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Lagrangian description of massive higher spin supermultiplets in AdS_3 space

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Abstract: We construct the Lagrangian formulation of massive higher spin on-shell (1,0) supermultiplets in three dimensional anti-de Sitter space. The construction is based on description of the massive three dimensional fields in terms of frame-like gauge invariant formalism and technique of gauge invariant curvatures. For the two possible massive supermultiplets $(s, s+1/2)$ and $(s, s-1/2)$ we derive explicit form of the supertransformations leaving the sum of bosonic and fermionic Lagrangians invariant.

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1 Introduction

Three dimensional field models attract much attention due to their comparatively simple structure and remarkable properties of three-dimensional flat and curved spaces. One of the achievements in this area is a construction of three-dimensional higher spin field theories (see the pioneer works [1–3], for modern development see e.g. the recent papers [4, 5] and references therein).

The aim of this work is to construct the massive higher spin supermultiplets in the three-dimensional anti-de Sitter space AdS_3. As is well known [6], the AdS_3 space possesses the special properties since all the AdS_3 superalgebras (as well as conventional AdS_3 algebra itself) factorize into "left" and "right" parts. For the case of simplest (1,0) superalgebra (the one we are working here with) such factorization has the form:

$$\mathrm{OSp}(1,2) \otimes \mathrm{Sp}(2)$$

so that we have supersymmetry in the "left" sector only. It means that the minimal massive supermultiplet must contain just one bosonic and one fermionic degrees of freedom.

Recently we constructed such supermultiplets using the unfolded formalism [7]. In this paper we develop the Lagrangian formulation for these supermultiplets. It generalizes the Lagrangian formulation for massive supermultiplets in three dimensional Minkowski space given in [8]. Note here that the off-shell superfield description for the supermultiplets containing topologically massive higher spin fields was constructed recently in [9].[1]

We develop the component approach[2] to Lagrangian construction for supersymmetric massive higher spin fields in AdS_3 on the base of gauge invariant formulation of massive higher spin fields. It was shown long enough [12, 13] that massive bosonic (fermionic) spin-s field can be treated as system of massless fields with spins $s, s-1, s-2, \ldots 0(1/2)$ coupled by the Stueckelberg symmetries. Later, a frame-like formulation of massive higher spin fields has been developed in the framework of the same approach [14]. It allows us to reformulate the massive higher spin theory in terms of gauge invariant objects (curvatures). In massless four-dimensional higher spin theory such curvatures are very convenient objects and allow, e.g, to built the cubic higher spin interactions [15–18]. However the construction of curvatures proposed in [15–18] is essentially adapted only for theories in four and higher dimensions. Nevertheless the formalism of gauge invariant objects can be successfully applied to massive higher spin fields in three dimensions as well. It was shown that in three dimensions the gauge invariant Lagrangians for massive higher spin fields [19, 20] can be rewritten in explicitly gauge invariant form [21]. In this work we elaborate this formalism for Lagrangian construction of massive higher spin supermultiplets with minimal (1,0) supersymmetry where the algebra of the supercharges has the form

$$\{Q_\alpha, Q_\beta\} \sim P_{\alpha\beta} + \frac{\lambda}{2} M_{\alpha\beta}$$

Here $P_{\alpha(2)}$ and $M_{\alpha(2)}$ are the generators of AdS_3 algebra with the following commutation relations (the conventions and notations for the indices are specified in the end of introduction)

$$[M_{\alpha(2)}, M_{\beta(2)}] \sim \varepsilon_{\alpha\beta} M_{\alpha\beta}, \qquad [P_{\alpha(2)}, P_{\beta(2)}] \sim \lambda^2 \varepsilon_{\alpha\beta} M_{\alpha\beta}$$
$$[M_{\alpha(2)}, P_{\beta(2)}] \sim \varepsilon_{\alpha\beta} P_{\alpha\beta}$$

Note that as in all three-dimensional AdS supergravities [6] (see also [22] for higher dimensions) in our frame-like formalism we consider supertransformations just as the part of local superalgebra acting in the fiber only. As a result the commutators of our supertransformations close off-shell without any need of some auxiliary fields (see section 6). In the massive supermultiplets case the price we have to pay is that the Lagrangians are invariant under the supertransformations up to the terms proportional to the spin-1 and spin-0 auxiliary fields equations only.

[1] The off-shell $3D, \mathcal{N}=2$ massless higher spin superfields and their massive deformations has been elaborated in [10]. The conditions defying the $\mathcal{N}=1$ massive superfield representations in AdS_3 were formulated in [11].

[2] List of references on component and superfield formulations of higher spin supersymmetric theories is given in our paper [7].

A general scheme for Lagrangian formulation of massive higher spin supermultiplets looks as follows. Let Ω^A, Φ^A is a set of fields and R^A, F^A is a set of curvatures corresponding to frame-like gauge invariant formulation of massive bosonic and fermionic fields respectively. The curvatures have the following structure

$$R^A = D\Omega^A + (e\Omega)^A, \qquad F^A = D\Phi^A + (e\Phi)^A$$

where $e \equiv e_\mu{}^{\alpha\beta}$ is a non-dynamical background AdS_3 frame. Lagrangian both for bosonic and for fermionic massive higher spins are presented as the quadratic forms in curvatures expression [21]

$$\mathcal{L}_{\mathcal{B}} = \sum R^A R^A, \qquad \mathcal{L}_{\mathcal{F}} = \sum F^A F^A \tag{1.1}$$

where $\mathcal{L}_B$ is a bosonic field Lagrangian and $\mathcal{L}_F$ is a fermionic one. Note that for massless higher spin fields in three dimensions such form of the Lagrangians is impossible. Then, in order to realize supersymmetry between the bosonic and fermionic massive fields we deform the curvatures by gravitino field $\Psi_\mu{}^\alpha$ with parameter of supertransformations ζ^α. In the case of global supersymmetry we consider gravitino field as a non-dynamical background and parameter of supertransformations as global (i.e. covariantly constant). Such a construction can be interpreted as a supersymmetric theory in terms of background fields of supergravity. Schematically the curvature deformations are written as

$$\Delta R^A = (\Psi\Phi)^A, \qquad \Delta F^A = (\Psi\Omega)^A$$

and supertransformations have the form

$$\delta\Omega^A \sim (\Phi\zeta)^A, \qquad \delta\Phi^A \sim (\Omega\zeta)^A$$

The restrictions on the form of deformations and supertransformations are imposed by requirement of covariant transformations for deformed curvatures $\hat{R}^A = R^A + \Delta R^A$

$$\delta\hat{R}^A \sim (F\zeta)^A, \qquad \delta\hat{F}^A \sim (R\zeta)^A$$

Finally the supersymmetric Lagrangian for given supermultiplets is the sum of Lagrangians (1.1) where initial curvatures are replaced by deformed ones $R, F \to \hat{R}, \hat{F}$. Possible arbitrariness is fixed by the condition that the Lagrangian must be invariant under the supertransformations:

$$\delta\hat{\mathcal{L}} = \sum[\hat{R}^A\delta\hat{R}^A + \hat{F}^A\delta\hat{F}^A] = \sum R^A(F\zeta)^A = 0$$

Aim of the current paper is to demonstrate how such a general scheme can actually be realized for supermultiplets in AdS_3.

The paper is organized as follows. First of all we fix the notations and conventions. In section 2, following the above scheme we consider as an example, the detailed Lagrangian construction of massive supermultiplet $(2, \frac{3}{2})$. Such an example illustrates all the key points of the construction. The next sections are devoted to generalizations of these results for arbitrary massive supermultiplets. In section 3 we consider the gauge invariant formulation of massive fields with spin s and spin $s+\frac{1}{2}$ which will be used for supersymmetric constructions. In particular, we write out all field variables and gauge-invariant curvatures and consider the Lagrangian in terms of these curvatures. Further, following the above general scheme, we study two massive higher spin supermultiplets. Supermultiplets $(s, s+\frac{1}{2})$ is studied in section 4 and $(s, s+\frac{1}{2})$ is studied in section 5. At last, in section 6 we show how the AdS_3 superalgebra is realized on our supermultiplets. In conclusion we summaries and discuss the results obtained. For completeness, we include in the paper two appendices devoted to frame-like description of massless fields in three dimensions. Appendix A contains the bosonic fields, appendix B contains the fermionic ones.

Notations and conventions. We use a frame-like multispinor formalism where all the objects (3,2,1,0-forms) have totally symmetric local spinor indices. To simplify the expressions we will use the condensed notations for the spinor indices such that e.g.

$$\Omega^{\alpha(2k)} = \Omega^{(\alpha_1\alpha_2\ldots\alpha_{2k})}$$

Also we will always assume that spinor indices denoted by the same letters and placed on the same level are symmetrized, e.g.

$$\Omega^{\alpha(2k)}\zeta^{\alpha} = \Omega^{(\alpha_1\ldots\alpha_{2k}}\zeta^{\alpha_{2k+1})}$$

AdS_3 space will be described by the background frame (one-form) $e^{\alpha(2)}$ and the covariant derivative D normalized so that

$$D \wedge D\zeta^{\alpha} = -\lambda^2 E^{\alpha}{}_{\beta}\zeta^{\beta}$$

Basis elements of $1,2,3$-form spaces are respectively $e^{\alpha(2)}$, $E^{\alpha(2)}$, E where the last two are defined as double and triple wedge product of $e^{\alpha(2)}$:

$$e^{\alpha\alpha} \wedge e^{\beta\beta} = \varepsilon^{\alpha\beta}E^{\alpha\beta}, \qquad E^{\alpha\alpha} \wedge e^{\beta\beta} = \varepsilon^{\alpha\beta}\varepsilon^{\alpha\beta}E.$$

Also we write some useful relations for these basis elements

$$E^{\alpha}{}_{\gamma} \wedge e^{\gamma\beta} = 3\varepsilon^{\alpha\beta}E, \qquad e^{\alpha}{}_{\gamma} \wedge e^{\gamma\beta} = 4E^{\alpha\beta}.$$

Further on the sign of wedge product $\wedge$ will be omitted.

2 Massive supermultiplet $(2, \frac{3}{2})$ example

In this section we consider in details the Lagrangian realization for massive supermultiplet $(2, 3/2)$ example using the method mentioned above in the introduction. We start with the gauge-invariant formulations of free massive fields with spin 2 and spin 3/2 separately. Using such formulations, we present the full set of gauge-invariant curvatures for them. Then we consider the deformations of these curvatures by background gravitino field Ψ^{α} and find suitable supertransformations. As a result we construct the supersymmetric Lagrangian.

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2.1 Free fields

Spin 2. In gauge invariant form the massive spin-2 field is described by system of massless fields with spins $2, 1, 0$. In frame-like approach the corresponding set of fields consists of $(\Omega^{\alpha(2)}, f^{\alpha(2)})$, $(B^{\alpha(2)}, A)$ and $(\pi^{\alpha(2)}, \varphi)$ (see appendix A for details). Lagrangian for free massive field in AdS_3 have the form [19]

$$\begin{aligned}\mathcal{L} = & \Omega_{\alpha\beta} e^\beta{}_\gamma \Omega^{\alpha\gamma} + \Omega_{\alpha(2)} D f^{\alpha(2)} + E B^{\alpha(2)} B_{\alpha(2)} \\ & - e^{\alpha(2)} B_{\alpha(2)} D A - E \pi^{\alpha(2)} \pi_{\alpha(2)} + E^{\alpha(2)} \pi_{\alpha(2)} D \varphi \\ & + 2m e_{\alpha(2)} \Omega^{\alpha(2)} A + m f_{\alpha\beta} E^\beta{}_\gamma B^{\alpha\gamma} + 4\tilde{m} E_{\alpha(2)} \pi^{\alpha(2)} A \\ & + \frac{M^2}{4} f_{\alpha\beta} e^\beta{}_\gamma f^{\alpha\gamma} - m\tilde{m} E_{\alpha(2)} f^{\alpha(2)} \varphi + \frac{3}{2} m^2 E \varphi\varphi \end{aligned} \tag{2.1}$$

It is invariant under the following gauge transformations

$$\begin{aligned} \delta\Omega^{\alpha(2)} &= D\eta^{\alpha(2)} + \frac{M^2}{4} e^\alpha{}_\beta \xi^{\alpha\beta} & \delta f^{\alpha(2)} &= D\xi^{\alpha(2)} + e^\alpha{}_\beta \eta^{\alpha\beta} - 2m e^{\alpha(2)} \xi \\ \delta B^{\alpha(2)} &= -2m\eta^{\alpha(2)} & \delta A &= D\xi - \frac{m}{4} e_{\alpha(2)} \xi^{\alpha(2)} \\ \delta\pi^{\alpha(2)} &= -m\tilde{m}\xi^{\alpha(2)} & \delta\varphi &= -4\tilde{m}\xi \end{aligned} \tag{2.2}$$

where m is the mass parameter and

$$\tilde{m}^2 = M^2, \quad M^2 = m^2 + \lambda^2$$

The curvatures invariant under gauge transformations (2.2) have the form

$$\begin{aligned} \mathcal{R}^{\alpha(2)} &= D\Omega^{\alpha(2)} + \frac{M^2}{4} e^\alpha{}_\beta f^{\alpha\beta} - m^2 E^\alpha{}_\beta B^{\alpha\beta} - m\tilde{m} E^{\alpha(2)} \varphi \\ \mathcal{T}^{\alpha(2)} &= D f^{\alpha(2)} + e^\alpha{}_\beta \Omega^{\alpha\beta} - 2m e^{\alpha(2)} A \\ \mathcal{B}^{\alpha(2)} &= D B^{\alpha(2)} - \Omega^{\alpha(2)} + \frac{M^2}{4} e^\alpha{}_\beta \pi^{\alpha\beta} \\ \mathcal{A} &= DA - \frac{m}{4} e_{\alpha(2)} f^{\alpha(2)} + 2m E_{\alpha(2)} B^{\alpha(2)} \\ \Pi^{\alpha(2)} &= D\pi^{\alpha(2)} - f^{\alpha(2)} + e^\alpha{}_\beta B^{\alpha\beta} + \frac{m}{2M} e^{\alpha(2)} \varphi \\ \Phi &= D\varphi + 4\tilde{m} A + mM e_{\alpha(2)} \pi^{\alpha(2)} \end{aligned} \tag{2.3}$$

Here we have changed a normalization for the two zero forms

$$B^{\alpha(2)} \to -2m B^{\alpha(2)} \qquad \pi^{\alpha(2)} \to -mM\pi^{\alpha(2)} \tag{2.4}$$

It is interesting to point out that, unlike massless theory in three dimensions, the Lagrangian for massive spin 2 (2.1) can be presented in manifestly gauge invariant form

$$\mathcal{L} = -\frac{1}{2} \mathcal{R}_{\alpha(2)} \Pi^{\alpha(2)} - \frac{1}{2} \mathcal{T}_{\alpha(2)} \mathcal{B}^{\alpha(2)} - \frac{m}{4\tilde{m}} e_{\alpha(2)} \mathcal{B}^{\alpha(2)} \Phi \tag{2.5}$$

Spin-$\frac{3}{2}$. In gauge invariant formulation, the massive spin-3/2 field is described by system of massless fields with the spins $3/2, 1/2$. In frame-like approach the corresponding set of fields consists of Φ^α, ϕ^α. (see appendix B for details). The free Lagrangian for mass m_1 field in AdS_3 has the following form

$$\begin{aligned}\mathcal{L} = & -\frac{i}{2}\Phi_\alpha D\Phi^\alpha + \frac{i}{2}\phi_\alpha E^\alpha{}_\beta D\phi^\beta - \\ & -i\frac{M_1}{2}\Phi_\alpha e^\alpha{}_\beta \Phi^\beta - i2m_1\Phi_\alpha E^\alpha{}_\beta \phi^\beta - i\frac{3M_1}{2}E\phi_\alpha\phi^\alpha \end{aligned} \tag{2.6}$$

It is invariant under gauge transformations

$$\delta\Phi^\alpha = D\xi^\alpha + M_1 e^\alpha{}_\beta \xi^\beta \qquad \delta\phi^\alpha = -2m_1\xi^\alpha \tag{2.7}$$

where

$$M_1{}^2 = m_1{}^2 + \frac{\lambda^2}{4}$$

One can construct the gauge invariant curvatures with respect (2.7). After change of normalization $\phi^\alpha \to -2m_1\phi^\alpha$ they look like

$$\begin{aligned}\mathcal{F}^\alpha &= D\Phi^\alpha + M_1 e^\alpha{}_\beta \Phi^\beta - 4m_1{}^2 E^\alpha{}_\beta \phi^\beta \\ \mathcal{C}^\alpha &= D\phi^\alpha - \Phi^\alpha + M_1 e^\alpha{}_\beta \phi^\beta \end{aligned} \tag{2.8}$$

As in the previous spin-2 case, one notes that Lagrangian (2.6) can be rewritten in terms of curvatures (2.8)

$$\mathcal{L} = \frac{i}{2}\mathcal{F}_\alpha \mathcal{C}^\alpha \tag{2.9}$$

The above results on spin-2 and spin-$\frac{3}{2}$ fields are building blocks for supersymmetrization. Full set of gauge invariant curvatures contains (2.3) and (2.8) respectively. Corresponding expressions for Lagrangians in terms of curvatures are (2.5) and (2.9).

2.2 Supersymmetric system

Before we turn to Lagrangian formulation for massive supermultiplet $(2, 3/2)$ let us consider this supermultiplet in massless flat limit. That is we consider the sum of Lagrangians (2.3) and (2.6) in the limit $m, m_1, \lambda \to 0$. It has the form

$$\begin{aligned}\hat{\mathcal{L}} = & \Omega_{\alpha\beta} e^\beta{}_\gamma \Omega^{\alpha\gamma} + \Omega_{\alpha(2)} Df^{\alpha(2)} + EB_{\alpha\beta}B^{\alpha\beta} - B_{\alpha\beta}e^{\alpha\beta}DA \\ & -E\pi_{\alpha\beta}\pi^{\alpha\beta} + \pi_{\alpha\beta}E^{\alpha\beta}D\varphi - \frac{i}{2}\Phi_\alpha D\Phi^\alpha + \frac{1}{2}\phi_\alpha E^\alpha{}_\beta D\phi^\beta \end{aligned} \tag{2.10}$$

Such a Lagrangian describes the system of free massless fields with spins $2, 3/2, 1, 1/2, 0$. One can show there exist global supertransformations that leave the Lagrangian (2.10) invariant. These supertransformations with the redefinition (2.4) have form

$$\begin{aligned}\delta f^{\alpha(2)} &= i\beta_1 \Phi^\alpha \zeta^\alpha \\ \delta A &= i\alpha_0 \Phi^\alpha \zeta_\alpha - i2m_1\beta_0 e_{\alpha\beta}\phi^\alpha\zeta^\beta \\ \delta\varphi &= im_1\tilde{\delta}_0 \phi^\gamma \zeta_\gamma \\ \delta\Phi^\alpha &= 2\beta_1 \Omega^{\alpha\beta}\zeta_\beta - 2m\alpha_0 e_{\beta(2)} B^{\beta(2)}\zeta^\alpha \\ \delta\phi^\alpha &= \frac{4m\beta_0}{m_1} B^{\alpha\beta}\zeta_\beta + \frac{mM\tilde{\delta}_0}{2m_1}\pi^{\alpha\beta}\zeta_\beta \end{aligned} \tag{2.11}$$

The parameters $\beta_1, \beta_0, \alpha_0, \tilde{\delta}_0$ are arbitrary but, as it will be seen later, they are fixed in massive case. The choice of such notations for them will be clear from in next section for arbitrary higher spin supermultiplets. To prove the invariance we need to use the Lagrangian equations of motion for auxiliary fields $\pi^{\alpha(2)}, B^{\alpha(2)}$ corresponding to spins 0 and 1

$$E^\alpha{}_\beta DB^{\beta\gamma} = E^\gamma{}_\beta DB^{\beta\alpha}, \qquad E^\alpha{}_\gamma d\pi^{\beta\gamma} = \frac{1}{2}\varepsilon^{\alpha\beta}E_{\gamma\delta}D\pi^{\gamma\delta} \tag{2.12}$$

Thus, we see that in the massless flat limit the Lagrangian formulation of pure massive supermultiplets $(2, \frac{3}{2})$ we must get the supertransformations (2.11). This requirement will provide us the unique possibility for construction of the correct supersymmetric massive theory.

Deformation of curvatures. Now we are ready to realize the massive supermultiplets. We do it deforming curvatures by background gravitino field Ψ^α with global transformation in AdS_3

$$\delta\Psi^\alpha = D\zeta^\alpha + \frac{\lambda}{2}e^\alpha{}_\beta\zeta^\beta \tag{2.13}$$

The main idea is to deform the curvatures so that they transform covariantly through themselves. It means that for all the deformed curvatures $\hat{R}^A = R^A + \Delta R^A$ the following relations should take place

$$\delta\hat{R}^A = \delta_\zeta R^A + \delta_0\Delta R^A = (R\zeta)^A \tag{2.14}$$

where δ_0 is the transformation (2.13) and δ_ζ is the linear supertransformation.

Let us consider the following ansatz for deformation of spin-2 curvatures

$$\begin{aligned}
\Delta R^{\alpha(2)} &= i\rho_1\Phi^\alpha\Psi^\alpha - i\hat{\rho}_0 e^{\alpha(2)}\phi_\gamma\Psi^\gamma & \Delta T^{\alpha(2)} &= i\beta_1\Phi^\alpha\Psi^\alpha \\
\Delta\mathcal{B}^{\alpha(2)} &= -i\hat{\rho}_1\phi^\alpha\Psi^\alpha & \Delta\mathcal{A} &= i\alpha_0\Phi^\gamma\Psi_\gamma - i2m_1\beta_0 e_{\alpha\beta}\phi^\alpha\Psi^\beta \\
\Delta\Pi^{\alpha(2)} &= -i\hat{\beta}_1\phi^\alpha\Psi^\alpha & \Delta\Phi &= im_1\tilde{\delta}_0\phi_\gamma\Psi^\gamma
\end{aligned} \tag{2.15}$$

Also one introduces the corresponding supertransformations

$$\begin{aligned}
\delta\Omega^{\alpha(2)} &= i\rho_1\Phi^\alpha\zeta^\alpha - i\hat{\rho}_0 e^{\alpha(2)}\phi_\gamma\zeta^\gamma & \delta f^{\alpha(2)} &= i\beta_1\Phi^\alpha\zeta^\alpha \\
\delta B^{\alpha(2)} &= i\hat{\rho}_1\phi^\alpha\zeta^\alpha & \delta A &= i\alpha_0\Phi^\gamma\zeta_\gamma - i2m_1\beta_0 e_{\alpha\beta}\phi^\alpha\zeta^\beta \\
\delta\pi^{\alpha(2)} &= i\hat{\beta}_1\phi^\alpha\zeta^\alpha & \delta\varphi &= -im_1\tilde{\delta}_0\phi_\gamma\zeta^\gamma
\end{aligned} \tag{2.16}$$

The form of the supertransformations is completely determined by the form of deformations for curvatures. Besides $\beta_1, \beta_0, \alpha_0, \tilde{\delta}_0$ parameters which remain arbitrary from the massless case, we have new arbitrary parameters $\rho_1, \hat{\rho}_1, \hat{\rho}_0, \hat{\beta}_1$. All the parameters will be fixed by requirement of covariant homogeneous curvature transformation (2.14). In addition there are two mass parameters M, M_1 an we will see that one is related to another.

Let us check the requirement (2.14) for curvatures $\mathcal{R}^{\alpha(2)}, \mathcal{T}^{\alpha(2)}$. One one hand

$$
\begin{aligned}
\delta\hat{\mathcal{R}}^{\alpha(2)} &= i\rho_1 D\Phi^\alpha\zeta^\alpha + i\hat{\rho}_0 e^{\alpha(2)} D\phi^\beta\zeta_\beta + i\frac{M^2}{2}\beta_1 e^{\alpha(2)}\Phi^\beta\zeta_\beta \\
&\quad + i\left(\frac{M^2}{2}\beta_1 - \frac{1}{2}\lambda\rho_1\right) e^\alpha{}_\gamma\Phi^\alpha\zeta^\gamma + i\left(-m^2\hat{\rho}_1 - \frac{1}{2}\lambda\hat{\rho}_0 + \frac{1}{2}mMm_1\tilde{\delta}_0\right) E^\alpha{}_\gamma\phi^\alpha\zeta^\gamma \\
&\quad + i\left(-m^2\hat{\rho}_1 - \frac{1}{2}\lambda\hat{\rho}_0 - \frac{1}{2}mMm_1\tilde{\delta}_0\right) E^\alpha{}_\gamma\phi^\gamma\zeta^\alpha \\
\delta\hat{\mathcal{T}}^{\alpha(2)} &= i\beta_1 D\Phi^\alpha\zeta^\alpha + i(-2m\alpha_0 + 2\rho_1) e^{\alpha(2)}\Phi^\beta\zeta_\beta + i\left(2\rho_1 - \frac{1}{2}\lambda\beta_1\right) e^\alpha{}_\gamma\Phi^\alpha\zeta^\gamma \\
&\quad + i(-4mm_1\beta_0 - 4\hat{\rho}_0) E^\alpha{}_\gamma\phi^\alpha\zeta^\gamma + i(-4mm_1\beta_0 + 4\hat{\rho}_0) E^\alpha{}_\gamma\phi^\gamma\zeta^\alpha
\end{aligned}
$$

and on the other hand

$$
\begin{aligned}
\delta\hat{\mathcal{R}}^{\alpha(2)} &= i\rho_1\mathcal{F}^\alpha\zeta^\alpha - i\hat{\rho}_0 e^{\alpha(2)}\mathcal{C}^\beta\zeta_\beta = i\rho_1 D\Phi^\alpha\zeta^\alpha + i\hat{\rho}_0 e^{\alpha(2)} D\phi^\beta\zeta_\beta \\
&\quad + i(\hat{\rho}_0 + 2M_1\rho_1) e^{\alpha(2)}\Phi^\beta\zeta_\beta + i(M_1\rho_1) e^\alpha{}_\gamma\Phi^\alpha\zeta^\gamma \\
&\quad - iM_1\hat{\rho}_0 E^\alpha{}_\gamma\phi^\alpha\zeta^\gamma + i(-M_1\hat{\rho}_0 - 4{m_1}^2\rho_1) E^\alpha{}_\gamma\phi^\gamma\zeta^\alpha \\
\delta\hat{\mathcal{T}}^{\alpha(2)} &= i\beta_1\mathcal{F}^\alpha\zeta^\alpha = i\beta_1 D\Phi^\alpha\zeta^\alpha + i(2M_1\beta_1) e^{\alpha(2)}\Phi^\beta\zeta_\beta \\
&\quad + i(M_1\beta_1) e^\alpha{}_\gamma\Phi^\alpha\zeta^\gamma - i4{m_1}^2\beta_1 E^\alpha{}_\gamma\phi^\gamma\zeta^\alpha
\end{aligned}
\tag{2.17}
$$

Comparing the above relations, we obtain the solution

$$
\begin{aligned}
M_1 &= M - \frac{\lambda}{2}, \qquad \rho_1 = \frac{M}{2}\beta_1, \qquad \hat{\rho}_0 = -\frac{M(M-\lambda)}{2}\beta_1, \qquad \alpha_0 = -\frac{(M-\lambda)}{2m}\beta_1 \\
\tilde{\delta}_0 &= 4\beta_0 = \frac{2m_1}{m}\beta_1
\end{aligned}
$$

For curvatures $\mathcal{B}^{\alpha(2)}, \Pi^{\alpha(2)}$ we have on one hand

$$
\begin{aligned}
\delta\hat{\mathcal{B}}^{\alpha(2)} &= i\hat{\rho}_1 D\phi^\alpha\zeta^\alpha - i\rho_1\Phi^\alpha\zeta^\alpha \\
&\quad + i\left(-\hat{\rho}_0 + \frac{M^2}{2}\hat{\beta}_1\right) e^{\alpha(2)}\phi^\beta\zeta_\beta + i\left(\frac{M^2}{2}\hat{\beta}_1 - \frac{1}{2}\lambda\hat{\rho}_1\right) e^\alpha{}_\gamma\phi^\alpha\zeta^\gamma \\
\delta\hat{\Pi}^{\alpha(2)} &= i\hat{\beta}_1 D\phi^\alpha\zeta^\alpha - i\beta_1\Phi^\alpha\zeta^\alpha \\
&\quad + i\left(\frac{mm_1\tilde{\delta}_0}{2M} + 2\hat{\rho}_1\right) e^{\alpha(2)}\phi^\beta\zeta_\beta + i\left(2\hat{\rho}_1 - \frac{1}{2}\lambda\hat{\beta}_1\right) e^\alpha{}_\gamma\phi^\alpha\zeta^\gamma
\end{aligned}
$$

and on the other hand

$$
\begin{aligned}
\delta\hat{\mathcal{B}}^{\alpha(2)} &= i\hat{\rho}_1\mathcal{C}^\alpha\zeta^\alpha = i\hat{\rho}_1 D\phi^\alpha\zeta^\alpha - i\hat{\rho}_1\Phi^\alpha\zeta^\alpha \\
&\quad + i(2M_1\hat{\rho}_1) e^{\alpha(2)}\phi^\beta\zeta_\beta + i(M_1\hat{\rho}_1) e^\alpha{}_\gamma\phi^\alpha\zeta^\gamma \\
\delta\hat{\Pi}^{\alpha(2)} &= i\hat{\beta}_1\mathcal{C}^\alpha\zeta^\alpha = i\hat{\beta}_1 D\phi^\alpha\zeta^\alpha - i\hat{\beta}_1\Phi^\alpha\zeta^\alpha \\
&\quad + i(2M_1\hat{\beta}_1) e^{\alpha(2)}\phi^\beta\zeta_\beta + i(M_1\hat{\beta}_1) e^\alpha{}_\gamma\phi^\alpha\zeta^\gamma
\end{aligned}
\tag{2.18}
$$

Comparison of above relations yield

$$
\hat{\beta}_1 = \beta_1, \qquad \hat{\rho}_1 = \rho_1
$$

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The transformation laws for curvatures $\mathcal{A}$ and Φ look like

$$\delta\hat{\mathcal{A}} = i\alpha_0\mathcal{F}^\alpha\zeta_\alpha + i2m_1\beta_0 e_{\alpha\beta}\mathcal{C}^\alpha\zeta^\beta, \quad \delta\hat{\Phi} = im_1\tilde{\delta}_0\mathcal{C}^\gamma\zeta_\gamma \tag{2.19}$$

Now we consider ansatz for deformation of spin-3/2 curvatures

$$\begin{aligned}
\Delta\mathcal{F}^\alpha &= 2\beta_1\Omega^{\alpha\beta}\Psi_\beta - 2m\alpha_0 e_{\gamma(2)}B^{\gamma(2)}\Psi^\alpha + \delta_0 f^{\alpha\beta}\Psi_\beta + \gamma_0 A\Psi^\alpha - \tilde{\gamma}_0 e^{\alpha\beta}\varphi\Psi_\beta \\
\Delta\mathcal{C}^\alpha &= -\frac{4m\beta_0}{m_1}B^{\alpha\beta}\Psi_\beta - \frac{mM\tilde{\delta}_0}{2m_1}\pi^{\alpha\beta}\Psi_\beta - \rho_0\varphi\Psi^\alpha
\end{aligned}$$

One introduces the corresponding supertransformations

$$\begin{aligned}
\delta\Phi^\alpha &= 2\beta_1\Omega^{\alpha\beta}\zeta_\beta - 2m\alpha_0 e_{\gamma(2)}B^{\gamma(2)}\zeta^\alpha + \delta_0 f^{\alpha\beta}\zeta_\beta + \gamma_0 A\zeta^\alpha - \tilde{\gamma}_0 e^{\alpha\beta}\varphi\zeta_\beta \\
\delta\phi^\alpha &= \frac{4m\beta_0}{m_1}B^{\alpha\beta}\zeta_\beta + \frac{mM\tilde{\delta}_0}{2m_1}\pi^{\alpha\beta}\zeta_\beta + \rho_0\varphi\zeta^\alpha
\end{aligned} \tag{2.20}$$

Let us require the conditions (2.14). The we have on one hand

$$\begin{aligned}
\delta\hat{\mathcal{F}}^\alpha &= 2\beta_1 D\Omega^{\alpha\beta}\zeta_\beta + 2m\alpha_0 e_{\beta(2)}DB^{\beta(2)}\zeta^\alpha + \delta_0 Df^{\alpha\beta}\zeta_\beta + \gamma_0 DA\zeta^\alpha + \tilde{\gamma}_0 D\varphi e^\alpha{}_\beta\zeta^\beta \\
&\quad +(-\lambda\beta_1 - 2M_1\beta_1)e_{\gamma(2)}\Omega^{\alpha\gamma}\zeta^\gamma + (2M_1\beta_1)e_{\gamma(2)}\Omega^{\gamma(2)}\zeta^\alpha \\
&\quad +\left(-\frac{1}{2}\lambda\delta_0 - M_1\delta_0\right)e_{\gamma(2)}f^{\alpha\gamma}\zeta^\gamma + (M_1\delta_0)e_{\gamma(2)}f^{\gamma(2)}\zeta^\alpha + \left(M_1\gamma_0 - \frac{1}{2}\lambda\gamma_0\right)e^\alpha{}_\gamma A\zeta^\gamma \\
&\quad +(-4mM_1\alpha_0 - 16mm_1\beta_0 + 2m\lambda\alpha_0)E^\alpha{}_\gamma B^{\gamma\beta}\zeta_\beta + (4mM_1\alpha_0 - 2m\lambda\alpha_0)E_{\gamma(2)}B^{\alpha\gamma}\zeta^\gamma \\
&\quad -2mMm_1\tilde{\delta}_0 E^\alpha{}_\gamma\pi^{\gamma\beta}\zeta_\beta + (4M_1\tilde{\gamma}_0 - 4{m_1}^2\rho_0 + 2\lambda\tilde{\gamma}_0)E^\alpha{}_\gamma\varphi\zeta^\gamma \\
\delta\hat{\mathcal{C}}^\alpha &= \frac{4m\beta_0}{m_1}DB^{\alpha\beta}\zeta_\beta + \frac{mM\tilde{\delta}_0}{2m_1}D\pi^{\alpha\beta}\zeta_\beta + \rho_0 D\varphi\zeta^\alpha - 2\beta_1\Omega^{\alpha\beta}\zeta_\beta - \delta_0 f^{\alpha\beta}\zeta_\beta - \gamma_0 A\zeta^\alpha \\
&\quad +\left(-\frac{2m\beta_0}{m_1}\lambda - \frac{4mM_1\beta_0}{m_1}\right)e_{\gamma(2)}B^{\alpha\gamma}\zeta^\gamma + \left(2m\alpha_0 + \frac{4mM_1\beta_0}{m_1}\right)e_{\gamma(2)}B^{\gamma(2)}\zeta^\alpha \\
&\quad +\left(-\frac{mM\tilde{\delta}_0}{4m_1}\lambda - \frac{mMM_1\tilde{\delta}_0}{2m_1}\right)e_{\gamma(2)}\pi^{\alpha\gamma}\zeta^\gamma \\
&\quad +\left(\frac{mMM_1\tilde{\delta}_0}{2m_1}\right)e_{\gamma(2)}\pi^{\gamma(2)}\zeta^\alpha + \left(-\tilde{\gamma}_0 + M_1\rho_0 - \frac{1}{2}\lambda\rho_0\right)e^\alpha{}_\gamma\varphi\zeta^\gamma
\end{aligned}$$

and on the other hand

$$\begin{aligned}
\delta\hat{\mathcal{F}}^\alpha &= 2\beta_1\mathcal{R}^{\alpha\beta}\zeta_\beta + 2m\alpha_0 e_{\beta(2)}\mathcal{B}^{\beta(2)}\zeta^\alpha + \delta_0\mathcal{T}^{\alpha\beta}\zeta_\beta + \gamma_0\mathcal{A}\zeta^\alpha + \tilde{\gamma}_0\Phi e^\alpha{}_\beta\zeta^\beta \\
&= 2\beta_1 D\Omega^{\alpha\beta}\zeta_\beta + 2m\alpha_0 e_{\beta(2)}DB^{\beta(2)}\zeta^\alpha + \delta_0 Df^{\alpha\beta}\zeta_\beta + \gamma_0 DA\zeta^\alpha + \tilde{\gamma}_0 D\varphi e^\alpha{}_\beta\zeta^\beta \\
&\quad -2\delta_0 e_{\gamma(2)}\Omega^{\alpha\gamma}\zeta^\gamma + (-2m\alpha_0 + \delta_0)e_{\gamma(2)}\Omega^{\gamma(2)}\zeta^\alpha - M^2\beta_1 e_{\gamma(2)}f^{\alpha\gamma}\zeta^\gamma \\
&\quad +\left(-\frac{1}{4}m\gamma_0 + \frac{M^2}{2}\beta_1\right)e_{\gamma(2)}f^{\gamma(2)}\zeta^\alpha + (2m\delta_0 - 4M\tilde{\gamma}_0)e^\alpha{}_\gamma A\zeta^\gamma \\
&\quad +(-2m^2\beta_1 + 2m\gamma_0)E^\alpha{}_\gamma B^{\gamma\beta}\zeta_\beta + (2m^2\beta_1 + 2m\gamma_0)E_{\gamma(2)}B^{\alpha\gamma}\zeta^\gamma \\
&\quad +(-2mM\tilde{\gamma}_0 + 4mM^2\alpha_0)E^\alpha{}_\gamma\pi^{\gamma\beta}\zeta_\beta + (2mM\tilde{\gamma}_0 + 4mM^2\alpha_0)E_{\gamma(2)}\pi^{\alpha\gamma}\zeta^\gamma \\
&\quad +2mM\beta_1 E^\alpha{}_\gamma\varphi\zeta^\gamma
\end{aligned} \tag{2.21}$$

$$
\begin{aligned}
\delta\hat{\mathcal{C}}^{\alpha} &= \frac{4m\beta_0}{m_1}\mathcal{B}^{\alpha\beta}\zeta_\beta + \frac{mM\tilde{\delta}_0}{2m_1}\Pi^{\alpha\beta}\zeta_\beta + \rho_0\Phi\zeta^\alpha \\
&= \frac{4m\beta_0}{m_1}DB^{\alpha\beta}\zeta_\beta + \frac{mM\tilde{\delta}_0}{2m_1}D\pi^{\alpha\beta}\zeta_\beta + \rho_0 D\varphi\zeta^\alpha - \frac{4m\beta_0}{m_1}\Omega^{\alpha\beta}\zeta_\beta - \frac{mM\tilde{\delta}_0}{2m_1}f^{\alpha\beta}\zeta_\beta \\
&\quad +4M\rho_0 A\zeta^\alpha - \frac{mM\tilde{\delta}_0}{m_1}e_{\gamma(2)}B^{\alpha\gamma}\zeta^\gamma + \frac{mM\tilde{\delta}_0}{2m_1}e_{\gamma(2)}B^{\gamma(2)}\zeta^\alpha - \frac{2mM^2\beta_0}{m_1c_0}e_{\gamma(2)}\pi^{\alpha\gamma}\zeta^\gamma \\
&\quad + \left(mM\rho_0 + \frac{mM^2\beta_0}{m_1}\right)e_{\gamma(2)}\pi^{\gamma(2)}\zeta^\alpha - \frac{m^2M\tilde{\delta}_0}{4m_1M}e^\alpha{}_\gamma\varphi\zeta^\gamma
\end{aligned} \tag{2.22}
$$

Comparison of the above relations gives us at $M_1 = M - \frac{\lambda}{2}$

$$
\delta_0 = M\beta_1, \qquad \gamma_0 = -2\tilde{\gamma}_0 = -\frac{2M(M-\lambda)}{m}\beta_1, \qquad \rho_0 = \frac{(M-\lambda)}{2m}\beta_1
$$

Thus imposing the condition of the covariant curvature transformations (2.14) we fix the supersymmetric deformations up to common parameter β_1 and relation between mass $M_1 = M - \frac{\lambda}{2}$. The law of supersymmetry transformations is given by (2.16), (2.20). The obtained result is in agreement with our recent work [7] where we studied the analogous supermultiplets $(2, 3/2)$ in unfolded formulation.

Invariant Lagrangian. Now we turn to construction of supersymmetric Lagrangian corresponding to supermultiplets $(2, \frac{2}{3})$. Actually in terms of curvatures it presents a sum of Lagrangians for spin 2 (2.5) and spin 3/2 (2.9), where the initial curvatures are replaced by deformed ones $R \to \hat{R}$

$$
\hat{\mathcal{L}} = -\frac{1}{2}\hat{\mathcal{R}}_{\alpha(2)}\hat{\Pi}^{\alpha(2)} - \frac{1}{2}\hat{\mathcal{T}}_{\alpha(2)}\hat{\mathcal{B}}^{\alpha(2)} - \frac{m}{4\tilde{m}}e_{\alpha(2)}\hat{\mathcal{B}}^{\alpha(2)}\hat{\Phi} + \frac{i}{2}\hat{\mathcal{F}}_\alpha\hat{\mathcal{C}}^\alpha
$$

Using the knowing supertransformations for curvatures (2.17), (2.18), (2.19), (2.21), (2.22) it is not very difficult to check the invariance of the Lagrangian. Indeed

$$
\begin{aligned}
\delta\hat{\mathcal{L}} &= -i\left(\rho_1 - \frac{mM}{4m_1}\tilde{\delta}_0\right)\mathcal{F}^\alpha\zeta^\alpha\Pi_{\alpha(2)} - i(\hat{\beta}_1 - \beta_1)\mathcal{R}_{\alpha(2)}\mathcal{C}^\alpha\zeta^\alpha \\
&\quad -i\left(\beta_1 - \frac{2m}{m_1}\beta_0\right)\mathcal{F}^\alpha\zeta^\alpha\mathcal{B}_{\alpha(2)} - i\left(\hat{\rho}_1 - \frac{1}{2}\delta_0\right)\mathcal{T}_{\alpha(2)}\mathcal{C}^\alpha\zeta^\alpha \\
&\quad -i\left(\frac{mm_1}{4\tilde{m}}\tilde{\delta}_0 + m\alpha_0\right)e_{\alpha(2)}\hat{\mathcal{B}}^{\alpha(2)}\mathcal{C}^\gamma\zeta_\gamma - \frac{i}{2}\left(\frac{m}{\tilde{m}}\hat{\rho}_1 + \tilde{\gamma}_0\right)e_{\alpha(2)}\mathcal{C}^\alpha\zeta^\alpha\Phi \\
&\quad -\frac{i\gamma_0}{2}\mathcal{A}\zeta^\alpha\mathcal{C}_\alpha + \frac{i\rho_0}{2}\hat{\mathcal{F}}_\alpha\Phi\zeta^\alpha + \frac{1}{2}i\hat{\rho}_0 e^{\alpha(2)}\mathcal{C}^\beta\Pi_{\alpha(2)}\zeta_\beta
\end{aligned}
$$

Taking into account the equations of equation for fields $B^{\alpha(2)}, \pi^{\alpha(2)}$ which are equivalent to the following relations

$$
\Phi = 0, \quad \mathcal{A} = 0 \quad \Rightarrow \quad e_{\gamma(2)}\Pi^{\gamma(2)} = D\Phi - 4M\mathcal{A} = 0
$$

we see that the variation $\delta\hat{\mathcal{L}}$ vanishes.

3 Free massive higher spin fields

For completeness in this section we discuss the gauge invariant formulation of free massive bosonic and fermionic higher spin fields in AdS_3 [19, 20]. Besides, we present the gauge invariant curvatures, the Lagrangians in explicit component form and the Lagrangians in terms of curvatures for bosonic spin s and fermionic spin $s+1/2$ fields.

3.1 Bosonic spin-s field

In gauge invariant form the massive spin s field is described by system of massless fields with spins $s,(s-1),\dots,0$. In frame-like approach the corresponding set of fields consists of (A)

$$(\Omega^{\alpha(2k)}, f^{\alpha(2k)}) \quad 1 \le k \le (s-1), \quad (B^{\alpha(2)}, A), \quad (\pi^{\alpha(2)}, \varphi)$$

Free Lagrangian for the fields with mass m in AdS_3 have the form

$$\begin{aligned}
\mathcal{L} = & \sum_{k=1}^{s-1}(-1)^{k+1}[k\Omega_{\alpha(2k-1)\beta}e^{\beta}{}_{\gamma}\Omega^{\alpha(2k-1)\gamma} + \Omega_{\alpha(2k)}Df^{\alpha(2k)}] \\
& + EB_{\alpha\beta}B^{\alpha\beta} - B_{\alpha\beta}e^{\alpha\beta}DA - E\pi_{\alpha\beta}\pi^{\alpha\beta} + \pi_{\alpha\beta}E^{\alpha\beta}D\varphi \\
& + \sum_{k=1}^{s-2}(-1)^{k+1}a_k\left[-\frac{(k+2)}{k}\Omega_{\alpha(2)\beta(2k)}e^{\alpha(2)}f^{\beta(2k)} + \Omega_{\alpha(2k)}e_{\beta(2)}f^{\alpha(2k)\beta(2)}\right] \\
& + 2a_0\Omega_{\alpha(2)}e^{\alpha(2)}A - a_0 f_{\alpha\beta}E^{\beta}{}_{\gamma}B^{\alpha\gamma} + 2sM\pi_{\alpha\beta}E^{\alpha\beta}A \\
& + \sum_{k=1}^{s-1}(-1)^{k+1}b_k f_{\alpha(2k-1)\beta}e^{\beta}{}_{\gamma}f^{\alpha(2k-1)\gamma} + b_0 f_{\alpha(2)}E^{\alpha(2)}\varphi + \frac{3a_0{}^2}{2}E\varphi^2
\end{aligned} \tag{3.1}$$

where

$$\begin{aligned}
a_k{}^2 &= \frac{k(s+k+1)(s-k-1)}{2(k+1)(k+2)(2k+3)}[M^2-(k+1)^2\lambda^2] \\
a_0{}^2 &= \frac{(s+1)(s-1)}{3}[M^2-\lambda^2] \\
b_k &= \frac{s^2M^2}{4k(k+1)^2}, \qquad b_0 = \frac{sMa_0}{2}, \qquad M^2 = m^2+(s-1)^2\lambda^2
\end{aligned} \tag{3.2}$$

It is invariant under the following gauge transformations

$$\begin{aligned}
\delta\Omega^{\alpha(2k)} &= D\eta^{\alpha(2k)} + \frac{(k+2)a_k}{k}e_{\beta(2)}\eta^{\alpha(2k)\beta(2)} \\
& \quad + \frac{a_{k-1}}{k(2k-1)}e^{\alpha(2)}\eta^{\alpha(2k-2)} + \frac{b_k}{k}e^{\alpha}{}_{\beta}\xi^{\alpha(2k-1)\beta} \\
\delta f^{\alpha(2k)} &= D\xi^{\alpha(2k)} + e^{\alpha}{}_{\beta}\eta^{\alpha(2k-1)\beta} + a_k e_{\beta(2)}\xi^{\alpha(2k)\beta(2)} \\
& \quad + \frac{(k+1)a_{k-1}}{k(k-1)(2k-1)}e^{\alpha(2)}\xi^{\alpha(2k-2)} \\
\delta\Omega^{\alpha(2)} &= D\eta^{\alpha(2)} + 3a_1 e_{\beta(2)}\eta^{\alpha(2)\beta(2)} + b_1 e^{\alpha}{}_{\gamma}\xi^{\alpha\gamma} \\
\delta f^{\alpha(2)} &= D\xi^{\alpha(2)} + e^{\alpha}{}_{\gamma}\eta^{\alpha\gamma} + a_1 e_{\beta(2)}\xi^{\alpha(2)\beta(2)} + 2a_0 e^{\alpha(2)}\xi
\end{aligned} \tag{3.3}$$

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$$\delta B^{\alpha(2)} = 2a_0\eta^{\alpha(2)}, \qquad \delta A = D\xi + \frac{a_0}{4}e_{\alpha(2)}\xi^{\alpha(2)}$$
$$\delta\pi^{\alpha(2)} = \frac{Msa_0}{2}\xi^{\alpha(2)}, \qquad \delta\varphi = -2Ms\xi$$

One can construct the curvatures invariant under these gauge transformation. After change of normalization

$$B^{\alpha(2)} \to 2a_0 B^{\alpha(2r)} \qquad \pi^{\alpha(2)} \to b_0\pi^{\alpha(2)} \tag{3.4}$$

the curvatures look like

$$\begin{aligned}
\mathcal{R}^{\alpha(2k)} &= D\Omega^{\alpha(2k)} + \frac{(k+2)a_k}{k}e_{\beta(2)}\Omega^{\alpha(2k)\beta(2)} \\
&\quad + \frac{a_{k-1}}{k(2k-1)}e^{\alpha(2)}\Omega^{\alpha(2k-2)} + \frac{b_k}{k}e^{\alpha}{}_{\beta}f^{\alpha(2k-1)\beta} \\
\mathcal{T}^{\alpha(2k)} &= Df^{\alpha(2k)} + e^{\alpha}{}_{\beta}\Omega^{\alpha(2k-1)\beta} + a_k e_{\beta(2)}f^{\alpha(2k)\beta(2)} \\
&\quad + \frac{(k+1)a_{k-1}}{k(k-1)(2k-1)}e^{\alpha(2)}f^{\alpha(2k-2)} \\
\mathcal{R}^{\alpha(2)} &= D\Omega^{\alpha(2)} + 3a_1 e_{\beta(2)}\Omega^{\alpha(2)\beta(2)} + b_1 e^{\alpha}{}_{\gamma}f^{\alpha\gamma} - a_0{}^2E^{\alpha}{}_{\beta}B^{\alpha\beta} + b_0E^{\alpha(2)}\varphi \\
\mathcal{T}^{\alpha(2)} &= Df^{\alpha(2)} + e^{\alpha}{}_{\gamma}\Omega^{\alpha\gamma} + a_1 e_{\beta(2)}f^{\alpha(2)\beta(2)} + 2a_0e^{\alpha(2)}A \\
\mathcal{B}^{\alpha(2)} &= DB^{\alpha(2)} - \Omega^{\alpha(2)} + b_1e^{\alpha}{}_{\beta}\pi^{\alpha\beta} + 3a_1e_{\beta(2)}B^{\alpha(2)\beta(2)} \\
\mathcal{A} &= DA + \frac{a_0}{4}e_{\alpha(2)}f^{\alpha(2)} - 2a_0E_{\gamma(2)}B^{\gamma(2)} \\
\Pi^{\alpha(2)} &= D\pi^{\alpha(2)} - f^{\alpha(2)} + e^{\alpha}{}_{\beta}B^{\alpha\beta} - \frac{a_0}{sM}e^{\alpha(2)}\varphi + a_1e_{\beta(2)}\pi^{\alpha(2)\beta(2)} \\
\Phi &= D\varphi + 2MsA - b_0e_{\alpha(2)}\pi^{\alpha(2)}
\end{aligned} \tag{3.5}$$

$$\begin{aligned}
\mathcal{B}^{\alpha(2k)} &= DB^{\alpha(2k)} - \Omega^{\alpha(2k)} + \frac{b_k}{k}e^{\alpha}{}_{\beta}\pi^{\alpha(2k-1)\beta} + \frac{a_{k-1}}{k(2k-1)}e^{\alpha(2)}B^{\alpha(2k-2)} \\
&\quad + \frac{(k+2)}{k}a_ke_{\beta(2)}B^{\alpha(2k)\beta(2)} \\
\Pi^{\alpha(2k)} &= D\pi^{\alpha(2k)} - f^{\alpha(2k)} + e^{\alpha}{}_{\beta}B^{\alpha(2k-1)\beta} + \frac{(k+1)a_{k-1}}{k(k-1)(2k-1)}e^{\alpha(2)}\pi^{\alpha(2k-2)} \\
&\quad + a_ke_{\beta(2)}\pi^{\alpha(2k)\beta(2)}
\end{aligned} \tag{3.6}$$

In comparison with massive spin 2 case, the construction of curvatures for higher spins has some peculiarities. Namely, in order to achieve gauge invariance for all curvatures we should introduce the so called extra fields $B^{\alpha(2k)}, \pi^{\alpha(2k)}$, $2 \le k \le s-1$ with the following gauge transformations

$$\delta B^{\alpha(2k)} = \eta^{\alpha(2k)} \qquad \delta\pi^{\alpha(2k)} = \xi^{\alpha(2k)}$$

As we already pointed out above, in three dimensions it is possible to write Lagrangian in terms of the curvatures only. In case of arbitrary integer spin field, the corresponding Lagrangian (3.1) can be rewritten in the form

$$\mathcal{L} = -\frac{1}{2}\sum_{k=1}^{s-1}(-1)^{k+1}[\mathcal{R}_{\alpha(2k)}\Pi^{\alpha(2k)} + \mathcal{T}_{\alpha(2k)}\mathcal{B}^{\alpha(2k)}] + \frac{a_0}{2sM}e_{\alpha(2)}\mathcal{B}^{\alpha(2)}\Phi \tag{3.7}$$

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3.2 Fermionic spin-$(s+\frac{1}{2})$ field

In gauge invariant form the massive spin $(s+1/2)$ field is described by system of massless fields with spins $(s+1/2),(s-1/2),\ldots,1/2$. In frame-like approach the corresponding set of fields consists of (B)

$$\Phi^{\alpha(2k+1)} \quad 0 \le k \le (s-1), \quad \phi^\alpha$$

Free Lagrangian for fields with mass m_1 in AdS_3 looks like

$$\begin{aligned}
\frac{1}{i}\mathcal{L} = & \sum_{k=0}^{s-1}(-1)^{k+1}\left[\frac{1}{2}\Phi_{\alpha(2k+1)}D\Phi^{\alpha(2k+1)}\right] + \frac{1}{2}\phi_\alpha E^\alpha{}_\beta D\phi^\beta \\
& + \sum_{k=1}^{s-1}(-1)^{k+1}c_k\Phi_{\alpha(2k-1)\beta(2)}e^{\beta(2)}\Phi^{\alpha(2k-1)} + c_0\Phi_\alpha E^\alpha{}_\beta\phi^\beta \\
& + \sum_{k=0}^{s-1}(-1)^{k+1}\frac{d_k}{2}\Phi_{\alpha(2k)\beta}e^\beta{}_\gamma\Phi^{\alpha(2k)\gamma} - \frac{3d_0}{2}E\phi_\alpha\phi^\alpha
\end{aligned} \tag{3.8}$$

where

$$\begin{aligned}
c_k{}^2 &= \frac{(s+k+1)(s-k)}{2(k+1)(2k+1)}\left[M_1{}^2 - (2k+1)^2\frac{\lambda^2}{4}\right] \\
c_0{}^2 &= 2s(s+1)\left[M_1{}^2 - \frac{\lambda^2}{4}\right] \\
d_k &= \frac{(2s+1)}{(2k+3)}M_1, \qquad M_1{}^2 = m_1{}^2 + \left(s-\frac{1}{2}\right)^2\lambda^2
\end{aligned} \tag{3.9}$$

The Lagrangian is invariant under the gauge transformations

$$\begin{aligned}
\delta\Phi^{\alpha(2k+1)} &= D\xi^{\alpha(2k+1)} + \frac{d_k}{(2k+1)}e^\alpha{}_\beta\xi^{\alpha(2k)\beta} \\
&\quad + \frac{c_k}{k(2k+1)}e^{\alpha(2)}\xi^{\alpha(2k-1)} + c_{k+1}e_{\beta(2)}\xi^{\alpha(2k+1)\beta(2)} \\
\delta\phi^\alpha &= c_0\xi^\alpha
\end{aligned} \tag{3.10}$$

Let us construct gauge invariant curvatures with respect (3.10). After change of normalization $\phi^\alpha \to c_0\phi^\alpha$ the curvatures have form

$$\begin{aligned}
\mathcal{F}^{\alpha(2k+1)} &= D\Phi^{\alpha(2k+1)} + \frac{d_k}{(2k+1)}e^\alpha{}_\beta\Phi^{\alpha(2k)\beta} \\
&\quad + \frac{c_k}{k(2k+1)}e^{\alpha(2)}\Phi^{\alpha(2k-1)} + c_{k+1}e_{\beta(2)}\Phi^{\alpha(2k+1)\beta(2)} \\
\mathcal{F}^\alpha &= D\Phi^\alpha + d_0e^\alpha{}_\beta\Phi^\beta + c_1e_{\beta(2)}\Phi^{\alpha\beta(2)} - c_0{}^2E^\alpha{}_\beta\phi^\beta \\
\mathcal{C}^\alpha &= D\phi^\alpha - \Phi^\alpha + d_0e^\alpha{}_\beta\phi^\beta + c_1e_{\beta(2)}\phi^{\alpha\beta(2)} \\
\mathcal{C}^{\alpha(2k+1)} &= D\phi^{\alpha(2k+1)} - \Phi^{\alpha(2k+1)} + \frac{d_k}{(2k+1)}e^\alpha{}_\beta\phi^{\alpha(2k)\beta} \\
&\quad + \frac{c_k}{k(2k+1)}e^{\alpha(2)}\phi^{\alpha(2k-1)} + c_{k+1}e_{\beta(2)}\phi^{\alpha(2k+1)\beta(2)}
\end{aligned} \tag{3.11}$$

As in the case of integer spins in order to achieve gauge invariance for all curvatures we should introduce the set of extra fields $\phi^{\alpha(2k+1)}$, $1 \leq k \leq (s-1)$ with the following gauge transformation

$$\delta\phi^{\alpha(2k+1)} = \xi^{\alpha(2k+1)}$$

Finally the Lagrangian (3.8) can be rewritten in terms of curvatures only as follows

$$\mathcal{L} = -\frac{i}{2}\sum_{k=0}^{s-2}(-1)^{k+1}\mathcal{F}_{\alpha(2k+1)}\mathcal{C}^{\alpha(2k+1)} \tag{3.12}$$

In section 5 we will need the description of massive spin-$(s-1/2)$ field. It can be obtained from the above description by replacement $s \to (s-1)$.

So we have considered the free massive fields with spins s and spins $s+\frac{1}{2}$. Also, we have formulated the gauge invariant curvatures (3.5), (3.6), (3.11) and gauge invariant Lagrangians (3.7), (3.12). In the next sections we will study a supersymmetrization of these results and construct the Lagrangian description of the supermultiplets $(s, s+\frac{1}{2})$ $(s, s-\frac{1}{2})$.

4 Massive supermultiplet $(s, s+\frac{1}{2})$

In this section we consider the massive higher spin supermultiplets when the highest spin is fermion. For these supermultiplets we construct the deformation of the curvatures, find the supertransformations and present the supersymmetric Lagrangian.

Massless flat limit. Before we turn to realization of given massive supermultiplets let us consider their structure at massless flat limit $m, m_1, \lambda \to 0$. In this case the Lagrangian will be described by the system of massless fields with spins $(s+\frac{1}{2}), s, \ldots, \frac{1}{2}, 0$ in three dimensional flat space

$$\begin{aligned}
\mathcal{L} = &\sum_{k=1}^{s-1}(-1)^{k+1}[k\Omega_{\alpha(2k-1)\beta}e^\beta{}_\gamma\Omega^{\alpha(2k-1)\gamma} + \Omega_{\alpha(2k)}Df^{\alpha(2k)}] \\
&+EB_{\alpha\beta}B^{\alpha\beta} - B_{\alpha\beta}e^{\alpha\beta}DA - E\pi_{\alpha\beta}\pi^{\alpha\beta} + \pi_{\alpha\beta}E^{\alpha\beta}D\varphi \\
&+\frac{i}{2}\sum_{k=0}^{s-1}(-1)^{k+1}\Phi_{\alpha(2k+1)}D\Phi^{\alpha(2k+1)} + \frac{1}{2}\phi_\alpha E^\alpha{}_\beta D\phi^\beta
\end{aligned} \tag{4.1}$$

One can show that this Lagrangian is supersymmetric. Indeed, if the equations of motion (2.12) are fulfilled, the Lagrangian is invariant under the following supertransformations

$$\begin{aligned}
\delta f^{\alpha(2k)} &= i\beta_k\Phi^{\alpha(2k-1)}\zeta^\alpha + i\alpha_k\Phi^{\alpha(2k)\beta}\zeta_\beta \\
\delta f^{\alpha(2)} &= i\beta_1\Phi^\alpha\zeta^\alpha + i\alpha_1\Phi^{\alpha(2)\beta}\zeta_\beta \\
\delta A &= i\alpha_0\Phi^\alpha\zeta_\alpha + ic_0\beta_0 e_{\alpha\beta}\phi^\alpha\zeta^\beta, \qquad \delta\varphi = -\frac{ic_0\tilde{\delta}_0}{2}\phi^\gamma\zeta_\gamma \\
\delta\Phi^{\alpha(2k+1)} &= \frac{\alpha_k}{(2k+1)}\Omega^{\alpha(2k)}\zeta^\alpha + 2(k+1)\beta_{k+1}\Omega^{\alpha(2k+1)\beta}\zeta_\beta
\end{aligned}$$

$$
\begin{aligned}
\delta\Phi^\alpha &= 2\beta_1\Omega^{\alpha\beta}\zeta_\beta + 2a_0\alpha_0 e_{\beta(2)}B^{\beta(2)}\zeta^\alpha \\
\delta\phi^\alpha &= \frac{8a_0\beta_0}{c_0}B^{\alpha\beta}\zeta_\beta + \frac{b_0\tilde{\delta}_0}{c_0}\pi^{\alpha\beta}\zeta_\beta
\end{aligned}
$$

Here we take into account the normalization (3.4). Thus, requiring that massive theory has a correct massless flat limit we partially fix an arbitrariness in the choice of the supertransformations. Parameters $\alpha_k, \beta_k, \beta_0, \alpha_0, \tilde{\delta}_0$ at this step are still arbitrary.

Deformation of curvatures. Again we will realize supersymmetry deforming the curvatures by the background gravitino field Ψ^α. We start with the construction of the deformations for bosonic fields

$$
\begin{aligned}
\Delta\mathcal{R}^{\alpha(2k)} &= i\rho_k\Phi^{\alpha(2k-1)}\Psi^\alpha + i\sigma_k\Phi^{\alpha(2k)\beta}\Psi_\beta \\
\Delta\mathcal{T}^{\alpha(2k)} &= i\beta_k\Phi^{\alpha(2k-1)}\Psi^\alpha + i\alpha_k\Phi^{\alpha(2k)\beta}\Psi_\beta \\
\Delta\mathcal{R}^{\alpha(2)} &= i\rho_1\Phi^\alpha\Psi^\alpha + i\sigma_1\Phi^{\alpha(2)\beta}\Psi_\beta + i\hat{\rho}_0 e^{\alpha(2)}\phi^\beta\Psi_\beta \\
\Delta\mathcal{T}^{\alpha(2)} &= i\beta_1\Phi^\alpha\Psi^\alpha + i\alpha_1\Phi^{\alpha(2)\beta}\Psi_\beta \\
\Delta\mathcal{A} &= i\alpha_0\Phi^\alpha\Psi_\alpha + ic_0\beta_0 e_{\alpha(2)}\phi^\alpha\Psi^\beta, \qquad \Delta\Phi = \frac{ic_0\tilde{\delta}_0}{2}\phi^\alpha\Psi_\alpha \\
\Delta\mathcal{B}^{\alpha(2k)} &= -i\hat{\rho}_k\phi^{\alpha(2k-1)}\Psi^\alpha - i\hat{\sigma}_k\phi^{\alpha(2k)\beta}\Psi_\beta \\
\Delta\Pi^{\alpha(2k)} &= -i\hat{\beta}_k\phi^{\alpha(2k-1)}\Psi^\alpha - i\hat{\alpha}_k\phi^{\alpha(2k)\beta}\Psi_\beta
\end{aligned}
$$

Corresponding ansatz for supertransformations has the form

$$
\begin{aligned}
\delta\Omega^{\alpha(2k)} &= i\rho_k\Phi^{\alpha(2k-1)}\zeta^\alpha + i\sigma_k\Phi^{\alpha(2k)\beta}\zeta_\beta \\
\delta f^{\alpha(2k)} &= i\beta_k\Phi^{\alpha(2k-1)}\zeta^\alpha + i\alpha_k\Phi^{\alpha(2k)\beta}\zeta_\beta \\
\delta\Omega^{\alpha(2)} &= i\rho_1\Phi^\alpha\zeta^\alpha + i\sigma_1\Phi^{\alpha(2)\beta}\zeta_\beta + i\hat{\rho}_0 e^{\alpha(2)}\phi^\beta\zeta_\beta \\
\delta f^{\alpha(2)} &= i\beta_1\Phi^\alpha\zeta^\alpha + i\alpha_1\Phi^{\alpha(2)\beta}\zeta_\beta \\
\delta A &= i\alpha_0\Phi^\alpha\zeta_\alpha + ic_0\beta_0 e_{\alpha(2)}\phi^\alpha\zeta^\beta, \qquad \delta\varphi = -\frac{ic_0\tilde{\delta}_0}{2}\phi^\gamma\zeta_\gamma \\
\delta B^{\alpha(2k)} &= i\hat{\rho}_k\phi^{\alpha(2k-1)}\zeta^\alpha + i\hat{\sigma}_k\phi^{\alpha(2k)\beta}\zeta_\beta \\
\delta\pi^{\alpha(2k)} &= i\hat{\beta}_k\phi^{\alpha(2k-1)}\zeta^\alpha + i\hat{\alpha}_k\phi^{\alpha(2k)\beta}\zeta_\beta
\end{aligned}
\tag{4.2}
$$

All parameters will be fixed from requirement of covariant transformations of the curvatures (2.14). First of all we consider

$$
\begin{aligned}
\delta\hat{\mathcal{R}}^{\alpha(2k)} &= i\rho_k\mathcal{F}^{\alpha(2k-1)}\zeta^\alpha + i\sigma_k\mathcal{F}^{\alpha(2k)\beta}\zeta_\beta \\
\delta\hat{\mathcal{T}}^{\alpha(2k)} &= i\beta_k\mathcal{F}^{\alpha(2k-1)}\zeta^\alpha + i\alpha_k\mathcal{F}^{\alpha(2k)\beta}\zeta_\beta
\end{aligned}
\tag{4.3}
$$

It leads to relation $M_1 = M + \frac{\lambda}{2}$ between mass parameters M_1 and M and defines the parameters

$$\begin{aligned}
{\alpha_k}^2 &= k(s+k+1)[M+(k+1)\lambda]\hat{\alpha}^2 \\
{\beta_k}^2 &= \frac{(k+1)(s-k)}{k(2k+1)}[M-k\lambda]\hat{\beta}^2 \\
{\sigma_k}^2 &= \frac{(s+k+1)}{k(k+1)^2}[M+(k+1)\lambda]\hat{\sigma}^2 \\
{\rho_k}^2 &= \frac{(s-k)}{k^3(k+1)(2k+1)}[M-k\lambda]\hat{\rho}^2
\end{aligned} \tag{4.4}$$

where

$$\hat{\beta} = \frac{\hat{\alpha}}{\sqrt{2}}, \qquad \hat{\rho} = \frac{sM}{2\sqrt{2}}\hat{\alpha}, \qquad \hat{\sigma} = \frac{sM}{2}\hat{\alpha}, \qquad \hat{\alpha}^2 = \frac{{\alpha_{s-1}}^2}{2s(s-1)[M+s\lambda]}$$

From the requirement that

$$\begin{aligned}
\delta\hat{\mathcal{B}}^{\alpha(2k)} &= i\hat{\rho}_k\mathcal{C}^{\alpha(2k-1)}\zeta^\alpha + i\hat{\sigma}_k\mathcal{C}^{\alpha(2k)\beta}\zeta_\beta \\
\delta\hat{\Pi}^{\alpha(2k)} &= i\hat{\beta}_k\mathcal{C}^{\alpha(2k-1)}\zeta^\alpha + i\hat{\alpha}_k\mathcal{C}^{\alpha(2k)\beta}\zeta_\beta
\end{aligned} \tag{4.5}$$

one gets

$$\hat{\rho}_k = \rho_k, \qquad \hat{\sigma}_k = \sigma_k, \qquad \hat{\beta}_k = \beta_k, \qquad \hat{\alpha}_k = \alpha_k$$

Requirement of covariant transformations for the other curvatures

$$\begin{aligned}
\delta\hat{\mathcal{R}}^{\alpha(2)} &= i\rho_1\mathcal{F}^\alpha\zeta^\alpha + i\sigma_1\mathcal{F}^{\alpha(2)\beta}\zeta_\beta - i\hat{\rho}_0 e^{\alpha(2)}\mathcal{C}^\beta\zeta_\beta \\
\delta\hat{\mathcal{T}}^{\alpha(2)} &= i\beta_1\mathcal{F}^\alpha\zeta^\alpha + i\alpha_1\mathcal{F}^{\alpha(2)\beta}\zeta_\beta \\
\delta\hat{\mathcal{A}} &= i\alpha_0\mathcal{F}^\alpha\zeta_\alpha - ic_0\beta_0 e_{\alpha\beta}\mathcal{C}^\alpha\zeta^\beta, \qquad \delta\hat{\Phi} = -\frac{ic_0\tilde{\delta}_0}{2}\mathcal{C}^\gamma\zeta_\gamma
\end{aligned} \tag{4.6}$$

gives the solution

$$\hat{\rho}_0 = -\frac{1}{8}{c_0}^2\beta_1, \qquad \tilde{\delta}_0 = 4\beta_0 = \frac{c_0}{a_0}\beta_1, \qquad \alpha_0 = \frac{{c_0}^2}{4sMa_0}\beta_1$$

Now let us consider the deformation of curvatures for fermion. We choose ansatz in the form

$$\begin{aligned}
\Delta\mathcal{F}^{\alpha(2k+1)} &= \frac{\alpha_k}{(2k+1)}\Omega^{\alpha(2k)}\Psi^\alpha + 2(k+1)\beta_{k+1}\Omega^{\alpha(2k+1)\beta}\Psi_\beta \\
&\quad +\gamma_k f^{\alpha(2k)}\Psi^\alpha + \delta_k f^{\alpha(2k+1)\beta}\Psi_\beta \\
\Delta\mathcal{F}^\alpha &= 2\beta_1\Omega^{\alpha\beta}\Psi_\beta + 2a_0\alpha_0 e_{\beta(2)}B^{\beta(2)}\Psi^\alpha + \delta_0 f^{\alpha\beta}\Psi_\beta + \gamma_0 A\Psi^\alpha + \tilde{\gamma}_0\varphi e^\alpha{}_\beta\Psi^\beta \\
\Delta\mathcal{C}^\alpha &= -\frac{8a_0\beta_0}{c_0}B^{\alpha\beta}\Psi_\beta - \frac{b_0\tilde{\delta}_0}{c_0}\pi^{\alpha\beta}\Psi_\beta - \rho_0\varphi\Psi^\alpha \\
\Delta\mathcal{C}^{\alpha(2k+1)} &= -\tilde{\beta}_k B^{\alpha(2k+1)\beta}\Psi_\beta - \tilde{\alpha}_k B^{\alpha(2k)}\Psi^\alpha - \tilde{\delta}_k\pi^{\alpha(2k+1)\beta}\Psi_\beta - \tilde{\gamma}_k\pi^{\alpha(2k)}\Psi^\alpha
\end{aligned}$$

and ansatz for supertransformations in the form

$$\begin{aligned}
\delta\Phi^{\alpha(2k+1)} &= \frac{\alpha_k}{(2k+1)}\Omega^{\alpha(2k)}\zeta^\alpha + 2(k+1)\beta_{k+1}\Omega^{\alpha(2k+1)\beta}\zeta_\beta \\
&\quad + \gamma_k f^{\alpha(2k)}\zeta^\alpha + \delta_k f^{\alpha(2k+1)\beta}\zeta_\beta \\
\delta\Phi^\alpha &= 2\beta_1\Omega^{\alpha\beta}\zeta_\beta + 2a_0\alpha_0 e_{\beta(2)}B^{\beta(2)}\zeta^\alpha + \delta_0 f^{\alpha\beta}\zeta_\beta + \gamma_0 A\zeta^\alpha + \tilde{\gamma}_0\varphi e^\alpha{}_\beta\zeta^\beta \\
\delta\phi^\alpha &= \frac{8a_0\beta_0}{c_0}B^{\alpha\beta}\zeta_\beta + \frac{b_0\tilde{\delta}_0}{c_0}\pi^{\alpha\beta}\zeta_\beta + \rho_0\varphi\zeta^\alpha \\
\delta\phi^{\alpha(2k+1)} &= \tilde{\beta}_k B^{\alpha(2k+1)\beta}\zeta_\beta + \tilde{\alpha}_k B^{\alpha(2k)}\zeta^\alpha + \tilde{\delta}_k\pi^{\alpha(2k+1)\beta}\zeta_\beta + \tilde{\gamma}_k\pi^{\alpha(2k)}\zeta^\alpha
\end{aligned} \tag{4.7}$$

From the requirement that

$$\begin{aligned}
\delta\hat{\mathcal{F}}^{\alpha(2k+1)} &= \frac{\alpha_k}{(2k+1)}\mathcal{R}^{\alpha(2k)}\zeta^\alpha + 2(k+1)\beta_{k+1}\mathcal{R}^{\alpha(2k+1)\beta}\zeta_\beta \\
&\quad + \gamma_k\mathcal{T}^{\alpha(2k)}\zeta^\alpha + \delta_k\mathcal{T}^{\alpha(2k+1)\beta}\zeta_\beta
\end{aligned} \tag{4.8}$$

we have the same relation between masses $M_1 = M + \frac{\lambda}{2}$. Besides, it leads to

$$\begin{aligned}
\gamma_k{}^2 &= \frac{(s+k+1)}{k(k+1)^2(2k+1)^2}[M+(k+1)\lambda]\hat{\gamma}^2 \\
\delta_k{}^2 &= \frac{(s-k-1)}{(k+1)(k+2)(2k+3)}[M-(k+1)\lambda]\hat{\delta}^2
\end{aligned}$$

where

$$\hat{\gamma} = \frac{sM}{2}\hat{\alpha}, \qquad \hat{\delta} = \frac{sM}{\sqrt{2}}\hat{\alpha}$$

Requirement

$$\delta\hat{\mathcal{C}}^{\alpha(2k+1)} = \tilde{\beta}_k\mathcal{B}^{\alpha(2k+1)\beta}\zeta_\beta + \tilde{\alpha}_k\mathcal{B}^{\alpha(2k)}\zeta^\alpha + \tilde{\delta}_k\Pi^{\alpha(2k+1)\beta}\zeta_\beta + \tilde{\gamma}_k\Pi^{\alpha(2k)}\zeta^\alpha \tag{4.9}$$

gives us

$$\tilde{\gamma}_k = \gamma_k, \quad \tilde{\delta}_k = \delta_k, \quad \tilde{\alpha}_k = \frac{\alpha_k}{(2k+1)}, \quad \tilde{\beta}_k = 2(k+1)\beta_{k+1}$$

At last, requirement for the other curvatures

$$\begin{aligned}
\delta\hat{\mathcal{F}}^\alpha &= 2\beta_1\mathcal{R}^{\alpha\beta}\zeta_\beta - 2a_0\alpha_0 e_{\beta(2)}\mathcal{B}^{\beta(2)}\zeta^\alpha + \delta_0\mathcal{T}^{\alpha\beta}\zeta_\beta + \gamma_0\mathcal{A}\zeta^\alpha + \tilde{\gamma}_0\Phi e^\alpha{}_\beta\zeta^\beta \\
\delta\hat{\mathcal{C}}^\alpha &= \frac{8a_0\beta_0}{c_0}\mathcal{B}^{\alpha\beta}\zeta_\beta + \frac{b_0\tilde{\delta}_0}{c_0}\Pi^{\alpha\beta}\zeta_\beta + \rho_0\Phi\zeta^\alpha
\end{aligned} \tag{4.10}$$

yields solution

$$\gamma_0 = -2\tilde{\gamma}_0 = \frac{c_0{}^2}{2a_0}\beta_1, \quad \rho_0 = -\frac{c_0{}^2}{4sMa_0}\beta_1$$

Now, all the arbitrary parameters are fixed.

Supersymmetric Lagrangian is the sum of free Lagrangians where the initial curvatures are replacement by deformed ones

$$\hat{\mathcal{L}} = -\frac{1}{2}\sum_{k=1}^{s-1}(-1)^{k+1}[\hat{\mathcal{R}}_{\alpha(2k)}\hat{\Pi}^{\alpha(2k)} + \hat{\mathcal{T}}_{\alpha(2k)}\hat{\mathcal{B}}^{\alpha(2k)}] + \frac{a_0}{2sM}e_{\alpha(2)}\hat{\mathcal{B}}^{\alpha(2)}\hat{\Phi}$$
$$-\frac{i}{2}\sum_{k=0}^{s-1}(-1)^{k+1}\hat{\mathcal{F}}_{\alpha(2k+1)}\hat{\mathcal{C}}^{\alpha(2k+1)} \tag{4.11}$$

The Lagrangian is invariant under the supertransformations (4.3), (4.5), (4.6), (4.8), (4.9), (4.10) up to equations of motion for the fields $B^{\alpha(2)}, \pi^{\alpha(2)}$

$$\Phi = 0, \quad \mathcal{A} = 0 \quad \Rightarrow \quad e_{\gamma(2)}\Pi^{\gamma(2)} = D\Phi - 2sM\mathcal{A} = 0 \tag{4.12}$$

The Lagrangian (4.11) is a final solution for the massive supermultiplet $(s, s+\frac{1}{2})$.

5 Massive supermultiplet $(s, s-\frac{1}{2})$

In this section we consider another massive higher spin supermultiplet when the highest spin is boson. The massive spin-s field was described in section 3.1 in terms of massless fields. The massive spin-$(s-1/2)$ field can be obtained for the results in section 3.2 if one makes the replacement $s \to (s-1)$. So the set of massless fields for the massive field with spin $s-1/2$ is $\Phi^{\alpha(2k+1)}$, $0 \le k \le s-2$ and ϕ^α. The gauge invariant curvatures and the Lagrangian have the forms (3.10) and (3.12) where the parameters are

$$\begin{aligned}
{c_k}^2 &= \frac{(s+k)(s-k-1)}{2(k+1)(2k+1)}\left[{M_1}^2 - (2k+1)^2\frac{\lambda^2}{4}\right] \\
{c_0}^2 &= 2s(s-1)\left[{M_1}^2 - \frac{\lambda^2}{4}\right] \\
d_k &= \frac{(2s-1)}{(2k+3)}M_1, \qquad {M_1}^2 = {m_1}^2 + \left(s-\frac{3}{2}\right)^2\lambda^2
\end{aligned} \tag{5.1}$$

Following our procedure we should construct the supersymmetric deformations for curvatures. Actually the structure of deformed curvatures and supertransformations have the same form as in previous section for supermultiplets $(s, s+1/2)$. There is a difference only in parameters (5.1). Therefore we present here only the supertransformations for curvatures. Requirement (2.14) for bosonic fields

$$\begin{aligned}
\delta\hat{\mathcal{R}}^{\alpha(2k)} &= i\rho_k\mathcal{F}^{\alpha(2k-1)}\zeta^\alpha + i\sigma_k\mathcal{F}^{\alpha(2k)\beta}\zeta_\beta \\
\delta\hat{\mathcal{T}}^{\alpha(2k)} &= i\beta_k\mathcal{F}^{\alpha(2k-1)}\zeta^\alpha + i\alpha_k\mathcal{F}^{\alpha(2k)\beta}\zeta_\beta \\
\delta\hat{\mathcal{R}}^{\alpha(2)} &= i\rho_1\mathcal{F}^\alpha\zeta^\alpha + i\sigma_1\mathcal{F}^{\alpha(2)\beta}\zeta_\beta - i\hat{\rho}_0 e^{\alpha(2)}\mathcal{C}^\beta\zeta_\beta \\
\delta\hat{\mathcal{T}}^{\alpha(2)} &= i\beta_1\mathcal{F}^\alpha\zeta^\alpha + i\alpha_1\mathcal{F}^{\alpha(2)\beta}\zeta_\beta \\
\delta\hat{\mathcal{A}} &= i\alpha_0\mathcal{F}^\alpha\zeta_\alpha - ic_0\beta_0 e_{\alpha\beta}\mathcal{C}^\alpha\zeta^\beta, \qquad \delta\hat{\Phi} = -\frac{ic_0\tilde{\delta}_0}{2}\mathcal{C}^\gamma\zeta_\gamma \\
\delta\hat{\mathcal{B}}^{\alpha(2k)} &= i\hat{\rho}_k\mathcal{C}^{\alpha(2k-1)}\zeta^\alpha + i\hat{\sigma}_k\mathcal{C}^{\alpha(2k)\beta}\zeta_\beta \\
\delta\hat{\Pi}^{\alpha(2k)} &= i\hat{\beta}_k\mathcal{C}^{\alpha(2k-1)}\zeta^\alpha + i\hat{\alpha}_k\mathcal{C}^{\alpha(2k)\beta}\zeta_\beta
\end{aligned}$$

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gives us the relation $M_1 = M - \frac{\lambda}{2}$ between masses M_1 and M. Besides, it leads to

$$\begin{aligned}
{\sigma_k}^2 &= \frac{(s-k-1)}{k(k+1)^2}[M-(k+1)\lambda]\hat{\sigma}^2 \\
{\rho_k}^2 &= \frac{(s+k)}{k^3(k+1)(2k+1)}[M+k\lambda]\hat{\rho}^2 \\
{\alpha_k}^2 &= k(s-k-1)[M-(k+1)\lambda]\hat{\alpha}^2 \\
{\beta_k}^2 &= \frac{(k+1)(s+k)}{k(2k+1)}[M+k\lambda]\hat{\beta}^2
\end{aligned} \tag{5.2}$$

$$\hat{\rho}_k = \rho_k, \qquad \hat{\sigma}_k = \sigma_k, \qquad \hat{\beta}_k = \beta_k, \qquad \hat{\alpha}_k = \alpha_k$$

$$\hat{\rho}_0 = -\frac{1}{8}{c_0}^2\beta_1, \qquad \tilde{\delta}_0 = 4\beta_0 = \frac{c_0}{a_0}\beta_1, \qquad \alpha_0 = \frac{{c_0}^2}{4sMa_0}\beta_1$$

where

$$\hat{\beta} = \frac{\hat{\alpha}}{\sqrt{2}}, \qquad \hat{\rho} = \frac{sM}{2\sqrt{2}}\hat{\alpha}, \qquad \hat{\sigma} = \frac{sM}{2}\hat{\alpha}, \qquad \hat{\alpha}^2 = \frac{{\alpha_{s-2}}^2}{(s-2)[M-(s-1)\lambda]}$$

From requirement of covariant supertransformations for fermionic curvatures

$$\begin{aligned}
\delta\hat{\mathcal{F}}^{\alpha(2k+1)} &= \frac{\alpha_k}{(2k+1)}\mathcal{R}^{\alpha(2k)}\zeta^\alpha + 2(k+1)\beta_{k+1}\mathcal{R}^{\alpha(2k+1)\beta}\zeta_\beta \\
&\quad + \gamma_k\mathcal{T}^{\alpha(2k)}\zeta^\alpha + \delta_k\mathcal{T}^{\alpha(2k+1)\beta}\zeta_\beta \\
\delta\hat{\mathcal{F}}^{\alpha} &= 2\beta_1\mathcal{R}^{\alpha\beta}\zeta_\beta - 2a_0\alpha_0 e_{\beta(2)}\mathcal{B}^{\beta(2)}\zeta^\alpha + \delta_0\mathcal{T}^{\alpha\beta}\zeta_\beta + \gamma_0\mathcal{A}\zeta^\alpha + \tilde{\gamma}_0\Phi e^\alpha{}_\beta\zeta^\beta \\
\delta\hat{\mathcal{C}}^{\alpha} &= \frac{8a_0\beta_0}{c_0}\mathcal{B}^{\alpha\beta}\zeta_\beta + \frac{b_0\tilde{\delta}_0}{c_0}\Pi^{\alpha\beta}\zeta_\beta + \rho_0\Phi\zeta^\alpha \\
\delta\hat{\mathcal{C}}^{\alpha(2k+1)} &= \tilde{\beta}_k\mathcal{B}^{\alpha(2k+1)\beta}\zeta_\beta + \tilde{\alpha}_k\mathcal{B}^{\alpha(2k)}\zeta^\alpha + \tilde{\delta}_k\Pi^{\alpha(2k+1)\beta}\zeta_\beta + \tilde{\gamma}_k\Pi^{\alpha(2k)}\zeta^\alpha
\end{aligned}$$

one gets

$$\begin{aligned}
{\gamma_k}^2 &= \frac{(s-k-1)}{k(k+1)^2(2k+1)^2}[M-(k+1)\lambda]\hat{\gamma}^2 \\
{\delta_k}^2 &= \frac{(s+k+1)}{(k+1)(k+2)(2k+3)}[M+(k+1)\lambda]\hat{\delta}^2
\end{aligned}$$

$$\tilde{\gamma}_k = \gamma_k, \qquad \tilde{\delta}_k = \delta_k, \qquad \tilde{\alpha}_k = \frac{\alpha_k}{(2k+1)}, \qquad \tilde{\beta}_k = 2(k+1)\beta_{k+1}$$

$$\gamma_0 = -2\tilde{\gamma}_0 = \frac{{c_0}^2}{2a_0}\beta_1, \qquad \rho_0 = -\frac{{c_0}^2}{4sMa_0}\beta_1$$

where

$$\hat{\gamma} = \frac{sM}{2}\hat{\alpha}, \qquad \hat{\delta} = \frac{sM}{\sqrt{2}}\hat{\alpha}$$

Supersymmetric Lagrangian have form (4.11) and it is invariant under the supertransformations up to equations of motion for auxiliary fields $B^{\alpha(2)}, \pi^{\alpha(2)}$ (4.12).

6 Realization of AdS_3 (super)algebra

In this section we analyze the commutators of (super)transformations and show how the (super)algebra is realized in our construction. All considerations are valid both for $(s, s+1/2)$ supermultiplets and for $(s, s-1/2)$ one.

6.1 AdS-transformations

Before we turn to supersymmetric theory let us discuss the conventional AdS_3 algebra. In the frame formalism, AdS space is described by background Lorentz connection fields $\omega^{\alpha(2)}$ and background frame field $e^{\alpha(2)}$. First of them enters implicitly through the covariant derivative D, second one enters explicitly. Let $\eta^{\alpha(2)}$ and $\xi^{\alpha(2)}$ be the parameters of Lorentz transformations and pseudo-translation respectively. The theory of massive spin-s field has the following laws under these transformations

$$\begin{aligned}
\delta_\eta \Omega^{\alpha(2k)} &= \eta^\alpha{}_\beta \Omega^{\alpha(2k-1)\beta} \qquad \delta_\eta f^{\alpha(2k)} = \eta^\alpha{}_\beta f^{\alpha(2k-1)\beta} \qquad (6.1)\\
\delta_\xi \Omega^{\alpha(2k)} &= \frac{(k+2)a_k}{k}\xi_{\beta(2)}\Omega^{\alpha(2k)\beta(2)} + \frac{a_{k-1}}{k(2k-1)}\xi^{\alpha(2)}\Omega^{\alpha(2k-2)} + \frac{b_k}{k}\xi^\alpha{}_\beta f^{\alpha(2k-1)\beta}\\
\delta_\xi f^{\alpha(2k)} &= \xi^\alpha{}_\beta \Omega^{\alpha(2k-1)\beta} + a_k \xi_{\beta(2)} f^{\alpha(2k)\beta(2)} + \frac{(k+1)a_{k-1}}{k(k-1)(2k-1)}\xi^{\alpha(2)} f^{\alpha(2k-2)} \qquad (6.2)
\end{aligned}$$

here a_k and b_k are defined by (3.2). For massive spin-$(s \pm 1/2)$ field the transformation laws look like

$$\begin{aligned}
\delta_\eta \Phi^{\alpha(2k+1)} &= \eta^\alpha{}_\beta \Phi^{\alpha(2k)\beta}\\
\delta_\xi \Phi^{\alpha(2k+1)} &= \frac{d_k}{(2k+1)}\xi^\alpha{}_\beta \Phi^{\alpha(2k)\beta} + \frac{c_k}{k(2k+1)}\xi^{\alpha(2)}\Phi^{\alpha(2k-1)}\\
&\quad + c_{k+1}\xi_{\beta(2)}\Phi^{\alpha(2k+1)\beta(2)} \qquad (6.3)
\end{aligned}$$

here c_k and d_k are defined by (3.9) for spin-$(s+1/2)$ and (5.1) for spin-$(s-1/2)$. To consider a structure of the AdS_3 algebra $\mathrm{Sp}(2) \otimes \mathrm{Sp}(2)$ only in left sector we introduce the new variables for bosonic fields

$$\hat{\Omega}^{\alpha(2k)} = \Omega^{\alpha(2k)} + \frac{sM}{2k(k+1)} f^{\alpha(2k)}, \qquad \hat{f}^{\alpha(2k)} = \Omega^{\alpha(2k)} - \frac{sM}{2k(k+1)} f^{\alpha(2k)} \qquad (6.4)$$

so that the variables $\hat{\Omega}^{\alpha(2k)}$ correspond to left sector. In terms of this variables the transformations (6.1), (6.2) have form

$$\begin{aligned}
\delta_\eta \hat{\Omega}^{\alpha(2k)} &= \eta^\alpha{}_\beta \hat{\Omega}^{\alpha(2k-1)\beta}\\
\delta_\xi \hat{\Omega}^{\alpha(2k)} &= \frac{(k+2)a_k}{k}\xi_{\beta(2)}\hat{\Omega}^{\alpha(2k)\beta(2)} + \frac{a_{k-1}}{k(2k-1)}\xi^{\alpha(2)}\hat{\Omega}^{\alpha(2k-2)}\\
&\quad + \frac{sM}{2k(k+1)}\xi^\alpha{}_\beta \hat{\Omega}^{\alpha(2k-1)\beta} \qquad (6.5)
\end{aligned}$$

Now let us consider the commutators of these transformations. The direct calculations lead to the following results

$$\begin{aligned}
[\delta_{\eta_1},\delta_{\eta_2}]\hat{\Omega}^{\alpha(2k)} &= (\eta_2{}^\alpha{}_\beta\eta_1{}^\beta{}_\gamma - \eta_1{}^\alpha{}_\beta\eta_2{}^\beta{}_\gamma)\hat{\Omega}^{\alpha(2k-1)\gamma},\\
[\delta_{\xi_1},\delta_{\xi_2}]\hat{\Omega}^{\alpha(2k)} &= \frac{\lambda^2}{4}(\xi_2{}^\alpha{}_\gamma\xi_1{}^\gamma{}_\beta - \xi_1{}^\alpha{}_\gamma\xi_2{}^\gamma{}_\beta)\hat{\Omega}^{\alpha(2k-1)\beta},\\
[\delta_\eta,\delta_\xi]\hat{\Omega}^{\alpha(2k)} &= 2\frac{(k+2)a_k}{k}\xi_{\beta(2)}\eta^\beta{}_\gamma\hat{\Omega}^{\alpha(2k)\beta\gamma} + \frac{a_{k-1}}{k(2k-1)}\xi^\alpha{}_\gamma\eta^{\alpha\gamma}\hat{\Omega}^{\alpha(2k-2)}\\
&\quad + \frac{sM}{2k(k+1)}(\xi^\alpha{}_\beta\eta^\beta{}_\gamma - \eta^\alpha{}_\gamma\xi^\gamma{}_\beta)\hat{\Omega}^{\alpha(2k-1)}
\end{aligned}$$

Comparison with (6.5) shows that we have the AdS-algebra

$$\begin{aligned}
&[M_{\alpha(2)},M_{\beta(2)}] \sim \varepsilon_{\alpha\beta}M_{\alpha\beta}, \qquad [P_{\alpha(2)},P_{\beta(2)}] \sim \lambda^2\varepsilon_{\alpha\beta}M_{\alpha\beta}\\
&[M_{\alpha(2)},P_{\beta(2)}] \sim \varepsilon_{\alpha\beta}P_{\alpha\beta}
\end{aligned}$$

An analogous results have place for the commutators in the fermionic sector.

6.2 Supertransformations

Let us consider the supersymmetric theory. The supertransformations for massive higher spin supermultiplets have form (4.2), (4.7)

$$\begin{aligned}
\delta\Omega^{\alpha(2k)} &= \frac{isM}{2k(k+1)}\beta_k\Phi^{\alpha(2k-1)}\zeta^\alpha + \frac{isM}{2k(k+1)}\alpha_k\Phi^{\alpha(2k)\beta}\zeta_\beta\\
\delta f^{\alpha(2k)} &= i\beta_k\Phi^{\alpha(2k-1)}\zeta^\alpha + i\alpha_k\Phi^{\alpha(2k)\beta}\zeta_\beta\\
\delta\Phi^{\alpha(2k+1)} &= \frac{\alpha_k}{(2k+1)}\Omega^{\alpha(2k)}\zeta^\alpha + 2(k+1)\beta_{k+1}\Omega^{\alpha(2k+1)\beta}\zeta_\beta\\
&\quad + \frac{sM}{2k(k+1)(2k+1)}\alpha_k f^{\alpha(2k)}\zeta^\alpha + \frac{sM}{(k+2)}\beta_{k+1}f^{\alpha(2k+1)\beta}\zeta_\beta
\end{aligned}$$

where the parameters α_k and β_k are defined by (4.4) for $(s,s{+}1/2)$ supermultiplets and (5.2) for $(s,s-1/2)$. In terms of new variables (6.4) the supertransformations look like

$$\begin{aligned}
\delta\hat{\Omega}^{\alpha(2k)} &= \frac{isM}{k(k+1)}\beta_k\Phi^{\alpha(2k-1)}\zeta^\alpha + \frac{isM}{k(k+1)}\alpha_k\Phi^{\alpha(2k)\beta}\zeta_\beta\\
\delta\hat{f}^{\alpha(2k)} &= 0\\
\delta\Phi^{\alpha(2k+1)} &= \frac{\alpha_k}{(2k+1)}\hat{\Omega}^{\alpha(2k)}\zeta^\alpha + 2(k+1)\beta_{k+1}\hat{\Omega}^{\alpha(2k+1)\beta}\zeta_\beta
\end{aligned}$$

One can see that the $\hat{f}^{\alpha(2k)}$ fields are inert under the supertransformations. It just means that we have (1,0) supersymmetry. Let us calculate the commutator of two supertransformations. We obtain

$$
\begin{aligned}
[\delta_1,\delta_2]\hat{\Omega}^{\alpha(2k)} &= isM\hat{\alpha}^2\bigg[\frac{a_{k-1}}{k(2k-1)}\hat{\Omega}^{\alpha(2k-1)}\zeta_1{}^{\alpha}\zeta_2{}^{\alpha}+\frac{2(k+2)a_k}{k}\hat{\Omega}^{\alpha(2k)\gamma\beta}\zeta_{1\beta}\zeta_{2\gamma}\\
&\quad+\frac{sM}{k(k+1)}\hat{\Omega}^{\alpha(2k-1)\gamma}\zeta_1{}^{\alpha}\zeta_{2\gamma}+\lambda\hat{\Omega}^{\alpha(2k-1)\gamma}\zeta_1{}^{\alpha}\zeta_{2\gamma}\bigg]-(1\leftrightarrow 2)\\
[\delta_1,\delta_2]\hat{\Phi}^{\alpha(2k+1)} &= isM\hat{\alpha}^2\bigg[\frac{c_k}{k(2k+1)}\Phi^{\alpha(2k-1)}\zeta_1{}^{\alpha}\zeta_2{}^{\alpha}+2c_{k+1}\Phi^{\alpha(2k+1)\gamma\beta}\zeta_{1\beta}\zeta_{2\gamma}\\
&\quad+\frac{2d_k}{(2k+1)}\Phi^{\alpha(2k)\gamma}\zeta_1^{\alpha}\zeta_{2\gamma}+\lambda\Phi^{\alpha(2k)\gamma}\zeta_1^{\alpha}\zeta_{2\gamma}\bigg]-(1\leftrightarrow 2)
\end{aligned}
$$

Here we use the explicit expressions for α_k and β_k and conditions

$$
\begin{aligned}
\frac{2\beta_k{}^2}{(k+1)}+\frac{\alpha_k{}^2}{k(k+1)(2k+1)} &= \hat{\alpha}^2\left[\frac{sM}{k(k+1)}+\lambda\right]\\
\frac{2\beta_{k+1}{}^2}{(k+2)}+\frac{\alpha_k{}^2}{k(k+1)(2k+1)} &= \hat{\alpha}^2\left[\frac{2d_k}{(2k+1)}+\lambda\right]
\end{aligned}
$$
$$
\alpha_{k-1}\beta_k=(k+1)\hat{\alpha}^2a_{k-1},\qquad \alpha_k\beta_{k+1}=(k+2)\hat{\alpha}^2a_k,\qquad \alpha_k\beta_k=(k+1)c_k\hat{\alpha}^2
$$

Comparing the commutators of supertransformations with (6.3) we obtain the (1,0) AdS_3 superalgebra with the commutation relation

$$
\{Q_\alpha,Q_\beta\}\sim P_{\alpha\beta}+\frac{\lambda}{2}M_{\alpha\beta} \tag{6.6}
$$

As we see, the algebra of the supertransformations is closed. It is worth emphasizing that we did not apply the equations of motion to obtain the relation (6.6) both in bosonic and in fermionic sectors. This situation is analogous to one for massless higher-spin fields in the three-dimensional frame-like formalism. Recall that in the massive supermultiplets case the invariance of the Lagrangians is achieved up to the terms proportional to the spin-1 and spin-0 auxiliary fields equations only. Note that in dimensions $d \geq 4$ one would have to use equations for the higher spins auxiliary fields as well (though in odd dimensions there exist examples of the theories where Lagrangians are invariant without any use of e.o.m [22]). The difference here comes from the well known fact that all massless higher spin fields in three dimensions do not have any local degrees of freedom.

7 Summary

Let us summarize the results. In this paper we have constructed the Lagrangian formulation for massive higher spin supermultiplets in the AdS_3 space in the case of minimal (1,0) supersymmetry. For description of the massive higher spin fields we have adapted for massive fields in three dimensions the frame-like gauge invariant formalism and technique of gauge invariant curvatures. The supersymmetrization is achieved by deformation of the curvatures by background gravitino field and hence the supersymmetric Lagrangians are formulated with help of background fields of three-dimensional supergravity. In AdS_3 the space the minimal (1,0) supersymmetry combines the massive fields in supermultiplets

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with one bosonic degree of freedom and one fermionic one. As a result we have derived the supersymmetric and gauge invariant Lagrangians for massive higher spin $(s, s+1/2)$ and $(s, s-1/2)$ supermultiplets.

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A Massless bosonic fields

In this appendix we consider the frame-like formulation for the massless bosonic fields in three dimensional flat space. For every spin we present field variables and write out the corresponding Lagrangian. All massless fields with spin $s \geq 1$ are gauge ones so that we also present the gauge transformations for them.

Spin 0. It is described by physical 0-form φ and auxiliary 0-form $\pi^{\alpha(2)}$. Lagrangian looks like

$$\mathcal{L} = -E\pi_{\alpha\beta}\pi^{\alpha\beta} + \pi_{\alpha\beta}E^{\alpha\beta}D\varphi$$

Spin 1. It is described by physical 1-form A and auxiliary 0-form $B^{\alpha(2)}$. Lagrangian looks like

$$\mathcal{L} = EB_{\alpha\beta}B^{\alpha\beta} - B_{\alpha\beta}e^{\alpha\beta}DA$$

It is invariant under gauge transformations

$$\delta A = D\xi$$

Spin 2. It is described by physical 1-form $f^{\alpha(2)}$ and auxiliary 1-form $\Omega^{\alpha(2)}$. Lagrangian looks like

$$\mathcal{L} = \Omega_{\alpha\beta}e^{\beta}{}_{\gamma}\Omega^{\alpha\gamma} + \Omega_{\alpha(2k)}Df^{\alpha(2k)}$$

The gauge transformations have the form

$$\delta\Omega^{\alpha(2)} = D\eta^{\alpha(2)}, \qquad \delta f^{\alpha(2)} = D\xi^{\alpha(2)} + e^{\alpha}{}_{\gamma}\eta^{\alpha\gamma}$$

Spin k. It is described by physical 1-form $f^{\alpha(2k-2)}$ and auxiliary 1-form $\Omega^{\alpha(2k-2)}$. Lagrangian looks like

$$\mathcal{L} = (-1)^{k+1}[k\Omega_{\alpha(2k-1)\beta}e^{\beta}{}_{\gamma}\Omega^{\alpha(2k-1)\gamma} + \Omega_{\alpha(2k)}Df^{\alpha(2k)}]$$

The gauge transformations have the form

$$\delta\Omega^{\alpha(2k)} = D\eta^{\alpha(2k)}, \qquad \delta f^{\alpha(2k)} = D\xi^{\alpha(2k)} + e^{\alpha}{}_{\beta}\eta^{\alpha(2k-1)\beta}$$

B Massless fermionic fields

In this appendix we consider the frame-like formulation for the massless fermionic fields in three dimensional flat space. For every spin we present field variables and write out the corresponding Lagrangian. All massless fields with spin $s \geq 3/2$ are gauge ones so that we also present gauge transformations for them.

Spin 1/2. It is described by master 0-form ϕ^α. Lagrangian looks like

$$\mathcal{L} = \frac{1}{2}\phi_\alpha E^\alpha{}_\beta D\phi^\beta$$

Spin 3/2. It is described by physical 1-form Φ^α. Lagrangian and gauge transformations have the form

$$\mathcal{L} = -\frac{i}{2}\Phi_\alpha D\Phi^\alpha, \qquad \delta\Phi^\alpha = D\xi^\alpha$$

Spin $k+1/2$. It is described by physical 1-form $\Phi^{\alpha(2k-1)}$. Lagrangian and gauge transformations have the form

$$\mathcal{L} = (-1)^{k+1}\frac{i}{2}\Phi_{\alpha(2k+1)}D\Phi^{\alpha(2k+1)}, \qquad \delta\Phi^{\alpha(2k+1)} = D\xi^{\alpha(2k+1)}$$

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