

The Weak Gravity Conjecture and the axionic black hole paradox

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ABSTRACT: In theories with a perturbatively massless 2-form (dual to an axion), a paradox may arise in the process of black hole evaporation. Schwarzschild black holes can support a non-trivial Wilson-line-type field, the integral of the 2-form around their horizon. After such an ‘axionic black hole’ evaporates, the Wilson line must be supported by the corresponding 3-form field strength in the region formerly occupied by the black hole. In the limit of small axion decay-constant f , the energy required for this field configuration is too large. Thus, energy can not be conserved in the process of black hole evaporation. The natural resolution of this paradox is through the presence of light strings, which allow the black hole to “shed” its axionic hair sufficiently early. This gives rise to a new Weak-Gravity-type argument in the 2-form context: small coupling, in this case f , enforces the presence of light strings or a low cutoff. We also discuss how this argument may be modified in situations where the weak coupling regime is achieved in the low-energy effective theory through an appropriate gauging of a model with a vector field and two 2-forms.

KEYWORDS: Black Holes, Gauge Symmetry, Global Symmetries

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1 Introduction

Very roughly speaking, the Weak Gravity Conjecture (WGC) says that a U(1) gauge theory with coupling $g \ll 1$ can only be consistently coupled to quantum gravity if there are charged states or even a cutoff at the scale $gM_P \ll M_P$ [1]. This is expected to extend to $(p + 1)$ -form gauge theories with charged p -branes in any number of dimensions.¹ The simplest way to formulate the analogous statement about a low cutoff is then to say that $\Lambda^{p+1} \sim g$ in Planck units (see [4–20] for a selection of recent related work).

Much of the recent phenomenological interest in the WGC derives from its potential power to constrain axion inflation [21–37]: to this end, one views axion models as 0-form gauge theories in the regime where the axionic coupling $1/f$ is small (i.e. $f \gg M_P$). However, fundamental justifications for the WGC both in the 1-form, the 0-form and various other cases and different regimes are difficult to obtain. Thus, we believe that supplying *any* arguments for or against it, even if in a slightly unusual setting, is important.

It is our aim to analyse possible arguments in favor of the WGC on the dual side of the more familiar axion / large-field-inflation case mentioned above. Indeed, we dualise the 0-form φ to a 2-form, $dB_2 = f^2 * d\varphi$, and consider the action

$$-\int d^4x \sqrt{-g} \frac{1}{f^2} |dB_2|^2 + \int_{\text{worldsheet}} B_2. \tag{1.1}$$

We want to understand whether we can have small f without light charged objects, i.e. without light strings. In the extreme, the tension σ of the lightest string might be $\sigma \sim$

¹Notice that our convention for designating generalized gauge theories differs from that of, e.g. [2, 3]. Here, standard theories of charged particles coupled to 1-form gauge fields are referred to as 1-form gauge theories, rather than 0-form ones.

M_P^2 , but still $f \ll M_P$ (see [28] for constraints on such regimes in perturbative string compactifications). Is this inconsistent in any tangible way?

The standard magnetic WGC would have to argue about the smallest instanton not yet being a ‘black hole’ (in this case wormhole or gravitational instanton). This is clearly questionable. The electric WGC would have to argue about the stability of extremal black strings. But such macroscopic objects appear rather pathological (having a deficit angle greater than 2π) and, moreover, possess no horizon.² What is worse, both routes rest simply on assumptions, such as ‘stable extremal black holes (BHs) must not exist’ (see however [18] for recent progress towards justifying this requirement rigorously).

Here we suggest a completely different approach and, with certain caveats to be explained, arrive at an actual physical inconsistency for parametrically small f in the absence of light strings.

The basic idea is to consider axionic BHs as in [38]. Such objects are distinguished from standard Schwarzschild BHs exclusively by having non-zero values of $2\pi b \equiv \int B_2$, where the integral is over any sphere homotopic to their horizon. Such a ‘Wilson-loop variable’ b is locally unobservable and carries no energy (the field strength $H_3 = dB_2$ vanishes), and so the axionic BH behaves at the semi-classical level exactly as a Schwarzschild one.

A paradox arises in the evaporation process of such a black hole: by causality, the B_2 boundary integral far away from the BH can not change. Once the BH is gone, spacetime has become non-singular and the interior of this boundary must, according to Gauss’ law, contain a minimal H_3 field strength. The energy carried by this field strength is too large compared to the mass of the black hole just before the low-energy effective theory has broken down. Thus, energy can not be conserved.

In slightly more detail, the BH is expected to shrink via Hawking radiation, maintaining the value of b , at least up to a radius r of the order of the Planck length. While the later stages in the life of the BH cannot be properly described without a UV complete theory, it is clear that the only way in which a complete evaporation can be consistent with a non-zero b is through a ‘leftover’ field strength:

$$\int_{V(r)} H_3 = \int_{S^2(r)} B_2 \sim b \sim \mathcal{O}(1). \tag{1.2}$$

Here the sphere is the boundary of the volume $V(r)$, the latter being a ball of radius r at the place where the BH used to be. But in the regime of small f , the prefactor $1/f^2$ of the field energy stored in H_3 is huge. It is in fact much larger than the available energy $\sim \mathcal{O}(M_P)$ of the smallest semiclassical BH which was still controlled in the low-energy effective theory. Unless there is a remnant, we have a contradiction.³

This contradiction is resolved if light strings are present in the spectrum of the theory. Strings ‘lassoing’ the black hole do interact with the B_2 integral, and indeed generate an effective potential for the Wilson loop variable of the form $V(b) \sim e^{-4\pi\sigma r^2} \cos b$. This is

²One might try to overcome this by demanding instead that two same-charge microscopic strings should always repel.

³Similar ideas have been used in [39, 40] in a somewhat different context to argue that certain types of global symmetries (carried by skyrmions/baryons) can be preserved in BH evaporation.

analogous to the familiar potential for the Wilson line variable $\oint A_1$ in a standard (1-form) gauge theory compactification from 5d to 4d. As long as the BH radius R is much larger than the length scale set by the string tension, $R \gg 1/\sqrt{\sigma}$, the strings are irrelevant: the effective potential induced by a virtual string ‘lassoing’ the BH has a suppression $\sim \exp(-4\pi\sigma R^2)$ and is negligible. However, once $R \sim 1/\sqrt{\sigma}$, the effective potential for the ‘Wilson loop variable’ b ceases to be exponentially suppressed. Now b is driven to zero dynamically near the BH, the value of b at larger radii is supported by a non-vanishing field strength H_3 near the BH, and the BH can eventually disappear without a trace. The problem with the evaporation of axionic BHs is resolved if light strings exist.⁴

The rest of this paper is organized as follows. In section 2 we estimate the specific parametric bounds on the string tension that result from different evaporation rates of small black holes. We spell out in section 3 several assumptions implicit in our arguments and the physical effects that motivate them. Finally, in section 4 we study a setup in which a massless 2-form field with small effective f arises via alignment, upon Higgsing a linear combination of two B_2 fields, in analogy to [14, 26]. Our arguments suggest the appearance of light monopoles in such a theory.

The simultaneous work [41] also discusses generalized (global) symmetries and their possible problems from a somewhat different perspective.

2 The final moments of an axionic black hole

As we have discussed in the Introduction, a contradiction arises in the process of evaporation of axionic black holes in the absence of light strings.⁵ This implies a parametric upper bound on the tension of strings (for a given gauge coupling f). In order to derive the parametric form of the bound, we need to make certain assumptions about the latest stages of the evaporation of axionic BHs. We consider next two possibilities and derive the parametric form of the constraints that arise in each case.

2.1 Immediate breakdown at critical radius

The simplest assumption, or at least the assumption leading to the simplest estimate, is a catastrophic, explosion-like evaporation of the BH at the moment when it reaches the

⁴The appearance of a potential due to lassoing strings is not undisputed [44]. Indeed, in a partition function calculation in the scalar field basis, a single lassoing string has infinite action. First, this does not preclude single-instanton contributions in the B_2 basis. Second, even staying in the scalar field basis, nothing speaks against contributions from even numbers of lassoing strings. This is analogous to calculating the familiar instanton-induced cosine potential in quantum mechanics, but using a basis of fixed discrete momenta, such that only even numbers of instantons occur. The toy model calculation of [34] shows that a potential can still be derived in this way. Moreover, if it still turned out that a potential in the strict sense does not arise, tunneling processes corresponding to lassoing strings are certainly possible and can give rise to a dynamical effect on b . In any event we can argue, without directly referring to light strings, that the dynamical activation of b kicks in at a scale $R \sim \Lambda^{-1}$ given by the cutoff of the theory. A bound on such a cutoff, analogous to the *magnetic* rather than *electric* WGC, would then be obtained. Although we mostly refer to light strings throughout this work, both points of view are related by associating the cutoff to the tension of charged objects, i.e. $\Lambda \sim \sqrt{\sigma}$.

⁵There are several important caveats to this simple reasoning that must be considered to reach our conclusions. In order to avoid obscuring our results, we postpone their discussion to later sections.

critical radius $R_c \equiv 1/\sqrt{\sigma}$. This is not a totally unnatural expectation: according to the arguments of [8, 12, 13], not just one but many string states should come in and an extreme growth of the number of degrees of freedom with energy may indeed lead to an instantaneous evaporation at a temperature $T_c \sim 1/R_c \sim \Lambda \equiv \sqrt{\sigma}$.

In this case, there is no time for any effect arising from the dynamical strings to propagate. The H_3 field they dynamically induce is limited to a ball of radius R_c , the total resulting field energy being

$$E \sim \frac{1}{f^2} \int_{V(R_c)} d^3x \sqrt{g} |H_3|^2 \sim \frac{b^2}{f^2 R_c^3} \sim \frac{1}{f^2 R_c^3}. \tag{2.1}$$

Now, we should require $E \lesssim M(R_c) \sim R_c M_p^2$ if energy is to be conserved in the final stage of the evaporation. This leads to a lower bound on the string tension σ of the form

$$\sigma \lesssim f \cdot M_p, \quad \text{or} \quad \Lambda^2 \lesssim f \cdot M_p \tag{2.2}$$

This simple parametric estimate coincides precisely with the standard WGC bound for strings.

The origin of this bound is conceptually very simple, and perhaps finds a generalization to other setups (in particular to different dimensions and for objects of different co-dimension). On the one hand, a non-vanishing generalized Wilson-line can be supported by a topologically non-trivial p -cycle of a gravitational object with energy $\sim M_p^{d-2}$. On the other hand, in a trivial topology, a field strength H_{p+1} that supports the same Wilson-line has an energy that scales with the coupling constant g as $\sim 1/g^2$. When both situations are related by a dynamical change in the topology, energy conservation imposes a bound $M_p^{d-2} \gtrsim 1/g^2$. Of course, one needs to complete this inequality to make it dimensionally consistent. If one can argue that there is only one other scale $\Lambda \sim R_c^{-1}$ involved in the process, the WGC is recovered.

2.2 Slow evaporation and spread flux

While the result (2.2) is suggestive, the assumption of immediate BH evaporation at a critical temperature is likely too naive. We may obtain a much more conservative bound if we assume that, after the effective potential is activated at a temperature $T_c \sim 1/R_c \sim \Lambda \equiv \sqrt{\sigma}$, the H_3 flux induced has a time t_{ev} to spread out radially at the speed of light before the BH completely evaporates. Assuming that in its latest stages the BH still radiates energy at rate $-dM/dt \sim M_p^4/M^2$, the evaporation time from the critical mass $M_c \sim R_c M_p^2$ to zero scales as⁶ $t_{ev} \sim M_c^3/M_p^4 \sim M_p^2/\sigma^{3/2} \gg R_c$. In this scenario, the left-over field H_3 that accounts for the Wilson loop variable b after the BH disappears has spread out to a ball of radius $R_c + t_{ev} \approx t_{ev} \sim M_p^2/\sigma^{3/2}$. The energy stored in such a flux scales as

$$E \sim \frac{1}{f^2} \int_{V(t_{ev})} d^3x \sqrt{g} |H_3|^2 \sim \frac{b^2}{f^2 t_{ev}^3} \sim \frac{\sigma^{9/2}}{f^2 M_p^6}. \tag{2.3}$$

⁶Given the assumptions we have to make about the latest stage of BH evaporation, our bounds on the tension of strings can only be considered estimates. In fact, the numerical factors we are neglecting can be very large. For example, the numerical factor in front of t_{ev} is 5120π .

Arguing as before that this energy should be less than the mass of the axionic BH of radius R_c , we obtain a bound on the string tension σ of the form

$$\sigma \sim \Lambda^2 \lesssim f^{2/5} \cdot M_p^{8/5}. \tag{2.4}$$

This bound is much weaker than the one obtained in (2.2) (which coincides with the WGC). It would become even weaker if the BH decay process slowed down for some reason at the latest stages of the evaporation, and the H_3 flux had more time to spread over a broader region. In fact, the bound would disappear entirely if the decay never reached an end (in this case there would be a remnant which could by itself support a non-zero value of b). We find it reasonable to assume that the decay will not slow down. By contrast, we think it is likely that the right bound derived this way will lie somewhere between (2.2) and (2.4).

3 Infrared divergences and quantum effects

We have so far glossed over some key issues that need to be addressed before accepting the conclusions of last section.

3.1 Non-perturbative effects

A first question one should ask when considering axionic BHs is whether the variable $b = \int B_2$ is measurable and has a physical meaning at all [43–45]. Since B_2 is locally pure gauge, it does not exist in a local sense. The way to measure it is via an Aharonov-Bohm (AB) type experiment, where strings lassoing the BH acquire a phase proportional to b which can then be manifested in an interference pattern.

A problem arises because the energy stored in an infinitely long string is logarithmically divergent in the IR. Strings are hence confining, and there is a limit in the maximum radius $R_{\max} \sim f^{-1} \exp(M_P^2/f^2)$ that a string loop can reach [2, 46, 47]. In order for the axionic hair on a BH to be measurable, we have to make sure that the AB experiment can be performed, i.e. that the BH’s size is smaller than R_{\max} . Given the exponential dependence of R_{\max} with respect to f (recall that we are interested in the $f \ll M_P$ limit), this condition can be easily satisfied.

Another effect that we have not yet considered is that induced by instantons. When non-perturbative effects are taken into account, there is no strictly massless propagating degree of freedom (other than the graviton) in the theory. The instantons induce a mass gap, albeit an exponentially suppressed one. Under certain conditions this can be described in terms of a coupling of B_2 to a non-dynamical 3-form (see [48–52]), but it is most easily seen in the dual description where instantons generate a periodic potential for the axion.

Recall that, upon circumnavigating a string, the axion field shifts by a period $\phi \rightarrow \phi + 2\pi$. In the absence of a potential, this shift will be uniformly distributed around the string, i.e. $\phi \sim \theta$, where θ is the angle that parametrizes the winding trajectory. When a potential $V(\phi)$ is present, however, ϕ will tend to remain at its minimum for most of the trajectory. In this case, the string will be the boundary of a domain wall where the field ϕ jumps discretely. As before, this puts a limit in the maximum size R_{\max} of the string loops one can consider. Beyond R_{\max} , strings loops are unstable due to nucleation

of smaller string loops which eat up the axionic domain wall. Also as before, this effect is non-perturbatively suppressed (the axionic potential scales typically as $e^{-M_P/f}$) and can be safely ignored for sufficiently small strings.

One may ask, still, whether axionic BHs can even exist once instantons are taken into account. One possibility would be that instanton effects induced a non-perturbative potential to the Wilson-line variable b . In this case, a non-zero b could not spread all the way to infinity, since this would carry an infinite potential energy, independently of how small the instanton effects are. However, one may simply consider a pair of axionic BHs, with opposite value of $\int B_2$ so that the system still has a finite energy. As long as the non-perturbative potential induced is small enough, it will take an exponentially long time for the system to relax to its stable configuration (by merging the two axionic BHs). In this note we implicitly work in a regime of parameters where instanton effects are highly suppressed and do not play a role for the relatively small black holes that we are considering.

It is finally also interesting to describe the B_2 Wilson-line in the dual axionic picture. Consider for simplicity a quantum mechanical model, obtained by compactifying a 4d theory with a massless B_2 field on a T^3 , parametrized by coordinates (x, y, z) . We can turn on a Wilson-line along any of the torus two-cycles, say T_{xy}^2 . The quantum mechanical canonical momentum of B_{xy} is given by $\partial_t B_{xy}$. By Heisenberg's uncertainty principle, a state with highly localized value of the Wilson line will correspond to a very broad superposition of states with fixed $\partial_t B_{xy}$. Upon 4d Hodge duality, $d\phi = *_4 B_2$, such states correspond to axion flux turned on along the one-cycle dual to T_{xy}^2 , i.e. our original state with Wilson-line B_{xy} will be described as a superposition of states with different values of $\partial_z \phi$. The value of the Wilson line corresponds to the phase, analogous to the θ -angle, introduced in this superposition. In the case of an axionic BH, the two-cycle is the (homology class of the) horizon, and its dual corresponds to the (non-compact) radial direction. Hence, in the scalar language, a non-trivial value of the axionic hair b corresponds to the phase in the superposition of states with non-trivial axionic gradient $\partial_r \phi$. Of course, once instanton effects are taken into account and a non-perturbative potential for ϕ is generated, the duality between ϕ and B_2 becomes more involved.

3.2 Quantum vs. classical

It is essential to understand under which conditions the Wilson line variable b of a BH can be thought of as a classical degree of freedom. To this end, consider the action

$$S \sim - \int d^4x \sqrt{-g} \frac{1}{f^2} |dB_2|^2 \tag{3.1}$$

in the BH background

$$ds^2 = -(1 - R/r)dt^2 + (1 - R/r)^{-1}dr^2 + r^2d\Omega^2. \tag{3.2}$$

Assuming that B_2 is proportional to the normalized harmonic 2-form on S^2 , parametrized by $b = b(t, r)$, gives

$$S \sim \int dt \int_R^\infty \frac{dr}{r^2} \cdot \frac{1}{f^2} [(1 - R/r)^{-1} (\partial_t b)^2 - (1 - R/r) (\partial_r b)^2]. \tag{3.3}$$

We see that the kinetic term diverges near the horizon. This agrees with the intuition that any dynamics near the horizon should be extremely slow in the time variable suitable for the observer at infinity. Furthermore, the gradient term goes to zero near the horizon. This is again intuitive since there should be no suppression of configurations where the values of b very near the horizon and further away differ significantly. Indeed, the (effectively frozen) value of b near the horizon should not be able to influence an observer at a certain distance. We thus take it for granted that, in total, the effect of the near-horizon region in the above action will be as follows: the BH horizon does not fix the value of b , in agreement with the intuitive expectation from the no-hair theorem.

This can be modelled by excising a sphere of some radius $\gtrsim R$ (we do not care about $\mathcal{O}(1)$ factors) and considering only the dynamics of b in the outer region. At our qualitative level of analysis we can then also set $(1 - R/r)$ to unity and simply write

$$S \sim \int dt \int_R^\infty \frac{dr}{r^2} \cdot \frac{1}{f^2} [(\partial_t b)^2 - (\partial_r b)^2] . \tag{3.4}$$

Here we took the lower limit of integration to be R for simplicity, although it should of course be slightly larger as explained above. Clearly, an action like (3.4) requires a boundary condition at $r = R$. We choose Neumann boundary conditions for consistency with the classical shift symmetry of b and our expectation, argued above, that the near horizon region does not break this invariance classically.⁷

Crucially, the r integral now converges, such the quantum mechanical model for the zero mode (the r -independent mode of b) reads

$$S \sim \int dt \frac{1}{f^2 R} (\partial_t b)^2 . \tag{3.5}$$

In other words, the dynamics is the same that one would obtain from a compactification to one dimension on a compact 3d space of typical radius R with one non-trivial two-cycle.

We are thus dealing with quantum mechanics of a variable with period 2π and a single mass scale $f^2 R$ introduced through the kinetic term. Adopting textbook knowledge to our setting, it is clear that this mass scale translates into a time-scale which governs the spread of an optimally localized gaussian wave packet to the maximal width of 2π . On time scales shorter than

$$t_{qm} \sim 1/(f^2 R) \tag{3.6}$$

we can then think of our effective potential, induced by lassoing strings, as of a classical force acting on the classical variable b .

Now, in our ‘immediate breakdown’ scenario, the typical time scale is $t_c \equiv R_c$. It is simply the time a signal needs to travel across the relevant region of space. For our classical analysis to be meaningful one should then require that this typical time is small enough:

$$t_c < t_{qm} \sim 1/(f^2 R_c) , \quad \text{that is} \quad \sigma > f^2 . \tag{3.7}$$

⁷A more careful modelling of the dynamics of b in the near horizon region would be interesting but is not necessary for the point we want to make in this section.

This comfortably contains the range of very high string tensions constrained by the WGC and by our analysis.

For the ‘slow-evaporation’ scenario, the much longer time scale of evaporation of a BH with radius R_c is the relevant one. As we saw, this is $t_{ev} \sim M_p^2/\sigma^{3/2}$. We now have to impose that

$$t_{ev} < t_{qm} \sim 1/(f^2 R_c), \quad \text{that is} \quad \sigma > f \cdot M_p. \quad (3.8)$$

We evaluate the implications of this bound for our analysis by considering three possible regimes: first, if $f^{2/5} M_p^{8/5} \lesssim \sigma$, our classical analysis is valid and hence the paradox derived in section 2.2 should be taken seriously. Second, if $f M_p \lesssim \sigma \lesssim f^{2/5} M_p^{8/5}$, our classical analysis is valid but no paradox arises according to section 2.2. The existence of this region exemplifies how, in the slow-evaporation scenario, our constraint on σ is weaker than that from the WGC. Third, if $\sigma \lesssim f M_p$ our classical analysis is not trustworthy. However, we were anyway not trying to make a new statement about this regime of low string tension, consistent with the WGC. (It might, independently, be interesting to study axionic black hole evaporation in this regime. But we do not need this for the present paper.)

Before closing this section we note that one might find it counterintuitive that a field-theoretic classical analysis breaks down at large rather than at short times. This apparent problem can be understood by thinking of our integrated field variable b of eq. (3.5) as of a non-relativistic quantum mechanical particle. Let us denote the position by x and the mass by m , with the appropriate translation in b , f and R easily derived. Now, if x is non-compact and the initial state of the particle has a certain Gaussian spread Δx , we can always think classically by asking long-distance questions, i.e. questions for which the spread in x is negligible (and the spread in the dual variable p is as usual ascribed to our imprecise knowledge of the initial velocity). Long time scales are helpful since they allow us to know the momentum and hence the position in future measurements rather precisely.

However, we can also consider (and in our case must consider) quantum mechanics with a periodic x , say $x \equiv x + L$. Now, to use the classical intuition of a particle on a circle we need $\Delta x \ll L$. But the width of an optimally localized Gaussian wave packet grows according to $\Delta x(t) \sim t/(m \Delta x(0))$ at late times t (which of course follows simply from the necessarily present momentum uncertainty). Requiring both $\Delta x(0)$ and $\Delta x(t)$ to be smaller than L implies $t \lesssim mL^2$, which is the analogue of our eq. (3.6) in this quantum mechanical model. Thus, intuitively speaking, there does indeed exist a maximal time scale beyond which it makes no sense to talk about a particle localized even approximately somewhere on a circle.

4 Systems with (aligned) multiple axions

A recurring question when addressing quantum gravity constraints on axion decay constants, is how these extend to winding trajectories [53] in the field space of axions. A particularly simple case of such trajectories arises in the presence of two axions upon Higgsing [48, 54] a linear combination of them [26]. It has been argued more generally in [14] that such theories, even if they satisfy the WGC in their Coulomb phase, could effectively

violate it (at least in some of its forms) in their Higgs phase. We would like to study next how the physical setups and constraints described above behave under Higgsing in a system with multiple B_2 forms.

Let us consider a particular system of two 2-form gauge fields B_i , with $i = 1, 2$, coupled to a single one-form field A in a Stueckelberg-like manner

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{f^2} (|H_1|^2 + |H_2|^2) - \frac{1}{4g^2} |dA + B_1 + NB_2|^2 \right]. \quad (4.1)$$

Here N is a (large) integer, $H_i = dB_i$ and, for simplicity, we have taken identical axion decay constants $f_1 = f_2 \equiv f$. The coupling between A and B_i gives a mass to the linear combination $B_a \propto B_1 + NB_2$. The orthogonal combination $B_b \propto -NB_1 + B_2$ remains (perturbatively) massless. It will feature the desired small coupling $f_{\text{eff}} \sim f/N$ and can hence be used to construct our axionic BHs. We can rewrite (4.1) in terms of these fields as

$$S_{A,B_a} = \int d^4x \sqrt{-g} \left[-\frac{1+N^2}{f^2} (|H_a|^2 + |H_b|^2) - \frac{1}{4g^2} |dA + (1+N^2)B_a|^2 \right] \quad (4.2)$$

where $B_1 = B_a - NB_b$ and $B_2 = NB_a + B_b$.

The Lagrangian (4.1) is invariant under the gauge transformations: $\{B_1, B_2, dA\} \rightarrow \{B_1 + d\Lambda_1, B_2 + d\Lambda_2, dA - d\Lambda_1 - Nd\Lambda_2\}$. For a spacetime with a non-trivial two-cycle σ , we can define the variables

$$a \equiv \frac{1}{2\pi} \int_{\sigma} dA, \quad b_i = \frac{1}{2\pi} \int_{\sigma} B_i. \quad (4.3)$$

Under gauge transformations with $\int_{\sigma} d\Lambda_i = 2\pi c_i$, these variables shift as $\{b_1, b_2, a\} \rightarrow \{b_1 + c_1, b_2 + c_2, a - c_1 - Nc_2\}$. These continuous shift symmetries are of course broken to discrete periodicities by the presence of strings.

As mentioned before, the axionic hair can only be measured if there exist strings in the spectrum charged under the B_i . We will assume the existence of strings Σ_1 and Σ_2 coupled to B_1 and B_2 with unit charge, i.e.

$$S_{\text{str}} = \int_{\Sigma_1} B_1 + \int_{\Sigma_2} B_2. \quad (4.4)$$

This spectrum determines the periodicity of the ‘‘Wilson line’’ type variables b_i . The path integral measure includes $e^{iS_{\text{str}}}$ (in turn the phase measured by Aharonov-Bohm interference experiments) which is only invariant under large gauge transformations with $c_i \in \mathbb{Z}$. Hence, the continuous shift symmetries are broken to discrete periodicities:

$$\{b_1, b_2, a\} \rightarrow \{b_1 + 1, b_2, a - 1\}, \quad \text{and} \quad \{b_1, b_2, a\} \rightarrow \{b_1, b_2 + 1, a - N\}. \quad (4.5)$$

The normalization of B_a and B_b in (4.2) has been chosen so that strings still carry integral charges, i.e.

$$S_{\text{str}} = \int_{\Sigma_1} (B_a - NB_b) + \int_{\Sigma_2} (NB_a + B_b). \quad (4.6)$$

A convenient basis for the space of large gauge transformations (4.5) is given by

$$\{b_a, b_b, a\} \rightarrow \{b_a, b_b + 1, a\}, \quad \text{and} \quad \{b_a, b_b, a\} \rightarrow \left\{ b_a - \frac{1}{1 + N^2}, b_b + \frac{N}{1 + N^2}, a + 1 \right\}, \quad (4.7)$$

where b_a and b_b are defined analogously to b_1 and b_2 . We see that b_b , the massless field, still has unit periodicity.

We have constructed an effective theory of a single light two-form B_b with small coupling f_{eff} . Following the arguments of previous sections, we would naively conclude that the strings in this setup should be extremely light, otherwise an inconsistency would arise. As we discuss next, however, there are extra ingredients in this theory that may allow for a different way out.

Consider a monopole of the theory (4.2), i.e. a particle that sources

$$\frac{1}{2\pi} \int_{S^2} dA = a. \quad (4.8)$$

For concreteness, consider an (anti-) monopole of charge $a = -1$. Because of the Stueckelberg coupling between the A and B_a , such a monopole carries a non-trivial value of b_a . From (4.2) one can see that minimum action configurations satisfy $dA \rightarrow -(1 + N^2)B_a$ as $r \rightarrow \infty$. This implies that, asymptotically, the magnetic monopole has

$$\left\{ b_a = \frac{1}{1 + N^2}, b_b = 0, a = -1 \right\}. \quad (4.9)$$

It looks as if the particle sources the ‘Wilson loop’ variable associated with the heavy field B_a , but this is actually just a matter of gauge choice. From (4.7), we see that (4.9) can be equivalently written as

$$\left\{ b_a = 0, b_b = \frac{N}{1 + N^2}, a = 0 \right\}. \quad (4.10)$$

Given that the full periodicity of b_b is unity, we see that the anti-monopole carries an axionic charge of about $1/N$ (of the maximum, which is of course the same as no axionic charge). Similar objects have been constructed in [45].

This can be intuitively understood by looking at the (b_1, b_2) field space. The monopole charge (4.9) can be rewritten as

$$\left\{ b_1 = \frac{1}{1 + N^2}, b_2 = \frac{N}{1 + N^2}, a = -1 \right\}. \quad (4.11)$$

As should be clear from figure 1, the point in (b_1, b_2) field space given by (4.11) can actually be viewed as a fractional value of the light B_b ‘Wilson line’.

Crucially, this implies that, from the perspective of the low energy effective theory, a BH with appropriate axion charge (roughly a multiple of $1/N$, e.g. K/N with $K \leq N/2$), can in principle decay. It would decay to K anti-monopoles of the 1-form theory which was used to create the small axion decay constant $f_{\text{eff}} \simeq f/N$ in the first place. However, this clearly requires the relevant monopoles to be light enough.

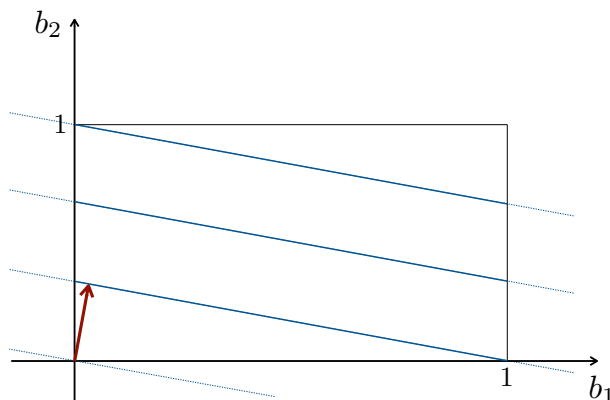


Figure 1. Field space of b_1 and b_2 . The subspace parametrized by the light axion for $N = 3$ is shown. The arrow denotes the point associated with an S^2 loop around an anti-monopole.

Let us consider an axionic BH with $b_b \approx 1/2$. If we denote the mass of the magnetic monopoles by m , these will start being emitted once the BH reaches a radius $R_m \sim 1/m$, which corresponds to a BH of mass $M_{bh} \sim M_P^2/m$. Since the BH is required to emit $N/2$ of these monopoles before its decay, we should require that $M_{bh} \gtrsim Nm/2$, that is, $m \lesssim M_P/\sqrt{N}$.

This is still a conservative estimate, since in principle both monopoles and anti-monopoles will be radiated. In the absence of axionic hair, the net axionic charge carried by these would be zero. The study of the dynamical effect that the axionic hair of the BH has in the process of (anti-)monopole emission, and how this leads to a discharge of the axionic BH is a complicated subject that we leave for future work. However, we expect that, once this effect is taken into account, the actual upper bound on the monopole masses will be significantly lower than $m \lesssim M_P/\sqrt{N}$.

To put our result in perspective, recall that in the un-Higgsed theory the existence of strings with tension $\sigma_{1,2} \sim \Lambda^2 \lesssim fM_P$ suffices to satisfy the WGC and to avoid the possible pathologies discussed in section 2. By contrast, the situation in the Higgs phase appears to be more involved. Indeed, at low energies one encounters a theory with a single light field B_b , with an effective coupling $f_{\text{eff}} \sim f/N$, while the strings are now much heavier than the expected cutoff $\Lambda_{\text{eff}} \lesssim \sqrt{f_{\text{eff}}M_P} \sim \sqrt{fM_P/N}$. Our pathology of axionic BH evaporation appears to occur. At the same time, a solution specific to this type of construction suggests itself: the possible lightness of monopoles of the gauged 1-form theory cures the problem if $m \lesssim M_P/\sqrt{N}$. Notice in particular that if $f \sim M_P$, one concludes that these monopoles have to reside precisely at the expected effective cutoff scale $m \lesssim \Lambda_{\text{eff}} \sim M_P/\sqrt{N}$. One may be tempted to conclude that the mechanisms of alignment and Higgsing are insufficient to generate an effective theory the cutoff of which evades the WGC: new low-scale physics may be forced to enter in a different sector.

Clearly, other conclusions are also possible. First of all, it may simply be impossible to construct the theory with A , B_1 and B_2 and the required large N in consistent quantum gravity models, even though the WGC does not directly forbid such a situation. Furthermore, it is conceivable that such models avoid our ‘axionic BH evaporation problem’ in

some other way, not related to light strings or light monopoles. Nevertheless, the possible way out through light monopoles looks intriguing.

5 Conclusions

In this note we have presented a novel argument that suggests a parametric bound on the tension of strings in theories with 2-form fields at weak coupling. While the exact form of the bound depends on the precise rate of evaporation of small axionic black holes, under certain assumptions we recover the constraints expected from the WGC. Our arguments are not air-tight, and allow for ways out that do not involve light strings. If the evaporation of black holes slowed down at late stages (with respect to the standard rate of Hawking radiation), or if for some reason the semiclassical description of the evaporation became invalid at a stage earlier than expected, our bounds would become weaker.

In this respect, one can ask whether paradigms such as the firewall hypothesis would drastically affect our results. While precise statements are hard to make, our preliminary conclusion is that they would not. As long as one can construct (young) black holes with axionic charge at about the critical radius R_c , our results follow exclusively from the requirement of black hole evaporation in a finite time. Of course, if firewalls were to significantly extend the axionic black hole lifetime, our results would change. If this modified lifetime were known, our analysis could easily be adapted and yield a new WGC-like bound.

Another escape to our conclusions would be the existence of long-lived remnants, which would by themselves resolve the apparent paradox we have discussed. The absence of remnants has been extensively used to motivate the WGC as well as other ‘folk theorems’ of quantum gravity, such as the absence of global symmetries. The latter conjecture is supported by convincing arguments against remnants that carry arbitrary conserved charges [42], but these do not directly apply to remnants that merely support a Wilson-line b (see nevertheless [41]). In this respect, our no-remnant (or equivalently topology change) hypothesis is a stronger requirement. This is not too surprising, the WGC is a much stronger statement than the simple no-global symmetry hypothesis, and is hence much harder to prove. In fact, we do not know of fully convincing arguments for the WGC based exclusively on semi-classical analysis (see however [55]).

We have further applied our arguments to a setup in which a light 2-form field with small effective decay constant f_{eff} arises upon Higgsing an ‘aligned’ linear combination of two 2-forms. Interestingly, rather than constraining the string spectrum, the evaporation of axionic black holes suggests the appearance of light states in a different sector, namely light monopoles which carry fractional amounts of axion hair. Given the prominent role played by similar models in the paradigm of large field (‘natural’) inflation, it would be interesting to understand whether our conclusions can be confirmed by alternative considerations and/or for objects of different dimensionality.

Concerning objects of different dimensionality, one encounters some obvious obstacles. First, in four or less flat space-time dimensions black holes are the only objects with horizon and a corresponding Hawking evaporation process. So our constraint on 2-forms does not

generalize in any obvious way. One might be tempted to consider BTZ black holes in $d = 3$ AdS and try to constrain the corresponding Wilson line and hence the 1-form gauge theory, but also here our logic does not go through straightforwardly. Crucially, we can not follow the evaporation of a macroscopic black hole all the way to empty space. Turning to space-time dimensions above four, we can in principle consider black branes in flat space, but then we would face the Gregory-Laflamme instability. Other higher dimensional black objects such as black rings, whose horizons have richer topologies ($S^1 \times S^{d-3}$), will also be interesting to consider. We leave the study of how to adopt our logic to these cases to future work.

Possibly, the right way to extend and generalize our argument is not via other black objects but via space-time topology changes. Indeed, the main idea we have used is the disappearance of an (effectively) non-trivial 2-cycle of our space time - the 2-cycle around the black hole horizon. Now, one might instead consider a geometry where two copies of \mathbb{R}^d are linked by a throat with cross-section S^p ($p \leq d - 2$), i.e., a wormhole, possibly fibered over an appropriate space of dimension $d - p - 2$. Dynamically, this wormhole should collapse in pure gravity. But now, one may consider putting a non-zero p -form ‘Wilson-line’ on its non-trivial cycle and demand a consistent dynamical transition to the trivial topology of two separate \mathbb{R}^d spaces. It is conceivable that, following the logic of section 2.1, one can recover the WGC bound. We leave a more detailed study of such possibilities to future work.

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