

On the questions of asymptotic recoverability of information and subsystems in quantum gravity

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ABSTRACT: A longstanding question in quantum gravity regards the localization of quantum information; one way to formulate this question is to ask how subsystems can be defined in quantum-gravitational systems. The gauge symmetry and necessity of solving the gravitational constraints appear to imply that the answers to this question here are different than in finite quantum systems, or in local quantum field theory. Specifically, the constraints can be solved by providing a “gravitational dressing” for the underlying field-theory operators, but this modifies their locality properties. It has been argued that holography itself may be explained through this role of the gauge symmetry and constraints, at the nonperturbative level, but there are also subtleties in constructing a holographic map in this approach. There are also claims that holography is implied even by perturbative solution of the constraints. This short note provides further examination of these questions, and in particular investigates to what extent perturbative or nonperturbative solution of the constraints implies that information naïvely thought to be localized can be recovered by asymptotic measurements, and the relevance of this in defining subsystems. In the leading perturbative case, the relevant effects are seen to be exponentially suppressed and asymptotically vanishing, for massive fields. These questions are, for example, important in sharply characterizing the unitarity problem for black holes.

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It has long been believed that localization of information in quantum gravity may behave differently than in local quantum field theory (LQFT). Certainly this has been one of the themes of the proposal of holography [1–3], and is also motivated by consideration of the diffeomorphism gauge symmetry of classical and perturbative quantum general relativity (GR). In trying to understand this localization, there has been increasing focus on the role of the constraints of perturbative GR and its possible nonperturbative generalization, and on properties of the gauge-invariant states and operators annihilated by the constraints. A particularly important question to answer, underlying a description of many information-theoretic aspects of quantum gravity, is in what sense one can precisely define *quantum subsystems* in the theory. For finite or locally finite quantum systems, subsystems are defined in terms of factorization of Hilbert spaces, and for LQFT systems they are defined in terms of commuting subalgebras of observables. But in perturbative GR, the situation is more subtle; the constraints can be solved by dressing underlying states or operators of LQFT [4–6]¹ (for some further development see [9–12]), and this dressing obstructs naïve extensions of such factorization or subalgebras.

Indeed, perhaps the leading candidate for an explanation of holography is that it arises precisely from the gauge symmetry and constraints of gravity; arguments in this direction have been given in [13–16]. There are certain subtleties [17, 18] in these arguments, so this remains an actively investigated question.

In trying to make the question of localization of information more concrete, a specific goal is to understand how to define subsystems in gravity, since in other quantum systems such a definition is at the basis of explaining how information can be localized. This question has in particular been preliminarily discussed in gravity, taking into account the preceding considerations, in [4, 10, 12]. It is also an important question in addressing concrete problems for quantum gravity such as that of explaining the reconciliation of black hole evolution with quantum mechanics; for example the assumption of the (approximate) existence of subsystems is one of the underlying assumptions in a “black hole theorem [19]” that sharply constrains the possible avenues for unitary evolution.

This short paper will examine some aspects and subtleties regarding the question of defining subsystems, in view of the constraints and properties of the dressing. One way to begin to address these is to ask a question: to what extent is information “recoverable” asymptotically in gravity, in situations where it would not be recoverable in LQFT, by making observations or performing experiments in an asymptotic region far from a region where a naïve field theory analysis would tell one it is localized?

The formal arguments for holography [13] do suggest that information is asymptotically recoverable, although these do ultimately seem to be based on assuming a non-perturbative solution of the constraints, among other subtleties [18]. A related argument was given in [9] (see below), that given a full solution of the constraints, one can use the fact that the

¹Earlier related work includes [7] and [8]. The former exhibited nontrivial commutators arising from the constraints but did not describe the dressed operators; the latter was focused on finding bulk operators that *commute*, and did not exhibit the bulk dressing described below.

translation operator is a boundary operator to translate a state to an asymptotic region, where it can be measured. There are also recent claims [20] that based on perturbative construction of dressings that solve the constraints, even at the perturbative level information is recoverable asymptotically. This paper will specifically explore some aspects of this perturbative vs. nonperturbative recoverability of information.

As a test case, consider perturbative quantization of two scalar fields, ϕ_a , $a = 1, 2$, coupled to gravity, preserving the global symmetry distinguishing them.² We will begin with the case of a spacetime M with general asymptotics (e.g. Minkowski or AdS), with simplifying restrictions in later examples.

To test recoverability of information, consider the two states

$$|J\rangle_a = e^{-i \int J(x)\phi_a(x)}|0\rangle = \mathcal{U}_{J_a}|0\rangle, \tag{1}$$

where $J(x)$ is a source function with compact support in some neighborhood U of M , or consider the dressed version of these states in the full gravitational theory.

First, in LQFT, we know that information about which state we choose is not recoverable at spacelike separation to U , through any measurements. To review the argument, if we consider measuring a correlator of some collection \mathcal{O}_A of operators spacelike to U , then

$${}_a\langle J | \prod_A \mathcal{O}_A | J \rangle_a = \langle 0 | \prod_A \mathcal{O}_A | 0 \rangle, \tag{2}$$

due to the operators \mathcal{O}_A commuting with $\phi(x)$ at spacelike separation. Thus such observations cannot distinguish the states $|J\rangle_a$, and cannot even distinguish them from vacuum.

The situation changes in gravity, once we solve the constraints to determine a gravitational dressing for the state; this is because in general this dressing must extend to infinity [6], affecting measurements there. In particular, if we consider a collection of Poincaré charges Q_α , we can use asymptotic measurements to determine ${}_a\langle J | \prod_\alpha Q_\alpha | J \rangle_a$. Such measurements *do* distinguish $|J\rangle_a$ from vacuum, but do not distinguish the $|J\rangle_a$, with $a = 1, 2$, from each other.

To see how this works, minimally couple the fields ϕ_a to gravity with the Einstein action. With a choice of time slicing, the constraints take the form

$$C_\mu(x) = \frac{1}{8\pi G}G_{0\mu}(x) - T_{0\mu}(x) = 0, \tag{3}$$

where in the AdS case T includes the cosmological term. The quantum version of these generalize the Wheeler-DeWitt equation, which corresponds to the $\mu = 0$ component. These may be solved by working with quantum deformations of a fixed background metric $g_{\mu\nu}$

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + \kappa h_{\mu\nu}, \tag{4}$$

with $\kappa^2 = 32\pi G$. At the quantum level, diffeomorphism-invariant operators are those commuting with the constraints, and states solving the constraints are annihilated by the

²There are arguments that such global symmetries are spoiled nonperturbatively in gravity, but this is not obviously true in the perturbative setting, and so let's remain agnostic on this point.

C_μ .³ For example, the states (1) may be promoted to such solutions,

$$|J\rangle_a = \mathcal{U}_{J_a}|0\rangle \rightarrow |\hat{J}\rangle_a = \hat{\mathcal{U}}_{J_a}[\phi, h]|0\rangle, \quad (5)$$

where the operators $\hat{\mathcal{U}}_{J_a}[\phi, h]$ now depend also on the metric perturbation, and are to be determined.

Finding complete solutions is a challenging problem, but perturbative solutions to linear order in κ have been studied in [4–6, 9, 10, 12, 22].⁴ These take the form

$$|\hat{J}\rangle_a \simeq e^{i \int d^3x V^\mu(x) T_{0\mu}(x)} |J\rangle_a, \quad (6)$$

where $V^\mu(x)$ are functionals of h that are linear at first order in κ , and are also a function of position x . Under a diffeomorphism transforming $\delta h_{\mu\nu} = -\partial_\mu \xi_\nu - \partial_\nu \xi_\mu + \mathcal{O}(\kappa)$, these vary according to the key relation

$$\delta V^\mu(x) = \kappa \xi^\mu(x), \quad (7)$$

but the V^μ are not uniquely fixed, and indeed differences correspond to adding a source free radiation field h to a given dressing.⁵ Working about Minkowski space, a broad class of these takes the form [12] (specified, say, at $t = 0$)

$$V_\mu(0, \vec{x}) = \int d^3x' \check{h}_x^{ij}(\vec{x}') \gamma_{\mu,ij}(0, \vec{x}') \quad (8)$$

with

$$\gamma_{\mu,ij} = \frac{\kappa}{2} (\partial_i h_{\mu j} + \partial_j h_{\mu i} - \partial_\mu h_{ij}) \quad (9)$$

the linearized Christoffel symbol, and with a classical tensor field \check{h}_x^{ij} satisfying

$$\partial_i \partial_j \check{h}_x^{ij}(\vec{x}') = -\delta^3(\vec{x}' - \vec{x}). \quad (10)$$

A special case is the line dressing of [5, 22],

$$V_{L\mu}(x, y) = -\frac{\kappa}{2} \int_y^x dx'^\nu \left\{ h_{\mu\nu}(x') - \int_y^{x'} dx''^\lambda [\partial_\mu h_{\nu\lambda}(x'') - \partial_\nu h_{\mu\lambda}(x'')] \right\}, \quad (11)$$

with $y = \infty$, giving a gravitational line running to infinity.

Properties of the dressing can be illustrated by calculating the expectation value of the leading order metric perturbation, using (6)

$${}_a \langle \hat{J} | h_{\mu\nu}(z) | \hat{J} \rangle_a \simeq i \int d^3x [h_{\mu\nu}(z), V^\lambda(x)] {}_a \langle J | T_{0\lambda}(x) | J \rangle_a. \quad (12)$$

³As in Gupta-Bleuler quantization [21], one actually requires that only half of the C_μ annihilate the state, as noted in [10].

⁴Ref. [20] likewise described perturbative solution of the constraints, but failed to note the direct relation of their perturbative analysis to this construction; one can see that the dressing constructions are in many respects simpler than their analysis. Ref. [23] also discussed implications of dressing for the “islands” discussion [24, 25].

⁵This corresponds to the dependence on h^{TT} described in [20].

This is clearly independent of a . The different choices of dressings lead to different values for the commutator and hence for the expectation value. However, much of this freedom arises precisely from adding the different free radiation fields on top of a dressing necessary to solve the constraints, e.g. by shifting \check{h}_x^{ij} by a source-free solution. Any such dressing and expectation value must however give the correct values (to leading order in κ) for the asymptotically-measured Poincaré charges, $P_\mu, M_{\mu\nu}$.

To see that this is the *only* necessary correlation with the choice of matter state,⁶ one can use the “standard dressing” construction as in [10, 12]. Suppose we choose a point $y \in U$ and *some* dressing $V_S^\mu(y)$ satisfying the key relation (7). Then, for general $x \in U$, define the dressing

$$V^\mu(x) = V_L^\mu(x, y) + V_S^\mu(y) + \frac{1}{2}(x - y)_\nu [\partial^\nu V_S^\mu(y) - \partial^\mu V_S^\nu(y)] . \tag{13}$$

Given the commutator

$$i[h_{\mu\nu}(z), V_S^\lambda(y)] = -h_{\mu\nu}^{S\lambda}(z, y) , \tag{14}$$

for z outside U , the commutator in (12) becomes

$$i[h_{\mu\nu}(z), V^\lambda(x)] = -h_{\mu\nu}^{S\lambda}(z, y) - \frac{1}{2}(x - y)_\sigma [\partial_y^\sigma h_{\mu\nu}^{S\lambda}(z, y) - \partial_y^\lambda h_{\mu\nu}^{S\sigma}(z, y)] , \tag{15}$$

since the metric perturbation outside U commutes with the line dressing connecting x and y . Then, the expectation value (12) becomes

$${}_a\langle \hat{J} | h_{\mu\nu}(z) | \hat{J} \rangle_a \simeq h_{\mu\nu}^{S\lambda}(z, y) {}_a\langle J | P_\lambda | J \rangle_a + \frac{1}{2} \partial_y^\sigma h_{\mu\nu}^{S\lambda}(z, y) {}_a\langle J | M_{\sigma\lambda} | J \rangle_a , \tag{16}$$

in terms of the total Poincaré charges P_μ and $M_{\mu\nu}$, with the latter defined with respect to an origin at y . This metric expectation value thus depends on the choice of point y and standard dressing V_S^μ , and on the total Poincaré charges of the matter configuration.

If one instead considers an n -point function ${}_a\langle \hat{J} | h_{\mu_1\nu_1}(z_1) \cdots h_{\mu_n\nu_n}(z_n) | \hat{J} \rangle_a$, then commuting the exponential in (6) through the $h_{\mu\nu}(z_A)$ yields a leading order contribution [12] from a product of terms like (16),

$${}_a\langle \hat{J} | h_{\mu_1\nu_1}(z_1) \cdots h_{\mu_n\nu_n}(z_n) | \hat{J} \rangle_a = {}_a\langle J | \prod_{A=1}^n \left[h_{\mu_A\nu_A}^{S\lambda_A}(z_A, y) P_{\lambda_A} + \frac{1}{2} \partial_y^{\sigma_A} h_{\mu_A\nu_A}^{S\lambda_A}(z_A, y) M_{\sigma_A\lambda_A} \right] | J \rangle_a , \tag{17}$$

now depending on the state $|J\rangle_a$ only through moments of its Poincaré charges. A special case of these expressions, where one computes soft charges from integrating the asymptotic $h_{\mu_A\nu_A}(z_A)$, was in [12] argued to show that the only required correlation of the soft charges with the state is likewise through the moments of the Poincaré charges.

While this shows that at the perturbative level measurements of the metric asymptotics are only sensitive to Poincaré charges, arguments have been given that more general asymptotic measurements can be used to determine the state given the full, rather than leading order, dressing. One is the argument of [13] (for further discussion see [16] and [18]),

⁶Perturbative gravitons can also be included.

and an even simpler one was given in [9]. These rely on the fact that in gravity the total momentum can be written as

$$P_\mu = P_\mu^{\text{ADM}}[h(\infty)] + \int d^3x C_\mu(x), \tag{18}$$

with P_μ^{ADM} the ADM expressions given in terms of surface integrals at infinity, and so for a solution of the constraints is given by just these ADM terms. So, following [9] we can consider the expectation value of the asymptotic operators

$${}_a \langle \widehat{J} | \phi_b(y) e^{iP_i^{\text{ADM}} c^i} | \widehat{J} \rangle_a \tag{19}$$

where y is now in the asymptotic region, and c^i describes a large translation that moves the support of J into the asymptotic region overlapping y . For a solution of the constraints the exponential in (19) is equivalent to one with the full momentum P_i , and this has the effect of translating the state $|\widehat{J}\rangle_a$ to this asymptotic region, where the operator $\phi_b(y)$ can then “register” the state with a result $\propto \delta_{ab}$ and distinguish the two possible states (1). This, thus, describes asymptotic recoverability of information, for a full nonperturbative solution of the constraints.

While these arguments have been illustrated in a Minkowski background, one expects them to straightforwardly generalize to the context of anti de Sitter space, with the same structure, using constructions of AdS dressings solving the constraints like those given in [11]. Thus, for example, one can construct a formal argument that by measuring combinations of matter and gravitational operators analogous to (19) at the AdS boundary, given a nonperturbative solution of the constraints, one can asymptotically recover information about the state.

Ref. [20] has recently argued that this asymptotic accessibility of information also extends to the perturbative context. Specifically, suppose we have perturbatively solved the constraints, to determine a dressing. Working again with the example of a Minkowski background, and generalizing eq. (17), or the preceding argument, one might expect that asymptotic measurements allow determination of the correlators

$${}_a \langle J | \prod_A \mathcal{O}_A \prod_\alpha Q_\alpha | J \rangle_a \tag{20}$$

where again Q_α are Poincaré charges, and \mathcal{O}_A are asymptotic operators, e.g. corresponding to matter operators. Let’s suppose that we can indeed measure such correlators via asymptotic measurements. Does that allow us to determine the difference between the $|J\rangle_a$? We emphasize that, once we have taken into account the use of the dressing in providing the means to make the asymptotic measurements, (20) is then regarded as an expression on a fixed background metric, e.g. AdS or Minkowski. Once again using the definition (1) of the state, and the commutativity of the asymptotic operators \mathcal{O}_A with \mathcal{U}_{J_i} , the correlator (20) becomes

$${}_a \langle J | \prod_A \mathcal{O}_A \prod_\alpha Q_\alpha | J \rangle_a = \langle 0 | \prod_A \mathcal{O}_A \prod_\alpha (\mathcal{U}_{J_a}^\dagger Q_\alpha \mathcal{U}_{J_a}) | 0 \rangle. \tag{21}$$

To understand the dependence of the latter expression on the state $|J\rangle_a$, consider the special case where the charge is the hamiltonian, as was considered in [20]. In this case, for free scalar fields, one finds

$$\mathcal{U}_{J_a}^\dagger H \mathcal{U}_{J_a} = H + \int d^3x \left(\dot{\phi}_J \dot{\phi}_a + \vec{\partial} \phi_J \cdot \vec{\partial} \phi_a \right) + E_J, \quad (22)$$

where ϕ_J is the classical solution with source J ,

$$\square \phi_J = J(x), \quad (23)$$

which has support only in the future lightcone of U , and E_J is the energy of this solution.

We now find that there can be dependence of the correlators (20), (21) on the choice of a . Once again, to take a simple example, consider the correlator

$${}_a \langle J | \phi_b(y) H | J \rangle_a \quad (24)$$

where y is in a region asymptotically far from U . The preceding steps then yield

$${}_a \langle J | \phi_b(y) H | J \rangle_a = \langle 0 | \phi_b(y) \int d^3x \left(\dot{\phi}_J \dot{\phi}_a + \vec{\partial} \phi_J \cdot \vec{\partial} \phi_a \right) | 0 \rangle, \quad (25)$$

which is proportional to δ_{ab} , seemingly registering the difference between the two states (1).

However, for massive fields the result (25) is exponentially small [19] and vanishes asymptotically. Specifically, let m be the mass of the fields ϕ_a , and let y be spatially separated from U with distance L , in the Minkowski example. Since ϕ_J only has support in the future lightcone of U , we have

$${}_a \langle J | \phi_b(y) H | J \rangle_a \sim \delta_{ab} e^{-mL}, \quad (26)$$

The recoverable distinction between the states is thus exponentially tiny in the asymptotic separation [19], and vanishes at infinity. A similar result holds with asymptotics of large-radius R AdS, with a suppression $\sim e^{-mR}$ near the boundary for fields with $R \gg 1/m$. From the expressions (21) and (22), we clearly expect similar exponential suppression for more general correlators.

Ref. [20] has alternately suggested using arbitrary powers of H in (21) to construct energy projectors. This has a similar result. Suppose we likewise assume that it is possible to make boundary measurements of such projected correlators. An example would be

$${}_a \langle J | \phi_b(y) | 0 \rangle \langle 0 | J \rangle_a. \quad (27)$$

But, such a correlator, while proportional to δ_{ab} , is again exponentially small in mL , or in mR in AdS.

It is an interesting property of gravity that there is such an in-principle asymptotic distinction between the states $|J\rangle_a$ also at the perturbative level discussed here. One way of describing what has happened is that access to measurements of local operators *and* Poincaré charges has given sensitivity to the small field correlations between the field in a neighborhood, and that in its complement. What is not clear is that such small effects

lead to a meaningful perturbative recoverability of the information about the identity of the state by asymptotic measurements.

For one thing, if one performs asymptotic measurements, e.g. by interacting or scattering of some system with the asymptotic gravitational field, one needs corresponding exponential sensitivity to register an effect with a probability that is not exponentially suppressed. This does not appear to indicate that the information is well recoverable asymptotically. Likewise, for example, transfer of entanglement associated with such information from the original system to a measuring system would be exponentially suppressed.

Moreover, the argument that the correlators (20) could be determined via asymptotic measurements relied on solving the constraints to determine the gravitationally-dressed state. If one only has a perturbative construction of the dressing solving the constraints, at some order, it is not clear that it will lead to precise enough determination of the correlators to distinguish $\exp\{-mL\}$ from zero. The question of perturbative solution of the constraints to determine the higher-order dressings, generalizing [5], and its accuracy and role in such arguments, is left to work in progress.

Similar arguments have been made for massless fields in [26]. Here, the correlators in (26) or (27) would only be power-law suppressed in the distance. With a nonperturbative construction of the dressing, enabling one to use asymptotic operators to *exactly* project on the vacuum, as assumed in [26], that appears to possibly allow asymptotic access to information, similarly to the case described in (19). However, if one only has a perturbatively-calculated dressing, there will be errors in its determination of the energy (e.g. proportional to the gravitational binding energy) which appear able to compete with even a power falloff $\sim 1/x^p$ in correlators at asymptotic distances, although detailed analysis of this question will be left for future work.

The preceding arguments are amplified by another argument [19] that, in the general situation, one needs a nonperturbative solution of the constraints if one hopes to recover information in asymptotic regions: the states (1) could differ by operators analogous to \mathcal{U}_{Ja} that act only within a black hole. This also suggests the need for a nonperturbative dressing, although one could try to formally run the preceding argument by constructing perturbative dressings, like those studied in [27], on the background geometry of the black hole.

In short, if there are indeed exponentially tiny (or even power law) effects by which information is delocalized, that leaves the question regarding to what extent one can still define an approximate notion of subsystem, in which information is for example dominantly localized. The preceding arguments suggest that standard measurements or scattering experiments don't necessarily have access to such information.

We can specifically return to the question of black holes, and to what extent a black hole is a subsystem, in which for example information can be localized. A black hole *is* such a system in the LQFT approximation, and that localization lies at the center of the problem of unitarity. It is certainly true that the small effects we have discussed make this picture more subtle, and in particular for example raise an additional subtlety in defining entropies of black holes or other candidate subsystems in gravity, but that is not clearly yet the resolution of the unitarity problem advocated in recent work (see, e.g., [20]).

Of course, the observation that effective couplings of the black hole state to its environment [28, 29] that are exponentially small in the black hole entropy can in principle restore unitarity is also suggestive, in the context of discussing an exponentially small delocalization of quantum information. But, it remains to be seen whether these effects are connected, and it is not clear that the latter provide the modifications to evolution necessary for unitarity. It would be very useful to more clearly understand the (approximate) localization of information in quantum gravity, and also the full evolution of that information, for which a nonperturbative completion may well be needed. Again the formal arguments for holography of [6, 13] appear to rely on a nonperturbative solution of the constraints, which is tantamount to starting with a nonperturbative description of evolution [18], suggesting that this doesn't lead to a direct solution to the problem.

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