Published for SISSA by Springer
Received: June 3, 2022
Accepted: June 21, 2022
Published: August 2, 2022

How many 1-loop neutrino mass models are there?

Carolina Arbeláez,[a] Ricardo Cepedello,[b] Juan Carlos Helo,[c,d] Martin Hirsch[e] and Sergey Kovalenko[d,f]

[a] *Department of Physics, Universidad Técnica Federico Santa María and Centro Científico Tecnológico de Valparaíso CCTVal, Avenida España 1680, Valparaíso, Chile*
[b] *Institut für Theoretische Physik und Astrophysik, University of Würzburg, Campus Hubland Nord, Würzburg D-97074, Germany*
[c] *Departamento de Física, Facultad de Ciencias, Universidad de la Serena, Avenida Cisternas 1200, La Serena, Chile*
[d] *Millennium Institute for Subatomic Physics at the High Energy Frontier (SAPHIR), Fernández Concha 700, Santiago, Chile*
[e] *Instituto de Física Corpuscular (CSIC-Universitat de València), C/ Catedrático José Beltrán 2, Paterna E-46980, València, Spain*
[f] *Departamento de Ciencias Físicas, Universidad Andrés Bello, Sazie 2212, Piso 7, Santiago, Chile*

E-mail: carolina.arbelaez@usm.cl, ricardo.cepedello@physik.uni-wuerzburg.de, jchelo@userena.cl, mahirsch@ific.uv.es, sergey.kovalenko@unab.cl

Abstract: It is well-known that at tree-level the $d = 5$ Weinberg operator can be generated in exactly three different ways, the famous seesaw models. In this paper we study the related question of how many phenomenologically consistent 1-loop models one can construct at d=5. First, we discuss that there are two possible classes of 1-loop neutrino mass models, that allow avoiding stable charged relics: (i) models with dark matter candidates and (ii) models with "exits". Here, we define "exits" as particles that can decay into standard model fields. Considering 1-loop models with new scalars and fermions, we find in the dark matter class a total of (115+203) models, while in the exit class we find (38+368) models. Here, 115 is the number of DM models, which require a stabilizing symmetry, while 203 is the number of models which contain a dark matter candidate, which maybe accidentally stable. In the exit class the 38 refers to models, for which one (or two) of the internal particles in the loop is a SM field, while the 368 models contain only fields beyond the SM (BSM) in the neutrino mass diagram. We then study the RGE evolution of the gauge couplings in all our 1-loop models. Many of the models in our list lead to Landau poles in some gauge coupling at rather low energies and there is exactly one model which unifies the gauge couplings at energies above 10^{15} GeV in a numerically acceptable way.

Keywords: Other Weak Scale BSM Models, Models for Dark Matter, Neutrino Interactions

ArXiv ePrint: 2205.13063

Open Access, © The Authors.
Article funded by SCOAP^3.

https://doi.org/10.1007/JHEP08(2022)023

JHEP08(2022)023

Contents

1 Introduction **1**

2 Setup and models **3**
- 2.1 Exit models 4
- 2.2 Dark matter models 7

3 Renormalization group running **11**

4 Discussion **14**

A Complete lists of 1-loop neutrino mass models **18**

JHEP08(2022)023

1 Introduction

It is well-known that the Weinberg operator, $\mathcal{O}_{\mathrm{W}}$, [1] can be generated at tree-level in exactly three different ways [2], the famous seesaw mechanisms [3–9]. The smallness of the observed neutrino masses has also motivated many papers on radiative neutrino mass models, starting with the classical papers [10–13]. For a recent review on loop models for neutrino mass, see [14]. One interesting question to ask then naturally is: how many possibilities actually exist to generate $\mathcal{O}_{\mathrm{W}}$ at 1-loop level? We will explore this question in the current paper.

Partial answers to our question already exist in the literature, starting with [2]. Ref. [15] worked within a diagrammatic approach: construct all possible topologies and from there derive all possible diagrams that can lead to $\mathcal{O}_{\mathrm{W}}$. Ref. [15] found a total of only four diagrams, descending from two topologies, which can yield "genuine" neutrino mass models, see figure 1.[1] Here, "genuine" models are defined as models for which at n-loop level all contributions to $\mathcal{O}_{\mathrm{W}}$ with n-1 loops or less (can) vanish, i.e. the list of diagrams in these constructions do not contain self-energies and other loop diagrams that are guaranteed to be only corrections or subdominant to lower order contributions. Using these criteria ref. [15] then provides lists of all 1-loop models up to electro-weak triplets. Note also that [19] lists 1-loop neutrino mass models with dark matter candidates up to electro-weak triplets.

An alternative approach to the problem is based on effective field theory. Here, one first constructs all lepton number violating operators, starting with $\mathcal{O}_{\mathrm{W}}$ and up to the desired dimension, allowed by the SM field content and symmetries. Ref. [20] lists all $\Delta(L) = 2$ operators up to dimension $d = 11$, see also [21]. "Opening up" or "exploding" the operators in all possible ways, together with adding the appropriate numbers of Higgses

[1] A similar approach was followed in [16, 17] for 2-loop models and in [18] for 3-loop models.

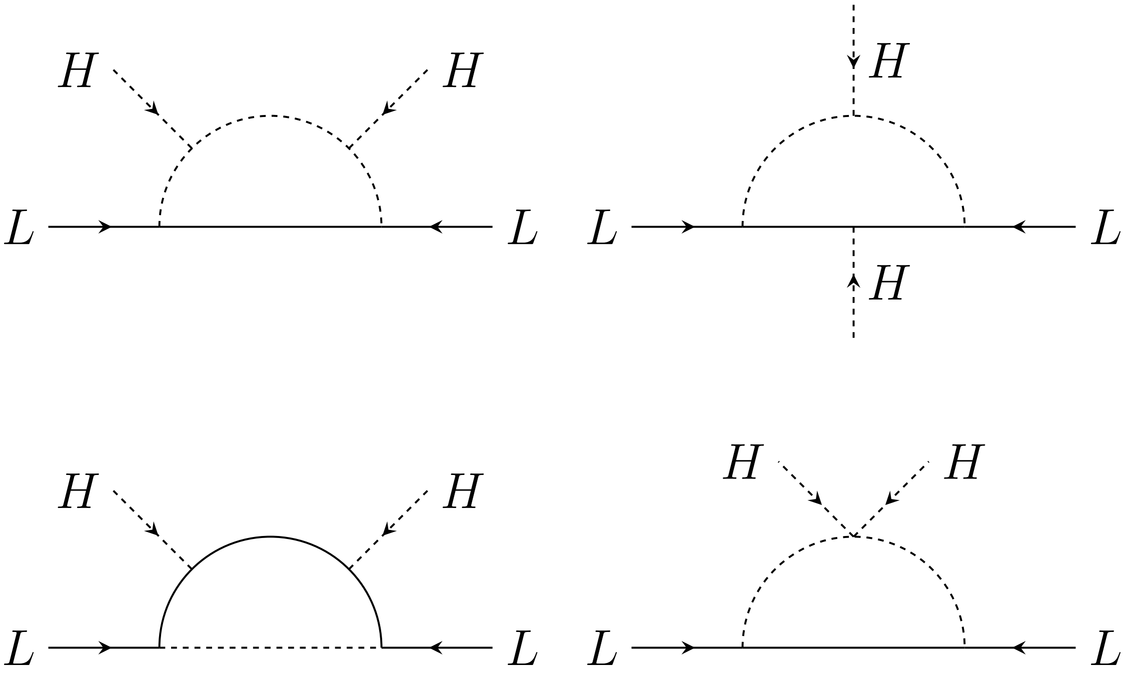

Figure 1. The four different 1-loop diagrams that can lead to genuine neutrino mass models [15]. Top line: T-I-1 (left) and T-I-2 (right), bottom T-I-3 (left) and T-3 (right).

to close loops, then, in principle, also allows a systematic construction of neutrino masses models [22].

None of the above papers, however, gives a "complete" list of 1-loop models. While ref. [15] provided a partial list of 1-loop neutrino mass models, here we aim at giving the complete list of *phenomenologically consistent* models. Of course, all models for neutrino mass should be able to reproduce experimental neutrino data, see for example [23], while at the same time obey upper limits on charged lepton flavour violation searches [24]. However, there are also other criteria that a successful model should fulfill and our main concern here is to avoid problems with cosmology.

Consider the following, very basic observation: in loop models of $\mathcal{O}_{\mathrm{W}}$ the particles internal to the n-loop diagram always couple in pairs to some external SM field, compare figure 1. If all particles in the loop are fields beyond the SM (BSM) and there are no other interactions in the model for the BSM fields than those appearing in the loop diagram, the corresponding model will have some accidental symmetry (in the simplest case a Z_2). The lightest of the loop particles (LLP) will then be absolutely stable.

Stable BSM fields, however, might lead to drastic changes in cosmology, in particular if they are electrically charged (and/or coloured). Experimental searches for stable charged relics put severe bounds on their abundance in the mass range $M \sim [1, 10^5]$ GeV, see for example [24–27]. To avoid problems with cosmology, (at least one of) the BSM particles in 1-loop models should therefore be able to decay to SM fields — unless the lightest of them is electrically neutral. This simple consideration limits the number of allowed, electrically

JHEP08(2022)023

charged BSM particles, see section 2.1, and thus one can have only a finite set of 1-loop neutrino mass models.

Electrically neutral BSM particles, on the other hand, can be candidates for dark matter in the form of WIMPs. Thus, one can construct 1-loop models of neutrino mass in which the lightest loop particles is a WIMP candidate, instead of decaying to SM fields. One can write down an infinite number of electro-weak multiplets, that contain one neutral state. However, the list of phenomenologically acceptable multiplets for WIMP candidates is rather short. To the best of our knowledge, this was first discussed in [28] and the subject has very recently been reconsidered in [29, 30], see also [31] and [32]. Note that the recent papers [29, 30] allows a larger list of acceptable multiplets than [28]. The possible connection between dark matter in SU(2) representations larger than triplets and loop models of neutrino mass has been discussed before in [33–37]. We will discuss more details in section 2.2.

We thus will discuss two types of loop models not disfavoured by cosmology:[2] (i) "exit" models, i.e. models in which there are no stable particles in the loop and (ii) dark matter models, i.e. models in which one of the loop particles can be a good WIMP candidate. For both cases we construct the complete list of models. We find that there are (38+368) exit models, while in the dark matter class there are a total of (115+203) models. In the exit class the 38 refers to models, for which one (or more) of the internal particles in the loop is a SM field, while the 368 models contain only fields beyond the SM. The 115 is the number of DM models, which require an additional symmetry to give an acceptable WIMP candidate, while 203 is the number of models which contain a dark matter candidate, which maybe accidentally stable.

The rest of the paper is organized as follows. In the next section we discuss the concrete criteria applied in the construction of our models. Subsection 2.1 deals with models in the exit class, while 2.2 discusses the dark matter class. In section 3 we then turn to RGE evolution of the SM gauge couplings. Since many of our models contain large SU(2) and/or coloured multiplets, Landau poles at rather low energies appear in many of these constructions. Interestingly in our long list of models there is exactly one variant, which leads to a near-perfect unification of the gauge couplings at a scale of roughly $m_{\rm G} \simeq 10^{17}$ GeV. In section 4, we summarise briefly our results and discuss how our lists would change, modifying or dropping some of the assumptions that went into their construction. The complete lists of models are relegated to the appendix.

2 Setup and models

Following [15] in our discussion we will concentrate on models with scalars and fermions. For models with new gauge vectors, the standard model gauge symmetry has to be extended and that symmetry needs to be broken to the SM. This implies that the scalar sector of the model needs to be discussed as well. This is beyond the scope of our present work. We will, however, briefly discuss loops with vectors in section 4. Note that models with new vectors will not require any additional diagram, beyond those shown in figure 1. This

[2] Somewhat fuzzily we call these "phenomenologically consistent" models, see discussion in section 4.

JHEP08(2022)023

Name	$\mathcal{S}$,[a]	$\mathcal{S}_1$	$\mathcal{S}_2$	φ	Ξ	Ξ_1/Δ,[a,b]	Θ_1,[c]	Θ_3,[c]
Irrep	$(1,1,0)$	$(1,1,1)$	$(1,1,2)$	$\left(1,2,\frac{1}{2}\right)$	$(1,3,0)$	$(1,3,1)$	$\left(1,4,\frac{1}{2}\right)$	$\left(1,4,\frac{3}{2}\right)$

Name	ω_1	ω_2	ω_4	Π_1	Π_7	ζ
Irrep	$\left(3,1,-\frac{1}{3}\right)$	$\left(3,1,\frac{2}{3}\right)$	$\left(3,1,-\frac{4}{3}\right)$	$\left(3,2,\frac{1}{6}\right)$	$\left(3,2,\frac{7}{6}\right)$	$\left(3,3,-\frac{1}{3}\right)$

Name	Ω_1	Ω_2	Ω_4	Υ	Φ
Irrep	$\left(6,1,\frac{1}{3}\right)$	$\left(6,1,-\frac{2}{3}\right)$	$\left(6,1,\frac{4}{3}\right)$	$\left(6,3,\frac{1}{3}\right)$	$\left(8,2,\frac{1}{2}\right)$

[a] The field does not appear in the list of valid 1-loop decompositions of $\mathcal{O}_{\mathrm{W}}$.
[b] Ref. [38] uses the symbol Ξ_1. In neutrino physics this field is usually denoted as Δ (seesaw type-II).
[c] Three Higgses.

Table 1. Scalar "exits": scalar bosons that can couple to a pair of standard model fields.

section is divided into two parts. We will examine first models with exits, before turning to dark matter models.

2.1 Exit models

In this subsection we will discuss the construction of models that contain no new stable particle, i.e. "exit" models. In this case, all particles in the neutrino mass loop can be charged and/or coloured. No new symmetry, beyond those of the SM, is needed to make these models genuine, if we also demand that the particle content of a given model does not generate O_{W} at tree-level. Thus, from our list of models in this class we delete all possible constructions containing either $F_{1,1,0} = N_R$, $F_{1,3,0} = \Sigma$ or $S_{1,3,1} = \Delta$.[3] Here and elsewhere in this paper, we use the notation F or S to denote fermions or scalars, with the subscript showing the transformation properties and quantum numbers for the SM gauge group, $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. Alternatively, for compactness, we use the notation introduced in [38], see also tables 1 and 2.

We can divide the exit class of models into two sub-classes: (i) the model contains at least one SM field in the loop. In this case the model by construction does not contain any stable BSM particles. And (ii) all particles appearing in the loop are BSM fields. In that case, there must be at least one particle among the BSM fields, which can decay to SM fields. A list of all BSM scalars that can decay to SM fields at tree-level are given in table 1. This table coincides with table 1 in reference [38]. Ref. [38] arrived at this table from a completely different consideration, namely, from the construction of all tree-level completions for the $d = 6$ SM effective field theory (SMEFT). The lists coincide simply because in both cases a BSM field must appear linearly in at least one term of the

[3] One can avoid the tree-level generation of O_{W} also using an additional (discrete) symmetry. Models with additional symmetries can contain N_R, Σ and Δ, they are discussed in the next subsection. Here, we only mention that one can construct, in principle, an additional 78 1-loop models in the exit class using these fields. We disregard all of them in the following as "non-genuine".

JHEP08(2022)023

Name	N,[a]	E	Δ_1	Δ_3	Σ,[a]	Σ_1
Irrep	$(1,1,0)$	$(1,1,-1)$	$\left(1,2,-\frac{1}{2}\right)$	$\left(1,2,-\frac{3}{2}\right)$	$(1,3,0)$	$(1,3,-1)$

Name	U	D	Q_1	Q_5	Q_7	T_1	T_2
Irrep	$\left(3,1,\frac{2}{3}\right)$	$\left(3,1,-\frac{1}{3}\right)$	$\left(3,2,\frac{1}{6}\right)$	$\left(3,2,-\frac{5}{6}\right)$	$\left(3,2,\frac{7}{6}\right)$	$\left(3,3,-\frac{1}{3}\right)$	$\left(3,3,\frac{2}{3}\right)$

[a] Field does not appear in the list of ordinary genuine 1-loop decompositions of $\mathcal{O}_{\mathrm{W}}$, since it mediates tree-level seesaw type-I/III. Symbols are again taken from [38].

Table 2. Fermion exits: new vector-like fermions that can couple to standard model fields.

Lagrangian. That term will allow to generate a $d = 6$ operator in SMEFT at tree-level and, at the same time, is responsible for the decay of the BSM field. In the table we give the quantum numbers of the scalars in the order $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ and also the symbols proposed in [38].

A few comments are in order. First of all, $\mathcal{S}$ does not appear in the list of genuine 1-loop exit models, we include it in the table only for completeness.[4] Second, the field Ξ_1/Δ does also not appear in our list of "genuine" exit models, since Δ is the mediator of the tree-level seesaw type-II. This field has been denoted as Ξ_1 in [38], but in the neutrino physics community it is more commonly known as Δ.[5] Also note that all scalars in the table can decay to SM fermion pairs, except $\mathcal{S}$, $\mathcal{S}_1$ and Ξ, which decay to pair of Higgses. Finally, Θ_1 and Θ_3 will decay to three Higgses. Particularly interesting, from the point of view of model building, is Θ_3, since this quadruplet scalar appears in the only genuine tree-level model [40, 41] for the operator $O_{7,W} = (H^\dagger H) \cdot O_{\mathrm{W}}$.

Table 2 contains all BSM fermion fields that can couple to a SM fermion plus a Higgs. In the mass eigenstate basis, in addition to decays to the physical Higgs and a SM fermion, these fields will also decay to a SM fermion plus a gauge boson. Again, we include $F_{1,1,0} = N$ and $F_{1,3,0} = \Sigma$ for completeness, although model constructions involving these fields are not included in our lists of genuine exit models, since they generate tree-level seesaws. There are five fields which have quantum numbers coinciding with some SM fermion. However, all fields in table 2 should be understood as vector-like fields (or self-conjugate Majorana fields, in case of N and Σ). We do not write the vector-like partners explicitly.

Given these lists of all possible BSM fields, that can decay directly to standard model particles via renormalisable interactions at tree-level, we can construct all possible 1-loop neutrino mass exit model variants, using the diagrams in figure 1. The task is in principle straight-forward, albeit tedious. We use our own code written in `Mathematica` to automatise the systematic generation of neutrino mass models. The diagrams in figure 1 can be represented as adjacency matrices by giving numbers to all the vertices. Each entry of the matrices will then correspond to a field in the diagram, i.e. entry (i, j) will be the field

[4] All 1-loop diagrams with $\mathcal{S}$ will either contain N or Ξ_1/Δ and thus are eliminated as non-genuine.

[5] We also note that the fields ω_1, ω_4, Π_1, Π_7 and ζ are known in the literature as scalar "leptoquarks". In the notation of the classic paper [39], these are called S_0, $\tilde{S}_0$, $\tilde{S}_{1/2}$, $S_{1/2}$ and S_1, respectively.

JHEP08(2022)023

#	Fields	#	Fields	#	Fields
1	$LF_{1,1,1}F_{1,2,3/2}S_{1,1,1}$	2	$Le_RF_{1,2,3/2}S_{1,1,1}$	3	$LF_{1,3,1}F_{1,2,3/2}S_{1,1,1}$
4	$LF_{1,1,1}S_{1,1,1}S_{1,2,1/2}$	5	$Le_RS_{1,1,1}S_{1,2,1/2}$	6	$LF_{1,3,1}S_{1,1,1}S_{1,2,1/2}$
7	$e_RF_{1,2,1/2}F_{1,2,3/2}S_{1,1,1}$	8	$e_RF_{1,2,1/2}S_{1,1,1}S_{1,2,1/2}$	9	$QF_{3,1,-1/3}F_{3,2,-5/6}S_{3,1,-1/3}$
10	$QF_{3,1,-1/3}F_{3,2,-5/6}S_{3,3,-1/3}$	11	$Qd_RF_{3,2,-5/6}S_{3,1,-1/3}$	12	$Qd_RF_{3,2,-5/6}S_{3,3,-1/3}$
13	$QF_{3,3,-1/3}F_{3,2,-5/6}S_{3,1,-1/3}$	14	$QF_{3,3,-1/3}F_{3,2,-5/6}S_{3,3,-1/3}$	15	$QF_{3,3,-1/3}F_{3,4,-5/6}S_{3,3,-1/3}$
16	$QF_{3,1,-1/3}F_{3,3,2/3}S_{3,2,1/6}$	17	$Qd_RF_{3,3,2/3}S_{3,2,1/6}$	18	$QF_{3,3,-1/3}F_{3,1,2/3}S_{3,2,1/6}$
19	$QF_{3,3,-1/3}F_{3,3,2/3}S_{3,2,1/6}$	20	$QF_{3,3,-1/3}F_{3,3,2/3}S_{3,4,1/6}$	21	$QF_{3,1,-1/3}S_{3,2,1/6}S_{3,1,-1/3}$
22	$QF_{3,1,-1/3}S_{3,2,1/6}S_{3,3,-1/3}$	23	$Qd_RS_{3,2,1/6}S_{3,1,-1/3}$	24	$Qd_RS_{3,2,1/6}S_{3,3,-1/3}$
25	$QF_{3,3,-1/3}S_{3,2,1/6}S_{3,1,-1/3}$	26	$QF_{3,3,-1/3}S_{3,2,1/6}S_{3,3,-1/3}$	27	$QF_{3,3,-1/3}S_{3,4,1/6}S_{3,3,-1/3}$
28	$u_RF_{3,2,7/6}F_{3,2,1/6}S_{3,1,2/3}$	29	$u_RF_{3,2,7/6}F_{3,2,1/6}S_{3,3,2/3}$	30	$u_RF_{3,2,7/6}F_{3,3,5/3}S_{3,2,7/6}$
31	$u_RF_{3,2,7/6}S_{3,2,7/6}S_{3,1,2/3}$	32	$u_RF_{3,2,7/6}S_{3,2,7/6}S_{3,3,2/3}$	33	$d_RF_{3,2,1/6}F_{3,2,-5/6}S_{3,1,-1/3}$
34	$d_RF_{3,2,1/6}F_{3,2,-5/6}S_{3,3,-1/3}$	35	$d_RF_{3,2,1/6}F_{3,3,2/3}S_{3,2,1/6}$	36	$d_RF_{3,2,1/6}S_{3,2,1/6}S_{3,1,-1/3}$
37	$d_RF_{3,2,1/6}S_{3,2,1/6}S_{3,3,-1/3}$	38	$HF_{1,3,1}S_{1,4,3/2}$		

Table 3. 1-loop neutrino mass models for which some internal field can be a SM fermion. For discussion, see text.

connecting vertices i and j. The power of this approach is twofold: (i) the contraction of all fields along a row or column should contain always a singlet, and (ii) with adjacency matrices one can then use tools from graph theory, for instance, to delete isomorphic diagrams. The external fields are already known, so once given the quantum number for one of the fields in the loop, the rest can be computed. This can be actually done for a general set of quantum numbers for the starting field (seed). As the external particles are colour blind, all the particles in the loop will have the same $\mathrm{SU}(3)_C$ representation as the seed, while $\mathrm{SU}(2)_L$ and hypercharge can be obtained by systematically solving the set of equations for each vertex, i.e. for each row/column of the adjacency matrix. Note that one should keep track of the several possibilities for the products of SU(2) representations, for example, a representation r times a doublet gives two possible representations $r \pm 1$, where only those representations larger than $\mathbf{1}$ are possible. Numbers can then be systematically given to the free charges of the seed to get a complete list of models, to which we apply our genuineness criteria and further classify them, as explained in the text. Chirality is also being tracked along the fermion line to afterwards check whether any of the internal fermions may be a SM fermion. It is worth noticing a slight subtlety in this approach first shown in [18]: the antisymmetric contractions of SU(2) implies that some couplings with identical particles vanishes exactly, for example, the coupling of two identical SU(2) doublets (like two Higgses) to a singlet. Diagrams with such couplings should be removed.[6]

The resulting lists are given in table 3 and in the appendix. The tables in the appendix are divided first into the four diagrams of figure 1. The models are then ordered first with respect to the scalar exits in table 1, then w.r.t. fermions as in table 2 and then sub-divided again into increasing number of exits that occur in each diagram.

[6]Even if this is a local feature of SU(2), diagrams with such non-local (effective) couplings may also vanish if, for example, the identical particles get a VEV, which is the case of the SM Higgs.

JHEP08(2022)023

We will discuss now a few, particular cases found in those tables. A subset of the models appearing in the diagrams T-I-2 and T-I-3 use fermions, with quantum numbers and couplings identical to one of the standard model fermion. In these cases, either one or two of the internal particles can be identified with SM quarks or leptons. These are particularly simple models, in the sense that fewer BSM fields are needed than in all other cases. We have identified a total of 38 possibilities in this special sub-class and list all of them in table 3. Note that in case there are two SM fields in the diagram, as for example in model #5, there are two more models (in this example #4 and #8) in which one of the two SM fields could also be a new, vector-like particles. These are counted in this table as extra models, since they contain a different number of degrees of freedom, see also section 3.

A number of the models listed in table 3 have appeared in the literature before. For example, model #5 is the famous Zee-model [10]. Models #23 and #24 are leptoquark models [42]. These are based on the idea to break lepton number in LQ models via LQ-Higgs interactions [43]. Note that supersymmetry with R-parity violation generates the same diagrams [44] with scalars that have the same quantum numbers as in the Zee model and the LQ model #23 of table 3. The particle content of models #11,#12 and #17, appeared first in tables 6 and 7 of [45], 1-loop neutrino masses in this setup were discussed then in [46].

Model #38 is special, first because it is the only model in this class based on T-3. Also, while this model is technically a "genuine" 1-loop model in the sense, that there is no tree-level $d = 5$ neutrino mass, this model generates actually a tree-level $d = 7$ mass. The model was first discussed in [41]. Whether the tree-level $d = 7$ or the 1-loop $d = 5$ contribution is numerically more important, depends essentially on the mass scale of $F_{1,3,1}$ and $S_{1,4,3/2}$. If these particles are heavier than, roughly $\Lambda \simeq 2\,\text{TeV}$, the loop tends to dominate, while for lighter masses the tree-level contribution is more important.

All models with only BSM particles in the loop are given in the appendix. We note, that all models in the diagram class T-I-1 will also have a contribution to the neutrino mass matrix via diagram T-3. In principle, one can find all T-3 models from the models in T-I-1, eliminating simply the "middle" scalar, compare with figure 1. Since the number of degrees of freedom in T-3 and T-I-1 models are different, however, we count these models as different.

Even though the size (and number) of the representations is limited in the exit class, very exotic states appear in our lists. For example, a model with Θ_3 allows SU(2) representations up to **6**-plets. As one can see from the tables, there are many models that have more than one exit particle in the diagram. In fact, there are several models in which all particles in the loop are one of the particle in the exit lists and this is possible within any of the four diagrams. Since none of the fields in these tables are singlets, one can expect interesting phenomenology at the LHC, if the mass scale of the BSM particles is around the electro-weak scale. A complete study of possible LHC signals is, however, beyond the scope of our present work.

2.2 Dark matter models

In this subsection we will discuss 1-loop neutrino mass models containing a WIMP dark matter candidate. The classical proto-type for this class of models is the scotogenic

JHEP08(2022)023

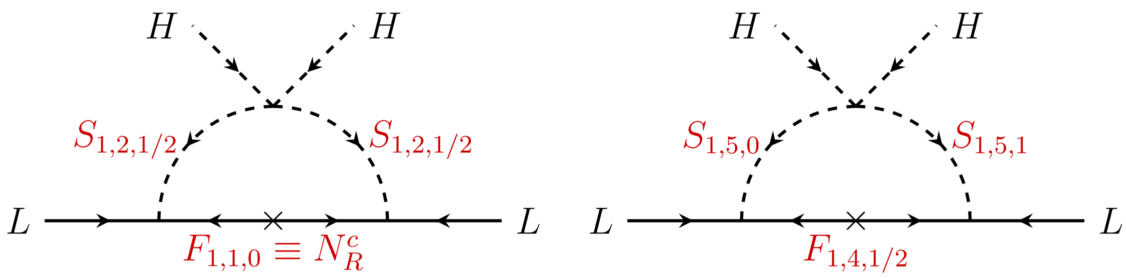

Figure 2. Two examples of dark matter models. To the left the original scotogenic model [47]; to the right an accidentally stable DM model, see text.

JHEP08(2022)023

model [47], see figure 2 (left). Models in the DM class can again be sub-divided into two sub-classes: (i) models that need a stabilizing (discrete) symmetry and (ii) models in which the DM candidate maybe accidentally stable [28], for an example see figure 2 (right). We will call these two classes (i) DM-E (since at least one of the particles in these models has quantum numbers coinciding with one of the exit particles) and (ii) DM-A (for accidental).

The division into these two classes can be easily understood. Consider the scotogenic model. The particle content of this model is such, that without any additional symmetry, beyond the gauge symmetries of the SM a tree-level type-I seesaw would exist and the loop particles would be unstable, i.e. they will decay to SM fields (a right-handed neutrino can decay to SM Higgses or gauge bosons plus SM leptons, for example). Ref. [47] solves these "problems" with the simple assumption that all particles in the loop transform odd under a new Z_2 symmetry. The lightest of the N_{R_i} and $S_{1,2,1/2}$ is then absolutely stable. On the other hand, for larger $SU(2)_L$ multiplets as DM candidates, the model might have an accidental Z_2 symmetry, like in the example model shown in figure 2 (right). For this model it is easy to see that $SU(2)_L$ dictates that, at the renormalisable level, the particles in the loop always couple in pairs to SM fields. Thus, the model has an accidental Z_2 and the LLP is stable automatically. We could call this an "accidental dark matter candidate" [48], in [28] this was named "minimal dark matter".

Note, that this reasoning assumes that there are no other BSM fields present in the model beyond those appearing in the 1-loop diagram. Consider again the example model figure 2 (right). If we add to this model a $S_{1,3,0}$, for example, then the vector-like fermion can decay to a SM L plus a $S_{1,3,0}$, while the latter decays to two Higgses and this extended model will have no DM candidate — unless we postulate an additional symmetry, which would put this model back into the first subclass. Thus, for the DM candidate to be accidental DM, there should be no particle from the exit class in the diagram and we always have to assume in our model constructions implicitly that there are no other BSM particles in the model, beyond those appearing in the 1-loop diagram. Otherwise, the model will belong to DM class (i) DM-E.

However, at this point we would like to stress that for us DM-E and DM-A are just a convenient classification scheme, dividing the model lists into those models with "small" and "large" SU(2) representations. For us, a model is specified from the particle content and the interactions of a given 1-loop neutrino mass diagrams. Nevertheless, any of the models may allow interactions beyond those appearing in the diagram and those additional interactions may put in danger the stability of the DM candidate. This issue has been discussed in several references [35–37]. See in particular [37], where it was shown that Yukawa interactions connecting the fermions of the neutrino mass diagram with their vector-like partners to the scalar DM candidates, $S_{1,n,0}$, are always possible. These interactions (together with those in the neutrino mass diagram) induce loop decays of the DM candidate. In essence, the stability of the DM can be interpreted as an upper limit on this new coupling. Adding a symmetry by hand, allows to eliminate this problem, of course, but the stability of the DM could no longer be considered accidental.

The above arguments about the stability of accidental dark matter are valid strictly speaking for interactions at the renormalizable level. One can always add some discrete symmetry to a model in our DM-A class and then our two subclasses are simply separating models containing potential exits from models which don't. However, in order to construct our lists of DM-A models, we need to identify all possible WIMP DM candidates first and here different DM-A candidates have to fulfill different theoretical constraints. The question of which electro-weak multiplets can be good dark matter candidates has been recently discussed in detail in two papers [29, 30]. Reference [29] treats all possible DM candidates with $Y = 0$, while [30] discusses the case $Y \neq 0$. The following discussion draws heavily from the results of these two papers.

Let us start with the case $Y = 0$. For multiplets with $Y = 0$ radiative corrections will generate a small mass splitting among the members of the multiplet and the sign of this splitting is such that the neutral member of the multiplet is the lightest particle [28]. Also, for multiplets with $Y = 0$, the neutral member of the multiplet does not couple to the Z-boson. Thus, all DM candidates with $Y = 0$ are usually "safe" from existing direct detection (DD) constraints, the best limits are currently from XENON-1t [49]. However, as stressed in [29], the situation will change in the next years and all multiplets $(S/F)_{1,n,0}$ with $n \geq 3$ might be ruled out by future DD experiments, such as DARWIN [50].

The authors of [29] then calculated the relic density, including the effects of Sommerfeld enhancement and bound state formation. The conclusion from this work is that all $(S/F)_{1,n,0}$, with $n = 3, \cdots, 13$ can give the correct relic density and are consistent with DD constraints. To this list, we also have to add the singlets $(S/F)_{1,1,0}$ for completeness.

Whether these DM candidates are accidental DM or need a stabilizing symmetry, however, depends on assumptions about possible non-renormalizable operators and also on whether the DM is scalar or fermion. For scalars, there always exists a $d = 5$ operator of the form $\mathcal{O}_5^S \propto \frac{1}{\Lambda} S^3 H^\dagger H$. This operator will lead to 1-loop DM decays and, for a coefficient of order one, Λ needs to be larger than the Planck scale, to guarantee a sufficiently long-lived DM. Thus, ref. [29] concludes that all $S_{1,n,0}$ (n odd) need a stabilizing symmetry.[7] For fermions, the corresponding operators are $d = 6$ for $n \leq 5$ or $d = 7$ for $n > 5$.

[7] Of course, unless the UV model that generates this operator is completely specified, the coefficient, c, of the operator is arbitrary. For $c \ll 1$ the stability of the DM looks accidental.

JHEP08(2022)023

In particular for $n > 5$ the resulting decay widths are sufficiently small, that the DM candidate is effectively stable, i.e. such fermions are "good" accidental DM, even if one takes into account NROs.

The discussion of the DM candidates with $Y \neq 0$ [30] follows along similar lines, but is slightly more involved. First of all, different from the case $Y = 0$, for multiplets with $Y \neq 0$ electro-weak radiative corrections do not guarantee that the neutral member of the multiplet is the lightest particle. In addition, multiplets with $Y \neq 0$ are strongly constrained by DD limits, since the coupling of the DM to the Z-boson has typically gauge strength. Thus, $Y \neq 0$ DM candidates must be inelastic dark matter, see below.

Again, we divide the discussion of the $Y \neq 0$ DM candidates into different sub-cases. We start with scalars with $Y = 1/2$, i.e. $S_{1,2n,1/2}$ $(n = 1, 2\dots)$. For scalars with quantum numbers $S_{1,2n,1/2}$ one can always write down the following quartic terms:

$$V \propto \lambda_5 (S^\dagger_{1,2n,1/2}(T^a) S^c_{1,2n,1/2})((H^c)^\dagger(\sigma^a/2)H) + \lambda_+ (S^\dagger_{1,2n,1/2}(T^a) S_{1,2n,1/2})(H^\dagger(\sigma^a/2)H). \tag{2.1}$$

After EWSB, these terms generate a mass splitting between the neutral and charged components of the multiplets and for sufficiently small DM mass one can always guarantee that the neutral state is the lightest, by the correct choice of λ_+. Importantly, the term proportional to λ_5 splits the neutral component of the multiplet into its real and imaginary part. Only the lighter of the two is the DM candidate and for sufficiently large mass splitting, m_0, the DD scattering through the Z^0-boson diagram is kinematically forbidden, since the coupling is of the form $Z^0 - \mathrm{Re}(S^0) - \mathrm{Im}(S^0)$. Such a near-degenerate system of dark matter accompanied by a slightly heavier partner is known in the literature as "inelastic dark matter".[8]

The scalar potential, however, also admits terms of the form $(S_{1,2n,1/2})^2 S^\dagger_{1,2n,1/2} H^\dagger$. These terms generate a decay width for $S_{1,2n,1/2}$ to SM Higgses and gauge bosons at 1-loop level. Since these interactions are not suppressed by a large energy scale, $S_{1,2n,1/2}$ can never be accidentally stable, although it is stable at tree-level for $n > 2$.

Consider next $F_{1,2n,Y}$ $(n = 1, 2\dots)$. Different from the scalar case, for fermions the mass splittings in the charged and the neutral sector, splitting in the neutral sector, as well as decay widths, appear only once terms at the non-renormalizable level are taken into account, see again [30]. The condition that a sufficiently large m_0 be generated, to have inelastic DM, leads to an upper bound on the new physics scale of the corresponding operator. If this upper bound is smaller than the mass of the DM candidate, required to give the correct relic density, the model clearly is inconsistent. This consideration rules out all $F_{1,2n,Y}$ for $Y > 1/2$ as good DM candidates, except $F_{1,3,1}$ and $F_{1,5,1}$. $F_{1,2n,1/2}$, on the other hand, are good accidental DM candidates, since the conditions of having a large enough half-live and sufficiently large m_0 can be simultaneously fulfilled for a wide range of new physics scales. (Here, "new" meaning beyond the fields defining the model.)

Finally, there are two more possible scalars that are consistent with the constraints discussed above: $S_{1,3,1}$ and $S_{1,5,1}$. Larger representations, $S_{1,n,1}$, $n > 5$, as well as larger

[8]Numerically the mass splitting has to be only order $\mathcal{O}(100)$ keV, because WIMPs move slowly through the galaxy.

hypercharges are ruled out by the same argument discussed above for $F_{1,2n,Y}$. For $S_{1,3,1}$ and $S_{1,5,1}$ only a "small" window in parameter space remains, where a large enough m_0 can actually be generated. At the same time, $S_{1,3,1}$ and $S_{1,5,1}$ can decay via "fast" interaction terms $S_{1,3,1}HH$ and $S^{\dagger}_{1,5,1}HH(H^{\dagger}H)$. Thus, a symmetry is needed in case of $S_{1,3,1}$ and, while $S_{1,5,1}$ is technically in the DM-A class, making it stable via a symmetry is probably the preferred situation.

Technically, $F_{1,3,1}$ and $S_{1,3,1}$ belong in our list of DM-E models, while $F_{1,5,1}$ and $S_{1,5,1}$ belongs to the class DM-A. However, for all of these four states new physics in the ultra-violet is needed for a consistent DM model. And the scale of this extra physics has to exist not too far above the DM mass. This is different from all the $Y = (1/2)$ (and $Y = 0$) candidates. We thus include models with these DM candidates in our counting, but in the appendix these models are separated off into their own table(s).

Most importantly, multiplets larger than $n = 13$ are excluded as WIMP candidates, because the cross section necessary to reproduce the relic density would violate unitarity bounds [29, 30]. This criterium is weaker than the one used originally in [28]. The authors of [28] argued that all multiplets that lead to a Landau pole in the running of α_2 below the GUT scale should be excluded from the list of valid candidates. Thus, the list of acceptable multiplets stops at $F_{1,5,0}$ and $S_{1,7,0}$ in [28]. In our scanning for valid models, we follow the weaker criterium of [29, 30] and consider multiplets up to $n = 13$.

In the appendix we give the full list of models in the dark matter class. There are four sets of tables, one for DM-E models, one for DM-A models and another two tables for the exceptional candidates with $Y = 1$ separating again exit models and accidental DM. Note that it is impossible to separate models into lists for $Y = 0$ and $Y = 1/2$ candidates, because they mostly appear in the same models. Having no criterium to decide, which of the particles in the loop is the lightest, the models listed in the tables guarantee only that at least one DM candidate is present in the model, but do not specify which is the true DM in case there are more than one candidate.

Because of the SU(2) contraction rule, $n \times 2 = \{(n+1), (n-1)\}$, the models contain multiplets up to **15**-plets. Most of these models have Landau poles in the evolution of the gauge couplings, at energies very close to the mass scale of the particles in the loop, as we are going to discuss in the next section.

3 Renormalization group running

In the previous section we have discussed the criteria for the construction of viable models. While there is only a finite number of these "phenomenologically acceptable" models, still we found more than 700 models in total. This number could be much further reduced, if we apply the additional requirement, that all models remain perturbative up to some large energy scale, say, for example, the grand unification scale, m_{G}. Note that RGE running of gauge couplings in a selected sample of 1-loop neutrino mass models has been studied also in [51]. Perturbativity, however, is clearly a condition expressing only a theoretical preference and thus there is a certain amount of arbitrariness in the formulation of the exact boundary conditions to be used. Most importantly, in the discussion in this section

JHEP08(2022)023

we will assume that the new physics scale, where our BSM particles live, is of the order of the electro-weak scale. Unless noted otherwise, $m_{\rm NP} = 1\,\text{TeV}$. Our motivation for this choice is simply that for larger values of $m_{\rm NP}$, there will not be any chance to test the models via observables outside the neutrino sector itself, in particular there won't be any signals at the LHC. Also, our calculations are done at 1-loop order and thus numbers quoted below should be understood as rough estimates only.

Consider the well known 1-loop solution of the renormalisation group equations, running from a scale t_0 to t is described by:

$$\alpha_i^{-1}(t) = \alpha_i^{-1}(t_0) - \frac{b_i}{2\pi}\log\left(\frac{t}{t_0}\right) \tag{3.1}$$

where $i = 1, 2, 3$ represents the $\mathrm{U}(1)_Y$, SU(2) and SU(3) couplings and the β-functions, b_i, contain the contributions of the SM particles plus the new fields integrated-in at the new physics scale, denoted here as $m_{\rm NP} = 1\,\text{TeV}$. Any α_i may diverge at an energy scale $\Lambda_{\mathrm{LP}_i} = t_0 \,\mathrm{Exp}\left[\frac{2\pi}{b_i}\alpha_i^{-1}(t_0)\right]$, which we call the "Landau pole". Clearly, with increasing b_i, Λ_{LP_i} will become lower, and larger multiplets give larger contributions to the b_i.

We have calculated this running for all our models. Let us discuss exit models first. In tables 5–7 in the appendix we highlighted with * all models where at least one of the Λ_{LP_i} is small, i.e. $\Lambda_{\mathrm{LP}_i} < 100\,\text{TeV}$. 53 models in the exit class fullfill this criterion. Note, in this set of low LPs models there are usually SU(3)-sextets, octets or triplet(s), some of them with large hypercharges. In figure 3, we show the numerical value for the Landau poles for a number of models. Here, we show only those models which have the lowest Λ_{LP_i} for a given exit particle. The numbers above the points refer to the specific model following the enumeration in the tables in the appendix. The topology containing most of the models with low LPs is T-I-3 (since it contains three fermions) while T-3 has the fewest "endangered" models. Of all the models, there is only one, where the minimal LP occurs in α_1 (Λ_{LP_1}).

In the tables in the appendix we have also marked all models, in which no Landau pole appears below the GUT scale (chosen to be $m_{\rm G} \simeq 10^{15}\,\text{GeV}$). There are a total of 57 of such "safe" models in the exit class. Thus, applying this strict criterion, would allow eliminating 87% of our 368 exit models.

We would also like to stress, of all the 368 models containing scalar or/and fermionic exit, there is only one, where the gauge couplings unify at a scale large enough to avoid constraints from proton decay. Figure 4 shows the running of the gauge couplings for this model. The model is marked in table 6 with a purple dagger †, it is model number 211. It belongs to diagram T-I-2 and contains 2 scalar and 2 fermionic exits: $(D, Q_1, \Pi_1, \omega_1)$. We note in passing that we have found 8 additional models, in which we observe a quantitatively acceptable unification of the gauge couplings, but at a rather low scale, where one would expect to have too short proton decay half-lives. Note however, that the authors of [52] recently discussed a mechanism to sufficiently suppress proton decay rates, even if the unification scale is below our (conservative) cutoff of $m_{\rm G} = 10^{15}\,\text{GeV}$. We have not worked out, whether such a suppression would work quantitatively in any of our 8 low-unification

JHEP08(2022)023

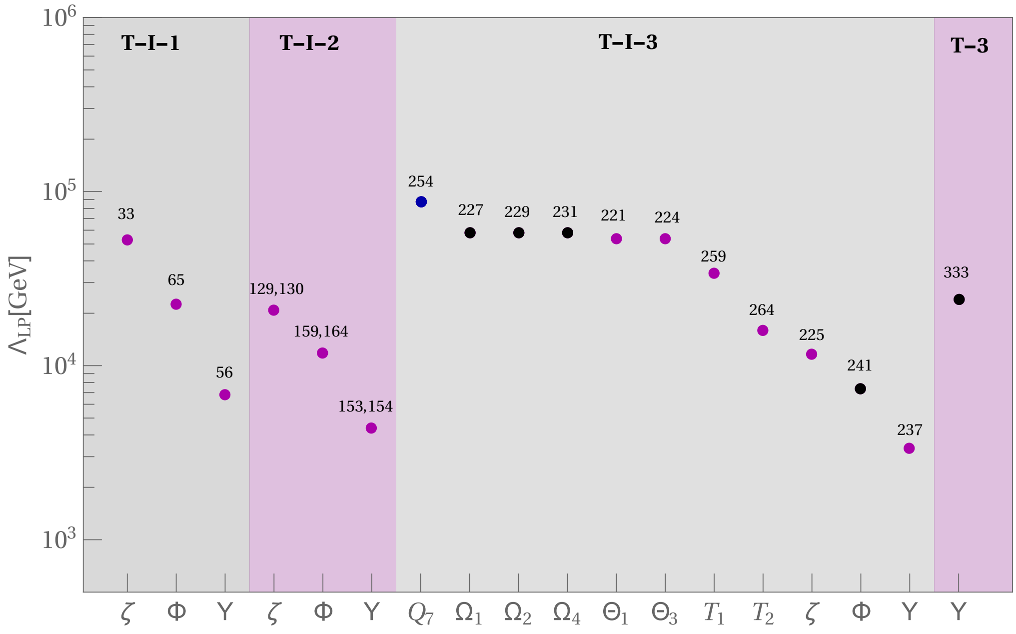

Figure 3. The fields shown on the x-axis correspond to the scalar/fermionic exit associated with the set of models where some Landau pole occurs at a low energy, $\Lambda_{\mathrm{LP}_i} < 100\,\mathrm{TeV}$. The points represent the model(s) with the minimal LP and colours blue, magenta and black if such minimal LP correspond to Λ_{LP_1}, Λ_{LP_2} or Λ_{LP_3}, respectively. The number above each point refers to the specific model as listed in the tables in the appendix.

models, since it depends on the details of the GUT model building, which is beyond the scope of our present work.

For completeness, we also checked the 38 models containing SM fermions, see table 3. Of these, 29 models do not have Landau poles below $m_{\mathrm{G}} = 10^{15}\,\mathrm{GeV}$. Also, in this class there are 4 models that give quantitatively good unification of gauge couplings but in all cases at energies below $10^{15}\,\mathrm{GeV}$.

Now we turn to models containing a DM candidate, shown in tables 8–10 of the appendix. As could be expected, models with large SU(2) multiplets DM fields lead also to Landau poles Λ_{LP_2} fairly below the GUT scale, as depicted in figure 5. In this figure, we show only the "extreme" models, i.e. those in which the LP scale takes either its minimal or maximal value for a given DM n-plet. (All other models for that DM n-plet have LPs in between the values shown.) Large DM representations, i.e. $n \geq 7$, lead Λ_{LP_2} values in the ballpark of a few TeVs for almost all topologies. Choosing a larger m_{NP} would give also larger values for Λ_{LP_i}. Of course, however the LPs are always only a factor of a few above the new physics scale.

For T-3, fewer new particles are needed, resulting in smaller b_i coefficients. Thus, the LPs appear at relatively larger energies. If we were to impose the criterion of having scenarios free of LPs below m_{G}, the list of acceptable models would stop at $n = 5$, thus eliminating most of the DM models. However, as discussed in section 2, in our scanning for valid models, we followed the weaker criterion of [29].

JHEP08(2022)023

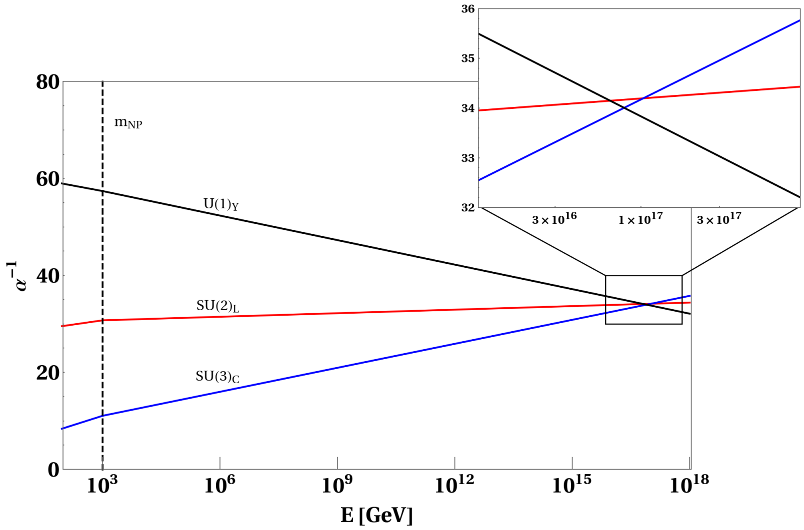

Figure 4. Running of the gauge couplings for the model $(D, Q_1, \Pi_1, \omega_1)$ from the T-I-2 class. This model unifies at a scale of $m_G \simeq 10^{17}$. In this plot all BSM fields are added at a scale of $m_{NP} = 1\,\text{TeV}$.

Also in the DM class we found some models which unify the gauge couplings. In total, there are 8 DM models with a quantitatively acceptable gauge coupling unification, but in all of these models the unification scale lies below $m_G < 10^{15}\,\text{GeV}$.

For the running of the gauge couplings, we have considered all the extra fields, including the DM candidates, to have a common mass of $m_{NP} = 1\,\text{TeV}$. For DM models this choice may not be completely realistic in the following sense: for a given representation one can calculate a mass scale, at which the chosen DM candidate would have the correct relic density to reproduce the measured DM abundance [29]. If we chose those values of the DM mass as m_{NP}, we obtain different numerical values for Λ_{LP_2}. In table 4 we show the corresponding values of the WIMP thermal masses (M_χ), taken from [29], for each DM representation and also the LP values for topology T-I-1. (For the rest of the topologies, the value of the LP is simply re-scaled by a multiplicative factor with respect to the values of figure 5.) The numerical values in the table differ, of course, from the values shown in the figure, but the principle observation remains unchanged: Landau poles appear at relatively low energies and thus most DM models would be excluded, if we require perturbativity up to the GUT scale.

In summary, the requirement that models remain perturbative up to some large energy scale (GUT scale) would exclude a large number of models from both, the exit and the DM class, if the new physics scale is of the order of $m_{NP} \sim 1\,\text{TeV}$.

4 Discussion

In the current paper we have attempted to give an answer to the question: how many 1-loop models for neutrino masses can be constructed at $d = 5$? We have presented complete lists of models based on certain selection criteria. Given the assumptions spelled out in

JHEP08(2022)023

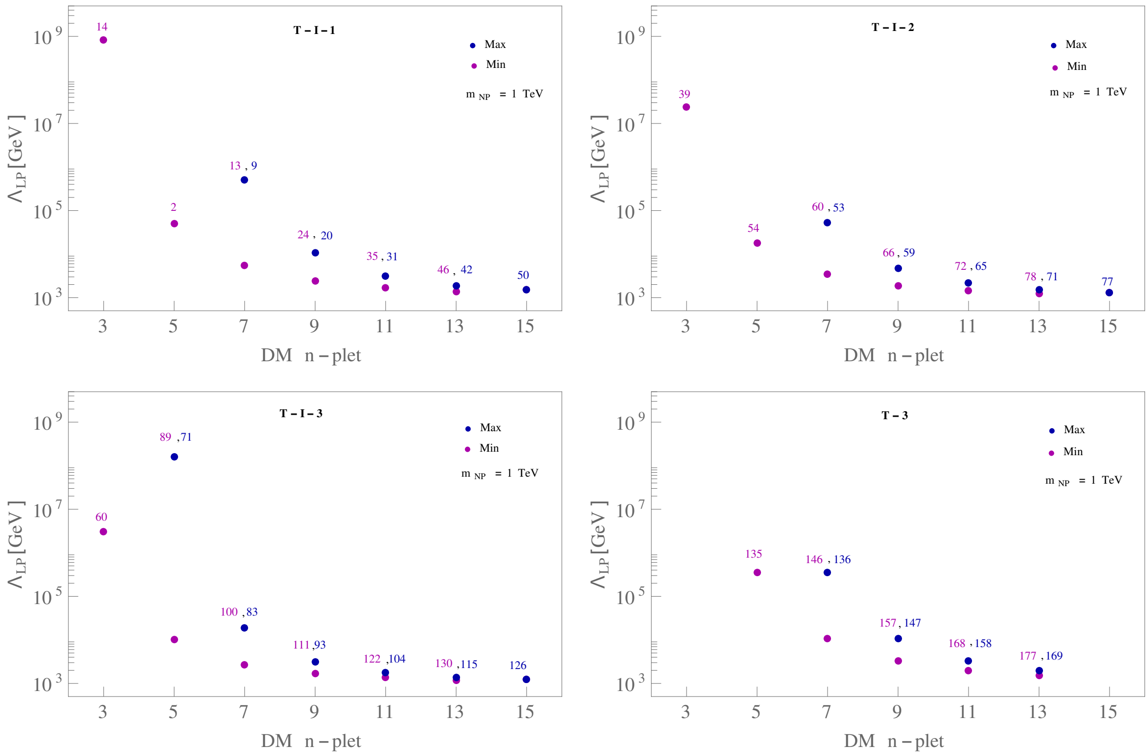

Figure 5. For each topology and electro-weak DM representation, we show the models with the minimal (magenta points) and maximal (blue points) SU(2)-Landau pole Λ_{LP_2} and its specific values. In the running of the gauge couplings all the additional fields beyond the SM are added at the scale $m_{\mathrm{NP}} = 1\,\mathrm{TeV}$. The numbers above the points refer to the specific model as listed in the DM tables in the appendix. All model numbers refer to DM-A in table 9, except the ones associated to 3-plet DM (and also model 71 of T-I-3) which belong to DM-E in table 8.

DM n-plet	M_χ	Min Λ_{LP_2}
3	2.5 TeV	3×10^6 TeV
5	15.4 TeV	9×10^2 TeV
7	54.2 TeV	3×10^2 TeV
9	117.8 TeV	3×10^2 TeV
11	199 TeV	4×10^2 TeV
13	338 TeV	5×10^2 TeV

Table 4. SU(2) n-plet WIMP thermal masses, for which the relic abundance of dark matter would be correctly reproduced and energy scale of the Landau pole, Λ_{LP_2}. In the running of the gauge couplings, all the extra fields up to the SM are added at the M_χ scale.

JHEP08(2022)023

detail in section 2, we have found a total of 724 1-loop neutrino mass models: 406 in the exit class and 318 in the dark matter class. While these are certainly uncomfortably big numbers, especially compared to the fact that there are only three tree-level seesaws, many of these models could actually be excluded in the future.

For the dark matter class, future DD experiments, such as DARWIN [50], will either finally detect WIMP dark matter or exclude most of the larger SU(2) multiplets [29, 30] as DM candidates. From our 318 DM models only 109 would survive non-observation of DM in DARWIN.

Also, there are theoretical considerations, such as perturbativity up to some large energy scales, that we have discussed in section 3. Conservatively, we have listed all possible models in our tables. However, if we require our new physics scale, at which the 1-loop neutrino mass is generated, to be around the electro-weak scale and add the condition that all gauge couplings remain perturbative up to the GUT scale, only 57 models (out of 406) in our exit class survive. Similarly, in the DM class only 59 out of the whole 318 would survive this constraint, eliminating in particular all models with representations larger than **5**-plets.

Two important assumptions on model building were used in all our constructions: (i) use only scalars and fermions as BSM fields; and (ii) avoid stable charged relics. Both of these assumptions can be questioned. Let us discuss 1-loop models with vectors first. Note that very few 1-loop models with gauge vectors do exist in the literature, a few examples are [53–55]. The two main problems with gauge vectors are that: (a) for many of the vectors, which appear in the 1-loop diagrams, it is not even possible to find a phenomenologically consistent or interesting gauge group [56]; and (b) complete gauge models in many cases also contain the ingredients for a tree-level seesaw, thus loops are most likely only a sub-dominant contribution to the neutrino mass in these constructions.

Disregarding these problems in the construction of valid gauge models, however, our automated diagram-based approach allows us, of course, to search also for valid 1-loop neutrino mass diagrams with vectors instead of scalars. From the list of valid "exit" vectors, see table 3 of [38], one can show that there are a total of 499 vector models in the exit class, out of which 34 models contain either one or two SM fermions. Two examples are shown in figure 6. The example on the left is from the diagram class T-I-2, while the one on the right is from T-I-3. Both diagrams have vector LQs as internal particles. Let's have a closer look to the diagram on the left first. The quantum numbers of the vectors are the same as in the scalar LQ model of table 3, model #23, except the hypercharge $Y = 2/3$ of the second vector in the diagram. Thus, the diagram contains u_R instead of d_R, but is otherwise very similar to the corresponding scalar LQ model. Many, but not all of our scalar models can be "vectorized" by such simple replacements.

The two examples shown in figure 6 can also serve to discuss the main problems one encounters in the construction of neutrino mass models based on extended gauge theories. Consider the model shown in the figure on the right. The vector in this diagram, $V_{3,1,2/3}$, can be generated from the adjoint of SU(4), when the Pati-Salam (PS) group [57], is broken to the standard model. Both, Q and u_R are of course present in Pati-Salam as members of the **4** and $\overline{\mathbf{4}}$. However, the $F_{3,2,7/6}$ is not part of a minimal PS model and thus another

JHEP08(2022)023

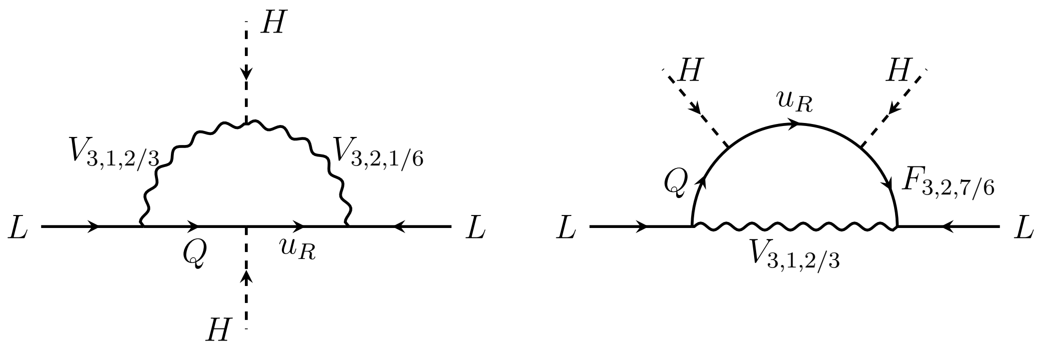

Figure 6. Two example 1-loop diagrams with vectors instead of scalars. For a discussion see text.

multiplet containing this fermion has to be added to complete the particle content of this 1-loop model. On the other hand, the $\overline{\mathbf{4}}$ necessarily contains a $F_{1,1,0}$, i.e. a right-handed neutrino. Since the 1-loop diagram necessarily violates lepton number, it seems reasonable that the model also generates a Majorana mass term for N_R. This could, for example, occur if the PS is first broken to the left-right group, which is then broken by a right-triplet to the SM group. The model then could generate the 1-loop diagram shown, but also has a tree-level seesaw to which the loop diagram would be only a minor correction in large parts of the parameter space. In other words, according to our criteria, in this setup the 1-loop diagram would not be considered "genuine". This problem — the presence of N_R or also other tree-level seesaws — occurs in many of the popular gauge groups, in which the SM group could be embedded. While it seems possible to construct a full model along the lines just discussed, in which the tree-level seesaw is sub-dominant (or absent entirely at tree-level), the model building required clearly is beyond our minimalistic approach to neutrino masses.

A second problem with vector diagrams is demonstrated by the diagram on the left of figure 6. Here, no BSM fermion appears, but two different vectors are needed to complete the diagram. Again, $V_{3,1,2/3}$ appears in the adjoint of SU(4). The other vector, $V_{3,2,1/6}$, appears, for example, in flipped SU(5) [58]. Flipped SU(5) has no N_R, so no tree-level seesaw type-I, but it induces proton decay and the "vector leptoquark" $V_{3,2,1/6}$ of the 1-loop diagram has also diquark couplings in this setup. Thus, when $V_{3,2,1/6}$ is interpreted as the gauge vector of flipped SU(5), its mass must lie at the grand unification scale. This mass scale is too large to generate the atmospheric neutrino mass scale with perturbative couplings from a 1-loop diagram. The problem is exacerbated by the fact, that the diagram needs two vectors. Thus, one would need to identify a group — or semisimple group [59] — which contains both, the Pati-Salam group and the SU(5), plus suitable model building to avoid proton decay and many other constraints in this extended theory. Finally, we would like to stress again, that for most of the exit vectors [38] it was shown in [56], that no suitable gauge group can be constructed at all, since they can not lead to models which contain the SM particle content.

Our second main assumption is to avoid stable charged relics. For the models in the exit class, one can actually question the validity of this criterion. From experimental

JHEP08(2022)023

data the absence of stable charged particles is established only for a certain mass window, roughly $M \sim [1, 10^5]$ GeV [24, 25]. 1-loop models for neutrino mass, however, can fit the observed data even for considerably more massive BSM states in the loop, roughly up to $10^{(12-13)}$ GeV for perturbative couplings. Such ultra-heavy particles would decouple very early in the history of the universe and therefore not be produced in any measurable quantities.[9] Thus, there is a window of parameter space for 1-loop neutrino mass models, where this criterion is not supported by experimental data. Clearly, we have disregarded this possibility. We note in passing, that such models would use, of course, even larger multiplets than what we have considered and thus Landau poles would exist in these constructions always not far above the mass scale of the BSM states.

For the dark matter models, on the other hand, the two most important constraints for valid WIMP candidates are (i) unitarity bounds on the annihilation cross section in the early universe and (ii) limits by direct detection experiments. Here we followed [29] and [30]. While unitarity bounds put a definitive upper limit on the size of the SU(2) multiplet, that can be a good WIMP candidate, the argument (ii) is slightly more fragile. We have considered models with $Y = 0$ DM candidates, as well as inelastic DM candidates. However, one could think about cooking up other ways to avoid the DD constraints and we have simply disregarded this possibility.

In summary, we provide "complete" lists of possible 1-loop models for neutrino masses. We have considered two possible classes of models, which can be consistent with cosmology: "Exit" models, with no stable particles in the loop and dark matter models, which assume that the lightest particle in the loop is neutral, stable and can be in agreement with known constraints. In the appendix we give the lists of all possible models, consistent with these assumptions. It would be interesting to study, whether some of these models can lead to phenomenology at colliders, say the LHC or FCC, that has not already been covered in previous work, see for example [14, 42, 60–68].

A Complete lists of 1-loop neutrino mass models

Here we give tables containing the 1-loop neutrino mass models as discussed in the previous sections. The models are divided in tables for each of the four 1-loop neutrino mass diagrams: T-I-1, T-I-2, T-I-3 and T-3, see figure 1. For each diagram, the models are classified into two large classes: "exit" (tables 5, 6, 7) and dark matter (tables 8, 9, 10) models. The exit models have been ordered in the tables from top to bottom by models with 1 exit to models with 4 exits. To identify the exits particles we have used the notation of ref. [38], shown in tables 1 and 2. The DM models have been separated in four class of models: models with exits that need a stabilizing symmetry to give an acceptable DM candidate (DM-E: table 8), models in which the DM is stable due to an accidental symmetry (DM-A: table 9) and another two cases for exceptional candidates with $Y = 1$ which are separated again in exit DM models (DM-E exceptional: table 10) and accidental DM models (DM-A exceptional: table 10). See section 2 for discussion.

[9] It is even conceivable such states are not produced at all, if the reheat temperature of the universe is sufficiently below the mass of these BSM states.

T-I-1 Exit								
1		$(S_{1,2,5/2}, S_{1,3,3}, \mathcal{S}_2, F_{1,2,5/2})$	2	-	$(\mathcal{S}_2, S_{1,2,5/2}, S_{1,2,3/2}, F_{1,1,2})$	3		$(\mathcal{S}_2, S_{1,2,5/2}, S_{1,2,3/2}, F_{1,3,2})$
4		$(\Theta_1, S_{1,5,1}, S_{1,5,0}, F_{1,4,1/2})$	5		$(\Theta_1, S_{1,5,1}, S_{1,5,0}, F_{1,6,1/2})$	6		$(S_{1,5,1}, S_{1,6,3/2}, \Theta_1, F_{1,5,1})$
7		$(S_{1,5,0}, \Theta_1, \Theta_1, F_{1,5,0})$	8		$(S_{1,5,0}, \Theta_1, S_{1,6,1/2}, F_{1,5,0})$	9		$(S_{1,3,2}, S_{1,2,5/2}, \Theta_3, F_{1,3,2})$
10		$(S_{1,3,2}, S_{1,4,5/2}, \Theta_3, F_{1,3,2})$	11		$(S_{1,3,2}, S_{1,4,5/2}, \Theta_3, F_{1,5,2})$	12		$(\Theta_3, S_{1,3,2}, S_{1,5,1}, F_{1,4,3/2})$
13		$(\Theta_3, S_{1,5,2}, S_{1,5,1}, F_{1,4,3/2})$	14		$(\Theta_3, S_{1,5,2}, S_{1,5,1}, F_{1,6,3/2})$	15		$(S_{1,5,2}, S_{1,4,5/2}, \Theta_3, F_{1,3,2})$
16		$(S_{1,5,2}, S_{1,4,5/2}, \Theta_3, F_{1,5,2})$	17		$(S_{1,5,2}, S_{1,6,5/2}, \Theta_3, F_{1,5,2})$	18		$(S_{1,5,1}, \Theta_3, S_{1,6,1/2}, F_{1,5,1})$
19		$(S_{3,2,-11/6}, S_{3,3,-7/3}, \omega_4, F_{3,2,-11/6})$	20		$(\omega_4, S_{3,2,-11/6}, S_{3,2,-5/6}, F_{3,1,-4/3})$	21		$(\omega_4, S_{3,2,-11/6}, S_{3,2,-5/6}, F_{3,3,-4/3})$
22		$(S_{3,1,5/3}, S_{3,2,13/6}, \Pi_7, F_{3,1,5/3})$	23		$(S_{3,1,5/3}, S_{3,2,13/6}, \Pi_7, F_{3,3,5/3})$	24		$(\Pi_7, S_{3,3,5/3}, S_{3,3,2/3}, F_{3,4,7/6})$
25		$(S_{3,3,5/3}, S_{3,2,13/6}, \Pi_7, F_{3,1,5/3})$	26		$(S_{3,3,5/3}, S_{3,2,13/6}, \Pi_7, F_{3,3,5/3})$	27		$(S_{3,3,5/3}, S_{3,4,13/6}, \Pi_7, F_{3,3,5/3})$
28		$(S_{3,4,1/6}, S_{3,3,2/3}, \zeta, F_{3,4,1/6})$	29		$(S_{3,4,1/6}, S_{3,5,2/3}, \zeta, F_{3,4,1/6})$	30		$(S_{3,2,-5/6}, S_{3,3,-4/3}, \zeta, F_{3,4,-5/6})$
31		$(S_{3,4,-5/6}, S_{3,3,-4/3}, \zeta, F_{3,4,-5/6})$	32		$(S_{3,4,-5/6}, S_{3,5,-4/3}, \zeta, F_{3,4,-5/6})$	33	*	$(\zeta, S_{3,4,1/6}, S_{3,4,-5/6}, F_{3,5,-1/3})$
34		$(S_{6,2,5/6}, S_{6,3,4/3}, \Omega_1, F_{6,2,5/6})$	35	-	$(\Omega_1, S_{6,2,5/6}, S_{6,2,-1/6}, F_{6,1,1/3})$	36		$(\Omega_1, S_{6,2,5/6}, S_{6,2,-1/6}, F_{6,3,1/3})$
37		$(S_{6,2,-1/6}, \Omega_1, S_{6,3,-2/3}, F_{6,2,-1/6})$	38		$(S_{6,2,-7/6}, S_{6,3,-5/3}, \Omega_2, F_{6,2,-7/6})$	39	-	$(\Omega_2, S_{6,2,-1/6}, S_{6,2,-7/6}, F_{6,1,-2/3})$
40		$(\Omega_2, S_{6,2,-1/6}, S_{6,2,-7/6}, F_{6,3,-2/3})$	41		$(S_{6,2,11/6}, S_{6,3,7/3}, \Omega_4, F_{6,2,11/6})$	42		$(\Omega_4, S_{6,2,11/6}, S_{6,2,5/6}, F_{6,1,4/3})$
43		$(\Omega_4, S_{6,2,11/6}, S_{6,2,5/6}, F_{6,3,4/3})$	44		$(S_{6,2,5/6}, S_{6,3,4/3}, \Upsilon, F_{6,2,5/6})$	45	*	$(S_{6,2,5/6}, S_{6,3,4/3}, \Upsilon, F_{6,4,5/6})$
46		$(S_{6,4,5/6}, S_{6,3,4/3}, \Upsilon, F_{6,2,5/6})$	47	*	$(S_{6,4,5/6}, S_{6,3,4/3}, \Upsilon, F_{6,4,5/6})$	48	*	$(S_{6,4,5/6}, S_{6,5,4/3}, \Upsilon, F_{6,4,5/6})$
49		$(S_{6,2,-1/6}, \Upsilon, S_{6,3,-2/3}, F_{6,2,-1/6})$	50	*	$(S_{6,2,-1/6}, \Upsilon, S_{6,3,-2/3}, F_{6,4,-1/6})$	51	-	$(\Upsilon, S_{6,2,5/6}, S_{6,2,-1/6}, F_{6,1,1/3})$
52		$(\Upsilon, S_{6,2,5/6}, S_{6,2,-1/6}, F_{6,3,1/3})$	53		$(\Upsilon, S_{6,2,5/6}, S_{6,4,-1/6}, F_{6,3,1/3})$	54		$(\Upsilon, S_{6,4,5/6}, S_{6,2,-1/6}, F_{6,3,1/3})$
55		$(\Upsilon, S_{6,4,5/6}, S_{6,4,-1/6}, F_{6,3,1/3})$	56	*	$(\Upsilon, S_{6,4,5/6}, S_{6,4,-1/6}, F_{6,5,1/3})$	57		$(S_{6,4,-1/6}, \Upsilon, S_{6,3,-2/3}, F_{6,2,-1/6})$
58	*	$(S_{6,4,-1/6}, \Upsilon, S_{6,3,-2/3}, F_{6,4,-1/6})$	59	*	$(S_{6,4,-1/6}, \Upsilon, S_{6,5,-2/3}, F_{6,4,-1/6})$	60		$(S_{8,1,1}, S_{8,2,3/2}, \Phi, F_{8,1,1})$
61		$(S_{8,1,1}, S_{8,2,3/2}, \Phi, F_{8,3,1})$	62		$(\Phi, S_{8,1,1}, S_{8,3,0}, F_{8,2,1/2})$	63		$(\Phi, S_{8,3,1}, S_{8,1,0}, F_{8,2,1/2})$
64		$(\Phi, S_{8,3,1}, S_{8,3,0}, F_{8,2,1/2})$	65	*	$(\Phi, S_{8,3,1}, S_{8,3,0}, F_{8,4,1/2})$	66		$(S_{8,3,1}, S_{8,2,3/2}, \Phi, F_{8,1,1})$
67		$(S_{8,3,1}, S_{8,2,3/2}, \Phi, F_{8,3,1})$	68		$(S_{8,3,1}, S_{8,4,3/2}, \Phi, F_{8,3,1})$	69	-	$(S_{8,1,0}, \Phi, \Phi, F_{8,1,0})$
70		$(S_{8,1,0}, \Phi, \Phi, F_{8,3,0})$	71		$(S_{8,3,0}, \Phi, \Phi, F_{8,1,0})$	72		$(S_{8,3,0}, \Phi, \Phi, F_{8,3,0})$
73		$(S_{8,3,0}, \Phi, S_{8,4,1/2}, F_{8,3,0})$	74		$(S_{3,4,7/6}, S_{3,3,5/3}, S_{3,3,2/3}, Q_7)$	75		$(S_{3,5,-1/3}, S_{3,4,1/6}, S_{3,4,-5/6}, T_1)$
76		$(S_{3,3,2/3}, S_{3,4,7/6}, S_{3,4,1/6}, T_2)$	77		$(S_{3,5,2/3}, S_{3,4,7/6}, S_{3,4,1/6}, T_2)$			
78	-	$(S_{1,2,3/2}, S_{1,3,2}, \mathcal{S}_1, \Delta_3)$	79		$(\Theta_1, S_{1,5,1}, \Xi, F_{1,4,1/2})$	80		$(\Xi, \Theta_1, \Theta_1, F_{1,5,0})$
81		$(S_{1,5,1}, \Theta_3, \Theta_1, F_{1,5,1})$	82	-	$(S_{3,2,-5/6}, S_{3,3,-4/3}, \omega_1, Q_5)$	83		$(\Pi_1, S_{3,3,2/3}, \zeta, F_{3,4,1/6})$
84		$(S_{3,3,2/3}, S_{3,4,7/6}, \Pi_1, T_2)$	85		$(\Pi_7, S_{3,1,5/3}, S_{3,3,2/3}, Q_7)$	86		$(\Pi_7, S_{3,3,5/3}, S_{3,3,2/3}, Q_7)$
87		$(S_{3,3,2/3}, \Pi_7, S_{3,4,1/6}, T_2)$	88		$(S_{3,4,1/6}, S_{3,3,2/3}, \zeta, Q_1)$	89	-	$(S_{3,2,-5/6}, S_{3,3,-4/3}, \zeta, Q_5)$
90		$(S_{3,4,-5/6}, S_{3,3,-4/3}, \zeta, Q_5)$	91		$(\zeta, S_{3,4,1/6}, S_{3,2,-5/6}, T_1)$	92		$(\zeta, S_{3,4,1/6}, S_{3,4,-5/6}, T_1)$
93		$(S_{6,2,-1/6}, \Upsilon, \Omega_2, F_{6,2,-1/6})$	94		$(S_{6,2,5/6}, \Omega_4, \Upsilon, F_{6,2,5/6})$			
95	-	$(\mathcal{S}_1, S_{1,2,3/2}, \varphi, E)$	96	-	$(\mathcal{S}_1, S_{1,2,3/2}, \varphi, \Sigma_1)$	97	-	$(S_{1,5,1}, \Theta_3, \Theta_1, \Sigma_1)$
98	-	$(\Pi_1, S_{3,3,2/3}, \omega_1, Q_1)$	99	-	$(\omega_1, \Pi_1, S_{3,2,-5/6}, D)$	100	-	$(\omega_1, \Pi_1, S_{3,2,-5/6}, T_1)$
101		$(\Pi_7, S_{3,3,5/3}, \omega_2, Q_7)$	102	-	$(S_{3,2,-5/6}, \omega_4, \zeta, Q_5)$	103	-	$(\Pi_1, S_{3,3,2/3}, \zeta, Q_1)$
104	-	$(S_{3,3,2/3}, \Pi_7, \Pi_1, U)$	105		$(S_{3,3,2/3}, \Pi_7, \Pi_1, T_2)$	106	-	$(\zeta, \Pi_1, S_{3,2,-5/6}, D)$
107		$(\zeta, \Pi_1, S_{3,2,-5/6}, T_1)$	108		$(\zeta, \Pi_1, S_{3,4,-5/6}, T_1)$			
109	-	$(\varphi, \mathcal{S}_1, \Xi, \Delta_1)$	110	-	$(\omega_2, \Pi_7, \Pi_1, U)$	111	-	$(\omega_2, \Pi_7, \Pi_1, T_2)$
112	-	$(\Pi_1, \omega_2, \zeta, Q_1)$						

Table 5. T-I-1 models with scalar and fermionic exits. The four horizontal divisions represent the models with 1, 2, 3 and 4 exit fields. The * and "−" represent the models where one of the Landau pole scales is very low, i.e. $\Lambda_{1,2,3} < 100\,\text{TeV}$, and where there is no Landau pole up to 10^{15} GeV.

JHEP08(2022)023

T-I-2 Exit					
113	$(F_{1,1,2}, F_{1,2,5/2}, S_{1,2,5/2}, \mathcal{S}_2)$	114	$(F_{1,3,2}, F_{1,2,5/2}, S_{1,2,5/2}, \mathcal{S}_2)$	115	$(F_{1,4,1/2}, F_{1,5,1}, S_{1,5,1}, \Theta_1)$
116	$(F_{1,6,1/2}, F_{1,5,1}, S_{1,5,1}, \Theta_1)$	117	$(F_{1,4,1/2}, F_{1,5,0}, S_{1,5,0}, \Theta_1)$	118	$(F_{1,5,0}, F_{1,6,1/2}, \Theta_1, S_{1,5,0})$
119	$(F_{1,4,3/2}, F_{1,3,2}, S_{1,3,2}, \Theta_3)$	120	$(F_{1,4,3/2}, F_{1,3,2}, S_{1,5,2}, \Theta_3)$	121	$(F_{1,4,3/2}, F_{1,5,2}, S_{1,3,2}, \Theta_3)$
122	$(F_{1,4,3/2}, F_{1,5,2}, S_{1,5,2}, \Theta_3)$	123	$(F_{1,6,3/2}, F_{1,5,2}, S_{1,5,2}, \Theta_3)$	124	$(F_{1,5,1}, F_{1,4,3/2}, \Theta_3, S_{1,5,1})$
125	$(F_{1,5,1}, F_{1,6,3/2}, \Theta_3, S_{1,5,1})$	126	$(F_{3,1,-4/3}, F_{3,2,-11/6}, S_{3,2,-11/6}, \omega_4)$	127	$(F_{3,3,-4/3}, F_{3,2,-11/6}, S_{3,2,-11/6}, \omega_4)$
128	$(F_{3,4,7/6}, F_{3,3,5/3}, S_{3,3,5/3}, \Pi_7)$	129 *	$(F_{3,4,-5/6}, F_{3,5,-1/3}, \zeta, S_{3,4,-5/6})$	130 *	$(F_{3,5,-1/3}, F_{3,4,1/6}, S_{3,4,1/6}, \zeta)$
131	$(F_{6,1,1/3}, F_{6,2,5/6}, S_{6,2,5/6}, \Omega_1)$	132	$(F_{6,2,-1/6}, F_{6,1,1/3}, \Omega_1, S_{6,2,-1/6})$	133	$(F_{6,2,-1/6}, F_{6,3,1/3}, \Omega_1, S_{6,2,-1/6})$
134	$(F_{6,3,1/3}, F_{6,2,5/6}, S_{6,2,5/6}, \Omega_1)$	135	$(F_{6,1,-2/3}, F_{6,2,-1/6}, S_{6,2,-1/6}, \Omega_2)$	136	$(F_{6,2,-7/6}, F_{6,1,-2/3}, \Omega_2, S_{6,2,-7/6})$
137	$(F_{6,2,-7/6}, F_{6,3,-2/3}, \Omega_2, S_{6,2,-7/6})$	138	$(F_{6,3,-2/3}, F_{6,2,-1/6}, S_{6,2,-1/6}, \Omega_2)$	139	$(F_{6,1,4/3}, F_{6,2,11/6}, S_{6,2,11/6}, \Omega_4)$
140	$(F_{6,2,5/6}, F_{6,1,4/3}, \Omega_4, S_{6,2,5/6})$	141	$(F_{6,2,5/6}, F_{6,3,4/3}, \Omega_4, S_{6,2,5/6})$	142	$(F_{6,3,4/3}, F_{6,2,11/6}, S_{6,2,11/6}, \Omega_4)$
143	$(F_{6,1,1/3}, F_{6,2,5/6}, S_{6,2,5/6}, \Upsilon)$	144	$(F_{6,2,-1/6}, F_{6,1,1/3}, \Upsilon, S_{6,2,-1/6})$	145	$(F_{6,2,-1/6}, F_{6,3,1/3}, \Upsilon, S_{6,2,-1/6})$
146 *	$(F_{6,2,-1/6}, F_{6,3,1/3}, \Upsilon, S_{6,4,-1/6})$	147	$(F_{6,3,1/3}, F_{6,2,5/6}, S_{6,2,5/6}, \Upsilon)$	148 *	$(F_{6,3,1/3}, F_{6,2,5/6}, S_{6,4,5/6}, \Upsilon)$
149 *	$(F_{6,3,1/3}, F_{6,4,5/6}, S_{6,2,5/6}, \Upsilon)$	150 *	$(F_{6,3,1/3}, F_{6,4,5/6}, S_{6,4,5/6}, \Upsilon)$	151 *	$(F_{6,4,-1/6}, F_{6,3,1/3}, \Upsilon, S_{6,2,-1/6})$
152 *	$(F_{6,4,-1/6}, F_{6,3,1/3}, \Upsilon, S_{6,4,-1/6})$	153 *	$(F_{6,4,-1/6}, F_{6,5,1/3}, \Upsilon, S_{6,4,-1/6})$	154 *	$(F_{6,5,1/3}, F_{6,4,5/6}, S_{6,4,5/6}, \Upsilon)$
155	$(F_{8,2,1/2}, F_{8,1,1}, S_{8,1,1}, \Phi)$	156	$(F_{8,2,1/2}, F_{8,1,1}, S_{8,3,1}, \Phi)$	157 *	$(F_{8,2,1/2}, F_{8,3,1}, S_{8,1,1}, \Phi)$
158 *	$(F_{8,2,1/2}, F_{8,3,1}, S_{8,3,1}, \Phi)$	159 *	$(F_{8,4,1/2}, F_{8,3,1}, S_{8,3,1}, \Phi)$	160	$(F_{8,1,0}, F_{8,2,1/2}, \Phi, S_{8,1,0})$
161	$(F_{8,1,0}, F_{8,2,1/2}, \Phi, S_{8,3,0})$	162 *	$(F_{8,2,1/2}, F_{8,3,0}, S_{8,1,0}, \Phi)$	163 *	$(F_{8,2,1/2}, F_{8,3,0}, S_{8,3,0}, \Phi)$
164 *	$(F_{8,3,0}, F_{8,4,1/2}, \Phi, S_{8,3,0})$	165	$(\Delta_3, F_{1,1,2}, S_{1,3,2}, S_{1,2,3/2})$	166	$(\Delta_3, F_{1,3,2}, S_{1,3,2}, S_{1,2,3/2})$
167	$(Q_5, F_{3,1,-4/3}, S_{3,3,-4/3}, S_{3,2,-5/6})$	168	$(Q_5, F_{3,3,-4/3}, S_{3,3,-4/3}, S_{3,2,-5/6})$	169	$(Q_5, F_{3,3,-4/3}, S_{3,3,-4/3}, S_{3,4,-5/6})$
170	$(Q_7, F_{3,3,5/3}, S_{3,3,5/3}, S_{3,4,7/6})$	171	$(T_1, F_{3,4,1/6}, S_{3,4,1/6}, S_{3,5,-1/3})$	172	$(F_{3,4,-5/6}, T_1, S_{3,5,-1/3}, S_{3,4,-5/6})$
173	$(T_2, F_{3,4,7/6}, S_{3,4,7/6}, S_{3,3,2/3})$	174	$(T_2, F_{3,4,7/6}, S_{3,4,7/6}, S_{3,5,2/3})$	175	$(F_{3,4,1/6}, T_2, S_{3,3,2/3}, S_{3,4,1/6})$
176	$(F_{3,4,1/6}, T_2, S_{3,5,2/3}, S_{3,4,1/6})$				
177 -	$(\Delta_3, F_{1,1,2}, \mathcal{S}_2, S_{1,2,3/2})$	178	$(\Delta_3, F_{1,3,2}, \mathcal{S}_2, S_{1,2,3/2})$	179	$(F_{1,4,1/2}, F_{1,5,0}, \Xi, \Theta_1)$
180	$(F_{1,4,1/2}, \Sigma_1, S_{1,5,1}, \Theta_1)$	181	$(\Delta_3, F_{1,3,2}, S_{1,3,2}, \Theta_3)$	182	$(\Sigma_1, F_{1,4,3/2}, \Theta_3, S_{1,5,1})$
183	$(Q_5, F_{3,1,-4/3}, \omega_4, S_{3,2,-5/6})$	184	$(Q_5, F_{3,3,-4/3}, \omega_4, S_{3,2,-5/6})$	185	$(F_{3,4,1/6}, T_2, S_{3,3,2/3}, \Pi_1)$
186	$(Q_7, F_{3,1,5/3}, S_{3,1,5/3}, \Pi_7)$	187	$(Q_7, F_{3,1,5/3}, S_{3,3,5/3}, \Pi_7)$	188	$(Q_7, F_{3,3,5/3}, S_{3,1,5/3}, \Pi_7)$
189	$(Q_7, F_{3,3,5/3}, S_{3,3,5/3}, \Pi_7)$	190	$(T_2, F_{3,4,7/6}, \Pi_7, S_{3,3,2/3})$	191	$(T_1, F_{3,4,1/6}, S_{3,4,1/6}, \zeta)$
192	$(F_{3,4,-5/6}, T_1, \zeta, S_{3,2,-5/6})$	193	$(F_{3,4,-5/6}, T_1, \zeta, S_{3,4,-5/6})$	194	$(Q_1, T_2, S_{3,3,2/3}, S_{3,4,1/6})$
195	$(T_2, Q_7, S_{3,4,7/6}, S_{3,3,2/3})$				
196 -	$(E, \Delta_3, S_{1,2,3/2}, \mathcal{S}_1)$	197 -	$(\Sigma_1, \Delta_3, S_{1,2,3/2}, \mathcal{S}_1)$	198 -	$(Q_5, D, \omega_1, S_{3,2,-5/6})$
199	$(Q_5, T_1, \omega_1, S_{3,2,-5/6})$	200 -	$(Q_1, U, S_{3,3,2/3}, \Pi_1)$	201	$(Q_1, T_2, S_{3,3,2/3}, \Pi_1)$
202	$(T_1, F_{3,4,1/6}, \Pi_1, \zeta)$	203	$(U, Q_7, \Pi_7, S_{3,3,2/3})$	204	$(T_2, Q_7, \Pi_7, S_{3,3,2/3})$
205 -	$(Q_5, D, \zeta, S_{3,2,-5/6})$	206	$(Q_5, T_1, \zeta, S_{3,2,-5/6})$	207	$(Q_5, T_1, \zeta, S_{3,4,-5/6})$
208	$(T_1, Q_1, S_{3,4,1/6}, \zeta)$				
209 -	$(\Delta_1, E, \mathcal{S}_1, \varphi)$	210 -	$(\Delta_1, \Sigma_1, \mathcal{S}_1, \varphi)$	211 †	$(D, Q_1, \Pi_1, \omega_1)$
212	$(T_1, Q_1, \Pi_1, \omega_1)$	213	$(U, Q_7, \Pi_7, \omega_2)$	214 -	$(Q_1, U, \omega_2, \Pi_1)$
215	$(Q_1, T_2, \omega_2, \Pi_1)$	216	$(T_2, Q_7, \Pi_7, \omega_2)$	217 -	(D, Q_1, Π_1, ζ)
218	(T_1, Q_1, Π_1, ζ)				

Table 6. T-I-2 models with scalar and fermionic exits: the model † unifies at a scale of $m_G \simeq 10^{17}$ GeV.

JHEP08(2022)023

T-I-3 Exit

219	$(F_{1,5,0}, F_{1,4,1/2}, F_{1,4,1/2}, \Xi)$	220	$(F_{1,4,1/2}, F_{1,5,1}, F_{1,5,0}, \Theta_1)$	221 *	$(F_{1,6,1/2}, F_{1,5,1}, F_{1,5,0}, \Theta_1)$
222	$(F_{1,4,3/2}, F_{1,3,2}, F_{1,5,1}, \Theta_3)$	223	$(F_{1,4,3/2}, F_{1,5,2}, F_{1,5,1}, \Theta_3)$	224 *	$(F_{1,6,3/2}, F_{1,5,2}, F_{1,5,1}, \Theta_3)$
225 *	$(F_{3,5,-1/3}, F_{3,4,1/6}, F_{3,4,-5/6}, \zeta)$	226	$(F_{6,1,1/3}, F_{6,2,5/6}, F_{6,2,-1/6}, \Omega_1)$	227 *	$(F_{6,3,1/3}, F_{6,2,5/6}, F_{6,2,-1/6}, \Omega_1)$
228	$(F_{6,1,-2/3}, F_{6,2,-1/6}, F_{6,2,-7/6}, \Omega_2)$	229 *	$(F_{6,3,-2/3}, F_{6,2,-1/6}, F_{6,2,-7/6}, \Omega_2)$	230	$(F_{6,1,4/3}, F_{6,2,11/6}, F_{6,2,5/6}, \Omega_4)$
231 *	$(F_{6,3,4/3}, F_{6,2,11/6}, F_{6,2,5/6}, \Omega_4)$	232	$(F_{6,1,1/3}, F_{6,2,5/6}, F_{6,2,-1/6}, \Upsilon)$	233 *	$(F_{6,3,1/3}, F_{6,2,5/6}, F_{6,2,-1/6}, \Upsilon)$
234 *	$(F_{6,3,1/3}, F_{6,2,5/6}, F_{6,4,-1/6}, \Upsilon)$	235 *	$(F_{6,3,1/3}, F_{6,4,5/6}, F_{6,2,-1/6}, \Upsilon)$	236 *	$(F_{6,3,1/3}, F_{6,4,5/6}, F_{6,4,-1/6}, \Upsilon)$
237 *	$(F_{6,5,1/3}, F_{6,4,5/6}, F_{6,4,-1/6}, \Upsilon)$	238 *	$(F_{8,2,1/2}, F_{8,1,1}, F_{8,3,0}, \Phi)$	239 *	$(F_{8,2,1/2}, F_{8,3,1}, F_{8,1,0}, \Phi)$
240 *	$(F_{8,2,1/2}, F_{8,3,1}, F_{8,3,0}, \Phi)$	241 *	$(F_{8,4,1/2}, F_{8,3,1}, F_{8,3,0}, \Phi)$	242	$(F_{1,1,2}, F_{1,2,5/2}, \Delta_3, S_{1,3,2})$
243	$(F_{1,3,2}, F_{1,2,5/2}, \Delta_3, S_{1,3,2})$	244	$(F_{1,3,2}, F_{1,4,5/2}, \Delta_3, S_{1,3,2})$	245	$(F_{1,4,3/2}, F_{1,3,2}, \Sigma_1, S_{1,2,3/2})$
246	$(\Sigma_1, F_{1,4,3/2}, F_{1,4,1/2}, S_{1,5,1})$	247	$(F_{3,1,-4/3}, F_{3,2,-11/6}, Q_5, S_{3,3,-4/3})$	248	$(F_{3,3,-4/3}, F_{3,2,-11/6}, Q_5, S_{3,3,-4/3})$
249	$(F_{3,3,-4/3}, F_{3,4,-11/6}, Q_5, S_{3,3,-4/3})$	250	$(F_{3,1,5/3}, F_{3,2,13/6}, Q_7, S_{3,1,5/3})$	251	$(F_{3,1,5/3}, F_{3,2,13/6}, Q_7, S_{3,3,5/3})$
252	$(F_{3,3,5/3}, F_{3,2,13/6}, Q_7, S_{3,1,5/3})$	253	$(F_{3,3,5/3}, F_{3,2,13/6}, Q_7, S_{3,3,5/3})$	254 *	$(F_{3,3,5/3}, F_{3,4,13/6}, Q_7, S_{3,3,5/3})$
255 *	$(F_{3,4,1/6}, F_{3,5,2/3}, T_1, S_{3,4,1/6})$	256	$(F_{3,4,-5/6}, F_{3,3,-4/3}, T_1, S_{3,2,-5/6})$	257	$(F_{3,4,-5/6}, F_{3,3,-4/3}, T_1, S_{3,4,-5/6})$
258 *	$(F_{3,4,-5/6}, F_{3,5,-4/3}, T_1, S_{3,4,-5/6})$	259 *	$(T_1, F_{3,4,1/6}, F_{3,4,-5/6}, S_{3,5,-1/3})$	260	$(F_{3,4,7/6}, F_{3,3,5/3}, T_2, S_{3,4,7/6})$
261 *	$(F_{3,4,7/6}, F_{3,5,5/3}, T_2, S_{3,4,7/6})$	262 *	$(T_2, F_{3,4,7/6}, F_{3,4,1/6}, S_{3,3,2/3})$	263 *	$(T_2, F_{3,4,7/6}, F_{3,4,1/6}, S_{3,5,2/3})$
264 *	$(F_{3,4,1/6}, T_2, F_{3,5,-1/3}, S_{3,4,1/6})$				
265	$(F_{1,1,2}, F_{1,2,5/2}, \Delta_3, \mathcal{S}_2)$	266	$(F_{1,3,2}, F_{1,2,5/2}, \Delta_3, \mathcal{S}_2)$	267	$(F_{1,4,1/2}, \Sigma_1, F_{1,5,0}, \Theta_1)$
268	$(F_{1,4,3/2}, F_{1,3,2}, \Sigma_1, \Theta_3)$	269	$(F_{1,4,3/2}, F_{1,5,2}, \Sigma_1, \Theta_3)$	270	$(F_{3,1,-4/3}, F_{3,2,-11/6}, Q_5, \omega_4)$
271	$(F_{3,3,-4/3}, F_{3,2,-11/6}, Q_5, \omega_4)$	272	$(F_{3,4,7/6}, F_{3,3,5/3}, T_2, \Pi_7)$	273 *	$(T_1, F_{3,4,1/6}, F_{3,4,-5/6}, \zeta)$
274	$(\Delta_3, F_{1,3,2}, E, S_{1,2,3/2})$	275	$(\Delta_3, F_{1,1,2}, \Sigma_1, S_{1,2,3/2})$	276	$(\Delta_3, F_{1,3,2}, \Sigma_1, S_{1,2,3/2})$
277	$(Q_5, F_{3,3,-4/3}, D, S_{3,2,-5/6})$	278	$(T_2, F_{3,4,7/6}, Q_1, S_{3,3,2/3})$	279	$(Q_5, F_{3,1,-4/3}, T_1, S_{3,2,-5/6})$
280	$(Q_5, F_{3,3,-4/3}, T_1, S_{3,2,-5/6})$	281	$(Q_5, F_{3,3,-4/3}, T_1, S_{3,4,-5/6})$	282	$(Q_7, F_{3,3,5/3}, T_2, S_{3,4,7/6})$
283	$(T_2, Q_7, F_{3,4,1/6}, S_{3,3,2/3})$	284	$(F_{3,4,1/6}, T_2, T_1, S_{3,4,1/6})$		
285	$(\Delta_3, F_{1,3,2}, \Sigma_1, \Theta_3)$	286	$(F_{3,4,1/6}, T_2, T_1, \Pi_1)$	287	$(Q_7, F_{3,1,5/3}, T_2, \Pi_7)$
288	$(Q_7, F_{3,3,5/3}, U, \Pi_7)$	289	$(Q_7, F_{3,3,5/3}, T_2, \Pi_7)$	290	$(T_1, Q_1, F_{3,4,-5/6}, \zeta)$
291	$(T_1, F_{3,4,1/6}, Q_5, \zeta)$	292 -	$(U, Q_7, Q_1, S_{3,3,2/3})$	293	$(Q_1, T_2, T_1, S_{3,4,1/6})$
294	$(T_2, Q_7, Q_1, S_{3,3,2/3})$				
295 -	$(E, \Delta_3, \Delta_1, \mathcal{S}_1)$	296 -	$(\Sigma_1, \Delta_3, \Delta_1, \mathcal{S}_1)$	297 -	(D, Q_1, Q_5, ω_1)
298	$(T_1, Q_1, Q_5, \omega_1)$	299 -	(U, Q_7, Q_1, ω_2)	300	$(T_2, Q_7, Q_1, \omega_2)$
301	(Q_1, U, T_1, Π_1)	302	(Q_1, T_2, D, Π_1)	303	(Q_1, T_2, T_1, Π_1)
304 -	(D, Q_1, Q_5, ζ)	305	(T_1, Q_1, Q_5, ζ)		

T-3 Exit

306	$(F_{1,2,5/2}, \mathcal{S}_2, S_{1,3,3})$	307	$(F_{1,4,1/2}, \Xi, S_{1,5,1})$	308	$(F_{1,5,1}, \Theta_1, S_{1,6,3/2})$
309	$(F_{1,5,0}, \Theta_1, \Theta_1)$	310	$(F_{1,5,0}, \Theta_1, S_{1,6,1/2})$	311	$(F_{1,3,2}, \Theta_3, S_{1,2,5/2})$
312	$(F_{1,3,2}, \Theta_3, S_{1,4,5/2})$	313	$(F_{1,5,2}, \Theta_3, S_{1,4,5/2})$	314	$(F_{1,5,2}, \Theta_3, S_{1,6,5/2})$
315	$(F_{1,5,1}, S_{1,6,1/2}, \Theta_3)$	316	$(F_{3,2,-11/6}, \omega_4, S_{3,3,-7/3})$	317	$(F_{3,1,5/3}, \Pi_7, S_{3,2,13/6})$
318	$(F_{3,3,5/3}, \Pi_7, S_{3,2,13/6})$	319	$(F_{3,3,5/3}, \Pi_7, S_{3,4,13/6})$	320	$(F_{3,4,1/6}, \zeta, S_{3,3,2/3})$
321	$(F_{3,4,1/6}, \zeta, S_{3,5,2/3})$	322	$(F_{3,4,-5/6}, \zeta, S_{3,3,-4/3})$	323	$(F_{3,4,-5/6}, \zeta, S_{3,5,-4/3})$
324	$(F_{6,2,5/6}, \Omega_1, S_{6,3,4/3})$	325	$(F_{6,2,-1/6}, S_{6,3,-2/3}, \Omega_1)$	326	$(F_{6,2,-7/6}, \Omega_2, S_{6,3,-5/3})$
327	$(F_{6,2,11/6}, \Omega_4, S_{6,3,7/3})$	328	$(F_{6,2,5/6}, \Upsilon, S_{6,3,4/3})$	329 *	$(F_{6,4,5/6}, \Upsilon, S_{6,3,4/3})$
330 *	$(F_{6,4,5/6}, \Upsilon, S_{6,5,4/3})$	331	$(F_{6,2,-1/6}, S_{6,3,-2/3}, \Upsilon)$	332 *	$(F_{6,4,-1/6}, S_{6,3,-2/3}, \Upsilon)$
333 *	$(F_{6,4,-1/6}, S_{6,5,-2/3}, \Upsilon)$	334	$(F_{8,1,1}, \Phi, S_{8,2,3/2})$	335	$(F_{8,3,1}, \Phi, S_{8,2,3/2})$
336	$(F_{8,3,1}, \Phi, S_{8,4,3/2})$	337 -	$(F_{8,1,0}, \Phi, \Phi)$	338	$(F_{8,3,0}, \Phi, \Phi)$
339	$(F_{8,3,0}, \Phi, S_{8,4,1/2})$	340	$(Q_7, S_{3,3,2/3}, S_{3,1,5/3})$	341	$(Q_7, S_{3,3,2/3}, S_{3,3,5/3})$
342	$(T_1, S_{3,2,-5/6}, S_{3,4,1/6})$	343	$(T_1, S_{3,4,-5/6}, S_{3,4,1/6})$	344	$(T_2, S_{3,4,1/6}, S_{3,4,7/6})$
345 -	$(\Delta_3, \mathcal{S}_1, S_{1,3,2})$	346 -	$(E, \varphi, S_{1,2,3/2})$	347 -	$(\Sigma_1, \varphi, S_{1,2,3/2})$
348 -	$(\Sigma_1, \Theta_1, S_{1,2,3/2})$	349	$(F_{1,5,1}, \Theta_1, \Theta_3)$	350 -	$(Q_1, \omega_1, S_{3,3,2/3})$
351 -	$(Q_5, \omega_1, S_{3,3,-4/3})$	352	$(Q_7, \omega_2, S_{3,3,5/3})$	353	$(T_2, \Pi_1, S_{3,4,7/6})$
354 -	$(D, S_{3,2,-5/6}, \Pi_1)$	355 -	$(T_1, S_{3,2,-5/6}, \Pi_1)$	356	$(T_1, S_{3,4,-5/6}, \Pi_1)$
357	$(T_2, S_{3,4,1/6}, \Pi_7)$	358 -	$(Q_1, \zeta, S_{3,3,2/3})$	359 -	$(Q_5, \zeta, S_{3,3,-4/3})$
360	$(F_{6,2,-1/6}, \Omega_2, \Upsilon)$	361	$(F_{6,2,5/6}, \Upsilon, \Omega_4)$		
362 -	$(\Delta_1, \Xi, \mathcal{S}_1)$	363 -	$(\Sigma_1, \varphi, \Theta_3)$	364 -	$(\Sigma_1, \Theta_1, \Theta_3)$
365 -	(Q_1, ζ, ω_2)	366 -	(Q_5, ζ, ω_4)	367 -	(U, Π_1, Π_7)
368 -	(T_2, Π_1, Π_7)				

Table 7. T-I-3 and T-3 models with scalar and fermionic exits.

JHEP08(2022)023

T-I-1 DM-E					
1	$(S_{1,2,1/2}, S_{1,3,1}, S_{1,1,0}, F_{1,2,1/2})$	2	$(S_{1,1,0}, S_{1,2,1/2}, S_{1,2,1/2}, F_{1,1,0})$	3	$(S_{1,1,0}, S_{1,2,1/2}, S_{1,2,1/2}, F_{1,3,0})$
4	$(S_{1,2,1/2}, S_{1,1,1}, S_{1,3,0}, F_{1,2,1/2})$	5	$(S_{1,3,0}, S_{1,2,1/2}, S_{1,2,1/2}, F_{1,1,0})$	6	$(S_{1,3,0}, S_{1,2,1/2}, S_{1,2,1/2}, F_{1,3,0})$
7	$(S_{1,3,0}, S_{1,2,1/2}, S_{1,4,1/2}, F_{1,3,0})$	8	$(S_{1,2,1/2}, S_{1,3,1}, S_{1,3,0}, F_{1,2,1/2})$	9	$(S_{1,2,1/2}, S_{1,3,1}, S_{1,3,0}, F_{1,4,1/2})$
10	$(S_{1,4,1/2}, S_{1,3,1}, S_{1,3,0}, F_{1,2,1/2})$	11	$(S_{1,4,1/2}, S_{1,3,1}, S_{1,3,0}, F_{1,4,1/2})$	12	$(S_{1,4,1/2}, S_{1,5,1}, S_{1,3,0}, F_{1,4,1/2})$
13	$(S_{1,3,0}, S_{1,4,1/2}, S_{1,4,1/2}, F_{1,3,0})$	14	$(S_{1,3,0}, S_{1,4,1/2}, S_{1,4,1/2}, F_{1,5,0})$	15	$(S_{1,4,1/2}, S_{1,3,1}, S_{1,5,0}, F_{1,4,1/2})$
16	$(S_{1,5,0}, S_{1,4,1/2}, S_{1,4,1/2}, F_{1,3,0})$	17	$(S_{1,5,0}, S_{1,4,1/2}, S_{1,4,1/2}, F_{1,5,0})$	18	$(S_{1,5,0}, S_{1,4,1/2}, S_{1,6,1/2}, F_{1,5,0})$
19	$(S_{1,3,1}, S_{1,4,3/2}, S_{1,2,1/2}, F_{1,3,1})$	20	$(S_{1,3,1}, S_{1,4,3/2}, S_{1,4,1/2}, F_{1,3,1})$	21	$(S_{1,3,1}, S_{1,4,3/2}, S_{1,4,1/2}, F_{1,5,1})$
22	$(S_{1,5,1}, S_{1,4,3/2}, S_{1,4,1/2}, F_{1,3,1})$	23	$(S_{1,5,1}, S_{1,4,3/2}, S_{1,4,1/2}, F_{1,5,1})$	24	$(S_{1,1,1}, S_{1,2,3/2}, S_{1,2,1/2}, F_{1,1,1})$
25	$(S_{1,1,1}, S_{1,2,3/2}, S_{1,2,1/2}, F_{1,3,1})$	26	$(S_{1,3,1}, S_{1,2,3/2}, S_{1,2,1/2}, F_{1,1,1})$	27	$(S_{1,3,1}, S_{1,2,3/2}, S_{1,2,1/2}, F_{1,3,1})$
28	$(S_{1,3,1}, S_{1,2,3/2}, S_{1,4,1/2}, F_{1,3,1})$	29	$(S_{1,4,1/2}, S_{1,5,1}, S_{1,5,0}, F_{1,4,1/2})$	30	$(S_{1,4,1/2}, S_{1,5,1}, S_{1,5,0}, F_{1,6,1/2})$
31	$(S_{1,5,1}, S_{1,6,3/2}, S_{1,4,1/2}, F_{1,5,1})$				
T-I-2					
32	$(F_{1,1,0}, F_{1,2,1/2}, S_{1,2,1/2}, S_{1,1,0})$	33	$(F_{1,2,1/2}, F_{1,3,0}, S_{1,1,0}, S_{1,2,1/2})$	34	$(F_{1,2,1/2}, F_{1,3,0}, S_{1,3,0}, S_{1,2,1/2})$
35	$(F_{1,1,0}, F_{1,2,1/2}, S_{1,2,1/2}, S_{1,3,0})$	36	$(F_{1,3,0}, F_{1,4,1/2}, S_{1,2,1/2}, S_{1,3,0})$	37	$(F_{1,2,1/2}, F_{1,3,0}, S_{1,3,0}, S_{1,4,1/2})$
38	$(F_{1,3,0}, F_{1,4,1/2}, S_{1,4,1/2}, S_{1,3,0})$	39	$(F_{1,4,1/2}, F_{1,5,0}, S_{1,3,0}, S_{1,4,1/2})$	40	$(F_{1,4,1/2}, F_{1,5,0}, S_{1,5,0}, S_{1,4,1/2})$
41	$(F_{1,2,1/2}, F_{1,1,1}, S_{1,1,1}, S_{1,2,1/2})$	42	$(F_{1,2,1/2}, F_{1,1,1}, S_{1,3,1}, S_{1,2,1/2})$	43	$(F_{1,2,1/2}, F_{1,3,1}, S_{1,1,1}, S_{1,2,1/2})$
44	$(F_{1,2,1/2}, F_{1,3,1}, S_{1,3,1}, S_{1,2,1/2})$	45	$(F_{1,4,1/2}, F_{1,3,1}, S_{1,3,1}, S_{1,2,1/2})$	46	$(F_{1,2,1/2}, F_{1,3,1}, S_{1,3,1}, S_{1,4,1/2})$
47	$(F_{1,4,1/2}, F_{1,3,1}, S_{1,3,1}, S_{1,4,1/2})$	48	$(F_{1,4,1/2}, F_{1,5,1}, S_{1,3,1}, S_{1,4,1/2})$	49	$(F_{1,4,1/2}, F_{1,3,1}, S_{1,5,1}, S_{1,4,1/2})$
50	$(F_{1,4,1/2}, F_{1,5,1}, S_{1,5,1}, S_{1,4,1/2})$	51	$(F_{1,6,1/2}, F_{1,5,1}, S_{1,5,1}, S_{1,4,1/2})$	52	$(F_{1,3,0}, F_{1,4,1/2}, S_{1,4,1/2}, S_{1,5,0})$
53	$(F_{1,5,0}, F_{1,6,1/2}, S_{1,4,1/2}, S_{1,5,0})$				
T-I-3					
54	$(F_{1,1,0}, F_{1,2,1/2}, F_{1,2,1/2}, S_{1,1,0})$	55	$(F_{1,3,0}, F_{1,2,1/2}, F_{1,2,1/2}, S_{1,1,0})$	56	$(F_{1,1,0}, F_{1,2,1/2}, F_{1,2,1/2}, S_{1,3,0})$
57	$(F_{1,3,0}, F_{1,2,1/2}, F_{1,2,1/2}, S_{1,3,0})$	58	$(F_{1,3,0}, F_{1,2,1/2}, F_{1,4,1/2}, S_{1,3,0})$	59	$(F_{1,3,0}, F_{1,4,1/2}, F_{1,4,1/2}, S_{1,3,0})$
60	$(F_{1,5,0}, F_{1,4,1/2}, F_{1,4,1/2}, S_{1,3,0})$	61	$(F_{1,2,1/2}, F_{1,1,1}, F_{1,3,0}, S_{1,2,1/2})$	62	$(F_{1,2,1/2}, F_{1,3,1}, F_{1,1,0}, S_{1,2,1/2})$
63	$(F_{1,2,1/2}, F_{1,3,1}, F_{1,3,0}, S_{1,2,1/2})$	64	$(F_{1,4,1/2}, F_{1,3,1}, F_{1,3,0}, S_{1,2,1/2})$	65	$(F_{1,2,1/2}, F_{1,3,1}, F_{1,3,0}, S_{1,4,1/2})$
66	$(F_{1,4,1/2}, F_{1,3,1}, F_{1,3,0}, S_{1,4,1/2})$	67	$(F_{1,4,1/2}, F_{1,3,1}, F_{1,5,0}, S_{1,4,1/2})$	68	$(F_{1,4,1/2}, F_{1,5,1}, F_{1,3,0}, S_{1,4,1/2})$
69	$(F_{1,4,1/2}, F_{1,5,1}, F_{1,5,0}, S_{1,4,1/2})$	70	$(F_{1,6,1/2}, F_{1,5,1}, F_{1,5,0}, S_{1,4,1/2})$	71	$(F_{1,3,0}, F_{1,4,1/2}, F_{1,4,1/2}, S_{1,5,0})$
T-3					
72	$(F_{1,2,1/2}, S_{1,1,0}, S_{1,3,1})$	73	$(F_{1,1,1}, S_{1,2,1/2}, S_{1,2,3/2})$	74	$(F_{1,3,1}, S_{1,2,1/2}, S_{1,2,3/2})$
75	$(F_{1,3,1}, S_{1,2,1/2}, S_{1,4,3/2})$	76	$(F_{1,2,1/2}, S_{1,3,0}, S_{1,1,1})$	77	$(F_{1,2,1/2}, S_{1,3,0}, S_{1,3,1})$
78	$(F_{1,4,1/2}, S_{1,3,0}, S_{1,3,1})$	79	$(F_{1,4,1/2}, S_{1,3,0}, S_{1,5,1})$	80	$(F_{1,3,1}, S_{1,4,1/2}, S_{1,2,3/2})$
81	$(F_{1,3,1}, S_{1,4,1/2}, S_{1,4,3/2})$	82	$(F_{1,5,1}, S_{1,4,1/2}, S_{1,4,3/2})$	83	$(F_{1,5,1}, S_{1,4,1/2}, S_{1,6,3/2})$
84	$(F_{1,1,0}, S_{1,2,1/2}, S_{1,2,1/2})$	85	$(F_{1,3,0}, S_{1,2,1/2}, S_{1,2,1/2})$	86	$(F_{1,3,0}, S_{1,2,1/2}, S_{1,4,1/2})$
87	$(F_{1,4,1/2}, S_{1,5,0}, S_{1,3,1})$	88	$(F_{1,3,0}, S_{1,4,1/2}, S_{1,4,1/2})$	89	$(F_{1,5,0}, S_{1,4,1/2}, S_{1,4,1/2})$
90	$(F_{1,5,0}, S_{1,4,1/2}, S_{1,6,1/2})$	91	$(F_{1,5,1}, S_{1,6,1/2}, S_{1,4,3/2})$		

Table 8. DM models with exits (DM-E) which need a stabilizing symmetry to give an acceptable WIMP candidate.

T-I-1 DM-A					
1	$(S_{1,5,0}, S_{1,6,1/2}, S_{1,6,1/2}, F_{1,5,0})$	2	$(S_{1,5,0}, S_{1,6,1/2}, S_{1,6,1/2}, F_{1,7,0})$	3	$(S_{1,6,1/2}, S_{1,5,1}, S_{1,5,0}, F_{1,4,1/2})$
4	$(S_{1,6,1/2}, S_{1,5,1}, S_{1,5,0}, F_{1,6,1/2})$	5	$(S_{1,6,1/2}, S_{1,5,1}, S_{1,7,0}, F_{1,6,1/2})$	6	$(S_{1,6,1/2}, S_{1,7,1}, S_{1,5,0}, F_{1,6,1/2})$
7	$(S_{1,6,1/2}, S_{1,7,1}, S_{1,7,0}, F_{1,6,1/2})$	8	$(S_{1,6,1/2}, S_{1,7,1}, S_{1,7,0}, F_{1,8,1/2})$	9	$(S_{1,7,0}, S_{1,6,1/2}, S_{1,6,1/2}, F_{1,5,0})$
10	$(S_{1,7,0}, S_{1,6,1/2}, S_{1,6,1/2}, F_{1,7,0})$	11	$(S_{1,7,0}, S_{1,6,1/2}, S_{1,8,1/2}, F_{1,7,0})$	12	$(S_{1,7,0}, S_{1,8,1/2}, S_{1,8,1/2}, F_{1,7,0})$
13	$(S_{1,7,0}, S_{1,8,1/2}, S_{1,8,1/2}, F_{1,9,0})$	14	$(S_{1,8,1/2}, S_{1,7,1}, S_{1,7,0}, F_{1,6,1/2})$	15	$(S_{1,8,1/2}, S_{1,7,1}, S_{1,7,0}, F_{1,8,1/2})$
16	$(S_{1,8,1/2}, S_{1,7,1}, S_{1,9,0}, F_{1,8,1/2})$	17	$(S_{1,8,1/2}, S_{1,9,1}, S_{1,7,0}, F_{1,8,1/2})$	18	$(S_{1,8,1/2}, S_{1,9,1}, S_{1,9,0}, F_{1,8,1/2})$
19	$(S_{1,8,1/2}, S_{1,9,1}, S_{1,9,0}, F_{1,10,1/2})$	20	$(S_{1,9,0}, S_{1,8,1/2}, S_{1,8,1/2}, F_{1,7,0})$	21	$(S_{1,9,0}, S_{1,8,1/2}, S_{1,8,1/2}, F_{1,9,0})$
22	$(S_{1,9,0}, S_{1,8,1/2}, S_{1,10,1/2}, F_{1,9,0})$	23	$(S_{1,9,0}, S_{1,10,1/2}, S_{1,10,1/2}, F_{1,9,0})$	24	$(S_{1,9,0}, S_{1,10,1/2}, S_{1,10,1/2}, F_{1,11,0})$
25	$(S_{1,10,1/2}, S_{1,9,1}, S_{1,9,0}, F_{1,8,1/2})$	26	$(S_{1,10,1/2}, S_{1,9,1}, S_{1,9,0}, F_{1,10,1/2})$	27	$(S_{1,10,1/2}, S_{1,9,1}, S_{1,11,0}, F_{1,10,1/2})$
28	$(S_{1,10,1/2}, S_{1,11,1}, S_{1,9,0}, F_{1,10,1/2})$	29	$(S_{1,10,1/2}, S_{1,11,1}, S_{1,11,0}, F_{1,10,1/2})$	30	$(S_{1,10,1/2}, S_{1,11,1}, S_{1,11,0}, F_{1,12,1/2})$
31	$(S_{1,11,0}, S_{1,10,1/2}, S_{1,10,1/2}, F_{1,9,0})$	32	$(S_{1,11,0}, S_{1,10,1/2}, S_{1,10,1/2}, F_{1,11,0})$	33	$(S_{1,11,0}, S_{1,10,1/2}, S_{1,12,1/2}, F_{1,11,0})$
34	$(S_{1,11,0}, S_{1,12,1/2}, S_{1,12,1/2}, F_{1,11,0})$	35	$(S_{1,11,0}, S_{1,12,1/2}, S_{1,12,1/2}, F_{1,13,0})$	36	$(S_{1,12,1/2}, S_{1,11,1}, S_{1,11,0}, F_{1,10,1/2})$
37	$(S_{1,12,1/2}, S_{1,11,1}, S_{1,11,0}, F_{1,12,1/2})$	38	$(S_{1,12,1/2}, S_{1,11,1}, S_{1,13,0}, F_{1,12,1/2})$	39	$(S_{1,12,1/2}, S_{1,13,1}, S_{1,11,0}, F_{1,12,1/2})$
40	$(S_{1,12,1/2}, S_{1,13,1}, S_{1,13,0}, F_{1,12,1/2})$	41	$(S_{1,12,1/2}, S_{1,13,1}, S_{1,13,0}, F_{1,14,1/2})$	42	$(S_{1,13,0}, S_{1,12,1/2}, S_{1,12,1/2}, F_{1,11,0})$
43	$(S_{1,13,0}, S_{1,12,1/2}, S_{1,12,1/2}, F_{1,13,0})$	44	$(S_{1,13,0}, S_{1,12,1/2}, S_{1,14,1/2}, F_{1,13,0})$	45	$(S_{1,13,0}, S_{1,14,1/2}, S_{1,14,1/2}, F_{1,13,0})$
46	$(S_{1,13,0}, S_{1,14,1/2}, S_{1,14,1/2}, F_{1,15,0})$	47	$(S_{1,14,1/2}, S_{1,13,1}, S_{1,13,0}, F_{1,12,1/2})$	48	$(S_{1,14,1/2}, S_{1,13,1}, S_{1,13,0}, F_{1,14,1/2})$
49	$(S_{1,14,1/2}, S_{1,15,1}, S_{1,13,0}, F_{1,14,1/2})$	50	$(S_{1,15,0}, S_{1,14,1/2}, S_{1,14,1/2}, F_{1,13,0})$		
T-I-2					
51	$(F_{1,4,1/2}, F_{1,5,0}, S_{1,5,0}, S_{1,6,1/2})$	52	$(F_{1,5,0}, F_{1,6,1/2}, S_{1,6,1/2}, S_{1,5,0})$	53	$(F_{1,5,0}, F_{1,6,1/2}, S_{1,6,1/2}, S_{1,7,0})$
54	$(F_{1,6,1/2}, F_{1,7,0}, S_{1,5,0}, S_{1,6,1/2})$	55	$(F_{1,6,1/2}, F_{1,7,0}, S_{1,7,0}, S_{1,6,1/2})$	56	$(F_{1,6,1/2}, F_{1,7,0}, S_{1,7,0}, S_{1,8,1/2})$
57	$(F_{1,7,0}, F_{1,8,1/2}, S_{1,6,1/2}, S_{1,7,0})$	58	$(F_{1,7,0}, F_{1,8,1/2}, S_{1,8,1/2}, S_{1,7,0})$	59	$(F_{1,7,0}, F_{1,8,1/2}, S_{1,8,1/2}, S_{1,9,0})$
60	$(F_{1,8,1/2}, F_{1,9,0}, S_{1,7,0}, S_{1,8,1/2})$	61	$(F_{1,8,1/2}, F_{1,9,0}, S_{1,9,0}, S_{1,8,1/2})$	62	$(F_{1,8,1/2}, F_{1,9,0}, S_{1,9,0}, S_{1,10,1/2})$
63	$(F_{1,9,0}, F_{1,10,1/2}, S_{1,8,1/2}, S_{1,9,0})$	64	$(F_{1,9,0}, F_{1,10,1/2}, S_{1,10,1/2}, S_{1,9,0})$	65	$(F_{1,9,0}, F_{1,10,1/2}, S_{1,10,1/2}, S_{1,11,0})$
66	$(F_{1,10,1/2}, F_{1,11,0}, S_{1,9,0}, S_{1,10,1/2})$	67	$(F_{1,10,1/2}, F_{1,11,0}, S_{1,11,0}, S_{1,10,1/2})$	68	$(F_{1,10,1/2}, F_{1,11,0}, S_{1,11,0}, S_{1,12,1/2})$
69	$(F_{1,11,0}, F_{1,12,1/2}, S_{1,10,1/2}, S_{1,11,0})$	70	$(F_{1,11,0}, F_{1,12,1/2}, S_{1,12,1/2}, S_{1,11,0})$	71	$(F_{1,11,0}, F_{1,12,1/2}, S_{1,12,1/2}, S_{1,13,0})$
72	$(F_{1,12,1/2}, F_{1,13,0}, S_{1,11,0}, S_{1,12,1/2})$	73	$(F_{1,12,1/2}, F_{1,13,0}, S_{1,13,0}, S_{1,12,1/2})$	74	$(F_{1,12,1/2}, F_{1,13,0}, S_{1,13,0}, S_{1,14,1/2})$
75	$(F_{1,13,0}, F_{1,14,1/2}, S_{1,12,1/2}, S_{1,13,0})$	76	$(F_{1,13,0}, F_{1,14,1/2}, S_{1,14,1/2}, S_{1,13,0})$	77	$(F_{1,13,0}, F_{1,14,1/2}, S_{1,14,1/2}, S_{1,15,0})$
78	$(F_{1,14,1/2}, F_{1,15,0}, S_{1,13,0}, S_{1,14,1/2})$				
T-I-3					
79	$(F_{1,4,1/2}, F_{1,5,1}, F_{1,5,0}, S_{1,6,1/2})$	80	$(F_{1,5,0}, F_{1,4,1/2}, F_{1,4,1/2}, S_{1,5,0})$	81	$(F_{1,5,0}, F_{1,4,1/2}, F_{1,6,1/2}, S_{1,5,0})$
82	$(F_{1,5,0}, F_{1,6,1/2}, F_{1,6,1/2}, S_{1,5,0})$	83	$(F_{1,5,0}, F_{1,6,1/2}, F_{1,6,1/2}, S_{1,7,0})$	84	$(F_{1,6,1/2}, F_{1,5,1}, F_{1,5,0}, S_{1,6,1/2})$
85	$(F_{1,6,1/2}, F_{1,5,1}, F_{1,7,0}, S_{1,6,1/2})$	86	$(F_{1,6,1/2}, F_{1,7,1}, F_{1,5,0}, S_{1,6,1/2})$	87	$(F_{1,6,1/2}, F_{1,7,1}, F_{1,7,0}, S_{1,6,1/2})$
88	$(F_{1,6,1/2}, F_{1,7,1}, F_{1,7,0}, S_{1,8,1/2})$	89	$(F_{1,7,0}, F_{1,6,1/2}, F_{1,6,1/2}, S_{1,5,0})$	90	$(F_{1,7,0}, F_{1,6,1/2}, F_{1,6,1/2}, S_{1,7,0})$
91	$(F_{1,7,0}, F_{1,6,1/2}, F_{1,8,1/2}, S_{1,7,0})$	92	$(F_{1,7,0}, F_{1,8,1/2}, F_{1,8,1/2}, S_{1,7,0})$	93	$(F_{1,7,0}, F_{1,8,1/2}, F_{1,8,1/2}, S_{1,9,0})$
94	$(F_{1,8,1/2}, F_{1,7,1}, F_{1,7,0}, S_{1,6,1/2})$	95	$(F_{1,8,1/2}, F_{1,7,1}, F_{1,7,0}, S_{1,8,1/2})$	96	$(F_{1,8,1/2}, F_{1,7,1}, F_{1,9,0}, S_{1,8,1/2})$
97	$(F_{1,8,1/2}, F_{1,9,1}, F_{1,7,0}, S_{1,8,1/2})$	98	$(F_{1,8,1/2}, F_{1,9,1}, F_{1,9,0}, S_{1,8,1/2})$	99	$(F_{1,8,1/2}, F_{1,9,1}, F_{1,9,0}, S_{1,10,1/2})$
100	$(F_{1,9,0}, F_{1,8,1/2}, F_{1,8,1/2}, S_{1,7,0})$	101	$(F_{1,9,0}, F_{1,8,1/2}, F_{1,8,1/2}, S_{1,9,0})$	102	$(F_{1,9,0}, F_{1,8,1/2}, F_{1,10,1/2}, S_{1,9,0})$
103	$(F_{1,9,0}, F_{1,10,1/2}, F_{1,10,1/2}, S_{1,9,0})$	104	$(F_{1,9,0}, F_{1,10,1/2}, F_{1,10,1/2}, S_{1,11,0})$	105	$(F_{1,10,1/2}, F_{1,9,1}, F_{1,9,0}, S_{1,8,1/2})$
106	$(F_{1,10,1/2}, F_{1,9,1}, F_{1,9,0}, S_{1,10,1/2})$	107	$(F_{1,10,1/2}, F_{1,9,1}, F_{1,11,0}, S_{1,10,1/2})$	108	$(F_{1,10,1/2}, F_{1,11,1}, F_{1,9,0}, S_{1,10,1/2})$
109	$(F_{1,10,1/2}, F_{1,11,1}, F_{1,11,0}, S_{1,10,1/2})$	110	$(F_{1,10,1/2}, F_{1,11,1}, F_{1,11,0}, S_{1,12,1/2})$	111	$(F_{1,11,0}, F_{1,10,1/2}, F_{1,10,1/2}, S_{1,9,0})$
112	$(F_{1,11,0}, F_{1,10,1/2}, F_{1,10,1/2}, S_{1,11,0})$	113	$(F_{1,11,0}, F_{1,10,1/2}, F_{1,12,1/2}, S_{1,11,0})$	114	$(F_{1,11,0}, F_{1,12,1/2}, F_{1,12,1/2}, S_{1,11,0})$
115	$(F_{1,11,0}, F_{1,12,1/2}, F_{1,12,1/2}, S_{1,13,0})$	116	$(F_{1,12,1/2}, F_{1,11,1}, F_{1,11,0}, S_{1,10,1/2})$	117	$(F_{1,12,1/2}, F_{1,11,1}, F_{1,11,0}, S_{1,12,1/2})$
118	$(F_{1,12,1/2}, F_{1,11,1}, F_{1,13,0}, S_{1,12,1/2})$	119	$(F_{1,12,1/2}, F_{1,13,1}, F_{1,11,0}, S_{1,12,1/2})$	120	$(F_{1,12,1/2}, F_{1,13,1}, F_{1,13,0}, S_{1,12,1/2})$
121	$(F_{1,12,1/2}, F_{1,13,1}, F_{1,13,0}, S_{1,14,1/2})$	122	$(F_{1,13,0}, F_{1,12,1/2}, F_{1,12,1/2}, S_{1,11,0})$	123	$(F_{1,13,0}, F_{1,12,1/2}, F_{1,12,1/2}, S_{1,13,0})$
124	$(F_{1,13,0}, F_{1,12,1/2}, F_{1,14,1/2}, S_{1,13,0})$	125	$(F_{1,13,0}, F_{1,14,1/2}, F_{1,14,1/2}, S_{1,13,0})$	126	$(F_{1,13,0}, F_{1,14,1/2}, F_{1,14,1/2}, S_{1,15,0})$
127	$(F_{1,14,1/2}, F_{1,13,1}, F_{1,13,0}, S_{1,12,1/2})$	128	$(F_{1,14,1/2}, F_{1,13,1}, F_{1,13,0}, S_{1,14,1/2})$	129	$(F_{1,14,1/2}, F_{1,15,1}, F_{1,13,0}, S_{1,14,1/2})$
130	$(F_{1,15,0}, F_{1,14,1/2}, F_{1,14,1/2}, S_{1,13,0})$				
T-3					
131	$(F_{1,4,1/2}, S_{1,5,0}, S_{1,5,1})$	132	$(F_{1,5,1}, S_{1,6,1/2}, S_{1,6,3/2})$	133	$(F_{1,5,0}, S_{1,6,1/2}, S_{1,6,1/2})$
134	$(F_{1,6,1/2}, S_{1,5,0}, S_{1,5,1})$	135	$(F_{1,6,1/2}, S_{1,5,0}, S_{1,7,1})$	136	$(F_{1,6,1/2}, S_{1,7,0}, S_{1,5,1})$
137	$(F_{1,6,1/2}, S_{1,7,0}, S_{1,7,1})$	138	$(F_{1,7,1}, S_{1,6,1/2}, S_{1,6,3/2})$	139	$(F_{1,7,1}, S_{1,6,1/2}, S_{1,8,3/2})$
140	$(F_{1,7,1}, S_{1,8,1/2}, S_{1,6,3/2})$	141	$(F_{1,7,1}, S_{1,8,1/2}, S_{1,8,3/2})$	142	$(F_{1,7,0}, S_{1,6,1/2}, S_{1,6,1/2})$
143	$(F_{1,7,0}, S_{1,6,1/2}, S_{1,8,1/2})$	144	$(F_{1,7,0}, S_{1,8,1/2}, S_{1,8,1/2})$	145	$(F_{1,8,1/2}, S_{1,7,0}, S_{1,7,1})$
146	$(F_{1,8,1/2}, S_{1,7,0}, S_{1,9,1})$	147	$(F_{1,8,1/2}, S_{1,9,0}, S_{1,7,1})$	148	$(F_{1,8,1/2}, S_{1,9,0}, S_{1,9,1})$
149	$(F_{1,9,1}, S_{1,8,1/2}, S_{1,8,3/2})$	150	$(F_{1,9,1}, S_{1,8,1/2}, S_{1,10,3/2})$	151	$(F_{1,9,1}, S_{1,10,1/2}, S_{1,8,3/2})$
152	$(F_{1,9,1}, S_{1,10,1/2}, S_{1,10,3/2})$	153	$(F_{1,9,0}, S_{1,8,1/2}, S_{1,8,1/2})$	154	$(F_{1,9,0}, S_{1,8,1/2}, S_{1,10,1/2})$
155	$(F_{1,9,0}, S_{1,10,1/2}, S_{1,10,1/2})$	156	$(F_{1,10,1/2}, S_{1,9,0}, S_{1,9,1})$	157	$(F_{1,10,1/2}, S_{1,9,0}, S_{1,11,1})$
158	$(F_{1,10,1/2}, S_{1,11,0}, S_{1,9,1})$	159	$(F_{1,10,1/2}, S_{1,11,0}, S_{1,11,1})$	160	$(F_{1,11,1}, S_{1,10,1/2}, S_{1,10,3/2})$
161	$(F_{1,11,1}, S_{1,10,1/2}, S_{1,12,3/2})$	162	$(F_{1,11,1}, S_{1,12,1/2}, S_{1,10,3/2})$	163	$(F_{1,11,1}, S_{1,12,1/2}, S_{1,12,3/2})$
164	$(F_{1,11,0}, S_{1,10,1/2}, S_{1,10,1/2})$	165	$(F_{1,11,0}, S_{1,10,1/2}, S_{1,12,1/2})$	166	$(F_{1,11,0}, S_{1,12,1/2}, S_{1,12,1/2})$
167	$(F_{1,12,1/2}, S_{1,11,0}, S_{1,11,1})$	168	$(F_{1,12,1/2}, S_{1,11,0}, S_{1,13,1})$	169	$(F_{1,12,1/2}, S_{1,13,0}, S_{1,11,1})$
170	$(F_{1,12,1/2}, S_{1,13,0}, S_{1,13,1})$	171	$(F_{1,13,1}, S_{1,12,1/2}, S_{1,12,3/2})$	172	$(F_{1,13,1}, S_{1,12,1/2}, S_{1,14,3/2})$
173	$(F_{1,13,0}, S_{1,12,1/2}, S_{1,12,1/2})$	174	$(F_{1,13,0}, S_{1,12,1/2}, S_{1,14,1/2})$	175	$(F_{1,13,0}, S_{1,14,1/2}, S_{1,14,1/2})$
176	$(F_{1,14,1/2}, S_{1,13,0}, S_{1,13,1})$	177	$(F_{1,14,1/2}, S_{1,13,0}, S_{1,15,1})$		

Table 9. Models in which the DM could be stable due to accidental symmetry (DM-A).

JHEP08(2022)023

T-I-2 DM-E (exceptional)					
1	$(F_{1,3,1}, F_{1,2,3/2}, S_{1,2,3/2}, S_{1,1,1})$	2	$(F_{1,3,1}, F_{1,2,3/2}, S_{1,2,3/2}, S_{1,3,1})$	3	$(F_{1,3,1}, F_{1,2,3/2}, S_{1,4,3/2}, S_{1,3,1})$
4	$(F_{1,3,1}, F_{1,4,3/2}, S_{1,2,3/2}, S_{1,3,1})$	5	$(F_{1,3,1}, F_{1,4,3/2}, S_{1,4,3/2}, S_{1,3,1})$	6	$(F_{1,3,1}, F_{1,4,3/2}, S_{1,4,3/2}, S_{1,5,1})$
7	$(F_{1,5,1}, F_{1,4,3/2}, S_{1,4,3/2}, S_{1,3,1})$				
T-I-3					
8	$(F_{1,2,3/2}, F_{1,1,2}, F_{1,3,1}, S_{1,2,3/2})$	9	$(F_{1,2,3/2}, F_{1,3,2}, F_{1,3,1}, S_{1,2,3/2})$	10	$(F_{1,2,3/2}, F_{1,3,2}, F_{1,3,1}, S_{1,4,3/2})$
11	$(F_{1,3,1}, F_{1,2,3/2}, F_{1,2,1/2}, S_{1,1,1})$	12	$(F_{1,3,1}, F_{1,2,3/2}, F_{1,2,1/2}, S_{1,3,1})$	13	$(F_{1,3,1}, F_{1,2,3/2}, F_{1,4,1/2}, S_{1,3,1})$
14	$(F_{1,3,1}, F_{1,4,3/2}, F_{1,2,1/2}, S_{1,3,1})$	15	$(F_{1,3,1}, F_{1,4,3/2}, F_{1,4,1/2}, S_{1,3,1})$	16	$(F_{1,3,1}, F_{1,4,3/2}, F_{1,4,1/2}, S_{1,5,1})$
17	$(F_{1,4,3/2}, F_{1,3,2}, F_{1,3,1}, S_{1,2,3/2})$	18	$(F_{1,4,3/2}, F_{1,3,2}, F_{1,3,1}, S_{1,4,3/2})$	19	$(F_{1,4,3/2}, F_{1,5,2}, F_{1,3,1}, S_{1,4,3/2})$
20	$(F_{1,5,1}, F_{1,4,3/2}, F_{1,4,1/2}, S_{1,3,1})$				
T-3					
21	$(F_{1,2,3/2}, S_{1,3,1}, S_{1,1,2})$	22	$(F_{1,2,3/2}, S_{1,3,1}, S_{1,3,2})$	23	$(F_{1,4,3/2}, S_{1,3,1}, S_{1,3,2})$
24	$(F_{1,4,3/2}, S_{1,3,1}, S_{1,5,2})$				

T-I-1 DM-A (exceptional)					
1	$(S_{1,5,1}, S_{1,4,3/2}, S_{1,6,1/2}, F_{1,5,1})$	2	$(S_{1,5,1}, S_{1,6,3/2}, S_{1,6,1/2}, F_{1,5,1})$	3	$(S_{1,7,1}, S_{1,6,3/2}, S_{1,6,1/2}, F_{1,5,1})$
T-I-2					
4	$(F_{1,4,1/2}, F_{1,5,1}, S_{1,5,1}, S_{1,6,1/2})$	5	$(F_{1,5,1}, F_{1,4,3/2}, S_{1,4,3/2}, S_{1,5,1})$	6	$(F_{1,5,1}, F_{1,4,3/2}, S_{1,6,3/2}, S_{1,5,1})$
7	$(F_{1,5,1}, F_{1,6,3/2}, S_{1,4,3/2}, S_{1,5,1})$	8	$(F_{1,5,1}, F_{1,6,3/2}, S_{1,6,3/2}, S_{1,5,1})$	9	$(F_{1,5,1}, F_{1,6,3/2}, S_{1,6,3/2}, S_{1,7,1})$
10	$(F_{1,6,1/2}, F_{1,5,1}, S_{1,5,1}, S_{1,6,1/2})$	11	$(F_{1,6,1/2}, F_{1,5,1}, S_{1,7,1}, S_{1,6,1/2})$		
T-I-3					
12	$(F_{1,4,3/2}, F_{1,3,2}, F_{1,5,1}, S_{1,4,3/2})$	13	$(F_{1,4,3/2}, F_{1,5,2}, F_{1,5,1}, S_{1,4,3/2})$	14	$(F_{1,4,3/2}, F_{1,5,2}, F_{1,5,1}, S_{1,6,3/2})$
15	$(F_{1,5,1}, F_{1,4,3/2}, F_{1,4,1/2}, S_{1,5,1})$	16	$(F_{1,5,1}, F_{1,4,3/2}, F_{1,6,1/2}, S_{1,5,1})$	17	$(F_{1,5,1}, F_{1,6,3/2}, F_{1,4,1/2}, S_{1,5,1})$
18	$(F_{1,5,1}, F_{1,6,3/2}, F_{1,6,1/2}, S_{1,5,1})$	19	$(F_{1,5,1}, F_{1,6,3/2}, F_{1,6,1/2}, S_{1,7,1})$	20	$(F_{1,6,3/2}, F_{1,5,2}, F_{1,5,1}, S_{1,4,3/2})$
21	$(F_{1,6,3/2}, F_{1,5,2}, F_{1,5,1}, S_{1,6,3/2})$	22	$(F_{1,6,3/2}, F_{1,7,2}, F_{1,5,1}, S_{1,6,3/2})$		
T-3					
23	$(F_{1,4,3/2}, S_{1,5,1}, S_{1,3,2})$	24	$(F_{1,4,3/2}, S_{1,5,1}, S_{1,5,2})$	25	$(F_{1,6,3/2}, S_{1,5,1}, S_{1,5,2})$
26	$(F_{1,6,3/2}, S_{1,5,1}, S_{1,7,2})$				

Table 10. Exceptional DM candidates with $Y = 1$. The table at the top corresponds to DM models with exits (DM-E) which need a stabilizing symmetry and the table at the bottom to accidental DM models (DM-A).

The symbol * has been placed next to each model where one of the Landau pole scales is very low, i.e $\Lambda_{1,2,3} < 100\,\text{TeV}$. On the other hand the symbol "$-$", placed next to a model, represents models where all Landau pole scales, larger than m_{NP}, are very large $\Lambda_{1,2,3} > 10^{15}\,\text{GeV}$. The symbol † is marked next to the only model that unifies at a scale of $m_{\text{G}} \simeq 10^{17}\,\text{GeV}$.

Acknowledgments

This work is supported by the Spanish grants PID2020-113775GB-I00 (AEI/10.13039/501100011033) and CIPROM/2021/054 (Generalitat Valenciana). J.C.H. acknowledge support from grant ANID FONDECYT-Chile No. 1201673. S.K. is supported by ANID PIA/APOYO AFB180002 (Chile) and by ANID FONDECYT (Chile) No. 1190845. J.C.H. and S.K. acknowledge support from ANID — Programa Milenio — code ICN2019_044. R.C. is supported by the Alexander von Humboldt Foundation Fellowship. C.A. is supported by FONDECYT-Chile grant No. 11180722 and ANID-Chile PIA/APOYO AFB 180002.

Open Access. This article is distributed under the terms of the Creative Commons Attribution License (CC-BY 4.0), which permits any use, distribution and reproduction in any medium, provided the original author(s) and source are credited. SCOAP^3 supports the goals of the International Year of Basic Sciences for Sustainable Development.

References

[1] S. Weinberg, *Baryon and Lepton Nonconserving Processes*, Phys. Rev. Lett. **43** (1979) 1566 [INSPIRE].

[2] E. Ma, *Pathways to naturally small neutrino masses*, Phys. Rev. Lett. **81** (1998) 1171 [hep-ph/9805219] [INSPIRE].

[3] P. Minkowski, *$\mu \to e\gamma$ at a Rate of One Out of 10^9 Muon Decays?*, Phys. Lett. B **67** (1977) 421 [INSPIRE].

[4] T. Yanagida, *Horizontal gauge symmetry and masses of neutrinos*, *Conf. Proc. C* **7902131** (1979) 95 [INSPIRE].

[5] R.N. Mohapatra and G. Senjanović, *Neutrino Mass and Spontaneous Parity Nonconservation*, Phys. Rev. Lett. **44** (1980) 912 [INSPIRE].

[6] M. Gell-Mann, P. Ramond and R. Slansky, *Complex Spinors and Unified Theories*, *Conf. Proc. C* **790927** (1979) 315 [arXiv:1306.4669] [INSPIRE].

[7] J. Schechter and J.W.F. Valle, *Neutrino Masses in* SU(2) × U(1) *Theories*, Phys. Rev. D **22** (1980) 2227 [INSPIRE].

[8] R.N. Mohapatra and G. Senjanović, *Neutrino Masses and Mixings in Gauge Models with Spontaneous Parity Violation*, Phys. Rev. D **23** (1981) 165 [INSPIRE].

[9] R. Foot, H. Lew, X.G. He and G.C. Joshi, *Seesaw Neutrino Masses Induced by a Triplet of Leptons*, Z. Phys. C **44** (1989) 441 [INSPIRE].

[10] A. Zee, *A Theory of Lepton Number Violation, Neutrino Majorana Mass, and Oscillation*, Phys. Lett. B **93** (1980) 389 [*Erratum ibid.* **95** (1980) 461] [INSPIRE].

[11] T.P. Cheng and L.-F. Li, *Neutrino Masses, Mixings and Oscillations in* SU(2) × U(1) *Models of Electroweak Interactions*, Phys. Rev. D **22** (1980) 2860 [INSPIRE].

[12] A. Zee, *Charged Scalar Field and Quantum Number Violations*, Phys. Lett. B **161** (1985) 141 [INSPIRE].

[13] K.S. Babu, *Model of 'Calculable' Majorana Neutrino Masses*, Phys. Lett. B **203** (1988) 132 [INSPIRE].

[14] Y. Cai, J. Herrero-García, M.A. Schmidt, A. Vicente and R.R. Volkas, *From the trees to the forest: a review of radiative neutrino mass models*, Front. Phys. **5** (2017) 63 [arXiv:1706.08524] [INSPIRE].

[15] F. Bonnet, M. Hirsch, T. Ota and W. Winter, *Systematic study of the $d = 5$ Weinberg operator at one-loop order*, JHEP **07** (2012) 153 [arXiv:1204.5862] [INSPIRE].

[16] D. Aristizabal Sierra, A. Degee, L. Dorame and M. Hirsch, *Systematic classification of two-loop realizations of the Weinberg operator*, JHEP **03** (2015) 040 [arXiv:1411.7038] [INSPIRE].

JHEP08(2022)023

[17] R. Cepedello, R. Fonseca and M. Hirsch, *Neutrino masses beyond the minimal seesaw*, *J. Phys. Conf. Ser.* **1468** (2020) 012171 [arXiv:1911.01125] [INSPIRE].

[18] R. Cepedello, R.M. Fonseca and M. Hirsch, *Systematic classification of three-loop realizations of the Weinberg operator*, *JHEP* **10** (2018) 197 [*Erratum JHEP* **06** (2019) 034] [arXiv:1807.00629] [INSPIRE].

[19] D. Restrepo, O. Zapata and C.E. Yaguna, *Models with radiative neutrino masses and viable dark matter candidates*, *JHEP* **11** (2013) 011 [arXiv:1308.3655] [INSPIRE].

[20] K.S. Babu and C.N. Leung, *Classification of effective neutrino mass operators*, *Nucl. Phys. B* **619** (2001) 667 [hep-ph/0106054] [INSPIRE].

[21] A. de Gouvêa and J. Jenkins, *A Survey of Lepton Number Violation Via Effective Operators*, *Phys. Rev. D* **77** (2008) 013008 [arXiv:0708.1344] [INSPIRE].

[22] J. Gargalionis and R.R. Volkas, *Exploding operators for Majorana neutrino masses and beyond*, *JHEP* **01** (2021) 074 [arXiv:2009.13537] [INSPIRE].

[23] P.F. de Salas et al., *2020 global reassessment of the neutrino oscillation picture*, *JHEP* **02** (2021) 071 [arXiv:2006.11237] [INSPIRE].

[24] Particle Data collaboration, *Review of Particle Physics*, *Prog. Theor. Exp. Phys.* **2020** (2020) 083C01 [INSPIRE].

[25] T.K. Hemmick et al., *A Search for Anomalously Heavy Isotopes of Low Z Nuclei*, *Phys. Rev. D* **41** (1990) 2074 [INSPIRE].

[26] A. Kudo and M. Yamaguchi, *Inflation with low reheat temperature and cosmological constraint on stable charged massive particles*, *Phys. Lett. B* **516** (2001) 151 [hep-ph/0103272] [INSPIRE].

[27] M. Taoso, G. Bertone and A. Masiero, *Dark Matter Candidates: A Ten-Point Test*, *JCAP* **03** (2008) 022 [arXiv:0711.4996] [INSPIRE].

[28] M. Cirelli, N. Fornengo and A. Strumia, *Minimal dark matter*, *Nucl. Phys. B* **753** (2006) 178 [hep-ph/0512090] [INSPIRE].

[29] S. Bottaro et al., *Closing the window on WIMP Dark Matter*, *Eur. Phys. J. C* **82** (2022) 31 [arXiv:2107.09688] [INSPIRE].

[30] S. Bottaro et al., *The last Complex WIMPs standing*, arXiv:2205.04486 [INSPIRE].

[31] T. Hambye, F.S. Ling, L. Lopez Honorez and J. Rocher, *Scalar Multiplet Dark Matter*, *JHEP* **07** (2009) 090 [*Erratum JHEP* **05** (2010) 066] [arXiv:0903.4010] [INSPIRE].

[32] A. Belyaev, G. Cacciapaglia, D. Locke and A. Pukhov, *Minimal Consistent Dark Matter models for systematic experimental characterisation: Fermion Dark Matter*, arXiv:2203.03660 [INSPIRE].

[33] Y. Cai, X.-G. He, M. Ramsey-Musolf and L.-H. Tsai, *RνMDM and Lepton Flavor Violation*, *JHEP* **12** (2011) 054 [arXiv:1108.0969] [INSPIRE].

[34] K. Kumericki, I. Picek and B. Radovcic, *Critique of Fermionic RνMDM and its Scalar Variants*, *JHEP* **07** (2012) 039 [arXiv:1204.6597] [INSPIRE].

[35] Y. Cai and M.A. Schmidt, *Revisiting the RνMDM Models*, *JHEP* **05** (2016) 028 [arXiv:1603.00255] [INSPIRE].

JHEP08(2022)023

[36] A. Ahriche, K.L. McDonald, S. Nasri and I. Picek, *A Critical Analysis of One-Loop Neutrino Mass Models with Minimal Dark Matter*, *Phys. Lett. B* **757** (2016) 399 [arXiv:1603.01247] [INSPIRE].

[37] D. Aristizabal Sierra, C. Simoes and D. Wegman, *Closing in on minimal dark matter and radiative neutrino masses*, *JHEP* **06** (2016) 108 [arXiv:1603.04723] [INSPIRE].

[38] J. de Blas, J.C. Criado, M. Pérez-Victoria and J. Santiago, *Effective description of general extensions of the Standard Model: the complete tree-level dictionary*, *JHEP* **03** (2018) 109 [arXiv:1711.10391] [INSPIRE].

[39] W. Buchmüller, R. Ruckl and D. Wyler, *Leptoquarks in Lepton-Quark Collisions*, *Phys. Lett. B* **191** (1987) 442 [*Erratum ibid.* **448** (1999) 320] [INSPIRE].

[40] F. Bonnet, D. Hernandez, T. Ota and W. Winter, *Neutrino masses from higher than $d = 5$ effective operators*, *JHEP* **10** (2009) 076 [arXiv:0907.3143] [INSPIRE].

[41] K.S. Babu, S. Nandi and Z. Tavartkiladze, *New Mechanism for Neutrino Mass Generation and Triply Charged Higgs Bosons at the LHC*, *Phys. Rev. D* **80** (2009) 071702 [arXiv:0905.2710] [INSPIRE].

[42] D. Aristizabal Sierra, M. Hirsch and S.G. Kovalenko, *Leptoquarks: Neutrino masses and accelerator phenomenology*, *Phys. Rev. D* **77** (2008) 055011 [arXiv:0710.5699] [INSPIRE].

[43] M. Hirsch, H.V. Klapdor-Kleingrothaus and S.G. Kovalenko, *New low-energy leptoquark interactions*, *Phys. Lett. B* **378** (1996) 17 [hep-ph/9602305] [INSPIRE].

[44] L.J. Hall and M. Suzuki, *Explicit R-Parity Breaking in Supersymmetric Models*, *Nucl. Phys. B* **231** (1984) 419 [INSPIRE].

[45] F. Bonnet, M. Hirsch, T. Ota and W. Winter, *Systematic decomposition of the neutrinoless double beta decay operator*, *JHEP* **03** (2013) 055 [*Erratum JHEP* **04** (2014) 090] [arXiv:1212.3045] [INSPIRE].

[46] J.C. Helo, M. Hirsch, T. Ota and F.A. Pereira dos Santos, *Double beta decay and neutrino mass models*, *JHEP* **05** (2015) 092 [arXiv:1502.05188] [INSPIRE].

[47] E. Ma, *Verifiable radiative seesaw mechanism of neutrino mass and dark matter*, *Phys. Rev. D* **73** (2006) 077301 [hep-ph/0601225] [INSPIRE].

[48] L. Di Luzio, R. Gröber, J.F. Kamenik and M. Nardecchia, *Accidental matter at the LHC*, *JHEP* **07** (2015) 074 [arXiv:1504.00359] [INSPIRE].

[49] XENON collaboration, *Dark Matter Search Results from a One Ton-Year Exposure of XENON1T*, *Phys. Rev. Lett.* **121** (2018) 111302 [arXiv:1805.12562] [INSPIRE].

[50] DARWIN collaboration, *DARWIN: towards the ultimate dark matter detector*, *JCAP* **11** (2016) 017 [arXiv:1606.07001] [INSPIRE].

[51] C. Hagedorn, T. Ohlsson, S. Riad and M.A. Schmidt, *Unification of Gauge Couplings in Radiative Neutrino Mass Models*, *JHEP* **09** (2016) 111 [arXiv:1605.03986] [INSPIRE].

[52] J.L. Evans, T.T. Yanagida and N. Yokozaki, *W boson mass anomaly and grand unification*, arXiv:2205.03877 [INSPIRE].

[53] E. Ma and J. Wudka, *Vector-Boson-Induced Neutrino Mass*, *Phys. Lett. B* **712** (2012) 391 [arXiv:1202.3098] [INSPIRE].

JHEP08(2022)023

[54] F.F. Deppisch, S. Kulkarni, H. Päs and E. Schumacher, *Leptoquark patterns unifying neutrino masses, flavor anomalies, and the diphoton excess*, *Phys. Rev. D* **94** (2016) 013003 [arXiv:1603.07672] [INSPIRE].

[55] R.M. Fonseca and M. Hirsch, *Lepton number violation in 331 models*, *Phys. Rev. D* **94** (2016) 115003 [arXiv:1607.06328] [INSPIRE].

[56] R.M. Fonseca and M. Hirsch, *Gauge vectors and double beta decay*, *Phys. Rev. D* **95** (2017) 035033 [arXiv:1612.04272] [INSPIRE].

[57] J.C. Pati and A. Salam, *Lepton Number as the Fourth Color*, *Phys. Rev. D* **10** (1974) 275 [*Erratum ibid.* **11** (1975) 703] [INSPIRE].

[58] S.M. Barr, *A New Symmetry Breaking Pattern for* SO(10) *and Proton Decay*, *Phys. Lett. B* **112** (1982) 219 [INSPIRE].

[59] B.C. Allanach, B. Gripaios and J. Tooby-Smith, *Semisimple extensions of the Standard Model gauge algebra*, *Phys. Rev. D* **104** (2021) 035035 [arXiv:2104.14555] [INSPIRE].

[60] Y. Cai, J.D. Clarke, M.A. Schmidt and R.R. Volkas, *Testing Radiative Neutrino Mass Models at the LHC*, *JHEP* **02** (2015) 161 [arXiv:1410.0689] [INSPIRE].

[61] K. Ghosh, S. Jana and S. Nandi, *Neutrino Mass Generation at TeV Scale and New Physics Signatures from Charged Higgs at the LHC for Photon Initiated Processes*, *JHEP* **03** (2018) 180 [arXiv:1705.01121] [INSPIRE].

[62] T. Nomura and H. Okada, *An Extended Colored Zee-Babu Model*, *Phys. Rev. D* **94** (2016) 075021 [arXiv:1607.04952] [INSPIRE].

[63] S. Khan, M. Mitra and A. Patra, *Neutrino and Collider Implications of a Left-Right Extended Zee Model*, *Phys. Rev. D* **98** (2018) 115038 [arXiv:1805.09844] [INSPIRE].

[64] M. Hirsch, *Neutrinos at colliders and lepton number violation*, *PoS* **ALPS2019** (2020) 015 [INSPIRE].

[65] C. Arbeláez, G. Cottin, J.C. Helo and M. Hirsch, *Long-lived charged particles and multi-lepton signatures from neutrino mass models*, *Phys. Rev. D* **101** (2020) 095033 [arXiv:2003.11494] [INSPIRE].

[66] Avnish and K. Ghosh, *Multi-charged TeV scale scalars and fermions in the framework of a radiative seesaw model*, arXiv:2007.01766 [INSPIRE].

[67] J. Gargalionis, I. Popa-Mateiu and R.R. Volkas, *Radiative neutrino mass model from a mass dimension-11* $\Delta L = 2$ *effective operator*, *JHEP* **03** (2020) 150 [arXiv:1912.12386] [INSPIRE].

[68] S. Ashanujjaman, D. Choudhury and K. Ghosh, *Search for exotic leptons in final states with two or three leptons and fat-jets at* 13 *TeV LHC*, *JHEP* **04** (2022) 150 [arXiv:2201.09645] [INSPIRE].

JHEP08(2022)023