Non-Abelian discrete R symmetries

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ABSTRACT: We discuss non-Abelian discrete R symmetries which might have some conceivable relevance for model building. The focus is on settings with $\mathcal{N} = 1$ supersymmetry, where the superspace coordinate transforms in a one-dimensional representation of the non-Abelian discrete symmetry group. We derive anomaly constraints for such symmetries and find that novel patterns of Green-Schwarz anomaly cancellation emerge. In addition we show that perfect groups, also in the non–R case, are always anomaly-free. An important property of models with non-Abelian discrete R symmetries is that superpartners come in different representations of the group. We present an example model, based on a $\mathbb{Z}_3 \rtimes \mathbb{Z}_8^R$ symmetry, to discuss generic features of models which unify discrete R symmetries, entailing solutions to the μ and proton decay problems of the MSSM, with non-Abelian discrete flavor symmetries.

Contents

1 Introduction and outline

Despite the lack of experimental evidence for superpartners at the LHC, supersymmetry is still one of the leading candidates for physics beyond the standard model. The so-called R symmetries, under which the superspace coordinate θ transforms non-trivially, play an important role both in the more formal aspects of supersymmetry as well as in model building. In the context of $\mathcal{N} = 1$ supersymmetry, the focus of the literature so far has been on Abelian symmetries, i.e. either a continuous $\mathrm{U}(1)_R$ or a discrete $\mathbb{Z}_N^R$ subgroup thereof.

It is, however, also possible to embed the R symmetry in a non-Abelian discrete symmetry group D. Since there is only one superspace coordinate in the $\mathcal{N} = 1$ case, θ has to furnish a one-dimensional representation of D. This means that the action of D on θ is Abelian, i.e. a $\mathbb{Z}_N^R$ symmetry. On the other hand, this $\mathbb{Z}_N^R$ symmetry can be part of a larger, in general non-Abelian symmetry group D.

The purpose of this study is to explore theoretical and phenomenological properties of such symmetries. This includes anomaly constraints and possible applications in flavor model building.

The outline of this paper is as follows. In section 2 we discuss anomaly constraints and anomaly cancellation by the Green-Schwarz (GS) mechanism. The proof that perfect groups are anomaly-free can be found in section 2.4. Next, in section 3 we survey possible symmetries and discuss specifically an extension of the minimal supersymmetric standard model (MSSM) based on a $\mathbb{Z}_3 \rtimes \mathbb{Z}_8^R$ symmetry. Finally, section 4 contains our conclusions.

2 Anomaly constraints

Anomaly constraints for discrete symmetries have been analyzed using various methods [1–4]. We will base our discussion on the path integral approach [5, 6], which can also be applied to discrete symmetries [7, 8]. A given symmetry operation is said to be anomalous if it implies a non-trivial transformation of the path integral measure. We start by reviewing the anomaly coefficients for Abelian discrete (R and non–R) symmetries.

2.1 Anomaly coefficients for discrete Abelian R and non-R symmetries

The anomaly conditions for discrete R symmetries depend on the charge of the superspace coordinate, q_θ; in the case of a non–R symmetry $q_\theta = 0$. Consider an operation u of order M, which generates a $\mathbb{Z}_M$ or a $\mathbb{Z}_M^R$ symmetry and might or might not be embedded in a non-Abelian symmetry group.

The superpotential transforms as

$$\mathscr{W} \;\to\; \mathrm{e}^{2\pi\,\mathrm{i}\,q_\mathscr{W}/M}\,\mathscr{W} \tag{2.1}$$

with $q_\mathscr{W} = 2q_\theta$ (such that $\int\mathrm{d}^2\theta\,\mathscr{W}$ is invariant). Superfields $\Phi^{(f)} = \phi^{(f)} + \sqrt{2}\,\theta\psi^{(f)} + \theta\theta\,F^{(f)}$ transform as

$$\Phi^{(f)} \;\to\; \mathrm{e}^{2\pi\,\mathrm{i}\,q^{(f)}/M}\,\Phi^{(f)}\;. \tag{2.2}$$

As a consequence, the (chiral) fermions acquire a phase

$$\psi^{(f)} \;\to\; \mathrm{e}^{2\pi\,\mathrm{i}\,(q^{(f)}-q_\theta)/M}\,\psi^{(f)}\;, \tag{2.3}$$

which induces a non-trivial transformation $\mathcal{D}\psi^{(f)}\,\mathcal{D}\overline{\psi}^{(f)} \;\to\; J^{-2}\,\mathcal{D}\psi^{(f)}\,\mathcal{D}\overline{\psi}^{(f)}$ of the path integral measure with non-vanishing Jacobian. In a setting with a non-Abelian gauge symmetry G the Jacobian is given by

$$J^{-2} \;=\; \exp\left\{\mathrm{i}\,\frac{2\pi}{M}\,A_{G-G-\mathbb{Z}_M^R}\int\mathrm{d}^4x\,\frac{1}{32\pi^2}\,F^{b,\mu\nu}\widetilde{F}^b_{\mu\nu}\right\}\;, \tag{2.4}$$

where F and $\widetilde{F}$ denote the field strength and its dual. For Abelian gauge factors and gravity one obtains analogous expressions. In the case of a non-Abelian gauge symmetry, the mixed anomaly coefficient reads [9] (see also [10, Appendix B])

$$A_{G-G-\mathbb{Z}_M^R} \;=\; \sum_f \ell(\boldsymbol{r}^{(f)})\cdot(q^{(f)}-q_\theta) + q_\theta\,\ell(\mathrm{adj}\,G)\;. \tag{2.5}$$

Here, $\boldsymbol{r}^{(f)}$ denotes a representation of the gauge group G, $\ell(\boldsymbol{r}^{(f)})$ is the Dynkin index of the gauge group representation $\boldsymbol{r}$, defined as

$$\delta_{ab}\,\ell(\boldsymbol{r}) \;=\; \mathrm{tr}\left[\mathsf{t}_a(\boldsymbol{r})\,\mathsf{t}_b(\boldsymbol{r})\right]\;, \tag{2.6}$$

and the sum goes over all fermions which transform non-trivially both under G and u. We work in conventions where $\ell(\boldsymbol{N}) = 1/2$ for $\mathrm{SU}(N)$. In this convention the Dynkin index of the adjoint is given as $\ell(\mathrm{adj}) = N$ for $\mathrm{SU}(N)$. In equation (2.5), $\ell(\mathrm{adj}\,G) = c_2(G)$ represents

the contribution from the gauginos. Here we have already allowed for R symmetries, i.e. we include the possibility that the superspace coordinate θ transforms non-trivially under the operation u. In what follows, we will mainly discuss the case of a setting with a non-Abelian gauge symmetry G, but the generalization to U(1) factors and gravity is straightforward.

Irrespective of the nature of the gauge group, all the anomaly coefficients are only defined modulo $M/2$. Notice that for odd M one can make all odd charges even by shifting them by M (cf. [8]). For such charges, the anomaly coefficients are then only defined modulo M.

If a symmetry appears anomalous, this is not necessarily a sign of inconsistency since there is the possibility of (discrete) Green-Schwarz anomaly cancellation, which, as we will discuss in what follows, can be employed for Abelian as well as non-Abelian discrete symmetries.

2.2 Discrete Green-Schwarz anomaly cancellation

The Green-Schwarz mechanism also works for discrete symmetries [9, 10]. The crucial ingredient is, as usual, the coupling of an 'axion' a to the field strength of the continuous gauge symmetry

$$\mathscr{L}_{\text{axion}} \supset -\frac{a}{8} F^b \widetilde{F}^b , \qquad (2.7)$$

and analogous terms for gravity (see e.g. [10] for details). Under a discrete transformation u the axion undergoes a shift

$$a \rightarrow a + \Delta^{(\mathsf{u})} , \qquad (2.8)$$

such that the change of $\mathscr{L}_{\text{axion}}$ compensates the phase induced by the non-trivial transformation of the path integral measure (2.4). This leads to a relation between $\Delta^{(\mathsf{u})}$ and the anomaly coefficients,

$$A_{\mathsf{u}} \equiv A_{G-G-\mathbb{Z}_M} = 2\pi M \Delta^{(\mathsf{u})} \mod \frac{M}{2} . \qquad (2.9)$$

In principle, one can have more than one axion, in which case

$$\mathscr{L}_{\text{axion}} \supset -\sum_\alpha \frac{c_\alpha}{8} a_\alpha F^b \widetilde{F}^b \qquad (2.10)$$

with some (real) coefficients c_α. In the case of a $\mathbb{Z}_M^{(R)}$ symmetry, however, there is always a unique linear combination of axions that shifts, i.e. one can 'diagonalize' the action on the axion fields, such that we are back at the one-axion case.

One can also have more than one gauge factor, i.e. $G = \prod_i G^{(i)}$. Then (2.7) generalizes to

$$\mathscr{L}_{\text{axion}} \supset -\sum_i c_i \frac{a}{8} F_b^{(i)} \widetilde{F}_b^{(i)} . \qquad (2.11)$$

In general, the coefficients c_i can be arbitrary (cf. [11]). However, in supersymmetric theories the axions are always accompanied by a superpartner 'saxion' field. In particular, in the MSSM non-universal c_i coefficients for the SM gauge factors will spoil the beautiful

picture of gauge coupling unification (see the discussion in [12]). This can be avoided by demanding 'anomaly universality', which amounts to requiring

$$A_{G^{(i)}-G^{(i)}-\mathbb{Z}_M^R} = \rho \mod \frac{M}{2} \quad \forall\, G^{(i)} \,, \tag{2.12}$$

and guarantees that we can use the Green-Schwarz mechanism to cancel possible anomalies. Let us now discuss anomaly constraints on non-Abelian discrete R symmetries.

2.3 Anomaly coefficients for non-Abelian discrete R and non-R symmetries

As pointed out in [7, 8], for non-Abelian discrete symmetries possible anomalies reside only in the Abelian parts, i.e. they can be attributed to a specific generator. Let us now focus to finite groups D. Then, for each group element $\mathsf{u} \in D$ there exists an integer M_{u} such that

$$\mathsf{u}^{M_{\mathsf{u}}} = \mathbb{1} \,, \tag{2.13}$$

i.e. u generates a $\mathbb{Z}_{M_{\mathsf{u}}}$ symmetry. In order to verify anomaly-freedom one has, therefore, only to check that the generators of the group generate anomaly-free $\mathbb{Z}_M$ groups.

To make this explicit, let $U_{\mathsf{u}}(\boldsymbol{d})$ be a matrix representation of an abstract group element $\mathsf{u} \in D$ in the representation $\boldsymbol{d}$. As a consequence of (2.13), one can always find a number M_{u} with $U_{\mathsf{u}}(\boldsymbol{d})^{M_{\mathsf{u}}} = \mathbb{1}$. This allows us to write

$$U_{\mathsf{u}}(\boldsymbol{d}) = \mathrm{e}^{2\pi\,\mathrm{i}\,\lambda_{\mathsf{u}}(\boldsymbol{d})\,/\,M_{\mathsf{u}}} \,, \tag{2.14}$$

where $\lambda_{\mathsf{u}}(\boldsymbol{d})$ in general has integer eigenvalues. A fermion charged under D and transforming in a representation $\boldsymbol{d}^{(f)}$, thus transforms under u as

$$\psi^{(f)} \to U_{\mathsf{u}}(\boldsymbol{d}^{(f)})\,\psi^{(f)} = \mathrm{e}^{2\pi\,\mathrm{i}\,\lambda_{\mathsf{u}}(\boldsymbol{d}^{(f)})\,/\,M_{\mathsf{u}}}\,\psi^{(f)} \,. \tag{2.15}$$

Whenever the meaning is clear from the context, we will suppress the subscript u and the representation $\boldsymbol{d}$ for brevity.

In the anomaly coefficient,

$$\delta_{\mathsf{u}}^{(f)} := \mathrm{tr}[\lambda_{\mathsf{u}}(\boldsymbol{d}^{(f)})] = \frac{M_{\mathsf{u}}}{2\pi\,\mathrm{i}} \ln \det U_{\mathsf{u}}(\boldsymbol{d}^{(f)}) \,, \tag{2.16}$$

now takes the role of the discrete $\mathbb{Z}_{M_{\mathsf{u}}}$ charge. This includes the usual modulo M behavior, as becomes explicit through the multi-valued logarithm in (2.16). Nevertheless, $\delta^{(f)}$ is, in general, *not* a one-to-one replacement for an Abelian charge. To see this consider, for example, the relation between the transformation behavior of a superfield Φ and the corresponding fermion, which, in analogy to equations (2.2) and (2.3), is given by

$$\boldsymbol{d}^{(\Phi)} = \boldsymbol{d}^{(\theta)} \otimes \boldsymbol{d}^{(\psi)} \,. \tag{2.17}$$

Here $\boldsymbol{d}^{(\theta)}$ denotes the representation of the superspace coordinate θ. In the case of $\mathcal{N} = 1$ SUSY, θ can only transform in a one-dimensional representation, i.e. $\dim(\boldsymbol{d}^{(\theta)}) = 1$ and $\dim(\boldsymbol{d}^{(\psi)}) = \dim(\boldsymbol{d}^{(\Phi)})$. Therefore, we can express the charge of a fermion component field in terms of the corresponding superfield charge as

$$\delta^{(\psi)} = \delta^{(\Phi)} - \dim(\boldsymbol{d}^{(\Phi)})\,\delta^{(\theta)} \,. \tag{2.18}$$

This illustrates that $\delta^{(f)}$ is *not* a one-to-one replacement for an Abelian charge: equation (2.18) only reduces to the usual addition of charges $q_\Phi^R = q_\theta^R + q_\psi^R$ for one-dimensional representations $\boldsymbol{d}^{(\Phi)}$.

For manifestly supersymmetric theories, it is convenient to make use of equation (2.18) to express the anomaly coefficients in terms of the charges of the superfield $\delta^{(s)}$ instead of the (fermion) component field charges $\delta^{(f)}$.

Using this convention, let us now present the anomaly coefficients. Assume that we have chiral superfields $\Phi^{(s)}$ which transform in representations $\boldsymbol{d}^{(s)}$ of a non-Abelian discrete R symmetry D, with charges $Q^{(s)}$ under the Abelian factors of a U(1) symmetry and as $\boldsymbol{r}^{(s)}$ under some non-Abelian gauge symmetry G. Then, the anomaly coefficients of the Abelian, u-generated subgroup $\mathbb{Z}_M$ of D are given by

$$A_{G-G-\mathbb{Z}_{M(\mathsf{u})}^R} = \sum_s \ell(\boldsymbol{r}^{(s)}) \cdot \left[\delta^{(s)} - \dim(\boldsymbol{d}^{(s)})\,\delta^{(\theta)}\right] + \ell(\mathrm{adj}\,G) \cdot \delta^{(\theta)}\,, \quad (2.19\text{a})$$

$$A_{\mathrm{U}(1)-\mathrm{U}(1)-\mathbb{Z}_{M(\mathsf{u})}^R} = \sum_s \left(Q^{(s)}\right)^2 \dim(\boldsymbol{r}^{(s)}) \cdot \left[\delta^{(s)} - \dim(\boldsymbol{d}^{(s)})\,\delta^{(\theta)}\right]\,, \quad (2.19\text{b})$$

$$\begin{aligned} A_{\mathrm{grav}-\mathrm{grav}-\mathbb{Z}_{M(\mathsf{u})}^R} = {} & -21\,\delta^{(\theta)} + \delta^{(\theta)} \sum_G \dim(\mathrm{adj}\,G) \\ & + \sum_s \dim(\boldsymbol{r}^{(s)}) \cdot \left[\delta^{(s)} - \dim(\boldsymbol{d}^{(s)})\,\delta^{(\theta)}\right]\,, \end{aligned} \quad (2.19\text{c})$$

where the sum goes over all chiral superfields. In the R symmetry case we have contributions not only from the matter fermions and higgsinos but also due to possible gauge singlets, gauginos and the gravitino. The charges of the latter two coincide with the charge of the superspace coordinate θ. The anomaly coefficients are in agreement with previous results: setting $\delta^{(\theta)} = 0$ one arrives at the coefficients for non–R, non-Abelian discrete symmetries [8], and setting $\delta^{(\phi)} = q_\phi^R$ and $\dim(\boldsymbol{d}^{(s)}) = 1$ leads to the coefficients for Abelian R symmetries (2.5) [10, 13].

In principle, one now would have to calculate the anomaly coefficients (2.19a)–(2.19c) for every single group element $\mathsf{u} \in D$ and check if they fulfill (2.12). As it has been argued, in [8], in the case $\rho = 0$ it is enough to check (2.12) only for the generators of D, since if $\rho = 0$ holds for two elements $\mathsf{u}, \mathsf{v} \in D$ it also holds for $\mathsf{w} = \mathsf{u} \cdot \mathsf{v}$. This is due to the nice properties of the determinant and the logarithm in (2.16). One has to be more careful in the general case $\rho \neq 0$ however, as shown in the following.

Let us assume that we have calculated the anomaly coefficients for any two group elements u of order M and v of order N as

$$A_{\mathsf{u}} = \rho \mod \frac{M}{2}\,, \quad (2.20\text{a})$$

$$A_{\mathsf{v}} = \sigma \mod \frac{N}{2}\,. \quad (2.20\text{b})$$

The anomaly coefficient of a third group element $\mathsf{w} = \mathsf{u} \cdot \mathsf{v}$ of order L then is given by[1]

$$A_{\mathsf{w}} = \sum_f \ell(\boldsymbol{r}^{(f)})\,\delta_{\mathsf{w}}^{(f)} + \ell(\mathrm{adj}\,G)\,\delta_{\mathsf{w}}^{(\theta)}$$

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[1]In the most general case (where we do not assume anything about the permuting properties or relative orders of u and v) we cannot say much about the relation of L, M and N.

$$= \sum_f \ell(\boldsymbol{r}^{(f)}) \left(\frac{L}{M} \delta_{\mathsf{u}}^{(f)} + \frac{L}{N} \delta_{\mathsf{v}}^{(f)} \right) + \ell(\operatorname{adj} G) \left(\frac{L}{M} \delta_{\mathsf{u}}^{(\theta)} + \frac{L}{N} \delta_{\mathsf{v}}^{(\theta)} \right)$$

$$= \frac{L}{M} \left(\rho \mod \frac{M}{2} \right) + \frac{L}{N} \left(\sigma \mod \frac{N}{2} \right) . \tag{2.21}$$

We can now distinguish three possible cases:

1. Neither u nor v generates an anomalous symmetry, i.e. $\rho = \sigma = 0$. We recover the trivial case as treated in [8]. The symmetry generated by $\{\mathsf{u}, \mathsf{v}\}$ is anomaly-free.

2. Without loss of generality, only u generates an anomalous symmetry, i.e. $\rho \neq 0 = \sigma$. It follows that also $\mathsf{w} = \mathsf{u} \cdot \mathsf{v}$ is anomalous, with an anomaly coefficient

$$A_{\mathsf{w}} = L \left(\frac{\rho}{M} \mod \frac{1}{2} \right) . \tag{2.22}$$

3. Both u and v generate anomalous symmetries. The anomaly coefficient for w is

$$A_{\mathsf{w}} = L \left[\left(\frac{\rho}{M} + \frac{\sigma}{N} \right) \mod \frac{1}{2} \right] . \tag{2.23}$$

   In this case, even though u and v appear anomalous, w might not. Note also the special case $\mathsf{u} = \mathsf{v}$ where $\mathsf{w} = \mathsf{u}^2$ appears anomalous if and only if $4\rho/M \notin \mathbb{Z}$.

A generalization of this discussion to three or more generators is possible in a straightforward way.

2.4 Green-Schwarz mechanism for non-Abelian discrete symmetries

In principle, the cancellation mechanism for Abelian discrete symmetries also works for the Abelian subgroups of non-Abelian symmetries. There are, however, some additional relations constraining possible axion transformations under the symmetry group. Consider two operations, u and v, in D. In general, those will induce shifts

$$\mathsf{u} : a \to a + \Delta^{(\mathsf{u})} , \tag{2.24a}$$

$$\mathsf{v} : a \to a + \Delta^{(\mathsf{v})} . \tag{2.24b}$$

In particular, the action of these shifts on the axion is Abelian — in other words: the chiral superfield containing the axion as a complex phase can only transform in a one-dimensional representation of our symmetry. As a consequence, axions are not allowed to shift under so-called commutator elements of the symmetry group[2] $\mathsf{x} := [\mathsf{u}, \mathsf{v}]$, since for such elements all fermion charges

$$\delta_{\mathsf{x}} = \frac{M}{2\pi \mathrm{i}} \ln \det U_{\mathsf{x}} , \tag{2.25}$$

and therefore the anomaly coefficients (2.19a)–(2.19c) trivially vanish. This immediately follows from the definition of a commutator element, whose representations always can be written as

$$U_{\mathsf{x}} = U_{\mathsf{u}\,\mathsf{v}\,\mathsf{u}^{-1}\,\mathsf{v}^{-1}} = U_{\mathsf{u}}\, U_{\mathsf{v}}\, U_{\mathsf{u}^{-1}}\, U_{\mathsf{v}^{-1}} = U_{\mathsf{u}}\, U_{\mathsf{v}}\, U_{\mathsf{u}}^{-1}\, U_{\mathsf{v}}^{-1} , \tag{2.26}$$

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[2] Here the group-theoretical definition of the commutator (cf. [14]) $[\mathsf{u}, \mathsf{v}] := \mathsf{u}\,\mathsf{v}\,\mathsf{u}^{-1}\,\mathsf{v}^{-1}$ is used.

which leads to a vanishing charge (2.25) and thus to vanishing anomaly coefficients. By noting that only the one-dimensional representations transform trivially under commutator elements, it is clear that axions can only transform as one-dimensional representations.

We would like to remark that, for the same reason, perfect groups, which are generated by commutator elements only, *always* are anomaly-free. Nevertheless, since they do not possess non-trivial one-dimensional representations, perfect groups are not relevant to this work.

Let us now discuss the cancellation of anomalies for the whole non-Abelian group. Consider two generating elements u and v of order M and N with their respective anomaly coefficients (cf. (2.20)). The combined operation $\mathsf{u} \cdot \mathsf{v}$ is assumed to have order L, and the anomaly coefficient of the combined operation is given by

$$A_{\mathsf{u} \cdot \mathsf{v}} = \omega \mod \frac{L}{2} . \tag{2.27}$$

As shown in equation (2.21), the combined anomaly coefficient can be rewritten as a non-trivial sum of the single anomaly coefficients. Let us now check whether it is always possible to cancel the combined anomaly. To do so, we impose an axion shift

$$\mathsf{u} \cdot \mathsf{v} : a \to a + \Delta^{(\mathsf{u} \cdot \mathsf{v})} , \tag{2.28}$$

which, due the Abelian nature of the axion transformation, must be given as

$$\Delta^{(\mathsf{u} \cdot \mathsf{v})} = \Delta^{(\mathsf{u})} + \Delta^{(\mathsf{v})} . \tag{2.29}$$

The condition for the cancellation of the combined anomaly, in analogy to (2.9), is

$$A_{\mathsf{u} \cdot \mathsf{v}} = 2 \pi L \Delta^{(\mathsf{u} \cdot \mathsf{v})} \mod \frac{L}{2} , \tag{2.30}$$

which can be rewritten as

$$
\begin{aligned}
A_{\mathsf{u} \cdot \mathsf{v}} &\overset{(2.29)}{=} 2 \pi L \left(\Delta^{(\mathsf{u})} + \Delta^{(\mathsf{v})} \right) \mod \frac{L}{2} \\
&\overset{(2.9)}{=} \frac{L}{M} \left(\rho \mod \frac{M}{2} \right) + \frac{L}{N} \left(\sigma \mod \frac{N}{2} \right) .
\end{aligned}
\tag{2.31}
$$

But this is exactly the same as (2.21). This means we do not have any constraints on the anomalies of combined elements, or in other words, if the single (Abelian) anomalies of the generator elements are vanishing (with or without employing the Green-Schwarz mechanism), the whole group is anomaly-free.

3 Non-Abelian discrete R symmetries in the MSSM

3.1 Symmetry search

In what follows, we will discuss specific examples of non-Abelian discrete R symmetries in the context of the MSSM. The non-Abelian discrete R symmetry will, in general, act non-trivially on flavor space. We will assume it to be partly broken by flavon VEVs at

a high scale, thus giving rise to a specific flavor structure. On the other hand, since we wish not to break supersymmetry at a high scale Λ, we require the R symmetry subgroup to be unbroken. Specifically, we will focus on settings in which there is a residual $\mathbb{Z}_4^R$ symmetry [15], which has recently been shown to be the unique Abelian discrete R symmetry which allows us to solve the μ problem and commutes with SO(10) in the matter sector [9, 10, 13]. An unbroken $\mathbb{Z}_2$ subgroup of this symmetry coincides with R parity.

We further demand that, after breaking the flavor symmetry D down to the residual $\mathbb{Z}_4^R$ symmetry, matter fields have R charge 1 and Higgs fields charge 0. This is because we assume a hierarchy between Λ and the scale of $\mathbb{Z}_4^R$ breaking, which is given by the gravitino mass $m_{3/2}$. In this case, a family-dependent $\mathbb{Z}_4^R$ charge assignment implies unrealistic mixing angles. Hence, in order to be consistent with this charge assignment while allowing for correlations in family space, the non-Abelian discrete R symmetry is required to have a multiplet representation whose components transform equally under the $\mathbb{Z}_4^R$ subgroup. In particular, this requires that the center of the group contains the $\mathbb{Z}_4^R$. One can see this with the help of an explicit representation: consider the representation matrix of the generating element of the $\mathbb{Z}_4^R$ subgroup in a basis in which it is diagonal. Since each component is required to transform equally under the subgroup, this matrix must be proportional to the unit matrix, therefore commuting with all other matrices of the representation, and thus a representation of an element of the center.

To summarize, we survey non-Abelian discrete R symmetries which satisfy the following criteria:

1. the symmetry contains, and can be spontaneously broken down to a $\mathbb{Z}_4$ symmetry by a multiplet VEV;

2. the symmetry contains a one-dimensional representation (for θ), which transforms non-trivially also under the unbroken subgroup;

3. the residual $\mathbb{Z}_4$ subgroup is part of the center of the symmetry group.

We have conducted a symmetry search in the SMALLGROUPS library of the GAP system for computational discrete algebra [16]. The results for the groups up to order 48 are shown in table 1. The smallest groups which fulfill all requirements are $\mathbb{Z}_3 \rtimes \mathbb{Z}_8$ and $\mathrm{S}_3 \times \mathbb{Z}_4$. The latter group contains the well known S_3, on which several working GUT flavor models are based [17–26]. Nevertheless, regarding an R symmetric extension of the MSSM, it would just be the trivial extension of any of the known S_3 models by a $\mathbb{Z}_4^R$. Such models should not concern us here. We will focus our considerations on the other possible lowest order group, namely $\mathbb{Z}_3 \rtimes \mathbb{Z}_8$. In contrast to S_3, we are not aware of any existing flavor model based on this group. For this reason, we have stated the necessary group theoretical details in appendix A.

3.2 $\mathbb{Z}_3 \rtimes \mathbb{Z}_8^R$ extension of the MSSM

Let us now discuss an example model for non-Abelian discrete R symmetries, which also act in flavor space, based on the particle spectrum of the the MSSM. Taking grand unification

$\mathcal{O}(D)$	Structure description	ID
24	$\mathbb{Z}_3 \rtimes \mathbb{Z}_8$	SG(24,1)
24	$\mathrm{S}_3 \times \mathbb{Z}_4$	SG(24,5)
32	$(\mathbb{Z}_8 \times \mathbb{Z}_2) \rtimes \mathbb{Z}_2$	SG(32,5)
32	$(\mathbb{Z}_4 \times \mathbb{Z}_4) \rtimes \mathbb{Z}_2$	SG(32,11)
32	$\mathbb{Z}_8 \rtimes \mathbb{Z}_4$	SG(32,12)
32	$D/\mathbb{Z}_4 = \mathrm{D}_8$	SG(32,15)
32	$(\mathbb{Z}_4 \times \mathbb{Z}_4) \rtimes \mathbb{Z}_2$	SG(32,24)
40	$\mathbb{Z}_5 \rtimes \mathbb{Z}_8$	SG(40,1)
40	$\mathbb{Z}_4 \times \mathrm{D}_{10}$	SG(40,3)
48	$\mathbb{Z}_{24} \rtimes \mathbb{Z}_2$	SG(48,5)
48	$(\mathbb{Z}_3 \rtimes \mathbb{Z}_8) \times \mathbb{Z}_2$	SG(48,6)
48	$(\mathbb{Z}_3 \rtimes \mathbb{Z}_8) \times \mathbb{Z}_2$	SG(48,7)
48	$(\mathbb{Z}_3 \rtimes \mathbb{Z}_4) \times \mathbb{Z}_4$	SG(48,8)
$\vdots$	$\vdots$	$\vdots$

Table 1. Result of the GAP scan, showing groups consistent with the requirements stated in the text up to order 48. We give order, name and/or structure description of the group as well as the SMALLGROUPS library ID of GAP.

seriously, we will arrange the matter fields in SU(5) multiplets. We further impose the condition of 'anomaly universality' (cf. the discussion in [12]) such that discrete anomalies can be cancelled by the GS mechanism without spoiling gauge coupling unification.

We will focus our discussion on the generic features of non-Abelian discrete R symmetry extensions rather than trying to enforce an entirely correct phenomenology. Thus, in the spirit of minimalism, we spare additional Abelian discrete 'shaping' symmetries and flavons other than the ones which are essential to symmetry breaking. The explicit construction of a possibly fully realistic model is left for future work. In the present work, we will employ a minimal example model to discuss the consistent assignment of representations, generalities of the symmetry breaking and VEV alignment, the construction of Yukawa coupling and mass matrices, and the explicit calculation of anomaly coefficients.

$\mathbb{Z}_3 \rtimes \mathbb{Z}_8$ and the $\mathbb{Z}_4$ subgroup. The group $\mathbb{Z}_3 \rtimes \mathbb{Z}_8$ is generated by the two elements u and v, which fulfill

$$\mathbb{Z}_3 \rtimes \mathbb{Z}_8 = \left\langle \mathsf{u}, \mathsf{v} \; ; \; \mathsf{u}^3 = \mathsf{v}^8 = \mathbb{1} \, , \; \mathsf{v}\,\mathsf{u}\,\mathsf{v}^{-1} = \mathsf{u}^{-1} \right\rangle \, . \tag{3.1}$$

The group is of order 24, it has 8 one-dimensional representations that we label as $\mathbf{1}_i$ ($i = 1 \ldots 8$) and 4 doublets, denoted by $\mathbf{2}_j$ ($j = 1 \ldots 4$). A more detailed discussion of the group is deferred to appendix A. As, by assumption, a $\mathbb{Z}_4$ subgroup of $\mathbb{Z}_3 \rtimes \mathbb{Z}_8$ will survive down to the SUSY breaking scale, we list the behavior of irreducible representations under this $\mathbb{Z}_4$ subgroup (which is the one generated by v^2) in table 2. Here $\mathbf{1}'$, $\mathbf{1}''$ and $\mathbf{1}'''$ label the representations of $\mathbb{Z}_4$, with the number of primes specifying the corresponding charge. For example, matter fields and the superspace coordinate θ will transform in the $\mathbf{1}'$ representation. As θ transforms non-trivially, the residual symmetry is an (order four) R symmetry,

$\mathbb{Z}_3 \rtimes \mathbb{Z}_8$	$\mathbf{1}_2$	$\mathbf{1}_3$	$\mathbf{1}_4$	$\mathbf{1}_5$	$\mathbf{1}_6$	$\mathbf{1}_7$	$\mathbf{1}_8$	$\mathbf{2}_1$	$\mathbf{2}_2$	$\mathbf{2}_3$	$\mathbf{2}_4$
	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
$\mathbb{Z}_4$	$\mathbf{1}$	$\mathbf{1}''$	$\mathbf{1}''$	$\mathbf{1}'$	$\mathbf{1}'''$	$\mathbf{1}'''$	$\mathbf{1}'$	$\mathbf{1}'' \oplus \mathbf{1}''$	$\mathbf{1} \oplus \mathbf{1}$	$\mathbf{1}''' \oplus \mathbf{1}'''$	$\mathbf{1}' \oplus \mathbf{1}'$

Table 2. Branching rules for $\mathbb{Z}_3 \rtimes \mathbb{Z}_8 \to \mathbb{Z}_4$.

$\left(Q, \overline{U}, \overline{E}\right)_{1,2}$	$\left(Q, \overline{U}, \overline{E}\right)_3$	$\left(\overline{D}, L\right)_{1,2}$	$\left(\overline{D}, L\right)_3$	H_u	H_d	χ	ϕ	θ
$\mathbf{1}_5$	$\mathbf{1}_8$	$\mathbf{2}_4$	$\mathbf{1}_5$	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{2}_2$	$\mathbf{1}_2$	$\mathbf{1}_5$

Table 3. Transformation of MSSM fields under $\mathbb{Z}_3 \rtimes \mathbb{Z}_8^R$. The notation for the MSSM fields is standard, θ is the superspace coordinate, χ and ϕ are (MS)SM singlet flavons.

denoted as $\mathbb{Z}_4^R$ in what follows. We observe that the $\mathbf{2}_2$ contains twice the trivial singlet of $\mathbb{Z}_4^R$. Thus a $\mathbf{2}_2$ VEV in *any* direction can break $\mathbb{Z}_3 \rtimes \mathbb{Z}_8^R \to \mathbb{Z}_4^R$, as desired. Note also that a $\mathbf{2}_2$ VEV aligned in the $(1,0)$ direction (in the basis specified in (A.1)) would break $\mathbb{Z}_3 \rtimes \mathbb{Z}_8^R \to \mathbb{Z}_8^R$.

Charge assignment. From the requirement that θ carries $\mathbb{Z}_4^R$ charge[3] 1 and the breaking pattern of $\mathbb{Z}_3 \rtimes \mathbb{Z}_8^R \to \mathbb{Z}_4^R$ in table 2, we infer that θ has to transform as a $\mathbf{1}_5$ (or as a $\mathbf{1}_8$, which would make no difference), the Higgs fields as $\mathbf{1}_1$ or $\mathbf{1}_2$, and matter can be assigned to $\mathbf{1}_5$, $\mathbf{1}_8$ or $\mathbf{2}_4$ under $\mathbb{Z}_3 \rtimes \mathbb{Z}_8^R$. For a general $\mathbb{Z}_M^R$ symmetry, in order to be anomaly universal, equation (2.5) applied to the non-Abelian gauge groups immediately leads to the requirement

$$q_{H_u} + q_{H_d} = 4\, q_\theta \mod M \,, \tag{3.2}$$

for the Higgs charges (cf. e.g. [13]). Applying this to the $\mathbb{Z}_8^R$ subgroup, we conclude that the Higgs fields have to transform in different representations. This will be important also in the explicit computation of anomaly coefficients in section 3.2.

To accomplish the breaking of the family symmetry $\mathbb{Z}_3 \rtimes \mathbb{Z}_8^R \to \mathbb{Z}_4^R$, we need at least one additional, SM singlet degree of freedom which transforms as a $\mathbf{1}_2$ or $\mathbf{2}_2$ and acquires a VEV. We therefore introduce two of such 'flavons', ϕ and χ, transforming as $\mathbf{1}_2$ and $\mathbf{2}_2$, respectively.

Different assignments either lead to a different breaking of $\mathbb{Z}_3 \rtimes \mathbb{Z}_8^R$ or to unfeasible $\mathbb{Z}_4^R$ charge assignments. The assignment we choose in accordance with all imposed requirements is listed in table 3. Of course, variations of the assignment of the matter and Higgs fields are possible. We have chosen our example such that one gets a glimpse on the variety of possible (leading order) mass matrix structures. There is one peculiar difference here with respect to traditional flavor models: since we are dealing with an R symmetry, the allowed superpotential terms may not be neutral but have to be charged instead. Since θ resides in a $\mathbf{1}_5$, the charge of the superspace integral measure is $\mathbf{1}_5^* \otimes \mathbf{1}_5^* = \mathbf{1}_4^* = \mathbf{1}_3$. Therefore, superpotential terms have to transform as $\mathbf{1}_4$. We wish to point out that, given the non-trivial transformation of θ, fermions and bosons furnish *different* representations under the

---

[3]As discussed for instance in [9], any $\mathbb{Z}_M^R$ symmetry solution to the μ problem requires $M = 4 \times \mathbb{N}$ and $q_\theta = M/4$.

flavor group. For instance, if a superfield transforms as $\mathbf{2}_4$, then the scalar components also furnishes this representations, but the fermions have to transform as $\mathbf{2}_2$ if θ transforms as $\mathbf{1}_5$ or $\mathbf{1}_8$.

Spontaneous breaking $\mathbb{Z}_3 \rtimes \mathbb{Z}_8^R \to \mathbb{Z}_4^R$. From the branching rules (cf. table 2) and the charges of the flavons, it is clear that a non-trivial VEV of either ϕ or χ will break $\mathbb{Z}_3 \rtimes \mathbb{Z}_8^R \to \mathbb{Z}_4^R$. In the general case, for the generation of potentially realistic fermion masses, we need to switch on both, $\langle \phi \rangle$ and $\langle \chi \rangle$. Note that, in order to achieve the breaking to the $\mathbb{Z}_4^R$, the doublet χ does not have to be aligned in any way since both components of the doublet transform trivially under this subgroup. The fact that multiplet VEVs do not have to be aligned for a desirable breaking pattern is a generic feature of the non-Abelian discrete R symmetries under discussion as can be inferred from the fact that the unbroken $\mathbb{Z}_4^R$ is required to be in the center of the non-Abelian group.

In the case of $\mathbb{Z}_3 \rtimes \mathbb{Z}_8^R$, however, an alignment of $\langle \chi \rangle$ along the $(1, 0)$ direction can arise due to the presence of a single additional field ξ transforming as $\mathbf{1}_4$ under the R symmetry and trivially under all other symmetries. At the renormalizable level, ξ couples only linearly to the flavon fields and does not possess couplings to the MSSM fields in the superpotential $\mathscr{W}$. Therefore, ξ automatically possesses the typical characteristics of a 'driving' field. In order to study the alignment, let us parameterize the VEVs as $\langle \chi \rangle = v \, (\cos \theta_\chi, \sin \theta_\chi)^T$ and $\langle \phi \rangle = v \, r_\phi$. Requiring SUSY to be unbroken at the flavor scale, one obtains the F-term condition

$$0 \overset{!}{=} \left. \frac{\partial \mathscr{W}}{\partial \xi} \right|_{\substack{\phi \to \langle \phi \rangle \\ \chi \to \langle \chi \rangle}} = -M^2 + g_1 \, v^2 \left(2 \cos^2 \theta_\chi - 1 \right) + g_2 \, v^2 \, r_\phi{}^2 \, , \qquad (3.3)$$

where we take $M^2, g_1, g_2 > 0$. What is crucial for the alignment is a choice of parameters such that there is a relative sign difference between the first and second terms. As one can check from (3.3), v and r_ϕ are minimized for $\theta_\chi = 0$, i.e. $\langle \chi \rangle \propto (1, 0)$. This corresponds to a breaking $\mathbb{Z}_3 \rtimes \mathbb{Z}_8^R \to \mathbb{Z}_8^R$ which would lead to the vanishing of two mixing angles since the residual $\mathbb{Z}_8^R$ symmetry is family dependent. We see that *this alignment has to be avoided* in order to obtain a correct phenomenology. However, a mild suppression of the leading-order contribution is enough to generate a small misalignment from the next-to-leading order terms of the superpotential, resulting in $\theta_\chi \approx \delta$, hence, modifying the VEV to $\langle \chi \rangle \propto (1, \delta)$. This then leads to a breaking $\mathbb{Z}_3 \rtimes \mathbb{Z}_8^R \to \mathbb{Z}_4^R$ with a slightly broken and hence approximate $\mathbb{Z}_8^R$. The small misalignment could, for instance, help to explain the small mixing to the third generation. In what follows, we will work with the VEVs

$$\langle \chi \rangle = v \begin{pmatrix} 1 \\ \delta \end{pmatrix} \quad \text{and} \quad \langle \phi \rangle = v \, r_\phi \, . \qquad (3.4)$$

Effective fermion mass matrices. We now use the direct product rules and the tensor structure of the decomposition (A.2) to identify terms consistent with all symmetries. The

effective neutrino mass operator is given by

$$
\begin{aligned}
\mathscr{W}_\nu^{\text{eff}} &= (H_u\,L^g)\,\kappa_{gf}\,(H_u\,L^f) \\
&= \frac{v_u^2}{\Lambda_\nu}\left\{ x_1\,(L_1\,L_1 - L_2\,L_2) + x_3\,L_3\,L_3 + 2\,x_4\,\frac{L_3}{\Lambda}\,(\chi_1\,L_2 - \chi_2\,L_1) \right. \\
&\qquad\qquad \left. + x_2\left[\frac{\chi_1}{\Lambda}\,(L_1\,L_1 + L_2\,L_2) + \frac{\chi_2}{\Lambda}\,(L_1\,L_2 + L_2\,L_1)\right]\right\}\,, \quad (3.5)
\end{aligned}
$$

where we have introduced dimensionless coupling coefficients x_i (in the following also y_i, z_i), the see-saw scale Λ_ν, as well as the flavor scale Λ, and set the Higgs fields to their VEVs. Terms involving more flavons are of higher order in $\varepsilon := v/\Lambda$ and are not discussed here. Setting the flavons to their VEVs, the emerging structure of the effective neutrino mass matrix is

$$
\kappa = \frac{v_u^2}{\Lambda_\nu}\begin{pmatrix} x_1 + x_2\,\varepsilon & x_2\,\varepsilon\,\delta & -x_4\,\varepsilon\,\delta \\ x_2\,\varepsilon\,\delta & -x_1 + x_2\,\varepsilon & x_4\,\varepsilon \\ -x_4\,\varepsilon\,\delta & x_4\,\varepsilon & x_3 \end{pmatrix}\,. \quad (3.6)
$$

The effective charged lepton mass is constrained to the form

$$
\begin{aligned}
\mathscr{W}_e &= \overline{E}^f Y_{fg}^{(e)}(H_d\,L^g) \\
&= v_d\left\{ y_1\,\frac{\overline{E}_1}{\Lambda}\,(\chi_1\,L_1 - \chi_2\,L_2) + y_2\,\frac{\overline{E}_2}{\Lambda}\,(\chi_1\,L_1 - \chi_2\,L_2) \right. \\
&\qquad \left. + y_3\,\frac{\overline{E}_3}{\Lambda}\,(\chi_1\,L_2 - \chi_2\,L_1) + y_4\,\frac{\phi}{\Lambda}\,\overline{E}_1\,L_3 + y_5\,\frac{\phi}{\Lambda}\,\overline{E}_2\,L_3 + y_6\,\overline{E}_3\,L_3\right\}\,, \quad (3.7)
\end{aligned}
$$

resulting in the structure

$$
Y^{(e)} = v_d\begin{pmatrix} y_1\,\varepsilon & -y_1\,\varepsilon\,\delta & y_4\,\varepsilon\,r_\phi \\ y_2\,\varepsilon & -y_2\,\varepsilon\,\delta & y_5\,\varepsilon\,r_\phi \\ -y_3\,\varepsilon\,\delta & y_3\,\varepsilon & y_6 \end{pmatrix}\,. \quad (3.8)
$$

As usual for settings with SU(5) relations, we have $Y^{(e)} \sim Y^{(d)T}$, which immediately fixes the structure of the down-quark Yukawa coupling. The up-quark Yukawa coupling has less structure since only one-dimensional representations are contracted. We find

$$
Y^{(u)} = v_u\begin{pmatrix} z_1 & z_2 & z_5\,\varepsilon\,r_\phi \\ z_3 & z_4 & z_6\,\varepsilon\,r_\phi \\ z_7\,\varepsilon\,r_\phi & z_8\,\varepsilon\,r_\phi & z_9 \end{pmatrix}\,. \quad (3.9)
$$

As already mentioned, it is possible to have variations of the charge assignment in table 3 which are consistent with all imposed requirements. Besides permutation in the family indices, such variations can only lead to mass matrices that are similar in structure to the ones of the example shown above. More precisely, one could, instead of the $\overline{\mathbf{5}}$–plets, combine two generations of the SU(5) $\mathbf{10}$–plets to a doublet, leading to a similar but transposed structure for $Y^{(e)}$ and $Y^{(d)}$, and to a swap in the structure of $Y^{(u)}$ and κ. Alternatively, also a setup in which two generations each of the $\overline{\mathbf{5}}$ and $\mathbf{10}$–plets get combined to doublets is possible, which is the only possibility in case of an SO(10) GUT. In this case, all mass matrices will take a form similar to (3.6).

$d^{(\Phi)}$	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{1}_5$	$\mathbf{1}_8$	$\mathbf{2}_2$	$\mathbf{2}_4$
$\delta_{\mathsf{v}}^{(\Phi)}$	0	4	5	1	4	6
$\delta_{\mathsf{v}^2}^{(\Phi)}$	0	0	1	1	0	2

Table 4. Charges of fields under the v and v^2 generated subgroups of $\mathbb{Z}_3 \rtimes \mathbb{Z}_8^R$, computed with equation (2.16). The charges are only defined modulo $M_{\mathsf{v}} = 8$ and $M_{\mathsf{v}^2} = 4$, respectively.

Model phenomenology. Even though we did not arrange our model to fit the experimental data, let us comment on the resulting phenomenology as it would be a starting point for the construction of possibly realistic models. Without imposing any additional symmetries, there are unsuppressed tree-level contributions to the mass matrices next to suppressed effective terms. As in other flavor models with non-Abelian discrete symmetries, it is clear that also in this case one needs to introduce further symmetries, such as, the so-called shaping symmetries or a U(1) of the Froggatt-Nielsen type, in order to obtain a completely natural and realistic model with hierarchical masses. For the particular model considered here, a Froggatt-Nielsen symmetry with $\lambda \sim \theta_c \sim 0.2$ may be used to explain the hierarchy among the parameters

$$y_1 \, : \, y_2 \, : \, y_3 \, : \, y_6 = \lambda^4 \, : \, \lambda^2 \, : \, \lambda^0 \, : \, \lambda^1 \, , \tag{3.10a}$$

$$z_1 \, : \, z_4 \, : \, z_9 = \lambda^8 \, : \, \lambda^4 \, : \, \lambda^0 \, , \tag{3.10b}$$

which can lead to a good agreement with the data as has been checked numerically using the MPT package [27]. However, as this is just a toy model with more parameters than observables, we refrain from fitting the model predictions to data. Yet our discussion shows that viable flavor models can, in principle, arise from non-Abelian discrete R symmetries, analogous to case of non–R, non-Abelian discrete symmetries (see e.g. [28–30] for reviews). This in turn affords the possibility of having a simultaneous solution to the μ problem and the flavor problem. In what follows we will use the toy model as a basis for an explicit calculation of the anomaly coefficients.

Anomalies of the $\mathbb{Z}_3 \rtimes \mathbb{Z}_8^R$ model. Finally, we can use formulae (2.19) to calculate the R-gauge-gauge anomaly coefficients of the $\mathbb{Z}_3 \rtimes \mathbb{Z}_8^R$ model. For this, we first have to calculate the charges of every representation. For the symmetry treated here, there are only two generators u and v. The representation matrix U equals the respective character for the one dimensional representations, and can be read off from equations (A.1a)–(A.1b) for the two dimensional representations. Since $\det U_{\mathsf{u}} = 1$ for all representations, the symmetry generated by u is trivially anomaly-free and we only have to care about v. The δ charges (2.16) for all relevant conjugacy classes are given in table 4. Here it pays off that we have expressed the anomaly coefficients in terms of the superfield charges via (2.18), such that in order to find the charges relevant for the anomaly coefficient we do not have to work out the representations of the fermion component fields and their respective charges, but instead take the superfield charge from table 4 and subtract the charge of θ times the dimensionality of the respective superfield's representation. We use the modulo M freedom

to shift all charges to positive values as a convention. Putting everything together, we find for the anomaly coefficients of the discussed model under the v generated subgroup of the discrete non-Abelian family symmetry $\mathbb{Z}_3 \rtimes \mathbb{Z}_8^R$ the expressions

$$A_{\mathrm{SU}(3)-\mathrm{SU}(3)-\mathbb{Z}_{8(\mathsf{v})}^R} = \frac{1}{2}\{[2+1]\cdot 4 + [1]\cdot 4\} + 3\cdot 5 = 3 \mod 4 , \qquad (3.11\mathrm{a})$$

$$A_{\mathrm{SU}(2)-\mathrm{SU}(2)-\mathbb{Z}_{8(\mathsf{v})}^R} = \frac{1}{2}\{[3]\cdot 4 + [1]\cdot 4 + 3 + 7\} + 2\cdot 5 = 3 \mod 4 , \qquad (3.11\mathrm{b})$$

$$\begin{aligned} A_{\mathrm{U}(1)-\mathrm{U}(1)-\mathbb{Z}_{8(\mathsf{v})}^R} = \frac{3}{5}&\left\{\left[3\cdot 2\cdot\left(\frac{1}{6}\right)^2 + 3\cdot\left(\frac{2}{3}\right)^2 + (1)^2\right]\cdot 4\right. \\ &+ \left[3\cdot\left(\frac{1}{3}\right)^2 + 2\cdot\left(\frac{1}{2}\right)^2\right]\cdot 4 + 2\cdot\left(\frac{1}{2}\right)^2\cdot 3 \qquad (3.11\mathrm{c}) \\ &\left.+ 2\cdot\left(\frac{1}{2}\right)^2\cdot 7\right\} = 3 \mod 4 . \end{aligned}$$

Here, we use square brackets to highlight the contributions arising from the **10** and $\overline{\mathbf{5}}$-plets, and GUT normalization for the U(1) charges. There is no contribution from the first and second family of the **10** as well as from the third family of the $\overline{\mathbf{5}}$-plets since their charge coincides with the superspace charge, i.e. the respective fermions are uncharged. Note that it is of fundamental importance that the Higgs fields are in different representations, otherwise the $\mathbb{Z}_8^R$ subgroup could *never* be anomaly universal in this setup (cf. the discussion around equation (3.2)). From the form of the anomaly and equation (2.23) we can immediately conclude that also the v^2, i.e. the unbroken $\mathbb{Z}_4^R$ subgroup appears anomalous with

$$A_{G-G-\mathbb{Z}_{4(\mathsf{v}^2)}^R} = 1 \mod 2 . \qquad (3.12)$$

Indeed, this anomaly is consistent with the findings of [10] as it should be, and the anomalies can be canceled by the Green-Schwarz mechanism.

Let us finally briefly comment on the $\mathbb{Z}_4^R$ phenomenology [10, 13]. The $\mathbb{Z}_4^R$ forbids the μ term in the MSSM but appears to be broken by non-perturbative effects. Since the order parameter of R symmetry breaking is the gravitino mass, a realistic effective μ term appears. Further, $\mathbb{Z}_4^R$ contains R or matter parity, such that dimension four proton decay operators are forbidden and dimension five operators are sufficiently suppressed.

As it is known that Abelian discrete R symmetries [31, 32] and non-Abelian discrete symmetries [33] can originate from orbifold compactifications it is tempting to speculate that non-Abelian discrete R symmetries may arise in non-Abelian orbifold compactifications, which have been studied recently in [34, 35].

3.3 Comments on R symmetries and the structure of soft terms

As is well known, the soft supersymmetry breaking terms are generated by appropriate effective operators involving a supersymmetry breaking spurion X. Specifically, for the scalar squared masses, the so-called A terms and the gaugino masses, these operators read

schematically (cf. e.g. [36])

$$\int \mathrm{d}^4\theta \, \frac{X^\dagger X}{\Lambda^2} \, Q^\dagger Q \xrightarrow{X \to F_X \, \theta^2} \widetilde{m}^2 \, |q|^2 \, , \tag{3.13a}$$

$$\int \mathrm{d}^2\theta \, \frac{X}{\Lambda} \, y \, Q^3 \xrightarrow{X \to F_X \, \theta^2} A \, y \, q^3 \, , \tag{3.13b}$$

$$\int \mathrm{d}^2\theta \, \frac{X}{\Lambda} \, W_\alpha W^\alpha \xrightarrow{X \to F_X \, \theta^2} M_\lambda \, \lambda\lambda \, . \tag{3.13c}$$

Here Λ is the cut-off scale, Q denotes a generic matter field and W_α is the multiplet containing the gaugino λ. If the matter fields furnish non-trivial representations under a non-Abelian (discrete) symmetry, one obtains from (3.13a) soft terms that are, at leading order, diagonal and get corrected by the flavor symmetry breaking terms. This leads to a structure that is somewhat similar to the one of 'minimal flavor violation' [37, 38] and can help to ameliorate or solve the supersymmetric flavor problems.

Let us now entertain the possibility that X has non-zero R charge under an appropriate, i.e. discrete or approximate, R symmetry. In fact, in the simplest scheme of supersymmetry breaking, such as the Polonyi model and the scenarios of meta-stable supersymmetry breaking [39], this situation is realized. Then the operator (3.13a) is still allowed while the A terms (3.13b) and gaugino masses (3.13c) are forbidden. Since the latter is phenomenologically excluded, one may introduce a second spurion X' with zero R charge. For $|F_X| \gg |F_{X'}|$ one then obtains heavy scalars and suppressed A terms and gaugino masses. This pattern is also obtained from KKLT-type moduli stabilization [40] with uplift by a matter field [41]. Here we see that this pattern can be enforced in a bottom-up approach by imposing R symmetries (but we have no explanation for the hierarchy $|F_X| \gg |F_{X'}|$). This discussion shows that R symmetries can be instrumental for engineering a certain pattern of soft terms.

Assume now that there is a non-Abelian discrete non–R symmetry H. If X is to furnish a higher-dimensional representation under H, there might be H-invariant contractions between X and the ingredients of the Yukawa couplings. In this case, provided the F-term VEVs of X and the flavon VEVs are not 'aligned', this will generically give rise to very dangerous flavor-violating operators via (3.13b). On the other hand, if the non-Abelian symmetry is also an R symmetry, these operators can be forbidden by assigning a non-zero R charge to the X field. One could then entertain the possibility that flavor and supersymmetry breaking is due to a single 'hidden sector'. Explicit model building in this direction is, however, beyond the scope of the present study.

4 Summary

In this paper we have discussed non-Abelian discrete R symmetries D. For phenomenological reasons we restricted ourselves to settings with $\mathcal{N} = 1$ supersymmetry in which the superspace coordinate θ furnishes a non-trivial one-dimensional representation of D. We have explored anomalies for such kinds of symmetries. In the course of this, we also have shown that perfect groups are always anomaly-free, which is of importance especially for the non–R case. It is instructive to compare GS anomaly cancellation for different kinds of

symmetries. In the case of an Abelian (continuous or discrete) symmetry, one can always cancel anomalies by the GS mechanism. In the case of a non-Abelian continuous (gauged) symmetries, an anomaly simply signals an inconsistency. Finally, for discrete non-Abelian symmetries, there is the possibility of multiple GS cancellation within one symmetry group. Here one can have different group operations associated with the shift of different (linear combinations of) axions. We have worked out the anomaly coefficients (equation (2.19)), and discussed GS anomaly cancellation in detail.

To illustrate our results, we discussed a toy model in which the MSSM gets amended by the discrete non-Abelian R symmetry $\mathbb{Z}_3 \rtimes \mathbb{Z}_8^R$. The model combines a flavor symmetry, which dictates certain relations between the Yukawa couplings, with an R symmetry that suppresses the μ term and dangerous proton decay operators. Moreover, due to the fact that it is an R symmetry, representations for so-called driving fields are automatically present in the spectrum, hence the question of 'VEV alignment' can be addressed without enlarging the symmetry group. Although the toy model is certainly not fully realistic, it illustrates the novel possibilities that arise once one promotes ordinary non-Abelian flavor symmetries to R symmetries: one can address the question of flavor and simultaneously solve the proton decay and μ problems with a single symmetry.

Acknowledgments

A The group $\mathbb{Z}_3 \rtimes \mathbb{Z}_8$

Let us briefly describe the relevant features of the group $\mathbb{Z}_3 \rtimes \mathbb{Z}_8$. A presentation of the group has already been given in (3.1). The character table is given in 5. The product rules for the irreducible representations are stated in tables 6 and 7. For the doublet representations $\mathbf{2}_j$, a possible form of the $\mathbb{Z}_3$ and $\mathbb{Z}_8$ generators u and v is given by

$$\widetilde{U}_j = \widetilde{U} = \frac{1}{2} \begin{pmatrix} -1 & \mathrm{i}\sqrt{3} \\ \mathrm{i}\sqrt{3} & -1 \end{pmatrix}, \tag{A.1a}$$

$$\widetilde{V}_1 = \begin{pmatrix} \mathrm{i} & 0 \\ 0 & -\mathrm{i} \end{pmatrix}, \qquad \widetilde{V}_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

$$\widetilde{V}_3 = \begin{pmatrix} \tau^* & 0 \\ 0 & -\tau^* \end{pmatrix} \quad \text{and} \quad \widetilde{V}_4 = \begin{pmatrix} \tau & 0 \\ 0 & -\tau \end{pmatrix}. \tag{A.1b}$$

	$\mathbb{1}$	v	v^2	v^4	u	v^3	v^5	v^6	uv^2	uv^4	v^7	uv^6
	1	3	1	1	2	3	3	1	2	2	3	2
$\mathbb{Z}_3 \rtimes \mathbb{Z}_8$	1a	8a	4a	2a	3a	8b	8c	4b	12a	6a	8d	12b
$\mathbf{1}_1$	1	1	1	1	1	1	1	1	1	1	1	1
$\mathbf{1}_2$	1	-1	1	1	1	-1	-1	1	1	1	-1	1
$\mathbf{1}_3$	1	$-i$	-1	1	1	i	$-i$	-1	-1	1	i	-1
$\mathbf{1}_4$	1	i	-1	1	1	$-i$	i	-1	-1	1	$-i$	-1
$\mathbf{1}_5$	1	$-\tau$	i	-1	1	τ^*	τ	$-i$	i	-1	$-\tau^*$	$-i$
$\mathbf{1}_6$	1	τ^*	$-i$	-1	1	$-\tau$	$-\tau^*$	i	$-i$	-1	τ	i
$\mathbf{1}_7$	1	$-\tau^*$	$-i$	-1	1	τ	τ^*	i	$-i$	-1	$-\tau$	i
$\mathbf{1}_8$	1	τ	i	-1	1	$-\tau^*$	$-\tau$	$-i$	i	-1	τ^*	$-i$
$\mathbf{2}_1$	2	0	-2	2	-1	0	0	-2	1	-1	0	1
$\mathbf{2}_2$	2	0	2	2	-1	0	0	2	-1	-1	0	-1
$\mathbf{2}_3$	2	0	$-2i$	-2	-1	0	0	$2i$	i	1	0	$-i$
$\mathbf{2}_4$	2	0	$2i$	-2	-1	0	0	$-2i$	$-i$	1	0	i

Table 5. Character table of $\mathbb{Z}_3 \rtimes \mathbb{Z}_8$. We define $\tau := e^{2\pi i/8}$. The conjugacy classes (c.c.) are labeled by the order of their elements and a letter. The first line gives a representative of the c.c. in the presentation specified in the text. The second line gives the cardinality of the corresponding c.c.

$\otimes$	$\mathbf{1}_2$	$\mathbf{1}_3$	$\mathbf{1}_4$	$\mathbf{1}_5$	$\mathbf{1}_6$	$\mathbf{1}_7$	$\mathbf{1}_8$	$\mathbf{2}_1$	$\mathbf{2}_2$	$\mathbf{2}_3$	$\mathbf{2}_4$
$\mathbf{1}_2$	$\mathbf{1}_1$	$\mathbf{1}_4$	$\mathbf{1}_3$	$\mathbf{1}_8$	$\mathbf{1}_7$	$\mathbf{1}_6$	$\mathbf{1}_5$	$\mathbf{2}_1$	$\mathbf{2}_2$	$\mathbf{2}_3$	$\mathbf{2}_4$
$\mathbf{1}_3$	$\mathbf{1}_4$	$\mathbf{1}_2$	$\mathbf{1}_1$	$\mathbf{1}_7$	$\mathbf{1}_5$	$\mathbf{1}_8$	$\mathbf{1}_6$	$\mathbf{2}_2$	$\mathbf{2}_1$	$\mathbf{2}_4$	$\mathbf{2}_3$
$\mathbf{1}_4$	$\mathbf{1}_3$	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{1}_6$	$\mathbf{1}_8$	$\mathbf{1}_5$	$\mathbf{1}_7$	$\mathbf{2}_2$	$\mathbf{2}_1$	$\mathbf{2}_4$	$\mathbf{2}_3$
$\mathbf{1}_5$	$\mathbf{1}_8$	$\mathbf{1}_7$	$\mathbf{1}_6$	$\mathbf{1}_4$	$\mathbf{1}_2$	$\mathbf{1}_1$	$\mathbf{1}_3$	$\mathbf{2}_3$	$\mathbf{2}_4$	$\mathbf{2}_2$	$\mathbf{2}_1$
$\mathbf{1}_6$	$\mathbf{1}_7$	$\mathbf{1}_5$	$\mathbf{1}_8$	$\mathbf{1}_2$	$\mathbf{1}_3$	$\mathbf{1}_4$	$\mathbf{1}_1$	$\mathbf{2}_4$	$\mathbf{2}_3$	$\mathbf{2}_1$	$\mathbf{2}_2$
$\mathbf{1}_7$	$\mathbf{1}_6$	$\mathbf{1}_8$	$\mathbf{1}_5$	$\mathbf{1}_1$	$\mathbf{1}_4$	$\mathbf{1}_3$	$\mathbf{1}_2$	$\mathbf{2}_4$	$\mathbf{2}_3$	$\mathbf{2}_1$	$\mathbf{2}_2$
$\mathbf{1}_8$	$\mathbf{1}_5$	$\mathbf{1}_6$	$\mathbf{1}_7$	$\mathbf{1}_3$	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{1}_4$	$\mathbf{2}_3$	$\mathbf{2}_4$	$\mathbf{2}_2$	$\mathbf{2}_1$
$\mathbf{2}_1$	$\mathbf{2}_1$	$\mathbf{2}_2$	$\mathbf{2}_2$	$\mathbf{2}_3$	$\mathbf{2}_4$	$\mathbf{2}_4$	$\mathbf{2}_3$				
$\mathbf{2}_2$	$\mathbf{2}_2$	$\mathbf{2}_1$	$\mathbf{2}_1$	$\mathbf{2}_4$	$\mathbf{2}_3$	$\mathbf{2}_3$	$\mathbf{2}_4$				
$\mathbf{2}_3$	$\mathbf{2}_3$	$\mathbf{2}_4$	$\mathbf{2}_4$	$\mathbf{2}_2$	$\mathbf{2}_1$	$\mathbf{2}_1$	$\mathbf{2}_2$				
$\mathbf{2}_4$	$\mathbf{2}_4$	$\mathbf{2}_3$	$\mathbf{2}_3$	$\mathbf{2}_1$	$\mathbf{2}_2$	$\mathbf{2}_2$	$\mathbf{2}_1$				

Table 6. Decomposition of the tensor products of irreducible representations of one-dimensional representations and doublets with one-dimensional representations.

$\otimes$	$\mathbf{2}_1$	$\mathbf{2}_2$	$\mathbf{2}_3$	$\mathbf{2}_4$
$\mathbf{2}_1$	$\mathbf{1}_1 \oplus \mathbf{1}_2 \oplus \mathbf{2}_2$	$\mathbf{1}_3 \oplus \mathbf{1}_4 \oplus \mathbf{2}_1$	$\mathbf{1}_5 \oplus \mathbf{1}_8 \oplus \mathbf{2}_4$	$\mathbf{1}_6 \oplus \mathbf{1}_7 \oplus \mathbf{2}_3$
$\mathbf{2}_2$	$\mathbf{1}_3 \oplus \mathbf{1}_4 \oplus \mathbf{2}_1$	$\mathbf{1}_1 \oplus \mathbf{1}_2 \oplus \mathbf{2}_2$	$\mathbf{1}_6 \oplus \mathbf{1}_7 \oplus \mathbf{2}_3$	$\mathbf{1}_5 \oplus \mathbf{1}_8 \oplus \mathbf{2}_4$
$\mathbf{2}_3$	$\mathbf{1}_5 \oplus \mathbf{1}_8 \oplus \mathbf{2}_4$	$\mathbf{1}_6 \oplus \mathbf{1}_7 \oplus \mathbf{2}_3$	$\mathbf{1}_3 \oplus \mathbf{1}_4 \oplus \mathbf{2}_1$	$\mathbf{1}_1 \oplus \mathbf{1}_2 \oplus \mathbf{2}_2$
$\mathbf{2}_4$	$\mathbf{1}_6 \oplus \mathbf{1}_7 \oplus \mathbf{2}_3$	$\mathbf{1}_5 \oplus \mathbf{1}_8 \oplus \mathbf{2}_4$	$\mathbf{1}_1 \oplus \mathbf{1}_2 \oplus \mathbf{2}_2$	$\mathbf{1}_3 \oplus \mathbf{1}_4 \oplus \mathbf{2}_1$

Table 7. Decomposition of the tensor products of two doublet representations.

Here we have used $\tau := \mathrm{e}^{2\pi\mathrm{i}/8}$ to denote the eight root of unity. We also state the explicit form of all the tensor products which one may need for the construction of the mass matrices of possible models. Let $(a_1, a_2)^T$ and $(b_1, b_2)^T$ each transform as a doublet and c be a one-dimensional representation. Then

$$(a_{\mathbf{2}_4} \otimes c_{\mathbf{1}_5}) = \begin{pmatrix} a_2\, c \\ a_1\, c \end{pmatrix}_{\mathbf{2}_1} , \tag{A.2a}$$

$$(a_{\mathbf{2}_4} \otimes c_{\mathbf{1}_8}) = \begin{pmatrix} a_1\, c \\ a_2\, c \end{pmatrix}_{\mathbf{2}_1} , \tag{A.2b}$$

$$(a_{\mathbf{2}_2} \otimes b_{\mathbf{2}_1}) = (a_1\, b_2 - a_2\, b_1)_{\mathbf{1}_3} \oplus (a_1\, b_1 - a_2\, b_2)_{\mathbf{1}_4} \oplus \begin{pmatrix} a_1\, b_1 + a_2\, b_2 \\ -(a_1\, b_2 + a_2\, b_1) \end{pmatrix}_{\mathbf{2}_1} , \tag{A.2c}$$

$$(a_{\mathbf{2}_2} \otimes b_{\mathbf{2}_4}) = (a_1\, b_2 - a_2\, b_1)_{\mathbf{1}_5} \oplus (a_1\, b_1 - a_2\, b_2)_{\mathbf{1}_8} \oplus \begin{pmatrix} a_1\, b_1 + a_2\, b_2 \\ -(a_1\, b_2 + a_2\, b_1) \end{pmatrix}_{\mathbf{2}_4} , \tag{A.2d}$$

$$(a_{\mathbf{2}_4} \otimes b_{\mathbf{2}_4}) = (a_1\, b_2 - a_2\, b_1)_{\mathbf{1}_3} \oplus (a_1\, b_1 - a_2\, b_2)_{\mathbf{1}_4} \oplus \begin{pmatrix} a_1\, b_1 + a_2\, b_2 \\ -(a_1\, b_2 + a_2\, b_1) \end{pmatrix}_{\mathbf{2}_1} , \tag{A.2e}$$

$$(a_{\mathbf{2}_2} \otimes b_{\mathbf{2}_2}) = (a_1\, b_1 - a_2\, b_2)_{\mathbf{1}_1} \oplus (a_1\, b_2 - a_2\, b_1)_{\mathbf{1}_2} \oplus \begin{pmatrix} a_1\, b_1 + a_2\, b_2 \\ -(a_1\, b_2 + a_2\, b_1) \end{pmatrix}_{\mathbf{2}_2} . \tag{A.2f}$$

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