Published for SISSA by Springer

Received: August 1, 2011
Accepted: August 20, 2011
Published: September 8, 2011

Index computation for 3d Chern-Simons matter theory: test of Seiberg-like duality

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Abstract: We work out the superconformal index for $\mathcal{N}=2$ supersymmetric Chern-Simons matter theories exhibiting Seiberg-like dualities proposed by Giveon and Kutasov. We consider $\mathrm{U}(N)/\,\mathrm{Sp}(2N)/\,\mathrm{O}(N)$ gauge theories of QCD type and find the perfect agreements for proposed dual pairs.

Keywords: Supersymmetry and Duality, Supersymmetric gauge theory, Chern-Simons Theories

ArXiv ePrint: 1107.4942[hep-th]

JHEP09(2011)037

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doi:10.1007/JHEP09(2011)037

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1 Introduction

Recently there have been tremendous progress in understanding of three-dimensional superconformal field theories (SCFT). The key observation was made by J. Schwarz that such theories could be described as Chern-Simons matter theories [1]. This led to the important development in AdS_4/CFT_3 correspondence for the supersymmetric theories with $\mathcal{N} \geq 4$ [2–11]. However the same insight can be used to understand the SCFT with $\mathcal{N} = 2$ supersymmetry [12]. For these theories, there have been intense studies in the context of AdS_4/CFT_3 correspondence [13–18]. We are interested in a subset of such theories, i.e., three-dimensional supersymmetric QCD with Chern-Simons couplings. In the IR limit, the Yang-Mills kinetic term is irrelevant and we are left with $\mathcal{N} = 2$ Chern-Simons matter theories. $\mathcal{N} = 1$ supersymmetric QCD in four-dimensions was intensively studied in relation to Seiberg duality [19]. Down to three dimensions there's an analogue of the Seiberg dualities in Chern-Simons matter theories with $\mathcal{N} = 3, \mathcal{N} = 2$ supersymmetry [20, 21]. Some of the evidences were presented in [22, 23], evaluating the partition function on S^3. The purpose of the paper is to give additional evidences by working out the superconformal index for dual pairs with $\mathcal{N} = 2$ supersymmetry. The index computation gives detailed information of BPS states of the SCFT of interest. Indeed the index matches perfectly and this provides a strong evidence that Seiberg-like duality holds for three-dimensional $\mathcal{N} = 2$ super Chern-Simons matter theories of QCD type. The superconformal index for QCD type theory without Chern-Simons term is computed by [24].

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The content of the paper is as follows. After introducing the essentials of superconformal index in three-dimensions, we appply this for $\mathcal{N} = 2$ $\mathrm{U}(N), \mathrm{Sp}(2N), \mathrm{O}(N)$ Chern-Simons theories with fundamental matters. It's important to have the gauge group $\mathrm{U}(N), \mathrm{O}(N)$ instead of $\mathrm{SU}(N), \mathrm{SO}(N)$ to have valid Seiberg-like dualities. In the main text, we just keep track of the energy of the state while in the appendix we turn on the chemical potentials for the flavor symmetries and redo the index computation.

2 Computation of the superconformal index

Let us discuss the general structures of the index. We consider the superconformal index for 3-d $\mathcal{N} = 2$ superconformal field theory (SCFT). Superconformal index for higher supersymmetric theory can be defined using their $\mathcal{N} = 2$ subalgebra. The bosonic subgroup of the 3-d $\mathcal{N} = 2$ superconformal algebra is $\mathrm{SO}(2,3) \times \mathrm{SO}(2)$. There are three Cartan elements denoted by ϵ, j_3 and R which come from three factors $\mathrm{SO}(2)_\epsilon \times \mathrm{SO}(3)_{j_3} \times \mathrm{SO}(2)_R$ in the bosonoic subalgebra. One can define the superconformal index for 3-d $\mathcal{N} = 2$ SCFT as follows [25],

$$I = \mathrm{Tr}(-1)^F \exp(-\beta'\{Q,S\}) x^{\epsilon+j_3} \prod_j y_j^{F_j} \tag{2.1}$$

where Q is a special supercharge with quantum numbers $\epsilon = \frac{1}{2}, j_3 = -\frac{1}{2}$ and $R = 1$ and $S = Q^\dagger$. They satisfy following anti-commutation relation,

$$\{Q,S\} = \epsilon - R - j_3 := \Delta. \tag{2.2}$$

In the index formula, the trace is taken over gauge-invariant local operators in the SCFT defined on $\mathbb{R}^{1,2}$ or over states in the SCFT on $\mathbb{R}\times S^2$. As is usual for Witten index , only BPS states satisfying the bound $\Delta = 0$ contributes to the index and the index is independent of β'. If we have additional conserved charges commuting with chosen supercharges (Q,S), we can turn on the associated chemical potentials and the index counts the number of BPS states with the specified quantum number of the conserved charges denoted by F_j in eq. (2.1).

The superconformal index is exactly caculable using localization technique [26, 27]. Following their works, the superconformal index can be written in the following form,

$$I(x) = \sum_m \int da \, \frac{1}{(\text{symmetry})} e^{-S^{(0)}_{\mathrm{CS}}} e^{ib_0(a)} y_j^{q_{0j}} x^{\epsilon_0} \exp\left[\sum_{n=1}^{\infty} \frac{1}{n} f_{\mathrm{tot}}(e^{ina}, y_j^n, x^n)\right]. \tag{2.3}$$

To take trace over Hilbert-space on S^2, we impose proper periodic boundary conditions on time direction $\mathbb{R}$. As a result, the base manifold becomes $S^1 \times S^2$. For saddle points in localization procedure, we need to turn on monopole fluxes on S^2 and holonomy along S^1. These configurations of the gauge fields are denoted by $\{m\}$ and $\{a\}$ collectively. Both variables take values in the Cartan subalgebra of G. S_0 denotes the classical action for the (monopole+holnomoy) configuration on $S^1 \times S^2$. ϵ_0 is called the Casmir energy. If the action contains the Chern-Simons terms, it gives the nonvanishing contribution,

$$S_0 = \frac{ik}{4\pi} \int \mathrm{tr}\left(A_0 \wedge dA_0 - \frac{2i}{3} A_0 \wedge A_0 \wedge A_0\right) = ik\,\mathrm{tr}(m\, a) \tag{2.4}$$

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where k is the Chern-Simons level. In (2.3), $\sum_m$ is over all integral magnetic monopoles charges, $f_{\text{tot}} = f_{\text{chiral}} + f_{\text{vector}}$ and (symmetry) = (theorderoftheWeylgroup). Each component in (2.3) is given by

$$
\begin{aligned}
S_{\text{CS}}^{(0)} &= i \sum_{\rho \in R_F} k\rho(m)\rho(a), \\
b_0(a) &= -\frac{1}{2} \sum_{\Phi} \sum_{\rho \in R_\Phi} |\rho(m)|\rho(a), \\
y_j^{q_{0j}} &= y_i^{-\frac{1}{2}\sum_\Phi \sum_{\rho\in R_\Phi} |\rho(m)| F_i(\Phi)}, \\
\epsilon_0 &= \frac{1}{2} \sum_{\Phi} (1-\Delta_\Phi) \sum_{\rho \in R_\Phi} |\rho(m)| - \frac{1}{2} \sum_{\alpha \in G} |\alpha(m)|, \\
f_{\text{chiral}}(e^{ia}, y_j, x) &= \sum_{\Phi} \sum_{\rho \in R_\Phi} \left[e^{i\rho(a)} y_j^{F_j} \frac{x^{|\rho(m)|+\Delta_\Phi}}{1-x^2} - e^{-i\rho(a)} y_j^{-F_j} \frac{x^{|\rho(m)|+2-\Delta_\Phi}}{1-x^2} \right]
\end{aligned}
\tag{2.5}
$$

where $\sum_{\rho\in R_F}$, $\sum_\Phi$ and $\sum_{\alpha\in G}$ represent the summations over all fundamental weights, all chiral multiplets, all weights and all roots, respectively. F_i are the Cartan generators acting only on the i-th Flavor. In addition, $\exp\left[\sum_{n=1}^\infty \frac{1}{n} f_{\text{vector}}(e^{ina}, x^n)\right]$ can be simplified as follows:

$$
\begin{aligned}
\exp\left[\sum_{n=1}^\infty \frac{1}{n} f_{\text{vector}}(e^{ina}, x^n)\right] &= \prod_{\alpha\in G} \exp\left[-\sum_{n=1}^\infty \frac{1}{n} e^{in\alpha(a)} x^{n|\alpha(m)|}\right] \\
&= \prod_{\alpha\in G} \exp\left[\ln\left(1 - e^{i\alpha(a)} x^{|\alpha(m)|}\right)\right] \\
&= \prod_{\alpha\in G} \left(1 - e^{i\alpha(a)} x^{|\alpha(m)|}\right).
\end{aligned}
\tag{2.6}
$$

2.1 Unitary case

We consider $\mathcal{N}=2$ $\mathrm{U}(N_c)$ gauge theory with N_f (anti)fundamental chiral muliplets $Q^a, \tilde{Q}_b$ and a Chern-Simons term at level k. It's magnetic dual is given by $\mathcal{N}=2$ $\mathrm{U}(|k|+N_f-N_c)$ gauge theory with N_f (anti)fundamental chiral multiplets $q_a, \tilde{q}^b$ and $N_f \times N_f$ matrix of singlets M_b^a with Chern-Simons term at level $-k$ and the superpotential

$$
W = M_b^a q_a \tilde{q}^b. \tag{2.7}
$$

The weights of the fundamental representation are ϵ_i where $i = 1, \cdots, N_c$, and the roots of $\mathrm{U}(N_c)$ are $\epsilon_i - \epsilon_j$ where $i, j = 1, \cdots, N_c$ and $i \neq j$. The superconformal index without the chemical potentials($y_j = 1$) is thus given by:

$$
S_{\text{CS}}^{(0)} = ik \sum_{i=1}^{N_c} a_i m_i, \tag{2.8}
$$

$$
b_0(a) = 0, \tag{2.9}
$$

$$
\epsilon_0 = \begin{cases} N_f(1-r) \sum_{i=1}^{N_c} |m_i| - \sum_{i<j}^{N_c} |m_i - m_j|, & \text{Electric} \\ N_f r \sum_{i=1}^{N_c} |m_i| - \sum_{i<j}^{N_c} |m_i - m_j|, & \text{Magnetic} \end{cases} \tag{2.10}
$$

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$$f_{\text{chiral}}(e^{ia},1,x) = \begin{cases} N_f \dfrac{x^r - x^{2-r}}{1-x^2}\left[\displaystyle\sum_{i=1}^{N_c} x^{|m_i|} 2\cos a_i\right], & \text{Electric} \\ N_f \dfrac{x^{1-r} - x^{1+r}}{1-x^2}\left[\displaystyle\sum_{i=1}^{N_c} x^{|m_i|} 2\cos a_i\right] + {N_f}^2 \dfrac{x^{2r}-x^{2-2r}}{1-x^2}, & \text{Magnetic} \end{cases} \tag{2.11}$$

$$\begin{aligned} \exp\left[\sum_{n=1}^{\infty}\frac{1}{n} f_{\text{vector}}(e^{jna}, x^n)\right] &= \prod_{i<j}^{N_c}\left(1 - e^{i(a_i-a_j)}x^{|m_i-m_j|}\right)\left(1 - e^{-i(a_i-a_j)}x^{|m_i-m_j|}\right) \\ &= \prod_{i<j}^{N_c}\left(1 - 2\cos(a_i-a_j)x^{|m_i-m_j|} + x^{2|m_i-m_j|}\right). \end{aligned} \tag{2.12}$$

Due to the flavor symmetry, one can assume that $Q^a, \tilde{Q}_b$ have the same R-charge r. Since M^a_b is quadratic in $Q, \tilde{Q}$ it has the R-charge $2r$. Since the superpotential has the dimension 2 in the IR limit, $q_a, \tilde{q}^b$ has the R-charge $1-r$.

The index formula can be expanded order by order in terms of variables p and q that are defined by

$$p = x^r, \qquad q = x^{1-r}. \tag{2.13}$$

The r dependence of the index can be restored by replacing p and q in the index formula expanded in terms of p and q by (2.13). We computed the indices of all possible dual pairs between the electric theory and the magnetic theory in the range $1 \le N_c, |k| + N_f - N_c \le 2$ with unfixed R-charge r, and confirmed the agreements up to at least $\mathcal{O}(p^{12})$ and $\mathcal{O}(q^{12})$. We list a part of the result in table 1.

Note that the index matches for arbitrary assignment of the R-charge for $Q, \tilde{Q}$. To determine the precise value of r we have to use the other method such as Z-maximization proposed by [28].

It's worthwhile to work out the gauge invariant operators of the first few lowest orders. We are working on U(N) case but similar argument can be given to other gauge groups. The easiest one is the chiral ring elements. For U(N) with N_f flavors, it is given by $Q^a_i \tilde{Q}^i_b$ where i is a gauge index running from 1 to N_c and a, b are flavor indices running from 1 to N_f. The total number of the chiral primaries is N_f^2. In the magnetic side, these are simply given by M^a_b. Due to the superpotential terms $q_a \tilde{q}^b$ turn out to be Q exact operators. The chiral ring elements contribute $+N_f^2 x^{2r}$ to the index. There are terms in the index which do not depend on R-charges such as $x^2, x^4 \cdots$. For lowest such term one can consider the operators involving fermions. The fermion operator $\psi^\dagger$ has R-charge $1-r$ and the spin $\frac{1}{2}$ as the lowest one. Thus, it gives the contribution of $x^{R+2j} = x^{2-r}$. For U(N) case, we have $Q^a \psi^\dagger_b$ or $\tilde{Q}_a \tilde{\psi}^{\dagger b}$ terms and each of which contributes $(x^r)(-x^{2-r})$ to the index. So the index get the contribution $-2N_f^2 x^2$. This explains the index for the gauge group U(2) and higher rank but for U(1) with $k = N_f = 1$ such term is missing. Thus we have to look for additional operators. For that purpose, one can consider monopole operators. For simplicity we consider U(1). One can consider the general U(N) but the resulting monopole operators will contribute to higher orders. We use the operator-state correspondence for conformal field theory and work out states on $S^2 \times R$. If we turn on

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(N_f,k,N_c)	Electric $\mathrm{U}(N_c)$	Magnetic $\mathrm{U}(\lvert k\rvert+N_f-N_c)$	Index (r is R-charge)
(1,1,1)	U(1)	U(1)	$1-x^4-2x^8+x^{2r}+x^{4r}+x^{6r}+x^{8r}+x^{-2r}\left(-x^4-x^8\right)+\cdots$
(1,2,1)	U(1)	U(2)	$1-2x^2-3x^4-2x^{5-r}+x^{4r}+x^{6r}+x^{2r}\left(1-2x^4\right)+x^r\left(2x^3+2x^5\right)+x^{-2r}\left(x^4+2x^6\right)+\cdots$
(2,1,1)	U(1)	U(2)	$1-8x^2+6x^4+48x^6-4x^{5-3r}+4x^{4-2r}+16x^{6r}+12x^{5+r}+x^{4r}\left(9-24x^2\right)+x^{2r}\left(4-16x^2-16x^4\right)+x^{-r}\left(4x^3+4x^5\right)+\cdots$
(1,2,2)	U(2)	U(1)	$1-2x^2-3x^4-2x^{6-3r}+x^{4r}+x^{6r}+x^{2r}\left(1-2x^4\right)+x^{-2r}\left(x^4+2x^6\right)+x^{-r}\left(2x^4+2x^6\right)+\cdots$
(2,1,2)	U(2)	U(1)	$1-8x^2+28x^4+32x^6+24x^{6-3r}+20x^{6r}+12x^{2+3r}+x^{4r}\left(10-48x^2\right)+x^r\left(4x^2-44x^4\right)+x^{2r}\left(4-24x^2+32x^4\right)+x^{-2r}\left(8x^4-24x^6\right)+x^{-4r}\left(-x^4-8x^6\right)+x^{-r}\left(-16x^4+52x^6\right)+\cdots$
(1,3,2)	U(2)	U(2)	$1-2x^2-2x^4+x^{4-2r}+x^{4r}+x^{2r}\left(1-3x^4\right)+\cdots$
(2,2,2)	U(2)	U(2)	$1-8x^2-2x^3+28x^4+4x^{4-2r}+10x^{4r}+x^{2r}\left(4-24x^2\right)+\cdots$
(3,1,2)	U(2)	U(2)	$1-18x^2+18x^3+198x^4+9x^{4-2r}+45x^{4r}+x^{2r}\left(9-144x^2\right)+\cdots$

Table 1. The index for various unitary groups.

the monopole flux n we have nonzero matter fields due to the Gauss constraints. The BPS state can be represented as

$$|Q_{a_1}Q_{a_2}\cdots Q_{a_{kn}}\rangle\,. \tag{2.14}$$

For each Q_{a_i} it has the R-charge r and the angular momentum $\frac{n}{2}$. This is due to the familiar fact that the charged scalar of charge e has the angular momentum $|en|$ in the presence of the monopole charge n on S^2. Thus for each Q^{a_i} we have $\epsilon=R+j=r+\frac{n}{2}$. We can count the number of such operators by the combination with repetition: ${}_{N_f}H_{kn}=\binom{N_f+kn-1}{kn}=\frac{(N_f+kn-1)!}{(N_f-1)!(kn)!}$. In addition, if the magnetic flux is negative, we have following gauge invariant operators in the same manner,

$$\left|\tilde{Q}^{a_1}\tilde{Q}^{a_2}\cdots\tilde{Q}^{a_{k|n|}}\right\rangle \tag{2.15}$$

Therefore, the contribution of this kind of operators to the superconformal index is given by

$$\frac{(N_f+k|n|-1)!}{(N_f-1)!(k|n|)!}x^{k|n|^2+N_f|n|+(k-N_f)|n|r}. \tag{2.16}$$

where the power of x is given by $\epsilon_0+\epsilon+j = N_f(1-r)|n|+k|n|(r+2\times\frac{|n|}{2}) = k|n|^2+N_f|n|+(k-N_f)|n|r$. For the $N_f = k = 1$ case, this contribution becomes $x^{|n|^2+|n|}$; two terms from $n = 1$ and $n = -1$ give $2x^2$, which exactly cancels the contribution $-2x^2$ from fermionic excitations $Q\psi^\dagger$ and $\tilde{Q}\tilde{\psi}^\dagger$. This explains the absence of x^2 term for U(1) gauge group with $N_f = k = 1$. Using the chiral ring elements and the monopole operators discussed above, one can understand the numerical value of the index of the few lowest orders in x.

2.2 Symplectic case

Now turn to $\mathcal{N} = 2$ Sp$(2N_c)$ gauge theory with $2N_f$ chiral muliplets Q^a and a Chern-Simons term at level k. Here k and N_f may be half-integral, but must sum to an integer. It's magnetic dual is given by $\mathcal{N} = 2$ Sp$(2(|k|+N_f-N_c-1))$ gauge theory with $2N_f$ chiral multiplets q_a and a Chern-Simons term at level $-k$. In addition, there are $N_f(2N_f-1)$ uncharged chiral multiplets M^{ab}, which couple through a superpotential that is given by

$$W = M^{ab}q_aq_b. \tag{2.17}$$

The weights of the fundamental representation are $\pm\epsilon_i$ where $i = 1,\cdots,N_c$, and the roots of Sp$(2N_c)$ are $\pm2\epsilon_i$ and $\pm\epsilon_i\pm\epsilon_j$ where $i,j = 1,\cdots,N_c$ and $i \neq j$. The computation is straightforward and we just list the results. We computed the indices of all dual pairs in the range $1 \leq N_c, |k|+N_f-N_c-1 \leq 2$ with unfixed R-charge r, and confirmed the agreements up to at least $\mathcal{O}(p^{12})$ and $\mathcal{O}(q^{12})$. Parts of them are listed in table 2.

2.3 Orthogonal case

The electric theory is given by $\mathcal{N} = 2$ O(N_c) gauge theory with N_f flavors of chiral superfields Q^a, $a = 1,\cdots,N_f$ in the vector representation and no superpotential. Its magnetic dual is given by O$(N_f-N_c+|k|+2)$ gauge theory with N_f flavors of chiral superfields q_a in the vector representation as well as a singlet chiral superfield M^{ab} which is a symmetric $N_f\times N_f$ matrix. The superpotential in the magnetic theory is

$$W = M^{ab}q_aq_b. \tag{2.18}$$

Let us first consider O$(2N)$ case. The index formula is given by (2.3). With facts that the weights of the fundamental representation are $\pm\epsilon_i$ where $i = 1,\cdots,N$ and that the roots of O$(2N)$ are $\pm\epsilon_i\pm\epsilon_j$ where $i,j = 1,\cdots,N$ and $i \neq j$,

$$S^{(0)}_{\rm CS} = i\frac{k}{2}\sum_{i=1}^{N}2a_im_i = ik\sum_{i=1}^{N}a_im_i, \tag{2.19}$$

$$b_0(a) = 0, \tag{2.20}$$

$$\epsilon_0 = \begin{cases} N_f(1-r)\sum_{i=1}^{N}|m_i|-\sum_{i<j}^{N}|m_i+m_j|-\sum_{i<j}^{N}|m_i-m_j|, & \text{Electric} \\ N_fr\sum_{i=1}^{N}|m_i|-\sum_{i<j}^{N}|m_i+m_j|-\sum_{i<j}^{N}|m_i-m_j|, & \text{Magnetic} \end{cases} \tag{2.21}$$

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(N_f, k, N_c)	Electric $\mathrm{Sp}(2N_c)$	Magnetic $\mathrm{Sp}(2(\lvert k\rvert + N_f - N_c - 1)$	Index (r is R-charge)
(1,2,1)	Sp(2)	Sp(2)	$1-4x^2-5x^4+4x^6+14x^8-x^{8-2r}+x^{4r}+x^{6r}+x^{8r}+x^{2r}\left(1-4x^4\right)+x^{-2r}\left(3x^4+4x^6\right)+\cdots$
(1,3,1)	Sp(2)	Sp(4)	$1-4x^2-2x^4+16x^6+3x^{4-2r}+x^{4r}+x^{6r}+x^{2r}\left(1-9x^4\right)+\cdots$
(2,2,1)	Sp(2)	Sp(4)	$1-16x^2+88x^4+19x^6+50x^{6r}+x^{4r}\left(20-160x^2\right)+x^{2r}\left(6-64x^2+156x^4\right)+x^{-2r}\left(10x^4-74x^6\right)+\cdots$
(1,3,2)	Sp(4)	Sp(2)	$1-4x^2-2x^4+12x^6+x^{4r}+x^{6r}+x^{2r}\left(1-5x^4\right)+x^{-2r}\left(3x^4-4x^6\right)+\cdots$
(1,4,2)	Sp(4)	Sp(4)	$1-4x^2+x^4+3x^{4-2r}+x^{4r}+x^{2r}\left(1-10x^4\right)+\cdots$
(2,3,2)	Sp(4)	Sp(4)	$1-16x^2+148x^4+10x^{4-2r}+21x^{4r}+x^{2r}\left(6-80x^2\right)+\cdots$
(3,2,2)	Sp(4)	Sp(4)	$1-36x^2+873x^4+21x^{4-2r}+120x^{4r}+x^{2r}\left(15-504x^2\right)+\cdots$
(4,1,2)	Sp(4)	Sp(4)	$1-64x^2+2896x^4+36x^{4-2r}+406x^{4r}+x^{2r}\left(28-1728x^2\right)+\cdots$

Table 2. Index for various symplectic groups.

$$f_{\mathrm{chiral}}(e^{ia},1,x) = \begin{cases} N_f \dfrac{x^r - x^{2-r}}{1-x^2}\left[\displaystyle\sum_{i=1}^{N} x^{|m_i|} 2\cos a_i\right], & \text{Electric} \\ N_f \dfrac{x^{1-r} - x^{1+r}}{1-x^2}\left[\displaystyle\sum_{i=1}^{N} x^{|m_i|} 2\cos a_i\right] & \\ \quad + \dfrac{N_f(N_f+1)}{2}\dfrac{x^{2r}-x^{2-2r}}{1-x^2}, & \text{Magnetic} \end{cases} \tag{2.22}$$

$$\begin{aligned} \exp\left[\sum_{n=1}^{\infty}\frac{1}{n} f_{\mathrm{vector}}(e^{jna},x^n)\right] &= \prod_{i<j}^{N}\left(1-e^{i(a_i+a_j)}x^{|m_i+m_j|}\right)\left(1-e^{-i(a_i+a_j)}x^{|m_i+m_j|}\right) \\ &\qquad \times\left(1-e^{i(a_i-a_j)}x^{|m_i-m_j|}\right)\left(1-e^{-i(a_i-a_j)}x^{|m_i-m_j|}\right) \\ &= \prod_{i<j}^{N}\left(1-2\cos(a_i+a_j)x^{|m_i+m_j|}+x^{2|m_i+m_j|}\right) \\ &\qquad \times\left(1-2\cos(a_i-a_j)x^{|m_i-m_j|}+x^{2|m_i-m_j|}\right). \end{aligned} \tag{2.23}$$

This index formula holds for $\mathrm{SO}(2N)$ case. We should consider the additional projection for Z_2 element of $\mathrm{O}(2N)$ not belonging to $\mathrm{SO}(2N)$ group. This kind of projection was

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considered before in the superconformal index computation for $\mathcal{N}=5$ super Chern-Simons matter theories [29] and we adopt the procedure to our purpose. We choose the specific Z_2 action,

$$Z_2 = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & 1 & \\ & & & \ddots \end{pmatrix}. \tag{2.24}$$

Under this Z_2 action, the eigenvalues of the holonomy and the monopole are projected into

$$e^{\pm i a_1} \to \pm 1, \qquad \pm m_1 \to 0. \tag{2.25}$$

The other variables are not affected. Thus, f_{chiral} turns into

$$f_{\text{chiral}}(e^{ia},1,x) = \begin{cases} N_f \dfrac{x^r - x^{2-r}}{1-x^2}\left[(1+(-1)^n) + \displaystyle\sum_{i=2}^{N} x^{|m_i|} 2\cos a_i\right], & \text{Electric} \\ N_f \dfrac{x^{1-r} - x^{1+r}}{1-x^2}\left[(1+(-1)^n) + \displaystyle\sum_{i=2}^{N} x^{|m_i|} 2\cos a_i\right] & \\ \quad + \dfrac{N_f(N_f+1)}{2}\dfrac{x^{2r} - x^{2-2r}}{1-x^2}, & \text{Magnetic} \end{cases} \tag{2.26}$$

and the vector term changes into

$$\begin{aligned} \exp\left[\sum_{n=1}^{\infty}\frac{1}{n} f_{\text{vector}}(e^{ina},x^n)\right] &= \prod_{i=2}^{N}\left(1-2i\sin(a_i)x^{|m_i|} - x^{2|m_i|}\right)\left(1+2i\sin(a_i)x^{|m_i|} - x^{2|m_i|}\right) \\ &\quad \prod_{1<i<j}^{N}\left(1-2\cos(a_i+a_j)x^{|m_i+m_j|} + x^{2|m_i+m_j|}\right) \\ &\quad \times \left(1-2\cos(a_i-a_j)x^{|m_i-m_j|} + x^{2|m_i-m_j|}\right). \end{aligned} \tag{2.27}$$

Other terms are obtained simply by setting $m_1 = 0$.

Let us turn to $\mathrm{O}(2N+1)$ theory. With facts that the weights of the fundamental representation are $\pm\epsilon_i$ where $i = 1,\cdots,N$ and that the roots of $\mathrm{O}(2N+1)$ are $\pm\epsilon_i$ and $\pm\epsilon_i \pm \epsilon_j$ where $i,j = 1,\cdots,N$ and $i \neq j$,

$$S^{(0)}_{\text{CS}} = ik\sum_{i=1}^{N} a_i m_i, \tag{2.28}$$

$$b_0(a) = 0, \tag{2.29}$$

$$\epsilon_0 = \begin{cases} N_f(1-r)\displaystyle\sum_{i=1}^{N}|m_i| - \sum_{i=1}^{N}|m_i| - \sum_{i<j}^{N}|m_i+m_j| - \sum_{i<j}^{N}|m_i-m_j|, & \text{Electric} \\ N_f r\displaystyle\sum_{i=1}^{N}|m_i| - \sum_{i=1}^{N}|m_i| - \sum_{i<j}^{N}|m_i+m_j| - \sum_{i<j}^{N}|m_i-m_j|, & \text{Magnetic} \end{cases} \tag{2.30}$$

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$$f_{\text{chiral}}(e^{ia},1,x)=\begin{cases} N_f\dfrac{x^r-x^{2-r}}{1-x^2}\left[\displaystyle\sum_{i=1}^{N}x^{|m_i|}2\cos a_i+1\right], & \text{Electric}\\ N_f\dfrac{x^{1-r}-x^{1+r}}{1-x^2}\left[\displaystyle\sum_{i=1}^{N}x^{|m_i|}2\cos a_i+1\right] & \\ \quad+\dfrac{N_f(N_f+1)}{2}\dfrac{x^{2r}-x^{2-2r}}{1-x^2}. & \text{Magnetic}\end{cases} \tag{2.31}$$

Note that we have to understand $e^{i\rho(a)}$ in the chiral letter index as the eigenvalues of the operator e^{ia}, which are $e^{\pm a_i}$ and 1 where $i=1,\cdots,N$.
In addition, $\exp\left[\sum_{n=1}^{\infty}\frac{1}{n}f_{\text{vector}}(e^{ima},x^m)\right]$ can be simplified as follows:

$$\begin{aligned}\exp\left[\sum_{n=1}^{\infty}\frac{1}{n}f_{\text{vector}}(e^{ima},x^m)\right]&=\prod_{i=1}^{N}\left(1-2\cos a_i x^{|m_i|}+x^{2|m_i|}\right)\\ &\quad\prod_{i<j}^{N}\left(1-2\cos(a_i+a_j)x^{|m_i+m_j|}+x^{2|m_i+m_j|}\right)\\ &\qquad\times\left(1-2\cos(a_i-a_j)x^{|m_i-m_j|}+x^{2|m_i-m_j|}\right).\end{aligned} \tag{2.32}$$

Again we have to consider the further projection due to proper O$(2N+1)$ elements. Under the Z_2 action,

$$Z_2=\begin{pmatrix}1&&&\\&\ddots&&\\&&1&\\&&&-1\end{pmatrix}, \tag{2.33}$$

an eigenvalue 1 of the holonomy in the fundamental representation is projected by

$$1\to-1 \tag{2.34}$$

while the others are not influenced. Furthermore, eigenvalues $e^{\pm ia_i}$ of the holonomy in the adjoint representation are projected by

$$e^{\pm ia_i}=e^{\pm ia_i}\cdot 1\to e^{\pm ia_1}\cdot(-1) \tag{2.35}$$

while the others, which are in the form of $e^{i(\pm a_i\pm a_j)}=e^{\pm ia_i}\cdot e^{\pm ia_i}$, are not influenced. Thus, the projected index is obtained from

$$f_{\text{chiral}}(e^{ia},1,x)=\begin{cases} N_f\dfrac{x^r-x^{2-r}}{1-x^2}\left[\displaystyle\sum_{i=1}^{N}x^{|m_i|}2\cos a_i+(-1)^n\right], & \text{Electric}\\ N_f\dfrac{x^{1-r}-x^{1+r}}{1-x^2}\left[\displaystyle\sum_{i=1}^{N}x^{|m_i|}2\cos a_i+(-1)^n\right] & \\ \quad+\dfrac{N_f(N_f+1)}{2}\dfrac{x^{2r}-x^{2-2r}}{1-x^2}, & \text{Magnetic}\end{cases} \tag{2.36}$$

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(N_f,k,N_c)	Electric $\mathrm{O}(N_c)$	Magnetic $\mathrm{O}(\lvert k\rvert+N_f-N_c+2)$	Index (r is R-charge)
(1,1,1)	O(1)	O(3)	$1-x^2-2x^4-2x^6-2x^8+x^{4r}+x^{6r}+x^{8r}+x^{2r}\left(1-x^6\right)+x^{-2r}\left(x^6+x^8\right)+\cdots$
(1,1,2)	O(2)	O(2)	$1-x^4-2x^8+x^{2r}+x^{4r}+x^{6r}+x^{8r}+x^{-2r}\left(-x^4-x^8\right)+\cdots$
(1,2,2)	O(2)	O(3)	$1-x^2-2x^4+x^{6-2r}-x^{5-r}+x^{4r}+x^{6r}+x^{2r}\left(1-x^4\right)+x^r\left(x^3+x^5\right)+\cdots$
(1,3,2)	O(2)	O(4)	$1-x^2-2x^4-x^6+x^{6-2r}+x^{2r}+x^{4r}+x^{6r}+\cdots$
(1,3,3)	O(3)	O(3)	$1-x^2-2x^4+x^5+x^{6-2r}+x^{4r}+x^{6r}+x^{2r}\left(1-x^4\right)+\cdots$
(1,4,3)	O(3)	O(4)	$1-x^2-2x^4+x^{6-2r}+x^{4r}+x^{6r}+x^{6+r}+x^{2r}\left(1-x^4-2x^6\right)+\cdots$
(2,1,4)	O(4)	O(1)	$1-4x^2+10x^4-x^{4-4r}+2x^{4-2r}+x^{4r}\left(6-12x^2-19x^4\right)+x^{2r}\left(3-8x^2-9x^4\right)+\cdots$
(5,1,4)	O(4)	O(4)	$1-25x^2+475x^4+10x^{4-2r}+120x^{4r}+x^{2r}\left(15-350x^2\right)+\cdots$

Table 3. Index for various orthogonal groups.

and

$$\exp\left[\sum_{n=1}^{\infty}\frac{1}{n}f_{\text{vector}}(e^{ina},x^n)\right]=\prod_{i=1}^{N}\left(1+2\cos a_i x^{|m_i|}+x^{2|m_i|}\right)\prod_{i<j}^{N}\left(1-2\cos(a_i+a_j)x^{|m_i+m_j|}+x^{2|m_i+m_j|}\right)\times\left(1-2\cos(a_i-a_j)x^{|m_i-m_j|}+x^{2|m_i-m_j|}\right). \tag{2.37}$$

We computed the indices of all dual pairs in the range $1\le N_c, |k|+N_f-N_c+2\le 4$ with unfixed R-charge r, and confirmed the agreements up to at least $\mathcal{O}(p^{12})$ and $\mathcal{O}(q^{12})$. It is crucial that we have $\mathrm{O}(N)$ gauge group instead of $\mathrm{SO}(N)$ to have the agreements in index for the proposed dual pairs. Parts of them are listed in table 3.

3 Conclusions

We work out the superconformal index for Seiberg-like dual pairs in three-dimensional Chern-Simons matter theories with gauge group $\mathrm{U}(N)/\,\mathrm{Sp}(2N)/\,\mathrm{O}(N)$ with matters with

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$N, 2N, N$-dimensional representation, respectively. We find perfect agreements as far as we can carry out the numerical computation. It would be interesting to attempt the analytic proof for the equality of the index for the dual pairs. Related discussion appears at [30–33]. Certainly the method adopted in the current work is applicable to other dualities. It would be interesting to carry out the similar index computation for various proposed dual pairs. The index computation is a useful tool to confirm proposed dualities as demonstrated in the current work.

Acknowledgments

We are grateful to Dongmin Gang for the discussion on the computation of the superconformal index. J.P. is supported by the KOSEF Grant R01-2008-000-20370-0, the National Research Foundation of Korea (NRF) Grants No. 2009-0085995 and 2005-0049409 through the Center for Quantum Spacetime (CQUeST) of Sogang University. J. P. also appreciates APCTP for its stimulating environment for research.

A Index computation with chemical potentials

In appendix, we list the results of index computations, turning on the chemical potentials for the flavor symmetry. When the chemical potentials are turned on, only the flavor charge terms $y_j^{q_{0j}}$ and the chiral letter index f_{chiral} are different from those in the main text. These can be read off from the universal formula eq. (2.5)

A.1 Unitary case

Besides the R symmetry, the (global) symmetries of the unitary case are given by $\mathrm{U}(N_f) \times \mathrm{U}(N_f) = \mathrm{U}(1)_A \times \mathrm{U}(1)_B \times \mathrm{SU}(N_f) \times \mathrm{SU}(N_f)$. We introduce the corresponding Cartan generators F_i and G_j for which the charge assignments of the matter contents on each of the electric side and the magnetic side are as follows:

$$F_i(Q^a) = F_i(\tilde{Q}_a) = \delta_{ia}, \qquad G_j(Q^a) = -G_j(\tilde{Q}_a) = \delta_{ja}, \tag{A.1}$$

$$F_i(q_a) = F_i(\tilde{q}^a) = -\delta_{ia}, \qquad F_i(M^a_b) = \delta_{ia} + \delta_{ib}, \tag{A.2}$$

$$G_j(q_a) = -G_j(\tilde{q}^a) = -\delta_{ja}, \qquad G_j(M^a_b) = \delta_{ja} - \delta_{jb}, \tag{A.3}$$

where $i, j = 1, 2, \cdots, N_f$. $\mathrm{U}(1)_A$ and $\mathrm{U}(1)_B$ are generated by $\Sigma_{i=1}^{N_f} F_i$ and $\Sigma_{j=1}^{N_f} G_j$ respectively. Note that $\mathrm{U}(1)_B$ is a gauged symmetry. The operators $y_i^{F_i}$ and $z_j^{G_j}$ then contribute

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to the superconformal index as follows:

$$
\begin{aligned}
y_j^{q_{0j}} &= y_i^{-\frac{1}{2}\sum_\Phi \sum_{\rho\in R_\Phi} |\rho(m)| F_i(\Phi)} \times z_j^{\frac{1}{2}\sum_\Phi \sum_{\rho\in R_\Phi} |\rho(m)| G_j(\Phi)} \\
&= \begin{cases} \prod_{j=1}^{N_f} y_j^{-\frac{1}{2}\sum_{i=1}^{N_c}|m_i|(1+1)} \times z_j^{-\frac{1}{2}\sum_{i=1}^{N_c}|m_i|(1-1)} = \prod_{j=1}^{N_f} y_j^{-\sum_{i=1}^{N_c}|m_i|}, & \text{Electric} \\ \prod_{j=1}^{N_f} y_j^{-\frac{1}{2}\sum_{i=1}^{N_c}|m_i|(-1-1)} \times z_j^{-\frac{1}{2}\sum_{i=1}^{N_c}|m_i|(1-1)} = \prod_{j=1}^{N_f} y_j^{\sum_{i=1}^{N_c}|m_i|}, & \text{Magnetic} \end{cases}
\end{aligned}
\tag{A.4}
$$

$$
f_{\text{chiral}}(e^{ia}, y_j z_j, x) = \begin{cases} \sum_{j=1}^{N_f} \dfrac{y_j^1 x^r - y_j^{-1} x^{2-r}}{1-x^2} \left[\sum_{i=1}^{N_c} x^{|m_i|}(z_j^1 e^{ia_i} - z_j^{-1} e^{-ia_i})\right], & \text{Electric} \\ \sum_{j=1}^{N_f} \dfrac{y_j^{-1} x^{1-r} - y_j^1 x^{1+r}}{1-x^2} \left[\sum_{i=1}^{N_c} x^{|m_i|}(z_j^1 e^{ia_i} - z_j^{-1} e^{-ia_i})\right] & \\ \quad + \sum_{i=1}^{N_f}\sum_{j=1}^{N_f} \dfrac{y_i^1 y_j^1 z_i^1 z_j^{-1} x^{2r} - y_i^{-1} y_j^{-1} z_i^{-1} z_j^1 x^{2-2r}}{1-x^2}. & \text{Magnetic} \end{cases}
\tag{A.5}
$$

We checked again every case discussed in the main text, turning on the chemical potentials. Here, we simply give one example since writing down the full results is rather cumbersome. For $(N_f, k, N_c) = (2, 1, 1)$, the electric U(1) and the magnetic U(2),

$$
\begin{aligned}
& I\ (x, y_1, y_2, z_1, z_2) \\
&= 1 + x^2\left(-4 - \frac{y_1 z_1}{y_2 z_2} - \frac{y_2 z_1}{y_1 z_2} - \frac{y_1 z_2}{y_2 z_1} - \frac{y_2 z_2}{y_1 z_1}\right) \\
&\quad + x^4\left(-2 + \frac{y_1^2}{y_2^2} + \frac{y_2^2}{y_1^2} + \frac{z_1^2}{z_2^2} + \frac{y_1 z_1}{y_2 z_2} + \frac{y_2 z_1}{y_1 z_2} + \frac{y_1 z_2}{y_2 z_1} + \frac{y_2 z_2}{y_1 z_1} + \frac{z_2^2}{z_1^2}\right) \\
&\quad + x^{4-2r}\left(\frac{1}{y_1^2} + \frac{1}{y_2^2} + \frac{z_1}{y_1 y_2 z_2} + \frac{z_2}{y_1 y_2 z_1}\right) + x^{3-r}\left(\frac{1}{y_2 z_1} + \frac{z_1}{y_2} + \frac{1}{y_1 z_2} + \frac{z_2}{y_1}\right) \\
&\quad + x^{2r}\left(y_1^2 + y_2^2 + \frac{y_1 y_2 z_1}{z_2} + \frac{y_1 y_2 z_2}{z_1} + x^2\left(-2y_1^2 - 2y_2^2 - \frac{y_1^2 z_1^2}{z_2^2} - \frac{y_2^2 z_1^2}{z_2^2} - \frac{y_1^3 z_1}{y_2 z_2}\right.\right. \\
&\qquad \left.\left. - \frac{2 y_1 y_2 z_1}{z_2} - \frac{y_2^3 z_1}{y_1 z_2} - \frac{y_1^3 z_2}{y_2 z_1} - \frac{2 y_1 y_2 z_2}{z_1} - \frac{y_2^3 z_2}{y_1 z_1} - \frac{y_1^2 z_2^2}{z_1^2} - \frac{y_2^2 z_2^2}{z_1^2}\right)\right) \\
&\quad + x^{4r}\left(y_1^4 + y_1^2 y_2^2 + y_2^4 + \frac{y_1^2 y_2^2 z_1^2}{z_2^2} + \frac{y_1^3 y_2 z_1}{z_2} + \frac{y_1 y_2^3 z_1}{z_2} + \frac{y_1^3 y_2 z_2}{z_1} + \frac{y_1 y_2^3 z_2}{z_1} + \frac{y_1^2 y_2^2 z_2^2}{z_1^2}\right) \\
&\quad + \cdots \\
&= 1 - x^2\left(\chi_1(u) + \chi_1(v) + 2\right) + x^4\left(\chi_1(u)\chi_1(v) - 3\right) \\
&\quad + x^{4-2r} y_0^{-2} \chi_{\frac{1}{2}}(u)\chi_{\frac{1}{2}}(v) + x^{3-r} y_0^{-1}\left(z_0 \chi_{\frac{1}{2}}(u) + z_0^{-1}\chi_{\frac{1}{2}}(v)\right) \\
&\quad + x^{2r} y_0^2 \chi_{\frac{1}{2}}(u)\chi_{\frac{1}{2}}(v)\left(1 + x^2\left(\chi_1(u) + \chi_1(v) - 2\right)\right) \\
&\quad + x^{4r} y_0^4\left(\chi_1(u)\chi_1(v) - 1\right) + \cdots
\end{aligned}
\tag{A.6}
$$

where $\chi_n(u) = u^{-n} + u^{-n+1} + \cdots + u^n$. $\chi_n(u)$ is the character of SU(2). A set of variables $y_0 = (y_1 y_2)^{1/2}$, $z_0 = (z_1 z_2)^{1/2}$, $u = \frac{y_1 z_1}{y_2 z_2}$ and $v = \frac{y_1 z_2}{y_2 z_1}$ correspond to the chemical potentials for the symmetries $\mathrm{U}(1)_A \times \mathrm{U}(1)_B \times \mathrm{SU}(2)_Q \times \mathrm{SU}(2)_{\tilde{Q}}$.

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A.2 Symplectic case

Besides the R symmtery, the global symmetries of the symplectic case are $\mathrm{U}(2N_f) = \mathrm{U}(1)_A \times \mathrm{SU}(2N_f)$. We introduce the Cartan generators F_i for which the charge assignments of the matter contents are the followings:

$$F_i(Q^a) = \delta_{ia}, \qquad F_i(q_a) = -\delta_{ia}, \qquad F_i(M^{ab}) = \delta_{ia} + \delta_{ib} \tag{A.7}$$

where $i = 1, 2, \cdots, 2N_f$. $\mathrm{U}(1)_A$ is generated by $\Sigma_{i=1}^{2N_f} F_i$. The operators $y_i^{F_i}$ then contribute to the index as follows:

$$y_j^{q_{0j}} = \begin{cases} \prod_{j=1}^{2N_f} y_j^{-\frac{1}{2}\sum_{i=1}^{N_c}|2m_i|(1)} = \prod_{j=1}^{2N_f} y_j^{-\sum_{i=1}^{N_c}|m_i|}, & \text{Electric} \\ \prod_{j=1}^{2N_f} y_j^{-\frac{1}{2}\sum_{i=1}^{N_c}|2m_i|(-1)} = \prod_{j=1}^{2N_f} y_j^{\sum_{i=1}^{N_c}|m_i|}, & \text{Magnetic} \end{cases} \tag{A.8}$$

$$f_{\text{chiral}}(e^{ia}, y_j, x) = \begin{cases} \sum_{j=1}^{2N_f} \frac{y_j^1 x^r - y_j^{-1} x^{2-r}}{1-x^2} \left[\sum_{i=1}^{N_c} x^{|m_i|} 2\cos a_i\right], & \text{Electric} \\ \sum_{j=1}^{2N_f} \frac{y_j^{-1} x^{1-r} - y_j^1 x^{1+r}}{1-x^2} \left[\sum_{i=1}^{N_c} x^{|m_i|} 2\cos a_i\right] & \\ \quad + \sum_{i=1}^{2N_f} \sum_{j=i+1}^{2N_f} \frac{y_i^1 y_j^1 x^{2r} - y_i^{-1} y_j^{-1} x^{2-2r}}{1-x^2}. & \text{Magnetic} \end{cases} \tag{A.9}$$

We checked every case discussed in the main text, turning on the chemical potentials, and give one example: $(N_f, k, N_c) = (1, 3, 1)$, the electric $\mathrm{Sp}(2)$ and the magnetic $\mathrm{Sp}(4)$,

$$\begin{aligned} I\ &(x, y_1, y_2) \\ &= 1 + x^2\left(-2 - \frac{y_1}{y_2} - \frac{y_2}{y_1}\right) + x^6\left(4 + \frac{2y_1^2}{y_2^2} + \frac{4y_1}{y_2} + \frac{4y_2}{y_1} + \frac{2y_2^2}{y_1^2}\right) \\ &\quad + x^{4-2r}\left(1 + \frac{1}{y_1^2} + \frac{1}{y_1}\right) + x^{2r}\left(y_1 y_2 + x^4\left(-2y_1^2 - \frac{y_1^3}{y_2} - 3y_1 y_2 - 2y_2^2 - \frac{y_2^3}{y_1}\right)\right) \\ &\quad + x^{4r} y_1^2 y_2^2 + x^{6r} y_1^3 y_2^3 + \cdots \\ &= 1 - x^2(\chi_1(u) + 1) - 2x^4 + 2x^6(\chi_2(u) + \chi_1(u)) + x^{4-2r} y_0^{-2} \chi_1(u) \\ &\quad + x^{2r} y_0^2 \left(1 - x^4(\chi_2(u) + \chi_1(u) + 1)\right) + x^{4r} y_0^4 + x^{6r} y_0^6 + \cdots \end{aligned} \tag{A.10}$$

where $y_0 = (y_1 y_2)^{1/2}$, $u = \frac{y_1}{y_2}$ correspond to the chemical potentials for global symmetries $\mathrm{U}(1)_A \times \mathrm{SU}(2)$ respectively.

A.3 Orthogonal case

Besides the R symmtery, the global symmetries of the orthogonal case are given by $\mathrm{U}(N_f) = \mathrm{U}(1)_A \times \mathrm{SU}(N_f)$. We introduce the Cartan generators F_i for which the charge assignments of the matter contents are the followings:

$$F_i(Q^a) = \delta_{ia}, \qquad F_i(q_a) = -\delta_{ia}, \qquad F_i(M^{ab}) = \delta_{ia} + \delta_{ib} \tag{A.11}$$

where $i = 1, 2, \cdots, N_f$. $\mathrm{U}(1)_A$ is generated by $\Sigma_{i=1}^{N_f} F_i$.

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A.3.1 O(2N) theory

The operators $y_i^{F_i}$ contribute to the index as follows:

$$y_j^{q_{0j}} = \begin{cases} \prod_{j=1}^{N_f} y_j^{-\frac{1}{2}\sum_{i=1}^{N}|2m_i|(1)} = \prod_{j=1}^{N_f} y_j^{-\sum_{i=1}^{N}|m_i|}, \text{ Electric} \\ \prod_{j=1}^{N_f} y_j^{-\frac{1}{2}\sum_{i=1}^{N}|2m_i|(-1)} = \prod_{j=1}^{N_f} y_j^{\sum_{i=1}^{N}|m_i|}, \text{ Magnetic} \end{cases} \tag{A.12}$$

$$f_{\text{chiral}}(e^{ia}, y_j, x) = \begin{cases} \sum_{j=1}^{N_f} \frac{y_j^1 x^r - y_j^{-1} x^{2-r}}{1-x^2} \left[\sum_{i=1}^{N} x^{|m_i|} 2\cos a_i\right], & \text{Electric} \\ \sum_{j=1}^{N_f} \frac{y_j^{-1} x^{1-r} - y_j^1 x^{1+r}}{1-x^2} \left[\sum_{i=1}^{N} x^{|m_i|} 2\cos a_i\right] \\ \quad + \sum_{i=1}^{N_f}\sum_{j=i}^{N_f} \frac{y_i^1 y_j^1 x^{2r} - y_i^{-1} y_j^{-1} x^{2-2r}}{1-x^2}. & \text{Magnetic} \end{cases} \tag{A.13}$$

By the projection, the chiral letter index changes into

$$f_{\text{chiral}}(e^{ia}, y_j, x) = \begin{cases} \sum_{j=1}^{N_f} \frac{y_j^1 x^r - y_j^{-1} x^{2-r}}{1-x^2} \left[(1+(-1)^n) + \sum_{i=2}^{N} x^{|m_i|} 2\cos a_i\right], & \text{Electric} \\ \sum_{j=1}^{N_f} \frac{y_j^{-1} x^{1-r} - y_j^1 x^{1+r}}{1-x^2} \left[(1+(-1)^n) + \sum_{i=2}^{N} x^{|m_i|} 2\cos a_i\right] \\ \quad + \sum_{i=1}^{N_f}\sum_{j=i}^{N_f} \frac{y_i^1 y_j^1 x^{2r} - y_i^{-1} y_j^{-1} x^{2-2r}}{1-x^2}. & \text{Magnetic} \end{cases} \tag{A.14}$$

A.3.2 O(2N + 1) theory

The operators $y_i^{F_i}$ contribute to the index as follows:

$$y_j^{q_{0j}} = \begin{cases} \prod_{j=1}^{N_f} y_j^{-\frac{1}{2}\sum_{i=1}^{N}|2m_i|(1)} = \prod_{j=1}^{N_f} y_j^{-\sum_{i=1}^{N}|m_i|}, \text{ Electric} \\ \prod_{j=1}^{N_f} y_j^{-\frac{1}{2}\sum_{i=1}^{N}|2m_i|(-1)} = \prod_{j=1}^{N_f} y_j^{\sum_{i=1}^{N}|m_i|}, \text{ Magnetic} \end{cases} \tag{A.15}$$

$$f_{\text{chiral}}(e^{ia}, y_j, x) = \begin{cases} \sum_{j=1}^{N_f} \frac{y_j^1 x^r - y_j^{-1} x^{2-r}}{1-x^2} \left[\sum_{i=1}^{N} x^{|m_i|} 2\cos a_i + 1\right], & \text{Electric} \\ \sum_{j=1}^{N_f} \frac{y_j^{-1} x^{1-r} - y_j^1 x^{1+r}}{1-x^2} \left[\sum_{i=1}^{N} x^{|m_i|} 2\cos a_i + 1\right] \\ \quad + \sum_{i=1}^{N_f}\sum_{j=i}^{N_f} \frac{y_i^1 y_j^1 x^{2r} - y_i^{-1} y_j^{-1} x^{2-2r}}{1-x^2}. & \text{Magnetic} \end{cases} \tag{A.16}$$

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By the projection, the chiral letter index changes into

$$f_{\text{chiral}}(e^{ia}, y_i, x) = \begin{cases} \displaystyle\sum_{j=1}^{N_f} \frac{y_j^1 x^r - y_j^{-1} x^{2-r}}{1-x^2} \left[\sum_{i=1}^{N} x^{|m_i|} 2\cos a_i + (-1)^n \right], & \text{Electric} \\ \displaystyle\sum_{j=1}^{N_f} \frac{y_j^{-1} x^{1-r} - y_j^1 x^{1+r}}{1-x^2} \left[\sum_{i=1}^{N} x^{|m_i|} 2\cos a_i + (-1)^n \right] & \\ \displaystyle\quad + \sum_{i=1}^{N_f} \sum_{j=i}^{N_f} \frac{y_i^1 y_j^1 x^{2r} - y_i^{-1} y_j^{-1} x^{2-2r}}{1-x^2}. & \text{Magnetic} \end{cases} \tag{A.17}$$

We checked every case discussed in the main text, turning on the chemical potentials, and give one example: $(N_f, k, N_c) = (2, 1, 2)$, the electric O(2) and the magnetic O(3),

$$\begin{aligned} & I\ (x, y_1, y_2) \\ &= 1 + x^2 \left(-2 - \frac{y_1}{y_2} - \frac{y_2}{y_1} \right) + x^4 \left(-1 + \frac{y_1}{y_2} + \frac{y_2}{y_1} \right) + \frac{x^{4-2r}}{y_1 y_2} + x^{3-r} \left(\frac{1}{y_1} + \frac{1}{y_2} \right) \\ &\quad + x^{2r} \left(y_1^2 + y_1 y_2 + y_2^2 + x^2 \left(-2y_1^2 - \frac{y_1^3}{y_2} - 2y_1 y_2 - 2y_2^2 - \frac{y_2^3}{y_1} \right) \right) \\ &\quad + x^{4r} \left(y_1^4 + y_1^3 y_2 + 2y_1^2 y_2^2 + y_1 y_2^3 + y_2^4 \right) + \cdots \\ &= 1 - x^2 \big(\chi_1(u) + 1 \big) + x^4 \big(\chi_1(u) - 2 \big) + x^{4-2r} y_0^{-2} + x^{3-r} y_0^{-1} \chi_{\frac{1}{2}}(u) \\ &\quad + x^{2r} y_0^2 \left(\chi_1(u) - x^2 (\chi_2(u) + \chi_1(u)) \right) + x^{4r} y_0^4 \big(\chi_2(u) + 1 \big) + \cdots \end{aligned} \tag{A.18}$$

where $y_0 = (y_1 y_2)^{1/2}$, $u = \frac{y_1}{y_2}$ correspond to the chemical potentials for global symmetries $U(1)_A \times SU(2)$ respectively.

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