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Fits to SO(10) grand unified models

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Abstract: We perform numerical fits of Grand Unified Models based on SO(10), using various combinations of 10-, 120- and 126-dimensional Higgs representations. Both the supersymmetric and non-supersymmetric versions are fitted, as well as both possible neutrino mass orderings. In contrast to most previous works, we perform the fits at the weak scale, i.e. we use RG evolution from the GUT scale, at which the GUT-relations between the various Yukawa coupling matrices hold, down to the weak scale. In addition, the right-handed neutrinos of the seesaw mechanism are integrated out one by one in the RG running. Other new features are the inclusion of recent results on the reactor neutrino mixing angle and the Higgs mass (in the non-SUSY case). As expected from vacuum stability considerations, the low Higgs mass and the large top-quark Yukawa coupling cause some pressure on the fits. A lower top-quark mass, as sometimes argued to be the result of a more consistent extraction from experimental results, can relieve this pressure and improve the fits. We give predictions for neutrino masses, including the effective one for neutrinoless double beta decay, as well as the atmospheric neutrino mixing angle and the leptonic CP phase for neutrino oscillations.

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Contents

1 Introduction

The origin of fermion masses and mixings is a long-standing question in elementary particle physics. Among different frameworks to address this problem, theories unifying strong and electroweak interactions as well as — partly or completely — quarks and leptons offer very attractive solutions. Particularly intriguing are models based on SO(10) symmetry.

In SO(10) all Standard Model (SM) particles of one generation plus a right-handed neutrino are assigned to a single 16-dimensional representation. The masses of the fermions arise from Yukawa interactions of two 16s with suitable Higgs fields when the latter develop vacuum expectation values (VEVs). Since [1, 2]

$$16 \otimes 16 = 10_{\mathrm{S}} \oplus \overline{126}_{\mathrm{S}} \oplus 120_{\mathrm{A}}, \tag{1.1}$$

the Higgs fields of renormalizable SO(10) models can belong to 10-, 126-, or 120-dimensional representations (denoted henceforth by 10_H, $\overline{126}_H$, and 120_H, respectively) yielding the Yukawa part of the Lagrangian,

$$\mathcal{L}_Y = 16\,(Y_{10} 10_H + Y_{126}\overline{126}_H + Y_{120} 120_H)\,16\,. \tag{1.2}$$

After spontaneous symmetry breaking the fermions obtain the masses [2–5]

$$\begin{aligned}
M_u &= v_{10}^u Y_{10} + v_{126}^u Y_{126} + v_{120}^u Y_{120}\,,\\
M_d &= v_{10}^d Y_{10} + v_{126}^d Y_{126} + v_{120}^d Y_{120}\,,\\
M_D &= v_{10}^u Y_{10} - 3v_{126}^u Y_{126} + v_{120}^D Y_{120}\,,\\
M_l &= v_{10}^d Y_{10} - 3v_{126}^d Y_{126} + v_{120}^l Y_{120}\,,\\
M_R &= v_{126}^R Y_{126}\,,\\
M_L &= v_{126}^L Y_{126}\,,
\end{aligned} \tag{1.3}$$

where $M_u, M_d, M_D, M_l, M_R, M_L$ are the up-quark, down-quark, Dirac neutrino, charged lepton, right-handed Majorana neutrino (type I seesaw), and left-handed Majorana neutrino (type II seesaw) mass matrices. Further, Y_{10}, Y_{126}, and Y_{120} are the Yukawa coupling matrices between the fermions and the 10-, $\overline{126}$-, and 120-dimensional representations, respectively. Here Y_{10} and Y_{126} are symmetric, while Y_{120} is anti-symmetric. The $v_{10/126/120}^{u/d/D/l/R/L}$ represent the parts of the VEV (or combination of VEVs) of the Higgs fields that are important for the respective particle type. Eq. (1.3) holds at the scale of SO(10) symmetry breakdown, henceforth referred to as $M_{\mathrm{GUT}} = 2 \times 10^{16}$ GeV. Fermion masses and mixings are measured at much lower energies, e.g., at $M_Z \approx 91.19$ GeV. Hence, to fit the parameters of a model to the data, one has to use renormalization group evolution (RGE) to obtain the model predictions at M_Z.

As obvious from eq. (1.3), in SO(10) all fermion mass matrices are related since they are combinations of the same Yukawa matrices. A numerical fit, taking into account these relations, and comparing to the experimentally determined observables, can test different SO(10) models for phenomenological viability. In this paper we will perform such tests. In this regard we consider several classes of SO(10) models, differing in the choice of Higgs representations. A minimal model with only one Higgs field is phenomenologically not viable since all fermions are then proportional to the same Yukawa matrix and are hence diagonal in the same basis, resulting in no mixing between up- and down-quarks or between charged leptons and neutrinos. A $\overline{126}_H$ field is required for neutrino mass generation via a seesaw mechanism. We consider models that additionally contain either a 10_H, or a 120_H, or both a 10_H and a 120_H. More model-dependent effects of intermediate scales are neglected in our study. We analyze supersymmetric versions of the models as well as models without SUSY.

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Fits of SO(10) models along the lines presented in this paper have of course been performed before [6–16]. Our approach is different from previous works in the following aspects:

1) Firstly, we perform a full renormalization group evolution and fit the models to the experimental data at $\mu = M_Z$. This means that we start from eq. (1.3), evolve the observables *down* to M_Z, and compare at that scale with available experimental data. In contrast, previous studies either did not include renormalization effects at all [6, 7], or evolved experimental values *up* to $M_{\rm GUT}$ [10–14] and fitted that data at $\mu = M_{\rm GUT}$. The latter procedure introduces the following issue: to evolve observables from M_Z to $M_{\rm GUT}$, certain high-energy model details (such as Yukawa couplings) have to be known, since they have an impact on the running of observables (this has been demonstrated long ago, e.g., for the m_b/m_τ ratio [17]). However, exactly these model details are varied while the fit is performed. Our approach is therefore more consistent.

2) Secondly, when performing our fits we take into account effects coming from non-degenerate right-handed neutrinos, ν_{R_i}, with mass M_i — an issue which is commonly neglected in fits to GUTs. When performing the RGE one has to integrate out the heavy neutrinos at appropriate energies. Since ν_{R_i} can be highly hierarchical one has to integrate them out one by one at $\mu = M_i(M_i)$ (as opposed to integrating out all at once at a common seesaw scale). This produces several effective field theories (EFT) during RGE — one EFT per heavy degree of freedom which is integrated out — with different running behavior of the parameters. Treating non-degenerate ν_{R_i} correctly can have sizable effects on neutrino parameters as has been demonstrated in ref. [18]. For our analysis we apply the method described in ref. [18] and integrate out ν_{R_i} at appropriate energies. Besides yielding more trustworthy results, a more precise analysis that takes into account RGE also leads to more reliable predictions of experimentally undetermined observables like the effective neutrino mass governing neutrinoless double beta decay or the leptonic CP violating phase $\delta^l_{\rm CP}$. To show the impact of our more rigorous treatment of RGEs, we will fit the models also when we evolve the experimental values up to $M_{\rm GUT}$, fit those values at that scale, use the low energy neutrino parameters and ignore the effects of non-degenerate right-handed neutrinos (denoted in what follows as "no RGE").

3) The inclusion of RGE allows us to consider the Higgs mass in the analysis of non-SUSY models, since it is related to the other observables via the RG equations (described in detail in section 3.2). As well-known, and expected, from vacuum stability bounds, requiring that the Higgs quartic coupling remains positive puts pressure on the fits due to the large value of the top-quark mass. We demonstrate the effects on the fits by using a lower value for the top-quark mass and by leaving out the Higgs mass, respectively. The fits are shown to improve considerably.

4) Finally, more precise data in the neutrino sector is now available, most notably through the discovery of a non-zero reactor mixing angle [19–21].

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Our fits will assume dominance of the type I seesaw, and will be performed for both possible neutrino mass orderings, as well as for the non-supersymmetric and supersymmetric cases.

The paper is build up as follows: in section 2 we will describe the different models that we fit, and in section 3 give some details on the fit procedure and the observables that need to be reproduced. Section 4 describes the results of the fits, before we conclude in section 5. The appendices contain for completeness the best-fit parameters of the Yukawa matrices, and the β-functions necessary for the RG evolution.

2 Model details and previous work

We now simplify the notation and rewrite eq. (1.3) as [3, 4, 13, 14]:

$$\begin{aligned} Y_u &= r(H + sF + it_u\, G)\,, \\ Y_d &= H + F + iG\,, \\ Y_D &= r(H - 3sF + it_D\, G)\,, \\ Y_l &= H - 3F + it_l\, G\,, \\ M_R &= r_R^{-1} F\,, \end{aligned} \tag{2.1}$$

where the Yukawa matrices Y_i are the mass matrices M_i from eq. (1.3) divided by the VEVs v or $v_{u/d}$ of the Standard Model (SM) or Minimal Supersymmetric Standard Model (MSSM), respectively. H, F, G correspond to Y_{10}, Y_{126}, and Y_{120}, respectively, i.e., H and F are symmetric and G is anti-symmetric. The parameters s, t_u, t_D, t_l are complex, whereas r, r_R can be chosen real without loss of generality [14]. We have omitted the type II seesaw term (compare to eq. (1.3)) since we assume for definiteness that the type I seesaw term dominates. We will consider supersymmetric (SUSY) as well as non-supersymmetric SO(10) models. In non-SUSY SO(10) the 10_H can be chosen real, but one can argue that this will not lead to a viable particle spectrum [22]. Taking the 10_H to be complex, its real and imaginary parts can couple separately to the fermionic 16 and will lead to two independent Yukawa matrices. To avoid this complication in the case of non-SUSY SO(10), we impose an additional Peccei-Quinn U(1) symmetry [23] as described in refs. [14, 22, 24]. In this case eq. (2.1) is valid both in the case of non-SUSY as well as SUSY SO(10).

We now define the different models which we want to test for viability using experimental data on fermion masses and mixings. The first differentiation between the models concerns their Higgs content. We consider two minimal setups with $10_H + \overline{126}_H$ or $120_H + \overline{126}_H$ and a setup with $10_H + \overline{126}_H + 120_H$. We refer to the $10_H + \overline{126}_H$ setup as "MN" ("MS") and to the $10_H + \overline{126}_H + 120_H$ setup as "FN" ("FS") in case we consider the non-SUSY (SUSY) versions of the models (M stands for "minimal", F for "full", N for "non-SUSY", S for "SUSY"). The non-SUSY $120_H + \overline{126}_H$ setup was argued, based on an analytical two generation approximation, to be an attractive minimal model to describe fermion masses and mixings [22]. The three generation non-SUSY setup case was later shown to be not successful [14]. We analyze this setup numerically within our approach, and confirm that it cannot reproduce the observed data, see below. The SUSY case is also analyzed by us, to the best of our knowledge for the first time, and shown not to be a valid model either. More details on the models are given in the following subsections, the main features are given in table 1.

Our analysis involving RGEs that integrate out the individual heavy Majorana masses is quite CPU-intensive. For this reason we are forced, in the present paper, to ignore some complications arising in SO(10) models:

The Higgs representations mentioned above are not enough to break SO(10) down to the SM. Therefore further Higgs representations are necessary (see also ref. [16, 25] for recent analyses). In case of non-SUSY models a minimal choice would be to add one 45_H [26] and in case of SUSY models one 210_H [27, 28]. Furthermore, in SUSY models one needs additionally a 126_H which keeps SUSY from being broken by D terms [28]. Since our analysis including the RGEs is already quite involved, we ignore details of different viable breaking chains, which would induce new scales, RGEs, parameters, etc. Effectively we therefore assume that intermediate symmetries are close to $M_{\rm GUT}$ and the running of parameters between these scales and $M_{\rm GUT}$ is not sizable. Hence, the relevant information for our analysis is the Higgs content given in table 1 and eqs. (1.3) or (2.1), together with the beta-functions of the SM or MSSM as given in the appendix.

Gauge coupling unification also depends on the breaking chain and the values of intermediate scales. E.g., in the minimal non-SUSY model based on $10_H + \overline{126}_H$, it has been shown that with Higgs VEVs of 45_H and $\overline{126}_H$ around 10^{14} GeV [26, 29] (i.e. suitable to reproduce the neutrino mass-squared differences via seesaw) gauge coupling unification can be achieved [30, 31]. In contrast, for the SUSY version of the minimal model it has been shown [32] that reproducing known values of neutrino mass-squared differences leads to light states which spoil gauge coupling unification. Since we do not analyze the details of SO(10) breaking, we will also not be concerned with the unification of gauge couplings. In our fit the gauge couplings (whose 1-loop RGE do only depend on themselves) are chosen at the GUT scale such that they reproduce their measured values at M_Z.

The results that we will obtain in this paper are therefore all under the assumption of negligible effects coming from intermediate scales M_I. Those are typically of order 10^{10} to 10^{11} GeV, and the running from the GUT scale to M_I involves 6 orders of magnitude, while from M_I to M_Z involves 8 orders of magnitude. Moreover, the gauge couplings influence the RGEs, and ignoring their unification will have an effect there as well. However, as emphasized in ref. [16], the contribution of the Higgs states with masses around M_I is only a sub-leading correction in the running from $M_{\rm GUT}$ to M_I, because the corresponding beta-function coefficients are small. Nevertheless, there are corrections to be expected, but their impact is hard to estimate, and would have to be made case-by-case. As an example, we can compare with ref. [16], in which a fit that takes into account intermediate scales and gauge coupling unification is performed. That scale is the one at which SO(10) is broken to the Pati-Salam group $\mathrm{SU}(4)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. As the values of the fitted observables are the same current values that we are using, this analysis is the one we should compare with. In our language, the fit in [16] is "no RGE" within model MN. With four free parameters less than we have (the charged lepton masses and r_R are not fitted), the χ^2-minimum is 17.4, compared to 1.1 in our case. Large part of the difference of the χ^2-minima can be attributed to the requirement that the baryon asymmetry as generated

Higgs content	SUSY	non-SUSY	free parameters
one of 10_H, 120_H, or $\overline{126}_H$	no mixing		—
$10_H + 120_H$	no type I seesaw		—
$120_H + \overline{126}_H$	not valid		17
$10_H + \overline{126}_H$	MS	MN	19
$10_H + 120_H + \overline{126}_H$	FS	FN	18

Table 1. Brief overview of considered models, names given to them in the text and the number of free parameters. Models with only one Higgs representation cannot produce mixing, and models without $\overline{126}_H$ do not have a seesaw mechanism. The number of observables that we fit is 18 or 19, depending on whether we include the Higgs mass or not. Models with $120_H + \overline{126}_H$ Higgs representations do not provide acceptable fits.

by thermal leptogenesis is included in the fit of ref. [16]. Performing the fit without the baryon asymmetry indeed gives $\chi^2_{\text{min}} \simeq 1$, in very good agreement with our result. As in our case, an inverted hierarchy is not possible. Regarding predictions, the atmospheric neutrino mixing parameter $\sin^2\theta_{23}$ is 0.353 in [16], and 0.406 here. Both approaches predict it to be somewhat low, and cause some pressure on the fits. With r_R not fitted in [16], the predictions for the neutrino masses are not really comparable, but agree within factors of a few. It seems that, at least in this particular example, the main features of the fits are stable with respect to the intermediate scales. There can apparently be shifts of the χ^2, but no dramatic shifts that cause a particular model to be ruled out with intermediate scales while being allowed without. We should stress however that we cannot guarante this for all models under study.

In what follows we will describe the properties of the models that we fit, summarizing shortly the results.

2.1 Minimal model with $10_H + \overline{126}_H$ (MN, MS)

In this model we do not have a 120_H, hence $G = 0$ in eq. (2.1). To count the number of free parameters we choose a basis in which H is real and diagonal, which leaves us with 19 real parameters: 3 in H, 12 in F (complex symmetric), and 4 in r (real), s (complex), and r_R (real) (assuming type I seesaw dominance).

There is a plethora of literature about the supersymmetric version of this model [14, 24, 32–49], which is often referred to as the "Minimal Supersymmetric Grand Unified Theory" (MSGUT).[1] Literature analyzing the non-supersymmetric version also exists [14, 15, 22, 26, 29, 51]. The predictivity of this model has been pointed out first in [24]. All authors analyzing the fermion spectrum neglect details of the RGE which affect the running of observables between M_Z and M_{GUT}, as described in section 1.

Results: in case of an inverted neutrino mass hierarchy both the non-SUSY as well as the SUSY versions of this model are not able to reproduce the data. In case of a normal hierarchy without including RGE, model MN (non-SUSY) gives a good fit to fermion

[1] An alternative approach containing two 10-dimensional Higgs representations can be found in [50].

masses and mixing angles. Including RGE and fitting in addition the Higgs mass leads to tension and a somewhat unsatisfactory fit. The top-quark mass is 3.4 σ too small at its best-fit point. This is related to its influence on the Higgs quartic coupling, whose positivity till the GUT scale requires a smaller top-quark mass than measured. If the top-quark mass was lower, as sometimes argued to be the result of a more consistent extraction from the data [52–54], the fit improves. In addition, the leptonic mixing angles θ^l_{23} and θ^l_{13} are not reproduced very well. The SUSY model MS can fit the data and fits including RGE are even somewhat better than fits without RGE. Further details will be discussed in section 4.1.

2.2 Alternative minimal model with $\mathbf{120}_H + \overline{\mathbf{126}}_H$

Due to absence of 10_H we have $H = 0$ in this model. Going to a basis with real diagonal F (3 parameters), we have 6 real parameters in G and 8 in s, r_R (real), t_u, t_l, t_D (complex), altogether 17 parameters (neglecting type II seesaw).

This model is analytically analyzed in ref. [22] in the case of only two fermion generations (second and third) and argued to be viable and predictive. A numerical three generation analysis finds the model to be unable to fit fermion masses and mixings [14]. To provide further evidence for this result we perform a fit of this model. In addition to the normal neutrino mass hierarchy considered in ref. [14], we also try to fit the inverted hierarchy. Further, we include full RGE into our analysis, which has not been done in previous studies. Moreover, we also attempt to fit the SUSY version of the model, which to the best of our knowledge has not been done before.

Results: we confirm with our more consistent fit approach that this class of models is not compatible with data on fermion masses and mixing angles, irrespective of the neutrino hierarchy or whether they are supersymmetric or non-supersymmetric.

2.3 Model with "full" Higgs content $\mathbf{10}_H + \overline{\mathbf{126}}_H + \mathbf{120}_H$

Without additional constraints, we would have the maximal number of parameters in this model. One can however considerably reduce the number of parameters by assuming all parameters to be real. This can be motivated or derived from an underlying parity symmetry [55] or spontaneous CP violation [56]. If CP is violated spontaneously solely by purely imaginary VEVs of the 120_H, this corresponds to taking all parameters in eq. (2.1) to be real. We will use the model with this reduced number of parameters and refer to it as "FN" in the non-supersymmetric case and as "FS" in the supersymmetric case. In a basis with real diagonal H (3 parameters) we count 6 parameters in F, 3 in G, and 6 in r, s, t_u, t_l, t_D, r_R, altogether 18 parameters. So in spite of introducing 120_H in addition to 10_H and $\overline{126}_H$, through the additional constraints this model has one parameter less than the "minimal" one. Therefore some authors refer to the SUSY version of this model as the "New Minimal Supersymmetric GUT" (NMSGUT) [57].

As in the minimal model there is a large amount of literature coping with the ability of the SUSY version of this model to reproduce the fermion spectrum and mixing [11, 13, 14, 41, 46, 55–65]. Without invoking supersymmetry, this model is analyzed only in refs. [14, 66].

Results: this class of models gives good fits to the data for both the normal and inverted neutrino mass hierarchy. For the fits of the non-SUSY version of this model the Higgs mass still leads to the top-quark mass being too small at its best-fit point, in this case 3.3 σ with normal neutrino mass hierarchy and 3.5 σ with inverted hierarchy, in analogy to the situation mentioned for model MN in section 2.1. Again, for a smaller top-quark mass the fits improve. If the fits include our rigorous treatment of RGEs, they worsen for the non-SUSY case, and to a lesser extent also in the SUSY case.

3 Details of the fitting procedure

We fit the models to experimental values of the masses of quarks and charged leptons, mass-squared differences of neutrinos, and mixing angles of quarks (including δ_{CKM}) and leptons. The quark and charged lepton masses at M_Z are taken from ref. [9]. Since the masses of charged leptons are measured with a very high accuracy that goes beyond our 1-loop RGE analysis, and since furthermore such precise values make a numerical fit very challenging, we assume an uncertainty of 5 % for these observables when fitting the models to the data. For the neutrino observables we neglect the running below M_Z and take the values from ref. [67].[2]

To check our numerical algorithm we also make fits without RGE as in ref. [14]. That means, as explained in section 1, we ignore the effect of non-degenerate right-handed neutrinos and take experimental values of observables at $\mu = M_{\text{GUT}}$ as given in tables 3 and 4, and fit the GUT-relations to those numbers. Note that to simplify the fitting procedure we symmetrized the error bars around the best-fit value whenever they were not symmetric. This will not have a large effect on the fits, since strongly non-symmetric errors are present only for the light quark masses where the uncertainty is large anyway. Finally, we perform separate fits for both a normal hierarchy (NH) and an inverted hierarchy (IH) of the neutrino masses (see section 3.1). We collect the values of observables underlying our analysis in table 2, 3, 4. To fit the model parameters to the observables we minimize

$$\chi^2 = \sum_{i=1}^{n} \left(\frac{y_i^{\text{theo}}(x) - y_i^{\text{exp}}}{\sigma_i^{\text{exp}}} \right)^2 \tag{3.1}$$

numerically with respect to $x = (x_1, \ldots, x_m)$, where y_i^{exp} are observables measured experimentally with uncertainty σ_i^{exp}, and $y_i^{\text{theo}}(x)$ is the corresponding theoretical prediction given the vector x of model parameters. We will later also look at χ^2 as a function of the atmospheric mixing angle, $\sin^2\theta_{23}^l$. To derive such a function for an observable O, one can add a term $(O^{\text{theo}}(x) - O)^2/(0.01\,O)^2$ to the definition of χ^2, where the denominator is a very small artificial uncertainty to let the minimization algorithm converge to a minimum. If O itself was part of the definition of χ^2 in eq. (3.1), then its term with the experimental uncertainty is removed. After performing the minimization of the so defined χ^2-function, one evaluates with the parameters obtained from that fit χ^2 as given in eq. (3.1), i.e. without the contribution of the artificial error, but including the contribution of the real experimental uncertainty. This method was previously used in refs. [14, 32, 56].

Observable	Exp. value
m_d [GeV]	0.0029 ± 0.001215
m_s [GeV]	0.055 ± 0.0155
m_b [GeV]	2.89 ± 0.09
m_u [GeV]	0.00127 ± 0.00046
m_c [GeV]	0.619 ± 0.084
m_t [GeV]	171.7 ± 3.0
$\sin\theta^q_{12}$	0.2246 ± 0.0011
$\sin\theta^q_{23}$	0.042 ± 0.0013
$\sin\theta^q_{13}$	0.0035 ± 0.0003
δ_{CKM}	1.2153 ± 0.0576
λ	0.521 ± 0.01

Observable	Exp. value
$\Delta m^2_{\odot}$ [eV^2]	$(7.5 \pm 0.185)\times 10^{-5}$
Δm^2_{31} (NH) [eV^2]	$(2.47 \pm 0.0685)\times 10^{-3}$
Δm^2_{31} (IH) [eV^2]	$(-2.355 \pm 0.0540)\times 10^{-3}$
$\sin^2\theta^l_{12}$	0.30 ± 0.013
$\sin^2\theta^{l,(\text{NH \& IH1})}_{23}$	0.41 ± 0.031
$\sin^2\theta^{l,(\text{IH2})}_{23}$	0.59 ± 0.022
$\sin^2\theta^l_{13}$	0.023 ± 0.0023
m_e [MeV]	0.48657 ± 0.02433
m_μ [GeV]	0.10272 ± 0.00514
m_τ [GeV]	1.74624 ± 0.08731

Table 2. Experimental values of observables at $\mu = M_Z$ used for our fits. The quark and charged lepton masses are taken from ref. [9], quark mixing parameters from ref. [14], neutrino mixing parameters from ref. [67] (table 1, second column). A 5 % uncertainty is assumed for the charged leptons, as explained in the text. The value of the Higgs quartic coupling λ is derived from the measurements of ATLAS [70] and CMS [71] as explained in section 3.2. Note that in our convention the Higgs self-interaction term in the Lagrangian is $-\frac{\lambda}{4}(\phi^\dagger\phi)^2$.

Obs.	Value [GeV]
m_d	0.00114 ± 0.000495
m_s	0.022 ± 0.0065
m_b	1.0 ± 0.04

Obs.	Value [GeV]
m_u	0.00048 ± 0.000185
m_c	0.235 ± 0.0345
m_t	74.0 ± 3.85

Obs.	Value [GeV]
$m_e \times 10^3$	0.46965 ± 0.02348
m_μ	0.09915 ± 0.00496
m_τ	1.68558 ± 0.08428

Table 3. Experimental values of observables at $\mu = M_{\text{GUT}}$ [14] used for non-SUSY fits without RGE. For mixing parameters as well as neutrino mass-squared differences the same values as in table 2 are used. A 5 % uncertainty is assumed for the charged leptons, as explained in the text.

For the minimization we use the downhill simplex algorithm [72, 73] in its implementation from the GNU Scientific Library [74], which also provides useful functions for numerical matrix diagonalization and for solving differential equations numerically. The parallelized computations are performed on the computer cluster of the Max-Planck-Institut für Kernphysik, Heidelberg, where up to 1700 CPU cores can be used.

Let us stress a general caveat of numerical minimization. The problem at hand is non-linear and multidimensional — therefore many local minima exist. With numerical algorithms it is impossible to determine whether a minimum is a global minimum of the function under consideration. A standard procedure to increase the confidence that a global minimum out of the many local ones has been found is to start the minimization many times with different initial parameters and to choose the lowest out of the many

[2] See also ref. [68, 69].

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Observable	$\tan\beta = 50$	$\tan\beta = 38$	$\tan\beta = 10$
m_u/m_c	0.0027 ± 0.0006	0.0027 ± 0.0006	0.0027 ± 0.0006
m_d/m_s	0.051 ± 0.007	0.051 ± 0.007	0.051 ± 0.007
m_c/m_t	0.0023 ± 0.0002	0.0024 ± 0.0002	0.0025 ± 0.0002
m_s/m_b	0.016 ± 0.002	0.017 ± 0.002	0.019 ± 0.002
m_e/m_μ	0.0048 ± 0.0002	0.0048 ± 0.0002	0.0048 ± 0.0002
m_μ/m_τ	0.05 ± 0.002	0.054 ± 0.002	0.059 ± 0.002
m_b/m_τ	0.73 ± 0.04	0.73 ± 0.04	0.73 ± 0.03
$\sin\theta^q_{12}$	0.227 ± 0.001	0.227 ± 0.001	0.227 ± 0.001
$\sin\theta^q_{23}$	0.0371 ± 0.0013	0.0386 ± 0.0014	0.04 ± 0.0014
$\sin\theta^q_{13}$	0.0033 ± 0.0007	0.0035 ± 0.0007	0.0036 ± 0.0007
δ_{CKM}	0.9828 ± 0.1784	0.9828 ± 0.1784	0.9828 ± 0.1787
$\Delta m^2_{21}/\Delta m^2_{31}$	0.03036 ± 0.0011	0.03036 ± 0.0011	0.03036 ± 0.0011
Δm^2_{31} (NH) [eV^2]	$(2.47 \pm 0.0685)\times 10^{-3}$	$(2.47 \pm 0.0685)\times 10^{-3}$	$(2.47 \pm 0.0685)\times 10^{-3}$
m_τ [GeV]	0.773 ± 0.0387	0.950 ± 0.0475	1.022 ± 0.0511
m_t [GeV]	94.7 ± 9.4	94.7 ± 9.4	92.2 ± 8.7

Table 4. Experimental values of observables at $\mu = M_{\mathrm{GUT}}$ [14] used for SUSY fits without RGE. The ratio of solar to atmospheric neutrino mass-squared difference is calculated from their values at $\mu = M_Z$ as given in table 2. The top-quark mass m_t and the tau mass m_τ at $\mu = M_{\mathrm{GUT}}$ for $\tan\beta = 50$, 10 are taken from ref. [9], the values for $\tan\beta = 38$ are interpolations. For the neutrino mixing angles as well as Δm^2_{31} the values at $\mu = M_Z$ as given in table 2 are used.

local minima that will be found. Furthermore one can perturb a minimum and restart the minimization from the perturbed point [73, 74]. These steps can be repeated many times until no improvement of the minimum is found any more. Both methods are used by our program.

3.1 Neutrino data

In the neutrino sector the absolute mass scale is experimentally not yet determined. At present, only the solar mass-squared difference Δm^2_{21} and the absolute value of the atmospheric mass-squared difference $|\Delta m^2_{31}|$ are known [67],

$$\begin{aligned}\Delta m^2_{21} &= 7.5 \pm 0.185 \times 10^{-5}\ \mathrm{eV}^2\,,\\ \Delta m^2_{31} &= 2.47 \pm 0.0685 \times 10^{-3}\ \mathrm{eV}^2\ (\mathrm{NH})\,,\\ \Delta m^2_{31} &= -2.355 \pm 0.0540 \times 10^{-3}\ \mathrm{eV}^2\ (\mathrm{IH})\,,\end{aligned}$$

where NH (normal hierarchy) and IH (inverted hierarchy) refer to two currently viable situations with $m_1 < m_2 < m_3$ and $m_3 < m_1 < m_2$, respectively. Furthermore, we symmetrized the uncertainties, as explained before.

Besides the different signs and values of Δm^2_{31}, also the neutrino mixing parameters have different preferred values depending on which mass hierarchy is assumed [67–69]. This hierarchy dependence is mostly pronounced for $\sin^2\theta^l_{23}$. Here, the best-fit value of $\sin^2\theta^l_{23}$ depends on aspects of the analysis, including the experiments that were considered.

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Comparing refs. [67–69] we notice that there currently exist two different equally valid best-fit values and corresponding $1\,\sigma$ regions for $\sin^2\theta^l_{23}$. We take the values to which the models will be fitted from ref. [67] and distinguish the following cases in our analysis:

$$\begin{aligned} \sin^2\theta^l_{23} &= 0.41 \pm 0.031 \quad \text{NH}\,, \\ \sin^2\theta^l_{23} &= 0.41 \pm 0.031 \quad \text{IH1}\,, \\ \sin^2\theta^l_{23} &= 0.59 \pm 0.022 \quad \text{IH2}\,. \end{aligned} \tag{3.2}$$

The quality of fits with the inverted neutrino mass hierarchy had the same quality for both IH1 and IH2. Hence, we will stick in our discussion of results in section 4 to the case IH2. While the other groups performing global neutrino fits do not have these two solutions, we decided to use the results from ref. [67], in order to have a test for the octant-dependence of the SO(10) fit results.

3.2 Higgs mass and quartic coupling

Although the Higgs boson mass $m_{\rm H}$ does not enter the SO(10) relations in eq. (1.3) there is interplay between $m_{\rm H}$ and the fermion observables during renormalization group evolution. In RGE in the non-supersymmetric case the Higgs quartic coupling λ appears, which in the SM is related to $m_{\rm H}$ by[3]

$$\lambda = \frac{2}{v^2}\, m_{\rm H}^2\,. \tag{3.3}$$

Recently, the ATLAS and CMS experiments at the Large Hadron Collider (LHC) have observed a new particle, which is in good agreement with a Standard Model Higgs boson, with the mass [70, 71]

$$\begin{aligned} m_{\rm H} &= 126.0 \pm 0.4\ (\text{stat}) \pm 0.4\ (\text{sys})\ \text{GeV} \qquad (\text{ATLAS}) \\ m_{\rm H} &= 125.3 \pm 0.4\ (\text{stat}) \pm 0.5\ (\text{sys})\ \text{GeV} \qquad (\text{CMS})\,. \end{aligned} \tag{3.4}$$

For our analysis we take a conservative estimate of the true Higgs mass, since there is no official combined analysis available. Our $1\,\sigma$ interval shall overlap exactly the $1\,\sigma$ intervals of the ATLAS and CMS experiments and we take the central value of this range as best-fit point. Thus, we take for our fits

$$m_{\rm H} = 125.6 \pm 1.2\ \text{GeV}\,. \tag{3.5}$$

The standard error propagation formula applied to eq. (3.3) then yields

$$\lambda = 0.521 \pm 0.010\,. \tag{3.6}$$

Note that for fits at $M_{\rm GUT}$, i.e. without RG evolution, we do not take into account the Higgs mass, since in that case there is no restriction on λ from the other observables.

[3] In our convention the Higgs self-interaction term in the Lagrangian is $-\frac{\lambda}{4}(\phi^\dagger\phi)^2$.

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Supersymmetric case. Above the supersymmetry breaking scale $M_{\rm SUSY}$, supersymmetry fixes λ to be[4] [75]

$$\lambda(\mu \geq M_{\rm SUSY}) = \frac{1}{4}\left(\frac{3}{5}\, g_1^2 + g_2^2\right)(\mu)\,. \tag{3.7}$$

Below $M_{\rm SUSY}$ the Higgs mass receives radiative corrections, the leading one given in a rough approximation (within the MSSM) by [75]

$$m_{\rm H}^2 = M_Z^2 + \frac{3\, g_2^2\, m_t^4(\mu_t)}{8\pi^2 M_W^2} \ln\left(\frac{M_{\rm SUSY}^2}{m_t^2(\mu_t)}\right), \tag{3.8}$$

with $\mu_t = \sqrt{m_t\, M_{\rm SUSY}}$ and all SUSY particles are assumed to have masses around $M_{\rm SUSY}$ in this approximation. By varying $M_{\rm SUSY}$ one can reproduce the measured value of $m_{\rm H}$ as given in eqs. (3.4) and (3.5). Solving eq. (3.8) for $M_{\rm SUSY}$ yields $M_{\rm SUSY} \approx 1\,\text{TeV}$. Since our main goal is to fit fermion masses and mixings within the SO(10) framework and not performing a detailed analysis of the MSSM, we do not specify $M_{\rm SUSY}$ or the specific SUSY spectrum. Hence, we will not try to fit $m_{\rm H}$ in the supersymmetric models.

3.3 Renormalization group evolution

The relations in eq. (2.1) have to be obeyed at $M_{\rm GUT}$. Therefore, for a given set of SO(10) parameters, in order to calculate the model predictions for the observables one has to use RGEs and evolve the parameters down to the energy scale at which the observables are known. In addition one has to integrate out heavy degrees of freedom during this process at their mass scale. In our case this applies to the right-handed neutrinos (ν_{R_i}), as their masses usually lie somewhere between 10^{10} GeV and $M_{\rm GUT}$. After integrating out a degree of freedom one ends up with an effective field theory (EFT) and has to match coefficients of effective operators with parameters from the full theory. Since ν_{R_i} are not degenerate in general, one has to integrate out several times and thus has to use different EFTs during the evolution from $M_{\rm GUT}$ to M_Z. This formalism is nicely described in refs. [18, 76]. We use the 1-loop β-functions as presented in ref. [76] for the SM and MSSM, respectively (see also [77]). The beta-functions are also presented in appendix B for reference. We should mention that we do not integrate out the top-quark below $\mu = m_t(m_t)$, since the energy scales m_t and M_Z are quite close. Furthermore we assume $M_{\rm SUSY} = M_Z$, i.e. we use the beta-functions of the MSSM for the evolution of parameters down to M_Z in case of SUSY models, since this is just a small effect as long as the SUSY breaking scale is not too far away from M_Z. We expect models being able to fit experimental data with $M_{\rm SUSY} = M_Z$ to be equally well suited to fit the data with $M_{\rm SUSY} = 500\,\text{GeV}$ or $1\,\text{TeV}$. Finally, since we do not specify the SUSY spectrum, we also do not consider SUSY threshold effects, which may have an impact for large $\tan\beta$ [10, 14, 78–83].

4 Results

In this section we present and discuss the results of our analysis. We quantify the deviation of model predictions by stating the pulls of all the observables considered. The pull of a

[4] We apply GUT normalization to the $\mathrm{U}(1)_{\rm Y}$ charge.

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model with respect to an observable y_i is defined as

$$\mathrm{pull}(y_i) = \frac{y_i^{\mathrm{theo}}(x) - y_i^{\mathrm{exp}}}{\sigma_i^{\mathrm{exp}}}, \tag{4.1}$$

with the variables as defined in eq. (3.1) on page 8. The pull measures the deviation of theoretical predictions (or best-fit values) from experimentally measured values in units of uncertainty of the observable. Its sign shows whether the theoretical prediction is too small or too large.

Different sets of observables. We will present the results of fits including RGE, where the input values are taken at $\mu = M_Z$, as well as the results of fits made without RGE, as explained in section 1. Our full set of observables to which the models are fitted are the masses of quarks and charged leptons, mass-squared differences of neutrinos, mixing angles of quarks and leptons, the CP phase δ_{CKM} in the CKM matrix and the Higgs quartic coupling λ. The full set of observables is used only in non-SUSY models with full RG analysis, λ is generally not fitted in SUSY models and in non-SUSY models without RG analysis (see section 3.2 for details). Hence the number of observables taken into account for the fits is 18 or 19. Numerical input values can be found in section 3. There we also pointed out that in case of the inverted neutrino mass hierarchy currently two numerically different best-fit solutions exist for the value of $\sin^2\theta^l_{23}$. Since good fits could be achieved for both possibilities, we restrict the discussion of our results to the case where $\sin^2\theta^l_{23} > 0.5$. For the normal hierarchy, the experimental best-fit value is $\sin^2\theta^l_{23} = 0.41 < 0.5$. We now proceed with the discussion of each model.

4.1 Minimal model with $\mathbf{10}_H + \overline{\mathbf{126}}_H$ (MN, MS)

Let us start by comparing our program to previous fit results. In case of no RGE, our minimal χ^2-value in the non-SUSY case is 1.1, to be compared with the value $\chi^2_{\mathrm{min}} \approx 0.7$ in ref. [14]. The SUSY model MS with normal neutrino mass hierarchy has been analyzed before (without RG evolution) [14, 32], albeit with older data underlying the analyses. The results lie in the range between $\chi^2_{\mathrm{min}} = 3.7$ and $\chi^2_{\mathrm{min}} = 5.1$ in case of type I seesaw dominance. Our fits yield $\chi^2_{\mathrm{min}} = 9.41$, 9.72, and 10.45 for $\tan\beta = 50$, 38, and 10, respectively. The results are summarized in table 5 and the best-fit values of observables and corresponding pulls, to be discussed below, are compiled in tables 6 and 7.

With inverted neutrino mass hierarchy it was impossible to produce a good fit of this model, with $\chi^2_{\mathrm{min}} > 200$ (400) in case of SUSY (non-SUSY) models. Within SUSY versions of such models this observation has already been made by other authors [32], however without including full RGE during the fitting procedure and hence with neglecting running of neutrino parameters which can be sizable in the inverted hierarchy case. Therefore, our conclusion is more stringent. Since with the inverted hierarchy it is impossible to fit the data, we present only results for the normal neutrino mass hierarchy.

We see that for the minimal models in the non-supersymmetric case, including the full RG analysis worsens the fit considerably, while doing the same for the SUSY model gives a better result than fitting without RG evolution (table 5). In case of non-SUSY models

Model	$\tan\beta$	Comment	$\chi^2_{\rm NH}$	$\chi^2_{\rm IH}$
MN	—	no RGE	1.103	395
	—	RGE	22.97	680
MS	50	no RGE	9.411	200
	50	RGE	3.294	420
	38	no RGE	9.715	300
	38	RGE	3.016	—
	10	no RGE	10.45	225
	10	RGE	2.889	—

Table 5. Fit results for models MN and MS (Higgs content $10_H + \overline{126}_H$, 19 free parameters), to 19 and 18 observables, respectively. We differentiate between fits to observables without RGE ("no RGE") and fits to observables including RGE. For the fits of MN including RGE the Higgs quartic coupling has also been fitted, as described in section 3.2. No fits are made for the time-consuming cases with RGE in the inverted hierarchy for $\tan\beta = 38$ and 10, because for NH the fits give essentially identical $\chi^2_{\rm min}$-values.

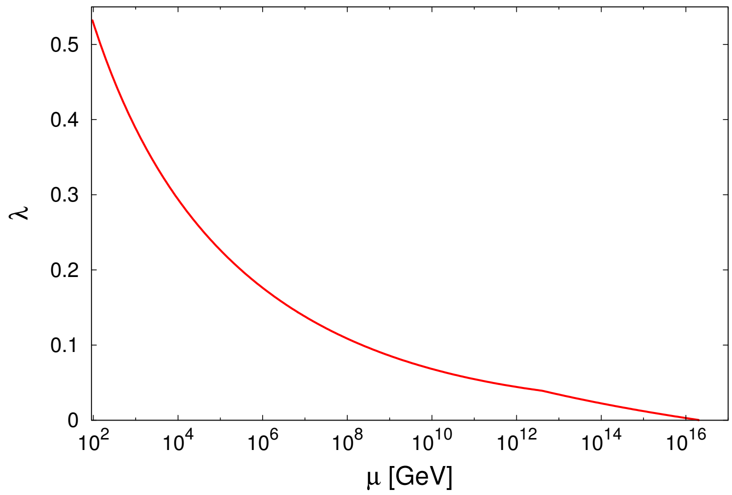

Figure 1. Running of λ for the best-fit parameters of model MN with normal neutrino mass hierarchy and RG evolution. The kink between $\mu = 10^{12}$ GeV and $\mu = 10^{13}$ GeV results from integrating out the heaviest right-handed neutrino.

there is an additional constraint when fitting with RG evolution, since in that case we also consider the Higgs mass (see section 3.2). Still, both non-SUSY and SUSY models can fit the data, the SUSY models being in better agreement. For the SUSY model we see no preferred value of $\tan\beta$ from our fits.

Let us now discuss the different contributions to $\chi^2_{\rm min}$. We show the best-fit values of observables and their corresponding pulls in tables 6 and 7. In case of non-SUSY fits without RGE one observes that the dominating contribution to $\chi^2_{\rm min}$ is due to the pull

	MN, no RGE		MN, with RGE	
Observable	best-fit	pull	best-fit	pull
m_d	0.00067	-0.9458	0.00298	0.0621
m_s	0.02406	0.3172	0.06887	0.8951
m_b	1.00309	0.0772	2.89370	0.0411
m_u	0.00048	0.0072	0.00131	0.0977
m_c	0.24243	0.2153	0.70754	1.0541
m_t	73.6931	-0.0797	161.411	-3.4295
$\sin\theta^q_{12}$	0.22462	0.0227	0.22476	0.1433
$\sin\theta^q_{23}$	0.04204	0.0304	0.04170	-0.2291
$\sin\theta^q_{13}$	0.00350	0.0091	0.00342	-0.2520
δ_{CKM}	1.21930	0.0699	1.25285	0.6525
Δm^2_{21}	7.50×10^{-5}	0.0180	7.53×10^{-5}	0.1626
Δm^2_{31}	2.47×10^{-3}	-0.0204	2.46×10^{-3}	-0.1858
$\sin^2\theta^l_{12}$	0.30039	0.0303	0.29864	-0.1044
$\sin^2\theta^l_{23}$	0.40631	-0.1189	0.34571	-2.0739
$\sin^2\theta^l_{13}$	0.02262	-0.1652	0.01847	-1.9678
m_e	4.697×10^{-4}	—	0.00049	0.0704
m_μ	9.914×10^{-2}	—	0.10143	-0.2508
m_τ	1.686	—	1.73804	-0.0939
λ	—	—	0.52731	0.6307
χ^2_{min}		**1.103**		**22.97**

Table 6. 19 Observables and pulls for model MN (minimal non-SUSY, $10_H + \overline{126}_H$, 19 free parameters) with and without considering RGE. Masses are given in GeV, mass-squared differences in eV^2.

of the down-quark mass. In case of SUSY fits without RGE, we fit for the sake of better comparison with ref. [14] mass ratios instead of masses. There the dominating contribution is the m_d/m_s ratio. In case of non-SUSY fits the main contribution to χ^2_{min} comes from the mass of the top-quark ($\sim 3.4\,\sigma$), followed by the pulls of $\sin^2\theta^l_{23}$ and $\sin^2\theta^l_{13}$.

Higgs mass vs. top-quark mass: the tension in the fit due to the top-quark mass is easily understood from the relatively light Higgs mass (and hence low quartic coupling λ). Namely, as well-known from the vacuum stability problem (for recent analyses, see refs. [84–88]), the beta-function governing the evolution of the Higgs quartic coupling λ, as given in appendix B.1, is dominated by the top-quark Yukawa, and drives λ towards negative values when going from low to high scale. Note that in our fit we start with $\lambda \geq 0$ at the GUT scale, and hence do not have a problem of negative λ. Nevertheless, the "too large" top-quark mass causes some pressure on the fits. The large top-quark Yukawa

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MS (with RGE)						
	$\tan\beta = 50$		$\tan\beta = 38$		$\tan\beta = 10$	
Observable	best-fit	pull	best-fit	pull	best-fit	pull
m_d	0.00087	-1.6714	0.00090	-1.6449	0.00091	-1.6381
m_s	0.04512	-0.6371	0.04711	-0.5089	0.04870	-0.4063
m_b	2.87626	-0.1526	2.88217	-0.0870	2.88499	-0.0557
m_u	0.00127	0.0018	0.00127	0.0068	0.00127	0.0064
m_c	0.62848	0.1129	0.62738	0.0997	0.62854	0.1135
m_t	171.453	-0.0823	171.522	-0.0593	171.539	-0.0537
$\sin\theta^q_{12}$	0.22460	-0.0040	0.22460	-0.0018	0.22460	-0.0009
$\sin\theta^q_{23}$	0.04191	-0.0675	0.04193	-0.0565	0.04193	-0.0543
$\sin\theta^q_{13}$	0.00351	0.0241	0.00351	0.0322	0.00351	0.0314
δ_{CKM}	1.21318	-0.0364	1.21398	-0.0225	1.21409	-0.0205
Δm^2_{21}	7.50×10^{-5}	0.0021	7.50×10^{-5}	0.0013	7.50×10^{-5}	0.0009
Δm^2_{31}	2.47×10^{-3}	-0.0022	2.47×10^{-3}	-0.0013	2.47×10^{-3}	-0.0010
$\sin^2\theta^l_{12}$	0.30015	0.0112	0.30007	0.0053	0.30004	0.0028
$\sin^2\theta^l_{23}$	0.40960	-0.0129	0.40977	-0.0073	0.40987	-0.0043
$\sin^2\theta^l_{13}$	0.02299	-0.0045	0.02297	-0.0138	0.02297	-0.0123
m_e	0.00049	0.0702	0.00049	0.0660	0.00049	0.0605
m_μ	0.10315	0.0839	0.10306	0.0665	0.10298	0.0513
m_τ	1.76204	0.1809	1.75740	0.1278	1.75528	0.1035
χ^2_{min}		**3.294**		**3.016**		**2.889**

Table 7. 18 Observables and pulls for model MS (minimal SUSY, $10_H + \overline{126}_H$, 19 free parameters) with RGE, for different values of $\tan\beta$. Masses are given in GeV, mass-squared differences in eV^2.

coupling[5] favors a larger Higgs mass than experimentally established. This ultimately results in relatively large pulls for the top-quark mass in fits with RG evolution. Therefore, we also performed a fit of model MN without including the Higgs mass. As expected, the fit improves from $\chi^2_{\mathrm{min}} = 22.97$ to $\chi^2_{\mathrm{min}} = 8.21$.

Recall, however, that there is a discussion on the correct value of the top-quark mass as determined via kinematic reconstruction [52–54]. The top-quark mass determination at the TeVatron is based on the final state of the decay products. Another possibility is to reconstruct the top-quark mass from the total cross section in the top-quark pair production. This method is more rigorous from a theoretical perspective and yields a smaller top-quark mass ($168.9 \pm 3.5\,\mathrm{GeV}$[6] [52]) than the world average and has larger error

[5]We neglect for this argument effects of higher loop or other corrections to the Higgs potential. Moreover, Dirac Yukawas have a similar, though somewhat weaker, effect than the top-quark Yukawa, see e.g. [89, 90].

[6]Note that this is the pole-mass $m_t(m_t)$, while the value given in table 2 is the mass at M_Z in the $\overline{MS}$ scheme. Converting to $\overline{MS}$ using 1-loop matching [91] conditions and evolving to $\mu = M_Z$ yields $m_t = 158.5 \pm 3.2\,\mathrm{GeV}$ as the observable to be used in the fits with light top-quark mass.

MN (NH, RGE)	with Higgs	w/o Higgs
heavy top	22.97	8.21
light top	10.06	6.70

FN (IH, RGE)	with Higgs	w/o Higgs
heavy top	13.3	0.67
light top	0.98	0.50

Table 8. Impact of top-quark mass and Higgs mass on fit results for models MN (normal hierarchy) and FN (inverted hierarchy). The χ^2-minima for the fit with a heavy top-quark mass and a light top-quark mass in combination with and without fitting the Higgs mass are shown.

bars. One can expect that the fit will improve when we use this lower top-quark mass. Indeed, $\chi^2_{\rm min}$ reduces from 22.97 to 10.06. We summarize different fits with heavy and light top-quark in combination with and without Higgs in table 8. Let us remark again that intermediate scales and other details of the full scalar sector are neglected in our analysis. Hence, a more correct but model-dependent treatment of the evolution of λ might lead to perfect compatibility of the measured Higgs mass and large top-quark mass (or make the problem worse).

The evolution of λ with energy for the best-fit parameters of model MN with normal neutrino mass hierarchy and RG evolution is shown in figure 1. Notice the kink between $\mu = 10^{12}$ GeV and $\mu = 10^{13}$ GeV, which results from integrating out the heaviest neutrino with a mass of $M_3 \simeq 3.6 \times 10^{12}$ GeV (see section 4.4). There is no further such kink at energies where the other heavy neutrinos are integrated out, since their contribution to the running of λ is negligible compared to the contribution of the top-quark.

4.2 Alternative minimal model with $\overline{\mathbf{126}}_H + \mathbf{120}_H$

This model was analyzed only in the non-SUSY version (as it was originally proposed to be attractive in that case [22]) and found to be unable to reproduce fermion masses and mixings. Although this result has been obtained previously [14], only the normal hierarchy was considered, and no detailed RGE analysis was performed. In this work we used full RGE to arrive at our results, thus our conclusion is stronger. Further, we also checked and excluded the possibility of the inverted hierarchy. The supersymmetric version, that to the best of our knowledge has not been analyzed before, is also not an option to save this model (to safe CPU-time, we only fitted the case of $\tan\beta = 10$). We find large χ^2-values for this case as well and therefore we only present a table with the large χ^2-values, see table 9. We note however that the normal neutrino mass hierarchy has significantly smaller, though still too large χ^2-values.

4.3 Model with $\mathbf{10}_H + \overline{\mathbf{126}}_H + \mathbf{120}_H$ (FN, FS)

This class of models, in spite of having one more Higgs representation, through the additional constraints (i.e. assuming spontaneous CP violation, see section 2) has one parameter less than the minimal models. Nevertheless, it is not only able to fit the data, but reproduces the data even much better than the other models. This is especially the case for the SUSY versions of this model. Furthermore, these models are also able to fit the data with inverted neutrino mass hierarchy, which differentiates them clearly from the previous models. Since we do not observe a significant difference in the quality of fits of SUSY models

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$\tan\beta$	Comment	$\chi^2_{\rm NH}$	$\chi^2_{\rm IH}$
—	no RGE	103	910
—	RGE	200	3859
10	no RGE	247	1861
10	RGE	224	4358

Table 9. Fit results of an alternative minimal model with $\overline{126}_H + 120_H$ Higgs representations, having 17 free parameters. No acceptable fit was found.

Model	$\tan\beta$	Comment	$\chi^2_{\rm NH}$	$\chi^2_{\rm IH}$
FN	—	no RGE	6.6×10^{-5}	2.5×10^{-4}
	—	RGE	11.2	13.3
FS	50	no RGE	9.0×10^{-10}	3.9×10^{-8}
	50	RGE	6.9×10^{-6}	0.602

Table 10. Values of $\chi^2_{\rm min}$ at the best-fit position for the model with $10_H + \overline{126}_H + 120_H$ (18 free parameters) in case of normal (NH) and inverted (IH) neutrino mass hierarchy. For IH we present the solution with $\sin^2\theta^l_{23} > 0.5$, but both possibilities yield equally good fits. Remarks as in table 5 apply analogously. Model FN (FS) contains 19 (18) free parameters.

with different values of $\tan\beta$, we fitted this model only for $\tan\beta = 50$ and $\tan\beta = 10$, which again yield very similar results, as in the case of the $10_H + \overline{126}_H$ model. Therefore we present here only the detailed results for $\tan\beta = 50$. Our results are tabulated in table 10 and the best-fit values of observables and their pulls are compiled in tables 11 and 12.

Let us first discuss the fits with normal hierarchy. This setup has been analyzed without RG evolution in the SUSY case in refs. [13, 14, 56] with $\chi^2_{\rm min}$ ranging between 0.01 and 0.33. The non-SUSY case has been fitted to data only in ref. [14] and results in $\chi^2_{\rm min} \sim 10^{-6}$. Again we observe that fitting the non-SUSY version of this model including RG evolution significantly worsens the fit. The SUSY fits turn out to be even better than the non-SUSY fits. Here, fits with RGE as well as fits without RGE yield χ^2-values that are essentially zero.

We now turn to the inverted hierarchy. In contrast to the $10_H + \overline{126}_H$ model an inverted neutrino mass hierarchy is viable here. The SUSY case with inverted hierarchy has been fitted to data in ref. [56], giving $\chi^2_{\rm min} = 0.011$, but RGE was not taken into account, which is especially important for the inverted hierarchy. We are not aware of any analysis of the non-SUSY case with inverted hierarchy. In our analysis, in the non-SUSY model the fit quality is approximately the same as in the normal hierarchy. Again, when fitting with RGE, inclusion of $m_{\rm H}$ severely constrains the model. For FN in case of inverted hierarchy we again, as for MN with normal hierarchy, performed an additional fit without including the Higgs mass. As expected from the discussion in section 4.1 the pull of the top-quark diminishes and we get $\chi^2_{\rm min} = 0.67$ to be compared to $\chi^2_{\rm min} = 13.3$ in case the

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Observable	best-fit	pull	best-fit	pull
m_d	0.00295	0.0414	0.00304	0.1167
m_s	0.06199	0.4512	0.06650	0.7417
m_b	2.88874	-0.0140	2.88956	-0.0049
m_u	0.00127	0.0003	0.00127	0.0008
m_c	0.62395	0.0590	0.62377	0.0568
m_t	161.943	-3.2525	161.207	-3.4977
Δm^2_{21}	7.50×10^{-5}	0.0015	7.50×10^{-5}	-0.0001
Δm^2_{31}	2.47×10^{-3}	-0.0037	-2.35×10^{-3}	0.0019
$\sin\theta^q_{12}$	0.22460	-0.0044	0.22460	0.0042
$\sin\theta^q_{23}$	0.04192	-0.0646	0.04182	-0.1347
$\sin\theta^q_{13}$	0.00350	-0.0031	0.00350	-0.0007
$\delta_{\rm CKM}$	1.21402	-0.0217	1.21650	0.0213
$\sin^2\theta^l_{12}$	0.30006	0.0048	0.30000	0.0002
$\sin^2\theta^l_{23}$	0.41029	0.0093	0.59025	0.0116
$\sin^2\theta^l_{13}$	0.02302	0.0078	0.02300	0.0015
m_e	0.00049	0.0001	0.00049	0.0005
m_μ	0.10232	-0.0777	0.10212	-0.1173
m_τ	1.74663	0.0045	1.74178	-0.0511
λ	0.52745	0.6455	0.52792	0.6917
$\chi^2_{\rm min}$		**11.2**		**13.3**

Table 11. 19 Observables and pulls for model FN (18 free parameters) fitted assuming normal (NH) or inverted (IH) hierarchy. Masses are given in GeV, mass-squared differences in eV^2.

Higgs mass is included in the fit. We also fit the model with the lower top-quark mass $168.9 \pm 3.5\,$GeV, finding $\chi^2_{\rm min} = 0.98$. We summarize different fits with heavy top-quark, with light top-quark and fits without Higgs mass in table 8.

For fits where $\chi^2_{\rm min} \sim 0$ the best-fit values of observables are essentially identical with the experimental values presented in section 3. Hence, we do not tabulate them. In the non-SUSY fits including RG evolution we have again the dominating contribution to $\chi^2_{\rm min}$ from m_t with a pull of ~ -3.25 (NH) or ~ -3.5 (IH) corresponding to $\Delta\chi^2_{m_t} \sim 10.6$ (NH) or $\Delta\chi^2_{m_t} \sim 12.2$ (IH) followed by the pulls of λ and the strange-quark mass m_s, 0.65 and 0.45 (NH) or 0.69 and 0.74 (IH), respectively. In the SUSY case with normal hierarchy the main contribution to $\chi^2_{\rm min}$ is the strange-quark mass with a pull of 0.37 followed by $\sin^2\theta^l_{13}$, m_d, and $\sin^2\theta^l_{23}$. Fitting the inverted hierarchy, again the pull of the strange-quark mass gives the main contribution to $\chi^2_{\rm min}$ with now the pull being -0.75 accounting for nearly the whole value of $\chi^2_{\rm min}$. The best-fit values of observables and their pulls for the non-SUSY and SUSY version of this model are summarized in tables 11 and 12, respectively.

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Observable	FS, NH, RGE best-fit	FS, NH, RGE pull	FS, IH, RGE best-fit	FS, IH, RGE pull
m_d	0.00290	-0.0001	0.00305	0.1255
m_s	0.05496	-0.0025	0.04337	-0.7500
m_b	2.89002	0.0003	2.88344	-0.0729
m_u	0.00127	-0.0000	0.00127	-0.0062
m_c	0.61903	0.0003	0.61551	-0.0416
m_t	171.699	-0.0003	171.655	-0.0150
Δm^2_{21}	7.50×10^{-5}	0.0000	7.50×10^{-5}	0.0000
Δm^2_{31}	2.47×10^{-3}	-0.0000	-2.36×10^{-3}	-0.0000
$\sin\theta^q_{12}$	0.22460	0.0000	0.22460	-0.0012
$\sin\theta^q_{23}$	0.04200	0.0003	0.04201	0.0114
$\sin\theta^q_{13}$	0.00350	-0.0000	0.00350	-0.0015
$\delta_{\rm CKM}$	1.21528	0.0001	1.21507	-0.0035
$\sin^2\theta^l_{12}$	0.30000	0.0000	0.30000	-0.0000
$\sin^2\theta^l_{23}$	0.41000	-0.0002	0.58972	-0.0129
$\sin^2\theta^l_{13}$	0.02300	-0.0003	0.02300	0.0011
m_e	0.00049	0.0000	0.00049	-0.0025
m_μ	0.10272	0.0001	0.10320	0.0937
m_τ	1.74622	-0.0002	1.75356	0.0838
$\chi^2_{\rm min}$		$\mathbf{6.89\times10^{-6}}$		**0.602**

Table 12. 18 Observables and pulls for model FS (18 free parameters) fitted assuming normal (NH) or inverted (IH) hierarchy. Masses are given in GeV, mass-squared differences in eV^2. The value $\tan\beta = 50$ is chosen here, with very little difference to other values.

4.4 Model predictions

There are observables which have not yet been measured experimentally but are fixed by the fits we performed, so they can be understood as predictions of the models we analyzed. For instance, the effective mass $\langle m_\nu\rangle$ that governs the lifetime of neutrinoless double beta decay ($0\nu\beta\beta$), defined as [92, 93],

$$\langle m_\nu\rangle = \left| U_{e1}^2\, m_1 + U_{e2}^2\, m_2\, e^{i\alpha} + U_{e3}^2\, m_3\, e^{i\beta} \right| \,, \tag{4.2}$$

is of interest. Here U is the leptonic mixing matrix, α, β are Majorana phases and m_i are the masses of light neutrinos. Additional parameters of interest are the leptonic CP violation phase $\delta^l_{\rm CP}$ as relevant for oscillation experiments, and the lightest neutrino mass m_0 ($m_0 = m_1$ for NH and $m_0 = m_3$ for IH). We also present the masses of heavy neutrinos M_i, though those are not really testable observables. We will discuss the non-SUSY case

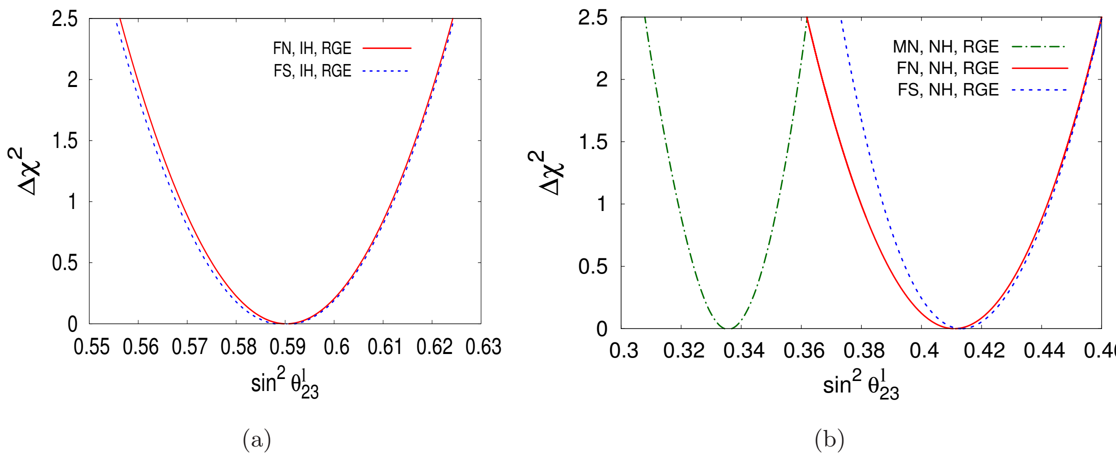

Figure 2. $\Delta\chi^2(\sin^2\theta^l_{23}) = \chi^2(\sin^2\theta^l_{23}) - \chi^2|_{\rm min}$ is shown (a) for models FN and FS in case of an inverted neutrino mass hierarchy and (b) for models MN, FN, and FS in case of a normal hierarchy.

as well as the SUSY case. In case of SUSY models we will restrict the discussion to models with $\tan\beta = 50$, since the results of models with other values of $\tan\beta$ are very similar. The numerical values are tabulated in table 13.

There is still the question whether the atmospheric neutrino mixing angle, θ^l_{23}, deviates from maximal mixing. While the best-fit points of global neutrino fits usually are away from maximal, this is typically only a less than $2\,\sigma$ effect (see table 2). In most of our fits the value of θ^l_{23} is fitted essentially at its best-fit point. The notable exception is model MN in case of a normal hierarchy, where $\sin^2\theta^l_{23}$ is off by about $2\,\sigma$, cf. Tbl. 6. We analyze the behaviour of χ^2 as a function of $\sin^2\theta^l_{23}$, employing the method described at the beginning of section 3. For the still viable models MN, FN, and FS[7] we plot $\Delta\chi^2(\sin^2\theta^l_{23}) = \chi^2(\sin^2\theta^l_{23}) - \chi^2_{\rm min}$ in figure 2. As can be seen from figure 2 neither model FN nor FS restricts the value of θ^l_{23} sizably beyond its experimental boundaries, independently of the neutrino mass hierarchy; $\Delta\chi^2(\sin^2\theta^l_{23})$ simply increases due to the deviation of $\sin^2\theta^l_{23}$ from the experimental best-fit value. Model MN in case of normal hierarchy, however, strongly favors a rather small value, $\sin^2\theta^l_{23} = 0.335 \pm 0.015$ at 68 % C.L. (corresponding to $\Delta\chi^2 = 1$), with a much steeper $\Delta\chi^2(\sin^2\theta^l_{23})$ function. Therefore, if after more precise measurements the value of θ^l_{23} contracts around its current experimental best-fit value, model MN will be strongly disfavored. For other cases, not shown in figure 2, especially the fits without RGE, the situation is similar to that of models FN and FS as presented in figure 2.

No general conclusions can be drawn for leptonic CP phase $\delta^l_{\rm CP}$ (this may be different if in future analyses the baryon asymmetry of the Universe as generated via thermal leptogenesis is also fitted). However, for neutrino masses, m_i, M_i and $\langle m_\nu\rangle$, one can observe that in the normal hierarchy, models FN and FS predict a higher seesaw scale (M_3) than models MN and MS. The light neutrino masses and the effective mass for neutrinoless double beta decay are also larger in models FN and FS.

[7]MN is not viable in the case of inverted hierarchy, as discussed in section 4.1.

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Mod	Comments	$\langle m_\nu \rangle$ [meV]	$\delta^l_{\rm CP}$ [rad]	$\sin^2\theta^l_{23}$	m_0 [meV]	M_3 [GeV]	M_2 [GeV]	M_1 [GeV]	$\chi^2_{\rm min}$
MN	no RGE, NH	0.35	0.7	0.406	3.03	$5.5{\times}10^{12}$	$7.2{\times}10^{11}$	$1.5{\times}10^{10}$	1.10
MN	RGE, NH	0.49	6.0	0.346	2.40	$3.6{\times}10^{12}$	$2.0{\times}10^{11}$	$1.2{\times}10^{11}$	23.0
MS	no RGE, NH	0.38	0.27	0.387	2.58	$3.9{\times}10^{12}$	$7.2{\times}10^{11}$	$1.6{\times}10^{10}$	9.41
MS	RGE, NH	0.44	2.8	0.410	6.83	$1.1{\times}10^{12}$	$5.7{\times}10^{10}$	$1.5{\times}10^{10}$	3.29
FN	no RGE, NH	4.96	1.7	0.410	8.8	$1.9{\times}10^{13}$	$2.8{\times}10^{12}$	$2.2{\times}10^{10}$	$6.6{\times}10^{-5}$
FN	RGE, NH	2.87	5.0	0.410	1.54	$9.9{\times}10^{14}$	$7.3{\times}10^{13}$	$1.2{\times}10^{13}$	11.2
FS	no RGE, NH	0.75	0.5	0.410	1.16	$1.5{\times}10^{13}$	$5.3{\times}10^{11}$	$5.7{\times}10^{10}$	$9.0{\times}10^{-10}$
FS	RGE, NH	0.78	5.4	0.410	3.17	$4.2{\times}10^{13}$	$4.9{\times}10^{11}$	$4.9{\times}10^{11}$	$6.9{\times}10^{-6}$
FN	no RGE, IH	35.37	5.4	0.590	35.85	$2.2{\times}10^{13}$	$4.9{\times}10^{12}$	$9.2{\times}10^{11}$	$2.5{\times}10^{-4}$
FN	RGE, IH	35.52	4.7	0.590	30.24	$1.1{\times}10^{13}$	$3.5{\times}10^{12}$	$5.5{\times}10^{11}$	13.3
FS	no RGE, IH	44.21	0.3	0.590	6.27	$1.2{\times}10^{13}$	$4.2{\times}10^{11}$	$3.5{\times}10^{7}$	$3.9{\times}10^{-8}$
FS	RGE, IH	24.22	3.6	0.590	11.97	$1.2{\times}10^{13}$	$3.1{\times}10^{11}$	$2.0{\times}10^{3}$	0.602

Table 13. Model predictions for effective $0\nu\beta\beta$ mass $\langle m_\nu \rangle$, leptonic CP violation $\delta^l_{\rm CP}$, atmospheric neutrino mixing parameter $\sin^2\theta^l_{23}$, lightest neutrino mass m_0 ($m_0 = m_1$ for NH and $m_0 = m_3$ for IH), and masses of heavy neutrinos M_i. For the SUSY models (MS, FS) the predictions with $\tan\beta = 50$ are shown which do not differ significantly from predictions with other values of $\tan\beta$. Models MN and MS have Higgs representations $10_H + \overline{126}_H$, models FN and FS have Higgs representations $10_H + 120_H + \overline{126}_H$. Models not included in this table do not give a good fit.

As can be seen in table 13, the values of essentially all parameters depend strongly on whether RG effects are taken into account or not. This shows quite strongly the necessity to include them.

5 Conclusions

In general, Grand Unified Theories and in particular models based on SO(10) offer intriguing frameworks to find an answer to the question of the origin of fermion masses and mixings. The various Yukawa coupling matrices of the fermions are related, and fits of those relations offer tests of the validity of the models. In this work we analyzed renormalizable SO(10) models based on $10_H + \overline{126}_H$, $120_H + \overline{126}_H$, and $10_H + 120_H + \overline{126}_H$ Higgs representations, assuming type I seesaw dominance. We considered non-supersymmetric as well as supersymmetric models with different values of $\tan\beta$. More model-dependent effects of intermediate scales were neglected in our study, possibly influencing the results.

In non-SUSY models there is a connection between the RGE of fermion parameters and the RGE of the Higgs quartic coupling, which is especially important for the top-quark Yukawa coupling. From the Higgs quartic coupling and the Yukawa couplings of fermions at $M_{\rm GUT}$, the quartic coupling and hence the Higgs mass gets determined at low energies as well. Therefore we included the Higgs mass into our list of observables, which has not been done in the literature so far. Through their RGE interplay, the somewhat low mass of the Higgs leads to a preference for a low top-quark mass, as discussed in section 4.1 in detail.

Further, we performed a complete 1-loop RGE from high scale, where the GUT relations for the various Yukawa matrices are valid, to the weak scale, at which the experimental data is available. This is a more consistent procedure than doing it the other way around. In addition, we treated right-handed neutrinos during RGE correctly and integrated them out one by one at their respective energy scales.

Finally, we gave the model predictions for several as yet unmeasured observables. These are the effective mass $\langle m_\nu \rangle$ relevant for neutrinoless double beta decay, the leptonic CP violating phase, $\delta^l_{\rm CP}$, the mass of the lightest neutrino, and, for completeness, the masses of the heavy neutrinos.

The results of our analysis are as follows:

- We showed that it is possible to fit the minimal setups MN and MS[8] (both with $10_H + \overline{126}_H$ Higgs representations responsible for fermion mass generation) in the case of the normal neutrino mass hierarchy, while both the non-SUSY (MN) and the SUSY (MS) cases do not work with the inverted hierarchy. The alternative minimal model $(120_H + \overline{126}_H)$ generates only very large $\chi^2_{\rm min}$-values, and is excluded for both possibilities of the neutrino mass hierarchy, as well as for the SUSY and non-SUSY cases. In contrast, models FN and FS $(10_H + 120_H + \overline{126}_H)$ have been shown to be able to reproduce both the normal and inverted hierarchy very well.

- For the non-SUSY models (MN and FN) we showed that fitting the Higgs mass leads to severe tensions for the top-quark mass, which is preferred to be more than $3\,\sigma$ smaller than the experimental value. For model FN this is the only observable that cannot be fitted close to its experimental value, while for model MN also $\sin^2\theta^l_{23}$ deviates significantly, i.e. it is about $2\,\sigma$ smaller than its experimentally measured value. The model sensitively depends on the value of $\sin^2\theta^l_{23}$.

- Regarding the impact of the Higgs mass, we have fitted the models also with the lower top-quark mass that is extracted from the total cross section in the top-quark pair production, which has been argued to be more consistent and theoretically more rigorous. As expected, with this lower value of m_t the fit improves considerably.

- An important conclusion is that predictions for the unknown parameters $\langle m_\nu \rangle$, $\delta^l_{\rm CP}$, θ^l_{23}, as well as light and heavy neutrino masses, and the value of the χ^2-minimum, depend on whether RGE is included or not. Thus we emphasize again the importance of inclusion of RGE when fitting SO(10) models defined at high energy scales.

We want to give a few comments on which questions remain open and could be addressed in the future. First of all, in our CPU-intensive analysis we neglected intermediate scales in the breaking scheme of the SO(10) GUT, as well as related gauge unification aspects. Moreover, the list of models we considered is not exhaustive, so one could analyze further models and compare analyses done with and without RGE. The models we considered either could or could not fit the data, irrespective of considering RGE or not.

[8] We remind the reader that model names containing the letter "S" refer to supersymmetric models, while those with "N" refer to non-supersymmetric models.

However, there may well be models where inclusion of RGE makes a difference between considering a model as viable or not. Since we restricted our analysis to the type I seesaw case it would be interesting to consider models in which either type II seesaw dominates or type I and type II seesaw contributions to neutrino mass are of equal order of magnitude. Further, the Yukawa sector is the most unsatisfactory part of gauge theories, as it comes along with a huge number of arbitrary parameters. Aspects such as Yukawa unification or the assumption of certain textures in the Yukawa couplings could be subject of future studies. This will help in unveiling structures in the Yukawa sector and provide hints to possible fundamental mechanisms governing the Yukawa structure of SO(10) gauge theories. Finally, the baryon asymmetry of the Universe as generated by thermal leptogenesis could be included as an additional observable in the fits. We leave these modifications and additions to future studies.

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A Best-fit parameters

A.1 Minimal models with $\mathbf{10}_H + \overline{\mathbf{126}}_H$

Since in these models only the normal neutrino mass hierarchy is viable we give the best-fit parameters only for that case.

MN, no RGE:

$$\begin{aligned} r &= 68.9624, \quad s = 0.370726 + 0.063044i, \quad r_R = 3.014 \times 10^{-16}\ \mathrm{GeV}^{-1} \\ H &= \begin{pmatrix} 1.22387{\times}10^{-6} & 0 & 0 \\ 0 & 5.92428{\times}10^{-5} & 0 \\ 0 & 0 & 6.29473{\times}10^{-3} \end{pmatrix} \\ F &= \begin{pmatrix} -2.95102{\times}10^{-6}{-}3.48291{\times}10^{-6}i & 1.27484{\times}10^{-5}{-}7.53714{\times}10^{-8}i & 1.07772{\times}10^{-4}{+}6.02931{\times}10^{-5}i \\ 1.27484{\times}10^{-5}{-}7.53714{\times}10^{-8}i & -1.538{\times}10^{-4}{+}6.75236{\times}10^{-5}i & -2.67281{\times}10^{-4}{+}2.48978{\times}10^{-4}i \\ 1.07772{\times}10^{-4}{+}6.02931{\times}10^{-5}i & -2.67281{\times}10^{-4}{+}2.48978{\times}10^{-4}i & -7.38503{\times}10^{-4}{-}1.44559{\times}10^{-3}i \end{pmatrix} \end{aligned} \tag{A.1}$$

MN, RGE:

$$\begin{aligned} r &= -63.9043, \quad s = 0.409807 - 0.0420522i, \quad r_R = 3.39236 \times 10^{-16}\ \mathrm{GeV}^{-1} \\ H &= \begin{pmatrix} 1.15249{\times}10^{-6} & 0 & 0 \\ 0 & 6.71983{\times}10^{-5} & 0 \\ 0 & 0 & 6.70159{\times}10^{-3} \end{pmatrix} \\ F &= \begin{pmatrix} -2.25817{\times}10^{-6}{+}7.40559{\times}10^{-6}i & 1.22057{\times}10^{-5}{-}1.39062{\times}10^{-5}i & -1.49653{\times}10^{-4}{+}8.30809{\times}10^{-5}i \\ 1.22057{\times}10^{-5}{-}1.39062{\times}10^{-5}i & -2.06635{\times}10^{-4}{-}1.34686{\times}10^{-5}i & 3.76355{\times}10^{-4}{+}2.15049{\times}10^{-4}i \\ -1.49653{\times}10^{-4}{+}8.30809{\times}10^{-5}i & 3.76355{\times}10^{-4}{+}2.15049{\times}10^{-4}i & -7.01333{\times}10^{-4}{-}7.53673{\times}10^{-4}i \end{pmatrix} \end{aligned} \tag{A.2}$$

MS, $\tan\beta = 50$, no RGE:

$$
\begin{aligned}
r &= 3.21051, \quad r_R = 1.46012 \times 10^{-16}\ \mathrm{GeV}^{-1}, \quad s = 0.34792 + 0.0110973i \\
H &= \begin{pmatrix} 1.83386{\times}10^{-5} & 0 & 0 \\ 0 & 1.11953{\times}10^{-3} & 0 \\ 0 & 0 & 0.1727 \end{pmatrix} \\
F &= \begin{pmatrix} -5.39376{\times}10^{-5}-6.82495{\times}10^{-5}i & 2.50369{\times}10^{-4}-4.11368{\times}10^{-5}i & 2.43574{\times}10^{-3}+1.41867{\times}10^{-3}i \\ 2.50369{\times}10^{-4}-4.11368{\times}10^{-5}i & -2.98376{\times}10^{-3}+1.03456{\times}10^{-3}i & -6.4605{\times}10^{-3}+6.2383{\times}10^{-3}i \\ 2.43574{\times}10^{-3}+1.41867{\times}10^{-3}i & -6.4605{\times}10^{-3}+6.2383{\times}10^{-3}i & -0.0106126-0.0257057i \end{pmatrix}
\end{aligned} \tag{A.3}
$$

MS, $\tan\beta = 50$, RGE:

$$
\begin{aligned}
r &= 1.87979, \quad r_R = -1.09758 \times 10^{-15}\ \mathrm{GeV}^{-1}, \quad s = 0.245295 + 0.0935775i \\
H &= \begin{pmatrix} 3.61945{\times}10^{-5} & 0 & 0 \\ 0 & 2.77898{\times}10^{-3} & 0 \\ 0 & 0 & 0.627274 \end{pmatrix} \\
F &= \begin{pmatrix} 4.13796{\times}10^{-6}+8.08833{\times}10^{-6}i & 5.12296{\times}10^{-4}-5.07815{\times}10^{-4}i & -1.66274{\times}10^{-3}-2.47433{\times}10^{-3}i \\ 5.12296{\times}10^{-4}-5.07815{\times}10^{-4}i & -8.8214{\times}10^{-3}-5.30048{\times}10^{-5}i & -0.0170155-0.019725i \\ -1.66274{\times}10^{-3}-2.47433{\times}10^{-3}i & -0.0170155-0.019725i & 4.46{\times}10^{-3}-0.0583555i \end{pmatrix}
\end{aligned} \tag{A.4}
$$

MS, $\tan\beta = 38$, no RGE:

$$
\begin{aligned}
r &= 3.43466, \quad r_R = 1.66182 \times 10^{-16}\ \mathrm{GeV}^{-1}, \quad s = 0.347871 + 7.81372 \times 10^{-3}i \\
H &= \begin{pmatrix} 1.969{\times}10^{-5} & 0 & 0 \\ 0 & 1.12891{\times}10^{-3} & 0 \\ 0 & 0 & 0.16146 \end{pmatrix} \\
F &= \begin{pmatrix} -5.95751{\times}10^{-5}-6.59382{\times}10^{-5}i & -2.40742{\times}10^{-4}+3.91345{\times}10^{-5}i & -2.42662{\times}10^{-3}-1.32616{\times}10^{-3}i \\ -2.40742{\times}10^{-4}+3.91345{\times}10^{-5}i & -3.05078{\times}10^{-3}+9.77839{\times}10^{-4}i & -6.40902{\times}10^{-3}+5.965{\times}10^{-3}i \\ -2.42662{\times}10^{-3}-1.32616{\times}10^{-3}i & -6.40902{\times}10^{-3}+5.965{\times}10^{-3}i & -9.73364{\times}10^{-3}-0.0241596i \end{pmatrix}
\end{aligned} \tag{A.5}
$$

MS, $\tan\beta = 38$, RGE:

$$
\begin{aligned}
r &= 2.96553, \quad r_R = -7.88987 \times 10^{-16}\ \mathrm{GeV}^{-1}, \quad s = 0.246797 + 0.0669722i \\
H &= \begin{pmatrix} 2.24626{\times}10^{-5} & 0 & 0 \\ 0 & 1.63169{\times}10^{-3} & 0 \\ 0 & 0 & 0.312163 \end{pmatrix} \\
F &= \begin{pmatrix} 4.70045{\times}10^{-6}+5.21251{\times}10^{-6}i & 3.33325{\times}10^{-4}-2.99098{\times}10^{-4}i & -7.94881{\times}10^{-4}-1.5865{\times}10^{-3}i \\ 3.33325{\times}10^{-4}-2.99098{\times}10^{-4}i & -5.46865{\times}10^{-3}-5.53341{\times}10^{-4}i & -8.40926{\times}10^{-3}-0.0113299i \\ -7.94881{\times}10^{-4}-1.5865{\times}10^{-3}i & -8.40926{\times}10^{-3}-0.0113299i & -7.01273{\times}10^{-4}-0.0414157i \end{pmatrix}
\end{aligned} \tag{A.6}
$$

MS, $\tan\beta = 10$, no RGE:

$$
\begin{aligned}
r &= 11.9008, \quad r_R = 1.66108 \times 10^{-16}\ \mathrm{GeV}^{-1}, \quad s = 0.352923 + 9.55355 \times 10^{-3}i \\
H &= \begin{pmatrix} 6.75094{\times}10^{-6} & 0 & 0 \\ 0 & 3.53643{\times}10^{-4} & 0 \\ 0 & 0 & 0.0455808 \end{pmatrix} \\
F &= \begin{pmatrix} -2.04956{\times}10^{-5}-1.99197{\times}10^{-5}i & 7.07769{\times}10^{-5}-8.71429{\times}10^{-6}i & -7.40964{\times}10^{-4}-3.79993{\times}10^{-4}i \\ 7.07769{\times}10^{-5}-8.71429{\times}10^{-6}i & -9.71448{\times}10^{-4}+2.86615{\times}10^{-4}i & 1.95589{\times}10^{-3}-1.69133{\times}10^{-3}i \\ -7.40964{\times}10^{-4}-3.79993{\times}10^{-4}i & 1.95589{\times}10^{-3}-1.69133{\times}10^{-3}i & -2.77458{\times}10^{-3}-7.05915{\times}10^{-3}i \end{pmatrix}
\end{aligned} \tag{A.7}
$$

MS, $\tan\beta = 10$, RGE:

$$
\begin{aligned}
r &= 13.1538, \quad r_R = -6.17321 \times 10^{-16}\ \mathrm{GeV}^{-1}, \quad s = 0.244325 + 0.0495071i \\
H &= \begin{pmatrix} 5.12266{\times}10^{-6} & 0 & 0 \\ 0 & 3.60146{\times}10^{-4} & 0 \\ 0 & 0 & 0.0622718 \end{pmatrix} \\
F &= \begin{pmatrix} 1.38859{\times}10^{-6}+1.34952{\times}10^{-6}i & -7.94633{\times}10^{-5}+6.55427{\times}10^{-5}i & -1.43753{\times}10^{-4}-3.70193{\times}10^{-4}i \\ -7.94633{\times}10^{-5}+6.55427{\times}10^{-5}i & -1.25786{\times}10^{-3}-2.04911{\times}10^{-4}i & 1.64299{\times}10^{-3}+2.45787{\times}10^{-3}i \\ -1.43753{\times}10^{-4}-3.70193{\times}10^{-4}i & 1.64299{\times}10^{-3}+2.45787{\times}10^{-3}i & -3.6772{\times}10^{-4}-0.0102974i \end{pmatrix}
\end{aligned} \tag{A.8}
$$

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A.2 Models with $\mathbf{10}_H + \overline{\mathbf{126}}_H + \mathbf{120}_H$

A.2.1 Normal neutrino mass hierarchy

FN, no RGE:

$$
\begin{aligned}
r &= 67.1992, \qquad r_R = -1.54145 \times 10^{-16}\ \mathrm{GeV}^{-1}, \quad s = -2.0155, \quad t_l = 1.09375,\\
t_u &= -0.973721, \quad t_D = -4.11394\\
H &= \begin{pmatrix} -1.44349\times10^{-3} & 0 & 0 \\ 0 & -2.12083\times10^{-4} & 0 \\ 0 & 0 & 8.38498\times10^{-6} \end{pmatrix}\\
F &= \begin{pmatrix} -3.52616\times10^{-3} & -1.87525\times10^{-5} & -3.01471\times10^{-5} \\ -1.87525\times10^{-5} & -4.43985\times10^{-4} & -2.33814\times10^{-5} \\ -3.01471\times10^{-5} & -2.33814\times10^{-5} & 1.94067\times10^{-6} \end{pmatrix}\\
G &= \begin{pmatrix} 0 & 1.98673\times10^{-3} & 1.36719\times10^{-4} \\ -1.98673\times10^{-3} & 0 & -1.67619\times10^{-5} \\ -1.36719\times10^{-4} & 1.67619\times10^{-5} & 0 \end{pmatrix}
\end{aligned}
\tag{A.9}
$$

FN, RGE:

$$
\begin{aligned}
r &= 63.4279, \qquad r_R = 1.08547 \times 10^{-18}\ \mathrm{GeV}^{-1}, \quad s = 0.450438, \quad t_l = 3.60171,\\
t_u &= -0.0648445, \quad t_D = -52.3076\\
H &= \begin{pmatrix} 4.1021\times10^{-6} & 0 & 0 \\ 0 & 1.29554\times10^{-4} & 0 \\ 0 & 0 & 6.78427\times10^{-3} \end{pmatrix}\\
F &= \begin{pmatrix} -7.62731\times10^{-6} & 7.68715\times10^{-6} & 3.06531\times10^{-5} \\ 7.68715\times10^{-6} & -2.21886\times10^{-4} & 5.05238\times10^{-4} \\ 3.06531\times10^{-5} & 5.05238\times10^{-4} & -7.89186\times10^{-4} \end{pmatrix}\\
G &= \begin{pmatrix} 0 & 3.65588\times10^{-5} & -2.26729\times10^{-5} \\ -3.65588\times10^{-5} & 0 & 1.19187\times10^{-5} \\ 2.26729\times10^{-5} & -1.19187\times10^{-5} & 0 \end{pmatrix}
\end{aligned}
\tag{A.10}
$$

FS, $\tan\beta = 50$, no RGE:

$$
\begin{aligned}
r &= -0.209965, \quad r_R = -2.06476 \times 10^{-16}\ \mathrm{GeV}^{-1}, \quad s = -3.15082, \quad t_l = 164.558\\
t_u &= -18.4887, \qquad t_D = 1.98859\\
H &= \begin{pmatrix} -0.0246111 & 0 & 0 \\ 0 & 2.11922 & 0 \\ 0 & 0 & 1.16561\times10^{-3} \end{pmatrix}\\
F &= \begin{pmatrix} -6.79587\times10^{-3} & 0.0141982 & 9.32493\times10^{-6} \\ 0.0141982 & -0.149691 & 4.06145\times10^{-3} \\ 9.32493\times10^{-6} & 4.06145\times10^{-3} & 4.49951\times10^{-4} \end{pmatrix}\\
G &= \begin{pmatrix} 0 & 3.58503\times10^{-3} & 4.93814\times10^{-5} \\ -3.58503\times10^{-3} & 0 & 1.87896\times10^{-4} \\ -4.93814\times10^{-5} & -1.87896\times10^{-4} & 0 \end{pmatrix}
\end{aligned}
\tag{A.11}
$$

FS, $\tan\beta = 50$, RGE:

$$
\begin{aligned}
r &= -2.26973, \quad r_R = -1.40822 \times 10^{-16}\ \mathrm{GeV}^{-1}, \quad s = 0.528664, \quad t_l = -1.31887,\\
t_u &= 0.598706, \qquad t_D = -0.206913\\
H &= \begin{pmatrix} 1.49174\times10^{-4} & 0 & 0 \\ 0 & 4.89692\times10^{-3} & 0 \\ 0 & 0 & 0.327351 \end{pmatrix}\\
F &= \begin{pmatrix} 9.74987\times10^{-4} & -3.17774\times10^{-3} & -6.38013\times10^{-3} \\ -3.17774\times10^{-3} & -5.84881\times10^{-4} & -8.33647\times10^{-3} \\ -6.38013\times10^{-3} & -8.33647\times10^{-3} & 0.302701 \end{pmatrix}\\
G &= \begin{pmatrix} 0 & -7.92375\times10^{-4} & 0.0323128 \\ 7.92375\times10^{-4} & 0 & -0.084999 \\ -0.0323128 & 0.084999 & 0 \end{pmatrix}
\end{aligned}
\tag{A.12}
$$

A.2.2 Inverted neutrino mass hierarchy

FN, no RGE:

$$\begin{aligned} r &= -71.6954, \quad r_R = 6.24335\times10^{-16}\ \mathrm{GeV}^{-1}, \quad s = 0.710962, \quad t_l = -11.9888 \\ t_u &= -0.049547, \quad t_D = 21.5488 \\ H &= \begin{pmatrix} 6.47261\times10^{-3} & 0 & 0 \\ 0 & 2.64518\times10^{-4} & 0 \\ 0 & 0 & 4.10329\times10^{-5} \end{pmatrix} \\ F &= \begin{pmatrix} -8.37936\times10^{-4} & -8.11918\times10^{-4} & 5.65936\times10^{-6} \\ -8.11918\times10^{-4} & -2.65901\times10^{-4} & 1.47332\times10^{-6} \\ 5.65936\times10^{-6} & 1.47332\times10^{-6} & -5.74189\times10^{-5} \end{pmatrix} \\ G &= \begin{pmatrix} 0 & -9.67226\times10^{-8} & -2.30032\times10^{-5} \\ 9.67226\times10^{-8} & 0 & -2.9926\times10^{-5} \\ 2.30032\times10^{-5} & 2.9926\times10^{-5} & 0 \end{pmatrix} \end{aligned} \tag{A.13}$$

FN, RGE:

$$\begin{aligned} r &= -65.5547, \quad r_R = 1.15340\times10^{-16}\ \mathrm{GeV}^{-1}, \quad s = 0.666694, \quad t_l = -9.90739, \\ t_u &= -0.0535721, \quad t_D = 15.6874 \\ H &= \begin{pmatrix} 4.28045\times10^{-5} & 0 & 0 \\ 0 & 2.67413\times10^{-4} & 0 \\ 0 & 0 & 6.59444\times10^{-3} \end{pmatrix} \\ F &= \begin{pmatrix} -6.35416\times10^{-5} & 2.97446\times10^{-6} & -2.10727\times10^{-6} \\ 2.97446\times10^{-6} & -2.90747\times10^{-4} & -8.51248\times10^{-4} \\ -2.10727\times10^{-6} & -8.51248\times10^{-4} & -6.38279\times10^{-4} \end{pmatrix} \\ G &= \begin{pmatrix} 0 & 3.89049\times10^{-5} & 3.20285\times10^{-5} \\ -3.89049\times10^{-5} & 0 & -7.38437\times10^{-7} \\ -3.20285\times10^{-5} & 7.38437\times10^{-7} & 0 \end{pmatrix} \end{aligned} \tag{A.14}$$

FS, $\tan\beta = 50$, no RGE:

$$\begin{aligned} r &= 4.05193, \quad r_R = 2.62768\times10^{-16}\ \mathrm{GeV}^{-1}, \quad s = 0.146437, \quad t_l = -1.88674 \\ t_u &= -0.143497, \quad t_D = -5.43533\times10^{-3} \\ H &= \begin{pmatrix} -5.64699\times10^{-5} & 0 & 0 \\ 0 & -0.111019 & 0 \\ 0 & 0 & -9.28316\times10^{-4} \end{pmatrix} \\ F &= \begin{pmatrix} 8.11456\times10^{-7} & 1.29532\times10^{-4} & 3.93343\times10^{-5} \\ 1.29532\times10^{-4} & -0.159366 & 0.0133342 \\ 3.93343\times10^{-5} & 0.0133342 & 4.41386\times10^{-3} \end{pmatrix} \\ G &= \begin{pmatrix} 0 & -3.1015\times10^{-3} & -7.83212\times10^{-4} \\ 3.1015\times10^{-3} & 0 & 1.02167\times10^{-3} \\ 7.83212\times10^{-4} & -1.02167\times10^{-3} & 0 \end{pmatrix} \end{aligned} \tag{A.15}$$

FS, $\tan\beta = 50$, RGE:

$$\begin{aligned} r &= -2.76923, \quad r_R = 5.24649\times10^{-16}\ \mathrm{GeV}^{-1}, \quad s = 0.239831, \quad t_l = -1.23885 \\ t_u &= -0.0166744, \quad t_D = 1.12799\times10^{-6} \\ H &= \begin{pmatrix} 0.327384 & 0 & 0 \\ 0 & -5.97675\times10^{-7} & 0 \\ 0 & 0 & 2.39637\times10^{-3} \end{pmatrix} \\ F &= \begin{pmatrix} 0.309553 & 3.84441\times10^{-6} & -0.0201878 \\ 3.84441\times10^{-6} & -3.51744\times10^{-10} & 1.65759\times10^{-6} \\ -0.0201878 & 1.65759\times10^{-6} & -6.76014\times10^{-3} \end{pmatrix} \\ G &= \begin{pmatrix} 0 & 1.98713\times10^{-3} & 0.0152033 \\ -1.98713\times10^{-3} & 0 & -1.47992\times10^{-3} \\ -0.0152033 & 1.47992\times10^{-3} & 0 \end{pmatrix} \end{aligned} \tag{A.16}$$

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B Beta-functions for RG evolution

To calculate the RGE of observables, RG equations for all parameters of the model under consideration have to be solved simultaneously. Here we summarize 1-loop RG equations for the SM and the MSSM extended by an arbitrary number of right-handed singlet neutrinos. The notation is as in refs. [18, 76]. In particular, we denote a quantity between the nth and $(n+1)$th mass threshold with a superscript (n). For further details including 2-loop beta-functions, we refer the reader to [8, 76, 94–97].

The beta-functions of the gauge couplings are not affected by the additional singlets at 1-loop order. They are given by

$$16\pi^2\, \beta_{g_A} \equiv 16\pi^2\, \mu \frac{g_A}{\mu} = b_A\, g_A^3\,, \tag{B.1}$$

with $(b_{\mathrm{SU(3)_C}}, b_{\mathrm{SU(2)_L}}, b_{\mathrm{U(1)_Y}}) = (-7, -\frac{19}{6}, \frac{41}{10})$ in the SM and $(-3, 1, \frac{33}{5})$ in the MSSM. For the $\mathrm{U(1)_Y}$ charge we use GUT normalization.

B.1 Beta-functions in the extended SM

The β-functions governing RG evolution in the SM extended by singlet neutrinos are given by [18, 76, 98]

$$\begin{aligned}
16\pi^2 \overset{(n)}{\beta}_\kappa &= -\frac{3}{2}(Y_e^\dagger Y_e)^T \overset{(n)}{\kappa} - \frac{3}{2}\overset{(n)}{\kappa}(Y_e^\dagger Y_e) + \frac{1}{2}\big(\overset{(n)}{Y}{}_D^\dagger \overset{(n)}{Y}_D\big)^T \overset{(n)}{\kappa} + \frac{1}{2}\overset{(n)}{\kappa}\big(\overset{(n)}{Y}{}_D^\dagger \overset{(n)}{Y}_D\big) \\
&\quad + 2\ \mathrm{Tr}(Y_e^\dagger Y_e)\overset{(n)}{\kappa} + 2\ \mathrm{Tr}\big(\overset{(n)}{Y}{}_D^\dagger \overset{(n)}{Y}_D\big)\overset{(n)}{\kappa} + 6\ \mathrm{Tr}(Y_u^\dagger Y_u)\overset{(n)}{\kappa} \\
&\quad + 6\ \mathrm{Tr}(Y_d^\dagger Y_d)\overset{(n)}{\kappa} - 3g_2^2\overset{(n)}{\kappa} + \lambda\overset{(n)}{\kappa}\,,
\end{aligned} \tag{B.2a}$$

$$16\pi^2 \overset{(n)}{\beta}_M = \big(\overset{(n)}{Y}_D \overset{(n)}{Y}{}_D^\dagger\big)\overset{(n)}{M} + \overset{(n)}{M}\big(\overset{(n)}{Y}_D \overset{(n)}{Y}{}_D^\dagger\big)^T\,, \tag{B.2b}$$

$$\begin{aligned}
16\pi^2 \overset{(n)}{\beta}_{Y_D} &= \overset{(n)}{Y}_D\left\{\frac{3}{2}\big(\overset{(n)}{Y}{}_D^\dagger \overset{(n)}{Y}_D\big) - \frac{3}{2}(Y_e^\dagger Y_e) + \mathrm{Tr}\big(\overset{(n)}{Y}{}_D^\dagger \overset{(n)}{Y}_D\big) + \mathrm{Tr}(Y_e^\dagger Y_e)\right. \\
&\qquad \left. + 3\ \mathrm{Tr}(Y_u^\dagger Y_u) + 3\ \mathrm{Tr}(Y_d^\dagger Y_d) - \frac{9}{20}g_1^2 - \frac{9}{4}g_2^2\right\},
\end{aligned} \tag{B.2c}$$

$$\begin{aligned}
16\pi^2 \overset{(n)}{\beta}_{Y_e} &= Y_e\left\{\frac{3}{2}Y_e^\dagger Y_e - \frac{3}{2}\overset{(n)}{Y}{}_D^\dagger \overset{(n)}{Y}_D - \frac{9}{4}g_1^2 - \frac{9}{4}g_2^2\right. \\
&\qquad \left. + \mathrm{Tr}\left[Y_e^\dagger Y_e + \overset{(n)}{Y}{}_D^\dagger \overset{(n)}{Y}_D + 3\,Y_d^\dagger Y_d\ + 3\,Y_u^\dagger Y_u\right]\right\},
\end{aligned} \tag{B.2d}$$

$$\begin{aligned}
16\pi^2 \overset{(n)}{\beta}_{Y_d} &= Y_d\left\{\frac{3}{2}Y_d^\dagger Y_d - \frac{3}{2}\,Y_u^\dagger Y_u - \frac{1}{4}g_1^2 - \frac{9}{4}g_2^2 - 8\,g_3^2\right. \\
&\qquad \left. + \mathrm{Tr}\left[Y_e^\dagger Y_e + \overset{(n)}{Y}{}_D^\dagger \overset{(n)}{Y}_D + 3\,Y_d^\dagger Y_d + 3\,Y_u^\dagger Y_u\right]\right\},
\end{aligned} \tag{B.2e}$$

$$\begin{aligned}
16\pi^2 \overset{(n)}{\beta}_{Y_u} &= Y_u\left\{\frac{3}{2}Y_u^\dagger Y_u - \frac{3}{2}\,Y_d^\dagger Y_d - \frac{17}{20}g_1^2 - \frac{9}{4}g_2^2 - 8\,g_3^2\right. \\
&\qquad \left. + \mathrm{Tr}\left[Y_e^\dagger Y_e + \overset{(n)}{Y}{}_D^\dagger \overset{(n)}{Y}_D + 3\,Y_d^\dagger Y_d + 3\,Y_u^\dagger Y_u\right]\right\},
\end{aligned} \tag{B.2f}$$

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$$
\begin{aligned}
16\pi^2 \overset{(n)}{\beta}_\lambda = {} & 6\,\lambda^2 - 3\,\lambda\left(3g_2^2+\frac{3}{5}g_1^2\right) + 3\,g_2^4 + \frac{3}{2}\left(\frac{3}{5}g_1^2+g_2^2\right)^2 \\
& + 4\,\lambda\ \mathrm{Tr}\left[Y_e^\dagger Y_e + \overset{(n)}{Y}{}_D^\dagger \overset{(n)}{Y}_D + 3\,Y_d^\dagger Y_d + 3\,Y_u^\dagger Y_u\right] \\
& - 8\ \mathrm{Tr}\left[Y_e^\dagger Y_e\, Y_e^\dagger Y_e + \overset{(n)}{Y}{}_D^\dagger \overset{(n)}{Y}_D\, \overset{(n)}{Y}{}_D^\dagger \overset{(n)}{Y}_D + 3\,Y_d^\dagger Y_d\, Y_d^\dagger Y_d + 3\,Y_u^\dagger Y_u\, Y_u^\dagger Y_u\right].
\end{aligned}
\tag{B.2g}
$$

Note that our convention that the Higgs self-interaction term in the Lagrangian is $-\frac{\lambda}{4}(\phi^\dagger\phi)^2$.

B.2 Beta-functions in the extended MSSM

The 1-loop beta-functions of the MSSM extended by heavy singlet neutrinos are given by [18, 76, 99]

$$
\begin{aligned}
16\pi^2 \overset{(n)}{\beta}_\kappa = {} & (Y_e^\dagger Y_e)^T \overset{(n)}{\kappa} + \overset{(n)}{\kappa}(Y_e^\dagger Y_e) + \left(\overset{(n)}{Y}{}_D^\dagger \overset{(n)}{Y}_D\right)^T \overset{(n)}{\kappa} + \overset{(n)}{\kappa}\left(\overset{(n)}{Y}{}_D^\dagger \overset{(n)}{Y}_D\right) \\
& + 2\,\mathrm{Tr}\left(\overset{(n)}{Y}{}_D^\dagger \overset{(n)}{Y}_D\right)\overset{(n)}{\kappa} + 6\,\mathrm{Tr}(Y_u^\dagger Y_u)\overset{(n)}{\kappa} - \frac{6}{5}g_1^2 \overset{(n)}{\kappa} - 6g_2^2 \overset{(n)}{\kappa}\,,
\end{aligned}
\tag{B.3a}
$$

$$
16\pi^2 \overset{(n)}{\beta}_M = 2\left(\overset{(n)}{Y}_D \overset{(n)}{Y}{}_D^\dagger\right)\overset{(n)}{M} + 2\,\overset{(n)}{M}\left(\overset{(n)}{Y}_D \overset{(n)}{Y}{}_D^\dagger\right)^T, \tag{B.3b}
$$

$$
16\pi^2 \overset{(n)}{\beta}_{Y_D} = \overset{(n)}{Y}_D \left\{3\overset{(n)}{Y}{}_D^\dagger \overset{(n)}{Y}_D + Y_e^\dagger Y_e + \mathrm{Tr}\left(\overset{(n)}{Y}{}_D^\dagger \overset{(n)}{Y}_D\right) + 3\,\mathrm{Tr}(Y_u^\dagger Y_u) - \frac{3}{5}g_1^2 - 3g_2^2\right\}, \tag{B.3c}
$$

$$
16\pi^2 \overset{(n)}{\beta}_{Y_d} = Y_d \left\{3Y_d^\dagger Y_d + Y_u^\dagger Y_u + 3\,\mathrm{Tr}(Y_d^\dagger Y_d) + \mathrm{Tr}(Y_e^\dagger Y_e) - \frac{7}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2\right\}, \tag{B.3d}
$$

$$
16\pi^2 \overset{(n)}{\beta}_{Y_u} = Y_u \left\{Y_d^\dagger Y_d + 3Y_u^\dagger Y_u + \mathrm{Tr}(\overset{(n)}{Y}{}_D^\dagger \overset{(n)}{Y}_D) + 3\,\mathrm{Tr}(Y_u^\dagger Y_u) - \frac{13}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2\right\}, \tag{B.3e}
$$

$$
16\pi^2 \overset{(n)}{\beta}_{Y_e} = Y_e \left\{3Y_e^\dagger Y_e + \overset{(n)}{Y}{}_D^\dagger \overset{(n)}{Y}_D + 3\,\mathrm{Tr}(Y_d^\dagger Y_d) + \mathrm{Tr}(Y_e^\dagger Y_e) - \frac{9}{5}g_1^2 - 3g_2^2\right\}. \tag{B.3f}
$$

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