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R_2 vertices for the effective ggH theory

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Abstract: We list all possible R_2 Feynman rules needed in NLO computations involving couplings of Higgs and gluons mediated by an infinitely heavy top loop. They provide the rational contribution generated by the $(d-4)$-dimensional part of the amplitude, paving the way for four-dimensional automatic NLO methods in Higgs phenomenology.

Keywords: QCD Phenomenology, NLO Computations

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1 Introduction

Automation of NLO calculations is an important issue in modern high energy particle physics. The complexity of the processes one has to deal with forbids a case by case approach, forcing the use of automatic tools [1–9].

In the last few years, a lot of progress has been achieved and, among the various available techniques [10–14], the Ossola-Papadopoulos-Pittau (OPP) method [15] appears to be a rather convenient one. OPP is a procedure which allows one to numerically extract, from dimensionally regulated one-loop amplitudes, the Cut-Constructible part plus a rational piece - called R_1 - strictly linked to the four-dimensional numerator of the Feynman diagrams. A second contribution to the rational part, dubbed R_2, cannot be obtained numerically in four-dimensions, and must be provided externally. A particularly convenient way to account for R_2 is via effective Feynman rules. This is because the R_2 contribution disappears when a sufficiently large number of particles undergo the scattering and, in renormalizable theories, only up to four-particle-vertices have to be considered.

The R_2 vertices for QCD [16], the Electroweak Model [17–19] and MSSM [20] have already been presented in the literature. In this paper, we list the R_2 Feynman rules generated by the interaction of one Higgs field H with two, three and four gluons - mediated by an infinitely heavy top loop - encoded in the effective Lagrangian [21, 22]

$$\mathcal{L}_{\rm eff} = -\frac{1}{4} A H G^a_{\mu\nu} G^{a,\mu\nu} \,, \tag{1.1}$$

where

$$A = \frac{\alpha_S}{3\pi v}\left(1 + \frac{11}{4}\frac{\alpha_S}{\pi}\right) \tag{1.2}$$

and v is the vacuum expectation value, $v^2 = (G_F\sqrt{2})^{-1}$.

There is a very important physical motivation that led us to study this problem: the main Higgs production mechanism at the LHC is via gluon fusion [23, 24], and considering NLO corrections to processes generated by the effective operator in eq. (1.1) provides an easy way to model, at two-loop precision, Higgs + n jets final states for jets with p_T not much larger than the top mass [25–27]. The Hjj and $Hjjj$ signatures have been recently considered [28, 29], in the context of a d-dimensional integrand reduction technique, within the GoSam [8]/Sherpa [30] framework. In this paper we provide the only missing ingredient needed by 4-dimensional NLO approaches, such as OPP, Open Loops [31] or FDR [32], to attack the problem in a fully automatic fashion.

In the next section we review the basics of R_2. In section 3 we list the contributing R_2 vertices and section 4 contains our conclusions.

2 R_2 in a nutshell

The full theory of R_2 can be found in [33, 34]. For the purposes of this paper, it is enough to recall that, starting from a one-loop amplitude computed in d dimensions, R_2 is generated by the explicit splits

$$
\begin{aligned}
d &= 4 + \epsilon\,, \\
\bar{q}^2 &= q^2 + \tilde{q}^2
\end{aligned}
\tag{2.1}
$$

in the numerators of the contributing Feynman diagrams, where $\bar{q}^2$ is the d-dimensional integration momentum squared, q^2 its four-dimensional part and $\tilde{q}^2$ the difference between the two. In renormalizable theories, in the limit $\epsilon \to 0$, the ϵ and $\tilde{q}^2$ pieces generate constant terms in one particle irreducible Green's functions up to four legs. The effective operator in eq. (1.1) has dimension five and so a non-vanishing contribution can survive in up to five-leg vertices.

In Dimensional Reduction like schemes [35], one is free to set $d = 4$ in eq. (2.1) right from the beginning, while the $\tilde{q}^2$ part is always necessary in order to keep gauge invariance. The transition rules between Dimensional Reduction and the usual 't Hooft-Veltman Dimensional Regularization [36], in which $d = 4 + \epsilon$, are well known [34, 37]. To allow the reading of our rules in both schemes we introduced a parameter $\lambda_{\rm HV}$ in our formulae, such that

$$
d = 4 + \lambda_{\rm HV}\,\epsilon\,. \tag{2.2}
$$

Thus, Dimensional Reduction corresponds to $\lambda_{\rm HV} = 0$ and Dimensional Regularization to $\lambda_{\rm HV} = 1$.

3 The R_2 vertices generated by the GGH operator

By power counting, a non-zero R_2 contribution is present only in the following five interactions

$$
Hgg,\quad Hggg,\quad Hgggg,\quad Hq\bar{q},\quad Hq\bar{q}g\,. \tag{3.1}
$$

We generated and computed, with the help of `QGRAF` [38] and `FORM` [39], all possible contributing diagrams[1] from which we extracted R_2 via eq. (2.1). Two independent calculations were performed, with slightly different strategies. In the first computation a Feynman

[1] We used the QCD Feynman rules in appendix B of [16], together with the effective Higgs-gluons couplings listed in [40].

parametrization was applied before eq. (2.1); in the second, eq. (2.1) was used to classify independent integrals, which were computed at a later stage of the calculation. Both procedures gave the same expressions, listed in figure 1, which represent the main result of this paper.

The $Hggg$ vertex is fully proportional to the QCD three gluon vertex

$$V^{\mu_1\mu_2\mu_3}_{m_1m_2m_3}(p_1,p_2,p_3) = \\ + g_s f_{m_1m_2m_3}\left[g^{\mu_1\mu_2}(p_2-p_1)^{\mu_3} + g^{\mu_2\mu_3}(p_3-p_2)^{\mu_1} + g^{\mu_3\mu_1}(p_1-p_3)^{\mu_2}\right], \tag{3.2}$$

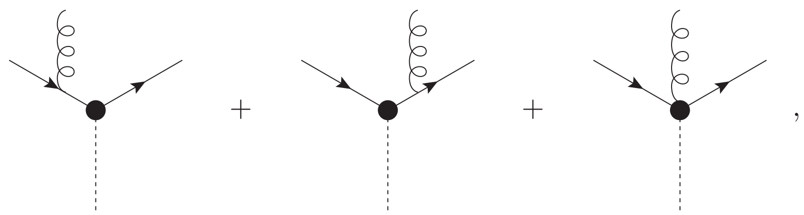
while the tensor involved in the Higgs/4-Gluons vertex reads

$$X^{\mu_1\mu_2\mu_3\mu_4}_{m_1m_2m_3m_4} = \\ + \mathrm{Tr}(T^{m_1}T^{m_2}T^{m_3}T^{m_4})\left[+21\, g^{\mu_1\mu_2}g^{\mu_3\mu_4} - 41\, g^{\mu_1\mu_3}g^{\mu_2\mu_4} + 21\, g^{\mu_1\mu_4}g^{\mu_2\mu_3}\right] \\ + \mathrm{Tr}(T^{m_1}T^{m_2}T^{m_4}T^{m_3})\left[+21\, g^{\mu_1\mu_2}g^{\mu_3\mu_4} + 21\, g^{\mu_1\mu_3}g^{\mu_2\mu_4} - 41\, g^{\mu_1\mu_4}g^{\mu_2\mu_3}\right] \\ + \mathrm{Tr}(T^{m_1}T^{m_3}T^{m_2}T^{m_4})\left[-41\, g^{\mu_1\mu_2}g^{\mu_3\mu_4} + 21\, g^{\mu_1\mu_3}g^{\mu_2\mu_4} + 21\, g^{\mu_1\mu_4}g^{\mu_2\mu_3}\right], \tag{3.3}$$

where the color structure is compactly expressed in terms of traces of color matrices T^a in the *adjoint* representation. The translation to traces of Gell-Mann matrices t^a in the fundamental representation of $\mathrm{SU}(N_c)$ is given by the formula

$$\mathrm{Tr}(T^aT^bT^cT^d) = N_c\left[\mathrm{Tr}(t^at^bt^ct^d) + \mathrm{Tr}(t^dt^ct^bt^a)\right] + \frac{\delta_{ab}\delta_{cd} + \delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc}}{2}. \tag{3.4}$$

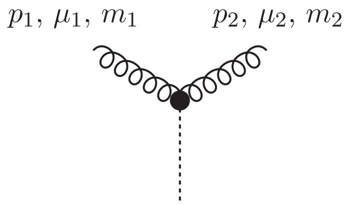
Any λ_{HV} contribution is entirely contained in the R_2 part. However, in physical combinations no dependence on λ_{HV} remains. Naturally this means that the vertices involving only gluons are devoid of λ_{HV}. Furthermore one can show that in the physical sum of diagrams, given by



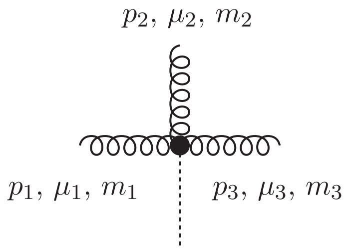
there is no dependence on λ_{HV}.

Finally, it is important to realize that, being one particle irreducible, the vertices in figure 1 are the building blocks from which the R_2 contribution to *any* amplitude involving one coupling of order A,[2] and *any* number of additional jets can be derived.

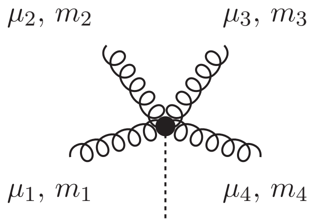
[2] See eq. (1.2).



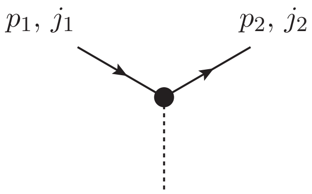
$$= \frac{iAg_s^2 N_c \delta^{m_1 m_2}}{384\pi^2} \Big\{ p_1^{\mu_1} p_2^{\mu_2} + 89\, p_1^{\mu_2} p_2^{\mu_1} + 14\,(p_1^{\mu_1} p_1^{\mu_2} + p_2^{\mu_1} p_2^{\mu_2}) - \left[17\,(p_1^2 + p_2^2) + 93\,(p_1 \cdot p_2)\right] g^{\mu_1 \mu_2} \Big\}$$



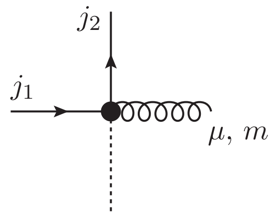
$$= -\frac{15 A g_s^2 N_c}{128\pi^2} V^{\mu_1\mu_2\mu_3}_{m_1 m_2 m_3}(p_1, p_2, p_3)$$



$$= \frac{iAg_s^4}{128\pi^2} X^{\mu_1\mu_2\mu_3\mu_4}_{m_1 m_2 m_3 m_4}$$



$$= \frac{iAg_s^2}{32\pi^2} \left(\frac{N_c^2 - 1}{2N_c} \right) \delta^{j_1 j_2} \lambda_{\rm HV} (\not p_1 - \not p_2)$$



$$= \frac{iAg_s^3}{64\pi^2} \gamma_\mu t^m_{j_2 j_1} \left[\frac{2\lambda_{\rm HV} + 1}{N_c} - (2\lambda_{\rm HV} + 3) N_c \right]$$

Figure 1. The R_2 vertices generated by the effective Lagrangian in eq. (1.1). All momenta are incoming, N_c is the number of colors and $\lambda_{\rm HV} = 1$ ($\lambda_{\rm HV} = 0$) in Dimensional Regularization (Reduction). Quarks are massless and the dashed line represents the Higgs field. The tensors $V^{\mu_1\mu_2\mu_3}_{m_1 m_2 m_3}(p_1, p_2, p_3)$ and $X^{\mu_1\mu_2\mu_3\mu_4}_{m_1 m_2 m_3 m_4}$ are defined in the text.

4 Conclusions

We have presented the complete set of effective R_2 Feynman rules generated by the dimension five operator GGH. They encode the missing analytical information needed to apply four-dimensional automatic integrand reduction techniques to Higgs physics.

Our result can be used to study both production and decay processes involving one Higgs particle and any number of jets. This is particularly useful for Higgs phenomenology at the LHC, where, due to the large amount of available energy, the Higgs particle is very often produced in association with a large number of jets.

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