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2D sigma models and differential Poisson algebras

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Abstract: We construct a two-dimensional topological sigma model whose target space is endowed with a Poisson algebra for differential forms. The model consists of an equal number of bosonic and fermionic fields of worldsheet form degrees zero and one. The action is built using exterior products and derivatives, without any reference to a worldsheet metric, and is of the covariant Hamiltonian form. The equations of motion define a universally Cartan integrable system. In addition to gauge symmetries, the model has one rigid nilpotent supersymmetry corresponding to the target space de Rham operator. The rigid and local symmetries of the action, respectively, are equivalent to the Poisson bracket being compatible with the de Rham operator and obeying graded Jacobi identities. We propose that perturbative quantization of the model yields a covariantized differential star product algebra of Kontsevich type. We comment on the resemblance to the topological A model.

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Contents

1 Introduction

There are two approaches to quantizing the algebra of functions on a Poisson manifold depending on whether one uses cohomological deformation techniques relying on algebraically well-defined structures [1, 2] or field theoretic methods based on path integrals for two-dimensional sigma models [3, 4].

Following the algebraic approach, the existence of a deformation of the Poisson bracket into a diffeomorphism invariant and associative bi-differential operator given in an $\hbar$ expansion, sometimes referred to as a star product, was first established in the symplectic case by Fedosov [1]. The existence of a unique star product on a Poisson manifold, fixed by the conditions of general covariance and associativity, was then demonstrated by Kontsevich [2] as part of a more general framework centered around his formality conjecture.

Inspired by string theory, Kontsevich also gave an explicit local form of the star product in the case of a general Poisson structure on $\mathbb{R}^n$. More precisely, Kontsevich's bi-differential operator is expanded into blocks, associated to graphs, built out of partial derivatives of the components of the Poisson bi-vector. On the other hand, Fedosov's star product algebra, which was later generalized to the Poisson case in [5], is realized in a manifestly covariant albeit less explicit fashion in terms of covariantly constant sections of an auxiliary bundle with flat connection built from the symplectic structure.

JHEP08(2015)095

Following the field theory approach, Kontsevich's local formula was derived using AKSZ path integral methods [6] from the perturbative expansion of the correlations functions of the Poisson sigma model subject to suitable boundary conditions [7] (see also [8]).

The main motivation behind this work is the fact that the Poisson algebra of functions, or zero-forms, admits a natural extension so as to include higher form degrees, sometimes referred to as differential Poisson algebras, which were defined and studied in the physics literature in [9]. Later, following the algebraic approach, the corresponding star product between forms has been studied in [10–13].[1] However, the manifestly generally covariant local formula remains to be given explicitly beyond $\hbar^2$-corrections except in the torsion free symplectic case [12] where a direct generalization of the Moyal formula applies.

In this paper, we provide a generalization of the two-dimensional Poisson sigma model in [3, 4] so as to include fermionic worldsheet zero-forms facilitating the mapping of target space differential forms to vertex operators of worldsheet form degree zero. Besides incorporating forms of higher degrees, drawing on the aforementioned partial results, we expect that the perturbative quantization of the model to be presented here will yield a manifestly covariant star product formula.

A key feature of differential Poisson algebras is the presence of a connection one-form $\widetilde{\Gamma}^{\alpha}{}_{\beta}$ that is compatible with the Poisson bi-vector $\Pi^{\alpha\beta}$. As we shall review in section 2, the covariantized Poisson bracket between two differential forms reads

$$\{\omega, \eta\} = \Pi^{\alpha\beta}\nabla_{\alpha}\omega \wedge \nabla_{\beta}\eta + (-1)^{|\omega|}\Pi^{\beta\gamma}\, \widetilde{R}^{\alpha}{}_{\gamma}\, i_{\alpha}\omega \wedge i_{\beta}\eta\,,$$

where ∇_{α} has connection coefficients $\Gamma^{\alpha}_{\beta\gamma} = \widetilde{\Gamma}^{\alpha}_{\gamma\beta}$ and $\widetilde{R}^{\alpha}{}_{\beta} = d\widetilde{\Gamma}^{\alpha}{}_{\beta} + \widetilde{\Gamma}^{\alpha}{}_{\gamma} \wedge \widetilde{\Gamma}^{\gamma}{}_{\beta}$. Since the connection plays a key role in covariantizing the star product in the algebraic approach, it is natural to seek an extension of the original Poisson sigma model that couples to it as well. Another problem that one would like to address is how to map target space p-forms $d\phi^{\alpha_1} \wedge \cdots \wedge d\phi^{\alpha_p}\omega_{\alpha_1\ldots\alpha_p}$ to vertex operators of form degree zero on the worldsheet. To this end, one observes that by introducing fermionic worldsheet zero-forms θ^{α}, the vertex operators can be taken to be $\theta^{\alpha_1}\cdots\theta^{\alpha_p}\omega_{\alpha_1\ldots\alpha_p}$. Thus, combining these two observations, we are lead to adding fermionic copies $(\theta^{\alpha}, \chi_{\alpha})$ of the original bosonic worldsheet zero- and one-forms $(\phi^{\alpha}, \eta_{\alpha})$. The proposed action, which we shall study in more detail in section 3, reads[2]

$$S[\phi, \eta, \theta, \chi] = \int_{M_2}\left(\eta_{\alpha} \wedge d\phi^{\alpha} + \frac{1}{2}\Pi^{\alpha\beta}\eta_{\alpha} \wedge \eta_{\beta} + \chi_{\alpha} \wedge \nabla\theta^{\alpha} + \frac{1}{4}\Pi^{\beta\epsilon}\widetilde{R}_{\gamma\delta}{}^{\alpha}{}_{\epsilon}\chi_{\alpha} \wedge \chi_{\beta}\theta^{\gamma}\theta^{\delta}\right)\,.$$

The role of the additional quartic fermion coupling is to ensure a rigid supersymmetry δ_{f} that in particular acts as $\delta_{\mathrm{f}}(\phi^{\alpha}, \theta^{\alpha}) = (\theta^{\alpha}, 0)$. Under additional conditions on the background, the action is also invariant under gauge transformations with one unconstrained parameter for each one-form. We shall show that these rigid and local symmetries, respectively, are equivalent to the bracket being compatible with the de Rham differential and

[1] In [14], the deformation quantization procedure has been set up and studied at order $\hbar$ in the case of more general vector bundles over Poisson manifolds.

[2] In the Conclusions, we shall comment on the resemblance between the action presented here and that of the topological A model.

obeying graded Jacobi identities. Thus, assuming that exist a gauge fixed action that is manifestly background covariant, we expect that the products of the aforementioned vertex operators contain the covariantized Kontsevich star product for differential forms of any degree, whose explicit construction we leave for a future work.

The plan of the paper is as follows. In section 2, we review the basic properties of differential Poisson algebras and the conditions on the Poisson bi-vector and curvature following from the Jacobi identities. In section 3, we present the sigma model action and show that its symmetries are equivalent to the salient features of the differential Poisson algebra. In section 4, we conclude and remark on the resemblance between our model and the topological A model, and its potential importance in higher spin theory. We give our conventions and some useful identities in appendix A.

2 Differential Poisson algebras

In this section we recall the defining relations of Poisson differential algebras [9, 10] and the resulting form of the Poisson bracket. The bracket consists of three compatible structures, namely a Poisson bi-vector $\Pi^{\alpha\beta}$, a connection $\Gamma^\alpha{}_\beta$ and a one-form $S^{\alpha\beta}$. The one-form contains the components of the bracket that are not contained in the pre-connection [10], that is, the covariant derivative along the Hamiltonian vector field defined using $\Pi^{\alpha\beta}$. In the symplectic case, one can set $S^{\alpha\beta} = 0$ by redefining $\Gamma^\alpha{}_\beta$, in which case the Poisson bracket is given by $\Pi^{\alpha\beta}$ and a curvature two-form $\widetilde{R}^\alpha{}_\beta$ constructed from the torsion. In what follows, we shall set $S^{\alpha\beta} = 0$, leaving for future work the analysis of whether there exists non-trivial S tensors in the non-symplectic case.

2.1 Definition

A differential Poisson algebra is a differential algebra Ω endowed with a graded skew-symmetric and degree preserving bilinear map $\{\cdot,\cdot\}$, called Poisson bracket, that is compatible with exterior differentiation and obeys the graded Leibniz rule, that is

$$\deg(\{\omega_1,\omega_2\}) = \deg(\omega_1) + \deg(\omega_2)\,, \tag{2.1}$$

$$\{\omega_1,\omega_2\} = (-1)^{\deg(\omega_1)\deg(\omega_2)+1}\{\omega_2,\omega_1\}\,, \tag{2.2}$$

$$\{\omega_1,\omega_2+\omega_3\} = \{\omega_1,\omega_2\} + \{\omega_1,\omega_3\}\,, \tag{2.3}$$

$$\{\omega_1,\omega_2\wedge\omega_3\} = \{\omega_1,\omega_2\}\wedge\omega_3 + (-1)^{\deg(\omega_1)\deg(\omega_2)}\omega_2\wedge\{\omega_1,\omega_3\}\,, \tag{2.4}$$

$$d\{\omega_1,\omega_2\} = \{d\omega_1,\omega_2\} + (-1)^{\deg(\omega_1)}\{\omega_1,d\omega_2\}\,, \tag{2.5}$$

and that obeys the graded Jacobi identity

$$\begin{aligned}\{\omega_1,\{\omega_2,\omega_3\}\} &+ (-1)^{\deg(\omega_1)(\deg(\omega_2)+\deg(\omega_3))}\{\omega_2,\{\omega_3,\omega_1\}\}\\ &+ (-1)^{\deg(\omega_3)(\deg(\omega_1)+\deg(\omega_2))}\{\omega_3,\{\omega_1,\omega_2\}\} = 0\,,\end{aligned} \tag{2.6}$$

where $\omega_i \in \Omega$ and $\deg(\cdot)$ is the form degree. We shall furthermore assume that Ω is realized as the algebra $\Omega(N)$ of differential forms on a manifold N. In what follows, we shall first use eqs. (2.1)–(2.5) to expand the Poisson bracket in terms of Π, S and the curvature $\widetilde{R}$, and then impose eq. (2.6).

2.2 Poisson bi-vector and compatible connection

Introducing local coordinates ϕ^α on N, we define

$$\Pi^{\alpha\beta} := \{\phi^\alpha, \phi^\beta\}\,, \tag{2.7}$$

which is thus an anti-symmetric tensor. The Poisson bracket between two zero-forms f and g can then be written as

$$\{f, g\} = \Pi^{\alpha\beta}\partial_\alpha f\, \partial_\beta g\ . \tag{2.8}$$

Next, to expand the Poisson bracket between a zero-form and a one-form in the coordinate basis, we define

$$\Upsilon^{\alpha\beta} := \{\phi^\alpha, d\phi^\beta\} = \frac{1}{2}d\Pi^{\alpha\beta} + \Sigma^{\alpha\beta}\,, \tag{2.9}$$

where $\Sigma^{\alpha\beta}$ is thus a symmetric one-form. From the Leibniz rule, it follows that

$$\{d\phi^\alpha, d\phi^\beta\} = d\Sigma^{\alpha\beta}\ . \tag{2.10}$$

Under a general coordinate transformation $\phi^\alpha = \phi^\alpha(\phi'^{\alpha'})$ with Jacobian $J^\alpha_{\beta'} = \partial\phi^\alpha/\partial\phi'^{\beta'}$ and inverse Jacobian $J'^{\alpha'}_\beta$, one has the non-tensorial transformation property

$$\Upsilon^{\alpha\beta} = J^\alpha_{\alpha'}J^\beta_{\beta'}\Upsilon'^{\alpha'\beta'} + J^\alpha_{\alpha'}\Pi'^{\alpha'\beta'}dJ^\beta_{\beta'}\ . \tag{2.11}$$

Introducing a connection one-form

$$\widetilde{\Gamma}^\alpha{}_\beta = d\phi^\gamma\widetilde{\Gamma}^\alpha_{\gamma\beta}\,, \tag{2.12}$$

one has the transformation law

$$\Pi^{\alpha\gamma}\widetilde{\Gamma}^\beta{}_\gamma = J^\alpha_{\alpha'}J^\beta_{\beta'}\Pi'^{\alpha'\gamma'}\widetilde{\Gamma}'^{\beta'}{}_{\gamma'} - \Pi^{\alpha\gamma}J'^{\beta'}_\gamma dJ^\beta_{\beta'}\ . \tag{2.13}$$

Using the tensorial transformation property of Π to rewrite (2.11) as

$$\Upsilon^{\alpha\beta} = J^\alpha_{\alpha'}J^\beta_{\beta'}\Upsilon'^{\alpha'\beta'} + \Pi^{\alpha\gamma}J'^{\beta'}_\gamma dJ^\beta_{\beta'}\,, \tag{2.14}$$

we can thus write

$$\Upsilon^{\alpha\beta} = U^{\alpha\beta} - \Pi^{\alpha\gamma}\widetilde{\Gamma}^\beta{}_\gamma\,, \tag{2.15}$$

where $U^{\alpha\beta}$ is a tensorial one-form. It follows that

$$\begin{aligned}\Upsilon^{\alpha\beta}_\gamma &= \frac{1}{2}\partial_\gamma\Pi^{\alpha\beta} + \Sigma^{\alpha\beta}_\gamma,\\ &= \frac{1}{2}\widetilde{\nabla}_\gamma\Pi^{\alpha\beta} + \Sigma^{\alpha\beta}_\gamma - \Pi^{\delta[\beta}\widetilde{\Gamma}^{\alpha]}_{\gamma\delta},\\ &= \frac{1}{2}\widetilde{\nabla}_\gamma\Pi^{\alpha\beta} + \Sigma^{\alpha\beta}_\gamma - \Pi^{\delta(\alpha}\widetilde{\Gamma}^{\beta)}_{\gamma\delta} - \Pi^{\alpha\delta}\widetilde{\Gamma}^\beta_{\gamma\delta}\ .\end{aligned} \tag{2.16}$$

Thus $U^{\alpha\beta} = \frac{1}{2}\widetilde{\nabla}\Pi^{\alpha\beta} + S^{\alpha\beta}$, where

$$S_\gamma^{\alpha\beta} := \Sigma_\gamma^{\alpha\beta} - \Pi^{\delta(\alpha}\widetilde{\Gamma}^{\beta)}_{\gamma\delta} \tag{2.17}$$

are the components of a tensorial one-form. In summary, we can write

$$\{\phi^\alpha, d\phi^\beta\} = \frac{1}{2}\widetilde{\nabla}\Pi^{\alpha\beta} + S^{\alpha\beta} - \Pi^{\alpha\gamma}\widetilde{\Gamma}^\beta{}_\gamma\,, \tag{2.18}$$

where the first two terms are tensorial and the last term, which is non-tensorial, is sometimes referred to as the pre-connection [10].

In the case that the Poisson manifold is regular, i.e. that the rank of its Poisson tensor is constant, one may choose the connection to belong to the equivalence class obeying

$$\widetilde{\nabla}_\alpha \Pi^{\beta\gamma} = 0\,, \tag{2.19}$$

which we shall assume henceforth, while leaving the more general case for a separate study.[3] It follows that

$$\{d\phi^\alpha, d\phi^\beta\} = -\widetilde{R}^{\alpha\beta} + \Pi^{\gamma\delta}\widetilde{\Gamma}^\alpha_\gamma \wedge \widetilde{\Gamma}^\beta_\delta + dS^{\alpha\beta}\,, \tag{2.20}$$

where the two-form

$$\widetilde{R}^{\alpha\beta} := \Pi^{\beta\gamma}\widetilde{R}^\alpha{}_\gamma = \widetilde{R}^{\beta\alpha}\,, \tag{2.21}$$

as a consequence of (2.19).

2.3 Manifestly covariant Poisson bracket

Let ω and η be differential forms of any degree. From the basic Poisson brackets (2.7), (2.18) and (2.20) in the coordinate basis, and invoking (2.2) and (2.4), it then follows that

$$\begin{aligned}\{\omega, \eta\} \; = \; & \Pi^{\alpha\beta}\nabla_\alpha\omega \wedge \nabla_\beta\eta + S^{\alpha\beta} \wedge \Big((-1)^{|\omega|}\nabla_\alpha\omega \wedge i_\beta\eta - i_\alpha\omega \wedge \nabla_\beta\eta\Big) \\ & + (-1)^{|\omega|}\Big(\widetilde{R}^{\alpha\beta} - \widetilde{\nabla}S^{\alpha\beta}\Big) \wedge i_\alpha\omega \wedge i_\beta\eta\,,\end{aligned} \tag{2.22}$$

where i denotes inner differentiation and ∇ uses the connection coefficients

$$\Gamma^\alpha_{\beta\gamma} := \widetilde{\Gamma}^\alpha_{\gamma\beta}\;. \tag{2.23}$$

By construction, the above manifestly covariant form of the Poisson bracket obeys (2.5), that is, it is compatible with the de Rham operator.

The equivalence class of compatible connections is generated by shifts $\delta\widetilde{\Gamma}^\alpha_{\beta\gamma}$ obeying $\delta(\widetilde{\nabla}_\alpha\Pi^{\beta\gamma}) = 0$ and $\delta\Pi^{\alpha\beta} = 0$, that is $\delta\widetilde{\Gamma}^{[\beta}_{\alpha\delta}\Pi^{\gamma]\delta} = 0$. Under such shifts, the Poisson bracket (2.18) is left invariant provided

$$\delta S^{\alpha\beta} = \Pi^{\alpha\gamma}\delta\widetilde{\Gamma}^\beta_\gamma\,, \tag{2.24}$$

which is indeed symmetric in α and β. The invariance of the full Poisson bracket (2.22) can then be verified using $\delta\nabla_\alpha\omega = -\delta\widetilde{\Gamma}^\beta_\alpha i_\beta\omega$ and $\delta\widetilde{R}^{\alpha\beta} = \Pi^{\alpha\gamma}\widetilde{\nabla}\delta\widetilde{\Gamma}^\beta_\gamma$. In the symplectic case, the shift symmetry (2.24) can be used to set $S = 0$. In what follows, we shall specialize to the case $S = 0$, which has been studied in detail in [9–12], leaving the analysis of the general case for future work.

[3] It is possible to relax the compatibility condition (2.19) at the expense of introducing additional terms to the Poisson bracket and corresponding Yukawa and quartic fermion couplings to the Poisson sigma model action.

JHEP08(2015)095

2.4 Jacobi identities

In order to analyze the Jacobi identities (2.6) (in the case that $S = 0$), one can use (2.4) to show that if they hold for functions and one-forms then they hold for forms of any degree. In the case of three functions f_1, f_2, f_3, one finds

$$0 = \{f_{[1}, \{f_2, f_{3]}\}\} = 3\nabla_\alpha f_{[1} \nabla_\beta f_2 \nabla_\gamma f_{3]} \Pi^{\alpha\delta} T^{\beta}_{\delta\epsilon} \Pi^{\epsilon\gamma} , \qquad (2.25)$$

from which it follows that

$$J_0^{\alpha\beta\gamma} := \Pi^{\delta[\alpha} T^{\beta}_{\delta\epsilon} \Pi^{\gamma]\epsilon} = 0 \ . \qquad (2.26)$$

In view of $\widetilde{\nabla}_\alpha \Pi^{\gamma\delta} = 0$, this condition is equivalent to that Π is a Poisson bi-vector, i.e.

$$\Pi^{\delta[\alpha} \partial_\delta \Pi^{\beta\gamma]} = 0 \ . \qquad (2.27)$$

In the case of two function f_1, f_2 and a one-form ω, the Jacobi identities read

$$0 = 2\{f_{[1}, \{f_{2]}, \omega\}\} + \{\omega, \{f_1, f_2\}\} = \nabla_\alpha f_{[1} \nabla_\beta f_{2]} \omega_\gamma \Pi^{\alpha\delta} \Pi^{\beta\epsilon} R_{\delta\epsilon}{}^{\gamma}{}_{\lambda} d\phi^\lambda \ , \qquad (2.28)$$

from which we obtain

$$J_1^{\alpha\beta,\gamma}{}_{\lambda} := \Pi^{\alpha\delta} \Pi^{\beta\epsilon} R_{\delta\epsilon}{}^{\gamma}{}_{\lambda} = 0 \ . \qquad (2.29)$$

Finally, for a single function f and two one-forms ω_1, ω_2, we have

$$0 = \{f, \{\omega_{(1}, \omega_{2)}\}\} + 2\{\omega_{(1}, \{\omega_{2)}, f\}\} = -\nabla_\alpha f i_\beta \omega_{(1} i_\gamma \omega_{2)} \Pi^{\alpha\delta} \nabla_\delta \widetilde{R}^{\beta\gamma} , \qquad (2.30)$$

which implies that

$$J_2^{\alpha,\beta\gamma}{}_{\delta\epsilon} := \Pi^{\alpha\lambda} \nabla_\lambda \widetilde{R}_{\delta\epsilon}{}^{\beta\gamma} = 0 \ . \qquad (2.31)$$

Finally, for three one-forms one finds that

$$J_3^{\alpha\beta\gamma}{}_{\rho\sigma\lambda} := \widetilde{R}_{\epsilon[\rho}{}^{(\alpha\beta} \widetilde{R}_{\sigma\lambda]}{}^{\gamma)\epsilon} = 0 \ . \qquad (2.32)$$

As observed in [10, 12], the compatibility between the Poisson bracket and the de Rham differential implies that the independent conditions are given by the following irreducible representations[4]

$$J_0^{\alpha\beta\gamma} = 0 \, , \qquad J_1^{\alpha(\beta,\gamma)}{}_{\lambda} = 0 \, , \qquad J_2^{(\alpha,\beta\gamma)}{}_{\delta\epsilon} = 0 \ . \qquad (2.33)$$

As for examples of non-trivial solutions, see [9, 10].

3 Poisson sigma model

In this section, we use the Poisson bi-vector and its compatible connection to construct the couplings in a two-dimensional topological sigma model action that exhibits an extra nilpotent rigid supersymmetry δ_{f} corresponding to the de Rham differential on N. As we shall see, the rigid symmetry fixes the coefficient of the quartic fermion coupling while the gauge symmetries require the background fields to obey the conditions (2.27), (2.29), (2.31) and (2.32), which we recall are equivalent to that the underlying differential Poisson algebra obeys the Jacobi identities.

[4] From (2.33) the remaining conditions follow by covariant differentiation, *viz.*

$$J_1^{[\alpha\beta,\gamma]}{}_{\lambda} \sim \nabla_\lambda J_0^{\alpha\beta\gamma} \, , \quad J_2^{[\alpha,\beta]\gamma}{}_{\delta\epsilon} \sim \nabla_{[\delta} J_1^{\alpha\beta,\gamma}{}_{\epsilon]} \, , \quad J_3^{\alpha\beta\gamma}{}_{\delta\epsilon\lambda} \sim \nabla_{[\delta} J_2^{(\alpha,\beta\gamma)}{}_{\epsilon\lambda]} \ .$$

3.1 The action

Our action, which is formulated on a two-dimensional manifold M_2, is given by

$$S = \int_{M_2} \left(\eta_\alpha \wedge d\phi^\alpha + \frac{1}{2}\Pi^{\alpha\beta}\eta_\alpha \wedge \eta_\beta + \chi_\alpha \wedge \nabla\theta^\alpha + \frac{1}{4}\widetilde{R}_{\gamma\delta}{}^{\alpha\beta}\chi_\alpha \wedge \chi_\beta\, \theta^\gamma\, \theta^\delta \right) , \tag{3.1}$$

where $\widetilde{R}_{\gamma\delta}{}^{\alpha\beta} = \Pi^{\beta\epsilon}\widetilde{R}_{\gamma\delta}{}^{\alpha}{}_{\epsilon}$ are the components of the two-form (2.21) obtained from the Poisson bi-vector and its compatible connection, and the covariant exterior derivative

$$\nabla\theta^\alpha := d\theta^\alpha + d\phi^\beta\Gamma^\alpha_{\beta\gamma}\theta^\gamma , \tag{3.2}$$

where the connection coefficients are defined in (2.23). The fields are assigned form degrees $\deg_2$ on M_2 and an additional Grassmann parity $\epsilon_{\mathrm{f}}(\cdot)$ as follows:

$$\deg_2(\phi^\alpha, \eta_\alpha; \theta^\alpha, \chi_\alpha) = (0; 1, 0, 1) , \qquad \epsilon_{\mathrm{f}}(\phi^\alpha; \eta_\alpha, \theta^\alpha, \chi_\alpha) = (0; 0, 1, 1) . \tag{3.3}$$

In what follows, we shall assume M_2 to be compact and that the pull-backs of $(\eta_\alpha, \chi_\alpha)$ to the boundary of M_2 vanish.

Geometrically, the action describes a sigma model with source M_2 and target space given by the $\mathbb{N}$-graded bundle

$$\widehat{N} = T^*[1,0]N \oplus T[0,1]N \oplus T^*[1,1]N , \tag{3.4}$$

coordinatized by $(\phi^\alpha; \eta_\alpha, \theta^\alpha, \chi_\alpha)$, where $T[p,\epsilon]N$ is the degree shift of the tangent bundle $T[0,\epsilon]N$ over N by p units *idem* $T^*[p,\epsilon]$. The Grassmann parity of $T[p,\epsilon]N$ is $\epsilon_{\mathrm{f}}(T[p,\epsilon]N) = \epsilon$. The sigma model map $\varphi : M_2 \to \widehat{N}$ has vanishing intrinsic degree and Grassmann parity, and the degree on M_2 is the form degree on M_2. Thus, if $\omega_{n,p,\epsilon}$ denotes an n-form on $\widehat{N}$ of degree p and Grassmann parity ϵ, then its pull-back $\omega_{(n+p),\epsilon} := \varphi^*\omega_{n,p,\epsilon}$ is a $(n+p)$-form on M_2 of Grassmann parity ϵ. The de Rham differential on $\widehat{N}$ has form degree one and degree one. We use the following Koszul sign convention, which is consistent with Leibniz' rule:

$$\omega_{n_1,p_1,\epsilon_1} \wedge \omega_{n_2,p_2,\epsilon_2} = (-1)^{(n_1+p_1)(n_2+p_2)+\epsilon_1\epsilon_2}\omega_{n_2,p_2,\epsilon_2} \wedge \omega_{n_1,p_1,\epsilon_1} . \tag{3.5}$$

The resulting sign convention for wedge products on M_2 reads

$$\omega_{p_1,\epsilon_1} \wedge \omega_{p_2,\epsilon_2} = (-1)^{p_1p_2+\epsilon_1\epsilon_2}\omega_{p_2,\epsilon_2} \wedge \omega_{p_1,\epsilon_1} . \tag{3.6}$$

The manifest target space covariance of the action amounts to the fact that target space diffeomorphisms

$$\delta_\xi\phi^\alpha = \xi^\alpha , \quad \delta_\xi\eta_\alpha = -\partial_\alpha\xi^\beta\eta_\beta , \quad \delta_\xi\theta^\alpha = \partial_\beta\xi^\alpha\theta^\beta , \quad \delta_\xi\chi_\alpha = -\partial_\alpha\xi^\beta\chi_\beta , \tag{3.7}$$

which act on the worldsheet fields, induce Lie derivatives acting on the background fields, i.e.

$$\delta_\xi S[\phi,\eta,\theta,\chi;\Pi,\Gamma] = \mathcal{L}_\xi S[\phi,\eta,\theta,\chi;\Pi,\Gamma] , \tag{3.8}$$

where

$$\mathcal{L}_\xi\Pi^{\alpha\beta} = \xi^\gamma\partial_\gamma\Pi^{\alpha\beta} + 2\partial_\gamma\xi^{[\alpha}\Pi^{\beta]\gamma} , \tag{3.9}$$

$$\mathcal{L}_\xi\Gamma^\alpha_{\beta\gamma} = \partial_\beta\partial_\gamma\xi^\alpha + \xi^\delta\partial_\delta\Gamma^\alpha_{\beta\gamma} - \partial_\delta\xi^\alpha\Gamma^\delta_{\beta\gamma} + \partial_\beta\xi^\delta\Gamma^\alpha_{\delta\gamma} + \partial_\gamma\xi^\delta\Gamma^\alpha_{\beta\delta} . \tag{3.10}$$

JHEP08(2015)095

3.2 Nilpotent rigid fermionic symmetry

The de Rham differential $d = d\phi^\alpha \partial_\alpha$ on N lifts to a holonomic vector field $\theta^\alpha\partial_\alpha$ on $T[0,1]N$, which in its turn can be extended to a nilpotent rigid supersymmetry $\delta_{\rm f}$ of the action as follows:

$$\begin{aligned}
\delta_{\rm f}\phi^\alpha &= \theta^\alpha\,,\\
\delta_{\rm f}\theta^\alpha &= 0\,,\\
\delta_{\rm f}\eta_\alpha &= \Gamma^\beta_{\alpha\gamma}\eta_\beta\theta^\gamma + \frac{1}{2}\widetilde{R}_{\beta\gamma}{}^\delta{}_\alpha\,\chi_\delta\,\theta^\beta\theta^\gamma\,,\\
\delta_{\rm f}\chi_\alpha &= -\eta_\alpha - \Gamma^\beta_{\alpha\gamma}\chi_\beta\theta^\gamma\,,
\end{aligned} \tag{3.11}$$

as can be seen using $\widetilde{\nabla}_\alpha\Pi^{\beta\gamma} = 0$ and the Bianchi identity $\widetilde{\nabla}_{[\alpha}\widetilde{R}_{\beta\gamma]}{}^\delta{}_\epsilon - \widetilde{T}^\lambda_{[\alpha\beta}\widetilde{R}_{\gamma]\lambda}{}^\delta{}_\epsilon = 0$. We note that $\deg_2(\delta_{\rm f}) = 0$, $\epsilon_{\rm f}(\delta_{\rm f}) = 1$ and that $\delta_{\rm f}^2\eta_\alpha = 0$ requires the aforementioned Bianchi identity. Moreover, just as the relative coefficient between the two terms in the Poisson bracket (2.22) is fixed by compatibility with the d operator, the rigid supersymmetry requirement fixes the relative strength between the kinetic terms and the quartic fermion term in the action (3.1). In fact, the $\delta_{\rm f}$-invariance of the action can be made manifest by observing that the Lagrangian is $\delta_{\rm f}$-exact, *viz.*

$$S \equiv \int_{M_2} L\,, \qquad L = \delta_{\rm f} V\,, \tag{3.12}$$

where

$$V = -\chi_\alpha \wedge \left(d\phi^\alpha + \frac{1}{2}\Pi^{\alpha\beta}\eta_\beta\right)\,, \tag{3.13}$$

as can easily be seen using $\widetilde{\nabla}_\alpha\Pi^{\beta\gamma} = 0$ and we note that there is no need to discard any total derivative in (3.12). The commutator between the rigid supersymmetry and the target space diffeomorphisms takes the form

$$[\delta_\xi,\delta_{\rm f}]\phi^\alpha = 0\,, \qquad [\delta_\xi,\delta_{\rm f}]\eta_\alpha = \mathcal{L}_\xi\Gamma^\gamma_{\alpha\beta}\,\eta_\gamma\theta^\beta + \mathcal{L}_\xi\widetilde{R}_{\beta\gamma}{}^\delta{}_\alpha\chi_\delta\,\theta^\beta\theta^\gamma\,, \tag{3.14}$$

$$[\delta_\xi,\delta_{\rm f}]\theta^\alpha = 0\,, \qquad [\delta_\xi,\delta_{\rm f}]\chi_\alpha = -\mathcal{L}_\xi\Gamma^\gamma_{\alpha\beta}\chi_\gamma\theta^\beta\,. \tag{3.15}$$

Thus the rigid supersymmetry commutes with background symmetries whose Lie derivatives annihilate $\Pi^{\alpha\beta}$ and $\Gamma^\alpha_{\beta\gamma}$ and hence the action as can be seen from (3.8).

3.3 Equations of motion

Applying the variational principle to the action (3.1) yields the following equations of motion:

$$\mathcal{R}^{\phi^\alpha} := d\phi^\alpha + \Pi^{\alpha\beta}\eta_\beta = 0\,, \tag{3.16}$$

$$\mathcal{R}^{\theta^\alpha} := \nabla\theta^\alpha + \frac{1}{2}\widetilde{R}_{\gamma\delta}{}^{\alpha\beta}\chi_\beta\theta^\gamma\theta^\delta = 0\,, \tag{3.17}$$

$$\mathcal{R}^{\chi_\alpha} := \nabla\chi_\alpha - \frac{1}{2}\widetilde{R}_{\alpha\delta}{}^{\beta\gamma}\chi_\beta\wedge\chi_\gamma\theta^\delta = 0\,, \tag{3.18}$$

$$\mathcal{R}^{\eta_\alpha} := \nabla\eta_\alpha + R_{\alpha\gamma}{}^\beta{}_\delta\chi_\beta\wedge d\phi^\gamma\theta^\delta + \frac{1}{4}\nabla_\alpha\widetilde{R}_{\delta\epsilon}{}^{\beta\gamma}\chi_\beta\wedge\chi_\gamma\theta^\delta\theta^\epsilon = 0\,, \tag{3.19}$$

where

$$\nabla\chi_\alpha := d\chi_\alpha - d\phi^\beta \Gamma^\gamma_{\beta\alpha}\wedge\chi_\gamma\,, \tag{3.20}$$

idem $\nabla\eta_\alpha$. We note that eqs. (3.16)–(3.18) are given by the functional derivatives of S with respect to $(\eta_\alpha, \chi_\alpha, \theta^\alpha)$, respectively, while eq. (3.19) has been obtained from

$$\begin{aligned}\frac{\delta S}{\delta\phi^\alpha} &= d\eta_\alpha + \frac{1}{2}\partial_\alpha\Pi^{\beta\gamma}\eta_\beta\wedge\eta_\gamma + (\Gamma^\gamma_{\alpha\beta}d\chi_\gamma - \chi_\gamma\wedge d\Gamma^\gamma_{\alpha\beta} + \partial_\alpha\Gamma^\gamma_{\delta\beta}\chi_\gamma\wedge d\phi^\delta)\theta^\beta \\ &\quad -\Gamma^\gamma_{\alpha\beta}\chi_\gamma\wedge d\theta^\beta + \frac{1}{4}\partial_\alpha\widetilde{R}_{\beta\gamma}{}^{\delta\epsilon}\chi_\delta\wedge\chi_\epsilon\theta^\beta\theta^\gamma\,,\end{aligned} \tag{3.21}$$

by rewriting $\partial_\alpha\Pi^{\beta\gamma}\eta_\beta\eta_\gamma$ using $\mathcal{R}^{\phi^\alpha}=0$ and $\widetilde{\nabla}_\alpha\Pi^{\beta\gamma}=0$, and the quantities $d\chi_\gamma\Gamma^\gamma_{\alpha\beta}\theta^\beta$ and $\chi_\gamma\Gamma^\gamma_{\alpha\beta}d\theta^\beta$ using $\mathcal{R}^{\chi_\alpha}=0$ and $\mathcal{R}^{\theta^\alpha}=0$, respectively.

3.4 Universal Cartan integrability

Let us demonstrate that the universal Cartan integrability of the equations of motion, which is required for the validity of Cartan gauge symmetries and on-shell integration, is equivalent to that the target space background obeys the conditions (2.27), (2.29), (2.31) and (2.32), i.e. that they can be used to define a differential Poisson algebra obeying the Jacobi identity (2.6). To this end, we derive the generalized Bianchi identities

$$\nabla\mathcal{R}^i + M^i_j\wedge\mathcal{R}^j + \mathcal{A}^i = 0\,, \tag{3.22}$$

where $\mathcal{R}^i := (\mathcal{R}^{\phi^\alpha}, \mathcal{R}^{\theta^\alpha}, \mathcal{R}^{\chi_\alpha}, \mathcal{R}^{\eta_\alpha})$ and M^i_j is a field dependent matrix, after which we require compatibility in the universal sense, that is, that the classical anomalies $\mathcal{A}^i$ vanishes on base manifolds of arbitrary dimensions.

As for $\nabla\mathcal{R}^{\phi^\alpha}$, and using $\nabla d\phi^\alpha = T^\alpha$, the resulting compatibility condition reads

$$\begin{aligned}\mathcal{A}^{\phi^\alpha} &= -\left(\frac{1}{2}\Pi^{\rho\delta}T^\alpha_{\rho\epsilon}\Pi^{\sigma\epsilon} + \nabla_\rho\Pi^{\alpha\sigma}\Pi^{\rho\delta}\right)\eta_\delta\wedge\eta_\sigma + \Pi^{\alpha\rho}\Pi^{\gamma\sigma}R_{\rho\gamma}{}^\beta{}_\delta\,\chi_\beta\wedge\eta_\sigma\theta^\delta \\ &\quad -\frac{1}{4}\Pi^{\alpha\rho}\nabla_\rho\widetilde{R}_{\delta\epsilon}{}^{\beta\gamma}\,\chi_\beta\wedge\chi_\gamma\theta^\delta\theta^\epsilon\,,\end{aligned} \tag{3.23}$$

that must thus hold without imposing any algebraic constraint on $(\phi^\alpha, \eta_\alpha; \theta^\alpha, \chi_\alpha)$. Thus, using also the identity $\nabla_\rho\Pi^{\alpha\sigma} = -2T^{[\alpha}_{\rho\epsilon}\Pi^{\sigma]\epsilon}$, which allows us to rewrite the first term as $\frac{3}{2}\Pi^{\rho[\delta}T^\alpha_{\rho\epsilon}\Pi^{\sigma]\epsilon}\eta_\delta\wedge\eta_\sigma$, the vanishing of $\mathcal{A}^{\phi^\alpha}$ requires

$$\Pi^{\delta[\alpha}T^\beta_{\delta\epsilon}\Pi^{\gamma]\epsilon} = 0\,,\qquad \Pi^{\alpha\rho}\Pi^{\sigma\beta}R_{\rho\sigma}{}^\gamma{}_\delta = 0\,,\qquad \Pi^{\alpha\lambda}\nabla_\lambda\widetilde{R}_{\beta\gamma}{}^{\rho\sigma} = 0\,, \tag{3.24}$$

which we identify as the complete set of conditions required for the Jacobi identity (2.6). In particular, the second condition in (3.24) is equivalent to that

$$\nabla^2 = 0 \qquad \text{on-shell}\,. \tag{3.25}$$

Next, taking into account (3.24), the vanishing of $\mathcal{A}^{\theta^\alpha}=0$ requires the additional condition

$$\widetilde{R}_{\epsilon\rho}{}^{(\alpha\beta}\widetilde{R}_{\sigma\lambda}{}^{\gamma)\epsilon}\,\chi_\beta\wedge\chi_\gamma\theta^\rho\theta^\sigma\theta^\lambda = 0\,, \tag{3.26}$$

or, equivalently,

$$\widetilde{R}_{\epsilon[\rho}{}^{(\alpha\beta}\widetilde{R}_{\sigma\lambda]}{}^{\gamma)\epsilon} = 0 \ . \tag{3.27}$$

which is a consequence of the previous conditions, as discussed below eq. (2.31). Turning to the integrability of $\mathcal{R}^{\chi_\alpha} = 0$, it follows from (3.24) and (3.27) that $\mathcal{A}^{\chi_\alpha}$ vanishes universally, noting that the $\chi^{\wedge 3}\theta^2$-terms in $\nabla\mathcal{R}^{\chi_\alpha}$ are proportional to $\widetilde{R}_{\epsilon[\alpha}{}^{(\rho\sigma}\widetilde{R}_{\beta\gamma]}{}^{\lambda)\epsilon}\chi_\rho \wedge \chi_\sigma \wedge \chi_\lambda \theta^\alpha\theta^\gamma$. Finally, using (3.25) one has

$$\begin{aligned}\mathcal{A}^{\eta_\alpha} = & -\frac{1}{4}\Pi^{\lambda\beta}(\nabla_\beta\nabla_\alpha\widetilde{R}_{\gamma\delta}{}^{\rho\sigma} + 2R_{\alpha\beta}{}^{\rho}{}_{\epsilon}\widetilde{R}_{\gamma\delta}{}^{\sigma\epsilon} + 2R_{\alpha\beta}{}^{\epsilon}{}_{\gamma}\widetilde{R}_{\delta\epsilon}{}^{\rho\sigma})\chi_\rho \wedge \chi_\sigma \wedge \eta_\lambda \theta^\gamma\theta^\delta \\ & +\frac{1}{4}\nabla_\alpha(\widetilde{R}_{\beta\delta}{}^{\rho\sigma}\widetilde{R}_{\epsilon\gamma}{}^{\lambda\beta})\,\chi_\rho \wedge \chi_\sigma \wedge \chi_\lambda \theta^\delta\theta^\epsilon\theta^\gamma \\ & +\Pi^{\sigma\beta}\Pi^{\delta\lambda}\left(\nabla_\beta R_{\delta\alpha}{}^{\rho}{}_{\gamma} - \frac{1}{2}T^{\epsilon}_{\beta\delta}R_{\alpha\epsilon}{}^{\rho}{}_{\gamma}\right)\chi_\rho \wedge \eta_\sigma \wedge \eta_\lambda \theta^\gamma \,, \end{aligned} \tag{3.28}$$

modulo $\mathcal{R}^{\chi_\alpha}$, $\mathcal{R}^{\phi^\alpha}$ and $\mathcal{R}^{\theta^\alpha}$. The second term is easily seen to be zero from condition (3.27). To show the vanishing of the first set of terms, we use the third condition in (3.24) and $\nabla_\alpha\Pi^{\lambda\beta} = -2T^{[\lambda}_{\alpha\epsilon}\Pi^{\beta]\epsilon}$ to compute

$$0 = \nabla_\alpha(\Pi^{\lambda\beta}\nabla_\beta\widetilde{R}_{\gamma\delta}{}^{\rho\sigma}) = \Pi^{\lambda\beta}(\nabla_\alpha\nabla_\beta\widetilde{R}_{\gamma\delta}{}^{\rho\sigma} + T^{\epsilon}_{\alpha\beta}\nabla_\epsilon\widetilde{R}_{\gamma\delta}{}^{\rho\sigma}) \ . \tag{3.29}$$

Employing the Ricci identity

$$[\nabla_\alpha, \nabla_\beta]\widetilde{R}_{\gamma\delta}{}^{\rho\sigma} = -T^{\epsilon}_{\alpha\beta}\nabla_\epsilon\widetilde{R}_{\gamma\delta}{}^{\rho\sigma} + 2R_{\alpha\beta}{}^{(\rho}{}_{\epsilon}\widetilde{R}_{\gamma\delta}{}^{\sigma)\epsilon} + 2R_{\alpha\beta}{}^{\epsilon}{}_{[\gamma}\widetilde{R}_{\delta]\epsilon}{}^{\rho\sigma} \,,$$

one has

$$\begin{aligned}&\Pi^{\lambda\beta}(\nabla_\beta\nabla_\alpha\widetilde{R}_{\gamma\delta}{}^{\rho\sigma} + 2R_{\alpha\beta}{}^{(\rho}{}_{\epsilon}\widetilde{R}_{\gamma\delta}{}^{\sigma)\epsilon} + 2R_{\alpha\beta}{}^{\epsilon}{}_{[\gamma}\widetilde{R}_{\delta]\epsilon}{}^{\rho\sigma}) \\ &\qquad = \Pi^{\lambda\beta}(\nabla_\alpha\nabla_\beta\widetilde{R}_{\gamma\delta}{}^{\rho\sigma} + T^{\epsilon}_{\alpha\beta}\nabla_\epsilon\widetilde{R}_{\gamma\delta}{}^{\rho\sigma}) = 0 \ . \end{aligned} \tag{3.30}$$

To show the vanishing of the third set of terms in (3.28), we rewrite the Bianchi identity $\nabla_{[\beta}R_{\delta\alpha]}{}^{\rho}{}_{\sigma} - T^{\epsilon}_{[\beta\delta}R_{\alpha]\epsilon}{}^{\rho}{}_{\gamma} = 0$ as

$$2\left(\nabla_{[\beta}R_{\delta]\alpha}{}^{\rho}{}_{\gamma} - \frac{1}{2}T^{\epsilon}_{\beta\delta}R_{\alpha\epsilon}{}^{\rho}{}_{\gamma}\right) + \nabla_\alpha R_{\beta\delta}{}^{\rho}{}_{\gamma} - 2T^{\epsilon}_{\alpha[\beta}R_{\delta]\epsilon}{}^{\rho}{}_{\gamma} = 0 \ . \tag{3.31}$$

On the other hand, the second condition in (3.24) together with $\nabla_\alpha\Pi^{\sigma\beta} = -2T^{[\sigma}_{\alpha\epsilon}\Pi^{\beta]\epsilon}$ implies

$$0 = \nabla_\alpha(\Pi^{\sigma\beta}\Pi^{\delta\lambda}R_{\beta\delta}{}^{\rho}{}_{\gamma}) = \Pi^{\sigma\beta}\Pi^{\delta\lambda}(\nabla_\alpha R_{\beta\delta}{}^{\rho}{}_{\gamma} - 2T^{\epsilon}_{\alpha[\beta}R_{\delta]\epsilon}{}^{\rho}{}_{\gamma}) \ . \tag{3.32}$$

Thus, contracting the above form of the Bianchi identity by $\Pi^{\sigma\beta}\Pi^{\delta\lambda}$ and using (3.32) it follows that the third term in $\mathcal{A}^{\eta_\alpha}$ vanishes as well.

In summary, we have showed that the universal Cartan integrability of the equations of motion eqs. (3.16)–(3.19) is equivalent to that the background fields Π and Γ can be used to define a differential Poisson algebra.

JHEP08(2015)095

3.5 Gauge transformations

Relying upon the general framework for Poisson sigma models [15–19], the universal Cartan integrability of the equations of motion implies that these as well as the action are invariant under suitably defined gauge transformations; for details, see section 2.2 of [18].

On-shell, the gauge transformations can be obtained by first rewriting the equations of motion eqs. (3.16)–(3.19) on the canonical form

$$\widehat{\mathcal{R}}^i := dZ^i + \widehat{\mathcal{Q}}^i(Z^j) = 0\,, \qquad Z^i := (\phi^\alpha, \eta_\alpha; \theta^\alpha, \chi_\alpha)\ . \tag{3.33}$$

Eliminating $d\phi^\alpha$ in ∇ using $\mathcal{R}^{\phi^\alpha} = 0$, we thus have

$$\widehat{\mathcal{R}}^{\phi^\alpha} = d\phi^\alpha + \Pi^{\alpha\beta}\eta_\beta\,, \tag{3.34}$$

$$\widehat{\mathcal{R}}^{\eta_\alpha} = d\eta_\alpha + \Pi^{\beta\gamma}\Gamma^\delta_{\beta\alpha}\,\eta_\gamma \wedge \eta_\delta + \Pi^{\gamma\lambda}R_{\alpha\gamma}{}^\beta{}_\delta\,\eta_\lambda \wedge \chi_\beta\theta^\delta + \frac{1}{4}\nabla_\alpha \widetilde{R}_{\delta\epsilon}{}^{\beta\gamma}\chi_\beta \wedge \chi_\gamma\theta^\delta\theta^\epsilon\,, \tag{3.35}$$

$$\widehat{\mathcal{R}}^{\theta^\alpha} = d\theta^\alpha - \Pi^{\beta\gamma}\Gamma^\alpha_{\beta\delta}\eta_\gamma\theta^\delta + \frac{1}{2}\widetilde{R}_{\gamma\delta}{}^{\alpha\beta}\chi_\beta\theta^\gamma\theta^\delta\,, \tag{3.36}$$

$$\widehat{\mathcal{R}}^{\chi_\alpha} = d\chi_\alpha + \Pi^{\beta\gamma}\Gamma^\delta_{\beta\alpha}\eta_\gamma \wedge \chi_\delta - \frac{1}{2}\widetilde{R}_{\alpha\delta}{}^{\beta\gamma}\chi_\beta \wedge \chi_\gamma\theta^\delta\ . \tag{3.37}$$

The on-shell gauge transformations are then given by

$$\delta Z^i = d\epsilon^i - \epsilon^j\frac{\partial}{\partial Z^j}\widehat{\mathcal{Q}}^i\,, \qquad \text{modulo } \widehat{\mathcal{R}}^i\,, \tag{3.38}$$

where ϵ^i denote the gauge parameters, of which there is one for each fields with strictly positive form degree, that is,

$$\epsilon^i = (0, \epsilon^{(\eta)}_\alpha; 0, \epsilon^{(\chi)}_\alpha)\,, \qquad \deg_2(\epsilon^i) = (-,0;-,0)\,, \qquad \epsilon_{\rm f}(\epsilon^i) = (-,0;-,1)\ . \tag{3.39}$$

Thus, the infinitesimal gauge transformations are given by

$$\delta\phi^\alpha = -\Pi^{\alpha\beta}\epsilon^{(\eta)}_\beta\,, \tag{3.40}$$

$$\begin{aligned}\delta\eta_\alpha = {}& \nabla\epsilon^{(\eta)}_\alpha - \Pi^{\beta\gamma}\Gamma^\delta_{\beta\alpha}\epsilon^{(\eta)}_\gamma\eta_\delta - \Pi^{\gamma\lambda}R_{\alpha\gamma}{}^\beta{}_\delta\,\epsilon^{(\eta)}_\lambda\chi_\beta\theta^\delta + \Pi^{\gamma\lambda}R_{\alpha\gamma}{}^\beta{}_\delta\,\eta_\lambda\epsilon^{(\chi)}_\beta\theta^\delta \\ & -\frac{1}{2}\nabla_\alpha\widetilde{R}_{\delta\epsilon}{}^{\beta\gamma}\epsilon^{(\chi)}_\beta\chi_\gamma\theta^\delta\theta^\epsilon\,,\end{aligned} \tag{3.41}$$

$$\delta\theta^\alpha = \Pi^{\beta\gamma}\Gamma^\alpha_{\beta\delta}\epsilon^{(\eta)}_\gamma\theta^\delta - \frac{1}{2}\widetilde{R}_{\gamma\delta}{}^{\alpha\beta}\epsilon^{(\chi)}_\beta\theta^\gamma\theta^\delta\,, \tag{3.42}$$

$$\delta\chi_\alpha = \nabla\epsilon^{(\chi)}_\alpha - \Pi^{\beta\gamma}\Gamma^\delta_{\beta\alpha}\epsilon^{(\eta)}_\gamma\chi_\delta + \widetilde{R}_{\alpha\delta}{}^{\beta\gamma}\epsilon^{(\chi)}_\beta\chi_\gamma\theta^\delta\,, \tag{3.43}$$

modulo $\mathcal{R}^i$.

Off-shell, the Lagrangian transforms under the on-shell gauge transformations (3.38) into a total derivative modulo a set of terms proportional to derivatives of the symplectic structure. The latter can be cancelled by using modified off-shell gauge transformations given by (see eqs. (33) and (34) of [18])

$$\delta Z^i = d\epsilon^i - \epsilon^j\frac{\partial}{\partial Z^j}\widehat{\mathcal{Q}}^i + \frac{1}{2}\epsilon^k\,\widehat{\mathcal{R}}^l\,\partial_l\,\widehat{\Omega}_{kj}\,\widehat{\mathcal{P}}^{ji}\,, \tag{3.44}$$

where we have introduced the symplectic two-form

$$\widehat{\Omega} = d\widehat{\Theta} = \frac{1}{2} dZ^i \widehat{\mathcal{O}}_{ij} dZ^j = \frac{1}{2} dZ^i dZ^j \widehat{\Omega}_{ij} \,, \qquad \widehat{\mathcal{P}}^{ik} \widehat{\mathcal{O}}_{kj} = -\delta^i_j \,, \tag{3.45}$$

of degree three on the target space $\widehat{N}$ given in (3.4), and the symplectic potential

$$\widehat{\Theta} = \eta_\alpha \wedge d\phi^\alpha + \chi_\alpha \wedge \nabla \theta^\alpha \,, \tag{3.46}$$

also known as the tautological one-form, here treated as a one-form of $\mathbb{N}$-degree two on $\widehat{N}$. Thus, the matrix $\widehat{\mathcal{O}}_{ij}$ can be read off from

$$\widehat{\Omega} = \frac{1}{2} \begin{pmatrix} d\phi^\rho & d\eta_\rho & d\theta^\rho & d\chi_\rho \end{pmatrix} \begin{pmatrix} 2\partial_{[\rho} \Gamma^\alpha_{\gamma]\beta} \chi_\alpha \theta^\beta & \delta_\rho{}^\gamma & -\Gamma^\alpha_{\rho\gamma} \chi_\alpha & -\Gamma^\gamma_{\rho\alpha} \theta^\alpha \\ \delta^\rho{}_\gamma & 0 & 0 & 0 \\ -\Gamma^\alpha_{\gamma\rho} \chi_\alpha & 0 & 0 & -\delta_\rho{}^\gamma \\ \Gamma^\rho_{\gamma\alpha} \theta^\alpha & 0 & \delta^\rho{}_\gamma & 0 \end{pmatrix} \begin{pmatrix} d\phi^\gamma \\ d\eta_\gamma \\ d\theta^\gamma \\ d\chi_\gamma \end{pmatrix} . \tag{3.47}$$

Moreover, the components $\widehat{\mathcal{P}}^{ji}$ of the Poisson structure on $\widehat{N}$ is given by

$$\widehat{\mathcal{P}}^{ik} = \begin{pmatrix} 0 & -\delta^\sigma{}_\rho & 0 & 0 \\ -\delta_\sigma{}^\rho & R_{\sigma\rho}{}^\alpha{}_\beta \chi_\alpha \theta^\beta & \Gamma^\rho_{\sigma\alpha} \theta^\alpha & -\Gamma^\alpha_{\sigma\rho} \chi_\alpha \\ 0 & \Gamma^\sigma_{\rho\alpha} \theta^\alpha & 0 & -\delta^\sigma{}_\rho \\ 0 & \Gamma^\alpha_{\rho\sigma} \chi_\alpha & \delta_\sigma{}^\rho & 0 \end{pmatrix} . \tag{3.48}$$

Using the above four by four matrices is simple to show that $\widehat{\mathcal{P}}^{ik} \widehat{\mathcal{O}}_{kj} = -\delta^i_j$. If the connection vanishes identically, then the off-shell modification of the gauge transformation (3.44) vanishes.

4 Conclusion and remarks

We have given an action of the covariant Hamiltonian form that describes a two-dimensional topological sigma model in a target space carrying the structure of a differential Poisson algebra. The kinetic term is given by the pull-back of a symplectic potential that is non-canonical and hence the off-shell gauge transformations contain an additional set of terms proportional to the Cartan curvatures. Besides the gauge symmetries, whose existence requires the background to obey conditions that are equivalent to those required by the Jacobi identities of the differential Poisson algebra, our action also exhibits a rigid supersymmetry corresponding to the de Rham differential on the Poisson manifold. Indeed, the latter symmetries fix the coefficients of the curvature terms in the action and the Poisson bracket, respectively.

We expect that the AKSZ quantization [7, 8] of the original Poisson sigma model can be generalized to the present model in a background diffeomorphism covariant fashion, i.e. such that there exists a generalization of (3.8) to the gauge fixed action (modulo BRST exact terms). Assuming furthermore that Kontsevich's formality theorem generalizes to the deformation of the graded Poisson bracket, the similarity between the Poisson

JHEP08(2015)095

bracket (2.22) and the quadratic part of the action (3.1) expanded around a constant background suggests that two-point correlation functions of suitable boundary vertex operators yield the extension of Kontsevich's star product formula to higher forms on a manifestly covariant as well as explicit form, at least in the case of $\mathbb{R}^n$ target space topology. Moreover, assuming the supercurrent to be anomaly free we expect its charge to be a deformation of the de Rham differential into a nilpotent operator that is compatible with the star product.

Clearly, the first steps in this direction are to reproduce the higher-form generalized Poisson bracket (2.22) at order $\hbar$ and then verify the bi-differential operator found in [11, 12] at order $\hbar^2$, which we leave for separate considerations.[5] More precisely, we propose that the BRST cohomology of the model contains a ring generated by the constant modes of $(\phi^\alpha, \theta^\alpha)$ that realizes the star product deformation of the space $\Omega(N)$ of differential forms on N. As already mentioned in the Introduction and using the notation of section 3.1, one can map the elements $d\phi^{\alpha_1} \wedge \cdots \wedge d\phi^{\alpha_p} \omega_{\alpha_1 \ldots \alpha_p}$ in $\Omega(N)$ to elements $\theta^{\alpha_1} \cdots \theta^{\alpha_p} \omega_{\alpha_1 \ldots \alpha_p}$ in the subspace $\Omega_{[0]}(T[0,1]N)$ of zero-forms in the space $\Omega(T[0,1]N)$ of differential forms on $T[0,1]N$. Likewise, in the gauged-fixed theory,[6] the ghosts $(c_\alpha, \gamma_\alpha)$ for $(\eta_\alpha, \chi_\alpha)$ have form degree zero, ghost number one and additional Grassmann parities $\epsilon_{\rm f}(c_\alpha, \gamma_\alpha) = (0,1)$. Their zero-modes yield realizations of star product deformations of the spaces $Poly^{(\pm)}(N)$ of symmetric $(+)$ and anti-symmetric $(-)$ polyvector fields on N by mapping anti-symmetric polyvectors $\Pi^{\alpha_1 \cdots \alpha_n}(\phi) \partial_{\alpha_1} \wedge \cdots \wedge \partial_{\alpha_n}$ to $\Pi^{\alpha_1 \cdots \alpha_n}(\phi) c_{\alpha_1} \cdots c_{\alpha_n}$ in $\Omega_{[0]}(T^*[1,0]N)$ and symmetric polyvectors $G^{\alpha_1 \cdots \alpha_n}(\phi) \partial_{\alpha_1} \odot \cdots \odot \partial_{\alpha_n}$ to $G^{\alpha_1 \cdots \alpha_n}(\phi) \gamma_{\alpha_1} \cdots \gamma_{\alpha_n}$ in $\Omega_{[0]}(T^*[1,1]N)$. It would be interesting to examine the resulting target space quantum geometries in more detail.

The action (3.1), which describes a classically topological theory that remains to be gauge fixed, bears a close resemblance to the complete action of the first order formulation [20–22] of the topological A model [23]. The latter is obtained by a topological twist of the $\mathcal{N} = (2,2)$ supersymmetric sigma model, and requires the target space to be Kähler, and hence symplectic, unlike our model, whose target space is only required to be a Poisson manifold. Moreover, the type A model refers to a worldsheet metric, which enters via additional couplings to the hermitian metric and its compatible curvature of the form $g^{\alpha\beta} \eta_\alpha \wedge * \eta_\beta$ and $g^{\alpha\epsilon} R_{\gamma\delta}{}^{\beta}{}_{\epsilon}(g) \chi_\alpha \wedge * \chi_\beta \theta^\gamma \theta^\delta$. Thus the type A model action is non-singular in the sense that it does not admit any local symmetries. Instead, the couplings are tuned such that the complete action is exact under a rigid nilpotent supersymmetry,[7] whose factorization yields a topological model. Our model, on the other hand, is classically topological without requiring the classical observables to be $\delta_{\rm f}$-closed. Thus, in the terminology of topological field theories, our model is of the Schwarz type, while the A model is of the Witten, or cohomological, type.

[5] Starting from a path integral weighted by $\exp(\frac{i}{\hbar} S)$, the perturbative expansion is obtained by rescaling $(\eta_\alpha, \chi_\alpha, \theta^\alpha) \to (\hbar \eta_\alpha, \sqrt{\hbar} \chi_\alpha, \sqrt{\hbar} \theta^\alpha)$ and expand the background fields covariantly around the two-dimensional vacuum in which $\langle \phi^\alpha \rangle$ and $\langle \theta^\alpha \rangle$ are constant and $\langle \eta_\alpha \rangle$ and $\langle \chi_\alpha \rangle$ vanish.

[6] In the gauge fixed theory all quantities are assigned form degrees, ghost numbers and additional Grassmann parities, and we choose the Koszul sign convention to be given by $AB = (-1)^{|A||B| + \epsilon_{\rm f}(A)\epsilon_{\rm f}(B)} BA$ where the total degree $|A| = \deg_2(A) + {\rm gh}(A)$.

[7] The rigid supersymmetry generator of the A model is sometimes referred to as a BRST operator, even though the twisting is not a gauge fixing procedure. Attempts to identify the type A model as a gauge fixed version of a classically topological theory have been made in [24].

It would be interesting to examine whether there are more robust relations between the type A and B models and also the interpolating A-I-B model [21], including their infinite (and possibly zero) volume limits, and our model and various deformations of it. As for the latter, one may consider adding Yukawa couplings formed out of the S-tensor defined in (2.17) and additional metric couplings $G^{\alpha\beta}\chi_\alpha \wedge \chi_\beta$ (which add terms of intrinsic degree minus two to the bracket). One may also seek ways to couple of our model to two-dimensional gravity, which may be of importance for the formulation of the theory on worldsheets of higher genus, and possibly new topological open strings. To this end, besides exploring the relations to the type A and B models, it may also be fruitful to explore another route, based on the observation that prior to adding the worldsheet fermions, the Poisson sigma model exhibits vacuum bubble cancelations in simple worldsheet topologies. Adding the fermions lead to that these cancellations generalize to arbitrary topologies [25] (including boundaries). Thus, including bubbles with external matter legs, one may expect anomaly-induced topological matter-gravity couplings. We plan to address these issues in a future work.

One motivation behind the present work is Vasiliev's higher spin gravity, whose field theoretic formulation is in terms of differential star product algebras [26]. The explicit models that have been constructed so far are formulated on products of commuting manifolds, containing spacetimes, and symplectic manifolds of simple topology, quantized using the Moyal star product. The covariantized Kontsevich formalism provides a tool facilitating the formulation of higher spin gravities on manifolds of more general topology, possibly as Frobenius-Chern-Simons theories (or BF analogs thereof) following [27]. Its extension to topological open strings, with non-trivial topological expansions, may lead to complementary first-quantized descriptions of higher spin gravity. The latter perspective is supported by the recent progress in computing higher spin tree amplitudes starting from traces over oscillator algebras [28–31], in its turn motivated by the proposal made in [32] for how Vasiliev's theory arise in tensionless limits of closed strings in anti-de Sitter spacetime.

Finally, a natural part of the application to higher spin gravity as well as discretized strings, and also more general constrained systems, is the gauging of Killing symmetries of our Poisson sigma model. In principle, this procedure ought to be straightforward and leads to a natural generalization of the original gauged Poisson sigma model, which we expect to report on in a forthcoming publication.

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A Conventions and notation

The covariant exterior derivatives of the components of a vector field $V = V^\alpha \partial_\alpha$ and a one-form $\omega = \omega_\alpha d\phi^\alpha$ are given by

$$\nabla V^\alpha = dV^\alpha + \Gamma^\alpha{}_\beta V^\beta \,, \qquad \nabla \omega_\alpha = d\omega_\alpha - \Gamma^\beta{}_\alpha \wedge \omega_\beta \,, \tag{A.1}$$

where $\Gamma^\alpha{}_\beta = d\phi^\gamma \Gamma^\alpha_{\gamma\beta}$ is the connection one-form. In terms of components, we have $\nabla V^\alpha = d\phi^\beta \nabla_\beta V^\alpha$ and $\nabla \omega_\alpha = d\phi^\beta \nabla_\beta \omega_\alpha$ where

$$\nabla_\alpha V^\beta = \partial_\alpha V^\beta + \Gamma^\beta_{\alpha\gamma} V^\gamma \,, \qquad \nabla_\alpha \omega_\beta = \partial_\alpha \omega_\beta - \Gamma^\gamma_{\alpha\beta} \omega_\gamma \,. \tag{A.2}$$

The basic Ricci identities read

$$[\nabla_\alpha, \nabla_\beta] V^\gamma = -T^\delta_{\alpha\beta} \nabla_\delta V^\gamma + R_{\alpha\beta}{}^\gamma{}_\delta V^\delta \,, \qquad [\nabla_\alpha, \nabla_\beta] \omega_\gamma = -T^\delta_{\alpha\beta} \nabla_\delta \omega_\gamma - R_{\alpha\beta}{}^\delta{}_\gamma \omega_\delta \,, \tag{A.3}$$

where the curvature and torsion tensors are

$$R_{\alpha\beta}{}^\gamma{}_\delta = 2\, \partial_{[\alpha} \Gamma^\gamma_{\beta]\delta} + 2\, \Gamma^\gamma_{[\alpha|\epsilon} \Gamma^\epsilon_{|\beta]\delta} \,, \qquad T^\gamma_{\alpha\beta} = 2\, \Gamma^\gamma_{[\alpha\beta]} \,. \tag{A.4}$$

The corresponding curvature and torsion two-forms are

$$R^\alpha{}_\beta = \frac{1}{2} d\phi^\gamma \wedge d\phi^\delta R_{\gamma\delta}{}^\alpha{}_\beta \,, \qquad T^\alpha = \frac{1}{2} d\phi^\gamma \wedge d\phi^\delta T^\alpha_{\gamma\delta} \,, \tag{A.5}$$

which can also be written as

$$R^\alpha{}_\beta = d\Gamma^\alpha{}_\beta + \Gamma^\alpha{}_\gamma \wedge \Gamma^\gamma{}_\beta \,, \qquad T^\alpha = \Gamma^\alpha{}_\beta \wedge d\phi^\beta \,. \tag{A.6}$$

The covariant exterior derivative of the one-form itself is given by

$$d\omega = \nabla \omega = (\nabla d\phi^\alpha) \omega_\alpha + (\nabla \omega_\alpha) d\phi^\alpha = d\phi^\alpha d\phi^\beta \left(\nabla_\alpha \omega_\beta + \frac{1}{2} T^\gamma_{\alpha\beta} \omega_\gamma \right) \,. \tag{A.7}$$

The Bianchi identities read

$$T^\alpha = \nabla d\phi^\alpha \,, \quad \nabla T^\alpha = R^\alpha{}_\beta \wedge d\phi^\beta \,, \qquad \nabla R^\alpha{}_\beta = 0 \,, \tag{A.8}$$

or in components

$$R_{[\alpha\beta}{}^\gamma{}_{\delta]} = \nabla_{[\alpha} T^\gamma_{\beta\delta]} - T^\epsilon_{[\alpha\beta} T^\gamma_{\delta]\epsilon} \,, \qquad \nabla_{[\alpha} R_{\beta\gamma]}{}^\delta{}_\epsilon - T^\lambda_{[\alpha\beta} R_{\gamma]\lambda}{}^\delta{}_\epsilon = 0 \,. \tag{A.9}$$

The square of the exterior covariant derivative acting on the components of a vector field and a one-form are given by

$$\nabla^2 V^\alpha = R^\alpha{}_\beta V^\beta \,, \qquad \nabla^2 \omega_\alpha = -R^\beta{}_\alpha \wedge \omega_\beta \,. \tag{A.10}$$

In analyzing the differential Poisson algebra, it is convenient to define a new connection $\widetilde{\nabla}$ with connection coefficients $\widetilde{\Gamma}^\alpha_{\gamma\beta} := \Gamma^\alpha_{\beta\gamma}$. We make repeated use of the identity

$$\nabla_\alpha \Pi^{\beta\gamma} = \widetilde{\nabla}_\alpha \Pi^{\beta\gamma} - 2T^{[\beta}_{\alpha\delta} \Pi^{\gamma]\delta} \,. \tag{A.11}$$

We denote the components of the curvature of $\widetilde{\nabla}$ by $\widetilde{R}_{\alpha\beta}{}^\gamma{}_\delta$ and define

$$\widetilde{R}^{\alpha\beta} := \Pi^{\beta\gamma} \widetilde{R}^\alpha{}_\gamma \,. \tag{A.12}$$

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JHEP08(2015)095