

Higgs pair productions in the CP-violating two-Higgs-doublet model

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ABSTRACT: In this work, we study the SM-like Higgs pair productions in the framework of the general CP-violating two-Higgs-doublet model. Several constraints are imposed to the model sequentially, including the SM-like Higgs boson signal fits, the precise measurements of the electric dipole moments, the perturbative unitarity and stability bounds to the Higgs potential, and the most recent LHC searches for the heavy Higgs bosons. We show how the CP-violating mixing angles are related to the Higgs cubic self couplings in this setup. Based on these constraints, we suggest benchmark models for the future high-energy collider searches for the Higgs pair productions. The e^+e^- colliders operating at $\sqrt{s} = (500 \text{ GeV}, 1 \text{ TeV})$ are capable of measuring the Higgs cubic self couplings of the benchmark models directly. Afterwards, we estimate the cross sections of the resonance contributions to the Higgs pair productions for the benchmark models at the future LHC and SppC/Fcc-hh runs. Other possible decay modes for the heavy Higgs bosons are also discussed.

KEYWORDS: Beyond Standard Model, CP violation, Higgs Physics

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1 Introduction

The discovery of the 125 GeV Higgs boson [1, 2] at the LHC runs at $7\oplus 8$ TeV validate Higgs mechanism for the spontaneous breaking of the electroweak gauge symmetry (EWSB). The current LHC measurements of the Higgs boson couplings to the SM fermions, gauge bosons, and loop-induced couplings to photons and gluons reach the precision of $\sim 10 - 20\%$ level. Besides, it is important to probe the Higgs self couplings to confirm the mechanism of the EWSB. This can be done by looking for the Higgs pair productions at both high-energy e^+e^- and pp colliders. The current LHC searches for the Higgs pair productions focus on the leading production channel of gluon-gluon fusion (ggF), as well as the promising final states of $b\bar{b}\gamma\gamma$. Some of the detailed studies at the LHC can be found in refs. [3–13]. From the experimental side, it is well-known that several future high-energy collider programs, such as the International Linear Collider (ILC) [14] in Japan, the Future eplus-eminus/hadron-hadron Cicular Collider (Fcc-ee/Fcc-hh) [15] at CERN, and the Circular electron-positron

Collider (CEPC)/ Super- pp -Collider(SppC) [16] in China, have been proposed in recent years. A key physical goal for these different high-energy collider programs is try to probe the shape of the Higgs potential. Some of the recent studies of the Higgs pair searches at the future colliders can be found in refs. [17–29].

In many of new physics models beyond the SM (BSM), the Higgs sector is extended with several scalar multiplets. The two-Higgs-doublet model (2HDM) is one attractive alternative to the SM, which allows for new phenomena in the scalar sector [30]. To discover another Higgs doublet in the future LHC experiments, a lot of efforts have been taken for the $t\bar{t}$ searches with the novel techniques [31–34], the decays of hZ final states [35], and the charged Higgs searches [36] as well.¹ Most of the current studies focus on the CP-conserving (CPC) version of 2HDM. Originally, the 2HDM was motivated to offer extra CP-violation (CPV) sources from the scalar sector [39]. Recently, it was also pointed out that the CPV 2HDM is likely to realize the EW baryogenesis [40], which is one of the most popular solutions to the baryon asymmetry in the Universe. Three neutral Higgs bosons, denoted as (h_1, h_2, h_3) , mix with each other in the CPV 2HDM. There are two angles of α_b and α_c to parametrize the size of the CPV effects, and the CPC limit can be easily restored by taking $\alpha_b = \alpha_c = 0$. The 125 GeV SM-like Higgs boson, often chosen to be h_1 in the spectrum, is a mixture of both CP-even and CP-odd states [41–45]. Such CPV couplings for the SM-like Higgs bosons are subject to the constraints from the searches for the electric dipole moments (EDMs) of the neutron, atoms, and molecules.² One of the most stringent one is from the ACME collaboration [48], where they reported an upper limit on the electron EDM (eEDM) of $|d_e/e| < 8.7 \times 10^{-29}$ cm. This bound can be translated to constrain the size of the CPV mixing through the Barr-Zee type diagrams. More specifically, we find that the sizes of the CPV mixings also determine the sizes of the Higgs cubic self couplings. Together with other existing constraints to the CPV 2HDM, which include the 125 GeV Higgs boson signal strengths, the perturbative unitarity and stability of the Higgs potential, and the constraints from the LHC searches for the heavy Higgs bosons, one can find the constraints to the heavy Higgs boson mass ranges and the sizes of the Higgs cubic self couplings. Therefore, the cross sections of the Higgs pair productions in the CPV 2HDM can be envisioned for the future experimental searches at the LHC and the SppC.

This paper aims to study the Higgs pair productions in the framework of the CPV 2HDM, including the precise measurement of the SM-like Higgs cubic self couplings at the e^+e^- colliders, and the resonance contributions in the gluon-gluon fusion (ggF) production channel at the pp colliders. The layout of this paper is described as follows. In section 2, we review the setup of the CPV 2HDM. With the assumptions of the degenerate heavy Higgs boson mass spectrum, we take the simplified parameter sets of $\alpha = -\pi/4$. We also obtain the gauge couplings, Yukawa couplings, and the self couplings for Higgs bosons in the physical basis. In section 3, we impose series of constraints to the CPV 2HDM parameter space. The combined constraints of 125 GeV Higgs signals and the eEDM bounds point

¹See also refs. [37, 38] for recent summaries of various search modes in the 2HDM at the LHC 14 TeV experiments.

²See, e.g., refs. [46, 47] for recent reviews.

to the $t_\beta \sim 1$ parameter choice. The size of the CPV mixing angle $|\alpha_b|$ is also bounded from above. For the CPV 2HDM-I, the CPV mixing is stringently constrained to be $|\alpha_b| \lesssim 5 \times 10^{-3}$, which is quite approaching to the CPC limit. For the CPV 2HDM-II, the constraints to the CPV mixing are much relaxed, and we focus on this case for the Higgs pair productions. The constraints from the unitarity, the stability, and the current LHC 8 TeV searches for the heavy Higgs bosons further restrict the allowed mass ranges of the heavy Higgs bosons and the soft \mathbb{Z}_2 -breaking mass term of m_{soft} . The main results of the Higgs pair productions in the CPV 2HDM are presented in section 4. By combining the current constraints, we show that the variations of the Higgs cubic self couplings are controlled by the size of the CPV mixing angle $|\alpha_b|$ and the soft mass term m_{soft} in the 2HDM potential. A set of benchmark models are given with the fixed CPV mixing angles and the maximally allowed soft mass terms. Under the small CPV limit, the Higgs cubic self coupling of λ_{111} for the SM-like Higgs boson tends to the SM predicted value of $\lambda_{hhh}^{\text{SM}} \simeq 32 \text{ GeV}$, and the resonance contributions become negligible as well. The corresponding Higgs pair production cross sections will tend to the predictions for the SM case. We estimate the physical opportunities of the precise measurement of the SM-like Higgs cubic self coupling λ_{111} at the future high-energy e^+e^- colliders, with focus on the $e^+e^- \rightarrow hhZ$ process at the $\sqrt{s} = 500 \text{ GeV}$ run. On the other hand, the heavy resonance contributions to the Higgs pair productions can become dominant at the pp colliders. The cross sections for the possible experimental search modes of $h_1h_1 \rightarrow (b\bar{b}\gamma\gamma, b\bar{b}WW)$ are estimated for both LHC 14 TeV and SppC/Fcc-hh 100 TeV runs. In addition, several other possible search modes of (W^+W^-, ZZ, hZ) are also mentioned. The conclusions and discussions are given in section 5.

2 The CPV 2HDM

2.1 The CPV 2HDM potential

In the general 2HDM, two Higgs doublets of $(\Phi_1, \Phi_2) \in 2_{+1}$ are introduced in the scalar sector. For simplicity, we consider the soft breaking of a discrete \mathbb{Z}_2 symmetry, under which two Higgs doublets transform as $(\Phi_1, \Phi_2) \rightarrow (-\Phi_1, \Phi_2)$. The corresponding Lagrangian is expressed as

$$\mathcal{L} = \sum_{i=1,2} |D\Phi_i|^2 - V(\Phi_1, \Phi_2), \tag{2.1a}$$

$$V(\Phi_1, \Phi_2) = m_{11}^2|\Phi_1|^2 + m_{22}^2|\Phi_2|^2 - (m_{12}^2\Phi_1^\dagger\Phi_2 + H.c.) + \frac{1}{2}\lambda_1|\Phi_1|^4 + \frac{1}{2}\lambda_2|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_1^\dagger\Phi_2|^2 + \frac{1}{2}\left[\lambda_5(\Phi_1^\dagger\Phi_2)^2 + H.c.\right], \tag{2.1b}$$

with (m_{12}^2, λ_5) being complex and all other parameters being real for the CPV 2HDM. After the EWSB, two Higgs doublets Φ_1 and Φ_2 in the unitarity gauge can be expressed as

$$\Phi_1 = \begin{pmatrix} -s_\beta H^+ \\ \frac{1}{\sqrt{2}}(v_1 + H_1^0 - is_\beta A^0) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} c_\beta H^+ \\ \frac{1}{\sqrt{2}}(v_2 e^{i\xi} + H_2^0 + ic_\beta A^0) \end{pmatrix}, \tag{2.2}$$

where $v_1^2 + v_2^2 = v^2 = (\sqrt{2} G_F)^{-1}$. The ratio between two Higgs VEVs is parametrized as

$$t_\beta \equiv \tan \beta = \frac{v_2}{v_1}, \quad (2.3)$$

and ξ represents the relative phase between two Higgs doublets. The imaginary components of m_{12}^2 and λ_5 are the source of CP violation, which lead to the mixings among three neutral states as $(h_1, h_2, h_3)^T = \mathcal{R}(H_1^0, H_2^0, A^0)^T$. Explicitly, the 3×3 mixing matrix is expressed as [50]

$$\begin{aligned} \mathcal{R} &= \mathcal{R}_{23}(\alpha_c) \mathcal{R}_{13}(\alpha_b) \mathcal{R}_{12}\left(\alpha + \frac{\pi}{2}\right) \\ &= \begin{pmatrix} -s_\alpha c_{\alpha_b} & c_\alpha c_{\alpha_b} & s_{\alpha_b} \\ s_\alpha s_{\alpha_b} s_{\alpha_c} - c_\alpha c_{\alpha_c} & -s_\alpha c_{\alpha_c} - c_\alpha s_{\alpha_b} s_{\alpha_c} & c_{\alpha_b} s_{\alpha_c} \\ s_\alpha s_{\alpha_b} c_{\alpha_c} + c_\alpha s_{\alpha_c} & s_\alpha s_{\alpha_c} - c_\alpha s_{\alpha_b} c_{\alpha_c} & c_{\alpha_b} c_{\alpha_c} \end{pmatrix}. \end{aligned} \quad (2.4)$$

The angle α parametrizes the mixing between two CP-even states of (H_1^0, H_2^0) . The CPV mixing angles of α_b and α_c parametrize the CP mixings between (H_1^0, A^0) and (H_2^0, A^0) , respectively. Their ranges are taken as

$$-\frac{\pi}{2} \leq \alpha_b \leq \frac{\pi}{2}, \quad -\frac{\pi}{2} \leq \alpha_c \leq \frac{\pi}{2}. \quad (2.5)$$

In the CPC limit, one has $\alpha_b = \alpha_c = 0$. Correspondingly, \mathcal{R} becomes block diagonal, and (h_1, h_2) are purely CP-even states.

By minimizing the CPV 2HDM potential, one obtains the following relations for the mass parameters

$$m_{11}^2 = \text{Re}(m_{12}^2 e^{i\xi}) t_\beta - \frac{1}{2} \left[\lambda_1 v^2 c_\beta^2 + (\lambda_3 + \lambda_4) v^2 s_\beta^2 + \text{Re}(\lambda_5 e^{2i\xi}) v^2 s_\beta^2 \right], \quad (2.6a)$$

$$m_{22}^2 = \text{Re}(m_{12}^2 e^{i\xi}) / t_\beta - \frac{1}{2} \left[\lambda_2 v^2 s_\beta^2 + (\lambda_3 + \lambda_4) v^2 c_\beta^2 + \text{Re}(\lambda_5 e^{2i\xi}) v^2 c_\beta^2 \right], \quad (2.6b)$$

$$\text{Im}(m_{12}^2 e^{i\xi}) = \frac{1}{2} v^2 s_\beta c_\beta \text{Im}(\lambda_5 e^{2i\xi}). \quad (2.6c)$$

The physical masses of (M_1, M_2, M_3, M_\pm) in the scalar spectrum are obtained from the 2HDM potential together with the minimization conditions given in eqs. (2.6). The charged Higgs boson mass squared reads

$$M_\pm^2 = \frac{1}{s_\beta c_\beta} \text{Re}(m_{12}^2 e^{i\xi}) - \frac{1}{2} \left[\lambda_4 + \text{Re}(\lambda_5 e^{2i\xi}) \right] v^2. \quad (2.7)$$

The mass squared matrix for the neutral sector can be expressed as

$$\mathcal{M}_0^2 = \begin{pmatrix} \lambda_1 c_\beta^2 + \nu s_\beta^2 & (\lambda_{345} - \nu) s_\beta c_\beta & -\frac{1}{2} \text{Im}(\lambda_5 e^{2i\xi}) s_\beta \\ (\lambda_{345} - \nu) s_\beta c_\beta & \lambda_2 s_\beta^2 + \nu c_\beta^2 & -\frac{1}{2} \text{Im}(\lambda_5 e^{2i\xi}) c_\beta \\ -\frac{1}{2} \text{Im}(\lambda_5 e^{2i\xi}) s_\beta & -\frac{1}{2} \text{Im}(\lambda_5 e^{2i\xi}) c_\beta & -\text{Re}(\lambda_5 e^{2i\xi}) + \nu \end{pmatrix} v^2, \quad (2.8)$$

with the short-handed notations of

$$\lambda_{345} \equiv \lambda_3 + \lambda_4 + \text{Re}(\lambda_5 e^{2i\xi}), \quad \nu \equiv \frac{\text{Re}(m_{12}^2 e^{i\xi})}{v^2 s_\beta c_\beta}. \quad (2.9)$$

By diagonalizing the mass squared matrix with the mixing matrix in eq. (2.4), one has

$$\mathcal{M}_0^2 = \mathcal{R}^T \text{diag}(M_1^2, M_2^2, M_3^2) \mathcal{R}, \quad (2.10)$$

from which one further obtains the relations to trade the quartic Higgs self couplings into the physical inputs as follows

$$\lambda_1 = \frac{M_1^2 \mathcal{R}_{11}^2 + M_2^2 \mathcal{R}_{21}^2 + M_3^2 \mathcal{R}_{31}^2}{v^2 c_\beta^2} - \nu t_\beta^2, \quad (2.11a)$$

$$\lambda_2 = \frac{M_1^2 \mathcal{R}_{12}^2 + M_2^2 \mathcal{R}_{22}^2 + M_3^2 \mathcal{R}_{32}^2}{v^2 s_\beta^2} - \nu / t_\beta^2, \quad (2.11b)$$

$$\lambda_3 = -\nu + \frac{2M_\pm^2}{v^2} + \frac{M_1^2 \mathcal{R}_{11} \mathcal{R}_{12} + M_2^2 \mathcal{R}_{21} \mathcal{R}_{22} + M_3^2 \mathcal{R}_{31} \mathcal{R}_{32}}{v^2 s_\beta c_\beta}, \quad (2.11c)$$

$$\lambda_4 = 2\nu - \frac{2M_\pm^2}{v^2} - \text{Re}(\lambda_5 e^{2i\xi}), \quad (2.11d)$$

$$\text{Re}(\lambda_5 e^{2i\xi}) = \nu - \frac{M_1^2 \mathcal{R}_{13}^2 + M_2^2 \mathcal{R}_{23}^2 + M_3^2 \mathcal{R}_{33}^2}{v^2}, \quad (2.11e)$$

$$\begin{aligned} \text{Im}(\lambda_5 e^{2i\xi}) = & -\frac{1}{v^2 s_\beta c_\beta} \left[(M_1^2 \mathcal{R}_{11} \mathcal{R}_{13} + M_2^2 \mathcal{R}_{21} \mathcal{R}_{23} + M_3^2 \mathcal{R}_{31} \mathcal{R}_{33}) c_\beta \right. \\ & \left. + (M_1^2 \mathcal{R}_{12} \mathcal{R}_{13} + M_2^2 \mathcal{R}_{22} \mathcal{R}_{23} + M_3^2 \mathcal{R}_{32} \mathcal{R}_{33}) s_\beta \right]. \end{aligned} \quad (2.11f)$$

For simplicity, we can always work in the basis where $\xi = 0$ by using the rephasing invariance. We also assume that $\text{Re}(m_{12}^2) \geq 0$, and use the notation for the soft mass term as

$$m_{\text{soft}}^2 \equiv \text{Re}(m_{12}^2). \quad (2.12)$$

The elements of $(\mathcal{M}_0^2)_{13}$ and $(\mathcal{M}_0^2)_{23}$ in eq. (2.8) provide the CPV mixings, which are related via t_β as

$$(\mathcal{M}_0^2)_{13} = (\mathcal{M}_0^2)_{23} t_\beta. \quad (2.13)$$

This leads to one additional constraint between mixing angles and mass eigenvalues as follows [50]

$$(M_1^2 - M_2^2 s_{\alpha_c}^2 - M_3^2 c_{\alpha_c}^2) s_{\alpha_b} (1 + t_\alpha) = (M_2^2 - M_3^2) (t_\alpha t_\beta - 1) s_{\alpha_c} c_{\alpha_c}. \quad (2.14)$$

In the analysis below, we always identify h_1 as the SM-like Higgs boson with mass of 125 GeV. We further simplify the parameter inputs by requiring all heavy Higgs boson masses are degenerate, i.e., $M_2 = M_3 = M_\pm \equiv M$. This was usually taken to relax the constraints from the electroweak precision measurements. The constraint of eq. (2.14) among the mixing angles becomes

$$\alpha_b = 0, \quad \text{or} \quad t_\alpha = -1. \quad (2.15)$$

Below, we will always take $\alpha = -\pi/4$.³ The input parameters of (β, α_b) will be determined through other constraints. Since α_c determines the size of the CPV mixing between two

³The study of the phenomenology with the CPV mixings of $|\alpha_b| \ll |\alpha_c|$ is carried out in a separate work [51].

mass-degenerate Higgs bosons of h_2 and h_3 in our setup, one can anticipate that α_c becomes unphysical in physical processes to be studied below. Without loss of generality, we always take $\alpha_c = 0$ for simplicity.

Thus, the set of input parameters can be summarized as follows

$$\begin{aligned}
 M_1 = 125 \text{ GeV}, \quad M_2 = M_3 = M_{\pm} = M, \quad m_{\text{soft}} \\
 \alpha = -\frac{\pi}{4}, \quad t_{\beta}, \quad \alpha_b, \quad \alpha_c = 0.
 \end{aligned}
 \tag{2.16}$$

Analogous to the CPC version of the general 2HDM, the parameter choice of $\beta - \alpha = \pi/2$ corresponds to the so-called ‘‘alignment limit’’. This can be achieved when taking into account the signal fit to the 125 GeV SM-like Higgs boson h_1 , as shown later. By further combining with the eEDM constraints, we will fix the parameters of t_{β} and α_b and constrain two other mass parameters of M and m_{soft} for our later discussions.

2.2 The couplings in the CPV 2HDM

For simplicity, we focus on the 2HDMs where the Yukawa sector has a \mathbb{Z}_2 symmetry and Φ_1 and Φ_2 each only gives mass to up-type quarks or down-type quarks and charged leptons. This is sufficient to suppress tree-level flavor changing processes mediated by the neutral Higgs bosons. The Yukawa couplings for the 2HDM-I and 2HDM-II read (and suppressing the CKM mixing),

$$\mathcal{L} = \begin{cases} -\left(\frac{c_{\alpha} m_u}{s_{\beta} v}\right) \bar{Q}_L \tilde{\Phi}_2 u_R - \left(\frac{c_{\alpha} m_d}{s_{\beta} v}\right) \bar{Q}_L \Phi_2 d_R + \text{h.c.} & \text{2HDM - I} \\ -\left(\frac{c_{\alpha} m_u}{s_{\beta} v}\right) \bar{Q}_L \tilde{\Phi}_2 u_R + \left(\frac{s_{\alpha} m_d}{c_{\beta} v}\right) \bar{Q}_L \Phi_1 d_R + \text{h.c.} & \text{2HDM - II,} \end{cases}
 \tag{2.17}$$

where $Q_L^T = (u_L, d_L)$ and $\tilde{\Phi}_2 \equiv i\sigma_2 \Phi_2^*$. For both cases, the charged lepton Yukawa coupling has the same form as that of the down-type quarks. Therefore, we can express the couplings between neutral Higgs bosons and the fermions and gauge bosons in the mass eigenbasis

$$\mathcal{L} = \sum_{i=1}^3 [-m_f (c_{f,i} \bar{f} f + \tilde{c}_{f,i} \bar{f} i \gamma_5 f) + a_i (2m_W^2 W_{\mu} W^{\mu} + m_Z^2 Z_{\mu} Z^{\mu})] \frac{h_i}{v}.
 \tag{2.18}$$

When $c_{f,i} \tilde{c}_{f,i} \neq 0$ or $a_i \tilde{c}_{f,i} \neq 0$, the mass eigenstate h_i couples to both CP-even and CP-odd operators, so the CP symmetry is violated. The coefficients of $c_{f,i}$, $\tilde{c}_{f,i}$ and a_i can be derived from the elements of the rotation matrix \mathcal{R} defined in eq. (2.4), which were also previously obtained in refs. [52–54]. Here, we summarize their explicit expressions under the alignment limit in table. 1. In this alignment limit of $\beta - \alpha = \pi/2$, the Higgs Yukawa couplings and Higgs gauge couplings are determined by the CPV mixing angles of (α_b, α_c) and t_{β} . By taking the CPC limit of $\alpha_b = \alpha_c = 0$, it is evident that (h_1, h_2) have the purely CP-even Yukawa couplings of $c_{f,i}$, while h_3 has the purely CP-odd Yukawa couplings of $\tilde{c}_{f,i}$. The previous studies of the collider measurements of the CPV in the Higgs Yukawa couplings can be found in refs. [43, 55–62].

	2HDM-I	2HDM-II
$c_{u,1}$	c_{α_b}	c_{α_b}
$c_{d,1} = c_{\ell,1}$	c_{α_b}	c_{α_b}
$\tilde{c}_{u,1}$	$-s_{\alpha_b}/t_\beta$	$-s_{\alpha_b}/t_\beta$
$\tilde{c}_{d,1} = \tilde{c}_{\ell,1}$	s_{α_b}/t_β	$-s_{\alpha_b} t_\beta$
a_1	c_{α_b}	c_{α_b}
$c_{u,2}$	$c_{\alpha_c}/t_\beta - s_{\alpha_b} s_{\alpha_c}$	$c_{\alpha_c}/t_\beta - s_{\alpha_b} s_{\alpha_c}$
$c_{d,2} = c_{\ell,2}$	$c_{\alpha_c}/t_\beta - s_{\alpha_b} s_{\alpha_c}$	$-s_{\alpha_b} s_{\alpha_c} - c_{\alpha_c} t_\beta$
$\tilde{c}_{u,2}$	$-c_{\alpha_b} s_{\alpha_c}/t_\beta$	$-c_{\alpha_b} s_{\alpha_c}/t_\beta$
$\tilde{c}_{d,2} = \tilde{c}_{\ell,2}$	$c_{\alpha_b} s_{\alpha_c}/t_\beta$	$-c_{\alpha_b} s_{\alpha_c} t_\beta$
a_2	$-s_{\alpha_b} s_{\alpha_c}$	$-s_{\alpha_b} s_{\alpha_c}$
$c_{u,3}$	$-s_{\alpha_c}/t_\beta - s_{\alpha_b} c_{\alpha_c}$	$-s_{\alpha_c}/t_\beta - s_{\alpha_b} c_{\alpha_c}$
$c_{d,3} = c_{\ell,3}$	$-s_{\alpha_c}/t_\beta - s_{\alpha_b} c_{\alpha_c}$	$-s_{\alpha_b} c_{\alpha_c} + s_{\alpha_c} t_\beta$
$\tilde{c}_{u,3}$	$-c_{\alpha_b} c_{\alpha_c}/t_\beta$	$-c_{\alpha_b} c_{\alpha_c}/t_\beta$
$\tilde{c}_{d,3} = \tilde{c}_{\ell,3}$	$c_{\alpha_b} c_{\alpha_c}/t_\beta$	$-c_{\alpha_b} c_{\alpha_c} t_\beta$
a_3	$-s_{\alpha_b} c_{\alpha_c}$	$-s_{\alpha_b} c_{\alpha_c}$

Table 1. The SM fermion and gauge boson couplings to Higgs mass eigenstates in the alignment of $\beta - \alpha = \pi/2$.

By extracting the cubic terms in the scalar potential eq. (2.1b), we can obtain the Higgs cubic self-interacting terms. The neutral part of the cubic terms are expressed as follows in the basis of (H_1^0, H_2^0, A^0)

$$\begin{aligned}
 -\mathcal{L}_{3s}/v &= \frac{1}{2}\lambda_1 c_\beta (H_1^0)^3 + \frac{1}{2}\lambda_2 s_\beta (H_2^0)^3 + \frac{1}{2}\lambda_{345} \left[c_\beta H_1^0 (H_2^0)^2 + s_\beta H_2^0 (H_1^0)^2 \right] \\
 &+ \frac{1}{2} \left\{ c_\beta \left[\lambda_1 s_\beta^2 + \lambda_{345} c_\beta^2 - 2\text{Re}(\lambda_5) \right] H_1^0 + s_\beta \left[\lambda_2 c_\beta^2 + \lambda_{345} s_\beta^2 - 2\text{Re}(\lambda_5) \right] H_2^0 \right\} (A^0)^2 \\
 &- \frac{1}{2} \text{Im}(\lambda_5) \left\{ 2 H_1^0 H_2^0 A^0 + s_\beta c_\beta \left[(H_1^0)^2 + (H_2^0)^2 - (A^0)^2 \right] \right\} A^0. \quad (2.19)
 \end{aligned}$$

From these terms, one can readily obtain the cubic interactions in terms of the mass eigenstates of (h_1, h_2, h_3) by using the orthogonal mixing matrix \mathcal{R} from eq. (2.4). Throughout our discussions, we define the Higgs cubic self couplings of λ_{ijk} ($i, j, k = 1, 2, 3$) to be the coefficients of the $h_i h_j h_k$ term from eq. (2.19)

$$\lambda_{ijk} \equiv \frac{1}{S!} \frac{\partial^3 \mathcal{L}_{3s}}{\partial h_i \partial h_j \partial h_k}, \quad (2.20)$$

where the symmetry factors are such that $S! = 3! = 6$ for $i = j = k$, $S! = 2$ for $i = j \neq k$, and $S = 1$ for $i \neq j \neq k$. A general derivation of the Higgs cubic self couplings in the CPV 2HDM was previously studied in refs. [63–65]. The explicit expressions of λ_{ijk} are tedious, while they can be greatly simplified with the fixed parameters through the following discussions.

3 The constraints in The CPV 2HDM

3.1 The 125 GeV SM-like Higgs boson constraint

In the CPV 2HDM, the productions and decay rates of the 125 GeV SM-like Higgs boson h_1 are controlled by both CP-even couplings of $c_{f,1}$ and CP-odd couplings of $\tilde{c}_{f,1}$. The production cross sections and decay rates are rescaled from the SM one as follows,

$$\frac{\sigma[gg \rightarrow h_1]}{\sigma[gg \rightarrow h_{\text{SM}}]} \approx \frac{(1.03 c_{u,1} - 0.06 c_{d,1})^2 + (1.57 \tilde{c}_{u,1} - 0.06 \tilde{c}_{d,1})^2}{(1.03 - 0.06)^2}, \quad (3.1a)$$

$$\frac{\Gamma[h_1 \rightarrow \gamma\gamma]}{\Gamma[h_{\text{SM}} \rightarrow \gamma\gamma]} \approx \frac{(0.23 c_{u,1} - 1.04 a_1)^2 + (0.35 \tilde{c}_{u,1})^2}{(0.23 - 1.04)^2}, \quad (3.1b)$$

$$\frac{\sigma[VV \rightarrow h_1]}{\sigma[VV \rightarrow h_{\text{SM}}]} = \frac{\sigma[V^* \rightarrow V h_1]}{\sigma[V^* \rightarrow V h_{\text{SM}}]} = \frac{\Gamma[h_1 \rightarrow VV^*]}{\Gamma[h_{\text{SM}} \rightarrow VV^*]} = a_1^2, \quad (3.1c)$$

$$\frac{\Gamma[h_1 \rightarrow b\bar{b}]}{\Gamma[h_{\text{SM}} \rightarrow b\bar{b}]} = \frac{\Gamma[h_1 \rightarrow \tau\tau]}{\Gamma[h_{\text{SM}} \rightarrow \tau\tau]} \approx c_{d,1}^2 + \tilde{c}_{d,1}^2. \quad (3.1d)$$

For the production cross sections and decay rates of the SM Higgs boson, we use the results from the LHC Higgs Working Group given in refs. [66, 67]. The LHC signal strengths of the SM-like Higgs boson in the presence of the CPV were discussed in refs. [52–54, 68–75]. From table. 1, one notes that the relevant Yukawa couplings of $(c_{f,1}, \tilde{c}_{f,1})$ and the Higgs gauge couplings of a_1 are only controlled by the Higgs VEV ratio of t_β as well as the CPV mixing angle of α_b . The heavy Higgs bosons in the spectrum are either irrelevant or negligible for the signal fit of h_1 . Based on the most recent LHC measurements of the 125 GeV signal strengths [76–81], we fit the signal strength of h_1 on the $(t_\beta, |\alpha_b|)$ plane and present the results with the eEDM constraints later.

3.2 The eEDM constraints

The ACME experiment [48], which searches for an energy shift of ThO molecules due to an external electric field, set stringent experimental bound to the eEDM.⁴ The bound reads

$$\left| \frac{d_e}{e} \right| < 8.7 \times 10^{-29} \text{ cm}. \quad (3.2)$$

The eEDM constraints to the CPV 2HDM-II were previously studied in the refs. [40, 53]. The effective Lagrangian term is given as follows

$$\mathcal{L}_{\text{eff}} = -\frac{i}{2} d_e \bar{e} \sigma_{\mu\nu} \gamma_5 e F^{\mu\nu} = i \frac{e \delta_e m_e}{v^2} \bar{e} \sigma_{\mu\nu} \gamma_5 e F^{\mu\nu}, \quad (3.3)$$

after integrating out the internal heavy degrees of freedoms. The constraint in eq. (3.2) can be converted to the bounds of the dimensionless Wilson coefficient of δ_e in eq. (3.3) such as

$$\frac{2m_e}{v^2} |\delta_e| < 8.7 \times 10^{-29} \text{ cm}. \quad (3.4)$$

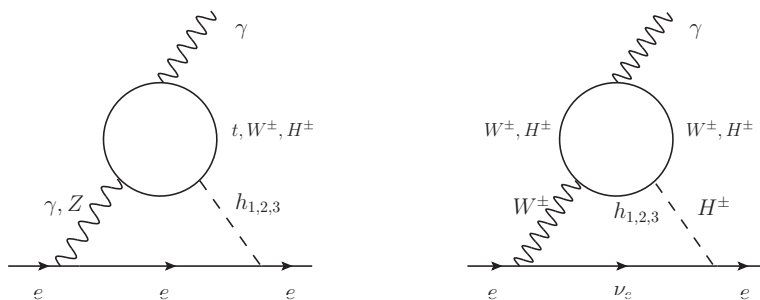


Figure 1. Left: the eEDM from the Barr-Zee type diagrams with the $h_i V_{\mu\nu} V^{\mu\nu}$ or $h_i V_{\mu\nu} \tilde{V}^{\mu\nu}$ operators (with $V_{\mu\nu} = F_{\mu\nu}/Z_{\mu\nu}$), and the CPV couplings between the neutral Higgs bosons h_i and the electron. Right: the eEDM from the $W^\pm H^\mp$ interactions and the CPV couplings for the charged Higgs bosons.

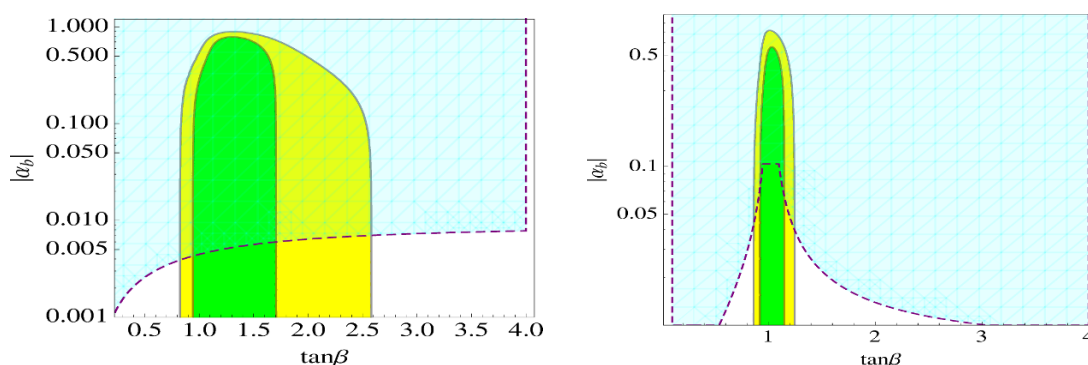


Figure 2. The signal strength fit to the 125 GeV Higgs boson h_1 and the eEDM constraint (light-blue shaded region) on the $(t_\beta, |\alpha_b|)$ plane, left panel: CPV 2HDM-I, right panel: CPV 2HDM-II. The green and yellow regions correspond to the 1σ and 2σ allowed regions for the LHC $7\oplus 8$ TeV signal fit to the h_1 in the CPV 2HDM.

In the CPV 2HDM, the Wilson coefficient δ_e are contributed by the two-loop Barr-Zee type $h_i \gamma \gamma (h_i Z \gamma)$ diagrams [82], and the $H^\pm W^\mp \gamma$ diagrams, as depicted in figure 1. The $h_i \gamma \gamma (h_i Z \gamma)$ diagrams include the contributions from: (i) the top-quark loops, (ii) the W -boson and the NGB loops, and (iii) the charged Higgs boson loops. The total contributions can be summarized as follows

$$\delta_e = (\delta_e)_t^{h_i \gamma \gamma} + (\delta_e)_W^{h_i \gamma \gamma} + (\delta_e)_{H^\pm}^{h_i \gamma \gamma} + (\delta_e)_t^{h_i Z \gamma} + (\delta_e)_W^{h_i Z \gamma} + (\delta_e)_{H^\pm}^{h_i Z \gamma} + (\delta_e)_{h_i}^{H^\pm W^\mp \gamma}. \quad (3.5)$$

Here, the superscripts of $h_i \gamma \gamma$, $h_i Z \gamma$, and $H^\pm W^\mp \gamma$ represent the operators for the specific Barr-Zee type diagrams. The subscripts of (t, W, H^\pm, h_i) represent the particles in the loops. Explicit expression for each term can be found in refs. [83–85], and summarized in

⁴As noted by [49] that current limits on the hadronic EDMs might provide similar sensitivities as the electron EDM, roughly $d_e/d_n \sim 10^{-2}$, thus one could expect that ^{199}Hg measurement [50] would give rises to complementary constraints on CP phases though hadronic EDMs are subjected to uncertainties of hadronic matrix elements [47].

the appendix of ref. [53]. Numerically, the leading contributions to the Wilson coefficient δ_e are mainly due to the $(\delta_e)^{h_1\gamma\gamma}$ and $(\delta_e)^{h_1Z\gamma}$ terms, while the contributions from the other heavy Higgs bosons of $(h_{2,3}, H^\pm)$ can be safely neglected. These terms are proportional to the CP-odd couplings of $\tilde{c}_{f,1}$, and further proportional to the CPV mixing angle α_b according to the Yukawa couplings listed in table. 1.

The eEDM upper bound from the ACME is converted to the constraints to the CPV 2HDM parameters on the $(t_\beta, |\alpha_b|)$ plane. The combined 125 GeV Higgs boson signal constraints and the eEDM constraints are shown in figure 2. It is clear that the eEDM bound is the leading one to set upper bounds to the CPV mixing angle of $|\alpha_b|$, as compared to the fits of the SM-like Higgs boson signal strengths. For the CPV 2HDM-I (left panel), the size of CPV mixing angle is significantly bound as $|\alpha_b| \lesssim 5 \times 10^{-3}$, and the 1σ allowed range of t_β is within $(0.9, 1.7)$. For the CPV 2HDM-II (right panel), the allowed region of the CPV mixing angle can be extended to $|\alpha_b| \lesssim 0.1$, while the 1σ allowed range of t_β is basically around 1.0. It has been noted in ref. [40] that the maximal cancellations between the $h_i F^{\mu\nu} V_{\mu\nu}$ operator and the $h_i F^{\mu\nu} \tilde{V}_{\mu\nu}$ operator can be achieved with the input of $t_\beta \sim 1$ in the CPV 2HDM-II. In order to highlight the CPV effects in the Higgs self couplings in the following discussions, we will focus on the CPV 2HDM-II with the fixed inputs of $\alpha = -\pi/4$ and $t_\beta = 1.0$. Furthermore, we also find that the Higgs cubic self couplings almost approach to the SM limit when the CPV mixing angle can be constrained as small as $|\alpha_b| \lesssim 0.01$. As stated in the previous paragraph, the Wilson coefficient of δ_e depends on the CPV mixings almost linearly. Therefore, if the future measurements of the eEDM can improve the precisions to an order of magnitude or more, they can be very useful to constrain the benchmark models for the Higgs pair productions in this setup.

3.3 The unitarity and stability constraints

To have a self-consistent description of the 2HDM potential, two other theoretical constraints should be taken into account, namely, the perturbative unitarity and the stability.

Very roughly speaking, the perturbative unitarity constraint means that the theory cannot be strongly coupled. According to the relations listed in eqs. (2.11), the constraints to the self couplings of λ_i can be converted to upper bounds to the Higgs boson masses and the soft mass term of m_{soft} in the 2HDM. In practice, the necessary and sufficient condition of the tree-level unitarity bounds can be obtained by evaluating the eigenvalues of the S -matrices for the scattering processes of the scalar fields in the 2HDM [86, 87]. Due to the Nambu-Goldstone theorem, the S -matrices can be expressed in terms of 2HDM quartic couplings λ_i . Explicitly, the unitarity conditions to be satisfied are that the eigenvalues of each S -wave amplitude matrix should be $\in (-1/2, 1/2)$. The S -wave amplitude matrices are due to fourteen neutral, eight singly-charged, and three doubly-charged scalar channels. They read

$$\begin{aligned}
 \text{neutral } a_0^0 : & \quad |\pi_i^+ \pi_i^- \rangle, & \quad |\pi_1^\pm \pi_2^\mp \rangle, & \quad \frac{1}{\sqrt{2}} |\pi_i^0 \pi_i^0 \rangle, & \quad \frac{1}{\sqrt{2}} |h_i h_i \rangle, \\
 & \quad |h_i \pi_i^0 \rangle, & \quad |\pi_1^0 \pi_2^0 \rangle, & \quad |h_1 h_2 \rangle, \\
 & \quad |h_1 \pi_2^0 \rangle, & \quad |h_2 \pi_1^0 \rangle, & &
 \end{aligned} \tag{3.6a}$$

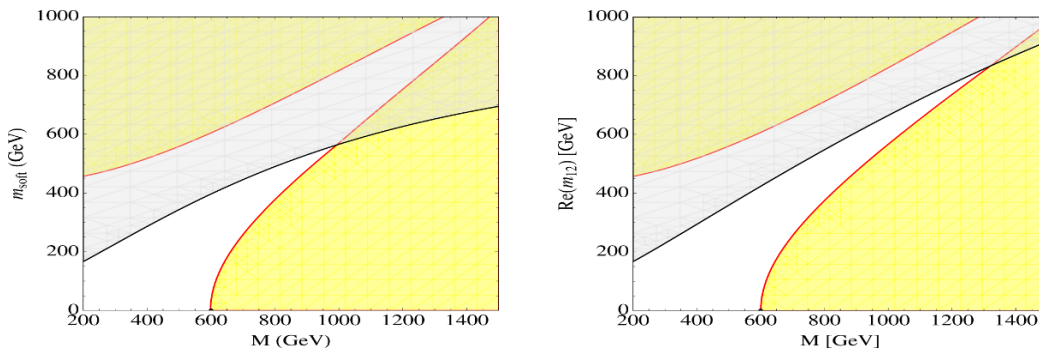


Figure 3. The combined perturbative unitarity and stability bounds on the (M, m_{soft}) plane for the CPV 2HDM-II. Left: the 2HDM-II with the fixed inputs of $(|\alpha_b|, t_\beta) = (0.1, 1.0)$, right: the 2HDM-II with the fixed inputs of $(|\alpha_b|, t_\beta) = (0.05, 1.0)$. The yellow shaded regions are excluded by the unitarity bounds, and the grey shaded regions are excluded by the stability bounds.

$$\text{singly-charged } a_0^+ : |\pi_i^+ \pi_i^0\rangle, \quad |\pi_i^+ h_i^0\rangle, \\ |\pi_1^+ \pi_2^0\rangle, \quad |\pi_2^+ \pi_1^0\rangle, \quad |\pi_1^+ h_2\rangle, \quad |\pi_2^+ h_1\rangle, \quad (3.6b)$$

$$\text{doubly-charged } a_0^{++} : \frac{1}{\sqrt{2}} |\pi_1^\pm \pi_1^\pm\rangle, \quad \frac{1}{\sqrt{2}} |\pi_2^\pm \pi_2^\pm\rangle, \quad |\pi_1^\pm \pi_2^\pm\rangle. \quad (3.6c)$$

The S -wave amplitude matrices for three different channels are expressed as

$$a_0^0 = \frac{1}{16\pi} \text{diag}(X_{4 \times 4}, Y_{4 \times 4}, Z_{3 \times 3}, Z_{3 \times 3}), \quad (3.7a)$$

$$a_0^+ = \frac{1}{16\pi} \text{diag}(Y_{4 \times 4}, Z_{3 \times 3}, \lambda_3 - \lambda_4), \quad (3.7b)$$

$$a_0^{++} = \frac{1}{16\pi} Z_{3 \times 3} \quad (3.7c)$$

where the expressions for the submatrices of $(X_{4 \times 4}, Y_{4 \times 4}, Z_{3 \times 3})$ are given in the ref. [87].

The stability constraints require a positive 2HDM potential for large values of Higgs fields along all field space directions. Collectively, they lead to the following conditions

$$\lambda_{1,2} > 0, \quad \lambda_3 > -\sqrt{\lambda_1 \lambda_2}, \quad \lambda_3 + \lambda_4 - |\lambda_5| > -\sqrt{\lambda_1 \lambda_2}, \quad (3.8)$$

with $\lambda_{6,7} = 0$ assumed. The combined constraints from the perturbative unitarity and stability to the (M, m_{soft}) parameter regions for the CPV 2HDM-II are shown in figure 3 with the fixed input parameters of $(|\alpha_b|, t_\beta) = (0.1, 1.0)$ (left panel) and $(|\alpha_b|, t_\beta) = (0.05, 1.0)$ (right panel). It turns out that the combined perturbative unitarity and stability put upper bounds to the heavy Higgs boson masses of $M \lesssim 1.0$ TeV for $|\alpha_b| = 0.1$, or $M \lesssim 1.2$ TeV for $|\alpha_b| = 0.05$. The stability constraints of (3.8) bound the soft mass term of m_{soft} from above. As seen from eqs. (2.11), very large values of m_{soft} will pull $\lambda_{1,2}$ into the negative regions, which violate the conditions described by eqs. (3.8). Later, we will find that the Higgs cubic self couplings, such as λ_{113} in our case, become enhanced with the large soft mass inputs of m_{soft} when they are close to the stability boundary.

3.4 The LHC searches for heavy Higgs bosons

The constraints to the signal strengths of the 125 GeV SM-like Higgs boson h_1 and the eEDM put bounds to the parameters of $(|\alpha_b|, t_\beta)$. The unitarity and stability constraints put upper bounds to the mass input parameters of (M, m_{soft}) . Below, we take into account the constraints from the $7 \oplus 8$ TeV LHC searches for the heavy Higgs bosons in the 2HDM spectrum. Such constraints were previously given in ref. [54], where authors included the constraints from $h_{2,3} \rightarrow WW/ZZ$ and $h_{2,3} \rightarrow Zh_1 \rightarrow \ell^+ \ell^- b \bar{b}$ final states. Additionally, there have been recent experimental searches to the $hh \rightarrow b \bar{b} \gamma \gamma$ final states from both ATLAS and CMS collaborations, which are included in our studies.

3.4.1 The heavy Higgs productions

The cross sections of the heavy Higgs bosons via the ggF channel can be rescaled from the SM-like Higgs production with the same mass as

$$\frac{\sigma[gg \rightarrow h_i]}{\sigma[gg \rightarrow h_{\text{SM}}]} = \frac{\left| c_{t,i} A_{1/2}^H(\tau_t^i) + c_{b,i} A_{1/2}^H(\tau_b^i) \right|^2 + \left| \tilde{c}_{t,i} A_{1/2}^A(\tau_t^i) + \tilde{c}_{b,i} A_{1/2}^A(\tau_b^i) \right|^2}{\left| A_{1/2}^H(\tau_t^i) + A_{1/2}^H(\tau_b^i) \right|^2}, \quad (3.9)$$

with the variable of

$$\tau_f^i \equiv \frac{M_i^2}{4m_f^2}, \quad f = t, b. \quad (3.10)$$

The cross sections of the heavy Higgs bosons via the VBF channel can be rescaled from the SM-like Higgs production with the same mass as

$$\frac{\sigma[qq \rightarrow qq h_i]}{\sigma[qq \rightarrow qq h_{\text{SM}}]} = a_i^2. \quad (3.11)$$

3.4.2 The heavy Higgs decays

Here, we list the partial decay widths of the heavy neutral Higgs bosons at the leading order (LO). The partial decay widths into the gauge bosons are

$$\frac{\Gamma[h_i \rightarrow VV]}{\Gamma[h_{\text{SM}} \rightarrow VV]} = a_i^2, \quad (3.12)$$

with $V = (W^\pm, Z)$. The partial decay widths into the SM fermions are

$$\frac{\Gamma[h_i \rightarrow f \bar{f}]}{\Gamma[h_{\text{SM}} \rightarrow f \bar{f}]} = (c_{f,i})^2 + (\tilde{c}_{f,i})^2. \quad (3.13)$$

We also consider the non-standard decay modes of the heavy Higgs bosons, which include $h_i \rightarrow h_1 Z$, $H^\pm \rightarrow h_1 W^\pm$, and $h_i \rightarrow h_1 h_1$. Their partial decay widths are

$$\Gamma[h_i \rightarrow h_1 Z] = \frac{|g_{i1z}|^2}{16\pi M_i} \sqrt{\left(1 - \frac{(M_1 + m_Z)^2}{M_i^2}\right) \left(1 - \frac{(M_1 - m_Z)^2}{M_i^2}\right)} \times \left[\frac{1}{m_Z^2} (M_i^2 - M_1^2)^2 - (2M_i^2 + 2M_1^2 - m_Z^2) \right], \quad (3.14a)$$

$$\Gamma[h_i \rightarrow h_1 h_1] = \frac{\lambda_{11i}^2}{4\pi M_i} \sqrt{1 - \frac{4M_1^2}{M_i^2}}, \quad (3.14b)$$

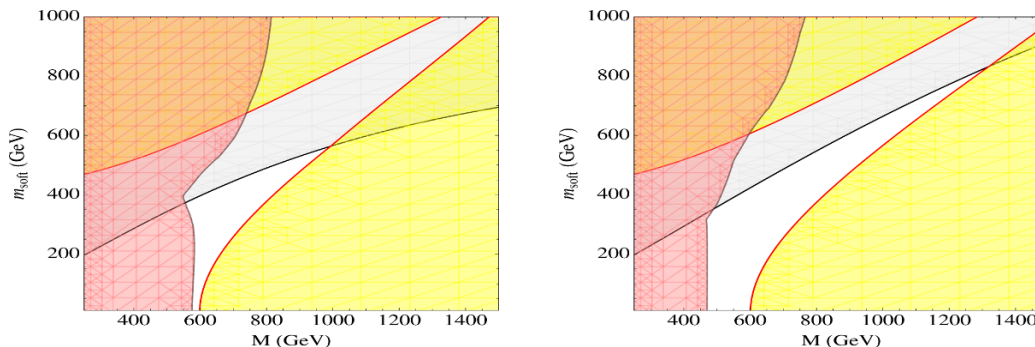


Figure 4. The combined unitarity and stability bounds on the (M, m_{soft}) plane for the CPV 2HDM-II, with fixed parameter of $t_\beta = 1.0$. Left: $|\alpha_b| = 0.1$, right: $|\alpha_b| = 0.05$. The pink shaded regions are excluded by the LHC searches for the heavy Higgs bosons.

where $g_{iz1} = (e/s_{2W})[(-s_\beta \mathcal{R}_{11} + c_\beta \mathcal{R}_{12})\mathcal{R}_{i3} - (-s_\beta \mathcal{R}_{i1} + c_\beta \mathcal{R}_{i2})\mathcal{R}_{13}]$. The cubic self couplings of λ_{11i} are obtained in eq. (2.20) from the Lagrangian terms in eq. (2.19), and their expansions in terms of the CPV mixing angle α_b are given in eqs. (4.2) later. By fixing the parameter choices of the alignment limit and $\alpha_c = 0$, we find the non-vanishing couplings of $g_{2z1} = -(e/s_{2W})s_{\alpha_b}$ and $\lambda_{113} \neq 0$.

3.4.3 The experimental search bounds

The current LHC experimental searches for the heavy Higgs bosons are performed via the (WW, ZZ) final states [88, 89], the $H \rightarrow hh \rightarrow b\bar{b} + \gamma\gamma$ [90, 91], and $A \rightarrow hZ \rightarrow (b\bar{b} + \ell^+\ell^-/\tau^+\tau^- + \ell^+\ell^-)$ [94, 95]. Since we always assume that $M_2 = M_3$, the constraints to the heavy Higgs boson searches at the LHC are imposed to the cross sections of $\sigma[pp \rightarrow h_2/h_3 \rightarrow XX]$

$$\begin{aligned} \sigma[pp \rightarrow h_2/h_3 \rightarrow XX] &= \sigma[gg \rightarrow h_2] \times \text{Br}[h_2 \rightarrow XX] \\ &+ \sigma[gg \rightarrow h_3] \times \text{Br}[h_3 \rightarrow XX], \end{aligned} \quad (3.15)$$

where we consider the leading production channel of ggF obtained from eq. (3.9). The decay branching ratios are obtained from the partial decay widths of eqs. (3.12), (3.13), (3.14a), and (3.14b) evaluated at the LO. We find the most stringent constraint to the heavy Higgs boson searches are from the recent CMS searches for the resonances with two SM-like Higgs bosons in ref. [91]. By converting all heavy Higgs boson constraints to the (M, m_{soft}) plane, we find the mass regions of $M_{2,3} \lesssim 600$ GeV are excluded for $|\alpha_b| = 0.1$, or $M_{2,3} \lesssim 500$ GeV are excluded for $|\alpha_b| = 0.05$, respectively. The current B -physics data also excludes the charged Higgs boson mass greater than $M_\pm \sim 340$ GeV for 2HDM-II [92, 93]. Combining with the previous unitarity and stability constraints, we display the allowed parameter regions of (M, m_{soft}) in figure 4. Accordingly, we consider two scenarios of

$$\begin{aligned} \text{(i)} : & |\alpha_b| = 0.1, \text{ with } M_{2,3} \in (600 \text{ GeV}, 1000 \text{ GeV}), \\ \text{(ii)} : & |\alpha_b| = 0.05, \text{ with } M_{2,3} \in (500 \text{ GeV}, 1200 \text{ GeV}), \end{aligned} \quad (3.16)$$

for the Higgs pair productions at the future high-energy e^+e^- and pp colliders. A set of benchmark models for the $|\alpha_b| = 0.1$ and $|\alpha_b| = 0.05$ cases are listed in table. 2, where the

$M_2 = M_3(\text{GeV})$	$ \alpha_b = 0.1$			$ \alpha_b = 0.05$		
	$m_{\text{soft}}(\text{GeV})$	$\lambda_{111}(\text{GeV})$	$\lambda_{113}(\text{GeV})$	$m_{\text{soft}}(\text{GeV})$	$\lambda_{111}(\text{GeV})$	$\lambda_{113}(\text{GeV})$
500	350	29.37	-70.33
600	400	19.45	-173.75	420	28.28	-102.66
700	440	16.80	-200.04	480	27.19	-133.01
800	480	13.89	-227.22	540	25.96	-167.14
900	520	10.74	-255.31	600	24.57	-205.05
1000	560	7.33	-284.30	660	23.05	-246.72
1100	710	21.66	-280.60
1200	770	19.87	-328.86

Table 2. The CPV 2HDM-II benchmark models for the CPV mixing angles of $|\alpha_b| = 0.1$ and $|\alpha_b| = 0.05$. The heavy Higgs boson mass ranges are chosen according to eq. (3.16). The non-vanishing Higgs cubic self couplings of λ_{111} and λ_{113} are listed for each model.

soft mass terms of m_{soft} are chosen to be close to the stability boundary for each heavy Higgs boson mass. In the next section, we will study the Higgs pair productions at the future e^+e^- and pp collider experiments based on these benchmark models.

3.5 The EW precision constraints

The Peskin-Takeuchi parameters of (S, T) for the EW precision tests were obtained in refs. [30, 96–100] for the 2HDM. In our simplified case with the alignment limit, the degenerate masses of $M_2 = M_3 = M_{\pm}$, and $\alpha_c = 0$, they read

$$\begin{aligned} \Delta S = \frac{1}{96 \pi^2 c_W} \frac{m_W^2}{v^2} & \left\{ c_{2w}^2 G(M_{\pm}^2, M_{\pm}^2, m_Z^2) + \left[G(M_1^2, M_2^2, m_Z^2) + \hat{G}(M_3^2, m_Z^2) \right] s_{\alpha_b}^2 \right. \\ & + \left[\hat{G}(M_1^2, m_Z^2) + G(M_2^2, M_3^2, m_Z^2) \right] c_{\alpha_b}^2 + \log \left(\frac{M_1^2 M_2^2 M_3^2}{M_{\pm}^6} \right) \\ & \left. - \left[\hat{G}(M_{H,\text{ref}}^2, m_Z^2) + \log \left(\frac{M_{H,\text{ref}}^2}{M_{\pm}^2} \right) \right] \right\}, \end{aligned} \quad (3.17a)$$

$$\alpha \Delta T = \frac{1}{6 \pi^2 v^2} \Delta_1 (m_W - m_Z) s_{\alpha_b}^2. \quad (3.17b)$$

for a reference value of the SM Higgs boson mass $M_{H,\text{ref}} = 125$ GeV. Here, we denote $\Delta_1 \equiv M_{\pm} - M_1$, and the functions of $G(x, y, z), \hat{G}(x, y)$ are given in [96]. By employing the current Gfitter fit to the EW data [101], the parameters are founded to be constrained by T parameter mostly for the CPV parameter α_b allowed by figure 2, and the degenerate masses of heavy Higgs bosons relax the constraints again.

4 Higgs pair productions at the colliders

In this section, we study the SM-like Higgs pair productions in the framework of the CPV 2HDM. The SM-like Higgs cubic self coupling of λ_{111} are modified due to the varying

inputs of the soft mass term and the CPV mixing angle. Therefore, we will discuss the precision measurement of λ_{111} at the future e^+e^- colliders for the benchmark models in table. 2. We will also focus on the most dominant channel for the resonance contributions, namely the ggF process at the hadron colliders, which include the LHC 14 TeV and the future SppC/Fcc-hh 100 TeV runs.

4.1 The Higgs cubic self couplings

Before evaluating the cross sections of the Higgs pair productions, it is necessary to look at the behaviors of the relevant Higgs cubic self couplings of λ_{11i} ($i = 1, 2, 3$). Following the previous constraints, we fix the parameters of $(\alpha, t_\beta) = (-\pi/4, 1.0)$, and keep the input of $\alpha_c = 0$. With these assumptions, we find that only λ_{111} and λ_{113} survive, and λ_{112} is always vanishing. Their explicit expressions in the mass eigenbasis read

$$\lambda_{111} = \frac{c_{\alpha_b}}{2v} \left[M_1^2 (s_{\alpha_b}^4 + c_{\alpha_b}^4 + s_{2\alpha_b}^2) + 2M_3^2 s_{\alpha_b}^4 - 4m_{\text{soft}}^2 s_{\alpha_b}^2 \right], \quad (4.1a)$$

$$\lambda_{113} = \frac{s_{\alpha_b}}{8v} \left[M_1^2 (3c_{4\alpha_b} + 8c_{2\alpha_b} - 3) - M_3^2 (3c_{4\alpha_b} - 4c_{2\alpha_b} - 3) - 8m_{\text{soft}}^2 (3c_{2\alpha_b} + 1) \right]. \quad (4.1b)$$

Since the CPV mixing angle of α_b is typically small by imposing the eEDM constraints, it is also useful to expand the cubic couplings in terms of the α_b angle as follows

$$\lambda_{111} \simeq \frac{M_1^2}{2v} + \frac{3M_1^2 - 8m_{\text{soft}}^2}{4v} \cdot \alpha_b^2 + \mathcal{O}(\alpha_b^4), \quad (4.2a)$$

$$\lambda_{113} \simeq \frac{2M_1^2 + M_3^2 - 8m_{\text{soft}}^2}{2v} \cdot \alpha_b + \mathcal{O}(\alpha_b^3). \quad (4.2b)$$

The Higgs cubic self coupling of λ_{111} starts with the SM predicted values of $\lambda_{hhh}^{\text{SM}} \simeq 32$ GeV, plus the higher order corrections of $\mathcal{O}(\alpha_b^2)$. The overall magnitude of λ_{113} is controlled by the size of the CPV mixing angle α_b . Hence, one can expect that the improvement in the precisions of the eEDM measurements will reduce the size of the heavy resonance contributions to the Higgs pair productions via the ggF process.

In figure 5, we plot the Higgs cubic self couplings of λ_{111} and λ_{113} for the $M_2 = M_3 = 600$ GeV case with different CPV mixing angles of α_b in the CPV 2HDM-II. The lower and upper bounds of the soft mass inputs m_{soft} in these plots are from the perturbative unitarity and the stability constraints, respectively. For a fixed input of α_b , the Higgs cubic self coupling of λ_{111} becomes smaller than the SM predicted value with the increasing inputs of m_{soft} . On the other hand, when the CPV mixing angle becomes as small as $\alpha_b = 0.01$, the Higgs cubic self coupling of λ_{111} is basically the same as $\lambda_{hhh}^{\text{SM}} \simeq 32$ GeV. The other Higgs cubic self coupling of λ_{113} also decreases from positive regions to negative regions with the increasing inputs of m_{soft} . Its variation is also controlled by the size of the CPV mixing angle of α_b , as seen from its behaviors with the different inputs of the CPV mixing angle of $\alpha_b = (0.1, 0.05, 0.01)$. For the $M_2 = M_3 = 600$ GeV case, λ_{113} tends to zero when the soft mass term is $m_{\text{soft}} \simeq 220$ GeV, as can be evaluated from eq. (4.2b). Thus, one would expect the corresponding resonance contributions to vanish. When the soft mass

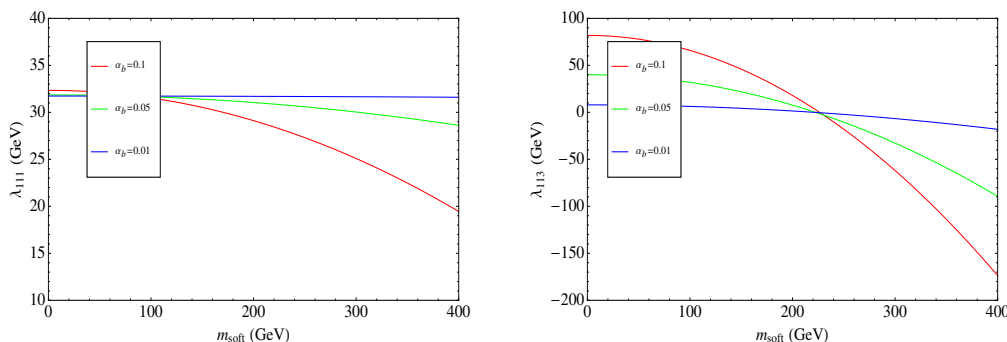


Figure 5. The Higgs cubic self couplings λ_{111} (left) and λ_{113} (right) versus m_{soft} for the $M_2 = M_3 = 600$ GeV case in the CPV 2HDM-II, with fixed inputs of $\alpha = -\pi/4$ and $t_\beta = 1.0$.

deviates from this value of $m_{\text{soft}} \simeq 220$ GeV, either increases to the stability boundary or decreases to zero, $|\lambda_{113}|$ increases. Correspondingly, one can expect large resonance contributions for such parameter inputs.

4.2 The precise measurement of λ_{111} at the future e^+e^- colliders

The future high-energy e^+e^- colliders provide opportunities of measuring the SM-like Higgs cubic self couplings. The direct measurements can be achieved via the $e^+e^- \rightarrow hhZ$ process with the center-of-mass energy of $\sqrt{s} = 500$ GeV, or via the vector boson fusion process of $e^+e^- \rightarrow hh\nu_e\bar{\nu}_e$ with the center-of-mass energy of $\sqrt{s} = 1$ TeV [14, 15, 17]. The first advantage of the e^+e^- colliders is that the relevant Higgs-gauge couplings for these processes can be precisely measured to the percentage level at the $\sqrt{s} = 240 - 250$ GeV runs [14–16]. For the CPV 2HDM, one has the Higgs-gauge couplings of

$$g_{1ZZ} = g_{hZZ}^{\text{SM}} c_{\alpha_b} = \frac{m_Z^2}{v} c_{\alpha_b}, \quad g_{11ZZ} = g_{hhZZ}^{\text{SM}} = \frac{m_Z^2}{2v^2}. \quad (4.3)$$

with $\delta g_{1ZZ} = |g_{1ZZ} - g_{hZZ}^{\text{SM}}| < \mathcal{O}(1\%)$ after imposing the eEDM constraints. The second advantage of the e^+e^- colliders is that the contributions to the total cross section from the heavy resonance of h_3 are typically less than $\mathcal{O}(10^{-4})$, hence they are negligible. Therefore, it is a good approximation to assume the SM predicted values for the Higgs-gauge couplings, and only vary the Higgs cubic self coupling of λ_{111} . The ratio of the total cross section of $\sigma[e^+e^- \rightarrow hhZ]$ to its SM counterpart can be parametrized as follows

$$\frac{\sigma[e^+e^- \rightarrow h_1h_1Z]}{\sigma[e^+e^- \rightarrow hhZ]_{\text{SM}}} = 0.097 \xi_{111}^2 + 0.369 \xi_{111} + 0.534, \quad (4.4)$$

at the TLEP and ILC 500 GeV runs, with $\xi_{111} \equiv \lambda_{111}/\lambda_{hhh}^{\text{SM}}$. The total cross sections at the TLEP and ILC 500 GeV runs versus the ratios of different Higgs cubic self couplings $\lambda_{111}/\lambda_{hhh}^{\text{SM}}$ are displayed on the left panel of figure 6. The ranges of λ_{111} in two set of benchmark models with $|\alpha_b| = 0.1$ and $|\alpha_b| = 0.05$ are also shown in the light-blue and light-green shaded regions, respectively. From the results given in table. 2 for the benchmark models, the Higgs cubic self couplings of λ_{111} are always smaller than the SM predicted

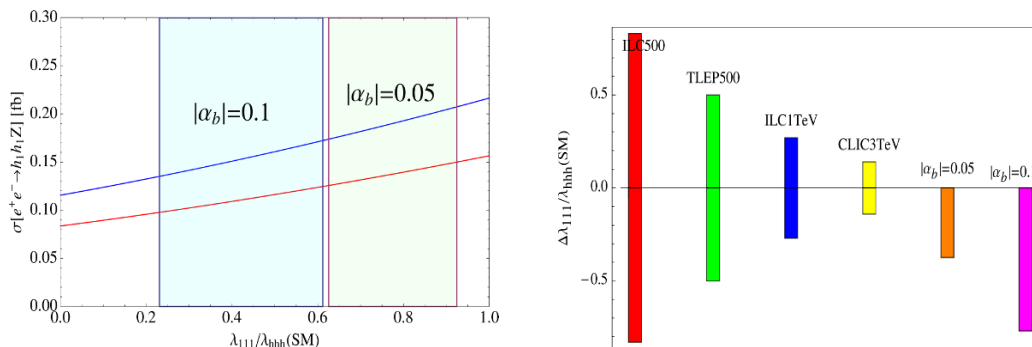


Figure 6. Left: the cross sections of $\sigma[e^+e^- \rightarrow h_1h_1Z]$ at the TLEP (red) and ILC (blue) 500 GeV versus the different Higgs cubic self couplings. Right: the expected accuracies on the Higgs cubic self couplings at the future e^+e^- colliders, and the $\Delta\lambda_{111}/\lambda_{hhh}^{\text{SM}}$ for the benchmark models of $|\alpha_b| = 0.1$ and $|\alpha_b| = 0.05$.

values. Thus, the corresponding cross sections of $\sigma[e^+e^- \rightarrow h_1h_1Z]$ are smaller than the SM predictions at the TLEP and the ILC. On the right panel of figure 6, we display the expected accuracies on the Higgs cubic self couplings for ILC500 (with $\int \mathcal{L}dt = 0.5 \text{ ab}^{-1}$), TLEP500 (with $\int \mathcal{L}dt = 1 \text{ ab}^{-1}$), ILC 1 TeV (with $\int \mathcal{L}dt = 1 \text{ ab}^{-1}$), and CLIC 3 TeV (with $\int \mathcal{L}dt = 2 \text{ ab}^{-1}$). The deviations of the Higgs cubic self couplings $\Delta\lambda_{111}/\lambda_{hhh}^{\text{SM}}$ corresponding to the benchmark models of $|\alpha_b| = 0.1$ and $|\alpha_b| = 0.05$ are shown for comparison. For the $|\alpha_b| = 0.1$ case, the largest deviations of λ_{111} can be probed with the accuracies reached by the TLEP 500 GeV; while for the smaller CPV mixing angle of $|\alpha_b| = 0.05$ case, the largest deviations of λ_{111} can be probed with the accuracies reached by the ILC 1 TeV.

4.3 The $pp \rightarrow h_1h_1$ in the CPV 2HDM

The parton-level differential cross sections of the Higgs pair production for both SM Higgs and BSM Higgs bosons via the ggF process were previously derived in refs. [102–105]. For the productions of the SM-like Higgs boson pairs, its differential cross section reads

$$\frac{d\hat{\sigma}}{d\hat{t}}[gg \rightarrow hh] = \frac{G_F^2 \alpha_s^2}{512(2\pi)^3} \sum_q \left[\left| (C_{\Delta}^h F_{\Delta}^h + C_{\square}^{hh} F_{\square}^{hh}) \right|^2 + \left| C_{\square}^{hh} G_{\square}^{hh} \right|^2 \right], \quad (4.5)$$

where the dominant contributions are due to the top-quark loops. The form factors of $(F_{\Delta}, F_{\square}, G_{\square})$ are from the loop integrals of the triangle diagrams, the $J = 0$ partial wave of the box diagrams, and the $J = 2$ partial wave of the box diagrams. Their explicit expressions are summarized in the appendix of ref. [103]. The relevant coefficients are given by

$$C_{\Delta}^h = \frac{(6\lambda_{hhh}v)\xi_h^q}{\hat{s} - M_h^2 + iM_h\Gamma_h}, \quad (4.6a)$$

$$C_{\square}^{hh} = (\xi_h^q)^2, \quad (4.6b)$$

with λ_{hhh} and ξ_h^q representing the Higgs cubic self couplings and the dimensionless Yukawa couplings of the SM-like Higgs boson, respectively. For the SM case, these couplings are

$$\lambda_{hhh}^{(\text{SM})} = \frac{M_h^2}{2v}, \quad (\xi_h^q)^{(\text{SM})} = 1. \quad (4.7)$$

The LO total cross sections for the LHC 14 TeV runs and the SppC 100 TeV runs can be estimated by using Madgraph 5 [106] as follows

$$\sigma_{\text{LO}}^{14}[pp \rightarrow hh] = 17.34 \text{ fb}, \quad \sigma_{\text{LO}}^{100}[pp \rightarrow hh] = 806.6 \text{ fb}. \quad (4.8)$$

For the most general case in the CPV 2HDM, all neutral Higgs bosons of h_i have both CP-even and CP-odd Yukawa couplings. Furthermore, the heavy resonances enter into the Higgs pair productions. The corresponding differential cross sections at the parton level can be generalized from the results in the appendix of ref. [103] for the different CP combinations of the final-state $h_1 h_1$, which are expressed as follows

$$\begin{aligned} \frac{d\hat{\sigma}}{dt}[gg \rightarrow h_1 h_1] &= \frac{G_F^2 \alpha_s^2}{512 (2\pi)^3} \sum_q \left[\left| \left(\sum_{h_i=h_1, h_3} C_{\Delta}^{h_i} \right) F_{\Delta}^h + C_{\square}^{hh} F_{\square}^{hh} + C_{\square}^{AA} F_{\square}^{AA} \right|^2 \right. \\ &\quad + \left| \left(\sum_{h_i=h_1, h_3} \tilde{C}_{\Delta}^{h_i} \right) F_{\Delta}^A + C_{\square}^{hA} F_{\square}^{hA} \right|^2 \\ &\quad \left. + \left| C_{\square}^{hh} G_{\square}^{hh} + C_{\square}^{AA} G_{\square}^{AA} \right|^2 + \left| C_{\square}^{hA} G_{\square}^{hA} \right|^2 \right]. \end{aligned} \quad (4.9)$$

The relevant couplings are

$$C_{\square}^{hh} = (c_{q,1})^2, \quad (4.10a)$$

$$C_{\square}^{AA} = (\tilde{c}_{q,1})^2, \quad (4.10b)$$

$$C_{\square}^{hA} = c_{q,1} \tilde{c}_{q,1}, \quad (4.10c)$$

$$C_{\Delta}^{h_i} = \frac{(g_{11i} v) c_{q,i}}{\hat{s} - M_i^2 + i M_i \Gamma_i}, \quad (4.10d)$$

$$\tilde{C}_{\Delta}^{h_i} = \frac{(g_{11i} v) \tilde{c}_{q,i}}{\hat{s} - M_i^2 + i M_i \Gamma_i}, \quad (4.10e)$$

with $g_{111} = 6\lambda_{111}$ and $g_{113} = 4\lambda_{113}$. To evaluate the cross sections, we implement all couplings given in eqs. (4.10) into the FeynRules [107], and pass the UFO model files into the Madgraph 5.

Now we present the results of the Higgs pair productions in the CPV 2HDM, by combining all previous constraints. As one can learn from eq. (4.9), the cross sections of $\sigma[pp \rightarrow h_1 h_1]$ get modified from their SM counterparts due to: (i) the modification of the Higgs cubic self coupling λ_{111} , (ii) the modifications of the top quark Yukawa couplings, and (iii) the additional resonance contributions. Through the signal fit to the 125 GeV SM-like Higgs boson h_1 and the eEDM constraints, the dimensionless Higgs Yukawa couplings are

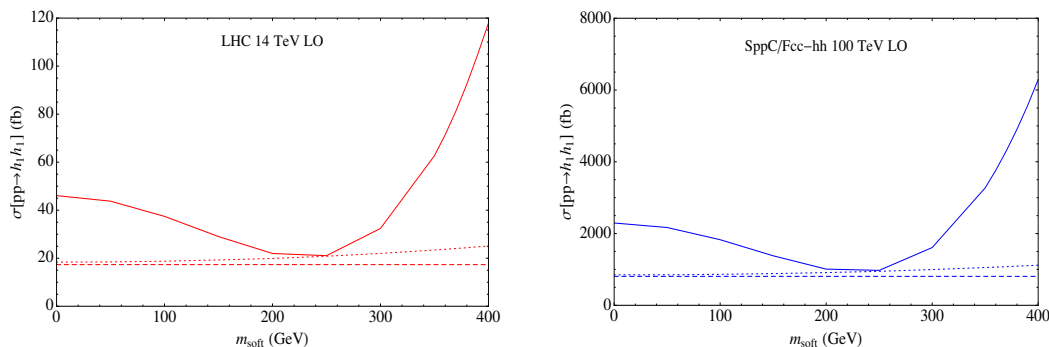


Figure 7. The cross sections of $\sigma[pp \rightarrow h_1 h_1]$ at the LHC 14 TeV (left) and SppC 100 TeV (right) versus the varying m_{soft} for the $M_2 = M_3 = 600$ GeV case in the CPV 2HDM-II, with fixed inputs of $|\alpha_b| = 0.1$.

bounded such that $\delta c_{f,1} < 1\%$ and $\tilde{c}_{f,1} \sim -0.1$. Therefore, the box diagram contributions are envisioned to approach to the SM predicted values. From the previous estimation of the Higgs cubic self couplings for the $M_2 = M_3 = 600$ GeV case, we may either have the large resonance contributions or go to the regions with the vanishing resonance contributions of $(\lambda_{111}, \lambda_{113}) \rightarrow (\lambda_{hh}^{\text{SM}}, 0)$. For these two limiting scenarios, further simplifications can be made for eq. (4.9), which are

$$\text{resonances : } \frac{d\hat{\sigma}}{dt} \approx \frac{G_F^2 \alpha_s^2}{512 (2\pi)^3} \left[\left| \sum_{h_i} C_{\Delta}^{h_i} F_{\Delta}^h \right|^2 + \left| \sum_{h_i} \tilde{C}_{\Delta}^{h_i} F_{\Delta}^A \right|^2 \right], \quad (4.11a)$$

$$\begin{aligned} \text{non - resonances : } \frac{d\hat{\sigma}}{dt} \approx \frac{G_F^2 \alpha_s^2}{512 (2\pi)^3} & \left[\left| C_{\Delta}^{h_1} F_{\Delta}^h + C_{\square}^{hh} F_{\square}^{hh} + C_{\square}^{AA} F_{\square}^{AA} \right|^2 \right. \\ & + \left| \tilde{C}_{\Delta}^{h_1} F_{\Delta}^A + C_{\square}^{hA} F_{\square}^{hA} \right|^2 \\ & \left. + \left| C_{\square}^{hh} G_{\square}^{hh} + C_{\square}^{AA} G_{\square}^{AA} \right|^2 + \left| C_{\square}^{hA} G_{\square}^{hA} \right|^2 \right]. \quad (4.11b) \end{aligned}$$

In figure 7, we display the LO cross sections of $\sigma[pp \rightarrow h_1 h_1]$ at the LHC and the SppC/Fcc-hh for the $M_2 = M_3 = 600$ GeV case. The solid curves represent the full results by combining every term in eq. (4.9). We also show the hypothetical cross sections of eq. (4.11b), where we turn off the Higgs cubic self coupling of λ_{113} while modify λ_{111} according to eq. (4.2a). Thus, it is evident that the total cross sections approach to the SM-like Higgs pair productions with the modified cubic self couplings. On the other hand, the LO cross sections at the LHC (SppC) can be as large as $\sim \mathcal{O}(100)$ fb ($\sim \mathcal{O}(6)$ pb) when the soft mass approaches to the stability boundary for this case.

Furthermore, we evaluate the LO cross sections for the benchmark models listed in table. 2. The typical cross sections subject all constraints in the previous context are $\mathcal{O}(10) - \mathcal{O}(100)$ fb at the LHC or $\mathcal{O}(1)$ pb at the SppC for the allowed mass ranges. The corresponding results are displayed in figure 8, for benchmark models with $|\alpha_b| = 0.1$ and $|\alpha_b| = 0.05$, respectively. We display the cross sections with the $h_1 h_1 \rightarrow b\bar{b} + \gamma\gamma$ and $h_1 h_1 \rightarrow b\bar{b} + W^+ W^-$ final states. From the experimental side, the $b\bar{b} + \gamma\gamma$ final states are the leading one to look for the Higgs pair productions at the hadron colliders, in that

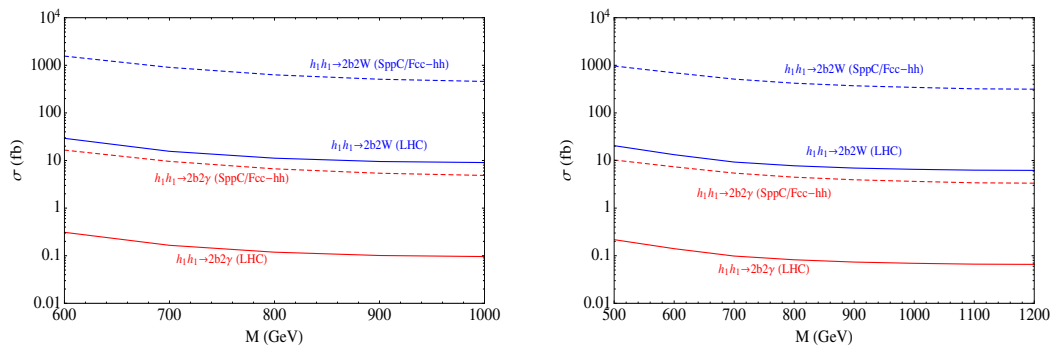


Figure 8. The total cross sections of $\sigma[pp \rightarrow h_1 h_1]$ via the ggF at the LHC 14 TeV (solid curves) and the SppC 100 TeV (dashed curves). Left: the cross sections for the benchmark models with the $\alpha_b = 0.1$ input, right: the cross sections for the benchmark models with the $\alpha_b = 0.05$ input.

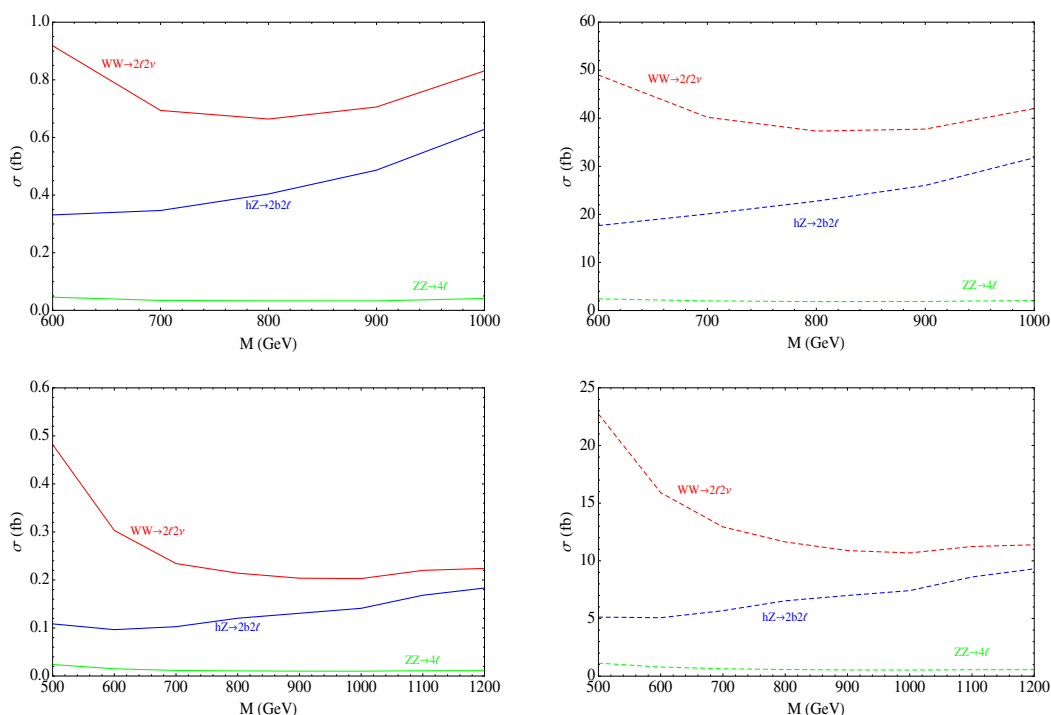


Figure 9. The cross sections of the other search modes of the heavy Higgs bosons at the LHC 14 TeV (left panels) and the SppC 100 TeV (right panels).

the relevant SM background is under control. The LO cross sections for the $b\bar{b} + \gamma\gamma$ of our benchmark models are $\sim \mathcal{O}(0.1)$ fb at the LHC, and they increase to $\mathcal{O}(10)$ fb at the SppC. In addition, one may also consider the $b\bar{b} + W_h W_\ell$ final states with the aid of the jet substructure technique [8]. The LO cross sections for the $b\bar{b} + WW$ of our benchmark models are $\sim \mathcal{O}(10)$ fb at the LHC, and they increase to $\mathcal{O}(1)$ pb at the SppC.

4.4 Other channels

Besides the Higgs pair productions, we also have the other search modes for the heavy Higgs bosons of h_2/h_3 , such as di-bosons and Higgs plus Z . In figure 9, we display the

cross sections of the other search modes of the heavy Higgs bosons, including $pp \rightarrow h_i \rightarrow (W^+W^- \rightarrow 2\ell 2\nu, ZZ \rightarrow 4\ell, hZ \rightarrow b\bar{b} + \ell^+\ell^-)$. The current LHC searches for the heavy Higgs bosons via these channels can be found in refs. [88, 89, 94, 95, 108–110]. The cross sections for these benchmark models are typically $\sim \mathcal{O}(0.01) - \mathcal{O}(0.1)$ fb at the LHC, and enhanced to $\mathcal{O}(1) - \mathcal{O}(10)$ fb at the SppC. Analogous to the Higgs pair production process at the resonance region, the decay branching ratios of $\text{Br}[h_i \rightarrow WW/ZZ/hZ] \propto \alpha_b^2$. Therefore, the improvements of the precise measurements of the future eEDM experiments can also suppress the expected cross sections for these final states.

5 Conclusion

The extended Higgs sector is a general setup with rich physical ingredients to address the issues that are beyond the SM. Particularly, the spontaneous CPV can be achieved with the general 2HDM setup. In this work, we study the Higgs pair productions in the framework of the CPV 2HDM, with the focus on the leading production channel of the ggF. The set of constraints to the CPV Higgs sector are taken into account, including the SM-like Higgs signal fit, the eEDM constraint, the perturbative unitarity and stability constraints, and the current LHC searches for the heavy Higgs bosons. Together with the simplification to the model, we focus on the CPV 2HDM-II, where a relatively large size of CPV mixing is possible at $t_\beta \sim 1$.

The Higgs cubic self couplings play the most crucial role for the Higgs pair production. For our case, two relevant cubic self couplings are λ_{111} and λ_{113} , which are controlled by the soft mass term m_{soft} and the CPV mixing angle of α_b . The precise measurement of the SM-like Higgs cubic coupling of λ_{111} can be achieved via the $e^+e^- \rightarrow h_1h_1Z$ and $e^+e^- \rightarrow h_1h_1\nu_e\bar{\nu}_e$ processes at the future high-energy e^+e^- colliders. The benchmark models in our discussions typically predict totally cross sections of $\sigma[e^+e^- \rightarrow h_1h_1Z]$ smaller than the SM predictions. The largest deviations of the SM-like Higgs cubic couplings λ_{111} are likely to be probed at the future TLEP 500 GeV and ILC 1 TeV runs. At the future high-energy pp collider runs, the Higgs pair productions are very likely to be controlled by the heavy resonance contributions. In the allowed mass range of the heavy Higgs bosons, we find the total production cross sections to be $\sigma[pp \rightarrow h_1h_1] \sim \mathcal{O}(10) - \mathcal{O}(100)$ fb at the LHC 14 TeV runs. They can be as large as $\sim \mathcal{O}(10^3)$ fb at the future SppC 100 TeV runs. Other search modes of di-bosons and Higgs plus Z that are currently probed at the LHC $7 \oplus 8$ TeV experiments are also estimated at the future LHC 14 TeV and the SppC 100 TeV experiments. The discovery of all these channels will manifest the structure of the Higgs sector. Therefore, it will be very helpful to further study the higher-order QCD corrections as well as the collider search capabilities for such heavy resonance contributions to the Higgs pairs.

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