

Integrating out heavy particles with functional methods: a simplified framework

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ABSTRACT: We present a systematic procedure to obtain the one-loop low-energy effective Lagrangian resulting from integrating out the heavy fields of a given ultraviolet theory. We show that the matching coefficients are determined entirely by the hard region of the functional determinant involving the heavy fields. This represents an important simplification with respect to the conventional matching approach, where the full and effective theory contributions have to be computed separately and a cancellation of the infrared divergent parts has to take place. We illustrate the method with a descriptive toy model and with an extension of the Standard Model with a heavy real scalar triplet. A comparison with other schemes that have been put forward recently is also provided.

KEYWORDS: Effective field theories, Beyond Standard Model

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1 Introduction

New physics searches at the LHC rely, namely, in the discovery of a new spectrum of particles with masses much larger than the electroweak scale though it is being seen that they can be rather elusive. Our present understanding of the laws of physics tells us that whether these are supersymmetric states or an extended scalar sector, for instance, their role at the electroweak scale should be weighted by inverse powers of their masses. This is the main tenet behind our concept and compelling use of effective field theories in particle physics: we obtain the low-energy theory by integrating out the heavier spectrum in the, up to now model-dependent, ultraviolet completion of the former. In this way we determine the marks of the underlying theory at higher scales on the low-energy couplings, i.e. Wilson coefficients, of the effective field theory (EFT). Upon comparison with the electroweak scale phenomenology we should be able to obtain information on new physics scenarios. This framework has pervaded the last fifty years of research in particle physics.

Although the rationale and the procedure has been well developed long ago in the literature (see for instance [1, 2]), the integration at next-to-leading order in the upper theory, that is to say at one loop, is undergoing lately an intense debate [3–8] that, as we put forward in this paper, still allows for simpler alternatives. There are two techniques to obtain the Wilson coefficients of the EFT. The most employed one amounts to matching the diagrammatic computation of given Green Functions with light particle external legs in the full theory, where heavy states can appear in virtual lines, and in the EFT, at energies where the EFT can describe the dynamics of the light particles as an expansion in inverse powers of the heavy particle mass scale. Alternatively one can perform the functional integration of

the heavier states without being concerned with specific Green Functions, and later extract the local contributions that are relevant for the description of the low-energy dynamics of the light fields. This last methodology was applied, for example, in refs. [9, 10], to obtain the non-decoupling effects of a heavy Higgs in the Standard Model (SM). The path integral formulation has obvious advantages over the matching procedure as, for instance, one does not need to handle Feynman diagrams nor symmetry factors, and one obtains directly the whole set of EFT operators together with their matching conditions, i.e. no prior knowledge about the specifics of the EFT operator structure, symmetries, etc., is required.

One of the issues recently arisen involves the widely used technique to perform the functional integration set up more than thirty years ago by the works of Aitchison and Fraser [11–14], Chan [15, 16], Gaillard [17] and Cheyette [18]. As implemented by refs. [3, 4], this technique did not include all the one-loop contributions from the integration, in particular those where heavy and light field quantum fluctuations appear in the same loop. This fact was noticed in ref. [5], and fixed later on in refs. [7, 8], by the use of variants of the functional approach which require additional ingredients in order to subtract the parts of the heavy-light loops which are already accounted for by the one-loop EFT contribution. Here we would like to introduce a more direct method to obtain the one-loop effective theory that builds upon the works of refs. [9, 10], and that uses the technique of “expansion by regions” [19–21] to read off the one-loop matching coefficients from the full theory computation, thus bypassing the need of subtracting any infrared contribution. In short, the determination of the one-loop EFT in the approach we propose reduces to the calculation of the *hard* part of the determinant of $\tilde{\Delta}_H$, where $\tilde{\Delta}_H$ arises from the diagonalization of the quadratic term in the expansion of the full theory Lagrangian around the classical field configurations, and the determinant is just the result of the Gaussian integration over the heavy quantum fluctuations. In this way, the terms that mix light and heavy spectra inside the loop get disentangled by means of a field transformation in the path integral that brings the quadratic fluctuation into diagonal form: the part involving only the light quantum fields remains untouched by the transformation and all heavy particle effects in the loops are shifted to the modified heavy quadratic form $\tilde{\Delta}_H$. This provides a conceptually simple and straightforward technique to obtain all the one-loop local EFT couplings from an underlying theory that can contain arbitrary interactions between the heavy and the light degrees of freedom.

The contents of the paper are the following. The general outline of the method is given in section 2, where we describe the transformation that diagonalizes the quadratic fluctuation which defines $\tilde{\Delta}_H$, and then discuss how to extract the contributions from $\tilde{\Delta}_H$ that are relevant for determining the one-loop EFT. In section 3 we compare our procedure with those proposed recently by [3, 7] and [4, 8]. The virtues of our method are better seen through examples: first we consider a simple scalar toy model in section 4, where we can easily illustrate the advantages of our procedure with respect the conventional matching approach; then we turn to an extension of the SM with a heavy real scalar triplet, that has been used as an example in recent papers. We conclude with section 5. Additional material concerning the general formulae for dimension-six operators, and the expression of the fluctuation operator in the SM case is provided in the appendices.

2 The method

We outline in this section the functional method to determine the EFT Lagrangian describing the dynamics of light particles at energies much smaller than m_H , the typical mass of a heavy particle, or set of particles, that reproduces the full-theory results at the one-loop level. The application of the method to specific examples is postponed to section 4.

Let us consider a general theory whose field content can be split into heavy (η_H) and light (η_L) degrees of freedom, that we collect generically in $\eta = (\eta_H, \eta_L)$. For charged degrees of freedom, the field and its complex conjugate enter as separate components in η_H and η_L . In order to obtain the one-loop effective action, we split each field component into a background field configuration, $\hat{\eta}$, which satisfy the classical equations of motion (EOM), and a quantum fluctuation η , i.e. we write $\eta \rightarrow \hat{\eta} + \eta$. Diagrammatically, the background part corresponds to tree lines in Feynman graphs while lines inside loops arise from the quantum fields; this means that terms higher than quadratic in the quantum fields yield vertices that can only appear in diagrams at higher loop orders. Therefore, at the one-loop level one has to consider only the Lagrangian up to terms quadratic in η :

$$\mathcal{L} = \mathcal{L}^{\text{tree}}(\hat{\eta}) + \mathcal{L}^{(\eta^2)} + \mathcal{O}(\eta^3) . \tag{2.1}$$

The zeroth order term, $\mathcal{L}^{\text{tree}}$, depends only on the classical field configurations and yields the tree-level effective action. At energies much lower than the mass of the heavy fields, the background heavy fields $\hat{\eta}_H$ can be eliminated from the tree-level action by using their EOM. The linear term in the expansion of \mathcal{L} around the background fields is, up to a total derivative, proportional to the EOM evaluated at $\eta = \hat{\eta}$, and thus vanishes. From the quadratic piece

$$\mathcal{L}^{(\eta^2)} = \frac{1}{2} \eta^\dagger \left. \frac{\partial^2 \mathcal{L}}{\partial \eta^* \partial \eta} \right|_{\eta=\hat{\eta}} \eta \equiv \frac{1}{2} \eta^\dagger \mathcal{O} \eta , \tag{2.2}$$

we identify the fluctuation operator \mathcal{O} , with generic form

$$\mathcal{O} = \begin{pmatrix} \Delta_H & X_{LH}^\dagger \\ X_{LH} & \Delta_L \end{pmatrix} , \tag{2.3}$$

and which depends only on the classical fields $\hat{\eta}$.

The one-loop effective action thus derives from the path integral

$$e^{iS} = \mathcal{N} \int \mathcal{D}\eta_L \mathcal{D}\eta_H \exp \left[i \int dx \mathcal{L}^{(\eta^2)} \right] , \tag{2.4}$$

which can be obtained by Gaussian integration. Our aim is to compute the one-loop heavy particle effects in the Green functions of the light fields as an expansion in the heavy mass scale m_H . In terms of Feynman diagrams, the latter corresponds to computing all one-loop diagrams involving heavy lines and expanding them in $1/m_H$. The latter can be formally achieved by doing the functional integration over the fields η_H . However, the presence of mixing terms among heavy and light quantum fields in $\mathcal{L}^{(\eta^2)}$ (equivalently, of

one-loop diagrams with both heavy and light lines inside the loop), makes it necessary to first rewrite the fluctuation operator in eq. (2.3) in an equivalent block-diagonal form. A way of achieving this is by performing shifts (with unit Jacobian determinant) in the quantum fields, which can be done in different ways. We choose a field transformation that shifts the information about the mixing terms X_{LH} in the fluctuation operator into a redefinition of the heavy-particle block Δ_H , while leaving Δ_L untouched. This has the advantage that all heavy particle effects in the one-loop effective action are thus obtained through the computation of the determinant that results from the path integral over the heavy fields. This shifting procedure was actually used in refs. [9, 10] for integrating out the Higgs field in the SU(2) gauge theory and in the SM. An alternative shift, which is implicitly used in ref. [7], will be discussed in section 3.

The explicit form of the field transformation that brings \mathcal{O} into the desired block-diagonal form reads

$$P = \begin{pmatrix} I & 0 \\ -\Delta_L^{-1} X_{LH} & I \end{pmatrix}, \tag{2.5}$$

and one immediately obtains

$$P^\dagger \mathcal{O} P = \begin{pmatrix} \tilde{\Delta}_H & 0 \\ 0 & \Delta_L \end{pmatrix}, \tag{2.6}$$

with

$$\tilde{\Delta}_H = \Delta_H - X_{LH}^\dagger \Delta_L^{-1} X_{LH}. \tag{2.7}$$

The functional integration over the heavy fields η_H can now be carried out easily,

$$e^{iS} = \left(\det \tilde{\Delta}_H\right)^{-c} \mathcal{N} \int \mathcal{D}\eta_L \exp \left[i \int dx \frac{1}{2} \eta_L^\dagger \Delta_L \eta_L \right], \tag{2.8}$$

with $c = 1/2, -1$ depending on the bosonic or fermionic nature of the heavy fields. For simplicity, we assume that all degrees of freedom in the heavy sector are either bosons or fermions. In the case of mixed statistics, one needs to further diagonalize $\tilde{\Delta}_H$ to decouple the bosonic and fermionic blocks. The remaining Gaussian integration in eq. (2.8) reproduces the one-loop contributions with light particles running inside the loop, and heavy fields can appear only as tree-level lines through the dependence of Δ_L in $\hat{\eta}_H$. We thus define the part of the one-loop effective action coming from loops involving heavy fields as

$$S_H = i c \ln \det \tilde{\Delta}_H. \tag{2.9}$$

In order to compute the determinant of $\tilde{\Delta}_H$ we use standard techniques developed in the literature [15, 22]. First it is rewritten as

$$S_H = i c \text{Tr} \ln \tilde{\Delta}_H, \tag{2.10}$$

where Tr denotes the full trace of the operator, also in coordinate space. It is convenient for our purposes to rewrite the functional trace using momentum eigenstates defined in d dimensions as

$$\begin{aligned}
 S_H &= i c \text{tr} \int \frac{d^d p}{(2\pi)^d} \langle p | \ln \tilde{\Delta}_H | p \rangle \\
 &= i c \text{tr} \int d^d x \int \frac{d^d p}{(2\pi)^d} e^{-ipx} \ln \left(\tilde{\Delta}_H(x, \partial_x) \right) e^{ipx} \\
 &= i c \text{tr} \int d^d x \int \frac{d^d p}{(2\pi)^d} \ln \left(\tilde{\Delta}_H(x, \partial_x + ip) \right) \mathbb{1}.
 \end{aligned} \tag{2.11}$$

The derivatives in $\tilde{\Delta}_H$ yields factors of ip upon acting on the exponentials.¹ The symbol tr denotes the trace over internal degrees of freedom only. Since $\tilde{\Delta}_H$ contains the kinetic term of the heavy fields, in the case of scalar fields it has the generic form

$$\tilde{\Delta}_H = -\hat{D}^2 - m_H^2 - U, \tag{2.12}$$

with \hat{D}_μ denoting the covariant derivative for the heavy fields with background gauge fields. Performing the shift $\partial_x \rightarrow \partial_x + ip$ we find

$$S_H = \frac{i}{2} \text{tr} \int d^d x \int \frac{d^d p}{(2\pi)^d} \ln \left(p^2 - m_H^2 - 2ip\hat{D} - \hat{D}^2 - U(x, \partial_x + ip) \right) \mathbb{1}. \tag{2.13}$$

For fermions, the same formula, eq. (2.13), applies but with an overall minus sign and with U replaced by

$$U_{\text{ferm.}} = -\frac{i}{2} \sigma^{\mu\nu} [\hat{D}_\mu, \hat{D}_\nu] - i [\hat{\not{D}}, \Sigma_e] + i \{ \hat{\not{D}}, \Sigma_o \} + 2m_H \Sigma_e + \Sigma (\Sigma_e - \Sigma_o). \tag{2.14}$$

Here $\Sigma \equiv \Sigma_e + \Sigma_o$ is defined by $\tilde{\Delta}_H = i\hat{\not{D}} - m_H - \Sigma$, and Σ_e (Σ_o) contains an even (odd) number of gamma matrices. Finally, we can Taylor expand the logarithm to get

$$S_H = \mp \frac{i}{2} \int d^d x \sum_{n=1}^{\infty} \frac{1}{n} \int \frac{d^d p}{(2\pi)^d} \text{tr} \left\{ \left(\frac{2ip\hat{D} + \hat{D}^2 + U(x, \partial_x + ip)}{p^2 - m_H^2} \right)^n \mathbb{1} \right\}, \tag{2.15}$$

where we have dropped an irrelevant constant term, and the negative (positive) global sign corresponds to the integration of boson (fermion) heavy fields.

The effective action eq. (2.15) generates all one-loop amplitudes with at least one heavy particle propagator in the loop. One-loop diagrams with n heavy propagators are reproduced from the n -th term in the expansion of eq. (2.15). In addition the diagram can contain light propagators, that arise upon expanding the term $X_{LH}^\dagger \Delta_L^{-1} X_{LH}$ in $\tilde{\Delta}_H$ using

$$\Delta_L^{-1} = \sum_{n=0}^{\infty} (-1)^n \left(\tilde{\Delta}_L^{-1} X_L \right)^n \tilde{\Delta}_L^{-1}, \tag{2.16}$$

¹Note that $\tilde{\Delta}_H$ can also depend in ∂_x^\dagger . Transpose derivatives are defined from the adjoint operator, which acts on the function at the left, and can be replaced by $-\partial_x$, the difference being a total derivative term. The identity $\mathbb{1}$ in eq. (2.11) serves as a reminder that derivatives at the rightmost disappear after acting on the exponential.

which corresponds to the Neumann series expansion of Δ_L^{-1} , and we have made the separation $\Delta_L = \tilde{\Delta}_L + X_L$, with $\tilde{\Delta}_L$ corresponding to the fluctuations coming from the kinetic terms, i.e. $\tilde{\Delta}_L^{-1}$ is the light field propagator. From the definition of the fluctuation operator \mathcal{O} , eq. (2.3), the terms in $\tilde{\Delta}_L$ are part of the diagonal components of \mathcal{O} . At the practical level, for the calculation of Δ_L^{-1} using eq. (2.16) it is simpler to define $\tilde{\Delta}_L$ directly as the whole diagonal of \mathcal{O} .

Loops with heavy particles receive contributions from the region of *hard* loop momenta $p \sim m_H$, and from the *soft* momentum region, where the latter is set by the low-energy scales in the theory, either $p \sim m_L$ or any of the light-particle external momenta, $p_i \ll m_H$. In dimensional regularization the two contributions can be computed separately by using the so-called “expansion by regions” [19–21]. In this method the contribution of each region is obtained by expanding the integrand into a Taylor series with respect to the parameters that are small there, and then integrating every region over the full d -dimensional space of the loop momenta. In the hard region, all the low-energy scales are expanded out and only m_H remains in the propagators. The resulting integrand yields local contributions in the form of a polynomial in the low-energy momenta and masses, with factors of $1/m_H$ to adjust the dimensions. This part is therefore fully determined by the short-distance behaviour of the full theory and has to be included into the EFT Lagrangian in order to match the amplitudes in the full and effective theories. Indeed, the coefficients of the polynomial terms from the hard contribution of a given (renormalized) amplitude provide the one-loop matching coefficients of corresponding local terms in the effective theory. This can be understood easily since the soft part of the amplitude results upon expanding the vertices and propagators according to $p \sim m_L \ll m_H$, with p the loop momentum. This expansion, together with the one-loop terms with light particles that arise from the Gaussian integral of Δ_L in eq. (2.8), yields the same one-loop amplitude as one would obtain using the Feynman rules of the effective Lagrangian for the light fields obtained by tree-level matching, equivalently the Feynman rules from $\mathcal{L}^{\text{tree}}$ in eq. (2.1) where the background heavy field $\hat{\eta}_H$ has been eliminated in favour of $\hat{\eta}_L$ using the classical EOM. Therefore, in the difference of the full-theory and EFT renormalized amplitudes at one-loop only the hard part of the full-theory amplitude remains, and one can read off the one-loop matching coefficients directly from the computation of the latter. Let us finally note that in the conventional matching approach, the same infrared regularization has to be used in the full and EFT calculations, in order to guarantee that the infrared behaviour of both theories is identical. This is of course fulfilled in the approach suggested here, since the one-loop EFT amplitude is defined implicitly by the full theory result. Likewise, the ultraviolet (UV) divergences of the EFT are determined by UV divergences in the soft part, that are regulated in d dimensions in our approach. For the renormalization of the amplitudes, we shall use the $\overline{\text{MS}}$ subtraction scheme.

Translated into the functional approach, the preceding discussion implies that the EFT Lagrangian at one-loop is then determined as

$$\int d^d x \mathcal{L}_{\text{EFT}}^{\text{1loop}} = S_H^{\text{hard}}, \tag{2.17}$$

where S_H^{hard} , containing only the hard part of the loops, can be obtained from the representation (2.15) by expanding the integrand in the hard loop-momentum limit, $p \sim m_H \gg m_L, \partial_x$. In order to identify the relevant terms in this expansion, it is useful to introduce the counting

$$p_\mu, m_H \sim \zeta, \tag{2.18}$$

and determine the order ζ^{-k} , $k > 0$, of each term in the integrand of eq. (2.15). For a given order in ζ only a finite number of terms in the expansion contributes because U is at most $\mathcal{O}(\zeta)$ and the denominator is $\mathcal{O}(\zeta^2)$.² For instance, to obtain the dimension-six effective operators, i.e. those suppressed by $1/m_H^2$, it is enough to truncate the expansion up to terms of $\mathcal{O}(\zeta^{-2})$, which means computing U up to $\mathcal{O}(\zeta^{-4})$ (recall that $d^4p \sim \zeta^4$). Though it was phrased differently, this prescription is effectively equivalent to the one used in refs. [9, 10] to obtain the non-decoupling effects (i.e. the $\mathcal{O}(m_H^0)$ terms) introduced by a SM-like heavy Higgs.

Finally we recall that, although the covariance of the expansion in eq. (2.15) is not manifest, the symmetry of the functional trace guarantees that the final result can be rearranged such that all the covariant derivatives appear in commutators [16, 23]. As a result, one can always rearrange the expansion of eq. (2.15) in a manifestly covariant way in terms of traces containing powers of U , field-strength tensors and covariant derivatives acting on them. As noted in refs. [17, 22, 23], this rearrangement can be easily performed when U does not depend on derivatives, as it is the case when only heavy particles enter in the loop.³ However, for the case where $U = U(x, \partial_x + ip)$, as it happens in general in theories with heavy-light loops, the situation is more involved and the techniques developed in refs. [17, 22, 23] cannot be directly applied. In this more general case it is convenient to separate U into momentum-dependent and momentum-independent pieces, i.e. $U = U_H(x) + U_{LH}(x, \partial_x + ip)$ which, at the diagrammatic level, corresponds to a separation into pure heavy loops and heavy-light loops. This separation presents two major advantages: first, the power counting for U_H and U_{LH} is generically different, with U_H at most $\mathcal{O}(\zeta)$ and U_{LH} at most $\mathcal{O}(\zeta^0)$, both for bosons and fermions, which allows for a different truncation of the series in eq. (2.15) for the terms involving only pure heavy contributions and those involving at least one power of U_{LH} . Second, universal expansions of eq. (2.15) in a manifestly covariant form for $U = U_H(x)$ have been derived in the literature up to $\mathcal{O}(\zeta^{-2})$, i.e. for the case of dimension-six operators [3, 22, 24, 25], that we reproduce in eq. (A.2). The evaluation of the remaining piece, corresponding to terms containing at least one power of U_{LH} can be done explicitly from eq. (2.17).

²The part of the operator U coming from Δ_H arises from interaction terms with at least three fields. If all three fields are bosons, the dimension-4 operator may contain a dimensionful parameter $\sim \zeta$ or a derivative, giving rise to a term in U of $\mathcal{O}(\zeta)$. If two of the fields are fermions the operator is already of dimension 4 and then $\Sigma \sim \zeta^0$, which yields a contribution in U of $\mathcal{O}(\zeta)$ upon application of eq. (2.14). Contributions from $X_{LH}^\dagger \Delta_L^{-1} X_{LH}$, in the following referred as heavy-light, appear from the product of two interaction terms and a light-field propagator and hence they generate terms in U of $\mathcal{O}(\zeta^0)$.

³With the exception of theories with massive vector fields and derivative couplings among two heavy and one light fields.

Let us end the section by summarizing the steps required to obtain the one-loop matching coefficients in our method:

1. We collect all field degrees of freedom in \mathcal{L} , light and heavy, in a field multiplet $\eta = (\eta_H, \eta_L)$, where η_i and $(\eta_i)^*$ must be written as separate components for charged fields. We split the fields into classical and quantum part, i.e $\eta \rightarrow \hat{\eta} + \eta$, and identify the fluctuation operator \mathcal{O} from the second order variation of \mathcal{L} with respect to η^* and η evaluated at the classical field configuration, see eqs. (2.2) and (2.3),

$$\mathcal{O}_{ij} = \left. \frac{\partial^2 \mathcal{L}}{\partial \eta_i^* \partial \eta_j} \right|_{\eta = \hat{\eta}}. \tag{2.19}$$

2. We then consider $U(x, \partial_x)$, given in eqs. (2.12) and (2.14), with $\tilde{\Delta}_H$ defined in eq. (2.7) in terms of the components of \mathcal{O} . Derivatives in U must be shifted as $\partial_x \rightarrow \partial_x + ip$. The computation of U requires the inversion of Δ_L : a general expression for the latter is provided in eq. (2.16). The operator $U(x, \partial_x + ip)$ has to be expanded up to a given order in ζ , with the counting given by $p, m_H \sim \zeta \gg m_L, \partial_x$. For deriving the dimension-six EFT operators, the expansion of U must be taken up to $\mathcal{O}(\zeta^{-4})$.
3. The final step consists on the evaluation of the traces of $U(x, \partial_x + ip)$ in eq. (2.15) up to the desired order — $\mathcal{O}(\zeta^{-2})$ for the computation of the one-loop dimension-six effective Lagrangian —. For this computation it is convenient to make the separation $U(x, \partial_x + ip) = U_H(x) + U_{LH}(x, \partial_x + ip)$ and apply the standard formulas for the traces of $U_H(x)$, see eq. (A.2). The remaining contributions consist in terms involving at least one power of $U_{LH}(x, \partial_x + ip)$: a general formula for the case of dimension-six operators can be found in eq. (A.3). Their computation only requires trivial integrals of the form:

$$\int \frac{d^d p}{(2\pi)^d} \frac{p_{\mu_1} \dots p_{\mu_{2k}}}{(p^2)^\alpha (p^2 - m_H^2)^\beta} = \frac{(-1)^{\alpha+\beta+k} i}{(4\pi)^{\frac{d}{2}}} \frac{\Gamma(\frac{d}{2} + k - \alpha) \Gamma(-\frac{d}{2} - k + \alpha + \beta)}{\Gamma(\beta) \Gamma(\frac{d}{2} + k)} \times \frac{g_{\mu_1 \dots \mu_{2k}}}{2^k} m_H^{d+2k-2\alpha-2\beta}, \tag{2.20}$$

where $g_{\mu_1 \dots \mu_{2k}}$ is the totally symmetric tensor with $2k$ indices constructed from $g_{\mu\nu}$ tensors.

Terms containing open covariant derivatives, i.e. derivatives acting only at the right-most of the traces, should be kept throughout the computation and will either vanish or combine in commutators, yielding gauge-invariant terms with field strength tensors. A discussion about such terms can be found in appendix A.

3 Comparison with previous approaches

In ref. [7], a procedure to obtain the one-loop matching coefficients also using functional integration has been proposed. We wish to highlight here the differences of that method, in the following referred as HLM, with respect to the one presented in this manuscript.

The first difference is how ref. [7] disentangles contributions from heavy-light loops from the rest. In the HLM method the determinant of the fluctuation operator \mathcal{O} which defines the complete one-loop action S is split using an identity (see their appendix B) that is formally equivalent in our language to performing a field transformation of the form

$$P_{\text{HLM}} = \begin{pmatrix} I & -\Delta_H^{-1} X_{LH}^\dagger \\ 0 & I \end{pmatrix}, \quad (3.1)$$

that block-diagonalizes the fluctuation operator as:

$$P_{\text{HLM}}^\dagger \mathcal{O} P_{\text{HLM}} = \begin{pmatrix} \Delta_H & 0 \\ 0 & \tilde{\Delta}_L \end{pmatrix}, \quad (3.2)$$

where now

$$\tilde{\Delta}_L = \Delta_L - X_{LH} \Delta_H^{-1} X_{LH}^\dagger. \quad (3.3)$$

The functional determinant is then separated in the HLM framework into two terms: the determinant of Δ_H , that corresponds to the loops with only heavy particles, and the determinant of $\tilde{\Delta}_L$, containing both the loops with only light propagators and those with mixed heavy and light propagators. The former contributes directly to U_H , and provides part of the one-loop matching conditions (namely those denoted as “heavy” in ref. [7]), upon using the universal formula valid for U not depending in derivatives, eq. (A.2), up to a given order in the expansion in $1/m_H$. On the other hand, to obtain the matching conditions that arise from $\tilde{\Delta}_L$ (called “mixed” contributions in the HLM terminology), one has to subtract those contributions already contained in the one-loop terms from the EFT theory matched at tree-level. To perform that subtraction without computing both the determinant of $\tilde{\Delta}_L$ and that of the quadratic fluctuation of $\mathcal{L}_{\text{EFT}}^{\text{tree}}$, HLM argues that one has to subtract to the heavy propagators that appear in the computation of $\det \tilde{\Delta}_L$ the expansion of the heavy propagator to a given order in the limit $m_H \rightarrow \infty$. According to HLM, the subtracted piece builds up the terms (“local counterparts”) that match the loops from $\mathcal{L}_{\text{EFT}}^{\text{tree}}$. These “local counterparts” have to be identified for each order in the EFT, and then dropped prior to the evaluation of the functional traces. This prescription resembles the one used in ref. [25] to obtain the one-loop effective Lagrangian from integrating out a heavy scalar singlet added to the SM.

While we do not doubt the validity of the HLM method, which the authors of ref. [7] have shown through specific examples, we believe the framework presented in this manuscript brings some important simplifications. Let us note first that in the method of ref. [7], contributions from heavy-light loops are incorporated into $\det \tilde{\Delta}_L$, which results from the functional integration over the light fields. If the light sector contains both bosonic and fermionic degrees of freedom that interact with the heavy sector (as it is the case in most extensions of the SM), a further diagonalization of $\tilde{\Delta}_L$ into bosonic and fermionic blocks is required in order to perform the Gaussian integral over the light fields. That step is avoided in our approach, where we shift all heavy particle effects into $\tilde{\Delta}_H$ and we only

need to perform the path integral over the heavy fields. Secondly, our method provides a closed formula (up to trivial integrations which depend on the structure of U_{LH}) valid for any given model, from which the matching conditions of all EFT operators of a given dimension are obtained. In this sense it is more systematic than the subtraction prescription of the HLM method, which requires some prior identification of the subtraction terms for the heavy particle propagators in the model of interest. Furthermore, in the HLM procedure the light particle mass in the light field propagators is not expanded out in the computation of the functional traces, and intermediate results are therefore more involved. In particular, non-analytic terms in the light masses can appear in intermediate steps of the calculation, and cancellations of such terms between different contributions have to occur to get the infrared-finite matching coefficients at one loop. Given the amount of algebra involved in the computation of the functional traces, automation is a prerequisite for integrating out heavy particles in any realistic model. In our method, such automation is straightforward (and indeed has been used for the heavy real scalar triplet example given in section 4). From the description of ref. [7], it seems to us that it is harder to implement the HLM method into an automated code that does not require some manual intervention.

An alternative framework to obtain the one-loop effective Lagrangian through functional integration, that shares many similarities with that of HLM, has been suggested in ref. [8]. The authors of ref. [8] have also introduced a subtraction procedure that involves the truncation of the heavy particle propagator. Their result for the dimension-6 effective Lagrangian in the case that the heavy-light quadratic fluctuation is derivative-independent has been written in terms of traces of manifestly gauge-invariant operators depending on the quadratic fluctuation $U(x)$, times coefficients where the EFT contributions have been subtracted. Examples on the calculation of such subtracted coefficients, which depend on the ultraviolet model, are provided in this reference. The approach is however limited, as stated by the authors, by the fact that it cannot be applied to cases where the heavy-light interactions contain derivative terms. That is the case, for instance, in extensions of the SM where the heavy fields have interactions with the SM gauge bosons (see the example we provide in subsection 4.2). Let us also note that the general formula provided in the framework of ref. [8] is written in terms of the components of the original fluctuation operator where no diagonalization to separate heavy- and light-field blocks has been performed. This implies that its application to models with mixed statistics in the part of the light sector that interacts with the heavy one, and even to models where the heavy and light degrees of freedom have different statistics, must require additional steps that are not discussed in ref. [8].

4 Examples

In this section we perform two practical applications of the framework that we have developed above. The first one is a scalar toy model simple enough to allow a comparison of our method with the standard matching procedure. Through this example we can also illustrate explicitly that matching coefficients arise from the hard region of the one-loop amplitudes in the full theory. The second example corresponds to a more realistic case where one integrates out a heavy real scalar triplet that has been added to the SM.

4.1 Scalar toy model

Let us consider a model with two real scalar fields, φ with mass m and ϕ with mass M , whose interactions are described by the Lagrangian

$$\mathcal{L}(\varphi, \phi) = \frac{1}{2} (\partial_\mu \phi \partial^\mu \phi - M^2 \phi^2) + \frac{1}{2} (\partial_\mu \varphi \partial^\mu \varphi - m^2 \varphi^2) - \frac{\kappa}{4!} \varphi^4 - \frac{\lambda}{3!} \varphi^3 \phi. \quad (4.1)$$

Assuming $M \gg m$ we wish to determine the effective field theory resulting from integrating out the ϕ field: $\mathcal{L}_{\text{EFT}}(\hat{\varphi})$. We perform the calculation up to and including $1/M^2$ -suppressed operators in the EFT. Within this model this implies that we have to consider up to six-point Green functions. This same model has also been considered in ref. [7].

At tree level we solve for the equation of motion of the ϕ field and we obtain

$$\hat{\phi} = -\frac{\lambda}{6M^2} \hat{\varphi}^3 + \mathcal{O}(M^{-4}), \quad (4.2)$$

that, upon substituting in eq. (4.1), gives the tree-level effective Lagrangian

$$\mathcal{L}_{\text{EFT}}^{\text{tree}} = \frac{1}{2} (\partial_\mu \hat{\varphi} \partial^\mu \hat{\varphi} - m^2 \hat{\varphi}^2) - \frac{\kappa}{4!} \hat{\varphi}^4 + \frac{\lambda^2}{72M^2} \hat{\varphi}^6. \quad (4.3)$$

To proceed at one loop we use the background field method as explained in section 2: $\phi \rightarrow \hat{\phi} + \phi$ and $\varphi \rightarrow \hat{\varphi} + \varphi$. We have $\eta = (\phi, \varphi)^\top$ and we consider the same counting as in eq. (2.18): $p_\mu, M \sim \zeta$. The fluctuation operator in eq. (2.3) is given by

$$\begin{aligned} \Delta_H &= -\partial^2 - M^2, \\ \Delta_L &= -\partial^2 - m^2 - \frac{\kappa}{2} \hat{\varphi}^2 - \lambda \hat{\varphi} \hat{\phi}, \\ X_{LH} &= -\frac{\lambda}{2} \hat{\varphi}^2, \end{aligned} \quad (4.4)$$

that only depends on the classical field configurations. In order to construct $\tilde{\Delta}_H(x, \partial_x + ip)$ in eq. (2.7) we need to determine $\Delta_L^{-1}(x, \partial_x + ip)$ up to, and including, terms of order ζ^{-4} :

$$\begin{aligned} \Delta_L(x, \partial_x + ip) &= p^2 - m^2 - 2i p_\mu \partial^\mu - \partial^2 - \frac{\kappa}{2} \hat{\varphi}^2 - \lambda \hat{\varphi} \hat{\phi}, \\ \Delta_L^{-1}(x, \partial_x + ip) &= \frac{1}{p^2} \left(1 + \frac{m^2}{p^2} \right) + \frac{1}{p^4} \left(2i p_\mu \partial^\mu + \partial^2 + \frac{\kappa}{2} \hat{\varphi}^2 \right) - 4 \frac{p_\mu p_\nu}{p^6} \partial^\mu \partial^\nu + \mathcal{O}(\zeta^{-5}). \end{aligned} \quad (4.5)$$

Using this result we get $U(x, \partial_x + ip)$ from eq. (2.12)

$$\begin{aligned} U(x, \partial_x + ip) &= \frac{\lambda^2}{4} \hat{\varphi}^2 \left[\frac{1}{p^2} \left(1 + \frac{m^2}{p^2} \right) + \frac{1}{p^4} \left(2i p_\mu \partial^\mu + \partial^2 + \frac{\kappa}{2} \hat{\varphi}^2 \right) - 4 \frac{p_\mu p_\nu}{p^6} \partial^\mu \partial^\nu \right] \hat{\varphi}^2 \\ &\quad + \mathcal{O}(\zeta^{-5}). \end{aligned} \quad (4.6)$$

Inserting this operator in eq. (2.15), we notice that at the order we are considering only the $n = 1$ term contributes, with

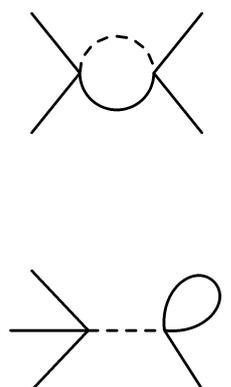
$$\mathcal{L}_{\text{EFT}}^{\text{1loop}} = -\frac{i}{2} \int \frac{d^d p}{(2\pi)^d} \frac{U(x, \partial_x + ip)}{p^2 - M^2}. \quad (4.7)$$

The momentum integration can be readily performed: in the $\overline{\text{MS}}$ regularization scheme with $\mu = M$ we finally obtain

$$\mathcal{L}_{\text{EFT}}^{\text{1loop}} = \frac{\lambda^2}{16(16\pi^2)} \left[2 \left(1 + \frac{m^2}{M^2} \right) \hat{\varphi}^4 - \frac{1}{M^2} \hat{\varphi}^2 \partial^2 \hat{\varphi}^2 + \frac{\kappa}{M^2} \hat{\varphi}^6 \right]. \quad (4.8)$$

Let us recover now this result through the usual matching procedure between the full theory $\mathcal{L}(\varphi, \phi)$ in eq. (4.1) and the effective theory without the heavy scalar field ϕ . Our goal is to further clarify the discussion given in section 2 on the hard origin of the matching coefficients of the effective theory by considering this purely academic case. In order to make contact with the result obtained in eq. (4.8) using the functional approach, we perform the matching off-shell and we use the $\overline{\text{MS}}$ regularization scheme with $\mu = M$. We do not consider in the matching procedure one-loop diagrams with only light fields, since they are present in both the full-theory and the effective theory amplitudes and, accordingly, cancel out in the matching.

For the model under discussion there is no contribution to the two- and three-point Green functions involving heavy particles in the loop. The diagrams contributing to the matching of the four-point Green function are given by



$$\begin{aligned}
 &= \frac{i}{16\pi^2} \lambda^2 \left[3 + 3 \frac{m^2}{M^2} + \frac{s+t+u}{2M^2} \right] \Big|_{\text{hard}} \\
 &\quad + \frac{i}{16\pi^2} \lambda^2 \left[-3 \frac{m^2}{M^2} + 3 \frac{m^2}{M^2} \ln \left(\frac{m^2}{M^2} \right) \right] \Big|_{\text{soft}} + \mathcal{O}(M^{-4}), \quad (4.9) \\
 &= \frac{i}{16\pi^2} \lambda^2 \left[-2 \frac{m^2}{M^2} + 2 \frac{m^2}{M^2} \ln \left(\frac{m^2}{M^2} \right) \right] \Big|_{\text{soft}} + \mathcal{O}(M^{-4}),
 \end{aligned}$$

where we have explicitly separated the contributions from the hard and soft loop-momentum regions. Note that a non-analytic term in m can only arise from the soft region, since in the hard region the light mass and the external momenta are expanded out from the propagators. For the corresponding EFT computation we need the effective Lagrangian matched at one-loop:

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{EFT}}^{\text{tree}} + \frac{\alpha}{4!} \hat{\varphi}^4 + \frac{\beta}{4!M^2} \hat{\varphi}^2 \partial^2 \hat{\varphi}^2 + \frac{\gamma}{6!M^2} \hat{\varphi}^6, \quad (4.10)$$

which now includes the dimension-6 operator with four light fields, and the one-loop matching coefficient for the 4- and 6-light field operators already present in $\mathcal{L}_{\text{EFT}}^{\text{tree}}$. The EFT

contributions to the four-point Green function read

$$\begin{aligned}
 \text{Diagram 1} &= \frac{i}{16\pi^2} \lambda^2 \left[-5 \frac{m^2}{M^2} + 5 \frac{m^2}{M^2} \ln \left(\frac{m^2}{M^2} \right) \right] + \mathcal{O}(M^{-4}), \\
 \text{Diagram 2} &= i \alpha - i \frac{\beta}{3M^2} (s + t + u).
 \end{aligned}
 \tag{4.11}$$

We see that the soft components of the full-theory amplitude match the one-loop diagram in the effective theory, and the matching coefficients of the φ^4 operators get thus determined by the hard part of the one-loop full-theory amplitude:

$$\alpha = \frac{3}{16\pi^2} \lambda^2 \left(1 + \frac{m^2}{M^2} \right), \quad \beta = -\frac{3}{16\pi^2} \frac{\lambda^2}{2}.
 \tag{4.12}$$

in agreement with the result for the φ^4 terms in eq. (4.8).

The next contribution to the one-loop effective theory comes from the six-point Green function. The full theory provides two diagrams for the matching:

$$\begin{aligned}
 \text{Diagram 3} &= \frac{i}{16\pi^2} 45 \frac{\kappa \lambda^2}{M^2} \Big|_{\text{hard}} + \frac{i}{16\pi^2} 45 \frac{\kappa \lambda^2}{M^2} \ln \left(\frac{m^2}{M^2} \right) \Big|_{\text{soft}} + \mathcal{O}(M^{-4}), \\
 \text{Diagram 4} &= \frac{i}{16\pi^2} 30 \frac{\kappa \lambda^2}{M^2} \ln \left(\frac{m^2}{M^2} \right) \Big|_{\text{soft}} + \mathcal{O}(M^{-4}),
 \end{aligned}
 \tag{4.13}$$

where once more we have explicitly separated the hard and soft contributions from each diagram. The six-point effective theory amplitude gives

$$\begin{aligned}
 \text{Diagram 5} &= \frac{i}{16\pi^2} 75 \frac{\kappa \lambda^2}{M^2} \ln \left(\frac{m^2}{M^2} \right) + \mathcal{O}(M^{-4}), \\
 \text{Diagram 6} &= i \frac{\gamma}{M^2}.
 \end{aligned}
 \tag{4.14}$$

Again, we note that the soft terms of the full theory are reproduced by the one-loop diagram in the effective theory. The local contribution is determined by the hard part of the full theory amplitude and thus reads

$$\gamma = \frac{45}{16\pi^2} \kappa \lambda^2,
 \tag{4.15}$$

that matches the result found in eq. (4.8) for the $\hat{\varphi}^6$ term.

4.2 Heavy real scalar triplet extension

As a second example, we consider an extension of the SM with an extra scalar sector comprised by a triplet of heavy scalars with zero hypercharge, Φ^a , $a = 1, 2, 3$, which interacts with the light Higgs doublet [26]. A triplet of scalars are ubiquitous in many extensions of the SM since the seminal article by Gelmini and Roncadelli [27]. However, we are not interested here in the phenomenology of the model but in how to implement our procedure in order to integrate out, at one loop, the extra scalar sector of the theory, assumed it is much heavier than the rest of the spectrum. Partial results for the dimension-6 operators involving the light Higgs doublet that are generated from this model have been provided in the functional approaches of refs. [7, 8].

The Lagrangian of the model is given by

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{2} D_\mu \Phi^a D^\mu \Phi^a - \frac{1}{2} M^2 \Phi^a \Phi^a - \frac{\lambda_\Phi}{4} (\Phi^a \Phi^a)^2 + \kappa \left(\phi^\dagger \tau^a \phi \right) \Phi^a - \eta \left(\phi^\dagger \phi \right) \Phi^a \Phi^a, \quad (4.16)$$

Here ϕ is the SM Higgs doublet and the covariant derivative acting on the triplet is defined as $D_\mu \Phi^a \equiv D_\mu^{ac} \Phi^c = (\partial_\mu \delta^{ac} + g \varepsilon^{abc} W_\mu^b) \Phi^c$. Within the background field method we split the fields into their classical (with hat) and quantum components: $\Phi^a \rightarrow \hat{\Phi}^a + \Phi^a$, $\phi \rightarrow \hat{\phi} + \phi$ and $W_\mu^a \rightarrow \hat{W}_\mu^a + W_\mu^a$. Given as an expansion in inverse powers of its mass, the classical field of the scalar triplet reads

$$\hat{\Phi}^a = \frac{\kappa}{M^2} \left(\hat{\phi}^\dagger \tau^a \hat{\phi} \right) - \frac{\kappa}{M^4} \left[\hat{D}^2 + 2\eta \left(\hat{\phi}^\dagger \hat{\phi} \right) \right] \left(\hat{\phi}^\dagger \tau^a \hat{\phi} \right) + \mathcal{O} \left(\frac{\kappa}{M^6} \right). \quad (4.17)$$

Following the procedure described in the section 2 we divide the fields into heavy and light, respectively, as $\eta_H = \Phi^a$ and $\eta_L = \{\phi, \phi^*, W_\mu^a\}$. The fluctuation matrix is readily obtained from eqs. (2.2) and (2.3),

$$\begin{aligned} \Delta_H &= \Delta_{\Phi\Phi}^{ab}, \\ X_{LH}^\dagger &= \left((X_{\phi^*\Phi}^a)^\dagger \quad (X_{\phi^*\Phi}^a)^\top \quad (X_{W\Phi}^{\nu da})^\top \right), \\ \Delta_L &= \begin{pmatrix} \Delta_{\phi^*\phi} & X_{\phi\phi}^\dagger & \left(X_{W\phi}^{\nu d} \right)^\dagger \\ X_{\phi\phi} & \Delta_{\phi^*\phi}^\top & \left(X_{W\phi}^{\nu d} \right)^\top \\ X_{W\phi}^{\mu c} & \left(X_{W\phi}^{\mu c} \right)^* & \Delta_W^{\mu\nu cd} \end{pmatrix}, \end{aligned} \quad (4.18)$$

with

$$\begin{aligned} \Delta_W^{\mu\nu ab} &= \left(\Delta_W^{\mu\nu ab} \right)_{\text{SM}} + g^2 g_{\mu\nu} \varepsilon^{acm} \varepsilon^{bdm} \hat{\Phi}^c \hat{\Phi}^d, \\ \Delta_{\phi^*\phi} &= \left(\Delta_{\phi^*\phi} \right)_{\text{SM}} + \kappa \tau^a \hat{\Phi}^a - \eta \hat{\Phi}^a \hat{\Phi}^a, \\ \Delta_{\Phi\Phi}^{ab} &= -\hat{D}_{ab}^2 + \delta_{ab} \left[-M^2 - \lambda_\Phi \hat{\Phi}^c \hat{\Phi}^c - 2\eta \left(\hat{\phi}^\dagger \hat{\phi} \right) \right] - 2\lambda_\Phi \hat{\Phi}^a \hat{\Phi}^b, \\ X_{W\Phi}^{\mu ab} &= g \varepsilon^{abc} \left(\hat{D}^\mu \hat{\Phi}^c \right) + g \varepsilon^{acd} \hat{\Phi}^c \hat{D}^\mu \hat{\Phi}^d, \\ X_{\phi^*\Phi}^a &= \kappa \tau^a \hat{\phi} - 2\eta \hat{\phi} \hat{\Phi}^a, \end{aligned} \quad (4.19)$$

and the rest of fluctuations in Δ_L involving only the light fields are contained in the quadratic piece of the SM Lagrangian, which we provide in eqs. (B.5) and (B.7). The quadratic term containing all fluctuations related to the heavy triplet is given by our formula (2.7),

$$\tilde{\Delta}_{\Phi\Phi} = \Delta_{\Phi\Phi} - X_{LH}^\dagger \Delta_L^{-1} X_{LH}. \quad (4.20)$$

The expansion in inverse powers of the heavy mass of the triplet requires a counting analogous to the one in eq. (2.18), i.e. $p_\mu \sim \zeta$ and $M \sim \zeta$. For the counting of the dimensionful parameter κ we choose $\kappa \sim \zeta$ and then, from eq. (4.17) we have $\hat{\Phi}^a \sim \zeta^{-1}$. As we are interested in dimension-six effective operators we can neglect contributions $\mathcal{O}(\zeta^{-5})$ and smaller. This is because in eq. (2.15) the propagator in the heavy particle provides an extra power ζ^{-2} . Hence we only need the numerator up to $\mathcal{O}(\zeta^{-4})$.

For practical reasons we choose to work in the Landau gauge for the quantum fluctuations, i.e. the renormalizable gauge with $\xi_W = 0$ in eqs. (B.8) and (B.11). The computation is much simpler in this gauge because the inverse of the propagators are transverse. Rearranging the expression in eq. (4.20), we can write

$$\begin{aligned} \tilde{\Delta}_{\Phi\Phi}^{ab} = \Delta_{\Phi\Phi}^{ab} - & \left[(X_{\phi^*\Phi}^a)^\dagger \bar{\Delta}_{\phi^*\phi}^{-1} X_{\phi^*\Phi}^b + (X_{\phi^*\Phi}^a)^\top \bar{X}_{\phi\phi} X_{\phi^*\Phi}^b + \text{c.c.} \right] \\ & - (\bar{X}_{W\Phi}^{\mu ca})^\top (\Delta_W^{\mu\nu cd})^{-1} \bar{X}_{W\Phi}^{\nu db} + \mathcal{O}(\zeta^{-5}), \end{aligned} \quad (4.21)$$

where c.c. is short for complex conjugation and we have used the following definitions:

$$\begin{aligned} \bar{\Delta}_{\phi^*\phi}^{-1} &= \Delta_{\phi^*\phi}^{-1} + \Delta_{\phi^*\phi}^{-1} X_{\phi\phi}^\dagger (\Delta_{\phi^*\phi}^{-1})^\top X_{\phi\phi} \Delta_{\phi^*\phi}^{-1}, \\ \bar{X}_{\phi\phi} &= - (\Delta_{\phi^*\phi}^{-1})^\top X_{\phi\phi} \Delta_{\phi^*\phi}^{-1}, \\ \bar{X}_{W\Phi}^{\mu ab} &= X_{W\Phi}^{\mu ab} - (X_{W\phi}^{\mu a} \Delta_{\phi^*\phi}^{-1} X_{\phi^*\Phi}^b + \text{c.c.}). \end{aligned} \quad (4.22)$$

To proceed we now come back to eq. (2.15) (with negative sign), with $m_H = M$ and $U = -\hat{D}^2 - M^2 - \tilde{\Delta}_{\Phi\Phi}$. Remember that the hat on the covariant derivatives indicates that only the classical field configuration for the gauge bosons is involved. Then by computing eq. (4.21) up to $\mathcal{O}(\zeta^{-4})$ one can obtain the one-loop effective theory that derives from the model specified in eq. (4.16) upon integrating out the triplet of heavy scalars.

We do not intend here to provide the complete result of the generated dimension-six operators. As a simple example and for illustrative purposes, we provide details on the computation of the heavy-light contributions arising from the quantum fluctuations of the electroweak gauge bosons. The latter provide the matching contributions to the dimension-six operators with Higgs fields and no field strength tensors proportional to g^2 , which were not obtained with the functional approach in ref. [8] due to the presence of “open” covariant derivatives. The computation of such contributions was also absent in the approach of ref. [7]. The relevant term in $U(x, \partial_x + ip)$ for this calculation is

$$\left[(\bar{X}_{W\Phi}^{\mu ca})^\top (\Delta_W^{\mu\nu cd})^{-1} \bar{X}_{W\Phi}^{\nu db} \right] (x, \partial_x + ip). \quad (4.23)$$

The first operator in eq. (4.23) simply reads

$$\begin{aligned}
\overline{X}_{W\Phi}^{\mu ab}(x, \partial_x + ip) &= -ig \epsilon^{abc} \hat{\Phi}^c p^\mu + \frac{\kappa}{p^2} ig \epsilon^{abc} \left(\hat{\phi}^\dagger \tau^c \hat{\phi} \right) p_\mu \\
&\quad + g \epsilon^{abc} \left(\hat{D}^\mu \hat{\Phi}^c \right) + g \epsilon^{acd} \hat{\Phi}^c \hat{D}^\mu db - \frac{1}{p^2} g \kappa \left\{ -\frac{i}{2} \left[\left(\hat{D}_\mu \hat{\phi} \right)^\dagger \tau^a \tau^b \hat{\phi} \right] \right. \\
&\quad \left. + \frac{i}{2} \left(\hat{\phi}^\dagger \tau^a \tau^b \hat{D}_\mu \hat{\phi} \right) + \frac{i}{2} \left(\hat{\phi}^\dagger \tau^a \tau^d \hat{\phi} \right) \hat{D}_\mu^{db} + \text{c.c.} \right\} + \mathcal{O}(\zeta^{-2}) \\
&= g \epsilon^{abc} \left(\hat{D}^\mu \hat{\Phi}^c \right) - \frac{g \kappa}{p^2} i \delta_{ab} \left(\hat{\phi}^\dagger \overleftrightarrow{D}_\mu \hat{\phi} \right) - g \frac{p^2 - M^2}{p^2} \epsilon^{acd} \hat{\Phi}^d \left(\hat{D}_\mu^{cb} + ip_\mu \delta^{cb} \right) \\
&\quad + \mathcal{O}(\zeta^{-2}) , \tag{4.24}
\end{aligned}$$

where, in the last line, we used the EOM for the heavy triplet, eq. (4.17), and we defined the hermitian derivative terms

$$\left(\phi^\dagger \overleftrightarrow{D}_\mu \phi \right) \equiv \left(\phi^\dagger D_\mu \phi \right) - \left[(D_\mu \phi)^\dagger \phi \right] , \tag{4.25}$$

with the covariant derivative acting on the Higgs field as specified in eq. (B.2). The contributions from the heavy triplet to the fluctuation Δ_W , see eq. (4.19), do not affect the computation of $\Delta_W^{-1}(x, \partial_x + ip)$ at leading order, and we can take the expression given in eq. (B.11) (with $\xi_W = 0$) for the latter. As a result we obtain

$$\begin{aligned}
&\left[\left(\overline{X}_{W\Phi}^{\mu ca} \right)^\top \left(\Delta_W^{\mu\nu cd} \right)^{-1} \overline{X}_{W\Phi}^{\nu db} \right] (x, \partial_x + ip) \\
&= g^2 \left[-\frac{g^{\mu\nu}}{p^2} + \frac{p^\mu p^\nu}{p^4} \right] \left[\delta_{ab} \left(\hat{D}_\mu \hat{\Phi}^c \right) \left(\hat{D}_\nu \hat{\Phi}^c \right) \right. \\
&\quad \left. - \left(\hat{D}_\mu \hat{\Phi}^a \right) \left(\hat{D}_\nu \hat{\Phi}^b \right) - \delta_{ab} \frac{\kappa^2}{p^4} \left(\hat{\phi}^\dagger \overleftrightarrow{D}_\mu \hat{\phi} \right) \left(\hat{\phi}^\dagger \overleftrightarrow{D}_\nu \hat{\phi} \right) \right] \\
&\quad + \mathcal{O}(\zeta^{-5}) , \tag{4.26}
\end{aligned}$$

and we dropped the terms proportional to $(p^2 - M^2)$ since they yield a null contribution in the momentum integration, as explained below.

Only the first term of the series in eq. (2.15) contributes in this case:

$$\mathcal{L}_{\text{EFT}}^{\text{1loop}} \Big|_W = -\frac{i}{2} \int \frac{d^d p}{(2\pi)^d} \frac{\left[\left(\overline{X}_{W\Phi}^{\mu ca} \right)^\top \left(\Delta_W^{\mu\nu cd} \right)^{-1} \overline{X}_{W\Phi}^{\nu da} \right] (x, \partial_x + ip)}{p^2 - M^2} . \tag{4.27}$$

From eq. (4.27) it is clear that terms proportional to $(p^2 - M^2)$ yield scaleless terms that are set to zero in dimensional regularization, which justifies having dropped them in eq. (4.26). After evaluating the integral in the $\overline{\text{MS}}$ regularization scheme, using the heavy triplet EOMs and rearranging the result through partial integration we finally get for $\mu = M$

$$\mathcal{L}_{\text{EFT}}^{\text{1loop}} \Big|_W = \frac{1}{16\pi^2} \frac{g^2 \kappa^2}{M^4} \left[-\frac{25}{16} \left(\hat{\phi}^\dagger \hat{\phi} \right) \partial^2 \left(\hat{\phi}^\dagger \hat{\phi} \right) + \frac{5}{4} \left[\left(\hat{\phi}^\dagger \hat{\phi} \right) \left(\hat{\phi}^\dagger \hat{D}^2 \hat{\phi} \right) + \text{h.c.} \right] - \frac{5}{4} \left| \hat{\phi}^\dagger \hat{D}_\mu \hat{\phi} \right|^2 \right] . \tag{4.28}$$

In order to compare this result with previous calculations done in the literature, we focus on the heavy triplet contributions to $Q_{\phi D} = |\phi^\dagger D^\mu \phi|^2$. From the result in eq. (4.28) we find for its one-loop matching coefficient

$$C_{\phi D}(\mu = M) \Big|_{\mathcal{O}(g^2)} = -\frac{1}{16\pi^2} \frac{\kappa^2}{M^4} \frac{5}{4} g^2, \quad (4.29)$$

which agrees with the result given in ref. [5] for the term proportional to g^2 . The remaining contributions to $C_{\phi D}(\mu = M)$ have also been calculated with our method. However their computation is lengthy and does not provide any new insight on the method. The final result reads

$$C_{\phi D}(\mu = M) = \frac{\kappa^2}{M^4} \left[-2 + \frac{1}{16\pi^2} \left(5 \frac{\kappa^2}{M^2} - \frac{5}{4} g^2 + 16\eta - 3\lambda - 20\lambda_\Phi \right) \right]. \quad (4.30)$$

In eq. (4.30) we have also included the term arising from the redefinition of ϕ that absorbs the one-loop contribution to the kinetic term, $\phi \rightarrow (1 - 3\kappa^2/64\pi^2 M^2)\phi$. This result is in agreement with the one provided in ref. [5] once we account for the different convention in the definition of λ : our λ equals 2λ in that reference.

5 Conclusions

The search for new physics in the next run at LHC stays as a powerful motivation for a systematic scrutiny of the possible extensions of the SM. The present status that engages both collider and precision physics has, on the theoretical side, a robust tool in the construction, treatment and phenomenology of effective field theories that are the remains of ultraviolet completions of the SM upon integration of heavy spectra.

Though, traditionally, there are two essential procedures to construct those effective field theories, namely functional methods and matching schemes, the latter have become the most frequently used. Recently there has been a rediscovery of the functional methods, initiated by the work of Henning et al. [3]. The latter work started a discussion regarding the treatment of the terms that mix heavy and light quantum fluctuations, that was finally clarified but which, in our opinion, was already settled in the past literature on the subject. In this article we have addressed this issue and we have provided a framework that further clarifies the treatment of the heavy-light contributions and simplifies the technical *modus operandi*.

The procedure amounts to a particular diagonalization of the quadratic form in the path integral of the full theory that leaves untouched the part that entails the light fields. In this way we can integrate, at one loop, contributions with only heavy fields inside the loop and contributions with mixed components of heavy and light fields, with a single computation and following the conventional method employed to carry out the first ones only. We have also showed that in the resulting determinant containing the heavy particle effects only the *hard* components are needed to derive the one-loop matching coefficients of the effective theory. Within dimensional regularization these hard contributions are obtained by expanding out the low-energy scales with respect the hard loop momentum

which has to be considered of the same order as the mass of the heavy particle. In this way, our determination of the EFT local terms that reproduce the heavy-particle effects does not require the subtraction of any one-loop contributions from the EFT, as opposed to the conventional (diagrammatic) matching approach or to the recently proposed methods that use functional techniques. We have included two examples in section 4: a scalar toy model, that nicely illustrates the simplicity of our approach as compared to the diagrammatic approach, and a heavy real scalar triplet extension of the SM, which shows that our method can be applied also to more realistic cases.

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A General expressions for dimension-six operators

In this appendix we work out $\mathcal{L}_{\text{EFT}}^{\text{1loop}}$ for the case of dimension-six operators. Following the guidelines in section 2, we make the separation $U(x, \partial_x + ip) = U_H(x) + U_{LH}(x, \partial_x + ip)$ and expand eq. (2.15) up to $\mathcal{O}(\zeta^{-2})$. The Lagrangian $\mathcal{L}_{\text{EFT}}^{\text{1loop}}$ then consists of two pieces:

$$\mathcal{L}_{\text{EFT}}^{\text{1loop}} = \mathcal{L}_{\text{EFT}}^{\text{1loop}} \Big|_{\text{H}} + \mathcal{L}_{\text{EFT}}^{\text{1loop}} \Big|_{\text{LH}} . \quad (\text{A.1})$$

The first term comes from contributions involving $U_H(x)$ only and, since $U_H(x)$ is momentum independent, it can be obtained from the universal formula provided in the literature [3, 22, 24, 25] (see also [4] for the case when several scales are involved) which, for completeness, we reproduce here:

$$\begin{aligned} \mathcal{L}_{\text{EFT}}^{\text{1loop}} \Big|_{\text{H}} = & \frac{c_s}{16\pi^2} \left\{ m_H^2 \left(1 + \ln \frac{\mu^2}{m_H^2} \right) \text{tr} \{ U_H \} \right. \\ & + \left[\frac{1}{2} \ln \frac{\mu^2}{m_H^2} \text{tr} \{ U_H^2 \} + \frac{1}{12} \ln \frac{\mu^2}{m_H^2} \text{tr} \{ \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \} \right] \\ & + \frac{1}{m_H^2} \left[-\frac{1}{6} \text{tr} \{ U_H^3 \} + \frac{1}{12} \text{tr} \{ (\hat{D}_\mu U_H)^2 \} - \frac{1}{12} \text{tr} \{ U_H \hat{F}^{\mu\nu} \hat{F}_{\mu\nu} \} \right. \\ & \quad \left. + \frac{1}{60} \text{tr} \{ (\hat{D}_\mu \hat{F}^{\mu\nu})^2 \} - \frac{1}{90} \text{tr} \{ \hat{F}^{\mu\nu} \hat{F}_{\nu\rho} \hat{F}^\rho{}_\mu \} \right] \\ & + \frac{1}{m_H^4} \left[\frac{1}{24} \text{tr} \{ U_H^4 \} - \frac{1}{12} \text{tr} \{ U_H (\hat{D}_\mu U_H)^2 \} + \frac{1}{60} \text{tr} \{ \hat{F}_{\mu\nu} (\hat{D}^\mu U_H) (\hat{D}^\nu U_H) \} \right. \\ & \quad \left. + \frac{1}{120} \text{tr} \{ (\hat{D}^2 U_H)^2 \} + \frac{1}{40} \text{tr} \{ U_H^2 \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} \} + \frac{1}{60} \text{tr} \{ (U_H \hat{F}_{\mu\nu})^2 \} \right] \end{aligned}$$

$$\begin{aligned}
 & + \frac{1}{m_H^6} \left[-\frac{1}{60} \text{tr} \{U_H^5\} + \frac{1}{20} \text{tr} \left\{ U_H^2 (\hat{D}_\mu U_H)^2 \right\} + \frac{1}{30} \text{tr} \left\{ (U_H \hat{D}_\mu U_H)^2 \right\} \right] \\
 & + \frac{1}{m_H^8} \frac{1}{120} \text{tr} \{U_H^6\} + \mathcal{O}(\zeta^{-3}) , \tag{A.2}
 \end{aligned}$$

where $c_s = 1/2, -1/2$ depending, respectively, on the bosonic or fermionic nature of the heavy fields. Here $F_{\mu\nu} \equiv [D_\mu, D_\nu]$ and the momentum integrals are regulated in d dimensions, with the divergences subtracted in the $\overline{\text{MS}}$ scheme. The second term in eq. (A.1) is built from pieces containing at least one power of U_{LH} . Given that U_H is at most $\mathcal{O}(\zeta)$ and U_{LH} at most $\mathcal{O}(\zeta^0)$ in our power counting, the series in eq. (2.15) has to be expanded up to $n = 5$ for the contributions to dimension-six operators

$$\begin{aligned}
 \mathcal{L}_{\text{EFT}}^{\text{1loop}} \Big|_{\text{LH}} &= -i c_s \int \frac{d^d p}{(2\pi)^d} \left\{ \frac{1}{p^2 - m_H^2} \text{tr}_s \{U\} + \frac{1}{2} \frac{1}{(p^2 - m_H^2)^2} \text{tr}_s \{U^2\} \right. \\
 & + \frac{1}{3} \frac{1}{(p^2 - m_H^2)^3} \left[\text{tr}_s \{U^3\} + \text{tr}_s \{U \hat{D}^2 U\} + 2i p^\mu \text{tr}_s \{U \hat{D}_\mu U\} \right] \\
 & + \frac{1}{4} \frac{1}{(p^2 - m_H^2)^4} \left[\text{tr}_s \{U^4\} + 2i p^\mu \text{tr}_s \{U^2 \hat{D}_\mu U\} + 2i p^\mu \text{tr}_s \{U \hat{D}_\mu U^2\} \right. \\
 & \quad + \text{tr}_s \{U^2 \hat{D}^2 U\} + \text{tr}_s \{U \hat{D}^2 U^2\} - 4 p^\mu p^\nu \text{tr}_s \{U \hat{D}_\mu \hat{D}_\nu U\} \\
 & \quad + 2i p^\mu \text{tr}_s \{U \hat{D}^2 \hat{D}_\mu U\} + 2i p^\mu \text{tr}_s \{U \hat{D}_\mu \hat{D}^2 U\} \\
 & \quad \left. + \text{tr}_s \{U (\hat{D}^2)^2 U\} \right] \\
 & + \frac{1}{5} \frac{1}{(p^2 - m_H^2)^5} \left[\text{tr}_s \{U^5\} + 2i p^\mu \text{tr}_s \{U^3 \hat{D}_\mu U\} + 2i p^\mu \text{tr}_s \{U^2 \hat{D}_\mu U^2\} \right. \\
 & \quad + 2i p^\mu \text{tr}_s \{U \hat{D}_\mu U^3\} - 4 p^\mu p^\nu \text{tr}_s \{U^2 \hat{D}_\mu \hat{D}_\nu U\} \\
 & \quad - 4 p^\mu p^\nu \text{tr}_s \{U \hat{D}_\mu U \hat{D}_\nu U\} - 4 p^\mu p^\nu \text{tr}_s \{U \hat{D}_\mu \hat{D}_\nu U^2\} \\
 & \quad \left. - 8i p^\mu p^\nu p^\rho \text{tr}_s \{U \hat{D}_\mu \hat{D}_\nu \hat{D}_\rho U\} \right] \Big\} \\
 & + \mathcal{L}_{\text{EFT}}^F + \mathcal{O}(\zeta^{-3}) . \tag{A.3}
 \end{aligned}$$

We have introduced a subtracted trace which is defined as

$$\text{tr}_s \{f(U, D_\mu)\} \equiv \text{tr} \{f(U, D_\mu) - f(U_H, D_\mu) - \Theta_f\} , \tag{A.4}$$

where f is an arbitrary function of U and covariant derivatives, and Θ_f generically denotes all the terms with covariant derivatives at the rightmost of the trace (i.e. open covariant derivative terms) contained in the original trace. The terms involving only U_H that are subtracted from the trace were already included in eq. (A.2) while all open derivative terms from the different traces are collected in $\mathcal{L}_{\text{EFT}}^F$. The latter combine into gauge invariant pieces with field-strength tensors, although the manner in which this occurs is not easily seen and involves the contribution from different orders in the expansion.

With the purpose of illustration, we compute $\mathcal{L}_{\text{EFT}}^F$ that results from the integration of the real scalar triplet extension of the SM presented in subsection 4.2. In this case, gauge invariance of the final result guarantees that the leading order contribution to $\mathcal{L}_{\text{EFT}}^F$ should contain at least four covariant derivatives, as terms with two covariant derivatives cannot be contracted to yield a gauge invariant term. As it is clear from eq. (2.15), traces with j derivatives and a number k of U operators have a power suppression of $\mathcal{O}(\zeta^{4-j-2k})$ (we recall that $d^d p \sim \zeta^4$). The expansion of the operator U_{LH} can yield in addition ℓ covariant derivatives, and each of these receives a further suppression of ζ^{-1} because they are accompanied with a light-field propagator, see eq. (A.6). Since U_{LH} is at most $\mathcal{O}(\zeta^0)$ we then find that, in general, terms with k insertions of U_{HL} and a total number of $j + \ell$ derivatives have a power counting of at most $\mathcal{O}(\zeta^{4-j-\ell-2k})$. As a result, the only gauge invariant object involving U_{LH} and four derivatives that one can construct at $\mathcal{O}(\zeta^{-2})$ includes only one power of U_{LH} (i.e. $j + \ell = 4$ and $k = 1$). Moreover, since U_{LH} has to be evaluated at leading order, the only relevant piece from U_{LH} for the computation of $\mathcal{L}_{\text{EFT}}^F$ reads

$$U_{LH}^F = X_{LH}^{(1)\dagger} \Delta_L^{-1} \Big|_{\hat{\eta}=0} X_{LH}^{(1)}. \quad (\text{A.5})$$

Here $X_{LH}^{(1)}$ is defined as the part of X_{LH} that is $\mathcal{O}(\zeta)$, and we remind that $\hat{\eta}$ stands for the classical field configurations. Using the expressions in eqs. (4.18) and (4.19) we have

$$U_{LH}^F(x, \partial_x + ip) \subset \frac{\kappa^2}{p^2} \sum_{m=0}^4 \left[\hat{\phi}^\dagger \tau^a \left(\frac{2ip\hat{D} + \hat{D}^2}{p^2} \right)^m \tau^b \hat{\phi} + \hat{\phi}^\dagger (\tau^a)^\dagger \left(\frac{2ip\hat{D}^* + \hat{D}^{*2}}{p^2} \right)^m (\tau^b)^* \hat{\phi}^* \right], \quad (\text{A.6})$$

where the covariant derivatives have to be expanded by applying the identities

$$\begin{aligned} D_\mu \tau^a \phi &= \tau^a (D_\mu \phi) + \tau^c \phi D_\mu^{ca}, \\ D_\mu^* (\tau^a)^* \phi^* &= (\tau^a)^* (D_\mu \phi)^* + (\tau^c)^* \phi^* D_\mu^{ca}, \end{aligned} \quad (\text{A.7})$$

with D_μ denoting the Higgs field covariant derivative, see eq. (B.2), and with D_μ^{ca} as defined in section 2. For the computation of $\mathcal{L}_{\text{EFT}}^F$ up to $\mathcal{O}(\zeta^{-2})$ we need to isolate the terms in eq. (2.15) with up to four open covariant derivatives and just one power of U_{LH}^F . These are given by

$$\begin{aligned} \mathcal{L}_{\text{EFT}}^F \subset & -\frac{i}{2} \int \frac{d^d p}{(2\pi)^d} \frac{1}{p^2 - M^2} \sum_{n=0}^4 \sum_{k=0}^n \frac{1}{n+1} \\ & \times \text{tr} \left\{ \left(\frac{2ip\hat{D} + \hat{D}^2}{p^2 - M^2} \right)^{n-k} U_{LH}^F(x, \partial_x + ip) \left(\frac{2ip\hat{D} + \hat{D}^2}{p^2 - M^2} \right)^k \right\}, \end{aligned} \quad (\text{A.8})$$

and using the cyclic property of the trace we get⁴

$$\mathcal{L}_{\text{EFT}}^F \subset -\frac{i}{2} \int \frac{d^d p}{(2\pi)^d} \frac{1}{p^2 - M^2} \sum_{n=0}^4 \text{tr} \left\{ U_{LH}^F(x, \partial_x + ip) \left(\frac{2ip\hat{D} + \hat{D}^2}{p^2 - M^2} \right)^n \right\}. \quad (\text{A.9})$$

Finally, keeping only terms with up to four covariant derivatives, performing the momentum integration (see eq. (2.20)) and evaluating the $SU(2)$ trace we arrive at the final result

$$\mathcal{L}_{\text{EFT}}^F = \frac{1}{16\pi^2} \frac{\kappa^2}{M^4} \left[-\frac{g^2}{3} (\hat{\phi}^\dagger \hat{\phi}) \hat{W}^{\mu\nu a} \hat{W}_{\mu\nu}^a + g (\hat{\phi}^\dagger i\hat{D}_\mu^{\leftrightarrow} \hat{\phi}) (\hat{D}_\nu \hat{W}^{\mu\nu})^a - \frac{gg'}{2} (\hat{\phi}^\dagger \tau^a \hat{\phi}) \hat{W}_{\mu\nu}^a \hat{B}^{\mu\nu} \right], \quad (\text{A.10})$$

with the field-strength tensors defined in eq. (B.3) and

$$(\hat{\phi}^\dagger i\hat{D}_\mu^{\leftrightarrow} \hat{\phi}) = i (\hat{\phi}^\dagger \tau^a D_\mu \hat{\phi}) - i [(D_\mu \hat{\phi})^\dagger \tau^a \hat{\phi}]. \quad (\text{A.11})$$

B The fluctuation operator of the SM

In this appendix we provide the fluctuation operator for the SM Lagrangian. The SM Lagrangian in compact notation is given by

$$\begin{aligned} \mathcal{L}_{\text{SM}} = & -\frac{1}{4} G_{\mu\nu}^\alpha G^{\mu\nu\alpha} - \frac{1}{4} W_{\mu\nu}^a W^{\mu\nu a} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + (D_\mu \phi)^\dagger D^\mu \phi - m_\phi^2 (\phi^\dagger \phi) - \frac{\lambda}{2} (\phi^\dagger \phi)^2 \\ & + \bar{\psi} i \not{D} \psi - \bar{\psi} \left(\tilde{\phi} y_u P_u P_R + \phi y_d P_d P_R + \text{h.c.} \right) \psi + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{ghost}}. \end{aligned} \quad (\text{B.1})$$

Here, $\psi = q, \ell$, $P_u (P_d)$ project into the up (down) sector, $y_{u,d}$ is a Yukawa matrix for up (down) fields, \mathcal{L}_{GF} and $\mathcal{L}_{\text{ghost}}$ are the gauge-fixing and ghost Lagrangians, respectively, and the covariant derivatives are defined as

$$\begin{aligned} \not{D} \psi &= (\not{\partial}_\mu - ig_c \not{G}^\alpha T^\alpha P_q - ig \not{W}^a T^a P_L - ig' \not{B} Y_\psi) \psi, \\ D_\mu \phi &= \left(\partial_\mu - ig W_\mu^a T^a - \frac{1}{2} ig' B_\mu \right) \phi. \end{aligned} \quad (\text{B.2})$$

In eq. (B.2), $T^a = \tau^a/2$ and $T^\alpha = \lambda^\alpha/2$ with τ^a and λ^α the Pauli and the Gell-Mann matrices, respectively, P_q denotes a projector into the quark sector, and the hypercharge reads $Y_\psi = Y_{\psi_L} P_L + Y_{\psi_R}^u P_u P_R + Y_{\psi_R}^d P_d P_R$. Accordingly, the field strength tensors are given by

$$\begin{aligned} G_{\mu\nu}^\alpha &= \partial_\mu G_\nu^\alpha - \partial_\nu G_\mu^\alpha + g f_{\alpha\beta\gamma} G_\mu^\beta G_\nu^\gamma, \\ W_{\mu\nu}^a &= \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g \epsilon_{abc} W_\mu^b W_\nu^c, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu. \end{aligned} \quad (\text{B.3})$$

⁴The use of the cyclic property when derivative terms are involved is only justified for the functional trace, that we denoted in this article as Tr . However, as noted in refs. [16, 22], in the evaluation of the functional determinant, which is a gauge invariant object, the trace over internal degrees of freedom ‘tr’ can be recast into the full trace through the use of the identity (we recall that $S = \int d^d x \mathcal{L}$)

$$\text{Tr}\{f(\hat{x})\} = \int d^d x \text{tr}\{\langle x|f(\hat{x})|x\rangle\} = \int d^d x \text{tr}\{f(x)\} \delta^d(0),$$

and then reverted to a trace over internal degrees of freedom after the application of the cyclic property.

Following the same procedure as in section 2, we separate the fields into background, $\hat{\eta}$, and quantum field configurations, η , and expand the SM Lagrangian to second order in the quantum fluctuation:

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{SM}}^{\text{tree}}(\hat{\eta}) + \mathcal{L}_{\text{SM}}^{(\eta^2)} + \mathcal{O}(\eta^3), \quad (\text{B.4})$$

where $\mathcal{L}_{\text{SM}}^{\text{tree}}$ is the tree-level SM effective Lagrangian, and $\mathcal{L}_{\text{SM}}^{(\eta^2)}$ is computed using eq. (2.2):

$$\mathcal{L}_{\text{SM}}^{(\eta^2)} = \frac{1}{2} \left(\phi^\dagger \ \phi^\top \ A_\mu^{\alpha\top} \ \bar{\psi} \ \psi^\top \right) \begin{pmatrix} \Delta_{\phi^*\phi} & X_{\phi\phi}^\dagger & (X_{A\phi}^{\nu b})^\dagger & \bar{X}_{\bar{\psi}\phi} & -X_{\bar{\psi}\phi^*}^\top \\ X_{\phi\phi} & \Delta_{\phi^*\phi}^\top & (X_{A\phi}^{\nu b})^\top & \bar{X}_{\bar{\psi}\phi^*} & -X_{\bar{\psi}\phi}^\top \\ X_{A\phi}^{\mu a} & (X_{A\phi}^{\mu a})^* & \Delta_A^{\mu\nu ab} & \bar{X}_{\bar{\psi}A}^{\mu a} & -(X_{\bar{\psi}A}^{\mu a})^\top \\ X_{\bar{\psi}\phi} & X_{\bar{\psi}\phi^*} & X_{\bar{\psi}A}^{\nu b} & \Delta_{\bar{\psi}\psi} & 0 \\ -\bar{X}_{\bar{\psi}\phi^*}^\top & -\bar{X}_{\bar{\psi}\phi}^\top & -(\bar{X}_{\bar{\psi}A}^{\nu b})^\top & 0 & -\Delta_{\bar{\psi}\psi}^\top \end{pmatrix} \begin{pmatrix} \phi \\ \phi^* \\ A_\nu^b \\ \psi \\ \bar{\psi}^\top \end{pmatrix} + \mathcal{L}_{\text{ghost}}^{(\eta^2)}, \quad (\text{B.5})$$

with $A_\mu^a = (G_\mu^\alpha \ W_\mu^a \ B_\mu)^\top$ denoting the gauge fields and

$$\Delta_A^{\mu\nu ab} = \begin{pmatrix} \Delta_G^{\mu\nu\alpha\beta} & 0 & 0 \\ 0 & \Delta_W^{\mu\nu ab} & \Delta_{BW}^{\mu\nu a} \\ 0 & \Delta_{BW}^{\mu\nu a} & \Delta_B^{\mu\nu} \end{pmatrix}, \quad X_{A\phi}^{\mu a} = \begin{pmatrix} 0 \\ X_{W\phi}^{\mu a} \\ X_{B\phi}^\mu \end{pmatrix}, \quad \bar{X}_{\bar{\psi}A}^{\mu a} = \begin{pmatrix} \bar{X}_{\bar{\psi}G}^{\mu\alpha} \\ \bar{X}_{\bar{\psi}W}^{\mu a} \\ \bar{X}_{\bar{\psi}B}^\mu \end{pmatrix}, \quad (\text{B.6})$$

where, generically, $\bar{X} = X^\dagger \gamma^0$. The pieces in the quadratic fluctuation are defined as

$$\begin{aligned} \Delta_{\phi^*\phi} &= -\hat{D}^2 - m_\phi^2 - \lambda(\hat{\phi}^\dagger \hat{\phi}) - \lambda \hat{\phi} \hat{\phi}^\dagger, \\ \Delta_G^{\mu\nu\alpha\beta} &= \delta_{\alpha\beta} \left[g^{\mu\nu} \hat{D}^2 + \frac{1-\xi_G}{\xi_G} \hat{D}^\mu \hat{D}^\nu \right] - g_c \epsilon_{\alpha\beta\gamma} \hat{G}^{\mu\nu\gamma}, \\ \Delta_W^{\mu\nu ab} &= \delta_{ab} \left\{ g^{\mu\nu} \left[\hat{D}^2 + \frac{1}{2} g^2 (\hat{\phi}^\dagger \hat{\phi}) \right] + \frac{1-\xi_W}{\xi_W} \hat{D}^\mu \hat{D}^\nu \right\} - g \epsilon_{abc} \hat{W}^{\mu\nu c}, \\ \Delta_B^{\mu\nu} &= g^{\mu\nu} \left[\partial^2 + \frac{1}{2} g'^2 (\hat{\phi}^\dagger \hat{\phi}) \right] + \frac{1-\xi_B}{\xi_B} \partial^\mu \partial^\nu, \\ \Delta_{BW}^{\mu\nu a} &= \frac{1}{2} g g' g_{\mu\nu} (\hat{\phi}^\dagger \tau^a \hat{\phi}), \\ \Delta_{\bar{\psi}\psi} &= i \hat{D} \not{\partial} - \left(i \tau_2 \hat{\phi}^* y_u P_u + \hat{\phi} y_d P_d + \text{h.c.} \right), \\ X_{\phi\phi} &= -\lambda \hat{\phi}^* \hat{\phi}^\dagger, \\ X_{W\phi}^{\mu a} &= \frac{1}{2} i g \left[\hat{\phi}^\dagger \tau^a \hat{D}^\mu - (\hat{D}^\mu \hat{\phi})^\dagger \tau^a \right], \\ X_{B\phi}^\mu &= \frac{1}{2} i g' \left[\hat{\phi}^\dagger \hat{D}^\mu - (\hat{D}^\mu \hat{\phi})^\dagger \right], \\ X_{\bar{\psi}G}^{\mu\alpha} &= \frac{1}{2} g_c \lambda^\alpha P_q \gamma^\mu \hat{\psi}, \\ X_{\bar{\psi}W}^{\mu a} &= \frac{1}{2} g \tau^a \gamma^\mu P_L \hat{\psi}, \end{aligned}$$

$$\begin{aligned}
X_{\bar{\psi}B}^\mu &= g' \gamma^\mu Y_\psi \hat{\psi}, \\
X_{\bar{\psi}\phi} &= -\overleftarrow{P}_u P_L y_u^\dagger \hat{\psi}^t i\tau_2 - y_d P_d P_R \hat{\psi}, \\
X_{\bar{\psi}\phi^*} &= -i\tau^2 y_u P_u P_R \hat{\psi} - \overleftarrow{P}_d P_L y_d^\dagger \hat{\psi}^t.
\end{aligned} \tag{B.7}$$

The superscript t in the fermion fields denotes transposition in isospin space. Additionally, we have fixed the gauge of the quantum fields using the *background field gauge*, which ensures that the theory remains invariant under gauge transformations of the background fields. This choice corresponds to the following gauge-fixing Lagrangian:

$$\mathcal{L}_{\text{GF}} = -\frac{1}{2\xi_G} \left(\hat{D}_\mu G^{\mu\alpha} \right)^2 - \frac{1}{2\xi_W} \left(\hat{D}_\mu W^{\mu a} \right)^2 - \frac{1}{2\xi_B} \left(\partial_\mu B^\mu \right)^2. \tag{B.8}$$

Finally we also provide the expansion for the inverse operators $\Delta_X(x, \partial_x + ip)^{-1}$, with $X = \{\phi^* \phi, B, W\}$, when $p_\mu \sim \zeta$. We have:

$$\begin{aligned}
\Delta_{\phi^* \phi}(x, \partial_x + ip) &= p^2 - m_\phi^2 - 2ip\hat{D} - \hat{D}^2 - \lambda \left(\hat{\phi}^\dagger \hat{\phi} \right) - \lambda \hat{\phi} \hat{\phi}^\dagger, \\
\Delta_W^{\mu\nu ab}(x, \partial_x + ip) &= \delta_{ab} \left[-g^{\mu\nu} p^2 - \frac{1 - \xi_W}{\xi_W} p^\mu p^\nu \right] + \mathcal{O}(\zeta), \\
\Delta_B^{\mu\nu}(x, \partial_x + ip) &= -g^{\mu\nu} p^2 - \frac{1 - \xi_B}{\xi_B} p^\mu p^\nu + \mathcal{O}(\zeta).
\end{aligned} \tag{B.9}$$

from where, and defining

$$\Omega = \hat{D}^2 + \lambda \left(\hat{\phi}^\dagger \hat{\phi} \right) + \lambda \hat{\phi} \hat{\phi}^\dagger, \tag{B.10}$$

it is straightforward to get

$$\begin{aligned}
\Delta_{\phi^* \phi}(x, \partial_x + ip)^{-1} &= \frac{1}{p^2} \left(1 + \frac{m_\phi^2}{p^2} + \frac{m_\phi^4}{p^4} \right) + 2i \frac{p_\mu}{p^4} \left(1 + 2 \frac{m_\phi^2}{p^2} \right) \hat{D}^\mu \\
&+ \frac{1}{p^4} \left(1 + 2 \frac{m_\phi^2}{p^2} \right) \Omega - 4 \frac{p_\mu p_\nu}{p^6} \left(1 + 3 \frac{m_\phi^2}{p^2} \right) \hat{D}^\mu \hat{D}^\nu \\
&+ 2i \frac{p_\mu}{p^6} \left\{ \hat{D}^\mu \Omega + \Omega \hat{D}^\mu \right\} + \frac{1}{p^6} \Omega^2 \\
&- 8i \frac{p_\mu p_\nu p_\rho}{p^8} \hat{D}^\mu \hat{D}^\nu \hat{D}^\rho + 16 \frac{p_\mu p_\nu p_\rho p_\sigma}{p^{10}} \hat{D}^\mu \hat{D}^\nu \hat{D}^\rho \hat{D}^\sigma \\
&- 4 \frac{p_\mu p_\nu}{p^8} \left\{ \hat{D}^\mu \hat{D}^\nu \Omega + \Omega \hat{D}^\mu \hat{D}^\nu + \hat{D}^\mu \Omega \hat{D}^\nu \right\} + \mathcal{O}(\zeta^{-7}), \\
\Delta_B^{\mu\nu}(x, \partial_x + ip)^{-1} &= -\frac{g^{\mu\nu}}{p^2} + (1 - \xi_B) \frac{p^\mu p^\nu}{p^4} + \mathcal{O}(\zeta^{-3}), \\
\Delta_W^{\mu\nu ab}(x, \partial_x + ip)^{-1} &= \delta_{ab} \left[-\frac{g^{\mu\nu}}{p^2} + (1 - \xi_W) \frac{p^\mu p^\nu}{p^4} \right] + \mathcal{O}(\zeta^{-3}),
\end{aligned} \tag{B.11}$$

and analogously for $\Delta_G^{\mu\nu\alpha\beta}(x, \partial_x + ip)^{-1}$. The inverse operator $[\Delta_{\phi^* \phi}^*(x, \partial_x + ip)]^{-1}$ can be obtained from $\Delta_{\phi^* \phi}(x, \partial_x + ip)^{-1}$ by making the substitution $\hat{D}_\mu \rightarrow \hat{D}_\mu^*$ while $[\Delta_{\phi^* \phi}^\dagger(x, \partial_x + ip)]^{-1}$ and $[\Delta_{\phi^* \phi}^*(x, \partial_x + ip)]^{-1}$ share the same expression, up to a total derivative term.

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