(s-1)\dot\beta\dot\alpha(s-2)}=0\,, \tag{4.2a}
$$

$$
\bar{\mathcal{D}}_{(\dot\alpha_1}G_{\alpha(s-1)\dot\alpha_2\ldots\dot\alpha_s)}=0\,. \tag{4.2b}
$$

These constraints are solved in terms of unconstrained prepotentials as follows:

$$\Gamma_{\alpha(s-1)\dot\alpha(s-1)} = \bar{\mathcal{D}}^{\dot\beta}\Phi_{\alpha(s-1)\,(\dot\beta\dot\alpha_1\cdots\dot\alpha_{s-1})}\,, \tag{4.3a}$$

$$G_{\alpha(s-1)\dot\alpha(s-1)} = \bar{\mathcal{D}}_{(\dot\alpha_1}\Psi_{\alpha(s-1)\,\dot\alpha_2\cdots\dot\alpha_{s-1})}\,. \tag{4.3b}$$

The prepotentials are defined modulo gauge transformations of the form:

$$\delta_\xi\Phi_{\alpha(s-1)\,\dot\alpha(s)} = \bar{\mathcal{D}}^{\dot\beta}\xi_{\alpha(s-1)\,(\dot\beta\dot\alpha_1\cdots\dot\alpha_s)}\,, \tag{4.4a}$$

$$\delta_\zeta\Psi_{\alpha(s-1)\,\dot\alpha(s-2)} = \bar{\mathcal{D}}_{(\dot\alpha_1}\zeta_{\alpha(s-1)\,\dot\alpha_2\cdots\dot\alpha_{s-2})}\,, \tag{4.4b}$$

with the gauge parameters $\xi_{\alpha(s-1)\,\dot\alpha(s+1)}$ and $\zeta_{\alpha(s-1)\,\dot\alpha(s-3)}$ being unconstrained.

The gauge transformations of the superfields H, Γ and G are

$$\delta_\Lambda H_{\alpha_1\dots\alpha_s\dot\alpha_1\dots\dot\alpha_s} = \bar{\mathcal{D}}_{(\dot\alpha_1}\Lambda_{\alpha_1\dots\alpha_s\dot\alpha_2\dots\dot\alpha_s)} - \mathcal{D}_{(\alpha_1}\bar\Lambda_{\alpha_2\dots\alpha_s)\dot\alpha_1\dots\dot\alpha_s}\,, \tag{4.5a}$$

$$\begin{aligned}\delta_\Lambda\Gamma_{\alpha_1\dots\alpha_{s-1}\dot\alpha_1\dots\dot\alpha_{s-1}} &= -\frac{s}{2(s+1)}\bar{\mathcal{D}}^{\dot\beta}\mathcal{D}^{\beta}\mathcal{D}_{(\beta}\bar\Lambda_{\alpha(s-1))\dot\beta\dot\alpha(s-1)}\\ &= -\frac{1}{4}\bar{\mathcal{D}}^{\dot\beta}\mathcal{D}^2\bar\Lambda_{\alpha_1\dots\alpha_{s-1}\dot\beta\dot\alpha_1\dots\dot\alpha_{s-1}}\\ &\quad -\frac{1}{2}\bar\mu(s-1)\bar{\mathcal{D}}^{\dot\beta}\bar\Lambda_{\alpha_1\dots\alpha_{s-1}\dot\beta\dot\alpha_1\dots\dot\alpha_{s-1}}\,,\end{aligned} \tag{4.5b}$$

$$\begin{aligned}\delta_\Lambda G_{\alpha_1\dots\alpha_{s-1}\dot\alpha_1\dots\dot\alpha_{s-1}} &= -\frac{1}{2}\bar{\mathcal{D}}_{(\dot\alpha_1}\bar{\mathcal{D}}^{|\dot\beta|}\mathcal{D}^{\beta}\Lambda_{\beta\alpha_1\dots\alpha_{s-1}\dot\alpha_2\dots\dot\alpha_{s-1})\dot\beta}\\ &\quad + \mathrm{i}(s-1)\bar{\mathcal{D}}_{(\dot\alpha_1}\mathcal{D}^{\beta|\dot\beta|}\Lambda_{\beta\alpha_1\dots\alpha_{s-1}\dot\alpha_2\dots\dot\alpha_{s-1})\dot\beta}\,.\end{aligned} \tag{4.5c}$$

Here the gauge parameter $\Lambda_{\alpha_1\dots\alpha_s\dot\alpha_1\dots\dot\alpha_{s-1}} = \Lambda_{(\alpha_1\dots\alpha_s)(\dot\alpha_1\dots\dot\alpha_{s-1})}$ is unconstrained. The symmetrisation in (4.5c) is extended only to the indices $\dot\alpha_1, \dot\alpha_2, \dots, \dot\alpha_{s-1}$. It follows from (4.5b) and (4.5c) that the transformation laws of the prepotentials $\Phi_{\alpha(s-1)\dot\alpha(s)}$ and $\Psi_{\alpha(s-1)\dot\alpha(s-2)}$ are

$$\delta_\Lambda\Phi_{\alpha_1\dots\alpha_{s-1}\dot\alpha_1\dots\dot\alpha_s} = -\frac{1}{4}\mathcal{D}^2\bar\Lambda_{\alpha_1\dots\alpha_{s-1}\dot\alpha_1\dots\dot\alpha_s} - \frac{1}{2}\bar\mu(s-1)\bar\Lambda_{\alpha_1\dots\alpha_{s-1}\dot\alpha_1\dots\dot\alpha_s}\,, \tag{4.6a}$$

$$\delta_\Lambda\Psi_{\alpha_1\dots\alpha_{s-1}\dot\alpha_1\dots\dot\alpha_{s-2}} = -\frac{1}{2}\Big(\bar{\mathcal{D}}^{\dot\beta}\mathcal{D}^{\beta} - 2\mathrm{i}(s-1)\mathcal{D}^{\beta\dot\beta}\Big)\Lambda_{\beta\alpha_1\dots\alpha_{s-1}\dot\beta\dot\alpha_1\dots\dot\alpha_{s-2}}\,. \tag{4.6b}$$

4.2 Non-conformal supercurrents: half-integer superspin

In the framework of the longitudinal formulation, let us couple the prepotentials $H_{\alpha(s)\dot\alpha(s)}$, $\Psi_{\alpha(s-1)\dot\alpha(s-2)}$ and $\bar\Psi_{\alpha(s-2)\dot\alpha(s-1)}$, to external sources

$$\begin{aligned}S^{(s+\frac12)}_{\rm source} = \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar\theta\,E\,\Big\{&H^{\alpha(s)\dot\alpha(s)}J_{\alpha(s)\dot\alpha(s)} + \Psi^{\alpha(s-1)\dot\alpha(s-2)}T_{\alpha(s-1)\dot\alpha(s-2)}\\ &+\bar\Psi_{\alpha(s-2)\dot\alpha(s-1)}\bar T^{\alpha(s-2)\dot\alpha(s-1)}\Big\}\,.\end{aligned} \tag{4.7}$$

Requiring $S^{(s+\frac12)}_{\rm source}$ to be invariant under (4.4b) gives

$$\bar{\mathcal{D}}^{\dot\beta}T_{\alpha(s-1)\dot\beta\dot\alpha_1\dots\dot\alpha_{s-3}} = 0\,, \tag{4.8a}$$

and therefore $T_{\alpha(s-1)\dot\alpha(s-2)}$ is a transverse linear superfield. Requiring $S^{(s+\frac12)}_{\rm source}$ to be invariant under the gauge transformations (4.5a) and (4.6b) gives the following conservation equation:

$$\bar{\mathcal{D}}^{\dot\beta}J_{\alpha_1\dots\alpha_s\dot\beta\dot\alpha_1\dots\dot\alpha_{s-1}} + \frac{1}{2}\Big(\mathcal{D}_{(\alpha_1}\bar{\mathcal{D}}_{(\dot\alpha_1} - 2\mathrm{i}(s-1)\mathcal{D}_{(\alpha_1(\dot\alpha_1}\Big)T_{\alpha_2\dots\alpha_s)\dot\alpha_2\dots\dot\alpha_{s-1})} = 0\,. \tag{4.8b}$$

For completeness, we also give the conjugate equation

$$\mathcal{D}^{\beta} J_{\beta\alpha_1\dots\alpha_{s-1}\dot\alpha_1\dots\dot\alpha_s} - \frac{1}{2}\Big(\bar{\mathcal{D}}_{(\dot\alpha_1}\mathcal{D}_{(\alpha_1} - 2\mathrm{i}(s-1)\mathcal{D}_{(\alpha_1(\dot\alpha_1}\Big)\bar{T}_{\alpha_2\dots\alpha_{s-1})\dot\alpha_2\dots\dot\alpha_s)} = 0\,. \tag{4.8c}$$

Similar considerations for the transverse formulation lead to the following non-conformal supercurrent multiplet

$$\bar{\mathcal{D}}^{\dot\beta}\mathbb{J}_{\alpha_1\dots\alpha_s\dot\beta\dot\alpha_1\dots\dot\alpha_{s-1}} - \frac{1}{4}(\bar{\mathcal{D}}^2 + 2\mu(s-1))\mathbb{F}_{\alpha_1\dots\alpha_s\dot\alpha_1\dots\dot\alpha_{s-1}} = 0\,, \tag{4.9a}$$

$$\mathcal{D}_{(\alpha_1}\mathbb{F}_{\alpha_2\dots\alpha_{s+1})\dot\alpha_1\dots\dot\alpha_{s-1}} = 0\,. \tag{4.9b}$$

Thus, the trace multiplet $\bar{\mathbb{F}}_{\alpha(s-1)\dot\alpha(s)}$ is longitudinal linear.

In the flat-superspace limit, the higher spin supercurrent multiplets (4.8) and (4.9) reduce to those described in [25].

As in [25], it is useful to introduce auxiliary complex variables $\zeta^\alpha \in \mathbb{C}^2$ and their conjugates $\bar\zeta^{\dot\alpha}$. Given a tensor superfield $U_{\alpha(m)\dot\alpha(n)}$, we associate with it the following field on $\mathbb{C}^2$

$$U_{(m,n)}(\zeta,\bar\zeta) := \zeta^{\alpha_1}\dots\zeta^{\alpha_m}\bar\zeta^{\dot\alpha_1}\dots\bar\zeta^{\dot\alpha_n}U_{\alpha_1\dots\alpha_m\dot\alpha_1\dots\dot\alpha_n}\,, \tag{4.10}$$

which is homogeneous of degree (m,n) in the variables ζ^α and $\bar\zeta^{\dot\alpha}$. We introduce operators that increase the degree of homogeneity in the variables ζ^α and $\bar\zeta^{\dot\alpha}$,

$$\mathcal{D}_{(1,0)} := \zeta^\alpha\mathcal{D}_\alpha\,, \tag{4.11a}$$

$$\bar{\mathcal{D}}_{(0,1)} := \bar\zeta^{\dot\alpha}\bar{\mathcal{D}}_{\dot\alpha}\,, \tag{4.11b}$$

$$\mathcal{D}_{(1,1)} := 2\mathrm{i}\zeta^\alpha\bar\zeta^{\dot\alpha}\mathcal{D}_{\alpha\dot\alpha} = -\big\{\mathcal{D}_{(1,0)},\bar{\mathcal{D}}_{(0,1)}\big\}\,. \tag{4.11c}$$

We also introduce two operators that decrease the degree of homogeneity in the variables ζ^α and $\bar\zeta^{\dot\alpha}$, specifically

$$\mathcal{D}_{(-1,0)} := \mathcal{D}^\alpha\frac{\partial}{\partial\zeta^\alpha}\,, \tag{4.12a}$$

$$\bar{\mathcal{D}}_{(0,-1)} := \bar{\mathcal{D}}^{\dot\alpha}\frac{\partial}{\partial\bar\zeta^{\dot\alpha}}\,. \tag{4.12b}$$

Making use of the above notation, the transverse linear condition (4.8a) and its conjugate become

$$\bar{\mathcal{D}}_{(0,-1)}T_{(s-1,s-2)} = 0\,, \tag{4.13a}$$

$$\mathcal{D}_{(-1,0)}\bar{T}_{(s-2,s-1)} = 0\,. \tag{4.13b}$$

The conservation equations (4.8b) and (4.8c) turn into

$$\frac{1}{s}\bar{\mathcal{D}}_{(0,-1)}J_{(s,s)} - \frac{1}{2}A_{(1,1)}T_{(s-1,s-2)} = 0\,, \tag{4.14a}$$

$$\frac{1}{s}\mathcal{D}_{(-1,0)}J_{(s,s)} - \frac{1}{2}\bar{A}_{(1,1)}\bar{T}_{(s-2,s-1)} = 0\,. \tag{4.14b}$$

where

$$A_{(1,1)} := -\mathcal{D}_{(1,0)}\bar{\mathcal{D}}_{(0,1)} + (s-1)\mathcal{D}_{(1,1)}\,, \quad \bar{A}_{(1,1)} := \bar{\mathcal{D}}_{(0,1)}\mathcal{D}_{(1,0)} - (s-1)\mathcal{D}_{(1,1)}\,. \tag{4.15}$$

Since $\bar{\mathcal{D}}^2_{(0,-1)} J_{(s,s)} = 0$, the conservation equation (4.14a) is consistent provided

$$\bar{\mathcal{D}}_{(0,-1)} \mathcal{A}_{(1,1)} T_{(s-1,s-2)} = 0 \,. \tag{4.16}$$

This is indeed true, as a consequence of the transverse linear condition (4.13a).

4.3 Improvement transformations

The conservation equations (4.8) and (4.9) define two consistent higher spin supercurrents in AdS. Similar to the two irreducible AdS supercurrents [12], with $(12+12)$ and $(20+20)$ degrees of freedom, the higher spin supercurrents (4.8) and (4.9) are equivalent in the sense that there always exists a well defined improvement transformation that converts (4.8) into (4.9). Such an improvement transformation is constructed below.

Since the trace multiplet $T_{\alpha(s-1)\dot{\alpha}(s-2)}$ is transverse, eq. (4.8a), there exists a well-defined complex tensor operator $X_{\alpha(s-1)\dot{\alpha}(s-1)}$ such that

$$T_{\alpha(s-1)\dot{\alpha}(s-2)} = \bar{\mathcal{D}}^{\dot{\beta}} X_{\alpha(s-1)(\dot{\beta}\dot{\alpha}_1\dots\dot{\alpha}_{s-2})} \,. \tag{4.17}$$

Let us introduce the real $U_{\alpha(s-1)\dot{\alpha}(s-1)}$ and imaginary $V_{\alpha(s-1)\dot{\alpha}(s-1)}$ parts of $X_{\alpha(s-1)\dot{\alpha}(s-1)}$,

$$X_{\alpha(s-1)\dot{\alpha}(s-1)} = U_{\alpha(s-1)\dot{\alpha}(s-1)} + \mathrm{i} V_{\alpha(s-1)\dot{\alpha}(s-1)} \,. \tag{4.18}$$

Then it may be checked that the operators

$$\mathbb{J}_{\alpha(s)\dot{\alpha}(s)} := J_{\alpha(s)\dot{\alpha}(s)} + \frac{s}{2}\big[\mathcal{D}_{(\alpha_1}, \bar{\mathcal{D}}_{(\dot{\alpha}_1}\big] U_{\alpha_2\dots\alpha_s)\dot{\alpha}_2\dots\dot{\alpha}_s)} + s\mathcal{D}_{(\alpha_1(\dot{\alpha}_1} V_{\alpha_2\dots\alpha_s)\dot{\alpha}_2\dots\dot{\alpha}_s)} \,, \tag{4.19a}$$

$$\mathbb{F}_{\alpha(s)\dot{\alpha}(s-1)} := \mathcal{D}_{(\alpha_1}\Big\{(2s+1) U_{\alpha_2\dots\alpha_s)\dot{\alpha}(s-1)} - \mathrm{i} V_{\alpha_2\dots\alpha_s)\dot{\alpha}(s-1)}\Big\} \tag{4.19b}$$

enjoy the conservation equation (4.9).

In accordance with the result obtained, for all applications it suffices to work with the longitudinal supercurrent (4.8). This is why in the integer superspin case, which will be studied in section 4.4, we will introduce only the longitudinal supercurrent.

There exists an improvement transformation for the supercurrent multiplet (4.8). Given a chiral scalar superfield Ω, introduce

$$\widetilde{J}_{(s,s)} := J_{(s,s)} + \mathcal{D}^s_{(1,1)}\big(\Omega + (-1)^s \bar{\Omega}\big) \,, \qquad \bar{\mathcal{D}}_{\dot{\alpha}} \Omega = 0 \,, \tag{4.20a}$$

$$\widetilde{T}_{(s-1,s-2)} := T_{(s-1,s-2)} + \frac{2(-1)^s}{s(s-1)} \bar{\mathcal{D}}_{(0,-1)} \mathcal{D}^{s-1}_{(1,1)} \bar{\Omega} + \frac{4(s+1)}{s} \mu \mathcal{D}^{s-2}_{(1,1)} \mathcal{D}_{(1,0)} \Omega \,. \tag{4.20b}$$

The operators $\widetilde{J}_{(s,s)}$ and $\widetilde{T}_{(s-1,s-2)}$ prove to obey the conservation equation (4.8).

4.4 Non-conformal supercurrents: integer superspin

We now make use of the new gauge formulation (3.10), or equivalently (3.16), for the integer superspin-s multiplet to derive the AdS analogue of the non-conformal higher spin supercurrents constructed in [26].

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Let us couple the prepotentials $H_{\alpha(s-1)\dot\alpha(s-1)}$, $Z_{\alpha(s-1)\dot\alpha(s-1)}$ and $\Psi_{\alpha(s)\dot\alpha(s-1)}$ to external sources

$$S^{(s)}_{\rm source} = \int {\rm d}^4x{\rm d}^2\theta{\rm d}^2\bar\theta\, E\,\Big\{\Psi^{\alpha(s)\dot\alpha(s-1)}J_{\alpha(s)\dot\alpha(s-1)} - \bar\Psi^{\alpha(s-1)\dot\alpha(s)}\bar J_{\alpha(s-1)\dot\alpha(s)} \\ +H^{\alpha(s-1)\dot\alpha(s-1)}S_{\alpha(s-1)\dot\alpha(s-1)} \\ +Z^{\alpha(s-1)\dot\alpha(s-1)}T_{\alpha(s-1)\dot\alpha(s-1)} + \bar Z^{\alpha(s-1)\dot\alpha(s-1)}\bar T_{\alpha(s-1)\dot\alpha(s-1)}\Big\}\,. \tag{4.21}$$

In order for $S^{(s)}_{\rm source}$ to be invariant under the ζ-transformation in (3.7a), the source $J_{\alpha(s)\dot\alpha(s-1)}$ must satisfy

$$\bar{\mathcal D}^{\dot\beta}J_{\alpha(s)\dot\beta\dot\alpha(s-2)} = 0 \quad\Longleftrightarrow\quad \mathcal D^\beta \bar J_{\beta\alpha(s-2)\dot\alpha(s)} = 0\,. \tag{4.22}$$

Next, requiring $S^{(s)}_{\rm source}$ to be invariant under the transformation (3.6) leads to

$$\bar{\mathcal D}_{(\dot\alpha_1}T_{\alpha(s-1)\dot\alpha_2\dots\dot\alpha_s)} = 0 \quad\Longleftrightarrow\quad \mathcal D_{(\alpha_1}\bar T_{\alpha_2\dots\alpha_s)\dot\alpha(s-1)} = 0\,. \tag{4.23}$$

We see that the superfields $J_{\alpha(s)\dot\alpha(s-1)}$ and $T_{\alpha(s-1)\dot\alpha(s-1)}$ are transverse linear and longitudinal linear, respectively. Finally, requiring $S^{(s)}_{\rm source}$ to be invariant under the $\mathfrak{V}$-transformation (3.7) gives the following conservation equation

$$-\frac12\mathcal D^\beta J_{\beta\alpha(s-1)\dot\alpha(s-1)} + S_{\alpha(s-1)\dot\alpha(s-1)} + \bar T_{\alpha(s-1)\dot\alpha(s-1)} = 0 \tag{4.24a}$$

as well as its conjugate

$$\frac12\bar{\mathcal D}^{\dot\beta}\bar J_{\alpha(s-1)\dot\beta\dot\alpha(s-1)} + S_{\alpha(s-1)\dot\alpha(s-1)} + T_{\alpha(s-1)\dot\alpha(s-1)} = 0\,. \tag{4.24b}$$

As a consequence of (4.23), from (4.24a) we deduce

$$\frac14\mathcal D^2 J_{\alpha(s)\dot\alpha(s-1)} - \frac12\bar\mu(s+2)J_{\alpha(s)\dot\alpha(s-1)} + \mathcal D_{(\alpha_1}S_{\alpha_2\dots\alpha_s)\dot\alpha(s-1)} = 0\,. \tag{4.25}$$

The equations (4.22) and (4.25) describe the conserved current supermultiplet which corresponds to our theory in the gauge (3.11).

Taking the sum of (4.24a) and (4.24b) leads to

$$\frac12\mathcal D^\beta J_{\beta\alpha(s-1)\dot\alpha(s-1)} + \frac12\bar{\mathcal D}^{\dot\beta}\bar J_{\alpha(s-1)\dot\beta\dot\alpha(s-1)} + T_{\alpha(s-1)\dot\alpha(s-1)} - \bar T_{\alpha(s-1)\dot\alpha(s-1)} = 0\,. \tag{4.26}$$

The equations (4.22), (4.23) and (4.26) describe the conserved current supermultiplet which corresponds to our theory in the gauge (3.13). As a consequence of (4.23), the conservation equation (4.26) implies

$$\frac12\mathcal D_{(\alpha_1}\Big\{\mathcal D^{|\beta|}J_{\alpha_2\dots\alpha_s)\beta\dot\alpha(s-1)} + \bar{\mathcal D}^{\dot\beta}\bar J_{\alpha_2\dots\alpha_s)\dot\beta\dot\alpha(s-1)}\Big\} + \mathcal D_{(\alpha_1}T_{\alpha_2\dots\alpha_s)\dot\alpha(s-1)} = 0\,. \tag{4.27}$$

Using our notation introduced in section 3, the transverse linear condition (4.22) turns into

$$\bar{\mathcal D}_{(0,-1)}J_{(s,s-1)} = 0\,, \tag{4.28}$$

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while the longitudinal linear condition (4.23) takes the form

$$\bar{\mathcal{D}}_{(0,1)}T_{(s-1,s-1)} = 0\,. \tag{4.29}$$

The conservation equation (4.24a) becomes

$$-\frac{1}{2s}\mathcal{D}_{(-1,0)}J_{(s,s-1)} + S_{(s-1,s-1)} + \bar{T}_{(s-1,s-1)} = 0 \tag{4.30}$$

and (4.27) takes the form

$$\frac{1}{2s}\mathcal{D}_{(1,0)}\left\{\mathcal{D}_{(-1,0)}J_{(s,s-1)} + \bar{\mathcal{D}}_{(0,-1)}\bar{J}_{(s-1,s)}\right\} + \mathcal{D}_{(1,0)}T_{(s-1,s-1)} = 0\,. \tag{4.31}$$

In the flat-superspace limit, the higher spin supercurrent multiplet described by eqs. (4.22) and (4.25) reduces to the one proposed in [26].

4.5 Improvement transformation

There exist an improvement transformation for the supercurrent multiplet (4.24). Given a chiral scalar superfield Ω, we introduce

$$\widetilde{J}_{(s,s-1)} := J_{(s,s-1)} + \mathcal{D}^{s-1}_{(1,1)}\mathcal{D}_{(1,0)}\Omega\,, \qquad \bar{\mathcal{D}}_{\dot\alpha}\Omega = 0\,, \tag{4.32a}$$

$$\begin{aligned}\widetilde{\bar{T}}_{(s-1,s-1)} &:= \bar{T}_{(s-1,s-1)} + \frac{s-1}{4s}\mathcal{D}^{s-1}_{(1,1)}(\mathcal{D}^2 - 4\bar{\mu})\Omega \\ &\quad + (-1)^s(s-1)\Big(\bar{\mu} + \frac{\mu}{s}\Big)\mathcal{D}^{s-1}_{(1,1)}\bar{\Omega}\,,\end{aligned} \tag{4.32b}$$

$$\begin{aligned}\widetilde{S}_{(s-1,s-1)} &:= S_{(s-1,s-1)} + \mu(s-1)\mathcal{D}^{s-1}_{(1,1)}\Omega + (-1)^{s-1}\bar{\mu}(s-1)\mathcal{D}^{s-1}_{(1,1)}\bar{\Omega} \\ &\quad + \bar{\mu}\frac{s-1}{s}\mathcal{D}^{s-1}_{(1,1)}\Omega + (-1)^{s-1}\mu\frac{s-1}{s}\mathcal{D}^{s-1}_{(1,1)}\bar{\Omega}\,.\end{aligned} \tag{4.32c}$$

It may be checked that the operators $\widetilde{J}_{(s,s-1)}$, $\widetilde{\bar{T}}_{(s-1,s-1)}$ and $\widetilde{S}_{(s-1,s-1)}$ obey the conservation equation (4.30), as well as (4.23) and (4.28).

5 Higher spin supercurrents for chiral superfields: half-integer superspin

In the remainder of this paper we will study explicit realisations of the higher spin supercurrents introduced above in various supersymmetric field theories in AdS.

5.1 Superconformal model for a chiral superfield

Let us consider the superconformal theory of a single chiral scalar superfield

$$S = \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\,E\,\bar{\Phi}\Phi\,, \tag{5.1}$$

where Φ is covariantly chiral, $\bar{\mathcal{D}}_{\dot\alpha}\Phi = 0$. We can define the conformal supercurrent $J_{(s,s)}$ in direct analogy with the flat superspace case [25, 27]

$$\begin{aligned}J_{(s,s)} = \sum_{k=0}^{s}(-1)^k\binom{s}{k}\bigg\{&\binom{s}{k+1}\mathcal{D}^k_{(1,1)}\mathcal{D}_{(1,0)}\Phi\;\mathcal{D}^{s-k-1}_{(1,1)}\bar{\mathcal{D}}_{(0,1)}\bar{\Phi} \\ &+\binom{s}{k}\mathcal{D}^k_{(1,1)}\Phi\;\mathcal{D}^{s-k}_{(1,1)}\bar{\Phi}\bigg\}\,.\end{aligned} \tag{5.2}$$

Making use of the massless equations of motion, $(\mathcal{D}^2 - 4\bar{\mu})\,\Phi = 0$, one may check that $J_{(s,s)}$ satisfies the conservation equation

$$\mathcal{D}_{(-1,0)} J_{(s,s)} = 0 \quad \Longleftrightarrow \quad \bar{\mathcal{D}}_{(0,-1)} J_{(s,s)} = 0\,. \tag{5.3}$$

The calculation of (5.3) in AdS is much more complicated than in flat superspace due to the fact that the algebra of covariant derivatives (2.4) is nontrivial. Let us sketch the main steps in evaluating the left-hand side of eq. (5.3) with $J_{(s,s)}$ given by (5.2). We start with the obvious relations

$$\frac{\partial}{\partial \zeta^\alpha} \mathcal{D}_{(1,1)} = 2\mathrm{i}\bar{\zeta}^{\dot\alpha} \mathcal{D}_{\alpha\dot\alpha}\,, \tag{5.4a}$$

$$\frac{\partial}{\partial \zeta^\alpha} \mathcal{D}^k_{(1,1)} = \sum_{n=1}^{k} \mathcal{D}^{n-1}_{(1,1)}\, 2\mathrm{i}\,\bar{\zeta}^{\dot\alpha} \mathcal{D}_{\alpha\dot\alpha}\, \mathcal{D}^{k-n}_{(1,1)}\,, \qquad k > 1\,. \tag{5.4b}$$

To simplify eq. (5.4b), we may push $\bar{\zeta}^{\dot\alpha}\mathcal{D}_{\alpha\dot\alpha}$, say, to the left provided that we take into account its commutator with $\mathcal{D}_{(1,1)}$:

$$[\bar{\zeta}^{\dot\alpha}\mathcal{D}_{\alpha\dot\alpha}\,, \mathcal{D}_{(1,1)}] = -4\mathrm{i}\,\bar{\mu}\mu\,\zeta_\alpha \bar{\zeta}^{\dot\alpha}\bar{\zeta}^{\dot\beta} \bar{M}_{\dot\alpha\dot\beta}\,. \tag{5.5}$$

Associated with the Lorentz generators are the operators

$$\bar{M}_{(0,2)} := \bar{\zeta}^{\dot\alpha}\bar{\zeta}^{\dot\beta}\bar{M}_{\dot\alpha\dot\beta}\,, \tag{5.6a}$$

$$M_{(2,0)} := \zeta^\alpha \zeta^\beta M_{\alpha\beta}\,, \tag{5.6b}$$

where $\bar{M}_{(0,2)}$ appears in the right-hand side of (5.5). These operators annihilate every superfield $U_{(m,n)}(\zeta,\bar{\zeta})$ of the form (4.10),[8]

$$\bar{M}_{(0,2)} U_{(m,n)} = 0\,, \qquad M_{(2,0)} U_{(m,n)} = 0\,. \tag{5.6c}$$

From the above consideration, it follows that

$$\left[\bar{\zeta}^{\dot\alpha}\mathcal{D}_{\alpha\dot\alpha}\,, \mathcal{D}^k_{(1,1)}\right] U_{(m,n)} = 0\,, \tag{5.7a}$$

$$\left(\frac{\partial}{\partial \zeta^\alpha}\mathcal{D}^k_{(1,1)}\right) U_{(m,n)} = 2\mathrm{i}k\,\bar{\zeta}^{\dot\alpha}\mathcal{D}_{\alpha\dot\alpha}\,\mathcal{D}^{k-1}_{(1,1)} U_{(m,n)}\,. \tag{5.7b}$$

We also state some other properties which we often use throughout our calculations

$$\mathcal{D}^2_{(0,1)} = -2\bar{\mu} M_{(2,0)}\,, \tag{5.8a}$$

$$\left[\mathcal{D}_{(1,0)}\,, \mathcal{D}_{(1,1)}\right] = \left[\bar{\mathcal{D}}_{(0,1)}\,, \mathcal{D}_{(1,1)}\right] = 0\,, \tag{5.8b}$$

$$\left[\mathcal{D}^\alpha, \mathcal{D}_{(1,1)}\right] = -2\bar{\mu}\,\zeta^\alpha \bar{\mathcal{D}}_{(0,1)}\,, \tag{5.8c}$$

$$\left[\mathcal{D}^\alpha, \mathcal{D}^k_{(1,1)}\right] = -2\bar{\mu}\,k\,\zeta^\alpha \mathcal{D}^{k-1}_{(1,1)} \bar{\mathcal{D}}_{(0,1)}\,, \tag{5.8d}$$

$$\left[\mathcal{D}^\alpha, \bar{\zeta}^{\dot\beta}\mathcal{D}_{\beta\dot\beta}\right] = \mathrm{i}\bar{\mu}\delta^\alpha_\beta\,\bar{\mathcal{D}}_{(0,1)}\,. \tag{5.8e}$$

The above identities suffice to prove that the supercurrent (5.2) does obey the conservation equation (5.3).

[8]These properties are analogous to those that play a fundamental role for the consistent definition of covariant projective supermultiplets in 5D $\mathcal{N} = 1$ [58, 59] and 4D $\mathcal{N} = 2$ [60] supergravity theories.

5.2 Non-superconformal model for a chiral superfield

Let us now add the mass term to (5.1) and consider the following action

$$S = \int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\, E\,\bar{\Phi}\Phi + \left\{\frac{1}{2}\int \mathrm{d}^4x\mathrm{d}^2\theta\,\mathcal{E}\, m\Phi^2 + \text{c.c.}\right\}, \tag{5.9}$$

with m a complex mass parameter. In the massive case $J_{(s,s)}$ satisfies a more general conservation equation (4.14a) for some superfield $T_{(s-1,s-2)}$. Making use of the equations of motion

$$-\frac{1}{4}(\mathcal{D}^2 - 4\bar{\mu})\Phi + \bar{m}\bar{\Phi} = 0, \qquad -\frac{1}{4}(\bar{\mathcal{D}}^2 - 4\mu)\bar{\Phi} + m\Phi = 0, \tag{5.10}$$

we obtain

$$\bar{\mathcal{D}}_{(0,-1)} J_{(s,s)} = F_{(s,s-1)}\,, \tag{5.11a}$$

where we have denoted

$$\begin{aligned} F_{(s,s-1)} = 2m(s+1)\sum_{k=0}^{s}(-1)^{s-1+k}\binom{s}{k}\binom{s}{k+1} \\ \times\left\{1+(-1)^s\frac{k+1}{s-k+1}\right\}\mathcal{D}^k_{(1,1)}\Phi\,\mathcal{D}^{s-k-1}_{(1,1)}\mathcal{D}_{(1,0)}\Phi\,. \end{aligned} \tag{5.11b}$$

We now look for a superfield $T_{(s-1,s-2)}$ such that (i) it obeys the transverse linear constraint (4.13a); and (ii) it satisfies the equation

$$F_{(s,s-1)} = \frac{s}{2}A_{(1,1)}T_{(s-1,s-2)}\,. \tag{5.12}$$

Our analysis will be similar to the one performed in [25] in flat superspace. We consider a general ansatz

$$T_{(s-1,s-2)} = (-1)^s m\sum_{k=0}^{s-2}c_k\mathcal{D}^k_{(1,1)}\Phi\,\mathcal{D}^{s-k-2}_{(1,1)}\mathcal{D}_{(1,0)}\Phi \tag{5.13}$$

with some coefficients c_k which have to be determined. For $k = 1, 2, \dots s-2$, condition (i) implies that the coefficients c_k must satisfy

$$kc_k = (s-k-1)c_{s-k-1}\,, \tag{5.14a}$$

while (ii) gives the following equation

$$c_{s-k-1}+sc_k+(s-1)c_{k-1} = -4(-1)^k\frac{s+1}{s}\binom{s}{k}\binom{s}{k+1}\left\{1+(-1)^s\frac{k+1}{s-k+1}\right\}. \tag{5.14b}$$

Condition (ii) also implies that

$$(s-1)c_{s-2}+c_0 = 4(-1)^s(s+1)\left\{1+(-1)^s\frac{s}{2}\right\}, \tag{5.14c}$$

$$c_0 = -\frac{4}{s}(s+1+(-1)^s)\,. \tag{5.14d}$$

It turns out that the equations (5.14) lead to a unique expression for c_k given by

$$c_k = -\frac{4(s+1)(s-k-1)}{s(s-1)} \sum_{l=0}^{k} \frac{(-1)^k}{s-l} \binom{s}{l} \binom{s}{l+1} \left\{ 1 + (-1)^s \frac{l+1}{s-l+1} \right\} , \qquad (5.15)$$
$$k = 0, 1, \dots s-2 .$$

If the parameter s is odd, $s = 2n+1$, with $n = 1, 2, \dots$, one can check that the equations (5.14a)–(5.14c) are identically satisfied. However, if the parameter s is even, $s = 2n$, with $n = 1, 2, \dots$, there appears an inconsistency: the right-hand side of (5.14c) is positive, while the left-hand side is negative, $(s-1)c_{s-2} + c_0 < 0$. Therefore, our solution (5.15) is only consistent for $s = 2n+1, n = 1, 2, \dots$.

Relations (5.2), (5.13), (5.14d) and (5.15) determine the non-conformal higher spin supercurrents in the massive chiral model (5.9). Unlike the conformal higher spin supercurrents (5.2), the non-conformal ones exist only for the odd values of s, $s = 2n+1$, with $n = 1, 2, \dots$.

In the flat-superspace limit, the above results reduce to those derived in [25] and in a revised version (v3, 26 Oct.) of ref. [29] (which appeared a few days before [25]).

5.3 Superconformal model with N chiral superfields

We now generalise the superconformal model (5.1) to the case of N covariantly chiral scalar superfields Φ^i, $i = 1, \dots N$,

$$S = \int \mathrm{d}^4x \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta}\, E\, \bar{\Phi}^i \Phi^i , \qquad \bar{\mathcal{D}}_{\dot{\alpha}} \Phi^i = 0 . \qquad (5.16)$$

The novel feature of the $N > 1$ case is that there exist two different types of conformal supercurrents, which are:

$$J^{+}_{(s,s)} = S^{ij} \sum_{k=0}^{s} (-1)^k \binom{s}{k} \left\{ \binom{s}{k+1} \mathcal{D}^k_{(1,1)} \mathcal{D}_{(1,0)} \Phi^i \; \mathcal{D}^{s-k-1}_{(1,1)} \bar{\mathcal{D}}_{(0,1)} \bar{\Phi}^j + \binom{s}{k} \mathcal{D}^k_{(1,1)} \Phi^i \; \mathcal{D}^{s-k}_{(1,1)} \bar{\Phi}^j \right\} , \qquad S^{ij} = S^{ji} \qquad (5.17)$$

and

$$J^{-}_{(s,s)} = \mathrm{i}\, A^{ij} \sum_{k=0}^{s} (-1)^k \binom{s}{k} \left\{ \binom{s}{k+1} \mathcal{D}^k_{(1,1)} \mathcal{D}_{(1,0)} \Phi^i \; \mathcal{D}^{s-k-1}_{(1,1)} \bar{\mathcal{D}}_{(0,1)} \bar{\Phi}^j + \binom{s}{k} \mathcal{D}^k_{(1,1)} \Phi^i \; \mathcal{D}^{s-k}_{(1,1)} \bar{\Phi}^j \right\} , \qquad A^{ij} = -A^{ji} . \qquad (5.18)$$

Here S and A are arbitrary real symmetric and antisymmetric constant matrices, respectively. We have put an overall factor $\sqrt{-1}$ in eq. (5.18) in order to make $J^{-}_{(s,s)}$ real. One can show that the currents (5.17) are (5.18) are conserved on-shell:

$$\mathcal{D}_{(-1,0)} J^{\pm}_{(s,s)} = 0 \quad \Longleftrightarrow \quad \bar{\mathcal{D}}_{(0,-1)} J^{\pm}_{(s,s)} = 0 . \qquad (5.19)$$

The above results can be recast in terms of the matrix conformal supercurrent $J_{(s,s)} = \big(J^{ij}_{(s,s)}\big)$ with components

$$J^{ij}_{(s,s)} := \sum_{k=0}^{s}(-1)^k \binom{s}{k} \left\{ \binom{s}{k+1} \mathcal{D}^k_{(1,1)} \mathcal{D}_{(1,0)} \Phi^i \; \mathcal{D}^{s-k-1}_{(1,1)} \bar{\mathcal{D}}_{(0,1)} \bar{\Phi}^j + \binom{s}{k} \mathcal{D}^k_{(1,1)} \Phi^i \; \mathcal{D}^{s-k}_{(1,1)} \bar{\Phi}^j \right\} , \tag{5.20}$$

which is Hermitian, ${J_{(s,s)}}^\dagger = J_{(s,s)}$. The chiral action (5.16) possesses rigid $\mathrm{U}(N)$ symmetry acting on the chiral column-vector $\Phi = (\Phi^i)$ by $\Phi \to g\Phi$, with $g \in \mathrm{U}(N)$, which implies that the supercurrent (5.20) transforms as $J_{(s,s)} \to g J_{(s,s)} g^{-1}$.

5.4 Massive model with N chiral superfields

Now let us consider a theory of N massive chiral multiplets with action

$$S = \int \mathrm{d}^4x \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta}\, E\, \bar{\Phi}^i \Phi^i + \left\{ \frac{1}{2} \int \mathrm{d}^4x \mathrm{d}^2\theta\, \mathcal{E}\, M^{ij} \Phi^i \Phi^j + \text{c.c.} \right\} , \tag{5.21}$$

where M^{ij} is a constant symmetric $N \times N$ mass matrix. The corresponding equations of motion are

$$-\frac{1}{4}(\mathcal{D}^2 - 4\bar{\mu})\Phi^i + \bar{M}^{ij}\bar{\Phi}^j = 0 , \qquad -\frac{1}{4}(\bar{\mathcal{D}}^2 - 4\mu)\bar{\Phi}^i + M^{ij}\Phi^j = 0 . \tag{5.22}$$

First we will consider the case where S is a real and symmetric matrix. Making use of the equations of motion, we obtain

$$\begin{aligned} \mathcal{D}_{(-1,0)} J_{(s,s)} = {} & 2(s+1)(S\bar{M})^{ji} \sum_{k=0}^{s} (-1)^k \binom{s}{k}\binom{s}{k} \\ & \times \frac{k}{k+1} \mathcal{D}^{k-1}_{(1,1)} \bar{\mathcal{D}}_{(0,1)} \bar{\Phi}^i \; \mathcal{D}^{s-k}_{(1,1)} \bar{\Phi}^j \\ & + 2(s+1)(S\bar{M})^{ji} \sum_{k=0}^{s-1} (-1)^k \binom{s}{k}\binom{s}{k+1} \\ & \times \mathcal{D}^k_{(1,1)} \bar{\Phi}^i \; \mathcal{D}^{s-k-1}_{(1,1)} \bar{\mathcal{D}}_{(0,1)} \bar{\Phi}^j . \end{aligned} \tag{5.23}$$

Now, suppose the product $S\bar{M}$ is symmetric, which implies $[S, \bar{M}] = 0$. Then, (5.23) becomes

$$\begin{aligned} \mathcal{D}_{(-1,0)} J_{(s,s)} = {} & 2(s+1)(S\bar{M})^{ij} \sum_{k=0}^{s-1} (-1)^k \binom{s}{k}\binom{s}{k+1} \\ & \times \left\{ 1 + (-1)^s \frac{k+1}{s-k+1} \right\} \mathcal{D}^k_{(1,1)} \bar{\Phi}^i \; \mathcal{D}^{s-k-1}_{(1,1)} \bar{\mathcal{D}}_{(0,1)} \bar{\Phi}^j . \end{aligned} \tag{5.24}$$

We now look for a superfield $\bar{T}_{(s-2,s-1)}$ such that (i) it obeys the transverse antilinear constraint (4.13b); and (ii) it satisfies the conservation equation (4.14b):

$$\mathcal{D}_{(-1,0)} J_{(s,s)} = \frac{s}{2} \bar{A}_{(1,1)} \bar{T}_{(s-2,s-1)} . \tag{5.25}$$

As in the single field case we consider a general ansatz

$$\bar{T}_{(s-2,s-1)} = (S\bar{M})^{ij} \sum_{k=0}^{s-2} c_k \mathcal{D}^k_{(1,1)} \bar{\Phi}^i \, \mathcal{D}^{s-k-2}_{(1,1)} \bar{\mathcal{D}}_{(0,1)} \bar{\Phi}^j \,. \tag{5.26}$$

Then for $k = 1, 2, \ldots s-2$, condition (i) implies that the coefficients c_k must satisfy

$$kc_k = (s-k-1)c_{s-k-1} \,, \tag{5.27a}$$

while (ii) gives the following equation

$$c_{s-k-1} + sc_k + (s-1)c_{k-1} = -4(-1)^k \frac{s+1}{s} \binom{s}{k} \binom{s}{k+1} \left\{ 1 + (-1)^s \frac{k+1}{s-k+1} \right\} . \tag{5.27b}$$

Condition (ii) also implies that

$$(s-1)c_{s-2} + c_0 = 4(-1)^s(s+1) \left\{ 1 + (-1)^s \frac{s}{2} \right\} , \tag{5.27c}$$

$$c_0 = -\frac{4}{s}(s+1+(-1)^s) \,. \tag{5.27d}$$

The above conditions coincide with eqs. (5.14a)–(5.14d) in the case of a single, massive chiral superfield, which are satisfied only for $s = 2n+1, n = 1, 2, \ldots$. Hence, the solution for the coefficients c_k is given by (5.15) for odd values of s and there is no solution for even s.

On the other hand, if $S\bar{M}$ is antisymmetric (which is equivalent to $\{S, \bar{M}\} = 0$), eq. (5.24) is slightly modified

$$\begin{aligned} \mathcal{D}_{(-1,0)} J_{(s,s)} = 2(s+1)(S\bar{M})^{ij} &\sum_{k=0}^{s-1} (-1)^k \binom{s}{k} \binom{s}{k+1} \\ &\times \left\{ -1 + (-1)^s \frac{k+1}{s-k+1} \right\} \mathcal{D}^k_{(1,1)} \bar{\Phi}^i \; \mathcal{D}^{s-k-1}_{(1,1)} \bar{\mathcal{D}}_{(0,1)} \bar{\Phi}^j \,. \end{aligned} \tag{5.28}$$

Starting with a general ansatz

$$\bar{T}_{(s-2,s-1)} = (S\bar{M})^{ij} \sum_{k=0}^{s-2} d_k \mathcal{D}^k_{(1,1)} \bar{\Phi}^i \, \mathcal{D}^{s-k-2}_{(1,1)} \bar{\mathcal{D}}_{(0,1)} \bar{\Phi}^j \tag{5.29}$$

and imposing conditions (i) and (ii) yield the following equations for the coefficients d_k

$$kd_k = -(s-k-1)d_{s-k-1} \,. \tag{5.30a}$$

$$-d_{s-k-1} + sd_k + (s-1)d_{k-1} = -4(-1)^k \frac{s+1}{s} \binom{s}{k} \binom{s}{k+1} \left\{ -1 + (-1)^s \frac{k+1}{s-k+1} \right\} . \tag{5.30b}$$

$$(s-1)d_{s-2} - d_0 = 4(-1)^s(s+1) \left\{ -1 + (-1)^s \frac{s}{2} \right\} . \tag{5.30c}$$

$$d_0 = \frac{4}{s}(s+1+(-1)^{s-1}) \,. \tag{5.30d}$$

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The equations (5.30) lead to a unique expression for d_k given by

$$d_k = -\frac{4(s+1)(s-k-1)}{s(s-1)} \sum_{l=0}^{k} \frac{(-1)^k}{s-l} \binom{s}{l} \binom{s}{l+1} \left\{ -1 + (-1)^s \frac{l+1}{s-l+1} \right\} , \tag{5.31}$$
$$k = 0, 1, \dots s-2 .$$

If the parameter s is even, $s = 2n$, with $n = 1, 2, \dots$, one can check that the equations (5.30a)–(5.30d) are identically satisfied. However, if the parameter s is odd, $s = 2n+1$, with $n = 1, 2, \dots$, there appears an inconsistency: the right-hand side of (5.30c) is positive, while the left-hand side is negative, $(s-1)d_{s-2} - d_0 < 0$. Therefore, our solution (5.31) is only consistent for $s = 2n, n = 1, 2, \dots$.

Finally, we consider $A^{ij} = -A^{ji}$ with the corresponding $J_{(s,s)}$ given by (5.18). The analysis in this case is similar to the one presented above and we will simply state the results. If s is odd the non-conformal higher spin supercurrents exist if $\{A, \bar{M}\} = 0$. The trace supercurrent $\bar{T}_{(s-2,s-1)}$ is given by (5.26) with the coefficients c_k given by

$$c_k = \mathrm{i}\frac{4(s+1)(s-k-1)}{s(s-1)} \sum_{l=0}^{k} \frac{(-1)^k}{s-l} \binom{s}{l} \binom{s}{l+1} \left\{ 1 + (-1)^s \frac{l+1}{s-l+1} \right\} , \tag{5.32}$$
$$k = 0, 1, \dots s-2 .$$

If s is even the non-conformal higher spin supercurrents exist if $[A, \bar{M}] = 0$. The trace supercurrent $\bar{T}_{(s-2,s-1)}$ is given by (5.29) with the coefficients d_k given by

$$d_k = \mathrm{i}\frac{4(s+1)(s-k-1)}{s(s-1)} \sum_{l=0}^{k} \frac{(-1)^k}{s-l} \binom{s}{l} \binom{s}{l+1} \left\{ -1 + (-1)^s \frac{l+1}{s-l+1} \right\} , \tag{5.33}$$
$$k = 0, 1, \dots s-2 .$$

Note that the coefficients c_k in (5.32) differ from similar coefficients in (5.15) by a factor of $-\mathrm{i}$. This means that for odd s we can define a more general supercurrent

$$J_{(s,s)} = H^{ij} \sum_{k=0}^{s} (-1)^k \binom{s}{k} \left\{ \binom{s}{k+1} \mathcal{D}^k_{(1,1)} \mathcal{D}_{(1,0)} \Phi^i \, \mathcal{D}^{s-k-1}_{(1,1)} \bar{\mathcal{D}}_{(0,1)} \bar{\Phi}^j \right.$$
$$\left. + \binom{s}{k} \mathcal{D}^k_{(1,1)} \Phi^i \, \mathcal{D}^{s-k}_{(1,1)} \bar{\Phi}^j \right\} , \tag{5.34}$$

where H^{ij} is a generic matrix which can be split into the symmetric and antisymmetric parts $H^{ij} = S^{ij} + \mathrm{i}A^{ij}$. Here both S and A are real and we put an i in front of A because $J_{(s,s)}$ must be real. From the above consideration it then follows that the corresponding more general solution for $\bar{T}_{(s-2,s-1)}$ reads

$$\bar{T}_{(s-2,s-1)} = (\bar{H}\bar{M})^{ij} \sum_{k=0}^{s-2} c_k \mathcal{D}^k_{(1,1)} \bar{\Phi}^i \, \mathcal{D}^{s-k-2}_{(1,1)} \bar{\mathcal{D}}_{(0,1)} \bar{\Phi}^j , \tag{5.35}$$

where $[S, \bar{M}] = 0$, $\{A, \bar{M}\} = 0$ and c_k are, as before, given by eq. (5.15). Similarly, the coefficients d_k in (5.33) differ from similar coefficients in (5.31) by a factor of $-\mathrm{i}$. This means that for even s we can define a more general supercurrent (5.34), where H^{ij} is a generic matrix which we can split as before into the symmetric and antisymmetric parts, $H^{ij} = S^{ij} + \mathrm{i}A^{ij}$. From the above consideration it then follows that the corresponding more general solution for $\bar{T}_{(s-2,s-1)}$ reads

$$\bar{T}_{(s-2,s-1)} = (\bar{H}\bar{M})^{ij} \sum_{k=0}^{s-2} d_k \mathcal{D}^k_{(1,1)} \bar{\Phi}^i \, \mathcal{D}^{s-k-2}_{(1,1)} \bar{\mathcal{D}}_{(0,1)} \bar{\Phi}^j \,, \tag{5.36}$$

where $\{S, \bar{M}\} = 0$, $[A, \bar{M}] = 0$ and d_k are given by eq. (5.31).

6 Higher spin supercurrents for chiral superfields: integer superspin

In this section we provide explicit realisations for the fermionic higher spin supercurrents (integer superspin) in models described by chiral scalar superfields.

6.1 Massive hypermultiplet model

Consider a free massive hypermultiplet in AdS[9]

$$S = \int \mathrm{d}^4x \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta} \, E \left(\bar{\Psi}_+ \Psi_+ + \bar{\Psi}_- \Psi_- \right) + \left\{ m \int \mathrm{d}^4x \mathrm{d}^2\theta \, \mathcal{E} \, \Psi_+ \Psi_- + \text{c.c.} \right\}, \tag{6.1}$$

where the superfields $\Psi_\pm$ are covariantly chiral, $\bar{\mathcal{D}}_{\dot\alpha} \Psi_\pm = 0$ and m is a complex mass parameter. By a change of variables it is possible to make m real. Let us introduce another set of fields $\Phi_\pm$, $\bar{\mathcal{D}}_{\dot\alpha} \Phi_\pm = 0$, related to $\Psi_\pm$ by the following transformations

$$\Phi_\pm = e^{\mathrm{i}\alpha/2} \Psi_\pm \,, \qquad m = M e^{\mathrm{i}\alpha} \,. \tag{6.2}$$

Under the transformations (6.2), the action (6.1) turns into

$$S = \int \mathrm{d}^4x \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta} \, E \left(\bar{\Phi}_+ \Phi_+ + \bar{\Phi}_- \Phi_- \right) + \left\{ M \int \mathrm{d}^4x \mathrm{d}^2\theta \, \mathcal{E} \, \Phi_+ \Phi_- + \text{c.c.} \right\}, \tag{6.3}$$

where the mass parameter M is now real. In the massless case, $M = 0$, the conserved fermionic supercurrent $J_{\alpha(s)\dot\alpha(s-1)}$ was constructed in [27] and is given by

$$J_{(s,s-1)} = \sum_{k=0}^{s-1} (-1)^k \binom{s-1}{k} \left\{ \binom{s}{k+1} \mathcal{D}^k_{(1,1)} \mathcal{D}_{(1,0)} \Phi_+ \, \mathcal{D}^{s-k-1}_{(1,1)} \Phi_- \right.$$
$$\left. - \binom{s}{k} \mathcal{D}^k_{(1,1)} \Phi_+ \, \mathcal{D}^{s-k-1}_{(1,1)} \mathcal{D}_{(1,0)} \Phi_- \right\} . \tag{6.4}$$

Making use of the massless equations of motion, $-\frac{1}{4}(\mathcal{D}^2 - 4\bar{\mu}) \, \Phi_\pm = 0$, one may check that $J_{(s,s-1)}$ obeys, for $s > 1$, the conservation equations

$$\mathcal{D}_{(-1,0)} J_{(s,s-1)} = 0, \qquad \bar{\mathcal{D}}_{(0,-1)} J_{(s,s-1)} = 0 \,. \tag{6.5}$$

[9]This model possesses off-shell $\mathcal{N} = 2$ AdS supersymmetry [17, 18, 61].

We will now construct fermionic higher spin supercurrents corresponding to the massive model (6.3). Making use of the massive equations of motion

$$-\frac{1}{4}(\mathcal{D}^2-4\bar{\mu})\Phi_+ + M\bar{\Phi}_- = 0, \qquad -\frac{1}{4}(\mathcal{D}^2-4\bar{\mu})\Phi_- + M\bar{\Phi}_+ = 0, \tag{6.6}$$

we obtain

$$\begin{aligned}
\mathcal{D}_{(-1,0)}J_{(s,s-1)} = 2M(s+1)&\sum_{k=0}^{s-1}(-1)^{k+1}\binom{s-1}{k}\binom{s}{k}\\
&\times\Big\{-\frac{s-k}{k+1}\mathcal{D}^k_{(1,1)}\bar{\Phi}_-\,\mathcal{D}^{s-k-1}_{(1,1)}\Phi_- + \mathcal{D}^k_{(1,1)}\Phi_+\,\mathcal{D}^{s-k-1}_{(1,1)}\bar{\Phi}_+\Big\}\\
&+2M(s+1)\sum_{k=1}^{s-1}(-1)^{k+1}\binom{s-1}{k}\binom{s}{k}\frac{k}{k+1}\\
&\times\mathcal{D}^{k-1}_{(1,1)}\bar{\mathcal{D}}_{(0,1)}\bar{\Phi}_-\,\mathcal{D}^{s-k-1}_{(1,1)}\mathcal{D}_{(1,0)}\Phi_-\\
&+2M(s+1)\sum_{k=0}^{s-2}(-1)^{k+1}\binom{s-1}{k}\binom{s}{k}\frac{s-1-k}{k+1}\\
&\times\mathcal{D}^k_{(1,1)}\mathcal{D}_{(1,0)}\Phi_+\,\mathcal{D}^{s-k-2}_{(1,1)}\bar{\mathcal{D}}_{(0,1)}\bar{\Phi}_+\,.
\end{aligned}\tag{6.7}$$

It can be shown that the massive supercurrent $J_{(s,s-1)}$ also obeys (4.28).

We now look for a superfield $T_{(s-1,s-1)}$ such that (i) it obeys the longitudinal linear constraint (4.29); and (ii) it satisfies (4.31), which is a consequence of the conservation equation (4.30). For this we consider a general ansatz

$$\begin{aligned}
T_{(s-1,s-1)} = &\sum_{k=0}^{s-1}c_k\,\mathcal{D}^k_{(1,1)}\Phi_-\,\mathcal{D}^{s-k-1}_{(1,1)}\bar{\Phi}_-\\
&+\sum_{k=0}^{s-1}d_k\,\mathcal{D}^k_{(1,1)}\Phi_+\,\mathcal{D}^{s-k-1}_{(1,1)}\bar{\Phi}_+\\
&+\sum_{k=1}^{s-1}f_k\,\mathcal{D}^{k-1}_{(1,1)}\mathcal{D}_{(1,0)}\Phi_-\,\mathcal{D}^{s-k-1}_{(1,1)}\bar{\mathcal{D}}_{(0,1)}\bar{\Phi}_-\\
&+\sum_{k=1}^{s-1}g_k\,\mathcal{D}^{k-1}_{(1,1)}\mathcal{D}_{(1,0)}\Phi_+\,\mathcal{D}^{s-k-1}_{(1,1)}\bar{\mathcal{D}}_{(0,1)}\bar{\Phi}_+\,.
\end{aligned}\tag{6.8}$$

Condition (i) implies that the coefficients must be related by

$$c_0=d_0=0\,, \qquad f_k=c_k\,, \qquad g_k=d_k\,, \tag{6.9a}$$

while for $k=1,2,\dots s-2$, condition (ii) gives the following recurrence relations:

$$\begin{aligned}
c_k+c_{k+1} = &\frac{M(s+1)}{s}(-1)^{s+k}\binom{s-1}{k}\binom{s}{k}\\
&\times\frac{1}{(k+2)(k+1)}\Big\{(2k+2-s)(s+1)-k-2\Big\}\,,
\end{aligned}\tag{6.9b}$$

$$\begin{aligned}
d_k+d_{k+1} = &\frac{M(s+1)}{s}(-1)^{k}\binom{s-1}{k}\binom{s}{k}\\
&\times\frac{1}{(k+2)(k+1)}\Big\{(2k+2-s)(s+1)-k-2\Big\}\,.
\end{aligned}\tag{6.9c}$$

Condition (ii) also implies that

$$c_1 = -(-1)^s \frac{M(s^2-1)}{2}\,, \qquad c_{s-1} = -\frac{M(s^2-1)}{s}\,; \tag{6.9d}$$

$$d_1 = -\frac{M(s^2-1)}{2}\,, \qquad d_{s-1} = -(-1)^s \frac{M(s^2-1)}{s}\,. \tag{6.9e}$$

The above conditions lead to simple expressions for c_k and d_k:

$$d_k = \frac{M(s+1)}{s}\frac{k}{k+1}(-1)^k \binom{s-1}{k}\binom{s}{k}\,, \tag{6.10a}$$

$$c_k = (-1)^s d_k\,, \tag{6.10b}$$

where $k = 1, 2, \dots s-1$.

6.2 Superconformal model with N chiral superfields

In this subsection we will generalise the above results for N chiral superfields Φ^i, $i = 1, \dots N$. We first consider the superconformal model (5.16). Let us construct the following fermionic supercurrent

$$J_{(s,s-1)} = C^{ij} \sum_{k=0}^{s-1} (-1)^k \binom{s-1}{k} \left\{ \binom{s}{k+1} \mathcal{D}^k_{(1,1)} \mathcal{D}_{(1,0)} \Phi^i \, \mathcal{D}^{s-k-1}_{(1,1)} \Phi^j - \binom{s}{k} \mathcal{D}^k_{(1,1)} \Phi^i \, \mathcal{D}^{s-k-1}_{(1,1)} \mathcal{D}_{(1,0)} \Phi^j \right\}, \tag{6.11}$$

where C^{ij} is a constant complex matrix. By changing the summation index it is not hard to show that $J_{(s,s-1)} = 0$ if (i) s is odd and C^{ij} is symmetric; and (ii) s is even and C^{ij} is antisymmetric, that is

$$C^{ij} = C^{ji}\,, \qquad s = 1, 3, \dots \implies J_{(s,s-1)} = 0\,; \tag{6.12a}$$

$$C^{ij} = -C^{ji}\,, \qquad s = 2, 4, \dots \implies J_{(s,s-1)} = 0\,. \tag{6.12b}$$

This means that we have to consider the two separate cases: the case of even s with symmetric C, and the case of odd s with antisymmetric C. Using the massless equation of motion, $-\frac{1}{4}(\mathcal{D}^2 - 4\bar{\mu})\,\Phi^i = 0$, one may check that $J_{(s,s-1)}$ satisfies the conservation equations (6.5)

$$\mathcal{D}_{(-1,0)} J_{(s,s-1)} = 0\,, \qquad \bar{\mathcal{D}}_{(0,-1)} J_{(s,s-1)} = 0\,. \tag{6.13}$$

In the case of a single chiral superfield, the supercurrent (6.11) exists for even s,

$$J_{(s,s-1)} = \sum_{k=0}^{s-1} (-1)^k \binom{s-1}{k} \left\{ \binom{s}{k+1} \mathcal{D}^k_{(1,1)} \mathcal{D}_{(1,0)} \Phi \, \mathcal{D}^{s-k-1}_{(1,1)} \Phi - \binom{s}{k} \mathcal{D}^k_{(1,1)} \Phi \, \mathcal{D}^{s-k-1}_{(1,1)} \mathcal{D}_{(1,0)} \Phi \right\}, \qquad s = 2, 4, \dots \tag{6.14}$$

The flat-superspace version of (6.14) can be extracted from the results of [26, 27].

6.3 Massive model with N chiral superfields

Now we move to the massive model (5.21). As was discussed in previous subsection, to construct the conserved currents we first have to calculate $\mathcal{D}_{(-1,0)}J_{(s,s-1)}$ using the equations of motion in the massive theory. The calculation depends on whether C^{ij} is symmetric or antisymmetric.

6.3.1 Symmetric C

If C^{ij} is a symmetric matrix, using the massive equation of motion, we obtain

$$\begin{aligned}\mathcal{D}_{(-1,0)}J_{(s,s-1)} = &-2(s+1)(C\bar{M})^{ji}\sum_{k=0}^{s-1}(-1)^{k+1}\binom{s-1}{k}\binom{s}{k}\frac{s-k}{k+1}\\ &\times \mathcal{D}^{k}_{(1,1)}\bar{\Phi}^i\,\mathcal{D}^{s-k-1}_{(1,1)}\Phi^j\\ &+2(s+1)(C\bar{M})^{ij}\sum_{k=0}^{s-1}(-1)^{k+1}\binom{s-1}{k}\binom{s}{k}\\ &\times \mathcal{D}^{k}_{(1,1)}\Phi^i\,\mathcal{D}^{s-k-1}_{(1,1)}\bar{\Phi}^j\\ &+2(s+1)(C\bar{M})^{ji}\sum_{k=1}^{s-1}(-1)^{k+1}\binom{s-1}{k}\binom{s}{k}\frac{k}{k+1}\\ &\times \mathcal{D}^{k-1}_{(1,1)}\bar{\mathcal{D}}_{(0,1)}\bar{\Phi}^i\;\mathcal{D}^{s-k-1}_{(1,1)}\mathcal{D}_{(1,0)}\Phi^j\\ &+2(s+1)(C\bar{M})^{ij}\sum_{k=0}^{s-2}(-1)^{k+1}\binom{s-1}{k}\binom{s}{k}\frac{s-1-k}{k+1}\\ &\times \mathcal{D}^{k}_{(1,1)}\mathcal{D}_{(1,0)}\Phi^i\;\mathcal{D}^{s-k-2}_{(1,1)}\bar{\mathcal{D}}_{(0,1)}\bar{\Phi}^j\,. \end{aligned} \tag{6.15}$$

Here we have two cases to consider:

1. $C\bar{M}$ is symmetric $\iff [C,\bar{M}]=0,\;\; s$ even.
2. $C\bar{M}$ is antisymmetric $\iff \{C,\bar{M}\}=0,\;\; s$ even.

Case 1. Eq. (6.15) can be simplified to yield

$$\begin{aligned}\mathcal{D}_{(-1,0)}J_{(s,s-1)} = &\,4(s+1)(C\bar{M})^{ij}\sum_{k=0}^{s-1}(-1)^{k+1}\binom{s-1}{k}\binom{s}{k}\\ &\times \mathcal{D}^{k}_{(1,1)}\Phi^i\,\mathcal{D}^{s-k-1}_{(1,1)}\bar{\Phi}^j\\ &+4(s+1)(C\bar{M})^{ij}\sum_{k=1}^{s-1}(-1)^{k+1}\binom{s-1}{k}\binom{s}{k}\frac{k}{k+1}\\ &\times \mathcal{D}^{k-1}_{(1,1)}\bar{\mathcal{D}}_{(0,1)}\bar{\Phi}^i\;\mathcal{D}^{s-k-1}_{(1,1)}\mathcal{D}_{(1,0)}\Phi^j\,. \end{aligned} \tag{6.16}$$

We now look for a superfield $T_{(s-1,s-1)}$ such that (i) it obeys the longitudinal linear constraint (4.29); and (ii) it satisfies (4.31), which is a consequence of the conservation equation (4.30). The precise form of eq. (4.31) in the present case is

$$\begin{aligned}
&\frac{1}{2s}\mathcal{D}_{(1,0)}\left\{\mathcal{D}_{(-1,0)}J_{(s,s-1)}+\bar{\mathcal{D}}_{(0,-1)}\bar{J}_{(s-1,s)}\right\}\\
&=\frac{2}{s+1}\mathcal{D}_{(1,0)}\sum_{k=0}^{s-1}(-1)^{k+1}\binom{s-1}{k}\binom{s}{k}\\
&\quad\times\left\{\frac{s}{k+1}(C\bar{M})^{ij}-\frac{(s+1)(s-k)}{(k+1)(k+2)}(\bar{C}M)^{ij}\right\}\\
&\quad\times\mathcal{D}^{k}_{(1,1)}\Phi^i\,\mathcal{D}^{s-k-1}_{(1,1)}\bar{\Phi}^j\\
&=-\mathcal{D}_{(1,0)}T_{(s-1,s-1)}\,.
\end{aligned}\tag{6.17}$$

To find $T_{(s-1,s-1)}$ we consider a general ansatz

$$\begin{aligned}
T_{(s-1,s-1)}=&\sum_{k=0}^{s-1}(c_k)^{ij}\,\mathcal{D}^{k}_{(1,1)}\Phi^i\;\mathcal{D}^{s-k-1}_{(1,1)}\bar{\Phi}^j\\
&+\sum_{k=1}^{s-1}(d_k)^{ij}\,\mathcal{D}^{k-1}_{(1,1)}\mathcal{D}_{(1,0)}\Phi^j\;\mathcal{D}^{s-k-1}_{(1,1)}\bar{\mathcal{D}}_{(0,1)}\bar{\Phi}^j\,.
\end{aligned}\tag{6.18}$$

It is possible to show that no solution for $T_{(s-1,s-1)}$ can be found unless we impose[10]

$$C\bar{M}=\bar{C}M\,.\tag{6.19}$$

Furthermore, condition (i) implies that the coefficients must be related by

$$(c_0)^{ij}=0\,,\qquad (c_k)^{ij}=(d_k)^{ij}\,,\tag{6.20a}$$

while for $k=1,2,\dots s-2$, while condition (ii) and eq. (6.19) gives the following recurrence relations

$$\begin{aligned}
(d_k)^{ij}+(d_{k+1})^{ij}=&-2\frac{(s+1)}{s}(C\bar{M})^{ij}(-1)^{k+1}\binom{s-1}{k}\binom{s}{k}\\
&\times\frac{1}{k+1}\left\{s-\frac{(s+1)(s-k)}{k+2}\right\}\,.
\end{aligned}\tag{6.20b}$$

Condition (ii) also implies that

$$(d_1)^{ij}=(1-s^2)(C\bar{M})^{ij}\,,\qquad (d_{s-1})^{ij}=\frac{2(1-s^2)}{s}(C\bar{M})^{ij}\,.\tag{6.20c}$$

The above conditions lead to simple expressions for d_k:

$$(d_k)^{ij}=\frac{2(s+1)}{s}(C\bar{M})^{ij}\frac{k}{k+1}(-1)^k\binom{s-1}{k}\binom{s}{k}\,,\tag{6.21}$$

where $k=1,2,\dots s-1$ and s is even.

[10]Since C and $\bar{M}$ commute we can take them both to be diagonal, $C=\operatorname{diag}(c_1,\dots,c_N)$, $M=\operatorname{diag}(m_1,\dots,m_N)$. Then the condition (6.19) means that $\arg(c_i)-\arg(m_i)=n_i\pi$ for some integers n_i.

Case 2. If we take $C\bar{M}$ to be antisymmetric, a similar analysis shows that no solution for $T_{(s-1,s-1)}$ exists for even values of s.

6.3.2 Antisymmetric C

If C^{ij} is antisymmetric we get:

$$\begin{aligned}\mathcal{D}_{(-1,0)}J_{(s,s-1)} = {} & 2(s+1)(C\bar{M})^{ji}\sum_{k=0}^{s-1}(-1)^{k+1}\binom{s-1}{k}\binom{s}{k}\frac{s-k}{k+1} \\ & \times \mathcal{D}^{k}_{(1,1)}\bar{\Phi}^i\,\mathcal{D}^{s-k-1}_{(1,1)}\Phi^j \\ & + 2(s+1)(C\bar{M})^{ij}\sum_{k=0}^{s-1}(-1)^{k+1}\binom{s-1}{k}\binom{s}{k} \\ & \times \mathcal{D}^{k}_{(1,1)}\Phi^i\,\mathcal{D}^{s-k-1}_{(1,1)}\bar{\Phi}^j \\ & - 2(s+1)(C\bar{M})^{ji}\sum_{k=1}^{s-1}(-1)^{k+1}\binom{s-1}{k}\binom{s}{k}\frac{k}{k+1} \\ & \times \mathcal{D}^{k-1}_{(1,1)}\bar{\mathcal{D}}_{(0,1)}\bar{\Phi}^i\;\mathcal{D}^{s-k-1}_{(1,1)}\mathcal{D}_{(1,0)}\Phi^j \\ & + 2(s+1)(C\bar{M})^{ij}\sum_{k=0}^{s-2}(-1)^{k+1}\binom{s-1}{k}\binom{s}{k}\frac{s-1-k}{k+1} \\ & \times \mathcal{D}^{k}_{(1,1)}\mathcal{D}_{(1,0)}\Phi^i\;\mathcal{D}^{s-k-2}_{(1,1)}\bar{\mathcal{D}}_{(0,1)}\bar{\Phi}^j\,. \end{aligned} \tag{6.22}$$

As in the symmetric C case, there are also two cases to consider:

1. $C\bar{M}$ is symmetric $\Longleftrightarrow \{C,\bar{M}\}=0,\ s$ odd.
2. $C\bar{M}$ is antisymmetric $\Longleftrightarrow [C,\bar{M}]=0,\ s$ odd.

Case 1. Using eq. (6.22) and keeping in mind that s is odd, we obtain

$$\begin{aligned}\mathcal{D}_{(-1,0)}J_{(s,s-1)} = {} & 4(s+1)(C\bar{M})^{ij}\sum_{k=0}^{s-1}(-1)^{k+1}\binom{s-1}{k}\binom{s}{k} \\ & \times \mathcal{D}^{k}_{(1,1)}\Phi^i\,\mathcal{D}^{s-k-1}_{(1,1)}\bar{\Phi}^j \\ & - 4(s+1)(C\bar{M})^{ij}\sum_{k=1}^{s-1}(-1)^{k+1}\binom{s-1}{k}\binom{s}{k}\frac{k}{k+1} \\ & \times \mathcal{D}^{k-1}_{(1,1)}\bar{\mathcal{D}}_{(0,1)}\bar{\Phi}^i\;\mathcal{D}^{s-k-1}_{(1,1)}\mathcal{D}_{(1,0)}\Phi^j\,. \end{aligned} \tag{6.23}$$

Then it follows that eq. (4.31) becomes

$$\begin{aligned}\frac{1}{2s}\mathcal{D}_{(1,0)}\left\{\mathcal{D}_{(-1,0)}J_{(s,s-1)}+\bar{\mathcal{D}}_{(0,-1)}\bar{J}_{(s-1,s)}\right\} = {} & \frac{2}{s+1}\mathcal{D}_{(1,0)}\sum_{k=0}^{s-1}(-1)^{k+1}\binom{s-1}{k}\binom{s}{k} \\ & \times\left\{\frac{s}{k+1}(C\bar{M})^{ij}-\frac{(s+1)(s-k)}{(k+1)(k+2)}(\bar{C}M)^{ij}\right\} \\ & \times \mathcal{D}^{k}_{(1,1)}\Phi^i\,\mathcal{D}^{s-k-1}_{(1,1)}\bar{\Phi}^j \\ = {} & -\mathcal{D}_{(1,0)}T_{(s-1,s-1)}\,. \end{aligned} \tag{6.24}$$

Note that it is the equation same as eq. (6.17) which means that the solution for $T_{(s-1,s-1)}$ is the same as in **Case 1**. That is, the matrices C and M must satisfy $C\bar{M} = \bar{C}M$, $T_{(s-1,s-1)}$ is given by eq. (6.18) and the coefficients $(c_k)^{ij}, (d_k)^{ij}$ are given by eqs. (6.20).

Case 2. If we take $C\bar{M}$ to be antisymmetric, a similar analysis shows that no solution for $T_{(s-1,s-1)}$ exists for odd values of s.

6.3.3 Massive hypermultiplet model revisited

As a consistency check of our general method, let us reconsider the case of a hypermultiplet studied previously. For this we will take $N = 2$, the mass matrix in the form

$$M = \begin{pmatrix} 0 & m \\ m & 0 \end{pmatrix} , \tag{6.25}$$

and denote $\Phi^i = (\Phi_+, \Phi_-)$. If s is even we will take C in the form

$$C = \begin{pmatrix} 0 & c \\ c & 0 \end{pmatrix} . \tag{6.26}$$

Note that C commutes with M. The condition $C\bar{M} = \bar{C}M$ is equivalent to $\arg(c) = \arg(m) + n\pi$. For simplicity, let us choose both c and m to be real. Under these conditions eq. (6.11) for $J_{(s,s-1)}$ becomes

$$\begin{aligned} J_{(s,s-1)} = c\sum_{k=0}^{s-1}(-1)^k \binom{s-1}{k}\binom{s}{k+1} \Big\{ & \mathcal{D}^k_{(1,1)}\mathcal{D}_{(1,0)}\Phi_+ \, \mathcal{D}^{s-k-1}_{(1,1)}\Phi_- \\ & +\mathcal{D}^k_{(1,1)}\mathcal{D}_{(1,0)}\Phi_- \, \mathcal{D}^{s-k-1}_{(1,1)}\Phi_+ \Big\} \\ + c\sum_{k=0}^{s-1}(-1)^{k+1} \binom{s-1}{k}\binom{s}{k} \Big\{ & \mathcal{D}^k_{(1,1)}\Phi_+ \, \mathcal{D}^{s-k-1}_{(1,1)}\mathcal{D}_{(1,0)}\Phi_- \\ & +\mathcal{D}^k_{(1,1)}\Phi_- \, \mathcal{D}^{s-k-1}_{(1,1)}\mathcal{D}_{(1,0)}\Phi_+ \Big\} . \end{aligned} \tag{6.27}$$

Introducing a new summation variable $k' = s - 1 - k$ for the second and fourth terms, we obtain

$$\begin{aligned} J_{(s,s-1)} = {}& c\sum_{k=0}^{s-1}(-1)^k \binom{s-1}{k}\binom{s}{k+1} \Big[(1+(-1)^s\Big]\mathcal{D}^k_{(1,1)}\mathcal{D}_{(1,0)}\Phi_+ \, \mathcal{D}^{s-k-1}_{(1,1)}\Phi_- \\ & - c\sum_{k=0}^{s-1}(-1)^k \binom{s-1}{k}\binom{s}{k} \Big[(1+(-1)^s\Big]\mathcal{D}^k_{(1,1)}\Phi_+ \mathcal{D}^{s-k-1}_{(1,1)}\mathcal{D}_{(1,0)}\Phi_- \ . \end{aligned} \tag{6.28}$$

We see that for even s it coincides with the hypermultiplet supercurrent given by (6.4) up to an overall coefficient $2c$. If s is odd we have to choose C to be antisymmetric

$$C = \begin{pmatrix} 0 & c \\ -c & 0 \end{pmatrix} . \tag{6.29}$$

Note that C now anticommutes with M. For simplicity, we again choose c and m to be real. Now the expression (6.11) for $J_{(s,s-1)}$ becomes

$$J_{(s,s-1)} = c\sum_{k=0}^{s-1}(-1)^k\binom{s-1}{k}\binom{s}{k+1}\Big[(1-(-1)^s\Big]\mathcal{D}^k_{(1,1)}\mathcal{D}_{(1,0)}\Phi_+\ \mathcal{D}^{s-k-1}_{(1,1)}\Phi_- \\ - c\sum_{k=0}^{s-1}(-1)^k\binom{s-1}{k}\binom{s}{k}\Big[(1-(-1)^s\Big]\mathcal{D}^k_{(1,1)}\Phi_+\mathcal{D}^{s-k-1}_{(1,1)}\mathcal{D}_{(1,0)}\Phi_- \,. \tag{6.30}$$

We see that for odd s it coincides with the hypermultiplet supercurrent given by (6.4) up to an overall coefficient $2c$. To summarise, we reproduced the hypermultiplet supercurrent (6.4) for both even and odd values of s. However, for even s it came from a symmetric matrix (6.26) and for odd s it came from an antisymmetric matrix (6.29).

Let us now consider $T_{(s-1,s-1)}$. First, we will note that the product $C\bar{M}$ is given by

$$C\bar{M} = cm\begin{pmatrix}1 & 0\\ 0 & (-1)^s\end{pmatrix}. \tag{6.31}$$

This means that $T_{(s-1,s-1)}$ is given by the following expression valid for all values of s

$$T_{(s-1,s-1)} = \sum_{k=0}^{s-1}(d_k)^{ij}\Big[\mathcal{D}^k_{(1,1)}\Phi^i\ \mathcal{D}^{s-k-1}_{(1,1)}\bar{\Phi}^j + \mathcal{D}^{k-1}_{(1,1)}\mathcal{D}_{(1,0)}\Phi^j\ \mathcal{D}^{s-k-1}_{(1,1)}\bar{\mathcal{D}}_{(0,1)}\bar{\Phi}^j\Big], \tag{6.32}$$

where the matrix $(d_k)^{ij}$ is given by

$$(d_k)^{ij} = 2cm\frac{s+1}{s}\frac{k}{k+1}(-1)^k\binom{s-1}{k}\binom{s}{k}\begin{pmatrix}1 & 0\\ 0 & (-1)^s\end{pmatrix}. \tag{6.33}$$

It is easy to see that this expression for $T_{(s-1,s-1)}$ coincides with the one obtained for the hypermultiplet in the previous subsections in eqs. (6.8), (6.9a), (6.10) up to an overall factor $2c$.

7 Summary and applications

In this paper, we have proposed higher spin conserved supercurrents for $\mathcal{N}=1$ supersymmetric theories in four-dimensional anti-de Sitter space. We have explicitly constructed such supercurrents in the case of N chiral scalar superfields with an arbitrary mass matrix M. The structure of the supercurrents depends on whether the superspin is integer or half-integer, as well as on the value of the superspin, and the mass matrix. Let us summarise our results.

In the case of half-integer superspin $s+1/2$, the supercurrent has the structure $J_{(s,s)} = H^{ij}J^{ij}_{(s,s)}$, where $i,j=1,\dots N$ and H^{ij} is a Hermitian matrix. The precise form of $J^{ij}_{(s,s)}$ was discussed in section 5. In massless theory it is conserved for all values of s. In massive theory, the conservation equation involves an additional complex multiplet $T_{(s-1,s-2)}$ whose existence depends on the value of s and the mass matrix. For odd values of s, it exists provided $[S,\bar{M}]=0$, $\{A,\bar{M}\}=0$, where S and A are the symmetric and antisymmetric parts of H, respectively. When s is even, it exists provided $\{S,\bar{M}\}=0$, $[A,\bar{M}]=0$.

In the case of integer superspin s, the fermionic supercurrent was discussed in section 6. It has the form $J_{(s,s-1)}=C^{ij}J^{ij}_{(s,s-1)}$. In massless theory it exists for even values of s if C is symmetric and for odd values of s if C is antisymmetric. In massive theory the conservation equation involves an additional complex multiplet $T_{(s-1,s-1)}$ and a real multiplet $S_{(s-1,s-1)}$. Their existence also depends on the value of s. For s even they exist provided $C\bar{M}=\bar{C}M$, $[C,\bar{M}]=0$ and for s odd provided $C\bar{M}=\bar{C}M$, $\{C,\bar{M}\}=0$.

In the rest of this section, we will discuss several applications of the results obtained in the paper.

7.1 Higher spin supercurrents for a massive chiral multiplet: integer superspin

Let us return to the model (5.9) describing the dynamics of a single massive chiral multiplet in AdS. It proves to possess conserved fermionic higher spin supercurrents. For even integer superspin, $s=2,4,\dots$, the supercurrent $J_{(s,s-1)}$ is given by (6.14). The corresponding trace multiplet is

$$T_{(s-1,s-1)}=\sum_{k=0}^{s-1}c_k\mathcal{D}^k_{(1,1)}\Phi\;\mathcal{D}^{s-k-1}_{(1,1)}\bar{\Phi}+\sum_{k=1}^{s-1}d_k\mathcal{D}^{k-1}_{(1,1)}\mathcal{D}_{(1,0)}\Phi\;\mathcal{D}^{s-k-1}_{(1,1)}\bar{\mathcal{D}}_{(0,1)}\bar{\Phi}\,, \tag{7.1}$$

where the coefficients c_k and d_k are given by (6.10). As an example, for $s=2$ we obtain

$$J_{(2,1)}=4\mathcal{D}_{(1,1)}\Phi\;\mathcal{D}_{(1,0)}\Phi-2\Phi\;\mathcal{D}_{(1,1)}\mathcal{D}_{(1,0)}\Phi\,, \tag{7.2a}$$

$$T_{(1,1)}=-3\bar{m}\Big(\bar{\Phi}\mathcal{D}_{(1,1)}\Phi+\mathcal{D}_{(1,0)}\Phi\;\bar{\mathcal{D}}_{(0,1)}\bar{\Phi}\Big)\,. \tag{7.2b}$$

It was claimed in [29] that the chiral model in Minkowski superspace

$$S_{\text{massive}}=\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\,\bar{\Phi}\Phi+\Big\{\frac{m}{2}\int \mathrm{d}^4x\mathrm{d}^2\theta\,\Phi^2+\text{c.c.}\Big\}\,,\qquad \bar{D}_{\dot{\alpha}}\Phi=0 \tag{7.3}$$

does not possess any conserved fermionic supercurrents $J_{(s,s-1)}$, for any value of the mass parameter m. Here we have demonstrated that they, in fact, do exist when s is even.

There is a simple explanation for why the conserved fermionic supercurrents were overlooked in the analysis of [29]. The point is that the authors of [29] considered only a particular ansatz for the Noether procedure to construct cubic vertices, $\delta_g\Phi=\mathcal{A}\Phi$, where $\mathcal{A}$ is a higher-derivative operator containing infinitely many local parameters. However, in order to generate the conserved fermionic supercurrents we constructed, it is necessary to deal with a more general ansatz $\delta_g\Phi=\mathcal{A}\Phi+\bar{D}^2\mathcal{B}\bar{\Phi}$, with $\mathcal{B}$ another higher-derivative operator.[11]

7.2 Higher spin supercurrents for a tensor multiplet

Let us consider a special case of the non-superconformal chiral model (5.9) with the mass parameter $m=\mu$,

$$S[\Phi,\bar{\Phi}]=\frac{1}{2}\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\,E\,(\Phi+\bar{\Phi})^2\,,\qquad \bar{\mathcal{D}}_{\dot{\alpha}}\Phi=0\,. \tag{7.4}$$

[11] We thank Konstantinos Koutrolikos for clarifying comments.

This theory is known to be dual to a tensor multiplet model [62]

$$S[L] = -\frac{1}{2}\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\, E\, L^2\,, \tag{7.5}$$

which is realised in terms of a real linear superfield $L = \bar{L}$, constrained by $(\bar{\mathcal{D}}^2 - 4\mu)L = 0$, which is the gauge-invariant field strength of a chiral spinor superfield

$$L = \mathcal{D}^\alpha \eta_\alpha + \bar{\mathcal{D}}_{\dot\alpha}\bar{\eta}^{\dot\alpha}\,, \qquad \bar{\mathcal{D}}_{\dot\beta}\eta_\alpha = 0\,. \tag{7.6}$$

We recall that the duality between (7.4) and (7.5) follows, e.g., from the fact the off-shell constraint

$$(\bar{\mathcal{D}}^2 - 4\mu)\mathcal{D}_\alpha(\Phi + \bar{\Phi}) = 0 \tag{7.7a}$$

and the equation of motion for Φ

$$(\bar{\mathcal{D}}^2 - 4\mu)(\Phi + \bar{\Phi}) = 0 \tag{7.7b}$$

are equivalent to the equation of motion for η_α

$$(\bar{\mathcal{D}}^2 - 4\mu)\mathcal{D}_\alpha L = 0 \tag{7.8a}$$

and the off-shell constraint

$$(\bar{\mathcal{D}}^2 - 4\mu)L = 0\,, \tag{7.8b}$$

respectively.

Higher spin supercurrents for the tensor model (7.5) can be obtained from the results derived in section 5.2 in conjunction with an improvement transformation of the type (4.20) with $\Omega = -\frac{1}{2}\Phi^2$. Given an odd $s = 3, 5\dots$, for the supercurrent we get

$$\begin{aligned} J_{(s,s)} = & -L\ \mathcal{D}^{s-1}_{(1,1)}\,[\mathcal{D}_{(1,0)}, \bar{\mathcal{D}}_{(0,1)}]L \\ & + \sum_{k=0}^{s-1}(-1)^k \binom{s}{k}\binom{s}{k+1}\mathcal{D}^k_{(1,1)}\mathcal{D}_{(1,0)}L\ \mathcal{D}^{s-k-1}_{(1,1)}\bar{\mathcal{D}}_{(0,1)}L \\ & + \frac{1}{2}\sum_{k=1}^{s-1}\left\{-1 + (-1)^k\binom{s}{k}\right\}\binom{s}{k}\mathcal{D}^{k-1}_{(1,1)}\,[\mathcal{D}_{(1,0)}, \bar{\mathcal{D}}_{(0,1)}]L\ \mathcal{D}^{s-k}_{(1,1)}L\,. \end{aligned} \tag{7.9}$$

The corresponding trace multiplet proves to be

$$\begin{aligned} T_{(s-1,s-2)} = & -\frac{4\mu}{s}L\ \mathcal{D}^{s-2}_{(1,1)}\mathcal{D}_{(1,0)}L + 4\mu\frac{s+1}{s}\mathcal{D}_{(1,0)}L\ \mathcal{D}^{s-3}_{(1,1)}\bar{\mathcal{D}}_{(0,1)}\mathcal{D}_{(1,0)}L \\ & - \frac{2}{s}\mathcal{D}^{s-2}_{(1,1)}\left\{\mathcal{D}_{(1,0)}\bar{\mathcal{D}}_{\dot\alpha}L\ \ \bar{\mathcal{D}}^{\dot\alpha}L\right\} \\ & + \mu\sum_{k=1}^{s-2}c_k\mathcal{D}^{k-1}_{(1,1)}\bar{\mathcal{D}}_{(0,1)}\mathcal{D}_{(1,0)}L\ \mathcal{D}^{s-k-2}_{(1,1)}\mathcal{D}_{(1,0)}L \\ & + \frac{4\mu}{s}\sum_{k=1}^{s-2}\binom{s-2}{k}\mathcal{D}^{k-1}_{(1,1)}\mathcal{D}_{(1,0)}\bar{\mathcal{D}}_{(0,1)}L\ \mathcal{D}^{s-k-2}_{(1,1)}\mathcal{D}_{(1,0)}L \\ & + 2\mu\frac{s+1}{s}\sum_{k=1}^{s-3}\binom{s-2}{k}\Big\{\mathcal{D}^k_{(1,1)}\mathcal{D}_{(1,0)}L\ \mathcal{D}^{s-k-3}_{(1,1)}\bar{\mathcal{D}}_{(0,1)}\mathcal{D}_{(1,0)}L \\ & \qquad\qquad + \mathcal{D}^{k-1}_{(1,1)}\bar{\mathcal{D}}_{(0,1)}\mathcal{D}_{(1,0)}L\ \mathcal{D}^{s-k-2}_{(1,1)}\mathcal{D}_{(1,0)}L\Big\}\,. \end{aligned} \tag{7.10}$$

The coefficient c_k is given by eq. (5.15), s is odd. The Ferrara-Zumino supercurrent ($s = 1$) for the model (7.5) in an arbitrary supergravity background was derived in section 6.3 of [3]. Modulo normalisation, the AdS supercurrent is

$$J_{\alpha\dot{\alpha}} = \bar{\mathcal{D}}_{\dot{\alpha}} L \mathcal{D}_\alpha L + L\big[\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\alpha}}\big] L\,, \tag{7.11a}$$

and the corresponding trace multiplet is

$$T = \frac{1}{4}(\bar{\mathcal{D}}^2 - 4\mu)L^2\,. \tag{7.11b}$$

The supercurrent obeys the conservation equation (A.1).

7.3 Higher spin supercurrents for a complex linear multiplet

The superconformal non-minimal scalar multiplet in AdS is described by the action

$$S[\Gamma, \bar{\Gamma}] = -\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\, E\,\bar{\Gamma}\Gamma\,, \tag{7.12}$$

where Γ is a complex linear scalar, $(\bar{\mathcal{D}}^2 - 4\mu)\Gamma = 0$. This is a dual formulation for the superconformal chiral model (5.1). As is well known, the duality between (5.1) and (7.12) follows from the fact that the off-shell constraint

$$(\mathcal{D}^2 - 4\bar{\mu})\bar{\Gamma} = 0 \tag{7.13a}$$

and the equation of motion for Γ

$$\bar{\mathcal{D}}_{\dot{\alpha}}\bar{\Gamma} = 0 \tag{7.13b}$$

are equivalent to the equation of motion for $\bar{\Phi}$, $(\mathcal{D}^2 - 4\bar{\mu})\Phi = 0$, and the off-shell constraint $\bar{\mathcal{D}}_{\dot{\alpha}}\Phi = 0$, respectively. In other words, on the mass shell we can identify $\bar{\Gamma}$ with Φ.

The higher spin supercurrents, $J_{(s,s)}$ and $J_{(s,s-1)}$, for the model (7.12) are obtained from (5.2) and (6.14), respectively, by replacing Φ with $\bar{\Gamma}$. The fermionic supercurrent $J_{(s,s-1)}$ exists for even values of s. In the flat-superspace limit, the expression for $J_{(s,s)}$ obtained coincides with the main result of [30].[12] It was claimed in [30] that the flat-superspace model

$$S[\Gamma, \bar{\Gamma}] = -\int \mathrm{d}^4x\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\bar{\Gamma}\Gamma\,, \qquad \bar{D}^2\Gamma = 0 \tag{7.14}$$

does not possess any conserved fermionic supercurrents $J_{(s,s-1)}$. Here we have demonstrated that they, in fact, do exist when s is even. Just like in the case of a massive chiral multiplet, the fermionic supercurrents were overlooked in [30] because only a particular ansatz for the Noether procedure was studied in [30].

[12] Actually the higher spin supercurrents derived in [30] are obtained from eq. (5.6) in [27] by replacing Φ with $\bar{\Gamma}$.

7.4 Gauge higher spin multiplets and conserved supercurrents

For each of the two off-shell formulations for the massless multiplet of half-integer superspin $s+1/2$, with $s=2,3,\ldots$, which we reviewed in section 4.1, it was shown in [22] that there exists a gauge-invariant field strength $W_{\alpha(2s+1)}$ which is covariantly chiral, $\bar{\mathcal{D}}_{\dot\beta}W_{\alpha(2s+1)}=0$, and is given by the expression

$$W_{\alpha(2s+1)}=-\frac{1}{4}(\bar{\mathcal{D}}^2-4\mu)\mathcal{D}_{(\alpha_1}{}^{\dot\beta_1}\cdots\mathcal{D}_{\alpha_s}{}^{\dot\beta_s}\mathcal{D}_{\alpha_{s+1}}H_{\alpha_{s+2}\cdots\alpha_{2s+1})\dot\beta_1\cdots\dot\beta_s}\,. \tag{7.15}$$

It was also shown in [22] that on the mass shell it holds that (i) $W_{\alpha(2s+1)}$ and its conjugate $\bar{W}_{\dot\alpha(2s+1)}$ are the only independent gauge-invariant field strengths; and (ii) $W_{\alpha(2s+1)}$ obeys the irreducibility condition

$$\mathcal{D}^\beta W_{\beta\alpha(2s)}=0\,. \tag{7.16}$$

The relations (7.15) and (7.16) also hold for the cases $s=0$ and $s=1$, which correspond to the vector multiplet and linearised supergravity, respectively. In terms of $W_{\alpha(2s+1)}$ and $\bar{W}_{\dot\alpha(2s+1)}$, we can define the following higher spin supercurrent

$$J_{\alpha(2s+1)\dot\alpha(2s+1)}=W_{\alpha(2s+1)}\bar{W}_{\dot\alpha(2s+1)}\,,\qquad s=0,1,\ldots\,, \tag{7.17}$$

which obeys the conservation equation

$$\bar{\mathcal{D}}_{(0,-1)}J_{(2s+1,2s+1)}=0\quad\Longleftrightarrow\quad\mathcal{D}_{(-1,0)}J_{(2s+1,2s+1)}=0\,. \tag{7.18}$$

In the case of the longitudinal formulation for the massless multiplet of integer superspin s, with $s=2,3,\ldots$, which we described in section 3, it was shown in [22] that there exists a gauge-invariant field strength $W_{\alpha(2s)}$ which is covariantly chiral, $\bar{\mathcal{D}}_{\dot\beta}W_{\alpha(2s)}=0$, and is given by the expression[13]

$$W_{\alpha(2s)}=-\frac{1}{4}(\bar{\mathcal{D}}^2-4\mu)\mathcal{D}_{(\alpha_1}{}^{\dot\beta_1}\cdots\mathcal{D}_{\alpha_{s-1}}{}^{\dot\beta_{s-1}}\mathcal{D}_{\alpha_s}\Psi_{\alpha_{s+1}\cdots\alpha_{2s})\dot\beta_1\cdots\dot\beta_{s-1}}\,. \tag{7.19}$$

As demonstrated in [22], on the mass shell it holds that (i) $W_{\alpha(2s)}$ and its conjugate $\bar{W}_{\dot\alpha(2s)}$ are the only independent gauge-invariant field strengths; and (ii) $W_{\alpha(2s)}$ obeys the irreducibility condition

$$\mathcal{D}^\beta W_{\beta\alpha(2s-1)}=0\,. \tag{7.20}$$

The relations (7.19) and (7.20) also hold for the case $s=1$, which corresponds to the gravitino multiplet. In terms of $W_{\alpha(2s)}$ and $\bar{W}_{\dot\alpha(2s)}$, we can define the higher spin supercurrent

$$J_{\alpha(2s)\dot\alpha(2s)}=W_{\alpha(2s)}\bar{W}_{\dot\alpha(2s)}\,,\qquad s=1,2,\ldots\,, \tag{7.21}$$

which obeys the conservation equation

$$\bar{\mathcal{D}}_{(0,-1)}J_{(2s,2s)}=0\quad\Longleftrightarrow\quad\mathcal{D}_{(-1,0)}J_{(2s,2s)}=0\,. \tag{7.22}$$

The conserved supercurrents $J_{\alpha(n)\dot\alpha(n)}=W_{\alpha(n)}\bar{W}_{\dot\alpha(n)}$, with $n=1,2,\ldots$, are the AdS extensions of those introduced many years ago by Howe, Stelle and Townsend [66].

[13] The flat-superspace version of (7.19) is given in section 6.9 of [3].

Now, for any positive integer $n > 0$, we can try to generalise the higher spin supercurrent (5.2) as follows:

$$\mathfrak{J}_{(s+n,s+n)} = \sum_{k=0}^{s} (-1)^k \frac{\binom{s}{k}\binom{s+n}{k}}{\binom{n+k}{n}} \Big\{ (-1)^n \frac{s-k}{n+k+1} \mathcal{D}^k_{(1,1)} \mathcal{D}_{(1,0)} W_{(n,0)} \ \mathcal{D}^{s-k-1}_{(1,1)} \bar{\mathcal{D}}_{(0,1)} \bar{W}_{(0,n)} \\ + \mathcal{D}^k_{(1,1)} W_{(n,0)} \ \mathcal{D}^{s-k}_{(1,1)} \bar{W}_{(0,n)} \Big\} \,. \tag{7.23}$$

Making use of the on-shell condition

$$\mathcal{D}_{(-1,0)} W_{(n,0)} = 0 \quad \Longleftrightarrow \quad (\mathcal{D}^2 - 2(n+2)\bar{\mu}) W_{(n,0)} = 0 \,, \tag{7.24}$$

one may check that

$$\mathcal{D}_{(-1,0)} \mathfrak{J}_{(s+n,s+n)} = 2n\bar{\mu} \sum_{k=0}^{s-1} (-1)^{n+k} \frac{s-k}{n+k+1} \frac{\binom{s}{k}\binom{s+n}{k}}{\binom{n+k}{n}} \\ \times \mathcal{D}^k_{(1,1)} W_{(n,0)} \ \mathcal{D}^{s-k-1}_{(1,1)} \bar{\mathcal{D}}_{(0,1)} \bar{W}_{(0,n)} \,. \tag{7.25}$$

This demonstrates that $\mathfrak{J}_{(s+n,s+n)}$ is not conserved in $\text{AdS}^{4|4}$.

In the flat-superspace limit, $\mu \to 0$, the right-hand side of (7.25) vanishes and $\mathfrak{J}_{(s+n,s+n)}$ becomes conserved. In Minkowski superspace, the conserved supercurrent $\mathfrak{J}_{(s+n,s+n)}$ was recently constructed in [31] as an extension of the non-supersymmetric approach [67].

As a generalisation of the conserved supercurrents $J_{\alpha(n)\dot{\alpha}(n)} = W_{\alpha(n)} \bar{W}_{\dot{\alpha}(n)}$, one can introduce

$$J_{\alpha(n)\dot{\alpha}(m)} = W_{\alpha(n)} \bar{W}_{\dot{\alpha}(m)} \,, \tag{7.26}$$

with $n \neq m$. They obey the conservation equations

$$\bar{\mathcal{D}}_{(0,-1)} J_{(n,m)} = 0 \,, \qquad \mathcal{D}_{(-1,0)} J_{(n,m)} = 0 \tag{7.27}$$

and can be viewed as Noether currents for the generalised superconformal higher spin multiplets introduced in [27]. Starting from the conserved supercurrents (7.26), one can construct a generalisation of (7.23). We will not elaborate on a construction here.

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A AdS supercurrents

There are only two irreducible AdS supercurrents, with $(12+12)$ and $(20+20)$ degrees of freedom [11].[14] The former is associated with minimal AdS supergravity (see, e.g., [3, 21]

[14] A supercurrent multiplet is called irreducible if it is associated with an off-shell formulation for pure supergravity.

for reviews) and the corresponding conservation equation is

$$\bar{\mathcal{D}}^{\dot\alpha} J_{\alpha\dot\alpha} = \mathcal{D}_\alpha T\,, \qquad \bar{\mathcal{D}}_{\dot\alpha} T = 0\,. \tag{A.1}$$

The latter corresponds to non-minimal AdS supergravity [12], and the conservation equation is

$$\bar{\mathcal{D}}^{\dot\alpha} \mathbb{J}_{\alpha\dot\alpha} = -\frac{1}{4}\bar{\mathcal{D}}^2 \zeta_\alpha\,, \qquad \mathcal{D}_{(\beta}\zeta_{\alpha)} = 0\,. \tag{A.2}$$

The vector superfields J_a and $\mathbb{J}_a$ are real.

The non-minimal supercurrent (A.2) is equivalent to the Ferrara-Zumino multiplet (A.1) in the sense that there always exists a well-defined improvement transformation that turns (A.2) into (A.1), as demonstrated in [12]. In AdS superspace, the constraint on the longitudinal linear compensator ζ_α is equivalent to

$$\zeta_\alpha = \mathcal{D}_\alpha(V + \mathrm{i}\,U)\,, \tag{A.3}$$

for well-defined real operators V and U. If we now introduce

$$J_{\alpha\dot\alpha} := \mathbb{J}_{\alpha\dot\alpha} + \frac{1}{6}[\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot\alpha}]V - \mathcal{D}_{\alpha\dot\alpha}U\,, \qquad T := \frac{1}{12}(\bar{\mathcal{D}}^2 - 4\mu)(V - 3\mathrm{i}U)\,, \tag{A.4}$$

then the operators $J_{\alpha\dot\alpha}$ and T prove to satisfy the conservation equation (A.1).

For the Ferrara-Zumino supercurrent (A.1), there exists an improvement transformation that is generated by a chiral scalar operator Ω. Specifically, using the operator Ω allows one to introduce new supercurrent $\widetilde{J}_{\alpha\dot\alpha}$ and chiral trace multiplet $\widetilde{T}$ defined by

$$\widetilde{J}_{\alpha\dot\alpha} = J_{\alpha\dot\alpha} + \mathrm{i}\mathcal{D}_{\alpha\dot\alpha}(\Omega - \bar\Omega)\,, \qquad \bar{\mathcal{D}}_{\dot\alpha}\Omega = 0\,, \tag{A.5a}$$

$$\widetilde{T} = T + 2\mu\Omega + \frac{1}{4}(\bar{\mathcal{D}}^2 - 4\mu)\bar\Omega\,. \tag{A.5b}$$

The operators $\widetilde{J}_{\alpha\dot\alpha}$ and $\widetilde{T}$ obey the conservation equation (A.1) for arbitrary Ω.[15]

B Conserved currents for free real scalars

In this appendix we will consider higher spin currents in free scalar field theory in flat space. Similar analysis for free fermions will be done in the next appendix.

Given an integer $s \geq 2$, the massless spin-s field [63] is described by real potentials $h_{\alpha(s)\dot\alpha(s)}$ and $h_{\alpha(s-2)\dot\alpha(s-2)}$ with the gauge freedom[16]

$$\delta h_{\alpha_1\dots\alpha_s\dot\alpha_1\dots\dot\alpha_s} = \partial_{(\alpha_1(\dot\alpha_1}\lambda_{\alpha_2\dots\alpha_s)\dot\alpha_2\dots\dot\alpha_s)}\,, \tag{B.1a}$$

$$\delta h_{\alpha_1\dots\alpha_{s-2}\dot\alpha_1\dots\dot\alpha_{s-2}} = \frac{s-1}{s^2}\partial^{\beta\dot\beta}\lambda_{\beta\alpha_1\dots\alpha_{s-2}\dot\beta\dot\alpha_1\dots\dot\alpha_{s-2}}\,, \tag{B.1b}$$

for an arbitrary real gauge parameter $\lambda_{\alpha(s-1)\dot\alpha(s-1)}$. The field $h_{\alpha(s)\dot\alpha(s)}$ may be interpreted as a conformal spin-s field [64, 65].

[15] Extension of the improvement transformation (A.5) to the case of supergravity is discussed in section 6.3 of [3].

[16] We follow the description of Fronsdal's theory [63] given in section 6.9 of [3].

To construct non-conformal higher spin currents, we couple $h_{\alpha(s)\dot\alpha(s)}$ and $h_{\alpha(s-2)\dot\alpha(s-2)}$ to external sources

$$S^{(s)}_{\text{source}} = \int \mathrm{d}^4x \left\{ h^{\alpha(s)\dot\alpha(s)} j_{\alpha(s)\dot\alpha(s)} + h^{\alpha(s-2)\dot\alpha(s-2)} t_{\alpha(s-2)\dot\alpha(s-2)} \right\} . \tag{B.2}$$

Requiring that $S^{(s)}_{\text{source}}$ be invariant under the λ-transformation in (B.1) gives the conservation equation

$$\partial^{\beta\dot\beta} j_{\beta\alpha_1\dots\alpha_{s-1}\dot\beta\dot\alpha_1\dots\dot\alpha_{s-1}} + \frac{s-1}{s^2}\partial_{(\alpha_1(\dot\alpha_1} t_{\alpha_2\dots\alpha_{s-1})\dot\alpha_2\dots\dot\alpha_{s-1})} = 0 . \tag{B.3}$$

Our derivation of (B.3) is analogous to that given in [36].

Let us introduce the following operators

$$\partial_{(1,1)} := 2\mathrm{i}\zeta^\alpha \bar\zeta^{\dot\alpha} \partial_{\alpha\dot\alpha} , \tag{B.4a}$$

$$\partial_{(-1,-1)} := 2\mathrm{i}\partial^{\alpha\dot\alpha} \frac{\partial}{\partial\zeta^\alpha}\frac{\partial}{\partial\bar\zeta^{\dot\alpha}} . \tag{B.4b}$$

The conservation equation (B.3) then becomes

$$\partial_{(-1,-1)} j_{(s,s)} + (s-1)\partial_{(1,1)} t_{(s-2,s-2)} = 0 \tag{B.5}$$

Note that both $j_{(s,s)}$ and $t_{(s-2,s-2)}$ are real.

Let us now consider the model for N massless real scalar fields ϕ^i, with $i = 1, \dots N$, in Minkowski space

$$S = -\frac{1}{2}\int \mathrm{d}^4x \, \partial_\mu \phi^i \partial^\mu \phi^i , \tag{B.6}$$

which admits conserved higher spin currents of the form

$$j_{(s,s)} = \mathrm{i}^s C^{ij} \sum_{k=0}^{s} (-1)^k \binom{s}{k}\binom{s}{k} \partial^k_{(1,1)}\phi^i \, \partial^{s-k}_{(1,1)}\phi^j , \tag{B.7}$$

where C^{ij} is a constant matrix. It can be shown that $j_{(s,s)} = 0$ if s is odd and C^{ij} is symmetric. Similarly, $j_{(s,s)} = 0$ if s is even and C^{ij} is antisymmetric. Thus, we have to consider two separate cases: the case of even s with symmetric C and, the case of odd s with antisymmetric C. Using the massless equation of motion $\Box\phi^i = 0$, one may show that $j_{(s,s)}$ satisfies the conservation equation

$$\partial_{(-1,-1)} j_{(s,s)} = 0 . \tag{B.8}$$

We now turn to the massive model

$$S = -\frac{1}{2}\int \mathrm{d}^4x \left\{ \partial_\mu\phi^i\partial^\mu\phi^i + (M^2)^{ij}\phi^i\phi^j \right\} , \tag{B.9}$$

where $M = (M^{ij})$ is a real, symmetric $N \times N$ mass matrix. In the massive theory, the conservation equation is described by (B.5) and so we first need to compute $\partial_{(-1,-1)} j_{(s,s)}$ using the massive equations of motion

$$\Box\phi^i - (M^2)^{ij}\phi^j = 0 . \tag{B.10}$$

For symmetric C, we obtain

$$\partial_{(-1,-1)} j_{(s,s)} = -8(s+1)^2 (CM^2)^{ij} \sum_{k=0}^{s-1} (-1)^k \binom{s}{k}\binom{s}{k} \times \frac{(s-k)^2}{(k+1)(k+2)} \partial^k_{(1,1)}\, \phi^j\; \partial^{s-k-1}_{(1,1)} \phi^i \,. \tag{B.11}$$

If C^{ij} is antisymmetric, we get

$$\partial_{(-1,-1)} j_{(s,s)} = 8\mathrm{i}(s+1)^2 (CM^2)^{ij} \sum_{k=0}^{s-1} (-1)^k \binom{s}{k}\binom{s}{k} \times \frac{(s-k)^2}{(k+1)(k+2)} \partial^k_{(1,1)}\, \phi^j\; \partial^{s-k-1}_{(1,1)} \phi^i \,. \tag{B.12}$$

Thus, in the massive real scalars there are four cases to consider:

1. Both C and CM^2 are symmetric $\Longleftrightarrow [C, M^2] = 0,\; s$ even.
2. C is symmetric; CM^2 is antisymmetric $\Longleftrightarrow \{C, M^2\} = 0,\; s$ even.
3. C is antisymmetric; CM^2 is symmetric $\Longleftrightarrow \{C, M^2\} = 0,\; s$ odd.
4. Both C and CM^2 are antisymmetric $\Longleftrightarrow [C, M^2] = 0,\; s$ odd.

Case 1. Eq. (B.11) is equivalent to

$$\partial_{(-1,-1)} j_{(s,s)} = -4(s+1)^2 (CM^2)^{ij} \sum_{k=0}^{s-1} (-1)^k \binom{s}{k}\binom{s}{k} (s-k) \times \left\{ \frac{s-k}{(k+1)(k+2)} + (-1)^{s-1} \frac{1}{s-k+1} \right\} \partial^k_{(1,1)} \phi^j\; \partial^{s-k-1}_{(1,1)} \phi^i \,. \tag{B.13}$$

We look for $t_{(s-2,s-2)}$ such that (i) it is real; and (ii) it satisfies the conservation equation (B.5). We consider a general ansatz

$$t_{(s-2,s-2)} = -(CM^2)^{ij} \sum_{k=0}^{s-2} d_k\, \partial^k_{(1,1)} \phi^j\, \partial^{s-k-2}_{(1,1)} \phi^i \,. \tag{B.14}$$

For $k = 1, 2, \ldots s-2$, condition (ii) gives

$$d_{k-1} + d_k = -4\frac{(s+1)^2}{s-1} (-1)^k \binom{s}{k}\binom{s}{k} (s-k) \times \left\{ \frac{s-k}{(k+1)(k+2)} + (-1)^{s-1} \frac{1}{s-k+1} \right\} . \tag{B.15a}$$

Condition (ii) also implies that

$$d_{s-2} + d_0 = -4s(s+1)(s+2) \,, \tag{B.15b}$$

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Equations (B.15) lead to the following expression for d_k, $k = 1, 2, \ldots s-2$

$$d_k = (-1)^k d_0 - \frac{4(s+1)^2}{s-1} \sum_{l=1}^{k} (-1)^k \binom{s}{l} \binom{s}{l} (s-l) \left\{ \frac{s-l}{(l+1)(l+2)} - \frac{1}{s-l+1} \right\}, \tag{B.16a}$$

$$d_0 = d_{s-2} = -2s(s+1)(s+2)\,. \tag{B.16b}$$

One can check that the equations (B.15a)–(B.15b) are identically satisfied if s is even.

Case 2. If we take CM^2 to be antisymmetric, a similar analysis shows that no solution for $t_{(s-2,s-2)}$ exists for even s.

Case 3. Now we consider the case where C is antisymmetric and CM^2 symmetric. Again, similar consideration shows that no solution for $t_{(s-2,s-2)}$ exists for odd s.

Case 4. Eq. (B.12) is equivalent to

$$\begin{aligned} \partial_{(-1,-1)} j_{(s,s)} = 4\mathrm{i}(s+1)^2 (CM^2)^{ij} \sum_{k=0}^{s-1} (-1)^k \binom{s}{k} \binom{s}{k} (s-k) \\ \times \left\{ \frac{s-k}{(k+1)(k+2)} - \frac{1}{s-k+1} \right\} \partial^k_{(1,1)} \phi^j \, \partial^{s-k-1}_{(1,1)} \phi^i\,. \end{aligned} \tag{B.17}$$

We consider a general ansatz

$$t_{(s-2,s-2)} = -\mathrm{i}(CM^2)^{ij} \sum_{k=0}^{s-2} d_k \, \partial^k_{(1,1)} \phi^j \, \partial^{s-k-2}_{(1,1)} \phi^i\,. \tag{B.18}$$

Imposing (i) and (ii) and keeping in mind that s is odd, we obtain the following conditions for d_k:

$$d_{k-1} + d_k = 4\frac{(s+1)^2}{s-1} (-1)^k \binom{s}{k} \binom{s}{k} (s-k) \times \left\{ \frac{s-k}{(k+1)(k+2)} - \frac{1}{s-k+1} \right\}. \tag{B.19a}$$

Condition (ii) also implies that

$$d_{s-2} - d_0 = -4s(s+1)(s+2)\,, \tag{B.19b}$$

Equations (B.19) lead to the following expression for d_k, $k = 1, 2, \ldots s-2$

$$d_k = (-1)^k d_0 + \frac{4(s+1)^2}{s-1} \sum_{l=1}^{k} (-1)^k \binom{s}{l} \binom{s}{l} \left\{ \frac{(s-l)^2}{(l+1)(l+2)} - \frac{s-l}{s-l+1} \right\}, \tag{B.20a}$$

$$d_0 = -d_{s-2} = 2s(s+1)(s+2)\,. \tag{B.20b}$$

One can check that the equations (B.19a)–(B.19b) are identically satisfied if s is odd.

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C Conserved currents for free Majorana fermions

Let us now consider N free massless Majorana fermions

$$S = -\mathrm{i} \int \mathrm{d}^4x \; \psi^{\alpha i} \partial_{\alpha\dot{\alpha}} \bar{\psi}^{\dot{\alpha} i} \,, \tag{C.1}$$

with the equation of motion

$$\partial_{\alpha\dot{\alpha}} \bar{\psi}^{\dot{\alpha} i} \Longrightarrow \Box \bar{\psi}^i_{\dot{\alpha}} = 0 \,, \qquad i = 1, \dots N \,. \tag{C.2}$$

We can construct the following higher spin currents

$$j_{(s,s)} = C^{ij} \sum_{k=0}^{s-1} (-1)^k \binom{s}{k} \binom{s}{k+1} \partial^k_{(1,1)} \zeta^\alpha \psi^i_\alpha \; \partial^{s-k-1}_{(1,1)} \bar{\zeta}^{\dot{\alpha}} \bar{\psi}^j_{\dot{\alpha}} \,, \qquad C^{ij} = C^{ji} \,, \tag{C.3}$$

$$j_{(s,s)} = \mathrm{i}\, C^{ij} \sum_{k=0}^{s-1} (-1)^k \binom{s}{k} \binom{s}{k+1} \partial^k_{(1,1)} \zeta^\alpha \psi^i_\alpha \; \partial^{s-k-1}_{(1,1)} \bar{\zeta}^{\dot{\alpha}} \bar{\psi}^j_{\dot{\alpha}} \,, \qquad C^{ij} = -C^{ji} \,, \tag{C.4}$$

where we put an extra i in eq. (C.4) since $j_{(s,s)}$ has to be real. Using the equation of motion (C.2), it can be shown that the currents (C.3), (C.4) are conserved

$$\partial_{(-1,-1)} j_{(s,s)} = 0 \,. \tag{C.5}$$

We now look at the massive model

$$S = -\int \mathrm{d}^4x \left\{ \mathrm{i} \psi^{\alpha i} \partial_{\alpha\dot{\alpha}} \bar{\psi}^{\dot{\alpha} i} + \left(\frac{1}{2} M^{ij} \psi^{\alpha i} \psi^j_\alpha + \frac{1}{2} \bar{M}^{ij} \bar{\psi}^i_{\dot{\alpha}} \bar{\psi}^{\dot{\alpha} j} \right) \right\} , \tag{C.6}$$

where M^{ij} is a constant symmetric $N \times N$ mass matrix. To construct the conserved currents, we compute $\partial_{(-1,-1)} j_{(s,s)}$ using the massive equations of motion ($i = 1, \dots, N$)

$$\mathrm{i} \partial_{\alpha\dot{\alpha}} \bar{\psi}^{\dot{\alpha} i} + M^{ij} \psi^j_\alpha = 0 \quad \Longrightarrow \quad \Box \bar{\psi}^i_{\dot{\alpha}} = (M\bar{M})^{ij} \bar{\psi}^j_{\dot{\alpha}} \,, \tag{C.7a}$$

$$-\mathrm{i} \partial_{\alpha\dot{\alpha}} \psi^{\alpha i} + \bar{M}^{ij} \bar{\psi}^j_{\dot{\alpha}} = 0 \quad \Longrightarrow \quad \Box \psi^i_\alpha = (\bar{M}M)^{ij} \psi^j_\alpha \,. \tag{C.7b}$$

If C^{ij} is a real symmetric matrix, we find

$$\begin{aligned}
\partial_{(-1,-1)} j_{(s,s)} = {} & -2(s+1) \sum_{k=0}^{s-1} \frac{k+1}{s-k+1} (-1)^k \binom{s}{k} \binom{s}{k+1} \\
& \times \left\{ (CM)^{ij} \partial^k_{(1,1)} \psi^{\alpha i} \; \partial^{s-k-1}_{(1,1)} \psi^j_\alpha + (-1)^s (C\bar{M})^{ij} \partial^k_{(1,1)} \bar{\psi}^i_{\dot{\alpha}} \; \partial^{s-k-1}_{(1,1)} \bar{\psi}^{\dot{\alpha} j} \right\} \\
& + 4(s+1)(s+2) \sum_{k=1}^{s-1} k (-1)^k \binom{s}{k} \binom{s}{k+1} \\
& \times \left\{ \frac{1}{k+2} (M\bar{M}C)^{ij} - \frac{k+1}{(s-k+2)(s-k+1)} (CM\bar{M})^{ij} \right\} \\
& \times \partial^{k-1}_{(1,1)} \zeta^\alpha \psi^i_\alpha \; \partial^{s-k-1}_{(1,1)} \bar{\zeta}^{\dot{\alpha}} \bar{\psi}^j_{\dot{\alpha}} \,.
\end{aligned} \tag{C.8}$$

If C^{ij} is antisymmetric, we have

$$\begin{aligned}\partial_{(-1,-1)}j_{(s,s)} &= -2\mathrm{i}(s+1)\sum_{k=0}^{s-1}\frac{k+1}{s-k+1}(-1)^k\binom{s}{k}\binom{s}{k+1}\\ &\quad\times\Big\{(CM)^{ij}\partial^k_{(1,1)}\psi^{\alpha i}\,\partial^{s-k-1}_{(1,1)}\psi^j_\alpha+(-1)^{s-1}(C\bar{M})^{ij}\partial^k_{(1,1)}\bar{\psi}^i_{\dot\alpha}\,\partial^{s-k-1}_{(1,1)}\bar{\psi}^{\dot\alpha j}\Big\}\\ &\quad+4\mathrm{i}(s+1)(s+2)\sum_{k=1}^{s-1}k(-1)^k\binom{s}{k}\binom{s}{k+1}\\ &\quad\times\left\{\frac{1}{k+2}(M\bar{M}C)^{ij}-\frac{k+1}{(s-k+2)(s-k+1)}(CM\bar{M})^{ij}\right\}\\ &\quad\times\partial^{k-1}_{(1,1)}\zeta^\alpha\psi^i_\alpha\,\partial^{s-k-1}_{(1,1)}\bar{\zeta}^{\dot\alpha}\bar{\psi}^j_{\dot\alpha}\,. \end{aligned}\tag{C.9}$$

There are four cases to consider:

1. $C, CM, CM\bar{M}$ are symmetric $\Longleftrightarrow [C,M]=[C,\bar{M}]=0, [M,\bar{M}]=0$.
2. $C, CM\bar{M}$ symmetric; CM antisymmetric $\Longleftrightarrow \{C,M\}=\{C,\bar{M}\}=0, [M,\bar{M}]=0$.
3. $C, CM\bar{M}$ antisymmetric; CM symmetric $\Longleftrightarrow \{C,M\}=\{C,\bar{M}\}=0, [M,\bar{M}]=0$.
4. $C, CM, CM\bar{M}$ are antisymmetric $\Longleftrightarrow [C,M]=[C,\bar{M}]=0, [M,\bar{M}]=0$.

Case 1. Eq. (C.8) becomes

$$\begin{aligned}\partial_{(-1,-1)}j_{(s,s)} &= -(s+1)\sum_{k=0}^{s-1}(-1)^k\binom{s}{k}\binom{s}{k+1}\\ &\quad\times\left\{\frac{k+1}{s-k+1}+(-1)^{s-1}\frac{s-k}{k+2}\right\}(CM)^{ij}\;\partial^k_{(1,1)}\psi^{\alpha i}\;\partial^{s-k-1}_{(1,1)}\psi^j_\alpha\\ &\quad+(-1)^{s-1}(s+1)\sum_{k=0}^{s-1}(-1)^k\binom{s}{k}\binom{s}{k+1}\\ &\quad\times\left\{\frac{k+1}{s-k+1}+(-1)^{s-1}\frac{s-k}{k+2}\right\}(C\bar{M})^{ij}\;\partial^k_{(1,1)}\bar{\psi}^i_{\dot\alpha}\;\partial^{s-k-1}_{(1,1)}\bar{\psi}^{\dot\alpha j}\\ &\quad+4(s+1)(s+2)\sum_{k=1}^{s-1}k(-1)^k\binom{s}{k}\binom{s}{k+1}\\ &\quad\times\left\{\frac{1}{k+2}-\frac{k+1}{(s-k+2)(s-k+1)}\right\}\\ &\quad\times(CM\bar{M})^{ij}\;\partial^{k-1}_{(1,1)}\zeta^\alpha\psi^i_\alpha\;\partial^{s-k-1}_{(1,1)}\bar{\zeta}^{\dot\alpha}\bar{\psi}^j_{\dot\alpha}\,. \end{aligned}\tag{C.10}$$

We look for $t_{(s-2,s-2)}$ such that (i) it is real; and (ii) it satisfies the conservation equation (B.5):

$$\partial_{(-1,-1)}j_{(s,s)}=-(s-1)\partial_{(1,1)}t_{(s-2,s-2)}\,.\tag{C.11}$$

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Consider a general ansatz

$$t_{(s-2,s-2)} = (CM)^{ij} \sum_{k=0}^{s-2} c_k\, \partial^k_{(1,1)} \psi^{\alpha i}\ \partial^{s-k-2}_{(1,1)} \psi^j_\alpha + (-1)^s (C\bar{M})^{ij} \sum_{k=0}^{s-2} c_k\, \partial^k_{(1,1)} \bar{\psi}^i_{\dot\alpha}\ \partial^{s-k-2}_{(1,1)} \bar{\psi}^{\dot\alpha j} + (CM\bar{M})^{ij} \sum_{k=1}^{s-2} g_k\, \partial^{k-1}_{(1,1)} \zeta^\alpha \psi^i_\alpha\ \partial^{s-k-1}_{(1,1)} \bar{\zeta}^{\dot\alpha} \bar{\psi}^j_{\dot\alpha}\,. \tag{C.12}$$

For $k = 1, 2, \dots s-2$, condition (i) gives

$$g_k = (-1)^{s-1} g_{s-1-k}\,, \tag{C.13a}$$

while condition (ii) gives

$$c_{k-1} + c_k = \frac{s+1}{s-1}(-1)^k \binom{s}{k}\binom{s}{k+1}\left\{\frac{k+1}{s-k+1} + (-1)^{s-1}\frac{s-k}{k+2}\right\}, \tag{C.13b}$$

$$g_{k-1} + g_k = -4\frac{(s+1)(s+2)}{s-1}(-1)^k \binom{s}{k}\binom{s}{k+1} k \left\{\frac{1}{k+2} - \frac{k+1}{(s-k+2)(s-k+1)}\right\}. \tag{C.13c}$$

Condition (ii) also implies that

$$c_{s-2} + c_0 = \frac{1}{s-1}\left\{2s + (-1)^{s-1} s^2 (s+1)\right\}, \tag{C.13d}$$

$$g_1 = \frac{2s(s-2)}{3}(s^2 + 5s + 6)\,, \tag{C.13e}$$

$$g_{s-2} = (-1)^{s-1}\frac{2s(s-2)}{3}(s^2 + 5s + 6)\,. \tag{C.13f}$$

The above conditions lead to the following expressions for c_k and g_k $(k = 1, 2, \dots s-2)$

$$c_k = (-1)^k c_0 + \frac{s+1}{s-1}\sum_{l=1}^{k} (-1)^k \binom{s}{l}\binom{s}{l+1}\left\{\frac{l+1}{s-l+1} + (-1)^{s-1}\frac{s-l}{l+2}\right\}, \tag{C.14a}$$

$$g_k = 4(-1)^k \frac{(s+1)(s+2)}{s-1}\sum_{l=1}^{k} \binom{s}{l}\binom{s}{l+1}\left\{\frac{l(l+1)}{(s-l+1)(s-l+2)} - \frac{l}{l+2}\right\}. \tag{C.14b}$$

If the parameter s is even, (C.14a) gives

$$c_{s-2} = c_0 = -\frac{1}{2}s(s+2) \tag{C.14c}$$

and (C.13a)–(C.13f) are identically satisfied. However, when s is odd, there appears an inconsistency: the right-hand side of (C.13d) is positive, while the left-hand side is negative, $c_{s-2} + c_0 < 0$. Therefore, our solution (C.14) is only consistent for $s = 2n, n = 1, 2, \dots$.

Case 2. If CM is antisymmetric while $CM\bar{M}$ symmetric, eq. (C.8) is slightly modified

$$\begin{aligned}
\partial_{(-1,-1)} j_{(s,s)} = &-(s+1)\sum_{k=0}^{s-1}(-1)^k \binom{s}{k}\binom{s}{k+1} \\
&\times \left\{\frac{k+1}{s-k+1}+(-1)^s \frac{s-k}{k+2}\right\}(CM)^{ij}\ \partial^k_{(1,1)}\psi^{\alpha i}\ \partial^{s-k-1}_{(1,1)}\psi^j_\alpha \\
&+(-1)^{s-1}(s+1)\sum_{k=0}^{s-1}(-1)^k \binom{s}{k}\binom{s}{k+1} \\
&\times \left\{\frac{k+1}{s-k+1}+(-1)^s \frac{s-k}{k+2}\right\}(C\bar{M})^{ij}\ \partial^k_{(1,1)}\bar{\psi}^i_{\dot\alpha}\ \partial^{s-k-1}_{(1,1)}\bar{\psi}^{\dot\alpha j} \\
&+4(s+1)(s+2)\sum_{k=1}^{s-1} k(-1)^k \binom{s}{k}\binom{s}{k+1} \\
&\times \left\{\frac{1}{k+2}-\frac{k+1}{(s-k+2)(s-k+1)}\right\} \\
&\times (CM\bar{M})^{ij}\ \partial^{k-1}_{(1,1)}\zeta^\alpha\psi^i_\alpha\ \partial^{s-k-1}_{(1,1)}\bar{\zeta}^{\dot\alpha}\bar{\psi}^j_{\dot\alpha}\,. \qquad \text{(C.15)}
\end{aligned}$$

Starting with a general ansatz

$$\begin{aligned}
t_{(s-2,s-2)} = &(CM)^{ij}\sum_{k=0}^{s-2} d_k\,\partial^k_{(1,1)}\psi^{\alpha i}\ \partial^{s-k-2}_{(1,1)}\psi^j_\alpha \\
&+(-1)^s (C\bar{M})^{ij}\sum_{k=0}^{s-2} d_k\,\partial^k_{(1,1)}\bar{\psi}^i_{\dot\alpha}\ \partial^{s-k-2}_{(1,1)}\bar{\psi}^{\dot\alpha j} \\
&+(CM\bar{M})^{ij}\sum_{k=1}^{s-2} g_k\,\partial^{k-1}_{(1,1)}\zeta^\alpha\psi^i_\alpha\ \partial^{s-k-1}_{(1,1)}\bar{\zeta}^{\dot\alpha}\bar{\psi}^j_{\dot\alpha} \qquad \text{(C.16)}
\end{aligned}$$

and imposing conditions (i) and (ii) yield

$$g_k = (-1)^{s-1} g_{s-1-k}\,, \qquad \text{(C.17a)}$$

$$d_{k-1}+d_k = \frac{s+1}{s-1}(-1)^k\binom{s}{k}\binom{s}{k+1}\left\{\frac{k+1}{s-k+1}-(-1)^{s-1}\frac{s-k}{k+2}\right\}, \qquad \text{(C.17b)}$$

$$g_{k-1}+g_k = -4\frac{(s+1)(s+2)}{s-1}(-1)^k\binom{s}{k}\binom{s}{k+1}k\left\{\frac{1}{k+2}-\frac{k+1}{(s-k+2)(s-k+1)}\right\}, \qquad \text{(C.17c)}$$

$$d_0-d_{s-2} = \frac{1}{s-1}\left\{2s+(-1)^s s^2(s+1)\right\}, \qquad \text{(C.17d)}$$

$$g_1 = \frac{2s(s-2)}{3}(s^2+5s+6), \qquad \text{(C.17e)}$$

$$g_{s-2} = (-1)^{s-1}\frac{2s(s-2)}{3}(s^2+5s+6)\,. \qquad \text{(C.17f)}$$

As a result, the coefficients d_k and g_k are given by ($k = 1, 2, \ldots s-2$)

$$d_k = (-1)^k d_0 + \frac{s+1}{s-1}\sum_{l=1}^{k}(-1)^k\binom{s}{l}\binom{s}{l+1}\left\{\frac{l+1}{s-l+1}-(-1)^{s-1}\frac{s-l}{l+2}\right\}, \qquad \text{(C.18a)}$$

$$g_k = 4(-1)^k\frac{(s+1)(s+2)}{s-1}\sum_{l=1}^{k}\binom{s}{l}\binom{s}{l+1}\left\{\frac{l(l+1)}{(s-l+1)(s-l+2)}-\frac{l}{l+2}\right\}. \qquad \text{(C.18b)}$$

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When the parameter s is odd, (C.18a) gives

$$d_{s-2} = -d_0 = \frac{1}{2}s(s+2) \tag{C.18c}$$

and (C.17a)–(C.17f) are identically satisfied. However, when s is even, there appears an inconsistency: the right-hand side of (C.17d) is positive, while the left-hand side is negative, $d_0 - d_{s-2} < 0$. Therefore, our solution (C.18) is only consistent for $s = 2n+1, n = 1, 2, \dots$.

Finally, we consider $C^{ij} = -C^{ji}$ with the corresponding $j_{(s,s)}$ given by (C.4). Similar considerations show that in **Case 3**, the non-conformal currents exist only if s is even. The trace $t_{(s-2,s-2)}$ is given by (C.12) with the coefficients c_k and g_k given by

$$c_k = \mathrm{i}(-1)^k c_0 + \mathrm{i}\frac{s+1}{s-1}\sum_{l=1}^{k}(-1)^k\binom{s}{l}\binom{s}{l+1}\left\{\frac{l+1}{s-l+1} + (-1)^{s-1}\frac{s-l}{l+2}\right\}, \tag{C.19a}$$

$$g_k = 4\mathrm{i}\,(-1)^k\frac{(s+1)(s+2)}{s-1}\sum_{l=1}^{k}\binom{s}{l}\binom{s}{l+1}\left\{\frac{l(l+1)}{(s-l+1)(s-l+2)} - \frac{l}{l+2}\right\}. \tag{C.19b}$$

In **Case 4**, the non-conformal currents exist only for odd values of s. The trace $t_{(s-2,s-2)}$ is given by (C.16) with the coefficients d_k and g_k given by

$$d_k = \mathrm{i}(-1)^k d_0 + \mathrm{i}\frac{s+1}{s-1}\sum_{l=1}^{k}(-1)^k\binom{s}{l}\binom{s}{l+1}\left\{\frac{l+1}{s-l+1} - (-1)^{s-1}\frac{s-l}{l+2}\right\}, \tag{C.20a}$$

$$g_k = 4\mathrm{i}\,(-1)^k\frac{(s+1)(s+2)}{s-1}\sum_{l=1}^{k}\binom{s}{l}\binom{s}{l+1}\left\{\frac{l(l+1)}{(s-l+1)(s-l+2)} - \frac{l}{l+2}\right\}. \tag{C.20b}$$

We observe that the coefficients c_k and g_k in eq. (C.19a) and (C.19b), respectively differ from similar coefficients in (C.14a) and (C.14b) by a factor of i. Hence, for even s we may define a more general supercurrent

$$j_{(s,s)} = C^{ij}\sum_{k=0}^{s-1}(-1)^k\binom{s}{k}\binom{s}{k+1}\,\partial^k_{(1,1)}\zeta^\alpha\psi^i_\alpha\;\partial^{s-k-1}_{(1,1)}\bar\zeta^{\dot\alpha}\bar\psi^j_{\dot\alpha}\,, \tag{C.21}$$

where C^{ij} is a generic matrix which can be split into the symmetric and antisymmetric parts: $C^{ij} = S^{ij} + \mathrm{i}A^{ij}$. Here both S and A are real and we put an i in front of A because $j_{(s,s)}$ must be real. From the above consideration it then follows that the corresponding more general solution for $t_{(s-2,s-2)}$ reads

$$\begin{aligned} t_{(s-2,s-2)} = (CM)^{ij}&\sum_{k=0}^{s-2}c_k\,\partial^k_{(1,1)}\psi^{\alpha i}\;\partial^{s-k-2}_{(1,1)}\psi^j_\alpha \\ &+(-1)^s(\bar C\bar M)^{ij}\sum_{k=0}^{s-2}c_k\,\partial^k_{(1,1)}\bar\psi^i_{\dot\alpha}\;\partial^{s-k-2}_{(1,1)}\bar\psi^{\dot\alpha j} \\ &+(CM\bar M)^{ij}\sum_{k=1}^{s-2}g_k\,\partial^{k-1}_{(1,1)}\zeta^\alpha\psi^i_\alpha\;\partial^{s-k-1}_{(1,1)}\bar\zeta^{\dot\alpha}\bar\psi^j_{\dot\alpha}\,, \end{aligned} \tag{C.22}$$

where $[S,M]=[S,\bar{M}]=0$, $\{A,M\}=\{A,\bar{M}\}=0$ and $[M,\bar{M}]=0$. The coefficients c_k and g_k are given by eqs. (C.14a) and (C.14b), respectively. Similarly, the coefficients d_k and g_k in (C.20a) and (C.20b) differ from similar coefficients in (C.18a) and (C.18b) by a factor of i. This means that for odd s we can define a more general supercurrent (C.21), where C^{ij} is a generic matrix which we can split as before into the symmetric and antisymmetric parts, $C^{ij}=S^{ij}+\mathrm{i}A^{ij}$. From the above consideration it then follows that the corresponding more general solution for $t_{(s-2,s-1)}$ reads

$$\begin{aligned} t_{(s-2,s-1)} = (CM)^{ij}\sum_{k=0}^{s-2} d_k\,\partial^k_{(1,1)}\psi^{\alpha i}\;\partial^{s-k-2}_{(1,1)}\psi^j_\alpha \\ +(-1)^s(\bar{C}\bar{M})^{ij}\sum_{k=0}^{s-2} d_k\,\partial^k_{(1,1)}\bar{\psi}^i_{\dot\alpha}\;\partial^{s-k-2}_{(1,1)}\bar{\psi}^{\dot\alpha j} \\ +(CM\bar{M})^{ij}\sum_{k=1}^{s-2} g_k\,\partial^{k-1}_{(1,1)}\zeta^\alpha\psi^i_\alpha\;\partial^{s-k-1}_{(1,1)}\bar{\zeta}^{\dot\alpha}\bar{\psi}^j_{\dot\alpha}\,, \end{aligned} \tag{C.23}$$

where $\{S,M\}=\{S,\bar{M}\}=0$, $[A,M]=[A,\bar{M}]=0$ and $[M,\bar{M}]=0$. The coefficients d_k and g_k are given by eqs. (C.18a) and (C.18b), respectively.

References

[1] G. Festuccia and N. Seiberg, *Rigid Supersymmetric Theories in Curved Superspace*, JHEP **06** (2011) 114 [arXiv:1105.0689] [INSPIRE].

[2] S.M. Kuzenko, *Symmetries of curved superspace*, JHEP **03** (2013) 024 [arXiv:1212.6179] [INSPIRE].

[3] I.L. Buchbinder and S.M. Kuzenko, *Ideas and Methods of Supersymmetry and Supergravity or a Walk Through Superspace*, IOP, Bristol, U.K. (1995), revised edition: (1998).

[4] S.M. Kuzenko and G. Tartaglino-Mazzucchelli, *Nilpotent chiral superfield in $N=2$ supergravity and partial rigid supersymmetry breaking*, JHEP **03** (2016) 092 [arXiv:1512.01964] [INSPIRE].

[5] S.M. Kuzenko, *Maximally supersymmetric solutions of R^2 supergravity*, Phys. Rev. **D 94** (2016) 065014 [arXiv:1606.00654] [INSPIRE].

[6] V.P. Akulov and D.V. Volkov, *Goldstone fields with spin 1/2*, Theor. Math. Phys. **18** (1974) 28 [*Teor. Mat. Fiz.* **18** (1974) 39] [INSPIRE].

[7] A. Salam and J.A. Strathdee, *Supergauge Transformations*, Nucl. Phys. **B 76** (1974) 477 [INSPIRE].

[8] B.W. Keck, *An Alternative Class of Supersymmetries*, J. Phys. **A 8** (1975) 1819 [INSPIRE].

[9] B. Zumino, *Nonlinear Realization of Supersymmetry in de Sitter Space*, Nucl. Phys. **B 127** (1977) 189 [INSPIRE].

JHEP09(2018)027

[10] E.A. Ivanov and A.S. Sorin, *Superfield formulation of* OSp(1, 4) *supersymmetry*, *J. Phys.* **A 13** (1980) 1159 [INSPIRE].

[11] D. Butter and S.M. Kuzenko, $N = 2$ *AdS supergravity and supercurrents*, *JHEP* **07** (2011) 081 [arXiv:1104.2153] [INSPIRE].

[12] D. Butter and S.M. Kuzenko, *A dual formulation of supergravity-matter theories*, *Nucl. Phys.* **B 854** (2012) 1 [arXiv:1106.3038] [INSPIRE].

[13] M. Magro, I. Sachs and S. Wolf, *Superfield Noether procedure*, *Annals Phys.* **298** (2002) 123 [hep-th/0110131] [INSPIRE].

[14] S.M. Kuzenko, *Variant supercurrent multiplets*, *JHEP* **04** (2010) 022 [arXiv:1002.4932] [INSPIRE].

[15] S. Ferrara and B. Zumino, *Transformation Properties of the Supercurrent*, *Nucl. Phys.* **B 87** (1975) 207 [INSPIRE].

[16] Z. Komargodski and N. Seiberg, *Comments on Supercurrent Multiplets, Supersymmetric Field Theories and Supergravity*, *JHEP* **07** (2010) 017 [arXiv:1002.2228] [INSPIRE].

[17] D. Butter and S.M. Kuzenko, $N = 2$ *supersymmetric* σ*-models in AdS*, *Phys. Lett.* **B 703** (2011) 620 [arXiv:1105.3111] [INSPIRE].

[18] D. Butter and S.M. Kuzenko, *The structure of* $N = 2$ *supersymmetric nonlinear* σ*-models in* AdS_4, *JHEP* **11** (2011) 080 [arXiv:1108.5290] [INSPIRE].

[19] D. Butter, S.M. Kuzenko, U. Lindström and G. Tartaglino-Mazzucchelli, *Extended supersymmetric* σ*-models in* AdS_4 *from projective superspace*, *JHEP* **05** (2012) 138 [arXiv:1203.5001] [INSPIRE].

[20] A. Adams, H. Jockers, V. Kumar and J.M. Lapan, $N = 1$ σ*-models in* AdS_4, *JHEP* **12** (2011) 042 [arXiv:1104.3155] [INSPIRE].

[21] S.J. Gates, M.T. Grisaru, M. Roček and W. Siegel, *Superspace Or One Thousand and One Lessons in Supersymmetry*, *Front. Phys.* **58** (1983) 1 [hep-th/0108200] [INSPIRE].

[22] S.M. Kuzenko and A.G. Sibiryakov, *Free massless higher superspin superfields on the anti-de Sitter superspace*, *Phys. Atom. Nucl.* **57** (1994) 1257 [*Yad. Fiz.* **57** (1994) 1326] [arXiv:1112.4612] [INSPIRE].

[23] S.M. Kuzenko, A.G. Sibiryakov and V.V. Postnikov, *Massless gauge superfields of higher half integer superspins*, *JETP Lett.* **57** (1993) 534 [*Pisma Zh. Eksp. Teor. Fiz.* **57** (1993) 521] [INSPIRE].

[24] S.M. Kuzenko and A.G. Sibiryakov, *Massless gauge superfields of higher integer superspins*, *JETP Lett.* **57** (1993) 539 [*Pisma Zh. Eksp. Teor. Fiz.* **57** (1993) 526] [INSPIRE].

[25] J. Hutomo and S.M. Kuzenko, *Non-conformal higher spin supercurrents*, *Phys. Lett.* **B 778** (2018) 242 [arXiv:1710.10837] [INSPIRE].

[26] J. Hutomo and S.M. Kuzenko, *The massless integer superspin multiplets revisited*, *JHEP* **02** (2018) 137 [arXiv:1711.11364] [INSPIRE].

[27] S.M. Kuzenko, R. Manvelyan and S. Theisen, *Off-shell superconformal higher spin multiplets in four dimensions*, *JHEP* **07** (2017) 034 [arXiv:1701.00682] [INSPIRE].

[28] P. Van Nieuwenhuizen, *Supergravity*, *Phys. Rept.* **68** (1981) 189 [INSPIRE].

[29] I.L. Buchbinder, S.J. Gates and K. Koutrolikos, *Higher Spin Superfield interactions with the Chiral Supermultiplet: Conserved Supercurrents and Cubic Vertices*, *Universe* **4** (2018) 6 [arXiv:1708.06262] [INSPIRE].

[30] K. Koutrolikos, P. Kočí and R. von Unge, *Higher Spin Superfield interactions with Complex linear Supermultiplet: Conserved Supercurrents and Cubic Vertices*, *JHEP* **03** (2018) 119 [arXiv:1712.05150] [INSPIRE].

[31] I.L. Buchbinder, S.J. Gates and K. Koutrolikos, *Conserved higher spin supercurrents for arbitrary spin massless supermultiplets and higher spin superfield cubic interactions*, *JHEP* **08** (2018) 055 [arXiv:1805.04413] [INSPIRE].

[32] A.A. Migdal, *Multicolor QCD as Dual Resonance Theory*, *Annals Phys.* **109** (1977) 365 [INSPIRE].

[33] Yu. M. Makeenko, *Conformal operators in quantum chromodynamics*, *Sov. J. Nucl. Phys.* **33** (1981) 440 [*Yad. Fiz.* **33** (1981) 842] [INSPIRE].

[34] N.S. Craigie, V.K. Dobrev and I.T. Todorov, *Conformally Covariant Composite Operators in Quantum Chromodynamics*, *Annals Phys.* **159** (1985) 411 [INSPIRE].

[35] F.A. Berends, G.J.H. Burgers and H. van Dam, *Explicit Construction of Conserved Currents for Massless Fields of Arbitrary Spin*, *Nucl. Phys.* **B 271** (1986) 429 [INSPIRE].

[36] D. Anselmi, *Theory of higher spin tensor currents and central charges*, *Nucl. Phys.* **B 541** (1999) 323 [hep-th/9808004] [INSPIRE].

[37] D. Anselmi, *Higher spin current multiplets in operator product expansions*, *Class. Quant. Grav.* **17** (2000) 1383 [hep-th/9906167] [INSPIRE].

[38] R. Manvelyan and W. Rühl, *Conformal coupling of higher spin gauge fields to a scalar field in AdS_4 and generalized Weyl invariance*, *Phys. Lett.* **B 593** (2004) 253 [hep-th/0403241] [INSPIRE].

[39] R. Manvelyan and K. Mkrtchyan, *Conformal invariant interaction of a scalar field with the higher spin field in AdS(D)*, *Mod. Phys. Lett.* **A 25** (2010) 1333 [arXiv:0903.0058] [INSPIRE].

[40] A. Fotopoulos, N. Irges, A.C. Petkou and M. Tsulaia, *Higher-Spin Gauge Fields Interacting with Scalars: The Lagrangian Cubic Vertex*, *JHEP* **10** (2007) 021 [arXiv:0708.1399] [INSPIRE].

[41] A. Fotopoulos and M. Tsulaia, *Current Exchanges for Reducible Higher Spin Modes on AdS*, in *19th International Colloquium on Integrable Systems and Quantum Symmetries (ISQS-19) Prague, Czech Republic, June 17–19, 2010*, arXiv:1007.0747 [INSPIRE].

[42] X. Bekaert and E. Meunier, *Higher spin interactions with scalar matter on constant curvature spacetimes: conserved current and cubic coupling generating functions*, *JHEP* **11** (2010) 116 [arXiv:1007.4384] [INSPIRE].

[43] N. Sakai and Y. Tanii, *Supersymmetry and Vacuum Energy in Anti-de Sitter Space*, *Phys. Lett.* **B 146** (1984) 38 [INSPIRE].

[44] C.P. Burgess, *Supersymmetry Breaking and Vacuum Energy on Anti-de Sitter Space*, *Nucl. Phys.* **B 259** (1985) 473 [INSPIRE].

[45] C.P. Burgess and C.A. Lütken, *Propagators and Effective Potentials in Anti-de Sitter Space*, *Phys. Lett.* **B 153** (1985) 137 [INSPIRE].

[46] C.J.C. Burges, D.Z. Freedman, S. Davis and G.W. Gibbons, *Supersymmetry in Anti-de Sitter Space*, *Annals Phys.* **167** (1986) 285 [INSPIRE].

[47] O. Aharony, D. Marolf and M. Rangamani, *Conformal field theories in anti-de Sitter space*, *JHEP* **02** (2011) 041 [arXiv:1011.6144] [INSPIRE].

JHEP09(2018)027

[48] O. Aharony, M. Berkooz, D. Tong and S. Yankielowicz, *Confinement in Anti-de Sitter Space*, *JHEP* **02** (2013) 076 [arXiv:1210.5195] [INSPIRE].

[49] O. Aharony, M. Berkooz, A. Karasik and T. Vaknin, *Supersymmetric field theories on $AdS_p \times S^q$*, *JHEP* **04** (2016) 066 [arXiv:1512.04698] [INSPIRE].

[50] A.A. Tseytlin, *On limits of superstring in $AdS_5 \times S^5$*, *Theor. Math. Phys.* **133** (2002) 1376 [*Teor. Mat. Fiz.* **133** (2002) 69] [hep-th/0201112] [INSPIRE].

[51] A.Y. Segal, *Conformal higher spin theory*, *Nucl. Phys.* **B 664** (2003) 59 [hep-th/0207212] [INSPIRE].

[52] X. Bekaert, E. Joung and J. Mourad, *Effective action in a higher-spin background*, *JHEP* **02** (2011) 048 [arXiv:1012.2103] [INSPIRE].

[53] P.C. West, *Introduction to Supersymmetry and Supergravity*, World Scientific, Singapore, (1986), extended revised edition: (1990).

[54] W. Siegel, *Solution to Constraints in Wess-Zumino Supergravity Formalism*, *Nucl. Phys.* **B 142** (1978) 301 [INSPIRE].

[55] B. Zumino, *Supergravity and superspace*, in *Recent Developments in Gravitation - Cargèse 1978*, M. Lévy and S. Deser eds., Plenum Press, N.Y., U.S.A., (1979), pp. 405.

[56] S.J. Gates Jr. and W. Siegel, $(3/2, 1)$ *Superfield of $O(2)$ Supergravity*, *Nucl. Phys.* **B 164** (1980) 484 [INSPIRE].

[57] U. Lindström and M. Roček, *Scalar Tensor Duality and $N = 1$, $N = 2$ Nonlinear σ-models*, *Nucl. Phys.* **B 222** (1983) 285 [INSPIRE].

[58] S.M. Kuzenko and G. Tartaglino-Mazzucchelli, *Five-dimensional Superfield Supergravity*, *Phys. Lett.* **B 661** (2008) 42 [arXiv:0710.3440] [INSPIRE].

[59] S.M. Kuzenko and G. Tartaglino-Mazzucchelli, *5D Supergravity and Projective Superspace*, *JHEP* **02** (2008) 004 [arXiv:0712.3102] [INSPIRE].

[60] S.M. Kuzenko, U. Lindström, M. Roček and G. Tartaglino-Mazzucchelli, *4D $N = 2$ Supergravity and Projective Superspace*, *JHEP* **09** (2008) 051 [arXiv:0805.4683] [INSPIRE].

[61] S.M. Kuzenko and G. Tartaglino-Mazzucchelli, *Field theory in 4D $N = 2$ conformally flat superspace*, *JHEP* **10** (2008) 001 [arXiv:0807.3368] [INSPIRE].

[62] W. Siegel, *Gauge Spinor Superfield as a Scalar Multiplet*, *Phys. Lett.* **B 85** (1979) 333 [INSPIRE].

[63] C. Fronsdal, *Massless Fields with Integer Spin*, *Phys. Rev.* **D 18** (1978) 3624 [INSPIRE].

[64] E.S. Fradkin and A.A. Tseytlin, *Conformal supergravity*, *Phys. Rept.* **119** (1985) 233 [INSPIRE].

[65] E.S. Fradkin and V. Ya. Linetsky, *Superconformal Higher Spin Theory in the Cubic Approximation*, *Nucl. Phys.* **B 350** (1991) 274 [INSPIRE].

[66] P.S. Howe, K.S. Stelle and P.K. Townsend, *Supercurrents*, *Nucl. Phys.* **B 192** (1981) 332 [INSPIRE].

[67] O.A. Gelfond, E.D. Skvortsov and M.A. Vasiliev, *Higher spin conformal currents in Minkowski space*, *Theor. Math. Phys.* **154** (2008) 294 [hep-th/0601106] [INSPIRE].