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Ghostbusters in higher derivative supersymmetric theories: who is afraid of propagating auxiliary fields?

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Abstract: We present for the first time a ghost-free higher-derivative chiral model with a propagating auxiliary F-term field (highest component of the chiral multiplet). We obtain this model by removing a ghost in a higher derivative chiral model, with Higgsing it in terms of an auxiliary vector superfield. Depending on the sign of the quadratic derivative term of the chiral superfield, the model contains two ghost free branches of the parameter regions. We find that supersymmetry is spontaneously broken in one branch while it is preserved in the other branch. As a consequence of dynamical F-term field, a conserved $U(1)$ charge corresponding to the number density of F appears, which can be regarded as a generalization of the R-symmetry.

Keywords: Superspaces, Supersymmetric Effective Theories, Supersymmetry Breaking

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1 Introduction

Supersymmetry (SUSY) is an interesting extension beyond the standard model of particle physics (SM) from various viewpoints. The SUSY extension of SM has dark matter candidates and a nice property controlling quantum corrections to the Higgs mass. It is also known that local SUSY, called supergravity (SUGRA), is the effective theory of superstring, which is a possible quantum gravity theory. Once we start with SUGRA, the renormalizability of a theory disappears and we cannot control such terms. Although such a non-renormalizability may be restored in the UV complete theory, we need to consider higher-order terms in the effective theory. If this is the case, higher-order derivative interactions would appear as the non-derivative higher-order terms exist.

It has been well known that higher-order derivative interactions may lead to the socalled Ostrogradski instability (see ref. [\[1](#page-12-0)] as a review) because the energy of such a system cannot be bounded. Therefore, it is important to specify a class of ghost-free higherderivative interactions. For non-SUSY scalar-tensor theory, Horndeski found a class of higher-derivative Lagrangian without ghosts [\[2](#page-12-1)] (see also refs. [\[3,](#page-12-2) [4\]](#page-12-3)).

In SUSY cases, ghost-free higher-derivative interactions have been studied in various contexts. The properties of such higher-derivative terms have been studied from theoretical and phenomenological viewpoints. For vector and real linear superfield, Dirac-Born-Infeld action was constructed in refs. [\[5](#page-12-4)[–7\]](#page-12-5). Their generalizations to SUGRA have also been

discussed in refs. [\[8](#page-12-6)[–13\]](#page-12-7). For a single chiral superfield, some types of ghost-free higherderivative terms have been known [\[14](#page-12-8), [15\]](#page-12-9). Their cosmological applications, especially to the inflationary universe, have also been discussed in refs. [\[16](#page-12-10)[–21](#page-13-0)]. BPS states in supersymmetric higher derivative chiral models were also studied, such as baby-Skyrmions [\[22](#page-13-1)[–26\]](#page-13-2) and various BPS topological solitons in the higher derivative chiral model [\[24\]](#page-13-3) coupled with gauge theory [\[27\]](#page-13-4). The other examples are low-energy effective action of supersymmetric QCD [\[28](#page-13-5)[–31\]](#page-13-6), the Wess-Zumino-Witten (WZW) term in the supersymmetric chiral Lagrangian [\[32](#page-13-7)[–38\]](#page-14-0), supersymmetric nonlinear realizations of spontaneously broken global symmetry [\[39](#page-14-1)], the low-energy effective theory on BPS solitons [\[40](#page-14-2)], k-field theories [\[41](#page-14-3), [42\]](#page-14-4), the Faddeev-Skyrme model [\[43,](#page-14-5) [44](#page-14-6)], and (pure) Skyrme model [\[45](#page-14-7)]. The low-energy effecitive action with loop effects has infinite numbers of derivative operators [\[46](#page-14-8)[–48](#page-14-9)]. A ghost free higher derivative theory is also studied recently in the framework of non-local field theories [\[49\]](#page-14-10).

In general, higher derivative terms in supersymmetric theories suffer from another problem specific for supersymmetry, namely the auxiliary field problem: the auxiliary field F (F-term) of a chiral superfield may propagate because of space-time derivative terms acting on it, and consequently it cannot be eliminated algebraically. This problem was seriously recognized [\[33](#page-13-8)[–37\]](#page-14-11) for the WZW term. As systematically studied in ref. [\[14](#page-12-8)], the models mentioned in the above paragraph are free from this problem (except for that in ref. $[44]$). In ref. $[50]$ (see also $[51]$), a very important model with a single chiral superfield with higher derivative term was studied, in which the auxiliary field F of the chiral superfield in fact becomes dynamical. In this model, a ghost chiral superfield is induced due to the higher derivative term. Because of this model, it is widely believed without any proof that the above mentioned two problems are related: when an auxiliary field becomes dynamical, a ghost should be present and the theory is pathological.

In this paper, we present a first counterexample to such a conjecture, i.e. a ghost-free higher-derivative chiral model with a propagating auxiliary field $F¹$ $F¹$ $F¹$. We achieve this by removing a ghost in the higher derivative chiral model in ref. [\[50\]](#page-14-12); the ghost is Higgsed away by a non-dynamical auxiliary vector superfield V associated with a $U(1)$ gauge symmetry.^{[2](#page-2-1)} As we will show, with appropriate couplings of chiral and gauge superfields, the ghost degrees of freedom can be removed thanks to the constraints and gauge degree of freedom. Depending on the sign of the quadratic derivative term of the chiral superfield, the model contains two ghost free branches of the parameter regions. We find that supersymmetry is spontaneously broken in one branch while it is preserved in the other branches, which are totally unexpected in the original Lagrangian. The auxiliary field F in the original chiral superfield is now in the lowest component of another chiral superfield after "unfolding" the

¹The propagating auxiliary fields have been discussed in the context of the higher-curvature SUGRA model [\[52](#page-14-14)[–56\]](#page-15-0), in which the kinetic term of auxiliary fields in the gravity multiplet exists due to higherderivative terms.

²Such auxiliary vector superfields were used to formulate supersymmetric nonlinear sigma models in terms of gauge theories [\[57](#page-15-1)[–59](#page-15-2)]. When V has no kinetic term, its vector component plays a role of an auxiliary field, and gaugino and auxiliary components behave as Lagrange multiplier fields, which give rise to constraints on the coupled superfields. Also, we need to fix the gauge degree of freedom.

higher derivative term to two chiral superfields with second derivative terms. As a consequence, a dynamical auxiliary field allows an unexpected $U(1)$ symmetry and associated conserved charge, which are not manifest in the original Lagrangian. This can be regarded as a generalization of the R-symmetry, so we may call it an R'-symmetry.

The remaining part is organized as follows. In section [2,](#page-3-0) we review a SUSY model with a higher-derivative term, which produces a ghost mode. We also find that an auxiliary field of a chiral superfield obtain its kinetic term, which is an additional mode due to a SUSY higher derivative. We extend the higher-derivative action to that coupled to a gauge superfield in section [3.](#page-4-0) We show how the ghost can be removed by such an extension, and find that, even after eliminating a ghost superfield, the superfield originated from a dynamical auxiliary field remains in the resultant system. In section [4,](#page-8-1) we briefly discuss some features of the resultant system. Finally, we conclude in section [5.](#page-10-0)

2 SUSY higher-derivative ghost

In this section, we review a model with a higher-derivative term inducing ghost modes. As an illustration of our new proposal, we will extend the model to that with a gauge superfield in the next section. Here, let us consider the following higher-derivative Lagrangian,

$$
\mathcal{L} = \int d^4\theta \left[\Phi \overline{\Phi} + \frac{\alpha}{\Lambda^2} D^2 \Phi \overline{D}^2 \overline{\Phi} \right]
$$
 (2.1)

where Λ is a real constant of mass dimension one, and α is a dimensionless real parameter. A chiral superfield Φ is defined as

$$
\Phi = \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y)
$$
\n
$$
= \phi(x) + \sqrt{2}\theta\psi(x) + \theta\theta F(x) + i\theta\sigma^m\overline{\theta}\partial_m\phi(x) - \frac{i}{\sqrt{2}}\theta\theta\partial_m\psi(x)\sigma^m\overline{\theta} + \frac{1}{4}\theta\theta\overline{\theta}\overline{\theta}\Box\phi(x),
$$
\n(2.2)

where $y^m = x^m + i\theta\sigma^m\overline{\theta}$. In terms of the component fields, the explicit form of the Lagrangian is given by

$$
\mathcal{L} = -\partial_{\mu}\phi\partial^{\mu}\bar{\phi} - i\bar{\psi}\bar{\sigma}^{\mu}\partial_{\mu}\psi + |F|^{2} + \frac{16\alpha}{\Lambda^{2}}\left(-\partial_{\mu}F\partial^{\mu}\bar{F} - i\Box\bar{\psi}\bar{\sigma}^{\mu}\partial_{\mu}\psi + |\Box\phi|^{2}\right). \tag{2.3}
$$

This Lagrangian has the U(1)³ symmetry corresponding to the phases of (ϕ, ψ, F) . The rotation of overall phase is the $U(1)$ symmetry which commutes with SUSY, whereas the R-symmetry is the phase rotation with the charges $(0, 1, 2)$. The other U(1) symmetry, which we call the R'-symmetry, is the phase rotation of F , whose conserved charge is nontrivial due to the presence of the kinetic term of F . Note that this symmetry exists in theories without the dynamical F-term field but its conserved charge vanishes on-shell.

With the technique called "unfolding" [\[50,](#page-14-12) [51](#page-14-13)], we can rewrite this Lagrangian into that without higher-derivative terms as follows: using a Lagrange multiplier chiral superfield Φ_1 , the Lagrangian (2.1) can be rewritten as

$$
\mathcal{L} = \int d^4\theta \left[\Phi \overline{\Phi} + \alpha \Phi_2 \overline{\Phi}_2 \right] + \left\{ \frac{\Lambda}{4} \int d^2\theta \, \Phi_1 \left(\Phi_2 - \frac{1}{\Lambda} \overline{D}^2 \overline{\Phi} \right) + \text{h.c.} \right\},\tag{2.4}
$$

where Φ_2 is a chiral superfield. For later convenience, we have chosen the normalization of Φ_1 so that the overall coefficient of the superpotential becomes $\Lambda/4$. The variation with respect to Φ_1 gives the constraint $\Phi_2 = \frac{1}{\Lambda} \overline{D}^2 \overline{\Phi}$, which reproduces the original Lagrangian [\(2.1\)](#page-3-1). Instead, if we use the following identity,

$$
\int d^2\theta \, \overline{D}^2(\Phi_1 \overline{\Phi}) + \text{h.c.} = \int d^4\theta \, (-4\Phi_1 \overline{\Phi} + \text{h.c.}) \quad \text{(up to total derivative terms)}, \quad (2.5)
$$

the Lagrangian [\(2.4\)](#page-3-2) becomes

$$
\mathcal{L} = \int d^4 \theta \left[\Phi \overline{\Phi} + (\Phi_1 \overline{\Phi} + \text{h.c.}) + \alpha \Phi_2 \overline{\Phi}_2 \right] + \left\{ \frac{\Lambda}{4} \int d^2 \theta \left(\Phi_1 \Phi_2 \right) + \text{h.c.} \right\}
$$

\n
$$
= \int d^4 \theta \left[|\Phi + \Phi_1|^2 - \Phi_1 \overline{\Phi}_1 + \alpha \Phi_2 \overline{\Phi}_2 \right] + \left\{ \frac{\Lambda}{4} \int d^2 \theta (\Phi_1 \Phi_2) + \text{h.c.} \right\}
$$

\n
$$
= \int d^4 \theta \left[\tilde{\Phi} \overline{\Phi} - \Phi_1 \overline{\Phi}_1 + \alpha \Phi_2 \overline{\Phi}_2 \right] + \left\{ \frac{\Lambda}{4} \int d^2 \theta (\Phi_1 \Phi_2) + \text{h.c.} \right\},
$$
 (2.6)

where

$$
\tilde{\Phi} \equiv \Phi + \Phi_1. \tag{2.7}
$$

From eq. [\(2.6\)](#page-4-2), we find that Φ_1 has a negative definite kinetic coefficient, that is, Φ_1 is a ghost superfield. Depending on the sign of α , Φ_2 is either a ghost or regular superfield.

Let us focus on Φ_2 . In our discussion above, Φ_2 came from $\overline{D}^2\overline{\Phi}$, whose lowest component is F^{Φ} . Indeed, the component expression of the second term in the action (2.1) is given by

$$
\int d^4\theta \left(D^2 \Phi \overline{D}^2 \overline{\Phi} \right) = 16 \Big(F \Box \overline{F} + \Box \phi \Box \overline{\phi} - i \Box \overline{\psi} \overline{\sigma}^n \partial_n \psi \Big). \tag{2.8}
$$

Thus, we can identify Φ_2 as the "dynamical" F-component due to the SUSY higherderivative contribution. It is important to note again that the presence of the higherderivative term here is problematic since at least one ghost mode appears, irrespective of the value of α .

3 Removing ghost and dynamical F-term

3.1 Gauged model

In this section, we discuss a possible modification of the higher-derivative system. A gauge symmetry is an important notion to remove some degree of freedom. We consider the case that the chiral superfield Φ is gauged under a U(1) symmetry. We introduce a gauge superfield V for the U(1) gauge symmetry under which the superfields Φ and V transform as

$$
\Phi \to \Phi e^{-2i\Lambda},\tag{3.1}
$$

$$
V \to V + i(\Lambda - \overline{\Lambda}), \tag{3.2}
$$

where Λ is a gauge parameter chiral superfield. The component expression of V in Wess-Zumino gauge is

$$
V = -\theta \sigma^m \overline{\theta} v_m(x) + i\theta \theta \overline{\theta} \overline{\lambda}(x) - i\overline{\theta} \overline{\theta} \theta \lambda(x) + \frac{1}{2} \theta \theta \overline{\theta} \overline{\theta} \mathcal{D}(x).
$$
 (3.3)

The $U(1)$ invariant extension of the Lagrangian (2.1) is given by

$$
\mathcal{L}' = \int d^4\theta \left[\Phi e^{2V} \overline{\Phi} + \frac{\alpha}{\Lambda^2} \left(D^2 (\Phi e^{2V}) \right) e^{-2V} \left(\overline{D}^2 (\overline{\Phi} e^{2V}) \right) + 2CV \right],\tag{3.4}
$$

where we have introduced a possible Fayet-Iliopoulos (FI) parameter C. The higherderivative superfield $\overline{D}^2(\overline{\Phi}e^{2V})$ is a chiral superfield, whose component is given by

$$
\overline{D}^2(\overline{\Phi}e^{2V}) = 4\Big[-\overline{F}(y) + \sqrt{2}\theta\Big(i\sigma^\mu D_\mu \overline{\psi}(y) - \sqrt{2}\lambda\overline{\phi}\Big) - \theta\theta\Big(\Box\overline{\phi}(y) + \mathcal{D}(y)\overline{\phi}(y) - \sqrt{2}i\overline{\lambda}(y)\overline{\psi}(y)\Big)\Big],\tag{3.5}
$$

where the covariant derivatives are $D_{\mu}\overline{\psi} = \partial_{\mu}\overline{\psi} - iv_{\mu}\overline{\psi}$, and $\Box \overline{\phi} = D^{\mu}D_{\mu}\overline{\phi}$. Rescaling $V \to gV$ and taking the limit of $g \to 0$, we obtain the component of $\overline{D}^2 \overline{\Phi}$.

3.2 Component expression

First, we illustrate how the ghost mode in the Lagrangian [\(2.1\)](#page-3-1) can be removed by the extension [\(3.4\)](#page-5-1). For simplicity, we focus on the bosonic part of the Lagrangian [\(3.4\)](#page-5-1), which is given by

$$
\mathcal{L}'|_{B} = -D_{\mu}\phi D^{\mu}\overline{\phi} + |F|^{2} + \mathcal{D}\left(|\phi|^{2} + C\right) + \frac{16\alpha}{\Lambda^{2}} \Big[-D_{\mu}FD^{\mu}\overline{F} + |\Box\phi + \mathcal{D}\phi|^{2} - \mathcal{D}|F|^{2} \Big],\tag{3.6}
$$

where we have used the component expression of V in the Wess-Zumino gauge (3.3) , $D_{\mu}F = \partial_{\mu}F + iA_{\mu}F$ and $\Box \phi = D^{\mu}D_{\mu}\phi$. To extract the ghost mode, we use the following trick: the Lagrangian (3.6) can be rewritten as

$$
\mathcal{L}'|_B = -D_\mu \phi D^\mu \overline{\phi} + |F|^2 + \mathcal{D} \left(|\phi|^2 + C \right) + \frac{16\alpha}{\Lambda^2} \Big[-D_\mu F D^\mu \overline{F} - \mathcal{D} |F|^2 \Big] - \frac{\Lambda^2}{16\alpha} |\phi_1|^2 + \Big[\overline{\phi_1} (\Box \phi + \mathcal{D}\phi) + \text{h.c.} \Big],
$$
 (3.7)

where ϕ_1 is a scalar field with the same U(1) charge as ϕ . It can be easily shown that the variation of ϕ_1 reproduces the Lagrangian [\(3.6\)](#page-5-2). Here, we perform partial integration for the terms on the second line, which gives

$$
\mathcal{L}'|_{B} = -D_{\mu}\phi D^{\mu}\overline{\phi} - \left\{D_{\mu}\overline{\phi}_{1}D^{\mu}\phi + \text{h.c.}\right\} - \frac{16\alpha}{\Lambda^{2}}D_{\mu}FD^{\mu}\overline{F} \n+ \mathcal{D}\left[|\phi|^{2} + \left\{\phi\overline{\phi}_{1} + \text{h.c.}\right\} + \frac{16\alpha}{\Lambda^{2}}|F|^{2} + C\right] + |F|^{2} - \frac{\Lambda^{2}}{16\alpha}|\phi_{1}|^{2}.
$$
\n(3.8)

We find that the determinant of the kinetic coefficient matrix of ϕ and ϕ_1 is negative as with the case of the ungauged model. However, we also find that D appears only linearly, and its E.O.M. gives a constraint on the scalar fields. In addition, we have the $U(1)$ gauge symmetry which implies that there is a redundancy in the description of our model. Therefore, we can remove one complex scalar from the system by solving the constraint from \mathcal{D} , and eliminating the auxiliary vector field A_{μ} . As we will see below, such a procedure can be simplified by using the superfield formalism. In the next subsection, we show by solving the E.O.M for the auxiliary vector superfield that the apparent ghost can be eliminated by the gauge symmetry.

3.3 Superfield calculation

3.3.1 The first model

In this subsection, we show that the ghost mode can be gauged away by eliminating the auxiliary vector superfield. As with the case of the original model [\(2.4\)](#page-3-2), we introduce chiral superfields Φ_1 and Φ_2 . In this case, they should have U(1) charges so that they transform

$$
\Phi_1 \to \Phi_1 e^{-2i\Lambda},\tag{3.9}
$$

$$
\Phi_2 \to \Phi_2 e^{2i\Lambda}.\tag{3.10}
$$

Using these superfields, we can construct the gauged version of the Lagrangian [\(2.4\)](#page-3-2),

$$
\mathcal{L} = \int d^4 \theta \left[\Phi e^{2V} \overline{\Phi} + \alpha \Phi_2 e^{-2V} \overline{\Phi}_2 + 2CV \right] + \left[\frac{\Lambda}{4} \int d^2 \theta \Phi_1 \left\{ \Phi_2 - \frac{1}{\Lambda} \overline{D}^2 \left(\overline{\Phi} e^{2V} \right) \right\} + \text{h.c.} \right]
$$

=
$$
\int d^4 \theta \left[\tilde{\Phi} e^{2V} \overline{\tilde{\Phi}} - \Phi_1 e^{2V} \overline{\Phi}_1 + \alpha \Phi_2 e^{-2V} \overline{\Phi}_2 + 2CV \right] + \left\{ \frac{\Lambda}{4} \int d^2 \theta \left(\Phi_1 \Phi_2 \right) + \text{h.c.} \right\}, \quad (3.11)
$$

where $\tilde{\Phi} \equiv \Phi + \Phi_1$ is the same as that in eq. [\(2.6\)](#page-4-2). Note that this field redefinition is consistent with the U(1) symmetry since Φ and Φ_1 have the same U(1) charges.

The variation with respect to V yields a constraint equation,

$$
\tilde{\Phi}e^{2V}\overline{\tilde{\Phi}} - \Phi_1 e^{2V}\overline{\Phi}_1 - \alpha \Phi_2 e^{-2V}\overline{\Phi}_2 + C = 0.
$$
\n(3.12)

Note that we need to fix the $U(1)^{\mathbb{C}}$ gauge redundancy, by which we can set one of the superfields as a constant. From the lowest component of this superfield equation in Wess-Zumino gauge, we obtain the following constraint on scalar fields:

$$
|\tilde{\phi}|^2 - |\phi_1|^2 - \alpha |\phi_2|^2 + C = 0.
$$
\n(3.13)

For the consistency of this equation, at least one of the scalar field have to be nonzero. When ϕ_1 is nonzero, it is convenient to define the following gauge invariant superfields

$$
X = \frac{\tilde{\Phi}}{\Phi_1}, \qquad Y = \Phi_1 \Phi_2. \tag{3.14}
$$

Similarly, when ϕ or ϕ_1 is nonzero, we can define gauge invariant superfields which are related to (X, Y) by a field redefinition. For the moment, we assume that Φ_1 is nonzero.

Let us discuss solutions for eq. (3.12) . The formal solutions of eq. (3.12) are given by

$$
e^{2V} \equiv G_{\pm} = \frac{1}{2|\Phi_1|^2} \frac{C \pm f}{1 - |X|^2},\tag{3.15}
$$

where we have defined the function f as

$$
f = \sqrt{C^2 - 4\alpha |Y|^2 (1 - |X|^2)}.
$$
\n(3.16)

Substituting into the Kähler potential, we obtain

$$
K = \tilde{\Phi}e^{2V}\overline{\tilde{\Phi}} - \Phi_1e^{2V}\overline{\Phi}_1 + \alpha\Phi_2e^{-2V}\overline{\Phi}_2 + 2CV
$$

= $\mp f + C \log \frac{C \pm f}{1 - |X|^2} - C \log 2|\Phi_1|^2.$ (3.17)

Note that the last term is unphysical since it can be eliminated by a Kähler transformation. Since the Kähler potential is written in terms of $|X|^2$ and $|Y|^2$, it has a $U(1)^2$ holomorphic isometry which remains after gauging one $U(1)$ symmetry among the $U(1)³$ symmetry of the ungauged action [\(2.3\)](#page-3-3).

The Kähler metric for X and Y is given by

$$
K_{i\overline{j}} = \begin{pmatrix} K_{X\overline{X}} & K_{X\overline{Y}} \\ K_{Y\overline{X}} & K_{Y\overline{Y}} \end{pmatrix} = \pm \frac{\alpha}{f} \begin{pmatrix} H_{\pm} - |Y|^2 & -\overline{X}Y \\ -X\overline{Y} & 1 - |X|^2 \end{pmatrix},
$$
(3.18)

where the function H_+ is given by

$$
H_{\pm} = \frac{1}{4\alpha} \left(\frac{C \pm f}{1 - |X|^2} \right)^2.
$$
 (3.19)

To find out the condition for the positive definiteness of the Kähler metric, let us consider the following pair of linearly independent vectors

$$
\xi = \frac{\partial}{\partial Y}, \qquad \xi' = (1 - |X|^2) \frac{\partial}{\partial X} + \overline{X} Y \frac{\partial}{\partial Y}.
$$
 (3.20)

Since they are mutually orthogonal, the Kähler metric is positive definite if both of the following norms of the vectors are positive:

$$
K_{i\bar{j}}\xi^{i}\bar{\xi}^{\bar{j}} = \pm \frac{\alpha}{f}(1 - |X|^{2}), \qquad K_{i\bar{j}}\xi'^{i}\bar{\xi}'^{\bar{j}} = \frac{C \pm f}{2}.
$$
 (3.21)

In addition, the solution for the auxiliary vector superfield eq. [\(3.15\)](#page-6-3) also has to be positive, i.e.

$$
G_{\pm} = \pm \frac{\alpha f}{|\Phi_1|^2} \frac{K_{i\bar{j}} \xi'^i \bar{\xi}^j}{K_{i\bar{j}} \xi^i \bar{\xi}^j} > 0.
$$
 (3.22)

These conditions are satisfied if and only if we choose G_{+} and the following conditions are satisfied:

$$
\alpha > 0,
$$
 $C > 0,$ $|X|^2 < 1,$ $|Y|^2 \le \frac{C^2}{4\alpha} \frac{1}{1 - |X|^2}.$ (3.23)

The kinetic coefficients are positive around the region satisfying the conditions. In the discussion above, we have assumed that Φ_1 is nonzero. If $\Phi_1 = 0$, the condition [\(3.23\)](#page-7-0) is not satisfied $\Phi_1 \approx 0$ corresponds to the region $X \approx \infty$. Therefore, Φ_1 has to be nonzero in the physically consistent situation.

Let us discuss a particular region satisfying the conditions realizing a stable system. We focus on the field region around $X \simeq Y \simeq 0$ ($\tilde{\Phi} \simeq \Phi_2 \simeq 0$), where the Kähler potential [\(3.17\)](#page-6-4) is approximately given by

$$
K \simeq C|X|^2 + \frac{\alpha}{C}|Y|^2. \tag{3.24}
$$

Thus, both of superfields are not ghost-like, and the instability is completely removed. However, one needs also to check whether the resultant action is compatible with the conditions. In particular, the remaining superfields have a superpotential term, which gives rise to a scalar potential. In the following, we show two concrete regions satisfying the conditions and also consistent with the vacuum determined by the scalar potential.

Let us make a comment on the geometry of our model. The resultant target space M is a certain fiber bundle over a hyperbolic space,

$$
M \simeq D \ltimes \frac{\text{SU}(1,1)}{\text{U}(1)}\tag{3.25}
$$

where $F \ltimes B$ denotes a fiber bundle over a base B with a fiber F. The base is parameterized by X, while the fiber D is a disk parameterized by Y having a range determined by X in eq. [\(3.23\)](#page-7-0).

3.3.2 The second model

It is worth noting that the case with $\alpha < 0$, $C < 0$ has the same structure as the case with $\alpha > 0$, $C > 0$ if we exchange $\tilde{\Phi} \leftrightarrow \Phi_1$ and flip the sign of the Kähler potential $K \to -K$. Therefore, for $\alpha < 0$, $C < 0$, the Kähler metric is positive definite, i.e. the model is ghostfree. Since we have flipped the sign of the Kähler potential, the corresponding Lagrangian has to have the negative kinetic term for Φ

$$
\mathcal{L}'' = \int d^4\theta \left[-\Phi e^{2V} \overline{\Phi} + \frac{\tilde{\alpha}}{\Lambda^2} \left(D^2 (\Phi e^{2V}) \right) e^{-2V} \left(\overline{D}^2 (\overline{\Phi} e^{2V}) \right) + 2\tilde{C}V \right],\tag{3.26}
$$

where $\tilde{\alpha} = -\alpha > 0$ and $\tilde{C} = -C > 0$. This Lagrangian is similar to that in eq. [\(3.4\)](#page-5-1), but the signs of the first terms are opposite to each other. This is not difficult to understand by the following reason: from the procedure in section [2,](#page-3-0) we find that $\tilde{\Phi}$ and Φ_1 always have the opposite sign. The one with a negative sign is regarded as the ghost mode. In the case with [\(3.26\)](#page-8-3), we can identify $\tilde{\Phi}$ as the ghost, and remove it by our mechanism. The effective Kähler metric around $\tilde{X} \equiv \Phi_1/\tilde{\Phi} \simeq 0$, $\tilde{Y} \equiv \tilde{\Phi} \Phi_2 \simeq 0$ is given by

$$
K \simeq \tilde{C}|\tilde{X}|^2 + \frac{\tilde{\alpha}}{\tilde{C}}|\tilde{Y}|^2. \tag{3.27}
$$

This system clearly has no ghost modes, as is the case with $\alpha > 0$, $C > 0$ discussed above. Although it seems that there is no difference between these cases, the system has a completely different vacuum structure because of the difference of the superpotentials, as will be discussed in the next section.

4 Behaviour of dynamical F-term superfield and SUSY breaking/preserving vacua

In this section, we consider the structure of the vacuum in our model.

4.1 The first model: SUSY breaking vacuum

In the previous section, we have seen that there is no ghost mode if the conditions [\(3.23\)](#page-7-0) are satisfied. In this case, the superpotential is linear in the chiral superfield $Y = \Phi_1 \Phi_2$

$$
W = \frac{\Lambda}{4}Y.\t(4.1)
$$

Therefore, the supersymmetry is spontaneously broken due to the nonzero values of the F-terms of Y and the F-term scalar potential

$$
F^Y = -\frac{\Lambda}{4}, \qquad V = \frac{C\Lambda^2}{16\alpha}.
$$
\n(4.2)

It is worth noting that Φ_2 plays a role of a SUSY breaking field as the Polonyi model. This means that the "dynamical F-term" superfield Φ_2 breaks SUSY spontaneously, and the order of SUSY breaking is determined by the FI parameter C and the cut-off Λ .

Here, let us make a comment on SUSY breaking in different supermultiplets with higher derivative term. It is known that a complex linear superfield with its higher-derivative term is dual to a chiral and nilpotent-chiral superfield [\[60](#page-15-3)[–62](#page-15-4)] (see also refs. [\[63,](#page-15-5) [64](#page-15-6)]). The nilpotent-chiral superfield also spontaneously breaks SUSY, as Φ_2 in our case. The relation between the higher-derivative extension of complex linear and chiral superfield discussed here is quite interesting. To the best of our knowledge, our model gives the first example of such SUSY breaking by a higher derivative term in chiral multiplets. We will investigate extensions of this SUSY breaking mechanism elsewhere.

4.2 The second model: SUSY preserving vacuum

On the other hand, in the model [\(3.26\)](#page-8-3), the structure of the superpotential is essentially different. In terms of $\tilde{X} \equiv \Phi_1/\tilde{\Phi}$ and $\tilde{Y} \equiv \tilde{\Phi} \Phi_2$ the superpotential is given by

$$
W = \frac{\Lambda}{4} \tilde{X} \tilde{Y}.
$$
\n(4.3)

The F-term of \tilde{X} and \tilde{Y} are

$$
\overline{F}^{\tilde{X}} = -\frac{\Lambda}{4\tilde{C}} \tilde{Y},\tag{4.4}
$$

$$
\overline{F}^{\tilde{Y}} = -\frac{\Lambda \tilde{C}}{4\tilde{\alpha}} \tilde{X}.
$$
\n(4.5)

Since the scalar potential becomes

$$
V \sim \frac{\Lambda^2}{16\tilde{\alpha}} \left(\tilde{C} |\tilde{X}|^2 + \frac{\tilde{\alpha}}{\tilde{C}} |\tilde{Y}|^2 \right),\tag{4.6}
$$

the vacuum $\tilde{X} = \tilde{Y} = 0$ ($\Phi_1 = \Phi_2 = 0$) is stable and the F-terms do not have vacuum expectation values. Therefore, with the Lagrangian [\(3.26\)](#page-8-3), SUSY is preserved at the vacuum.

It is quite interesting that we can realize both SUSY preserving or breaking vacuum from the almost the same systems (3.4) and (3.26) . In the case (3.4) , the higher-derivative term induces the ghost, and after removing it, we obtain the SUSY breaking vacuum with a cosmological constant, as shown in eq. (4.2) . On the other hand, when we start with a ghost-like superfield with its higher-derivative term [\(3.26\)](#page-8-3), we finally obtain the model with a SUSY preserving vacuum. We will investigate this feature in more detail elsewhere.

Note that SUSY breaking in our model is different from that in SUSY ghost condensation [\[14](#page-12-8)], in which the violation of time translation invariance occurs. In our model, SUSY is broken in a Lorentz invariant manner.

5 Summary and discussion

In this paper, we have proposed a new mechanism to construct ghost-free higher-derivative models within global SUSY. The important notion is a non-dynamical gauge superfield, which "eats" the ghost mode in the system. We have illustrated our mechanism with an example shown in section [2,](#page-3-0) which has a ghost superfield. As shown in section [3,](#page-4-0) the ghost mode can be removed thanks to the non-dynamical gauge superfield. It has been shown that, independently of the sign of a kinetic term of Φ , the higher-derivative system has one normal and one ghost mode, and we can remove a ghost superfield in the both cases. Interestingly enough, however, the resultant systems after removing a ghost are completely different from each other as discussed in section [4.](#page-8-1) In particular, the vacuum structures are different: one gives a SUSY breaking vacuum, and the other gives a SUSY preserving vacuum. The former is the first example of SUSY breaking induced by a higher derivative term in chiral superfields. One of the most interesting features is that because of the higher derivative term including space-time derivative on the F-term in the original chiral superfield Φ, the F-term becomes dynamical and resides in the lowest component of the chiral superfield Φ_2 .

The remaining question is the physical meaning of the a propagating F -term field F . As mentioned, the F-term is now in the lowest component of the chiral superfield Φ_2 , and so the structures of the SUSY multiplets are completely different from the original multiplets in the absence of the higher derivative term $(\alpha = 0)$. One of consequences of the propagating F-term is, as shown in this paper, the existence of the $U(1)$ conserved charge associated with the phase of F. It should be important to study more consequences for instance the structure of SUSY algebra and so on. The physical meaning of the SUSY breaking vacuum is unclear.

We have illustrated our mechanism of eliminating a ghost and introducing a dynamical auxiliary field in the simplest example. One of straightforward extensions is multiple chiral superfields. When the superderivative D^2 acts on n chiral superfields, there will be at most n ghost fields. Therefore, we need at least $U(1)^n$ gauging. With this regards, a higher derivative $\mathbb{C}P^1$ model in ref. [\[44\]](#page-14-6) contains two chiral superfields and only one U(1) gauge field. Consequently only one of two ghosts would be removed but the rest would remain, and so the theory is pathological. Another possible extension is a non-Abelian extension. For instance, if the original Lagrangian contains an N by N matrix chiral superfield, a "non-Abelian" (matrix) ghost superfield appears. This could be removed by $U(N)$ gauging. It is desired to construct a general framework by classifying how many ghosts non-gauged theories have, and which gauging $(U(1), U(N))$ or other gauge groups) can remove those ghosts. In other words, it would be very important to construct "generalized Nambu-Goldstone theorem" including ghosts and "generalized Higgs mechanism."

In the language of the Kähler geometry, eliminating vector superfield is known as the Kähler quotient. Usually this has been studied very well for positive norm metrics. Generalizing the Kähler quotient to include negative norm metrics should be a key point to understand the whole theory geometrically.

Our vector superfield is a non-dynamical and auxiliary field behaving as a Lagrange multiplier. If we add a kinetic term of the vector superfield, a gauge field absorbing a ghost will have a tachyonic mass, so still having the instability (our case can be understood as sending away such the tachyonic mass to infinity). The auxiliary field formulation of nonlinear sigma models in lower $(1+1 \text{ or } 2+1)$ dimensions often results in a kinetic term of the gauge field by the quantum effect, as can be explicitly shown in the large- N limit. If it was the same for our model in lower dimensions, there would be the quantum mechanically induced instability which is absent at the tree level. It is very interesting to study whether the instability exists or not in the quantization of dimensionally reduced model (or even the 3+1 dimensional theory as a cut-off theory).

In the formulation of (supersymmetric) $\mathbb{C}P^n$ model in terms of an auxiliary $U(1)$ gauge field (vector superfield), a vortex (flux tube) carrying the $U(1)$ gauge magnetic field is nothing but a $\mathbb{C}P^n$ sigma model lump (instanton). Whether our model admits such a lump and its stability (if it exists) are an interesting question.

As a non-dynamical gauge superfield and a propagating F-term regard, it is worth mentioning the similarity between our mechanism and a compensator in conformal SUGRA. In conformal SUGRA, we usually introduce a ghost-like superfield called a compensator [\[65](#page-15-7), [66\]](#page-15-8). The compensator is removed by the conformal gauge degrees of freedom, and finally the system does not have any ghost-like mode. The gauge fields of conformal symmetries are non-propagating as the gauge superfield in our mechanism. From this viewpoint, the ghost mode in our model is similar to the compensator, and the non-dynamical gauge superfield to the conformal gauge fields. In addition, the propagating auxiliary fields have been discussed in the context of the higher-curvature SUGRA model [\[52](#page-14-14)[–56\]](#page-15-0), where auxiliary fields in the gravity multiplet obtain the kinetic term due to higher-derivative terms of the gravity multiplet. Therefore, the presence of dynamical auxiliary fields may not be problematic.

Finally, the coupling of our model to SUGRA should be interesting for applications to cosmology such as inflationary models.

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