

## $B - L$ as a gauged Peccei-Quinn symmetry

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**ABSTRACT:** The gauged Peccei-Quinn (PQ) mechanism provides a simple prescription to embed the global PQ symmetry into a gauged U(1) symmetry. As it originates from the gauged PQ symmetry, the global PQ symmetry can be protected from explicit breaking by quantum gravitational effects once appropriate charge assignment is given. In this paper, we identify the gauged PQ symmetry with the  $B - L$  symmetry, which is obviously attractive as the  $B - L$  gauge symmetry is the most authentic extension of the Standard Model. As we will show, a natural  $B - L$  charge assignment can be found in a model motivated by the seesaw mechanism in the SU(5) Grand Unified Theory. As a notable feature of this model, it does not require extra SU(5) singlet matter fields other than the right-handed neutrinos to cancel the self and the gravitational anomalies.

**KEYWORDS:** Beyond Standard Model, Gauge Symmetry, Global Symmetries

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## 1 Introduction

The strong CP problem is longstanding and probably one of the most puzzling issues in particle physics. Although the Peccei-Quinn (PQ) mechanism [1–4] provides a successful solution to the problem, it is not very satisfactory from a theoretical point of view, as it relies on a global Peccei-Quinn  $U(1)$  symmetry. The Peccei-Quinn symmetry is required to be almost exact but explicitly broken by the QCD anomaly. Even tiny explicit breaking terms of the PQ symmetry spoil the PQ mechanism. It is, on the other hand, conceived that any global symmetries are broken by quantum gravity effects [5–10].<sup>1</sup> Thus, the PQ mechanism brings up another question, the existence of such an almost but not exact global symmetry.

In [16], a general prescription to achieve a desirable PQ symmetry is proposed in which the PQ symmetry originates from a gauged  $U(1)$  symmetry,  $U(1)_{\text{gPQ}}$ . The anomalies of  $U(1)_{\text{gPQ}}$  are canceled between the contributions from two (or more) PQ charged sectors, while the inter-sector interactions between the PQ charged sectors are highly suppressed by appropriate  $U(1)_{\text{gPQ}}$  charge assignment. As a result of the separation, a global PQ symmetry exist in addition to  $U(1)_{\text{gPQ}}$  as an accidental symmetry. The accidental PQ symmetry is highly protected from explicit breaking by quantum gravitational effects as it originates from the gauge symmetry. The gauged PQ mechanism is a generalization of

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<sup>1</sup>The direct non-perturbative effects on the axion potential of quantum gravity in ref. [11] do not spoil the gauged-PQ mechanism as the vacuum expectation values of the PQ-symmetry is much smaller enough than the Planck scale. It should be also noted that the non-perturbative explicit breaking effects of quantum gravity can be suppressed by the higher curvature topological terms if the coefficients of them are large [12–15].

the mechanisms which achieve the PQ symmetry as an accidental symmetry resulting from (discrete) gauge symmetries [17–25].

In this paper, we discuss whether the  $B-L$  symmetry can play a role of the gauged PQ symmetry. The  $B-L$  gauge symmetry is the most authentic extension of the Standard Model (SM) which explains the tiny neutrino masses via the seesaw mechanism [26–28] (see also [29]). Therefore, the identification of the gauged PQ symmetry with  $B-L$  makes the gauged PQ mechanism more plausible. An intriguing coincidence between the right-handed neutrino mass scale appropriate for the thermal leptogenesis [30] (see [31–33], for review) and the PQ breaking scale which avoids astrophysical constraints also motivates this identification [34].

As will be shown, we find a natural  $B-L$  charge assignment motivated by the seesaw mechanism in the SU(5) Grand Unified Theory (GUT), with which the gauged PQ mechanism is achieved. Notably, the charge assignment we find does not require extra SU(5) singlet matter fields other than the right-handed neutrinos to cancel the  $[U(1)_{\text{gPQ}}]^3$  and the gravitational anomalies.

The organization of the paper is as follows. In section 2, we discuss an appropriate  $B-L$  charge assignment so that it plays a role of  $U(1)_{\text{gPQ}}$ . In section 3, we discuss the properties of the axion and the global PQ symmetry. In section 4, we briefly discuss the domain wall problem. In section 5, we discuss supersymmetric (SUSY) extension of the model. The final section is devoted to our conclusions.

## 2 $B-L$ as a gauged PQ symmetry

Among the various extension of the Standard Model,  $B-L$  is the most plausible addition. The anomalies of the  $B-L$  gauge symmetry are canceled by simply introducing three SM singlet right-handed neutrinos  $\bar{N}_R$ . The  $B-L$  extended Standard Model naturally implements the seesaw mechanism by the spontaneous breaking of  $B-L$  at the intermediate scale.

Having the SU(5) GUT in mind, it is more convenient to consider “fiveness”,  $5(B-L) - 4Y$ , instead of  $B-L$ , as it commutes with the SU(5) gauge group. The fiveness charges of the matter fields are given by

$$\mathbf{10}_{\text{SM}}(+1), \quad \bar{\mathbf{5}}_{\text{SM}}(-3), \quad \bar{N}_R(+5), \quad (2.1)$$

while the Higgs doublet,  $h$ , has a charge  $+2$  (i.e.  $B-L = 0$ ).<sup>2</sup> Here, we use the SU(5) GUT representations for the matter fields, i.e.  $\mathbf{10}_{\text{SM}} = (q_L, \bar{u}_R, \bar{e}_R)$  and  $\bar{\mathbf{5}}_{\text{SM}} = (\bar{d}_R, \ell_L)$ , while  $\bar{N}_R$  denotes the right-handed neutrinos.

The seesaw mechanism is implemented by assuming that the right-handed neutrinos obtain Majorana masses from spontaneous breaking of fiveness. In this paper, we assume that the Majorana masses are provided by the vacuum expectation value (VEV) of a gauge singlet scalar field with fiveness,  $-10$ , i.e.,

$$\phi(-10), \quad (2.2)$$

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<sup>2</sup>The colored Higgs is assumed to obtain a mass of the GUT scale.

which couples to the right handed neutrinos,

$$\mathcal{L} = -\frac{1}{2}y_N\phi\bar{N}_R\bar{N}_R + \text{h.c.} . \tag{2.3}$$

Here,  $y_N$  denotes a coupling constant, with which the Majorana mass is given by  $M_N = y_N \langle \phi \rangle$ . By integrating out the right-handed neutrinos, the tiny neutrino masses are obtained, via

$$\mathcal{L} = y_\ell \bar{\mathbf{5}}_{\text{SM}} \bar{N}_R h^* + \text{h.c.}, \tag{2.4}$$

where  $y_\ell$  also denotes a coupling constant.

Now, let us identify the gauged PQ symmetry with  $B - L$ , i.e., fiveness. Following the general prescription of the gauged PQ mechanism in [16], let us introduce extra matter multiplets which obtain a mass from the VEV of  $\phi$ ;

$$\mathcal{L} = y_K \phi^* \mathbf{5}_K \bar{\mathbf{5}}_K + \text{h.c.}, \tag{2.5}$$

with  $y_K$  being a coupling constant.<sup>3</sup> Here, the extra multiplets ( $\mathbf{5}_K, \bar{\mathbf{5}}_K$ ) are assumed to form the  $\mathbf{5}$  and  $\bar{\mathbf{5}}$  representations of the SU(5) gauge group, respectively. As in the KSVZ axion model [35, 36], the Ward identity of the fiveness current,  $j_5$ , obtains an anomalous contribution from the extra multiplets,

$$\partial j_5|_{\text{SM+N+K}} = -\frac{g_a^2}{32\pi^2} 10 F^a \tilde{F}^a . \tag{2.6}$$

Here,  $F^a$  ( $a = 1, 2, 3$ ) are the gauge field strengths of the Standard Model and  $g_a$  the corresponding SM gauge coupling constants. The Lorentz indices and the gauge group representation indices are suppressed. The factor  $-10$  corresponds to the charge of the bi-linear,  $\mathbf{5}_K \bar{\mathbf{5}}_K$  (see eq. (2.5)).

In the gauged PQ mechanism, the  $U(1)_{\text{gPQ}}$  gauge anomalies are canceled by a contribution from another set of the PQ charged sector. For that purpose, let us also introduce 10-flavors of extra matter multiplets ( $\mathbf{5}'_K, \bar{\mathbf{5}}'_K$ ). We assume that they obtain masses from a VEV of a complex scalar field  $\phi'$  whose fiveness charge is  $+1$ ;

$$\mathcal{L} = y'_K \phi'^* \mathbf{5}'_K \bar{\mathbf{5}}'_K + \text{h.c.}, \tag{2.7}$$

where the charge of the bi-linear,  $\mathbf{5}'_K \bar{\mathbf{5}}'_K$ , is set to be  $+1$ . With this choice, the anomalous contributions of the Ward identity in (2.6) are canceled by the one from ( $\mathbf{5}'_K, \bar{\mathbf{5}}'_K$ ), i.e.,

$$\partial j_5|_{\text{SM+N+K+K}'} = 0 . \tag{2.8}$$

The fiveness charges of the respective extra multiplets are chosen as follows. To avoid stable extra matter fields, we assume that  $\bar{\mathbf{5}}_K$  and  $\bar{\mathbf{5}}'_K$  can mix with  $\bar{\mathbf{5}}_{\text{SM}}$ , so that

$$\mathbf{5}_K(-7), \bar{\mathbf{5}}_K(-3), \mathbf{5}'_K(+4), \bar{\mathbf{5}}'_K(-3), \tag{2.9}$$

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<sup>3</sup>The reason why the extra multiplets couple not to  $\phi$  but  $\phi^*$  will become clear shortly.

respectively. As a notable feature of this charge assignment, it cancels the  $[U(1)_{\text{gPQ}}]^3$  and the gravitational anomalies automatically without introducing additional SM singlet fields. In fact, the  $[U(1)_{\text{gPQ}}]^3$  and the gravitational anomalies are proportional to

$$\begin{aligned} [U(1)_{\text{gPQ}}]^3 &\propto ((-10 - \bar{q}_K)^3 + (\bar{q}_K)^3) + 10((1 - \bar{q}'_K)^3 + (\bar{q}'_K)^3), \\ [\text{gravitational}] &\propto ((-10 - \bar{q}_K) + (\bar{q}_K)) + 10((1 - \bar{q}'_K) + (\bar{q}'_K)), \end{aligned} \quad (2.10)$$

with  $\bar{q}_K$  and  $\bar{q}'_K$  are the charges of  $\bar{\mathbf{5}}_K$  and  $\bar{\mathbf{5}}'_K$ , respectively. By substituting  $\bar{q}_K = \bar{q}'_K = -3$ , we find that both the anomalies are vanishing.

The anomaly cancellation without singlet fields other than the right-handed neutrinos is by far advantageous compared with the previous models [16, 17, 37]. The singlet fields required for the anomaly cancellation tend to be rather light and longlived, which make the thermal history of the universe complicated [37]. The anomaly cancellation of the present model is, therefore, a very important success as it is partly motivated by thermal leptogenesis which requires a high reheating temperature after inflation, i.e.,  $T_R \gtrsim 10^9$  GeV [31–33].

Under the fineness symmetry, the interactions are restricted to

$$\begin{aligned} \mathcal{L} = & \mathbf{10}_{\text{SM}}\mathbf{10}_{\text{SM}}h^* + \mathbf{10}_{\text{SM}}\bar{\mathbf{5}}h + \bar{\mathbf{5}}\bar{N}_R h^* - \frac{1}{2}\phi\bar{N}_R\bar{N}_R + \phi^*\mathbf{5}_K\bar{\mathbf{5}} + \phi'^*\mathbf{5}'_K\bar{\mathbf{5}} + \text{h.c.}, \\ & -V(\phi, \phi', h). \end{aligned} \quad (2.11)$$

Here,  $\bar{\mathbf{5}}$  collectively denotes  $(\bar{\mathbf{5}}_{\text{SM}}, \bar{\mathbf{5}}_K, \bar{\mathbf{5}}'_K)$ , and  $V(\phi, \phi', h)$  is the scalar potential. The coupling coefficients are omitted for notational simplicity. At the renormalizable level, the above Lagrangian possesses a global  $U(1)$  symmetry, which is identified with the global PQ symmetry. The global PQ symmetry corresponds to a phase rotation of a gauge invariant combination,  $\phi\phi'^{10}$ , while the other fields are rotated appropriately. The global PQ charges of the individual fields are generically given by

$$Q = -\frac{Q_\phi}{10} \times q_5, \quad Q' = Q_{\phi'} - \frac{3}{10}Q_\phi, \quad (2.12)$$

for  $\{\text{SM}, \bar{N}_R, \mathbf{5}_K, \bar{\mathbf{5}}\}$  and  $\{\mathbf{5}'_K\}$ , respectively. Here,  $q_5$  denotes the fineness charge of each field, and  $Q_{\phi, \phi'}$  are the global PQ charges of  $\phi$  and  $\phi'$  with  $Q_\phi/Q_{\phi'} \neq -10$ , respectively.

The global PQ symmetry is broken by the QCD anomaly. In fact, under the global PQ rotation with a rotation angle  $\alpha_{\text{PQ}}$ ,

$$\phi\phi'^{10} \rightarrow e^{i\alpha_{\text{PQ}}} \times \phi\phi'^{10}, \quad (2.13)$$

the Lagrangian shifts by,

$$\delta\mathcal{L}_{\text{PQ}} = \frac{\alpha_{\text{PQ}}g_a^2}{32\pi^2} F^a \tilde{F}^a. \quad (2.14)$$

It should be noted that the normalization factor of eq. (2.14) is independent of the choice of the global PQ charge assignment for the individual fields.

Since the global PQ symmetry is just an accidental one, it is also broken by the Planck suppressed operators explicitly. However, due to the gauged fineness symmetry, no PQ-symmetry breaking operators such as  $\phi^n$  or  $\phi'^m$  ( $n > 0$ ) are allowed. As a result, the

explicit breaking terms of the global PQ symmetry are highly suppressed, and the lowest dimensional ones are given by,

$$\mathcal{L}_{\mathcal{PQ}} \sim \frac{1}{10!} \frac{\phi\phi'^{10}}{M_{\text{PL}}^7} + \text{h.c.}, \quad (2.15)$$

where  $M_{\text{PL}} \simeq 2.44 \times 10^{18}$  is the reduced Planck scale. As we will see in the next section, the breaking terms are acceptably small not to spoil the PQ mechanism in a certain parameter space.

### 3 Axion and global PQ symmetry

To see the properties of the accidental global PQ symmetry, let us decompose the axion from the would-be Goldstone boson of fiveness. Both of them originate from the phase components of  $\phi$  and  $\phi'$ ;

$$\phi = \frac{1}{\sqrt{2}} f_a e^{ia/f_a}, \quad \phi' = \frac{1}{\sqrt{2}} f_b e^{ib/f_b}, \quad (3.1)$$

where  $f_{a,b}$  are the decay constants and we keep only the Goldstone modes,  $a$  and  $b$ . The domains of the phase components are given

$$\theta_a \equiv a/f_a = [0, 2\pi), \quad \theta_b \equiv b/f_b = [0, 2\pi), \quad (3.2)$$

respectively.

In terms of  $\theta_{a,b}$ , fiveness is realized by,

$$\theta_{a,b} \rightarrow \theta_{a,b} + q_{a,b}\alpha(x) \quad (3.3)$$

$$Y_\mu \rightarrow Y_\mu - \partial_\mu\alpha(x)/g. \quad (3.4)$$

Here,  $\alpha(x)$  denotes a gauge parameter field with  $q_a = -10$  and  $q_b = +1$ ,  $Y_\mu$  the gauge field, and  $g$  the coupling constant, respectively. The gauge invariant effective Lagrangian of the Goldstone modes is given by,

$$\mathcal{L} = \frac{1}{2} f_a^2 D_a^\mu D_{a\mu} + \frac{1}{2} f_b^2 D_b^\mu D_{b\mu}, \quad (3.5)$$

where the covariant derivatives are defined by

$$D_{i\mu} = \partial_\mu\theta_i + gq_i Y_\mu, \quad (i = a, b). \quad (3.6)$$

The gauge invariant axion,  $A$  ( $\propto q_b\theta_a - q_a\theta_b$ ), and the would-be Goldstone mode,  $B$ , are given by

$$\begin{pmatrix} A \\ B \end{pmatrix} = \frac{1}{\sqrt{q_a^2 f_a^2 + q_b^2 f_b^2}} \begin{pmatrix} q_b f_b & -q_a f_a \\ q_a f_a & q_b f_b \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}. \quad (3.7)$$

By using  $A$  and  $B$ , the effective Lagrangian is reduced to,

$$\mathcal{L} = \frac{1}{2} (\partial A)^2 + \frac{1}{2} m_Y^2 \left( Y_\mu - \frac{1}{m_Y} \partial_\mu B \right)^2. \quad (3.8)$$

The second term is the Stückelberg mass term of the gauge boson with  $m_Y$  being the gauge boson mass,

$$m_Y^2 = g^2(q_a^2 f_a^2 + q_b^2 f_b^2). \quad (3.9)$$

Through the mass term, the would-be Goldstone mode  $B$  is absorbed into  $Y_\mu$  by the Higgs mechanism. The effective decay constant of the axion  $A$  is given by,

$$F_A = \frac{f_a f_b}{\sqrt{q_a^2 f_a^2 + q_b^2 f_b^2}}. \quad (3.10)$$

Given  $F_A$ , the domain of the gauge invariant axion is given by

$$\frac{A}{F_A} = [0, 2\pi). \quad (3.11)$$

when  $|q_a|$  and  $|q_b|$  are relatively prime integers [16].

The global PQ symmetry defined in the previous section is realized by a shift of

$$\frac{A}{F_A} \rightarrow \frac{A}{F_A} + \alpha_{\text{PQ}}, \quad (3.12)$$

where  $\alpha_{\text{PQ}}$  ranges from 0 to  $2\pi$ . In fact, the phase of the gauge invariant combination  $\phi\phi'^{10}$  rotates by

$$\phi\phi'^{10} \propto e^{iA/F_A} \rightarrow e^{i\alpha_{\text{PQ}}} e^{iA/F_A}, \quad (3.13)$$

as in eq. (2.13).

After integrating out the extra multiplets, the axion obtains anomalous couplings to the SM gauge fields,

$$\begin{aligned} \mathcal{L} &= \frac{g_a^2}{32\pi^2} (N_a \theta_a + N_b \theta_b) F^a \tilde{F}^a \\ &= \frac{g_a^2}{32\pi^2} (q_b \theta_a - q_a \theta_b) F^a \tilde{F}^a, \end{aligned} \quad (3.14)$$

Here, we have used the fact that the numbers of extra multiplets coupling to  $\phi$  and  $\phi'$  are giving by  $N_a = q_b = 1$  and  $N_b = -q_a = 10$ . By substituting eq. (3.7), the anomalous coupling is reduce to,

$$\mathcal{L} = \frac{g_a^2}{32\pi^2} \frac{A}{F_A} F^a \tilde{F}^a, \quad (3.15)$$

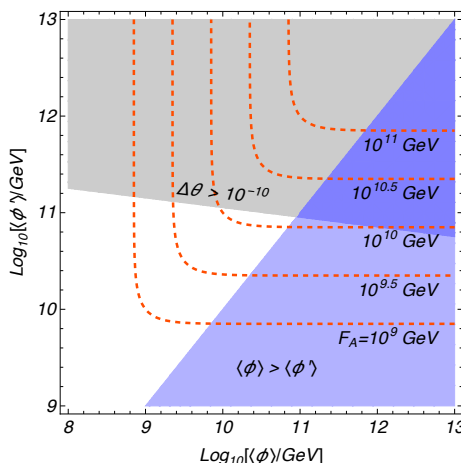
which reproduces the axial anomaly of eq. (2.14) by the shift of the axion in eq. (3.12). Through this term, the axion obtains a mass from the anomalous coupling below the QCD scale, with which the QCD vacuum angle is erased.

In the presence of the explicit breaking terms in eq. (2.15), the QCD vacuum angle is slightly shifted by<sup>4</sup>

$$\Delta\theta \sim 2 \frac{1}{10!} \frac{\langle\phi\rangle \langle\phi'\rangle^{10}}{M_{\text{PL}}^7 m_a^2 F_A^2} \sim 3 \times 10^{-11} \left( \frac{\langle\phi\rangle}{10^{10} \text{ GeV}} \right) \left( \frac{\langle\phi'\rangle}{10^{11} \text{ GeV}} \right)^{10}. \quad (3.16)$$

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<sup>4</sup>Hereafter,  $\langle\phi\rangle$  and  $\langle\phi'\rangle$  denote the absolute values of the VEVs of  $\phi$  and  $\phi'$ .



**Figure 1.** The constraint on the VEVs of  $\phi$  and  $\phi'$ . The gray shaded region is excluded by  $\Delta\theta < 10^{-10}$  for the non-SUSY model (see eq. (3.16)). The orange lines are the contours of the effective decay constant  $F_A$ . In the blue shaded region,  $\langle\phi\rangle > \langle\phi'\rangle$ .

where  $m_a$  denotes the axion mass. Such a small shift should be consistent with the current experimental upper limit on the  $\theta$  angle,  $\theta \lesssim 10^{-10}$  [38].

In figure 1, we show the constraint on the VEVs of  $\phi$  and  $\phi'$  from the experimental upper limit on  $\Delta\theta$ . In the gray shaded region, the explicit breaking effect in eq. (3.16) is too large to be consistent with  $\Delta\theta \lesssim 10^{-10}$ . The orange lines show the contours of the effective decay constant in eq. (3.10), which is mainly determined by the smaller one between  $\langle\phi\rangle$  and  $\langle\phi'\rangle$ . The figure shows that the model is consistent with the the experimental upper limit on  $\Delta\theta$  for  $\langle\phi'\rangle \lesssim 10^{11}$  GeV. As a result, we find that the gauged PQ mechanism based on the fiveness can solve the strong CP problem while satisfying the astrophysical constraint from the observation of supernova 1987A,  $F_A \gtrsim 10^8$  GeV [39], and the condition for successful thermal leptogenesis,  $M_N = y_N \langle\phi\rangle \gtrsim 10^9$  GeV [31–33].

Several comments are in order. Since  $(\bar{\mathbf{5}}_{\text{SM}}, \bar{\mathbf{5}}_K, \bar{\mathbf{5}}'_K)$  have identical gauge charges, they are indistinguishable from each other. Once  $\phi$  and  $\phi'$  obtain VEVs in the intermediate scale, 11-flavors of them become mass partners of  $\mathbf{5}$ 's, and 3-flavors of them remain massless. The SM 3-flavors of  $\bar{\mathbf{5}}$  are identified with those massless  $\bar{\mathbf{5}}$ 's.

It should also be noted that the “inter-sector” interactions via  $\bar{\mathbf{5}}$  do not lead to explicit breaking of the global PQ symmetry. To see this, it is most convenient to choose  $Q_\phi = 0$  and  $Q_{\phi'} = 1$  (see eq. (2.12)), which leads to the global PQ charges,

$$\phi'(+1), \quad \mathbf{5}'_K(+1), \quad (3.17)$$

with the charges of  $\{\text{SM}, \bar{N}_R, \phi, \mathbf{5}_K, \bar{\mathbf{5}}\}$  vanishing. As the fiveness invariant interactions of  $\bar{\mathbf{5}}$  in eq. (2.11) are also invariant under the global PQ symmetry in eq. (3.17), no explicit breaking terms are generated from the “inter-sector” interactions.<sup>5</sup>

<sup>5</sup>Note that  $\phi\phi'^{10}$  is the lowest dimensional operators among all the global PQ breaking operators. In this case, no larger explicit breaking terms are generated by radiative corrections other than the anomalous breaking terms given in eq. (3.15).



In the low energy effective theory, the axion couplings to the SM fields are the same with those in the KSVZ axion model except for those to the neutrinos. As  $B - L$  is an accidental symmetry of the SM except for the neutrino masses, the current couplings to the axion proportional to the fineness can be absorbed by the  $B - L$  rotation and  $U(1)_Y$  rotation. The non-vanishing couplings to the neutrinos can also be understood from the fact that the axion in the present model also plays a role of the Majoron [40] which is obvious in the limit of  $\langle\phi'\rangle \gg \langle\phi\rangle$ . However, it seems very difficult to test the direct couplings between the axion and the neutrinos in laboratory experiments.

#### 4 Domain wall problem

Here, let us briefly discuss the domain wall problem and axion dark matter. As discussed in [37], the model suffers from the domain wall problem for  $\langle\phi\rangle \gg \langle\phi'\rangle$  when global PQ symmetry breaking takes place after inflation. To avoid the domain wall problem, we assume either one of the following possibilities;

- (i) Both phase transitions of  $\langle\phi\rangle \neq 0$  and  $\langle\phi'\rangle \neq 0$  take place before inflation.
- (ii) The phase transition,  $\langle\phi'\rangle \neq 0$ , takes place before inflation while the transition,  $\langle\phi\rangle \neq 0$ , occurs after inflation.

The latter possibility is available as the fineness charges of  $\phi$  and  $\phi'$  are relatively prime and  $|q_a| : |q_b| = 10 : 1$ .<sup>6</sup>

For the first possibility, the cosmic axion abundance is given by,

$$\Omega_a h^2 \simeq 0.18 \theta_a^2 \left( \frac{F_A}{10^{12} \text{ GeV}} \right)^{1.19}, \tag{4.1}$$

for the initial misalignment angle  $\theta_a = \mathcal{O}(1)$  [41]. Thus, in the allowed parameter region in figure 1, i.e.,  $F_A \lesssim 10^{10}$  GeV, relic axion abundance is a subdominant component of dark matter. It should be also noted that the Hubble constant during inflation is required to satisfy,

$$H_{\text{inf}} \lesssim 10^8 \text{ GeV} \times \theta_a^{-1} \left( \frac{F_A}{10^{10} \text{ GeV}} \right)^{-0.19}. \tag{4.2}$$

to avoid the axion isocurvature problem (see refs. [42, 43]).<sup>7</sup>

For the second possibility, the cosmic axion abundance is dominated by the one from the decay of the string-domain wall networks [44],

$$\Omega_a h^2 \simeq 0.035 \pm 0.012 \left( \frac{F_A}{10^{10} \text{ GeV}} \right)^{1.19}. \tag{4.3}$$

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<sup>6</sup>The domain wall problem might also be solved for  $\langle\phi\rangle \sim \langle\phi'\rangle$  even if both the phase transitions take place after inflation. To confirm this possibility, detailed numerical simulations are required.

<sup>7</sup>Here, we do not assume that the axion is the dominant component of dark matter but use the axion relic abundance in eq. (4.1) to derive the constraint.

Thus, the relic axion from the string-domain wall network can be the dominant component of dark matter at the corner of the parameter space in figure 1. To avoid symmetry restoration after inflation, we also require that the maximum temperature during reheating [45],

$$T_{\text{MAX}} \simeq g_*^{-1/8} T_R^{1/2} H_{\text{inf}}^{1/4} M_{\text{PL}}^{1/4}, \quad (4.4)$$

does not exceed  $\langle \phi' \rangle$ , which leads to

$$H_{\text{inf}} \lesssim 5 \times 10^8 \text{ GeV} \left( \frac{\langle \phi' \rangle}{10^{11} \text{ GeV}} \right)^4 \left( \frac{10^9 \text{ GeV}}{T_R} \right)^2. \quad (4.5)$$

Here, we use the effective massless degrees of freedom  $g_* \simeq 200$ , though the condition does not depend on  $g_*$  significantly.

## 5 Supersymmetric extension

The SUSY extension of the present model is straightforward. The SM matter fields, the right-handed neutrinos, and the extra multiplets are simply extended to corresponding supermultiplets with the same fineness charges given in eqs. (2.1) and (2.9). The Higgs doublets are extended to the two Higgs doublet supermultiplets  $H_u$  and  $H_d$  as in the minimal SUSY Standard Model (MSSM). The fineness charges are assigned to be  $H_u(-2)$  and  $H_d(+2)$ , respectively. The complex scalars  $\phi$  and  $\phi'$  are also extended to corresponding supermultiplets which are accompanied by supermultiplets with opposite fineness charges,  $\bar{\phi}$  and  $\bar{\phi}'$  (see table 1).

Under the fineness symmetry, the superpotential is restricted to<sup>8</sup>

$$W = \mathbf{10}_{\text{SM}} \mathbf{10}_{\text{SM}} H_u + \mathbf{10}_{\text{SM}} \bar{\mathbf{5}} H_d + \bar{\mathbf{5}} \bar{N}_R H_u - \frac{1}{2} \phi \bar{N}_R \bar{N}_R + \bar{\phi} \mathbf{5}_K \bar{\mathbf{5}} + \bar{\phi}' \mathbf{5}'_K \bar{\mathbf{5}} \\ + X(2\phi\bar{\phi} - v^2) + Y(2\phi'\bar{\phi}' - v'^2). \quad (5.1)$$

Here,  $X$  and  $Y$  are introduced to make  $\phi$  and  $\phi'$  obtain non-vanishing VEVs, which are neutral under fineness.<sup>9</sup> The coupling coefficients are again omitted for notational simplicity. The SUSY extension again possesses the global PQ symmetry as in the case of the non-SUSY model.

In addition to fineness, we also assume that a discrete subgroup of  $U(1)_R$ ,  $\mathbb{Z}_{NR}$  ( $N > 2$ ), is an exact discrete gauge symmetry. This assumption is crucial to allow the VEV of the superpotential, and hence, the supersymmetry breaking scale much smaller than the Planck scale.<sup>10</sup> In the following, we take the simplest possibility,  $\mathbb{Z}_{4R}$  with the charge assignment given in table 1, which is free from  $\mathbb{Z}_{4R}$ -SU(5)<sup>2</sup> anomaly and the gravitational anomaly.<sup>11</sup> It should be noted that the mixed anomalies of  $\mathbb{Z}_{4R}$  and fineness do not put constraints on charges since they depend on the normalization of the heavy spectrum [52–61].<sup>12</sup>

<sup>8</sup>More generally, the Higgs bi-linear,  $H_u H_d$ , also couples to  $X$  and  $Y$ . We assume that the soft masses of the Higgs doublets are positive and larger than those of  $\phi$ 's and  $\phi'$ 's, so that the Higgs doublets do not obtain VEVs from the couplings to  $X$  and  $Y$ . We may also restrict those couplings by some symmetry.

<sup>9</sup>See [37] for details of the SUSY extension of the gauged PQ mechanism.

<sup>10</sup> $R$ -symmetry is also relevant for SUSY breaking vacua to be stable [50, 51].

<sup>11</sup>It should be noticed that there is no need to add extra SU(5) singlet fields to cancel the anomalies.

<sup>12</sup>GUT models consistent with the  $\mathbb{Z}_{4R}$  symmetry are discussed in, e.g. [62, 63].

	$\mathbf{10}_{\text{SM}}$	$\bar{\mathbf{5}}$	$\bar{N}_R$	$H_u$	$H_d$	$\mathbf{5}_K$	$\mathbf{5}'_K$	$\phi$	$\bar{\phi}$	$\phi'$	$\bar{\phi}'$	$X$	$Y$	$\mathbf{5}_E$	$\bar{\mathbf{5}}_E$
fiveness	+1	-3	+5	-2	+2	-7	+4	-10	+10	+1	-1	0	0	+3	-3
$\mathbb{Z}_{4R}$	+1	+1	+1	0	0	+1	+1	0	0	0	0	+2	+2	-1	+1

**Table 1.** The charge assignment of the fiveness symmetry and the gauged  $\mathbb{Z}_{4R}$  symmetry. Here, we fix the  $\mathbb{Z}_{4R}$  charges of the Higgs doublets to 0 which is motivated by pure gravity mediation model [46–48]. An extra multiplet ( $\mathbf{5}_E, \bar{\mathbf{5}}_E$ ) is introduced to cancel the  $\mathbb{Z}_{4R}$ -SU(5)<sup>2</sup> anomaly [49].

Under fiveness and the gauged  $\mathbb{Z}_{4R}$  symmetry, the lowest dimensional operators which break the global PQ symmetry are given by,

$$W_{\text{P}\mathcal{Q}} = \frac{m_{3/2}}{10!M_{\text{PL}}} \frac{\phi\phi'^{10}}{M_{\text{PL}}^8} + \frac{m_{3/2}}{10!M_{\text{PL}}} \frac{\bar{\phi}\bar{\phi}'^{10}}{M_{\text{PL}}^8}. \quad (5.2)$$

It should be noted that a lower dimensional PQ breaking term,  $\bar{\phi}'^5\bar{N}_R$ , is forbidden by the  $\mathbb{Z}_{4R}$  symmetry. The above superpotential contributes to the shift of the QCD vacuum angle mainly through the scalar potential,

$$\mathcal{L}_{\text{P}\mathcal{Q}} \sim \frac{8m_{3/2}^2}{10!M_{\text{PL}}} \frac{\phi\phi'^{10}}{M_{\text{PL}}^8} + \frac{8m_{3/2}^2}{10!M_{\text{PL}}} \frac{\bar{\phi}\bar{\phi}'^{10}}{M_{\text{PL}}^8} + \text{h.c.}, \quad (5.3)$$

where  $m_{3/2}$  denotes the gravitino mass. Compared with eq. (2.15), the explicit breaking is suppressed by a factor of  $(m_{3/2}/M_{\text{PL}})^2$ . Accordingly, the shift of the QCD vacuum angle is given by,

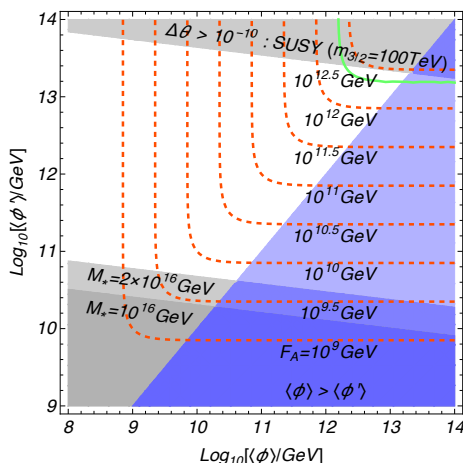
$$\Delta\theta \sim 2 \frac{1}{10!} \frac{8m_{3/2}^2 \langle\phi\rangle \langle\phi'\rangle^{10}}{M_{\text{PL}}^9 m_a^2 F_A^2} \sim 10^{-25} \left(\frac{m_{3/2}}{100 \text{ TeV}}\right)^2 \left(\frac{\langle\phi\rangle}{10^{11} \text{ GeV}}\right) \left(\frac{\langle\phi'\rangle}{10^{12} \text{ GeV}}\right)^{10}, \quad (5.4)$$

where we assume  $\langle\phi\rangle = \langle\bar{\phi}\rangle$  and  $\langle\phi'\rangle = \langle\bar{\phi}'\rangle$  for simplicity.<sup>13</sup>

In figure 2, we show the constraints on the VEVs of  $\phi$  and  $\phi'$  from the experimental upper limit on  $\Delta\theta$ . Here, we take the gravitino mass,  $m_{3/2} \simeq 100 \text{ TeV}$ , which is favored to avoid the cosmological gravitino problem for  $T_R \gtrsim 10^9 \text{ GeV}$  [65–67]. For  $m_{3/2} \simeq 100 \text{ TeV}$ , the scalar partner and the fermionic partner of the axion also do not cause cosmological problems as they obtain the masses of the order of the gravitino mass and decay rather fast [68]. In the figure, the gray shaded region is excluded by the constraint on  $\Delta\theta \lesssim 10^{-10}$ . Due to the suppression of the breaking term in eq. (5.3), the higher value of  $\langle\phi'\rangle$  is allowed compared with the non-SUSY model. The higher  $\langle\phi'\rangle$  is advantageous to avoid symmetry restoration after inflation (see eq. (4.5)), with which the domain wall problem is avoided in the possibility (ii) (see section 3). Accordingly, the decay constant can also be as high as about  $10^{11-12} \text{ GeV}$ , which also allows the axion to be the dominant dark matter component (see eq. (4.3)). Therefore, we find that the SUSY extension of the model is more successful.<sup>14</sup>

<sup>13</sup>The following argument can be easily extended to the cases with  $\langle\phi\rangle \neq \langle\bar{\phi}\rangle$  and  $\langle\phi'\rangle \neq \langle\bar{\phi}'\rangle$ .

<sup>14</sup>As in [37], we will discuss a possibility where SUSY and  $B-L$  are broken simultaneously elsewhere.



**Figure 2.** The constraint on the VEVs of  $\phi$  and  $\phi'$  for the SUSY extension. The gray shaded upper region is excluded for the SUSY model with  $m_{3/2} = 100$  TeV (see eq. (5.4)). The orange lines are the contours of the effective decay constant  $F_A$ . In the blue shaded region,  $\langle\phi\rangle > \langle\phi'\rangle$ . The gray shaded lower regions are excluded as the gauge coupling constants become non-perturbative below the GUT scale. The thin green region is excluded by the Axion Dark Matter eXperiment (ADMX) [64] where the dark matter density is assumed to be dominated by the relic axion.

It should be noted that the 11-flavors of extra multiplets at the intermediate scale make the renormalization group running of the MSSM gauge coupling constants asymptotic non-free. Thus, the masses of them are bounded from below so that perturbative unification is achieved. In the figure, the gray shaded lower region shows the contour of the renormalization scale  $M_*$  at which at least one of  $g_{1,2,3}$  becomes  $4\pi$ . Here, we use the one-loop renormalization group equations assuming that the extra quarks obtain masses of  $\langle\phi\rangle$  and  $\langle\phi'\rangle$ , respectively.<sup>15</sup> The result shows that the perturbative unification can be easily achieved for  $\langle\phi'\rangle \gtrsim 10^{9-10}$  GeV even in the presence of 11-flavors of the extra multiplets.

## 6 Conclusions and discussions

In this paper, we consider the gauged PQ mechanism where the gauged PQ symmetry is identified with the  $B - L$  symmetry (fiveness). As the  $B - L$  gauge symmetry is the most plausible extension of the SM, the identification of the gauged PQ symmetry with  $B - L$  is very attractive. An intriguing coincidence between the  $B - L$  breaking scale appropriate for the thermal leptogenesis and the favored PQ breaking scale from the astrophysical constraints also motivates this identification.

We found a natural  $B - L$  charge assignment motivated by the seesaw mechanism in the SU(5) GUT, with which the gauged PQ mechanism is achieved. There, the global PQ

<sup>15</sup>The masses of the sfermions, the heavy charged/neutral Higgs boson, the Higgsinos, and  $(\mathbf{5}_E, \bar{\mathbf{5}}_E)$  are at the gravitino mass scale,  $m_{3/2} \simeq 100-1000$  TeV. The gaugino masses are, on the other hand, assumed to be in the TeV scale as expected by anomaly mediation [69, 70]. This is motivated by the pure gravity mediation model in [46–48] (see also refs. [71–74] for similar models), where the Higgsino mass is generated from the  $R$ -symmetry breaking [75].

symmetry breaking effects are suppressed by the gauged fiveness symmetry so that the successful PQ mechanism is realized. As a notable feature, the fiveness charge assignment does not require extra  $SU(5)$  singlet matter fields other than the right-handed neutrinos to cancel the  $[U(1)_{\text{gPQ}}]^3$  anomaly and the gravitational anomaly. This feature is advantageous since the singlet fields required for anomaly cancellation tend to be rather light and longlived, and hence, often cause cosmological problems. As a result, we find that the gauged PQ mechanism based on the  $B - L$  symmetry is successfully consistent with thermal leptogenesis.

We also discussed the SUSY extension where the  $\mathbb{Z}_{4R}$  symmetry is also assumed. As has been shown, a larger effective decay constant is allowed in the SUSY model, as explicit breaking of the global PQ symmetry is more suppressed. Resultantly, the upper limit on the effective decay constant is extended to

$$F_A \lesssim 10^{12.5} \text{ GeV}, \tag{6.1}$$

which corresponds to the axion mass,

$$m_a \gtrsim 1.9 \mu\text{eV} \left( \frac{10^{12.5} \text{ GeV}}{F_A} \right). \tag{6.2}$$

The dark matter axion in this mass range can be detected by the ongoing ADMX-G2 experiment [76] and future ADMX-HF experiment [77].

In the SUSY model, it should be also noted that  $\mathbb{Z}_{4R}$  is spontaneously broken down to the  $\mathbb{Z}_{2R}$  symmetry.<sup>16</sup> Thus, the lightest supersymmetric particle in the MSSM sector also contributes to the dark matter density. Therefore, the model predicts a wide range of dark matter scenario from axion dominated dark matter to the LSP dominated dark matter, which can be tested by future extensive dark matter searches.

As emphasized above, the fiveness anomalies are canceled without introducing singlet fields other than the right-handed neutrinos. Although this feature is advantageous from the cosmological point of view, the fundamental reason for the cancellation remains an open question. In the case of the SM and the right-handed neutrino sector, the anomaly cancellation of the fiveness can be explained by the  $SO(10)$  unification in which the fiveness becomes a part of the  $SO(10)$  gauge symmetry. In the present model, however, it cannot be unified into the  $SO(10)$  or larger unified groups simply. At this point, the anomaly cancellation is a mere accident, and we have not found any deeper insights on the anomaly cancellation (see discussion in the appendix A).

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<sup>16</sup>For cosmological implication of the spontaneous discrete  $R$ -symmetry breaking, see ref. [78].

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## A Uniqueness of the fiveness charge

In this appendix, we discuss the uniqueness of the charge assignment in eq. (2.9), which might help to understand the origin of the anomaly free fiveness. First, let us suppose that there are  $(n + 3)$ -flavors of  $\bar{\mathbf{5}}$  fermions with the fiveness charge  $-3$ .<sup>17</sup> For  $n = 0$ , anomaly free fiveness and SU(5) gauge symmetries are achieved by introducing 3-flavors of  $\mathbf{10}_{\text{SM}}$  and  $\bar{N}_R$  with the fiveness charges  $+1$  and  $+5$ , respectively while allowing the first four Yukawa interactions in eq. (2.11). For  $n > 0$ , on the other hand, the anomaly free conditions require more fermions. Given the fact that the SM consists of 3 flavors, it is simplest to add  $n$ -flavors of  $\mathbf{5}$  fermions to cancel self- and gravitational anomalies of SU(5). When a  $\mathbf{5}$  fermion has fiveness charge  $+3$ , it becomes a mass partner of one of the  $n$ -flavors of  $\bar{\mathbf{5}}$ , which ends up with a model with  $(n - 1)$ -flavors of  $\bar{\mathbf{5}}$ . Thus, we assume that the charge of  $\mathbf{5}$ 's are not equal to  $+3$ .

The anomaly free charge assignment of  $\mathbf{5}$ 's is fixed in the following way. For all the  $n$ -flavors of  $(\mathbf{5}, \bar{\mathbf{5}})$  to have masses in the intermediate scale, they need to couple to the order parameters of fiveness. As we assume the seesaw mechanism, we have a natural candidate of such an order parameter, a complex scalar field,  $\phi$ , with a fiveness charge  $-10$ . In order to make all the  $n$ -flavors of  $(\mathbf{5}, \bar{\mathbf{5}})$  massive while achieving anomaly free fiveness, however, it is required to introduce one more complex scalar,  $\phi'$ , with the fiveness charge  $q_{\phi'}$ .

In the presence of  $\phi$  and  $\phi'$ , the mass terms of  $(\mathbf{5}, \bar{\mathbf{5}})$  are generated from

$$\mathcal{L} = \phi \mathbf{5} \bar{\mathbf{5}} + \phi^* \mathbf{5}' \bar{\mathbf{5}} + \phi' \mathbf{5}'' \bar{\mathbf{5}} + \phi'^* \mathbf{5}''' \bar{\mathbf{5}}, \quad (\text{A.1})$$

where the coupling coefficients are again omitted. Here,  $\mathbf{5}$ 's are divided into  $\{\mathbf{5}, \mathbf{5}', \mathbf{5}'', \mathbf{5}'''\}$  whose fiveness charges are given by,

$$\mathbf{5}(+13), \quad \mathbf{5}'(-7), \quad \mathbf{5}''(-q_{\phi'} + 3), \quad \mathbf{5}'''(q_{\phi'} + 3), \quad (\text{A.2})$$

respectively. We allocate  $N_5, N'_5, N''_5$  and  $N'''_5$  flavors to  $\{\mathbf{5}, \mathbf{5}', \mathbf{5}'', \mathbf{5}'''\}$  with  $N_5 + N'_5 + N''_5 + N'''_5 = n$ . The anomaly free conditions of fiveness are given by,

$$13^3 N_5 - 7^3 N'_5 + (q_{\phi'} + 3)^3 N''_5 + (-q_{\phi'} + 3)^3 N'''_5 - 3^3 n = 0, \quad (\text{A.3})$$

$$13 N_5 - 7 N'_5 + (q_{\phi'} + 3) N''_5 + (-q_{\phi'} + 3) N'''_5 - 3 n = 0. \quad (\text{A.4})$$

By solving the anomaly free conditions, we find only two sets of solutions,

$$N_5 = 0, \quad N'_5 = 10, \quad N''_5 = 0, \quad N'''_5 = 1, \quad q_{\phi'} = 1, \quad (\text{A.5})$$

or

$$N_5 = 7, \quad N'_5 = 1, \quad N''_5 = 3, \quad N'''_5 = 0, \quad q_{\phi'} = 20, \quad (\text{A.6})$$

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<sup>17</sup>The choice of  $-3$  just defines the normalization of fiveness.

both of which corresponds to  $n = 11$ .<sup>18</sup> Here, we restrict ourselves to  $n < 22$ . The first charge assignment is nothing but the fiveness charges assumed in this paper, while the later is another possibility. In this sense, we find that the number of the flavors,  $n = 11$ , is a unique choice within  $n < 22$ , and the fiveness charge assignment in this paper is one of the only two possibilities, where the second possibility is not suitable for the gauged PQ mechanism.

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<sup>18</sup>We take  $q_{\phi'} > 0$  without loss of generality.



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