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Effective action of the baryonic branch in string theory flux throats

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Abstract: We discuss consistent truncations of type IIB supergravity on resolved warped deformed conifolds with fluxes. These actions represent the gravitational duals to the baryonic branch deformation of the Klebanov-Strassler cascading gauge theory. As an application, we demonstrate that the baryonic branch is lifted in cascading gauge theory plasma.

Keywords: Gauge-gravity correspondence, Black Holes in String Theory, Holography and quark-gluon plasmas

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Contents

1 Introduction and summary

A conifold Y_6 is a simplest non-compact Calabi-Yau three-fold [1]. It is a cone over a homogeneous five dimensional Einstein manifold $T^{1,1} = (\mathrm{SU}(2) \times \mathrm{SU}(2))/\mathrm{U}(1)$, with the U(1) being a diagonal subgroup of the maximal torus of SU(2) × SU(2). When a large number $N \gg 1$ of D3-branes are placed at its tip, for large 't Hooft coupling $g_s N \gg 1$ their backreaction *warps* the conifold:

$$R^{3,1} \times Y_6 \quad \rightarrow \quad AdS_5 \times T^{1,1} \,. \tag{1.1}$$

Along with N-units of 5-form flux through $T^{1,1}$, the resulting geometry is a consistent background of type IIB string theory, holographically dual to $\mathcal{N} = 1$ four-dimensional superconformal $\mathrm{SU}(N) \times \mathrm{SU}(N)$ gauge theory [2]. The warped conifold can be *deformed* (without further breaking the supersymmetry) by wrapping $M \gg 1$ D5-branes over the two-cycle of $T^{1,1}$. In this case the supergravity background realizes the holographic dual to non-conformal $\mathcal{N} = 1$ supersymmetric $\mathrm{SU}(N + M) \times \mathrm{SU}(N)$ cascading gauge theory [3] (KS). One the geometry side, the $\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ global symmetry of $T^{1,1}$ is broken to $\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathbb{Z}_2$. The conifold deformation parameter breaking $\mathrm{U}(1) \rightarrow \mathbb{Z}_2$ represents the spontaneous chiral symmetry breaking in the confining vacuum of cascading gauge theory. The vacuum structure of $\mathcal{N} = 1$ cascading gauge theories was studied in [4]. Precisely when N is an integer multiple of M, there is a baryonic branch of confining vacua. In fact, the KS vacuum (without mobile D3-branes) corresponds to a special $\mathbb{Z}_2$ symmetric point on this branch. A generic point on the baryonic branch breaks $\mathbb{Z}_2$. The supergravity dual to the baryonic branch of cascading gauge theory was constructed in [5] (BGMPZ):

moving away from the $\mathbb{Z}_2$ symmetric solution corresponds to a *resolution* of the KS warped deformed conifold.

The type IIB supergravity backgrounds constructed in [3] and [5] are supersymmetric, and thus are not suitable to address nonsupersymmetric questions in cascading gauge theory. Likewise, given the prominent role the KS warped throat geometries play in constructing de-Sitter vacua in string theory [6], one needs to understand generic nonsupersymmetric deformations of BGMPZ *resolved warped deformed* conifolds. The first step in this direction was taken in [7], where a five dimensional effective action describing the $SU(2) \times SU(2) \times U(1)$ invariant sector of the warped conifold was constructed. This action includes five dimensional metric coupled to four bulk scalar fields. It was used to prove the renomalizability of cascading gauge theory [7], and detailed studies of thermodynamics and hydrodynamics of chirally symmetric phase of cascading gauge theory plasma [8–10]. In [9] it was shown that cascading gauge theory undergoes the first order confinement-deconfinement phase transition at a certain critical temperature T_c. Furthermore, there is a critical point at $T_u = 0.8749(0)T_c$ where the chirally symmetric phase becomes perturbatively unstable towards condensation of hydrodynamic (sound) modes [10]. To understand chiral symmetry breaking in cascading gauge theory plasma, in [11] we derived effective action corresponding to $SU(2) \times SU(2) \times \mathbb{Z}_2$ invariant sector of the warped deformed conifold — here, three additional scalar fields are included compare to [7]. This effective action[1] was used to establish that chiral symmetry breaking fluctuations in cascading gauge theory plasma become tachyonic at $T_{\chi SB} = 0.882503(0)T_c$; as a result, both confinement and the chiral symmetry breaking in cascading plasma occur simultaneously via the first-order phase transition at T_c.

Comparing to the warped deformed conifold consistent truncation [11], the BGMPZ supersymmetric holographic renormalization group (RG) flow contains two additional scalar fields (a mode dual to a dimension two operator and a mode mode dual to a dimension four operator of the boundary cascading gauge theory). It is straightforward to perform Kaluza-Klein reduction of this enlarged gravity-scalar sector and produce a five-dimensional truncation of the resolved warped deformed conifold [15].[2] Unfortunately, this action is not a consistent truncation away from the origin of the baryonic branch [15];[3] at the origin of the baryonic branch the truncation is consistent and is identical to [11].

The fully consistent $SU(2) \times SU(2)$ truncation of type IIB supergravity on resolved warped deformed conifold was constructed in [17][4] (CF). In this paper we reproduce the derivation of the effective action [17], and point further consistent truncation to effective action [11]. We further discuss linearized fluctuations of CF effective action about $SU(2) \times SU(2) \times U(1)$ symmetric warped conifold with fluxes consistent truncations of [7]. We recover consistent truncation of chiral symmetry breaking sector in cascading gauge

[1] Additional applications were considered in [12, 13].

[2] See also [16].

[3] I would like to thank Davide Cassani and Anton Faedo for bringing reference [15] to my attention, and pointing out the inconsistency of the truncation [16].

[4] Related discussion appeared in [18]. We will not attempt to verify [18] and relate it to earlier work, partly because the authors did not present the Chern-Simons part of the action in full generality.

theory plasma [11]. Lastly, we present linearized effective action describing baryonic branch deformation about SU(2)×SU(2)×U(1) symmetric states of cascading gauge theory plasma. We show that unlike $\mathbb{Z}_2$-invariant chiral symmetry breaking fluctuations, $\mathbb{Z}_2$-non-invariant baryonic branch fluctuations remain massive up to T_u in cascading gauge theory plasma, i.e., the baryonic branch is lifted by the finite temperature effects.

2 Effective action

In this section, following [17] and [19], we reproduce the derivation of CF effective action of the resolved warped deformed conifold with fluxes. The offshoot is that the effective action derived in [17] is correct; moreover, we did not find any typos in the presentation.

We will work in the gravitational approximation to type IIB string theory, using the type IIB supergravity action. This action takes the form (in the Einstein frame)

$$
\begin{aligned}
S_{10} =&\frac{1}{2\kappa_{10}^2}\int_{\mathcal{M}_{10}}\biggl(R_{10}\wedge\star_{10}1-\frac12 d\phi\wedge\star_{10}d\phi-\frac12 e^{-\phi}H_3\wedge\star_{10}H_3-\frac12 e^{\phi}F_3\wedge\star_{10}F_3\\
&-\frac12 e^{2\phi}F_1\wedge\star_{10}F_1-\frac14 F_5\wedge\star_{10}F_5\biggr)\\
&-\frac{1}{8\kappa_{10}^2}\int_{\mathcal{M}_{10}}(B_2\wedge d(C_2)-C_2\wedge d(B_2))\wedge d(C_4),
\end{aligned}
\tag{2.1}
$$

where $\mathcal{M}_{10}$ is the ten dimensional bulk space-time, κ_{10} is the ten dimensional gravitational constant. The form-field strengths, determined from the potentials $\{C_0\ ,B_2\ ,C_2\ ,C_4\}$, satisfy the Bianchi identities:

$$
d(F_1)=0\,,\qquad d(H_3)=0\,,\qquad d(F_3)=H_3\wedge F_1\,,\qquad d(F_5)=H_3\wedge F_3\,. \tag{2.2}
$$

The equations of motion following from the action (2.1) have to be supplemented by the self-duality condition

$$
\star_{10}F_5=F_5\,. \tag{2.3}
$$

It is important to remember that the self-duality condition (2.3) can not be imposed at the level of the action, as this would lead to wrong equations of motion.

Appendix A contains our conventions regarding differential forms.

2.1 Left-invariant forms on the $T^{1,1}$ coset

We use explicit parametrization of the coset $T^{1,1}=(\mathrm{SU}(2)\times\mathrm{SU}(2))/\mathrm{U}(1)$ in terms of angular coordinates $\{\theta_1,\phi_1,\theta_2,\phi_2,\psi\}$ with ranges $0\le\theta_{1,2}<\pi$, $0\le\phi_{1,2}<2\pi$, and $0\le\psi<4\pi$. As in [17] we choose the coframe 1-forms as

$$
\begin{aligned}
e^1&=-\sin\theta_1\ d(\phi_1)\,,\qquad e^2=d(\theta_1)\,,\\
e^3&=\cos\psi\ \sin\theta_2\ d(\phi_2)-\sin\psi\ d(\theta_2)\,,\\
e^4&=\sin\psi\ \sin\theta_2\ d(\phi_2)+\cos\psi\ d(\theta_2)\,,\\
e^5&=d(\psi)+\cos\theta_1\ d(\phi_1)+\cos\theta_2 d(\phi_2)\,.
\end{aligned}
\tag{2.4}
$$

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All left-invariant 1- and 2-forms on the coset are given by [17]:

$$\eta = -\frac{1}{3}e^5\,, \qquad \Omega = \frac{1}{6}(e^1+ie^2)\wedge(e^3-ie^4)\,,$$
$$J = \frac{1}{6}(e^1\wedge e^2 - e^3\wedge^4)\,, \qquad \Phi = \frac{1}{6}(e^1\wedge e^2+e^3\wedge e^4)\,. \tag{2.5}$$

2.2 Metric ansatz and its dimensional reduction

We take the ten-dimensional spacetime $\mathcal{M}_{10}$ to be a direct warped product $\mathcal{M}_5\times T^{1,1}$. The most general $\mathrm{SU}(2)\times\mathrm{SU}(2)$ invariant metric on $\mathcal{M}_{10}$ is parameterized by five 0-forms $\{u,v,\tau,\omega,\theta\}$, and a single 1-form A on $\mathcal{M}_5$, [17]:

$$\begin{aligned}
ds^2_{\mathcal{M}_{10}} &= \sum_I \underline{E}^I\underline{E}^I\,, \qquad ds^2_{\mathcal{M}_5} = \sum_i E^iE^i\,,\\
\underline{E}^I &= e^{-\frac{4}{3}u-\frac{1}{3}v}E^i\,, \qquad \text{for}\ \ I=i=1,\cdots 5\,,\\
\underline{E}^6 &= \frac{1}{\sqrt{6\cosh\tau}}\,e^{u+w}\,e^1\,, \qquad \underline{E}^7 = \frac{1}{\sqrt{6\cosh\tau}}\,e^{u+w}\,e^2\,,\\
\underline{E}^8 &= \sqrt{\frac{\cosh\tau}{6}}\,e^{u-w}\left(e^3+\tanh\tau\,e^{2\omega}\,\mathrm{Re}\left(e^{i\theta}(e^1+ie^2)\right)\right)\,,\\
\underline{E}^9 &= \sqrt{\frac{\cosh\tau}{6}}\,e^{u-w}\left(e^4+\tanh\tau\,e^{2\omega}\,\mathrm{Im}\left(e^{i\theta}(e^1+ie^2)\right)\right)\,,\\
\underline{E}^{10} &= e^v(\eta+A)\,.
\end{aligned} \tag{2.6}$$

Given (2.6), it is straightforward (albeit tedious) to reduce ten-dimensional Einstein-Hilbert term in (2.1). We find

$$\begin{aligned}
\frac{1}{2\kappa_{10}^2}\int_{\mathcal{M}_{10}}(R\star_{10}1) = \frac{1}{2\kappa_5^2}\int_{\mathcal{M}_5}\bigg[&R-\frac{1}{2}e^{\frac{8}{3}u+\frac{8}{3}v}\,(dA)^2+e^{-\frac{8}{3}u-\frac{2}{3}v}\,R_{T^{1,1}}-\frac{28}{3}du^2\\
&-\frac{4}{3}dv^2-\frac{8}{3}dudv-d\tau^2-4\cosh^2\tau\,d\omega^2-\sinh^2\tau\,(d\theta-3A)^2\bigg]\star 1,
\end{aligned} \tag{2.7}$$

where

$$\begin{aligned}
R_{T^{1,1}} =&\, 4e^{-4u+2v}\,\left[\sinh^2\tau-\cosh^2\tau\,\cosh(4\omega)\right]\\
&+24e^{-2u}\cosh\tau\,\cosh(2\omega)-9e^{-2v}\,\sinh^2\tau\,,
\end{aligned} \tag{2.8}$$

and

$$\kappa_5^2 = \frac{\kappa_{10}^2}{V_Y}\,, \qquad V_Y = -\frac{1}{2}\int_{T^{1,1}} J\wedge J\wedge\eta\,, \tag{2.9}$$

with V_Y being the volume of unit size $T^{1,1}$.

$\mathrm{SU}(2)\times\mathrm{SU}(2)$ symmetry requires that both the dilaton ϕ and the axion C_0 are 0-forms on $\mathcal{M}_5$. Their reduction on $T^{1,1}$ is trivial:

$$\frac{1}{2\kappa_{10}^2}\int_{\mathcal{M}_{10}}\left(-\frac{1}{2}(d\phi)^2-\frac{1}{2}e^{2\phi}F_1^2\right)\star_{10}1 = -\frac{1}{2\kappa_5^2}\int_{\mathcal{M}_5}\left[\frac{1}{2}d\phi^2+\frac{1}{2}e^{2\phi}dC_0^2\right]\star 1\,. \tag{2.10}$$

2.3 3-forms ansatz and their dimensional reduction

Most general $SU(2) \times SU(2)$ symmetric ansatz of NSNS 3-form field strength H_3 (solving the Bianchi identity (2.2)) is parameterized by a 2-form b_2, a one form b_1, two real 0-forms b^J and b^Φ, a complex 0-form b^Ω on $\mathcal{M}_5$ and a constant p, [17]:

$$\begin{aligned} H_3 &= p\,\Phi \wedge \eta + d(B_2)\,, \\ B_2 &= b_2 + b_1 \wedge (\eta + A) + b^J J + \mathrm{Re}\,(b^\Omega \Omega) + b^\Phi \Phi\,. \end{aligned} \tag{2.11}$$

The field strength H_3 can be decomposed in a basis of left-invariant forms on $T^{1,1}$ (2.5):

$$\begin{aligned} H_3 = {}& h_3 + h_2 \wedge (\eta + A) + h_1^J \wedge J + \mathrm{Re}\left[h_1^\Omega \wedge \Omega + h_0^\Omega\, \Omega \wedge (\eta + A)\right] \\ & + h_1^\Phi \wedge \Phi + p\,\Phi \wedge (\eta + A)\,, \end{aligned} \tag{2.12}$$

where we defined

$$\begin{aligned} h_3 &= db_2 - b_1 \wedge d(A)\,, & h_1^\Omega &= db^\Omega - 3i\,A\,b^\Omega \equiv Db^\Omega\,, \\ h_2 &= db_1\,, & h_0^\Omega &= 3i\,b^\Omega\,, \\ h_1^J &= db^J - 2b_1 \equiv Db^J\,, & h_1^\Phi &= db^\Phi - p\,A \equiv Db^\Phi\,. \end{aligned} \tag{2.13}$$

Reducing NSNS 3-form contribution in (2.1) on $T^{1,1}$ results in

$$\begin{aligned} &\frac{1}{2\kappa_{10}^2}\int_{\mathcal{M}_{10}} \left(-\frac{1}{2}e^{-\phi}H_3^2\right) \star_{10} 1 = -\frac{1}{2\kappa_5^2}\int_{\mathcal{M}_5} \bigg\{ e^{-4u-\phi}\Big[(\cosh^2\tau\cosh(4\omega) - \sinh^2\tau)(h_1^J)^2 \\ &+ (\cosh^2\tau\cosh(4\omega) + \sinh^2\tau)(h_1^\Phi)^2 + \cosh^2\tau |h_1^\Omega|^2 - \sinh^2 \mathrm{Re}\,(e^{-2i\theta}(h_1^\Omega)^2) \\ &- 2\cosh^2\tau\sinh(4\omega) h_1^J h_1^\Phi - 2\sinh(2\tau)\left(\sinh(2\omega)h_1^J - \cosh(2\omega)h_1^\Phi\right)\mathrm{Re}\,(ie^{-i\theta}h_1^\Omega)\Big] \\ &+ \frac{1}{2}e^{\frac{8}{3}u - \frac{4}{3}v - \phi}\, h_2^2 + \frac{1}{2}e^{\frac{16}{3}u + \frac{4}{3}v - \phi}\, h_3^2 + e^{-\frac{20}{3}u - \frac{8}{3}v - \phi}\Big[\mathrm{Re}(-e^{-2i\theta}\sinh^2\tau(h_0^\Omega)^2 \\ &+ 2ipe^{-i\theta}\sinh(2\tau)\cosh(2\omega)h_0^\Omega) + \cosh^2\tau|h_0^\Omega|^2 + p^2(\cosh^2\tau\cosh(4\omega) + \sinh^2\tau)\Big]\bigg\} \star 1\,. \end{aligned} \tag{2.14}$$

Similarly, most general $SU(2) \times SU(2)$ symmetric ansatz of RR 3-form field strength F_3 (solving the Bianchi identity (2.2)) is parameterized by a 2-form c_2, a one form c_1, two real 0-forms c^J and c^Φ, a complex 0-form c^Ω on $\mathcal{M}_5$ and a constant q, [17]:

$$\begin{aligned} F_3 &= q\,\Phi \wedge \eta + d(C_2) - C_0 H_3\,, \\ C_2 &= c_2 + c_1 \wedge (\eta + A) + c^J J + \mathrm{Re}\,(c^\Omega \Omega) + c^\Phi \Phi\,. \end{aligned} \tag{2.15}$$

The field strength F_3 can be decomposed in a basis of left-invariant forms on $T^{1,1}$ (2.5):

$$\begin{aligned} F_3 = {}& g_3 + g_2 \wedge (\eta + A) + g_1^J \wedge J + \mathrm{Re}\left[g_1^\Omega \wedge \Omega + g_0^\Omega\, \Omega \wedge (\eta + A)\right] \\ & + g_1^\Phi \wedge \Phi + (q - C_0 p)\,\Phi \wedge (\eta + A)\,, \end{aligned} \tag{2.16}$$

where we defined

$$\begin{aligned} g_3 &= dc_2 - c_1 \wedge d(A) - C_0 h_3\,, & g_1^\Omega &= dc^\Omega - 3i\,A\,c^\Omega - C_0 Db^\Omega \equiv Dc^\Omega - C_0 Db^\Omega\,, \\ g_2 &= dc_1 - C_0 db_1\,, & g_0^\Omega &= 3i\,(c^\Omega - C_0 b^\Omega)\,, \\ g_1^J &= dc^J - 2c_1 - C_0 Db^J \equiv Dc^J - C_0 Db^J\,, && \\ g_1^\Phi &= dc^\Phi - q\,A - C_0 Db^\Phi \equiv Dc^\Phi - C_0 Db^\Phi\,. && \end{aligned} \tag{2.17}$$

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Reducing RR 3-form contribution in (2.1) on $T^{1,1}$ results in expression equivalent to the r.h.s. of (2.14) with the obvious substitutions:

$$\frac{1}{2\kappa_{10}^2}\int_{\mathcal{M}_{10}}\left(-\frac{1}{2}e^{\phi}F_3^2\right)\star_{10}1=-\frac{1}{2\kappa_5^2}\int_{\mathcal{M}_5}\left\{\phi\to-\phi\,,h\to g\,,p\to(q-C_0p)\right\}. \tag{2.18}$$

2.4 5-form ansatz and its reduction reduction

Because of the self-duality condition (2.3), special care should be taken in dealing with the reduction of the 5-form; furthermore, to reproduce correct type IIB supergravity equations of motion the 5-form topological term (the second line in (2.1)) must be replaced with [17]

$$\begin{aligned}S_{\rm IIB,top}=&-\frac{1}{8\kappa_{10}^2}\int_{\mathcal{M}_5}\Bigg[\left(B_2\wedge(d(C_2)+2F_3^{fl})-C_2\wedge(d(B_2)+2H_3^{fl})\right)\wedge(d(C_4)+F_5^{fl})\\&+\frac{1}{2}\left(B_2\wedge B_2\wedge d(C_2)\wedge F_3^{fl}+C_2\wedge C_2\wedge d(B_2)\wedge H_3^{fl}\right)\Bigg]\\\equiv&-\frac{1}{8\kappa_{10}^2}\int_{\mathcal{M}_5}\left[L_5\wedge(d(C_4)+F_5^{fl})+L_{10}\right],\end{aligned} \tag{2.19}$$

where the third line is used to define L_5 and L_{10}, and

$$F_3^{fl}=q\,\Phi\wedge\eta\,,\qquad H_3^{fl}=p\,\Phi\wedge\eta\,,\qquad F_5^{fl}=k\,J\wedge J\wedge(\eta+A)\,, \tag{2.20}$$

for a constant k. Note that neither L_5 nor L_{10} contain 5-form degrees of freedom. The proper strategy in dealing with the 5-form self-duality condition was developed in [19], which we apply here.

Let's focus first on 5-from degrees of freedom. 5-form Bianchi identity (2.2) is solved with

$$F_5=d(C_4)+F_5^{fl}+\frac{1}{2}L_5\,, \tag{2.21}$$

and the 5-form part of the action (2.1) can be written as

$$S_{F_5}=-\frac{1}{8\kappa_{10}^2}\int_{\mathcal{M}_{10}}\left[F_5\wedge\star_{10}F_5+L_5\wedge F_5\right]. \tag{2.22}$$

As with 3-forms, we can decompose 5-from into the basis of left invariant forms on $T^{1,1}$:

$$\begin{aligned}F_5=&f_5+f_4\wedge(\eta+A)+f_3^J\wedge J+f_2^J\wedge J\wedge(\eta+A)+{\rm Re}\left[f_3^\Omega\wedge\Omega+f_2^\Omega\wedge\Omega\wedge(\eta+A)\right]\\&+f_3^\Phi\wedge\Phi+f_2^\Phi\wedge\Phi\wedge(\eta+A)+f_1\wedge J\wedge J+f_0\,J\wedge J\wedge(\eta+A)\,,\end{aligned} \tag{2.23}$$

with

$$f_0 = k + pc^\Phi - qb^\Phi + 3\mathrm{Im}\left[b^\Omega \overline{c^\Omega}\right] \equiv k + \frac{1}{2}\ell_0\,, \tag{2.24}$$

$$\begin{aligned} f_1 &= Da + \frac{1}{2}(qb^\Phi - pc^\Phi)A + \frac{1}{2}\Big[b^J Dc^J - b^\Phi Dc^\Phi + \mathrm{Re}\left[b^\Omega \overline{Dc^\Omega}\right] - b \leftrightarrow c\Big] \\ &\equiv Da + \frac{1}{2}\ell_1\,, \end{aligned} \tag{2.25}$$

$$f_2^J = d(a_1^J) + \frac{1}{2}\Big[b^J d(c_1) - b_1 \wedge Dc^J - b \leftrightarrow c\Big] \equiv d(a_1^J) + \frac{1}{2}\ell_2^J\,, \tag{2.26}$$

$$f_2^\Omega = Da_1^\Omega + 3ia_2^\Omega + \frac{1}{2}\Big[b^\Omega d(c_1) - b_1 \wedge Dc^\Omega + 3ic^\Omega b_2 - b \leftrightarrow c\Big] \equiv Da_1^\Omega + 3ia_2^\Omega + \frac{1}{2}\ell_2^\Omega\,, \tag{2.27}$$

$$\begin{aligned} f_2^\Phi &= d(a_1^\Phi) + \frac{1}{2}(qb_1 - pc_1)A + qb_2 - pc_2 + \frac{1}{2}\Big[b^\Phi d(c_1) - b_1 \wedge Dc^\Phi - b \leftrightarrow c\Big] \\ &\equiv d(a_1^\Phi) + \frac{1}{2}\ell_2^\Phi\,, \end{aligned} \tag{2.28}$$

$$\begin{aligned} f_3^\Omega &= Da_2^\Omega - a_1^\Omega \wedge d(A) + \frac{1}{2}\Big[b_2 \wedge Dc^\Omega + b^\Omega(d(c_2) - c_1 \wedge d(A)) - b \leftrightarrow c\Big] \\ &\equiv Da_2^\Omega - a_1^\Omega \wedge d(A) + \frac{1}{2}\ell_3^\Omega\,, \end{aligned} \tag{2.29}$$

$$\begin{aligned} f_3^\Phi =&\ d(a_2^\Phi) - a_1^\Phi \wedge d(A) + \frac{1}{2}\Big[pc_2 \wedge A - qb_2 \wedge A\Big] \\ &+ \frac{1}{2}\Big[b_2 \wedge Dc^\Phi + b^\Phi(d(c_2) - c_1 \wedge d(A)) - b \leftrightarrow c\Big] \equiv d(a_2^\Phi) - a_1^\Phi \wedge d(A) + \frac{1}{2}\ell_3^\Phi\,, \end{aligned} \tag{2.30}$$

$$\begin{aligned} f_3^J &= d(a_2^J) - 2a_3 - a_1^J \wedge d(A) + \frac{1}{2}\Big[b_2 \wedge Dc^J + b^J(d(c_2) - c_1 \wedge d(A)) - b \leftrightarrow c\Big] \\ &\equiv d(a_2^J) - 2a_3 - a_1^J \wedge d(A) + \frac{1}{2}\ell_3^J\,, \end{aligned} \tag{2.31}$$

$$f_4 = d(a_3) + \frac{1}{2}\Big[b_2 \wedge d(c_1) - b_1 \wedge (d(c_2) - c_1 \wedge d(A)) - b \leftrightarrow c\Big] \equiv d(a_3) + \frac{1}{2}\ell_4\,, \tag{2.32}$$

$$\begin{aligned} f_5 &= f_5^{flux} + d(a_4) - a_3 \wedge d(A) + \frac{1}{2}\Big[b_2 \wedge (d(c_2) - c_1 \wedge d(A)) - b \leftrightarrow c\Big] \\ &\equiv f_5^{flux} + d(a_4) - a_3 \wedge d(A) + \frac{1}{2}\ell_5\,, \end{aligned} \tag{2.33}$$

where we defined

$$\begin{aligned} Da &= d(a) - 2a_1^J - kA\,, \\ Da_1^\Omega &= d(a_1^\Omega) - 3iA \wedge a_1^\Omega\,, \\ Da_2^\Omega &= d(a_2^\Omega) - 3iA \wedge a_2^\Omega\,. \end{aligned} \tag{2.34}$$

The last identities in (2.24)–(2.33) are used to define $\{\ell_0, \ell_1, \ell_2^J, \ell_2^\Omega, \ell_2^\Phi, \ell_3^\Omega, \ell_3^J, \ell_3^\Phi, \ell_4, \ell_5\}$. The form fields $\{a, a_1^J, a_1^\Phi, a_1^\Omega, a_2^J, a_2^\Phi, a_2^\Omega, a_3, a_4\}$ are degrees of freedom of C_4:

$$\begin{aligned} d(C_4) =&\ d(a_4) - a_3 \wedge d(A) + d(a_3) \wedge (\eta + A) + (d(a_2^J) - 2a_3 - a_1^J \wedge d(A)) \wedge J \\ &+ d(a_1^J) \wedge J \wedge (\eta + A) + \mathrm{Re}\left[(Da_2^\Omega - a_1^\Omega \wedge d(A)) \wedge \Omega + (Da_1^\Omega + 3ia_2^\Omega) \wedge \Omega \wedge (\eta + A)\right] \\ &+ (d(a_2^\Phi) - a_1^\Phi \wedge d(A)) \wedge \Phi + d(a_1^\Phi) \wedge \Phi \wedge (\eta + A) + (d(a) - 2a_1^J) \wedge J \wedge J\,. \end{aligned} \tag{2.35}$$

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Note that given (2.35), $d^2(C_4) = 0$. The self-duality of the 5-form (2.3) relates $\{f_5, f_4, f_3^J, f_3^\Phi, f_3^\Omega\}$ to the remaining 5-form components in (2.23) as follows:

$$
\begin{aligned}
f_5 &= 2e^{-\frac{32}{3}u-\frac{8}{3}v} \star f_0 \,, && (2.36)\\
f_4 &= -\,2e^{-8u} \star f_1 \,, && (2.37)\\
f_3^J &= e^{-\frac{4}{3}u-\frac{4}{3}v} \star \Big[(\cosh^2\tau\cosh(4\omega) - \sinh^2\tau) f_2^J - \cosh^2\tau\sinh(4\omega) f_2^\Phi \\
&\quad - \sinh(2\tau)\sinh(2\omega)\mathrm{Re}\left(ie^{-i\theta} f_2^\Omega\right)\Big] \,, && (2.38)\\
f_3^\Phi &= e^{-\frac{4}{3}u-\frac{4}{3}v} \star \Big[\cosh^2\tau\sinh(4\omega) f_2^J - (\cosh^2\tau\cosh(4\omega) + \sinh^2\tau) f_2^\Phi \\
&\quad - \sinh(2\tau)\cosh(2\omega)\mathrm{Re}\left(ie^{-i\theta} f_2^\Omega\right)\Big] \,, && (2.39)\\
f_3^\Omega &= e^{-\frac{4}{3}u-\frac{4}{3}v} \star \Big[ie^{i\theta}\sinh(2\tau)\sinh(2\omega) f_2^J - ie^{i\theta}\sinh(2\tau)\cosh(2\omega) f_2^\Phi + \cosh^2\tau f_2^\Omega \\
&\quad - \sinh^2\tau e^{2i\theta}\overline{f_2^\Omega}\Big] \,. && (2.40)
\end{aligned}
$$

We can not substitute (2.36)–(2.40) directly into (2.22); rather, we supplement it with the following term:[5]

$$
\begin{aligned}
S'_{F_5} = \frac{1}{2\kappa_{10}^2}\int_{\mathcal{M}_5} \Big\{ &\left(f_5 - \frac{1}{2}\ell_5\right) k - \left(f_4 - \frac{1}{2}\ell_4\right)\wedge Da \\
&+ \left(f_3^J + a_1^J \wedge d(A) - \frac{1}{2}\ell_3^J\right)\wedge d(a_1^J) \\
&+ \mathrm{Re}\left[\left(f_3^\Omega - Da_2^\Omega + d(A)\wedge a_1^\Omega - \frac{1}{2}\ell_3^\Omega\right)\wedge \overline{Da_1^\Omega + 3ia_2^\Omega}\right] \\
&- \left(f_3^\Phi + a_1^\Phi \wedge d(A) - \frac{1}{2}\ell_3^\Phi\right)\wedge d(a_1^\Phi)\Big\} \wedge \left\{\frac{1}{2} J\wedge J\wedge\eta\right\}.
\end{aligned}
\tag{2.41}
$$

In the modified action $S_{F_5} + S'_{F_5}$, the self-duality constraints (2.36)–(2.40) arise as equations of motion:

$$
\begin{aligned}
&\frac{\delta}{\delta f_5}\left(S_{F_5} + S'_{F_5}\right) = 0 \,, && \frac{\delta}{\delta f_4}\left(S_{F_5} + S'_{F_5}\right) = 0 \,, && \frac{\delta}{\delta f_3^J}\left(S_{F_5} + S'_{F_5}\right) = 0 \,, \\
&\frac{\delta}{\delta f_3^\Phi}\left(S_{F_5} + S'_{F_5}\right) = 0 \,, && \frac{\delta}{\delta \mathrm{Re}\,[f_3^\Omega]}\left(S_{F_5} + S'_{F_5}\right) = 0 \,, && \frac{\delta}{\delta \mathrm{Im}\,[f_3^\Omega]}\left(S_{F_5} + S'_{F_5}\right) = 0 \,.
\end{aligned}
\tag{2.42}
$$

The reduced 5-form action is then obtained from imposing the self-duality constraints (2.36)–(2.40) in

$$
S_{F_5}^{\text{reduced}} = \left\{ -\frac{1}{8\kappa_{10}^2}\int_{\mathcal{M}_{10}} L_5 \wedge F_5 + S'_{F_5}\right\}\Bigg|_{F_5=\star_{10}F_5} = S_{F_5}^{\text{kinetic}} + S_{F_5}^{\text{topological}} \,, \tag{2.43}
$$

[5]This term is a total derivative on-shell.

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where (up to total derivatives)

$$S_{F_5}^{\text{kinetic}} = -\frac{1}{2\kappa_5^2}\int_{\mathcal{M}_5}\Big\{2e^{-8u}f_1^2 + e^{-\frac{4}{3}u-\frac{4}{3}v}\Big[\left(\cosh^2\tau\cosh(4\omega)-\sinh^2\tau\right)(f_2^J)^2$$
$$+\left(\cosh^2\tau\cosh(4\omega)+\sinh^2\tau\right)(f_2^\Phi)^2 - \sinh^2\tau\,\mathrm{Re}\left(e^{-2i\theta}(f_2^\Omega)^2\right)+\cosh^2\tau|f_2^\Omega|^2$$
$$-2\cosh^2\tau\sinh(4\omega)f_2^Jf_2^\Phi - 2\sinh(2\tau)\left(\sinh(2\omega)f_2^J-\cosh(2\omega)f_2^\Phi\right)\mathrm{Re}\left(ie^{-i\theta}f_2^\Omega\right)\Big]$$
$$+2e^{-\frac{32}{3}u-\frac{8}{3}v}f_0^2\Big\}\star 1\,, \tag{2.44}$$

$$S_{F_5}^{\text{topological}} = \frac{1}{2\kappa_5^2}\int_{\mathcal{M}_5}\Big\{\frac{i}{3}\overline{(Da_1^\Omega+3ia_2^\Omega)}\wedge D\left(Da_1^\Omega+3ia_2^\Omega\right)+A\wedge d(a_1^J)\wedge d(a_1^J)$$
$$-A\wedge d(a_1^\Phi)\wedge d(a_1^\Phi)-\frac{1}{2}\mathrm{Re}\left[\left(Da_1^\Omega+3ia_2^\Omega+f_2^\Omega\right)\wedge\overline{\ell_3^\Omega}\right]-\frac{1}{2}(d(a_1^J)+f_2^J)\wedge\ell_3^J$$
$$+\frac{1}{2}(d(a_1^\Phi)+f_2^\Phi)\wedge\ell_3^\Phi+\frac{1}{2}(Da+f_1)\wedge\ell_4-\frac{1}{2}(k+f_0)\wedge\ell_5\Big\}\,, \tag{2.45}$$

where we defined

$$D\left(Da_1^\Omega+3ia_2^\Omega\right) = d\left(Da_1^\Omega+3ia_2^\Omega\right)-3iA\wedge\left(Da_1^\Omega+3ia_2^\Omega\right). \tag{2.46}$$

Additional contribution to five-dimensional topological couplings comes from L_{10} term in (2.19), which, up to total derivatives, takes form:

$$S_{F_5}^{\text{topological,extra}} = \frac{1}{2\kappa_5^2}\int_{\mathcal{M}_5}\frac{1}{2}\Big[p(c_2+c_1\wedge A)-q(b_2+b_1\wedge A)\Big]\wedge\Big[c^\Phi d(b_2+b_1\wedge A)$$
$$-b^\Phi d(c_2+c_1\wedge A)\Big]\,. \tag{2.47}$$

2.5 CF effective action

Collecting (2.7), (2.10), (2.14), (2.18), (2.44), (2.45) and (2.47) we obtain the CF effective action [17]:

$$S_{\text{eff}} = \frac{1}{2\kappa_2^2}\int_{\mathcal{M}_5}R\star 1 + S_{kin,scal}+S_{kin,vect}+S_{kin,forms}+S_{\text{top}}+S_{\text{pot}}\,, \tag{2.48}$$

with

$$S_{\text{kin,scal}} = -\frac{1}{2\kappa_5^2}\int_{\mathcal{M}_5}\Big\{\frac{28}{3}du^2+\frac{4}{3}dv^2+\frac{8}{3}dudv+d\tau^2+4\cosh^2\tau\,d\omega^2 \tag{2.49}$$
$$+\sinh^2\tau\,(d\theta-3A)^2+e^{-4u-\phi}\Big[(\cosh^2\tau\cosh(4\omega)-\sinh^2\tau)(h_1^J)^2$$
$$+(\cosh^2\tau\cosh(4\omega)+\sinh^2\tau)(h_1^\Phi)^2+\cosh^2\tau|h_1^\Omega|^2-\sinh^2\mathrm{Re}\,(e^{-2i\theta}(h_1^\Omega)^2)$$
$$-2\cosh^2\tau\sinh(4\omega)h_1^Jh_1^\Phi-2\sinh(2\tau)\left(\sinh(2\omega)h_1^J-\cosh(2\omega)h_1^\Phi\right)\mathrm{Re}\,(ie^{-i\theta}h_1^\Omega)\Big]$$
$$+e^{-4u+\phi}\Big[h\to g\Big]+\frac{1}{2}d\phi^2+\frac{1}{2}e^{2\phi}dC_0^2+2e^{-8u}f_1^2\Big\}\star 1\,,$$

$$
\begin{aligned}
S_{\rm kin,vect} = &-\frac{1}{2\kappa_5^2}\int_{\mathcal{M}_5}\left\{\frac{1}{2}e^{\frac{8}{3}u+\frac{8}{3}v}\,(dA)^2+\frac{1}{2}e^{\frac{8}{3}u-\frac{4}{3}v-\phi}\,h_2^2+\frac{1}{2}e^{\frac{8}{3}u-\frac{4}{3}v+\phi}\,g_2^2\right. \\
&+e^{-\frac{4}{3}u-\frac{4}{3}v}\Big[\left(\cosh^2\tau\cosh(4\omega)-\sinh^2\tau\right)(f_2^J)^2+\left(\cosh^2\tau\cosh(4\omega)+\sinh^2\tau\right)(f_2^\Phi)^2 \\
&-\sinh^2\tau{\rm Re}\left(e^{-2i\theta}(f_2^\Omega)^2\right)+\cosh^2\tau|f_2^\Omega|^2-2\cosh^2\tau\sinh(4\omega)f_2^J f_2^\Phi \\
&\left.-2\sinh(2\tau)\left(\sinh(2\omega)f_2^J-\cosh(2\omega)f_2^\Phi\right){\rm Re}\left(ie^{-i\theta}f_2^\Omega\right)\Big]\right\}\star 1\,,
\end{aligned}
\tag{2.50}
$$

$$
S_{\rm kin,forms} = -\frac{1}{4\kappa_5^2}\int_{\mathcal{M}_5}e^{\frac{16}{3}u+\frac{4}{3}v}\left(e^{-\phi}h_3^2+e^{\phi}g_3^2\right)\star 1\,,
\tag{2.51}
$$

$$
\begin{aligned}
S_{\rm top} = &\frac{1}{2\kappa_5^2}\int_{\mathcal{M}_5}\left\{\frac{i}{3}\overline{(Da_1^\Omega+3ia_2^\Omega)}\wedge D\left(Da_1^\Omega+3ia_2^\Omega\right)+A\wedge d(a_1^J)\wedge d(a_1^J)\right. \\
&-A\wedge d(a_1^\Phi)\wedge d(a_1^\Phi)-\frac{1}{2}{\rm Re}\left[\left(Da_1^\Omega+3ia_2^\Omega+f_2^\Omega\right)\wedge\overline{\ell_3^\Omega}\right]-\frac{1}{2}(d(a_1^J)+f_2^J)\wedge\ell_3^J \\
&+\frac{1}{2}(d(a_1^\Phi)+f_2^\Phi)\wedge\ell_3^\Phi+\frac{1}{2}(Da+f_1)\wedge\ell_4-\frac{1}{2}(k+f_0)\wedge\ell_5+\frac{1}{2}\Big[p(c_2+c_1\wedge A) \\
&\left.-q(b_2+b_1\wedge A)\Big]\wedge\Big[c^\Phi d(b_2+b_1\wedge A)-b^\Phi d(c_2+c_1\wedge A)\Big]\right\},
\end{aligned}
\tag{2.52}
$$

$$
\begin{aligned}
S_{\rm pot} = &\frac{1}{2\kappa_5^2}\int_{\mathcal{M}_5}\left\{e^{-\frac{8}{3}u-\frac{2}{3}v}R_{T^{1,1}}-2e^{-\frac{32}{3}u-\frac{8}{3}v}f_0^2-e^{-\frac{20}{3}u-\frac{8}{3}v-\phi}\Big[{\rm Re}(-e^{-2i\theta}\sinh^2\tau(h_0^\Omega)^2\right. \\
&+2ipe^{-i\theta}\sinh(2\tau)\cosh(2\omega)h_0^\Omega)+\cosh^2\tau|h_0^\Omega|^2+p^2(\cosh^2\tau\cosh(4\omega)+\sinh^2\tau)\Big] \\
&\left.-e^{-\frac{20}{3}u-\frac{8}{3}v+\phi}\Big[h\to g\,,p\to(q-pC_0)\Big]\right\}\star 1\,.
\end{aligned}
\tag{2.53}
$$

The equations of motion obtained from (2.48) are equivalent to type IIB supergravity equations of motion [20]. Thus, SU(2) × SU(2) symmetric effective action (2.48) provides consistent truncation of type IIB supergravity on resolved warped deformed conifolds with fluxes.

2.6 Consistent truncations to KS/KT effective actions

There is a consistent truncation of the SU(2) × SU(2) symmetric CF action to SU(2) × SU(2) × $\mathbb{Z}_2$ sector describing warped deformed conifold with fluxes obtained in [11, 12, 15] with the non-vanishing CF fields identified as

$$
\begin{aligned}
&e^{-\frac{8}{3}u-\frac{2}{3}v}g_{\mu\nu}dx^\mu dx^\nu = g_{\mu\nu}^{KS}dx^\mu dx^\nu\,, \qquad k=216\Omega_0^{KS}\,,\quad q=P^{KS}\,,\quad \phi=\phi^{KS}\,, \\
&\frac{1}{3}e^v=\Omega_1^{KS}\,,\quad \frac{1}{\sqrt{6}}e^{u-\tau/2}=\Omega_2^{KS}\,,\qquad \frac{1}{\sqrt{6}}e^{u+\tau/2}=\Omega_3^{KS}\,,\quad b^\Phi=-3\left(h_1^{KS}+h_3^{KS}\right)\,, \\
&{\rm Im}\,[b^\Omega]=3\left(h_3^{KS}-h_1^{KS}\right)\,, \qquad\qquad {\rm Re}\,[c^\Omega]=6\left(h_2^{KS}-\frac{P^{KS}}{18}\right)\,,
\end{aligned}
\tag{2.54}
$$

where the superscript ^{KS} corresponds to the parametrization of fields in [11].

Further (consistent) restriction to a $SU(2)\times SU(2)\times U(1)$ symmetric sector of (2.54) with

$$\begin{aligned}
&\tau=0\,, && \mathrm{Im}\,[b^\Omega]=0\,, && \mathrm{Re}\,[c^\Omega]=0\,, && \left(b^\Phi-\frac{k}{q}\right)=-\frac{K}{2P}\,,\\
&e^v=\Omega_1=f_2^{1/2}h^{1/4}\,, && e^u=\Omega_2=f_3^{1/2}h^{1/4}\,, && q=P\,, &&
\end{aligned}\tag{2.55}$$

leads to the warped conifold with fluxes effective action of [7].

2.7 Decoupling of linearized fluctuations of CF action around KT action

Here we characterize decoupled linearized fluctuation sectors about $SU(2)\times SU(2)\times U(1)$ truncation of CF effective action:

$$\begin{aligned}
S_{\rm KT}=&\frac{1}{2\kappa_5^2}\int_{\mathcal{M}_5}\biggl\{R-\frac{28}{3}du^2-\frac{4}{3}dv^2-\frac{8}{3}dudv-e^{-4u-\phi}(db^\Phi)^2-\frac{1}{2}d\phi^2\\
&-2e^{-\frac{32}{3}u-\frac{8}{3}v}(b^\Phi q-k)^2-e^{-\frac{20}{3}u-\frac{8}{3}v+\phi}q^2+24e^{-\frac{14}{3}u-\frac{2}{3}v}-4e^{-\frac{20}{3}u+\frac{4}{3}v}\biggr\}\star 1\,.
\end{aligned}\tag{2.56}$$

Analyzing bilinears of the remaining CF modes about (2.56) we find that there are six decoupled sectors involving:

- $\{\delta C_0,\delta A,\delta c^\Phi,\delta a,\delta a_1^J\}$;
- $\{\delta b_2,\delta c_2,\delta a_1^\Phi,\delta c_1,\delta c^J\}$;
- $\{\delta a_1^\Omega,\delta a_2^\Omega\}$;
- $\{\mathrm{Re}\,[\delta b^\Omega],\mathrm{Im}\,[\delta c^\Omega]\}$;
- $\{\delta\tau,\mathrm{Im}\,[\delta b^\Omega]\equiv\delta b_2^\Omega,\mathrm{Re}\,[\delta c^\Omega]\equiv\delta c_1^\Omega\}$;
- $\{\delta\omega,\delta b^J,\delta b_1\}$.

Notice that $\delta\theta$ does not couple to quadratic order in KT truncation of CF effective action.

In what follows we focus on the last two fluctuation sets: the chiral symmetry breaking sector,

$$\begin{aligned}
&S_{\chi cb}\left[\delta\tau,\delta b_2^\Omega,\delta c_1^\Omega\right]=\frac{1}{\kappa_5^2}\int_{\mathcal{M}_5}\biggl\{-\frac{1}{2}(d\delta\tau)^2-\frac{1}{2}e^{-4u+\phi}(d\delta c_1^\Omega)^2-\frac{1}{2}e^{-4u-\phi}(d\delta b_2^\Omega)^2\\
&+2e^{-4u-\phi}\delta\tau db^\Phi d\delta b_2^\Omega+6e^{-\frac{32}{3}u-\frac{8}{3}v}(b^\Phi q-k)\delta b_2^\Omega\delta c_1^\Omega+6e^{-\frac{20}{3}u-\frac{8}{3}v+\phi}q\delta\tau\delta c_1^\Omega\\
&-\frac{9}{2}e^{-\frac{20}{3}u-\frac{8}{3}v}\left(e^{-\phi}(\delta b_2^\Omega)^2+e^{\phi}(\delta c_1^\Omega)^2\right)-\frac{1}{2}\Bigl(2e^{-\frac{20}{3}u-\frac{8}{3}v+\phi}q^2+9e^{-\frac{8}{3}u-\frac{8}{3}v}-12e^{-\frac{14}{3}u-\frac{2}{3}v}\\
&+2e^{-4u-\phi}(db^\Phi)^2\Bigr)(\delta\tau)^2\biggr\}\star 1\,,
\end{aligned}\tag{2.57}$$

and the baryonic branch deformation sector,

$$S_{\text{baryonic}}\left[\delta\omega,\delta b^J,\delta b_1\right]=\frac{1}{\kappa_5^2}\int_{\mathcal{M}_5}\Biggl\{-\frac{1}{4}e^{\frac{8}{3}u-\frac{4}{3}v-\phi}(d\delta b_1)^2-e^{-4u-\phi}\biggl(\frac{1}{2}(d\delta b^J)^2+2(\delta b_1)^2$$
$$-2d\delta b^J\delta b_1-4\delta\omega(d\delta b^J-2\delta b_1)db^\Phi\biggr)-2(d\delta\omega)^2+\biggl(-4e^{-\frac{20}{3}u-\frac{8}{3}v+\phi}q^2-16e^{-\frac{20}{3}u+\frac{4}{3}v}$$
$$+24e^{-\frac{14}{3}u-\frac{2}{3}v}-4e^{-4u-\phi}(db^\Phi)^2\biggr)(\delta\omega)^2\Biggr\}\star 1\,. \tag{2.58}$$

We explicitly verified that with the identifications

$$\delta b_2^\Omega=-\frac{1}{2P}\delta k_1\,,\qquad c_1^\Omega=\frac{P}{3}\delta k_2\,,\qquad \delta\tau=-\frac{\delta f}{f_3}\,, \tag{2.59}$$

the effective action $S_{\chi cb}$ is equivalent to the effective action obtained in [11].

Effective action S_{baryonic} is a new result. Remarkably, consistent truncation of the baryonic branch deformations around generic $SU(2)\times SU(2)\times U(1)$ states of cascading gauge theory requires inclusion of a vector field δb_1, in addition to the supersymmetric scalar modes δw and δb^J identified in [5]. We also verified that effective action (2.58), reduced[6] with $\delta b_1=0$, is equivalent to the one discussed in [16]. Notice that S_{baryonic} is invariant under the λ-gauge symmetry:

$$\delta b^J\to\delta b^J+2\lambda\,,\qquad \delta\omega\to\delta\omega\,,\qquad \delta b_1\to\delta b_1+d\lambda\,, \tag{2.60}$$

for an arbitrary 0-form λ on $\mathcal{M}_5$. This gauge symmetry is simply a restriction of general λ-gauge transformations discussed in [17] to linearized (decoupled) fluctuations $\{\delta\omega,\delta b^J,\delta b_1\}$ about $SU(2)\times SU(2)\times U(1)$ states of cascading gauge theory. Gauge symmetry (2.60) can be used to completely eliminate δb^J fluctuations.

3 Baryonic branch in cascading gauge theory plasma

As an application of the effective action (2.58), we study stability of the baryonic branch fluctuations in cascading gauge theory plasma [9]. We focus on geometries dual to thermal states of cascading plasma, and study the spectrum of the baryonic branch quasinormal modes of Klebanov-Tseytlin black hole [9, 10]. We show that these modes remain massive for all accessible temperatures, i.e., for $T\ge T_u$.

First, we rewrite effective action (2.58) using the KS background metric (see (2.54)):

$$g_{\mu\nu}\to g_{\mu\nu}\Omega^{-2}\,,\qquad \Omega=e^{-\frac{4}{3}u-\frac{1}{3}v}\,. \tag{3.1}$$

As a result of a Weyl rescaling (3.1),

$$\star 1\to\Omega^{-5}\star 1\,,\qquad A_{(p)}B_{(p)}\to\Omega^{2p}A_{(p)}B_{(p)}\,, \tag{3.2}$$

[6] As we emphasized earlier, such a reduction is not a consistent truncation.

for any p-forms $A_{(p)}$ and $B_{(p)}$ on $\mathcal{M}_5$. Thus, (2.58) is modified to

$$
\begin{aligned}
\hat{S}_{\text{baryonic}}\left[\delta\omega, \delta b^J, \delta b_1\right] = & \frac{1}{\kappa_5^2}\int_{\mathcal{M}_5}\left\{ -\frac{1}{4}e^{4u-v-\phi}(d\delta b_1)^2 - 2e^{4u+v}(d\,\delta\omega)^2 \right. \\
& + e^{v-\phi}\Big(-2(\delta b_1)^2 - 8\delta\omega\delta b_1 db^\Phi - 4(\delta\omega)^2(db^\Phi)^2 + 2d\delta b^J\delta b_1 + 4\delta\omega d\delta b^J db^\Phi - \frac{1}{2}(d\,\delta b^J)^2\Big) \\
& \left. + \Big(-4e^{-v+\phi}q^2 + 24e^{2u+v} - 16e^{3v}\Big)(\delta\omega)^2\right\}\star 1\,. \qquad (3.3)
\end{aligned}
$$

The background geometry dual to the deconfined homogeneous and isotropic phase of the cascading plasma is given by

$$
\begin{aligned}
& ds_5^2 = h^{-1/2}(1-f_1^2)^{-1/2}\left(-f_1^2\,dt^2 + dx_1^2 + dx_2^2 + dx_3^2\right) + \frac{1}{9}h^{1/2}f_2\,\frac{dr^2}{\tilde{f}_2^2}\,, \\
& u = \ln\left(f_3^{1/2}h^{1/4}\right), \qquad v = \ln\left(f_2^{1/2}h^{1/4}\right), \qquad db^\Phi = -\frac{1}{2P}dK\,, \qquad q = P\,,
\end{aligned}
\qquad (3.4)
$$

with $\{f_1, \tilde{f}_2, K, h, f_2, f_3, g_s \equiv e^\phi\}$ being functions of r only. We focus on modes at the threshold of instability, thus, without loss of generality we assume[7]

$$
\begin{aligned}
& \delta b^J = 0\,, \qquad\qquad \delta w = -\frac{1}{2}e^{ikx_1}\,\mathcal{Z}\,, \\
& \delta b_{1,x_1} = ike^{ikx_1}\mathcal{B}_{x_1}\,, \qquad \delta b_{1,r} = e^{ikx_1}\mathcal{B}_r\,, \qquad \delta b_{1,t} = \delta b_{1,x_2} = \delta b_{1,x_3} = 0\,,
\end{aligned}
\qquad (3.5)
$$

where $\{\mathcal{Z}, \mathcal{B}_{x_1}, \mathcal{B}_r\}$ are functions of the radial coordinate only, satisfying the following equations of motion (obtained from (3.3))

$$
\begin{aligned}
0 = & k^2 f_1^2\,\mathcal{Z} - \frac{9\tilde{f}_2^2 f_1^2}{hf_2(1-f_1^2)^{1/2}}\,\mathcal{Z}'' - \frac{9\tilde{f}_2 f_1}{f_2f_3h(1-f_1^2)^{3/2}}\Big(\tilde{f}_2f_3f_1'f_1^2 - 2\tilde{f}_2f_3'f_1^3 \\
& - f_3\tilde{f}_2'f_1^3 + \tilde{f}_2f_3f_1' + 2\tilde{f}_2f_3'f_1 + f_3\tilde{f}_2'f_1\Big)\,\mathcal{Z}' - \frac{f_1^2}{2h^2f_3^2g_sP^2f_2(1-f_1^2)^{1/2}} \\
& \Big(24hf_3g_sP^2f_2 - 16hg_sP^2f_2^2 - 4g_s^2P^4 - 9\tilde{f}_2^2(K')^2\Big)\,\mathcal{Z} + \frac{18\tilde{f}_2^2f_1^2K'}{g_sf_2f_3^2(1-f_1^2)^{1/2}Ph^2}\mathcal{B}_r\,,
\end{aligned}
\qquad (3.6)
$$

$$
\begin{aligned}
0 = & \mathcal{B}_{x_1}'' - \frac{1}{\tilde{f}_2f_3f_1f_2g_s(1-f_1^2)}\Big(2\tilde{f}_2f_2g_sf_1^3f_3' + f_3f_2g_sf_1^3\tilde{f}_2' - 2\tilde{f}_2f_2g_sf_1f_3' - f_3\tilde{f}_2f_1'f_2g_s \\
& - f_3f_2g_sf_1\tilde{f}_2' - f_3\tilde{f}_2g_s'f_2f_1^3 + f_3\tilde{f}_2f_2'g_sf_1 + f_3\tilde{f}_2g_s'f_2f_1 - f_3\tilde{f}_2f_2'g_sf_1^3\Big)\mathcal{B}_{x_1}' \\
& - \frac{8f_2^2}{9f_3^2\tilde{f}_2^2}\mathcal{B}_{x_1} - \mathcal{B}_r' + \frac{1}{f_3\tilde{f}_2f_1f_2g_s(1-f_1^2)}\Big(2\tilde{f}_2f_2g_sf_1^3f_3' + f_3f_2g_sf_1^3\tilde{f}_2' - 2\tilde{f}_2f_2g_sf_1f_3' \\
& - f_3\tilde{f}_2f_1'f_2g_s - f_3f_2g_sf_1\tilde{f}_2' - f_3\tilde{f}_2g_s'f_2f_1^3 + f_3\tilde{f}_2f_2'g_sf_1 + f_3\tilde{f}_2g_s'f_2f_1 \\
& - f_3\tilde{f}_2f_2'g_sf_1^3\Big)\mathcal{B}_r\,,
\end{aligned}
\qquad (3.7)
$$

$$
0 = \frac{hf_3^2k^2(1-f_1^2)^{1/2}}{f_2}\mathcal{B}_{x_1}' - \frac{hf_3^2f_1^2k^2(1-f_1^2)^{1/2} + 8f_1^2f_2}{f_1^2f_2}\mathcal{B}_r - \frac{4K'}{P}\mathcal{Z}\,. \qquad (3.8)
$$

[7]Here, we use the gauge symmetry (2.60) to eliminate δb^J and assume propagation of quasinormal modes along x_1 direction.

Notice that equation (3.8) can be used to algebraically eliminate $\mathcal{B}_r$ from equations (3.6) and (3.7).

To make use of the results in [9, 10] we use a radial coordinate x as

$$x \equiv 1 - f_1(r)\,. \tag{3.9}$$

The physical fluctuations described by (3.6)–(3.8) must be regular at the horizon of the KT BH, and be normalizable at the asymptotic $x \to 0_+$ boundary. Introducing

$$\mathfrak{q} = \frac{k}{2\pi T}\,, \tag{3.10}$$

and using the asymptotic expansion for the KT BH developed in [9],[8] the normalizability condition for $\{\mathcal{Z}\,,\mathcal{B}_{x_1}\}$ at the $x \to 0_+$ boundary translates into the following asymptotic solution

$$\mathcal{Z} = z_1 x^{1/2} + \frac{\pi^2 T^2 \mathfrak{q}^2 z_1}{4\sqrt{2}}(2k_s + 9 - \ln x)x + \mathcal{O}(x^{3/2}\ln^2 x)\,, \tag{3.11}$$

$$\mathcal{B}_{x_1} = x\left(b_{2,0} + \frac{\pi^2 T^2 \mathfrak{q}^2 z_1 \sqrt{2}\ln x}{1152}(12k_s + 94 - 3\ln x)\right) + \mathcal{O}(x^{3/2}\ln^3 x)\,, \tag{3.12}$$

where we presented the expansions only to leading order in the normalizable UV coefficients

$$\Big\{z_1\,, b_{2,0}\Big\}\,. \tag{3.13}$$

The independent UV normalizable coefficients (3.13) imply that the baryonic branch deformation in cascading plasma is associated with the development of the expectation values of operators of dimension-2 and dimension-4.

Since the equations of motion (3.6)–(3.8) are homogeneous, without the loss of generality we can set $\mathcal{Z}(1) = 1$. The IR, i.e., as $y \equiv (1-x) \to 0_+$, asymptotic expansion then takes form

$$\mathcal{Z} = 1 + \mathcal{O}(y^2)\,, \qquad \mathcal{B}_{x_1} = b_0^h + \mathcal{O}(y^2)\,, \tag{3.14}$$

where we presented the expansions only to leading order in the normalizable IR coefficient

$$\Big\{b_0^h\Big\}\,. \tag{3.15}$$

The results of the analysis of the dispersion relation of the baryonic branch quasinormal modes are presented in figure 1. In principle, we expect discrete branches of the quasinormal modes distinguished by the number of nodes in radial profiles $\{\mathcal{Z}\,,\mathcal{B}_{x_1}\}$. In what follows we consider only the lowest quasinormal mode, which has monotonic radial profiles. We find that over all range of temperatures, the fluctuations (solid blue line) have $\mathfrak{q}^2 < 0$ — as a result, they are massive. The red dashed line

$$\mathfrak{q}^2\Big|_{\rm red,dashed} = -0.47(1) + 0.02(2)\,\ln^{-1}\frac{T}{\Lambda} + \mathcal{O}\left(\ln^{-2}\frac{T}{\Lambda}\right)\,, \tag{3.16}$$

represents the best fit to (the high-temperature tail of) the data. Notice that in the limit $T \gg \Lambda$ the cascading theory approaches a conformal theory with temperature being the only relevant scale, thus, in agreement with (3.16), $\mathfrak{q}^2$ must approach a constant in this limit.

[8] As explained in [9] we can set in numerical analysis $a_0 = 1$.

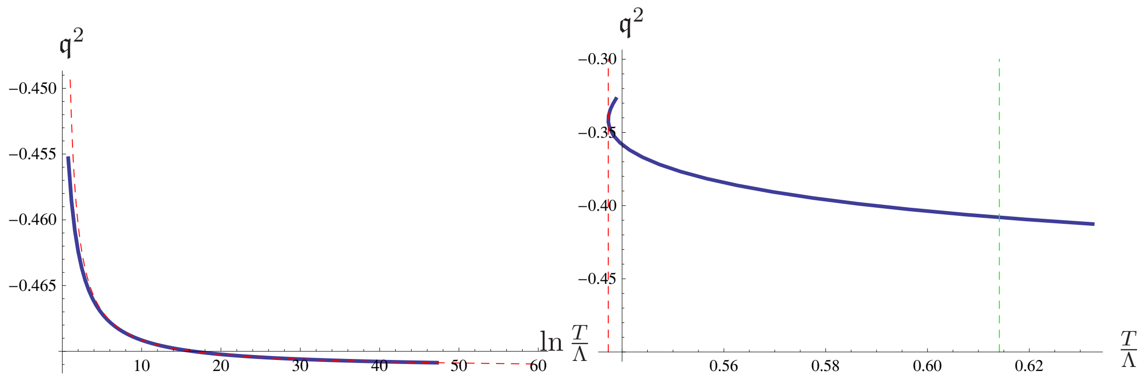

Figure 1. (Colour online) *Left panel:* dispersion relation of the baryonic branch quasinormal modes of the Klebanov-Tseytlin black hole as a function of $\ln \frac{T}{\Lambda}$ at high temperature. The solid blue line represents the dispersion relation of the baryonic branch fluctuations. The red dashed line is a fit (3.16) to the data. *Right panel:* dispersion relation at low temperatures. The vertical dashed green and red lines indicate $T = T_c$ (the confinement/deconfinement temperature) and $T = T_u$ (the hydrodynamic instability temperature) correspondingly.

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A Conventions

A differential p-form $A_{(p)}$ in ten dimensions is defined as

$$A_{(p)} = \frac{1}{p!} A_{(p)\, I_1 \cdots I_p}\, \underline{E}^{I_1} \wedge \cdots \wedge \underline{E}^{I_p} \,, \tag{A.1}$$

where $A_{(p)\, I_1 \cdots I_p}$ are form components in orthonormal ten-dimensional vielbein $\{\underline{E}^I\}$ basis. A Hodge dual is defined according to

$$\star_{10}\, \underline{E}^{I_1} \wedge \cdots \wedge \underline{E}^{I_p} = \frac{1}{(10-p)!} \epsilon^{I_1 \cdots I_p}{}_{I_{p+1} \cdots I_{10}}\, \underline{E}^{I_{p+1}} \wedge \cdots \wedge \underline{E}^{I_{10}} \,, \tag{A.2}$$

with

$$\epsilon_{1 \cdots 10} = +1 \,, \qquad \epsilon^{1 \cdots 10} = -1 \,. \tag{A.3}$$

Similarly, a differential p-form $A_{(p)}$ in five dimensions is defined as

$$A_{(p)} = \frac{1}{p!} A_{(p)\, i_1 \cdots i_p}\, E^{i_1} \wedge \cdots \wedge E^{i_p} \,, \tag{A.4}$$

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where $A_{(p)\,i_1\cdots i_p}$ are form components in orthonormal five-dimensional vielbein $\{E^i\}$ basis. A Hodge dual is defined according to

$$\star\ E^{i_1}\wedge\cdots\wedge E^{i_p}=\frac{1}{(5-p)!}\epsilon^{i_1\cdots i_p}{}_{i_{p+1}\cdots i_5}\,E^{i_{p+1}}\wedge\cdots\wedge E^{i_{10}}\,, \tag{A.5}$$

with

$$\epsilon_{1\cdots 5}=+1\,,\qquad \epsilon^{1\cdots 5}=-1\,. \tag{A.6}$$

Given two p-forms $A_{(p)}$ and $B_{(p)}$ we have

$$\begin{aligned} A_{(p)}\wedge\star_{10}B_{(p)}&=\left[\frac{1}{p!}\,A_{(p)I_1\cdots I_p}B_{(p)}^{I_1\cdots I_p}\right]\star_{10}1\equiv\left[A_{(p)}B_{(p)}\right]\star_{10}1\,,\\ A_{(p)}\wedge\star B_{(p)}&=\left[\frac{1}{p!}\,A_{(p)i_1\cdots i_p}B_{(p)}^{i_1\cdots i_p}\right]\star 1\equiv\left[A_{(p)}B_{(p)}\right]\star 1\,, \end{aligned} \tag{A.7}$$

in ten and five dimensions correspondingly.

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