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Multi-centered black hole flows

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Abstract: We describe the systematical construction of the first order formalism for multi-centered black holes with flat three dimensional base-space, within the so-called T^3 model of $N = 2$, $D = 4$ ungauged Maxwell-Einstein supergravity.

The three possible flow classes (BPS, composite non-BPS and almost BPS) are analyzed in detail, and various solutions, such as single-centered (static or under-rotating) and all known multi-centered black holes, are recovered in this unified framework. We also consider the possibility of obtaining new solutions.

The almost BPS class is proved to split into two general sub-classes, corresponding to a positive or negative value of the duality-invariant polynomial for the total charge; the well known almost BPS system is shown to be a particular solution of the second sub-class.

Keywords: Black Holes in String Theory, Black Holes, Supergravity Models

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Contents

1 Introduction

Multi-centered non supersymmetric extremal black holes have been the subject of a number of recent studies [3]–[14, 15], one possible reason being that supersymmetry actually plays a less crucial rôle than extremality in the investigation of these solutions.

The supersymmetric class for multi-centered black holes was investigated by Denef [16], in order to match the true BPS spectrum and the spectrum of spherically symmetric black holes in supergravity. The resulting multi-centered BPS first order equations, whose solutions and properties were then studied in [17], acquire the well-known form after integrating out the auxiliary phase field in the single-centered limit.

More than ten years passed before a systematical construction of solutions for the under-rotating non-BPS branch was worked out by exploiting group-theoretical techniques [3, 4].

The present paper is devoted to the determination of the most general flow equations underlying the first order formalism [18]–[25] for non-BPS multi-centered and/or rotating black holes with flat three dimensional base-space. For simplicity's sake, we will consider the so-called T^3 model of $N = 2$, $D = 4$ ungauged Maxwell-Einstein supergravity. But we expect all the results to be generalizable to the STU model, a task left to further forthcoming investigations.

The first attempts to write down flow equations for non-supersymmetric multi-centered configurations were made in [14, 15]. The method was based on the squaring action procedure, but unfortunately an explicit form of flow equations in terms of charges, fields and other parameters was missing. In the present investigation, we try to fill this gap, and write down first order equations for non-supersymmetric multi-centered configurations in an explicit form.

Besides its inner simplicity and elegance, the importance first order formalism relies in the possibility to switch from second order differential equations of motion to the first order ones, without doubling their number. Indeed, even if some auxiliary fields are added in the multi-centered case, they turn out to satisfy algebraic equations. This is due to the fact that the scalar charges are not independent, and the formalism automatically discards the blowing up solutions. Integrating first order differential equations is certainly easier, and thus the possibilities to find the corresponding attractor flows are considerably enhanced. In this paper, we will retrieve a number of known solutions in a unifying setting, and we will predict the existence of other new ones, at least at the level of the most general system of equations governing the flow.

The paper is organized as follows.

The setup and notations are defined in section 2.

Then, section 3 presents the general method of construction of first order equations, considering the well known BPS class.

In section 4, this method is then applied to the construction of the first order formalism for the composite non-BPS class, and well known examples, such as single-centered static and under-rotating black holes, and multi-centered electric and magnetic solutions, are recovered.

Section 5 exploits the same procedure for the broadest class of almost BPS multi-centered solutions. The existence of two general sub-classes of first order flow equations is proved, and the well known almost BPS system [5] is retrieved as a particular solution of the second class.

Finally, section 6 contains a summary and an outlook of results.

2 Setup and Notations

The bosonic part of the $N = 2$, $D = 4$ Maxwell-Einstein supergravity Lagrangian density action for the so-called T^3 model reads

$$\frac{\mathcal{L}}{\sqrt{-g}} = -\frac{1}{2}\,R + g^{\mu\nu}\,G\,\partial_\mu t \partial_\nu \bar{t} + \frac{1}{4}\,\mu_{\Lambda\Sigma} F^{\Lambda}_{\mu\nu} F^{\Sigma\,\mu\nu} + \frac{1}{4}\,\nu_{\Lambda\Sigma} F^{\Lambda}_{\mu\nu} * F^{\Sigma\,\mu\nu},$$

where $F^\Lambda = d\,A^\Lambda$ is the electromagnetic field, $G = -\frac{3}{(t-\bar{t})^2}$ is the metric on special Kähler symmetric scalar manifold $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$, and the scalar-vector couplings are given by

$$\mu_{\Lambda\Sigma} = \frac{i}{4}\begin{pmatrix} (t-\bar{t})\,(t^2+4t\bar{t}+\bar{t}^2) & -3(t^2-\bar{t}^2) \\ -3(t^2-\bar{t}^2) & 6(t-\bar{t}) \end{pmatrix}, \quad \nu_{\Lambda\Sigma} = \frac{1}{4}\begin{pmatrix} -(t+\bar{t})^3 & 3(t+\bar{t})^2 \\ 3(t+\bar{t})^2 & -12(t+\bar{t}) \end{pmatrix}.$$

The Ansatz of under-rotating multi-centered black hole (with flat $D = 3$ spatial slices), independently on the branches, reads

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = e^{2U}(dt + \omega_i dx^i)^2 - e^{-2U}\delta_{ij} dx^i dx^j, \tag{2.1}$$

where i denotes the Cartesian indices, and μ, ν are space-time indices. The functions U, ω_i and the complex modulus field $t = x - i\,y$, where x is axion and y is dilaton, are solutions of the following equations [1]:

$$
\begin{aligned}
\delta^{ij}\,\partial_i f_j = 0, \qquad \delta^{ij}\,\partial_i\left[e^{-2U} M_{\alpha\beta}\left(\partial_j b^\beta - f_j b^\beta\right)\right] &= 0, \\
G\,\partial_{(i} t \partial_{j)} \bar{t} + \partial_i U \partial_j U + \frac{1}{4}\, e^{4U} f_i f_j &= -\frac{1}{2}\, e^{-2U} \partial_i b^\alpha M_{\alpha\beta} \partial_j b^\beta, \\
\delta^{ij}\,\partial_i \partial_j U + \frac{1}{2}\, e^{4U} \delta^{ij} f_i f_j &= -\frac{1}{2}\, e^{-2U} \delta^{ij} \partial_i b^\alpha M_{\alpha\beta} \partial_j b^\beta, \\
\delta^{ij}\,\partial_i\left(G \partial_j t\right) - \frac{\partial G}{\partial \bar{t}}\, \delta^{ij} \partial_i t \partial_j \bar{t} &= \frac{1}{2}\, e^{-2U} \delta^{ij} \partial_i b^\alpha \frac{\partial M_{\alpha\beta}}{\partial t}\, \partial_j b^\beta,
\end{aligned}
\tag{2.2}
$$

where α, β are symplectic indices.

In the present framework, f_i corresponds to the rotation, and it is defined as follows:

$$
f_i \equiv -\delta_{ij} \varepsilon^{jkl} \partial_k \omega_l . \tag{2.3}
$$

On the other hand, b^α denotes the electromagnetic potential. As usual, we define the dual field strength $G_\Lambda = \partial \mathcal{L} / \partial F^\Lambda$ and the whole field strengths' symplectic vector $F^\alpha = \left(F^\Lambda, G_\Lambda\right)$, so that the electric component of the field strength F^α reads

$$
F^\alpha_{0i} = -\partial_i b^\alpha . \tag{2.4}
$$

As for $M_{\alpha\beta}$, it is a positive definite symplectic symmetric matrix, which defines the single-centered black hole potential [2] and reads

$$
M_{\alpha\beta} = \begin{pmatrix} \mu_{\Lambda\Sigma} + \nu_{\Lambda\Lambda'} \mu^{\Lambda'\Sigma'} \nu_{\Sigma'\Sigma} & \nu_{\Lambda\Lambda'} \mu^{\Lambda'\Sigma} \\ \mu^{\Lambda\Lambda'} \nu_{\Lambda'\Sigma} & \mu^{\Lambda\Sigma} \end{pmatrix},
$$

where $\mu_{\Lambda\Sigma} \mu^{\Sigma\Lambda'} \equiv \delta^{\Lambda'}_\Lambda$.

A crucial point is the integration of the electromagnetic field equations. In the following treatment, we will use the following Ansatz:

$$
e^{-2U} M_{\alpha\beta} \partial_i b^\beta = \Omega_{\alpha\beta} \hat{Q}^\beta_i , \tag{2.5}
$$

where $\hat{Q}^\beta_i = \left(\partial_i H^\beta + f_i b^\beta\right)$ and $\Omega_{\alpha\beta}$ is the bilinear symplectic invariant structure:

$$
\Omega_{\alpha\beta} = \begin{pmatrix} 0 & -\delta^\Sigma_\Lambda \\ \delta^\Lambda_\Sigma & 0 \end{pmatrix},
$$

and H^β are harmonic functions containing only appropriate electric and magnetic black hole charges (namely, fluxes of the 2-form field strengths).

It is worth remarking here that one may generally write $\hat{Q}^\beta_i = \left(H^\beta_i + f_i b^\beta\right)$, where H^β_i is the dual of the three dimensional field strength (not necessarily harmonic), and $\delta^{j\,i} \partial_j H^\beta_i = 0$. However, we will see below that (2.5) is effective for all classes of multi-centered black holes and models under consideration.

Substituting $\partial_i b^\beta$ given by (2.5) back into equations (2.2), one obtains

$$
\begin{aligned}
\delta^{ij}\,\partial_i f_j = 0, \qquad e^{-2U} M_{\alpha\beta}\partial_i b^\beta = \Omega_{\alpha\beta}\hat{Q}^\beta_i,\\
G\,\partial_{(i}t\partial_{j)}\bar{t} + \partial_i U\partial_j U + \frac{1}{4}\,e^{4U} f_i f_j = e^{2U}\hat{V}_{i\,j},\\
\delta^{ij}\,\partial_i\partial_j U + \frac{1}{2}\,e^{4U}\delta^{ij} f_i f_j = e^{2U}\delta^{ij}\,\hat{V}_{i\,j},\\
\delta^{ij}\,\partial_i\left(G\partial_j t\right) - \frac{\partial G}{\partial\bar{t}}\,\delta^{ij}\partial_i t\partial_j\bar{t} = e^{2U}\delta^{ij}\frac{\partial\hat{V}_{i\,j}}{\partial t}.
\end{aligned}
\tag{2.6}
$$

where

$$
\hat{V}_{i\,j} \equiv \hat{Z}_{(i}\,\hat{\bar{Z}}_{j)} + \hat{Z}_{t\,(i}\,\hat{\bar{Z}}_{\bar{t}\,j)}, \tag{2.7}
$$

is a generalized black hole potential, and (v^α denotes the covariantly holomorphic section of special Kähler geometry throughout)

$$
\hat{Z}_i = \hat{Q}^\alpha_i\,\Omega_{\alpha\beta}\,v^\beta, \quad \hat{Z}_{t\,i} = E^t\,D_t\hat{Z}_i = \hat{Q}^\alpha_i\,\Omega_{\alpha\beta}\,v^\beta_t, \tag{2.8}
$$

are Cartesian-indexed generalization of the $N = 2$ cental charge function and of its first Kähler-covariant derivative

$$
Z = Q^\alpha\,\Omega_{\alpha\beta}\,v^\beta, \quad Z_t = E^t\,D_t Z = Q^\alpha\,\Omega_{\alpha\beta}\,v^\beta_t, \tag{2.9}
$$

in which the charge vector $Q^\alpha = \{p^\Lambda, q_\Lambda\}$ is replaced by $\hat{Q}^\alpha_i$. $E^t \equiv E^t_{\hat{t}}$ is the (inverse) Vielbein of the scalar manifold ($\left|E^t\right|^{-2} = G$); and we henceforth refrain from hatting the subscript "t", denoting the flattened scalar coordinate.

Is worth mentioning here that an object like the generalized black hole potential $\hat{V}_{i\,j}$ appeared in [26], but our definition has a more transparent geometrical interpretation. As for single center static and spherically symmetric black holes, only the $V_{\tau\,\tau}$ (where $\tau = 1/r$ is the inverse radial coordinate) component survives in this case, and it equals the usual black hole potential V_{BH} [2].

We are now going to construct the first order flows for these equations in various branches. After [3], it is known that for under-rotating multi-centered black holes three possible flow classes exist, namely: BPS, non-BPS composite and almost BPS.

For all classes of multi-centered flows based on the known solutions [3, 4] and [16], one can consider the following Ansatz for the first order equations:

$$
\partial_i U = \hat{W}_i\left(\hat{Z}_i,\,\hat{Z}_{t\,i},\,f_i,\alpha_a\right)e^U, \tag{2.10}
$$

$$
\partial_i t = E^t\,\hat{\Pi}_i\left(\hat{Z}_i,\,\hat{Z}_{t\,i},\,f_i,\alpha_a\right)e^U, \tag{2.11}
$$

$$
\partial_i\alpha_a = \Gamma_{i\,a}\left(\hat{Z}_i,\,\hat{Z}_{t\,i},\,f_i,\alpha_a\right)e^U. \tag{2.12}
$$

Here $\hat{W}_i$ denotes a Cartesian-indexed generalization of the first order (fake) superpotential W for single center black hole [11]. $\hat{\Pi}_i$ is the corresponding generalization of the scalar charges ($2\bar{E}^{\bar{t}}\,\partial_{\bar{t}}W$), and α's are some auxiliary fields. *Ça va sans dire*, these functions will generally be different for the various classes.

For pedagogical reasons, we will start by considering the BPS class.

3 BPS flow

This is a well known case [16], and we will consider it in order to establish the procedure which we will exploit for the non-supersymmetric cases.

For single-centered BPS black holes in spherical coordinates, the first order superpotential reads

$$W = \sqrt{Z\,\bar{Z}}. \tag{3.1}$$

In order to re-express it in Cartesian coordinates, a mere replacement of charges by derivative of harmonic functions (namely, $Z \to Z_i$) is not convenient, because Cartesian indices would occur under square root, and covariance would be lost. Actually, manifest covariance can be preserved by introducing the phase α_0 and expressing W (3.1) as [16]

$$W = \mathrm{Re}\left[e^{-i\alpha_0}\,Z\right], \quad e^{-i\,\alpha_0} = \sqrt{\frac{\bar{Z}}{Z}}. \tag{3.2}$$

Then, one can easily switch to Cartesian coordinates by introducing the corresponding index:

$$W_i = \mathrm{Re}\left[e^{-i\alpha_0}\,Z_i\right]. \tag{3.3}$$

Of course, the definition of α_0 through Z and $\bar{Z}$ does not hold true anymore here, and the corresponding equation should be established. Rotation can be introduced into by prolonging central charge as $Z_i \to \hat{Z}_i$ (recall (2.8)). By exploiting such a procedure also for the scalar charges, one ends up with the following first order system:

$$\partial_i U = \mathrm{Re}\left[e^{-i\alpha_0}\,\hat{Z}_i\right] e^U, \tag{3.4}$$

$$\partial_i t = E^t\,e^{i\,\alpha_0}\,\hat{\bar{Z}}_{\bar{t}\,i}\,e^U. \tag{3.5}$$

By plugging these equations back into (2.6), one then gets equations for α_0 and f_i, namely:

$$\partial_i \alpha_0 = e^U\,\mathrm{Im}\left[e^{-i\,\alpha_0}\left(\hat{Z}_i + \sqrt{3}\,\hat{Z}_{t\,i}\right)\right], \tag{3.6}$$

$$f_i = 2\mathrm{Im}\left[e^{-i\alpha_0}\,\hat{Z}_i\right] e^{-U}. \tag{3.7}$$

We notice that eq. (3.7) contains f_i in both sides; thus, by recalling definition (2.8), it can be rewritten as follows:

$$(1 - 2\mathrm{Im}\left[e^{-i\alpha_0}\,b^\alpha\,\Omega_{\alpha\beta}\,v^\beta\right] e^{-U})f_i = 2\mathrm{Im}\left[e^{-i\alpha_0}\,Z_i\right] e^{-U}. \tag{3.8}$$

It is here worth noting that, due to the presence of the rotation, all equations contain the electromagnetic potential. One can easily show that the following Ansatz solves the equation for the electromagnetic potential in a model-independent way:

$$b^\beta = 2\mathrm{Re}\left[e^{-i\alpha_0}\,v^\beta\right] e^U. \tag{3.9}$$

We point out that this Ansatz can be constructed by replacing all $\hat{Z}_i$'s with the symplectic sections v^β in the superpotential W_i, and then by multiplying by $2e^U$. In the following sections, we will see this procedure to (partially) work also within non-supersymmetric classes of flows.

Plugging (3.9) back into all other equations, one finally gets the Denef system [16]:

$$\partial_i U = \mathrm{Re}\left[e^{-i\alpha_0}\, Z_i\right] e^U, \tag{3.10}$$

$$\partial_i t = E^t\, e^{i\,\alpha_0}\, \bar{Z}_{\bar{t}\,i}\, e^U, \tag{3.11}$$

$$\partial_i \alpha_0 = -\mathrm{Im}\left[\frac{\partial K}{\partial t}\,\partial_i t\right] + \frac{1}{2} f_i\, e^{2U}, \tag{3.12}$$

$$f_i = -2\mathrm{Im}\left[e^{-i\alpha_0}\, Z_i\right] e^{-U}, \tag{3.13}$$

where $K = -\log\left(-i(t-\bar{t})^3\right)$ is the Kähler potential.

The rotation should be divergence-free (namely, $\delta^{ij}\,\partial_i f_j = 0$), thus so from (3.13) one obtains

$$f_i = \partial_i H^\alpha\, \Omega_{\alpha\,\beta}\, H^\beta. \tag{3.14}$$

Let us now consider the static single-centered limit of the system (3.10)–(3.13). By setting $f_i = 0$ from the start, the standard BPS solution is obtained without any integration of the equation of electromagnetic fields; however, the condition

$$Q^\alpha\, \Omega_{\alpha\,\beta}\, h^\beta = 0, \tag{3.15}$$

where h^β are the asymptotical values of the harmonic functions H^β, should be introduced ad hoc.

On the other hand, after integrating electromagnetic fields out and finding (3.14), by setting $f_i = 0$ one obtains the condition (3.15) automatically.

4 Composite non-BPS flow

4.1 General analysis

In this section we are going to investigate the so-called composite non-BPS branch [3]. In order to determine the fake super potential and the corresponding scalar charges, in this case one can start from the single-centered non-BPS black hole fake superpotential [25], and introduce one or more phases. Due to the rather cumbersome explicit expression of the non-BPS W, it is not easy to guess how to consistently introduce phases, so we choose to proceed in another way. Namely, we will use the procedure described for the $N=2$, $D=4$ STU model in [3]. As treated in that paper, under-rotating solutions correspond to various nilpotent orbits, and thus one can try to switch from the BPS orbit to the non-BPS ones by performing the following steps (for the T^3 model):

1. Consider the scalar charges and the quantity whose real part is the BPS superpotential $W = \mathrm{Re}(w)$, and remove the phase α_0 therein:

$$w = Z, \quad \Pi = \bar{Z}_{\bar{t}}. \tag{4.1}$$

2. Switch to the duality frame (on spatial infinity) in which the axion x vanishes and the dilaton y is set to 1, thus establishing a one to one correspondence between the the charges and the central charge and its Kähler-covariant derivative. In such a way, all quantities can be rewritten solely in terms of the charges themselves.

3. For the composite non-BPS orbit, the signs of q_0 and of the combination $p^0 + q_1/3$ should be flipped, and such quantities should be written in terms of the central charge and of its derivative, thus yielding:

$$w = \frac{1}{2}\left(\bar{Z} - \sqrt{3}\,\bar{Z}_{\bar{t}}\right), \quad \Pi = -\frac{1}{2}\left(\sqrt{3}\,Z + Z_t\right). \tag{4.2}$$

4. Then, the phases should be introduced using the following rules under $[\mathrm{U}(1)]^2$ transformations [3]:

$$Z \to Z e^{\frac{i}{2}(\alpha_0+3\,\alpha_1)}, \quad Z_t \to Z_t e^{-\frac{i}{2}(\alpha_0+\alpha_1)}, \quad w \to w\, e^{-\frac{i}{2}(\alpha_0+3\,\alpha_1)}, \quad \Pi \to \Pi\, e^{\frac{i}{2}(\alpha_0-\alpha_1)}, \tag{4.3}$$

thus achieving the expressions for the fake superpotential and corresponding scalar charges:

$$W = \frac{1}{2}\mathrm{Re}\left[e^{-3i\alpha_1}\,\bar{Z} - \sqrt{3}e^{-i\alpha_1}\,\bar{Z}_{\bar{t}}\right], \tag{4.4}$$

$$\Pi = -\frac{1}{2}\left(\sqrt{3}\,e^{i\alpha_1}\,Z + e^{-i\alpha_1}\,Z_t\right). \tag{4.5}$$

Clearly, the expressions (4.4) and (4.5) are written only at spatial infinity. However, we choose to use them all along the flow, by just replacing the charges Q^α with the quantity $\hat{Q}^\alpha_i$, exhibiting the same symplectic covariance.This replacement can be understood by observing that $\hat{Q}^\alpha_i$ naturally occurs in the second order equations (2.6), and at infinity it equals Q^α when spherical symmetry is restored. After integrating the phase α_1 out, we will show below that the known expression of non-BPS W for the T^3 model [25] is retrieved.

By using the expressions of $\hat{W}_i$ and $\hat{\Pi}_i$, which as in the BPS class contain only one phase (in this case denoted as α_1), the following first order equations can be written down:

$$\partial_i U = \frac{1}{2}\mathrm{Re}\left[e^{-3i\alpha_1}\,\hat{\bar{Z}}_i - \sqrt{3}e^{-i\alpha_1}\,\hat{\bar{Z}}_{\bar{t}\,i}\right] e^U, \tag{4.6}$$

$$\partial_i t = -\frac{1}{2}E^t\left(\sqrt{3}\,e^{i\alpha_1}\,\hat{Z}_i + e^{-i\alpha_1}\,\hat{Z}_{t\,i}\right) e^U. \tag{4.7}$$

Thence, by plugging them back into the second order equations (2.6), one obtains

$$\partial_i \alpha_1 = -\frac{2}{\sqrt{3}}e^U\,\sin\alpha_1\,\mathrm{Re}\left[e^{i\alpha_1}\,\hat{\Pi}_i\right], \tag{4.8}$$

$$f_i = \mathrm{Im}\left[e^{-3i\alpha_1}\,\hat{\bar{Z}}_i - \sqrt{3}e^{-i\alpha_1}\,\hat{\bar{Z}}_{\bar{t}\,i}\right] e^{-U}. \tag{4.9}$$

It is here worth stressing that, as in the BPS class, f_i is the imaginary part of the quantity whose real part is W_i.

Moreover, f_i enters both sides of (4.9); thus, we would like to integrate out the electromagnetic potential completely, as for the BPS class. Following the procedure outlined in section 3, we replace the central charge and its flat derivative respectively by v^α and v^α_t, obtaining the following Ansatz:

$$b^\beta = \mathrm{Re}\left[e^{-3i\alpha_1}\,\bar v^\beta - \sqrt{3}e^{-i\alpha_1}\,\bar v^\beta_{\bar t}\right] e^U. \tag{4.10}$$

Unfortunately, this Ansatz does not work here well. Indeed, by inserting it into (4.9), it turns out that f_i drops out, and one ends up with the expression:

$$\mathrm{Im}\left[e^{-3i\alpha_1}\,\bar Z_i - \sqrt{3}e^{-i\alpha_1}\,\bar Z_{\bar t\,i}\right] = 0. \tag{4.11}$$

However, we can add to b^β (4.10) a new term proportional to a boundary function b. As we will see, b represents a real rotation, whereas in the BPS class only intrinsic rotation f_i is present. Consequently, the new Ansatz reads

$$b^\beta = \mathrm{Re}\left[e^{-3i\alpha_1}\,\bar v^\beta - \sqrt{3}e^{-i\alpha_1}\,\bar v^\beta_{\bar t}\right] e^U - b\,e^{3U}\mathrm{Im}\left[e^{-3i\alpha_1}\,\bar v^\beta - \sqrt{3}e^{-i\alpha_1}\,\bar v^\beta_{\bar t}\right]. \tag{4.12}$$

The new term added, proportional to b, is nothing but f_i (4.9)(namely, the imaginary part of the quantity whose real part is W_i) with the central charge and its flat derivative respectively replaced by v^β and v^β_t. Using this Ansatz, one finds that (4.11) holds true, whereas the equation for electromagnetic fields yields the following expression for f_i:

$$f_i = \partial_i b + 2e^{-U}\mathrm{Im}\left[e^{-3i\alpha_1}\bar Z_i\right] + \sqrt{3}\,b\,e^U\mathrm{Re}\left[\sqrt{3}e^{3i\alpha_1}\,Z_i - e^{i\alpha_1}\,Z_{t\,i}\right]. \tag{4.13}$$

Finally, by inserting (4.11) and (4.12) back into the other equations, one obtains full-fledged first order formalism for the composite non-BPS class of under-rotating multi-centered flows:

$$\partial_i U = \frac{1}{2}\mathrm{Re}\left[e^{-3i\alpha_1}\,\bar Z_i - \sqrt{3}e^{-i\alpha_1}\,\bar Z_{\bar t\,i}\right] e^U + \frac{1}{2}\,b\,f_i\,e^{4U} \tag{4.14}$$

$$\partial_i t = -\frac{1}{2}e^{U-2i\alpha_1}E^t\left(\sqrt{3}\,e^U\,\left(i - be^{2U}\right)f_i + \sqrt{3}\,e^{3i\alpha_1}\,Z_i + e^{i\alpha_1}\,Z_{t\,i}\right). \tag{4.15}$$

4.2 Some examples

In order to find explicit solutions, the system (4.14)–(4.15) should be supplemented by the condition (4.11),which is the integrated version of equation (4.8).

1. Let us consider the single-centered non-rotating non-supersymmetic black hole in the T^3 model. By setting $f_i = 0$ before the second integration of the electromagnetic field equations and bearing in mind that, by virtue of spherical symmetry, all three Cartesian equations in (4.11) can be replaced by the only radial coordinate, one obtains the following solution for α_1:

$$e^{2i\,\alpha_1} = -\frac{3\lambda + i\sqrt{-9\,\lambda^2 + 24\,\lambda\,Z_t\,\bar Z_{\bar t} + 16 Z_t\,\bar Z_{\bar t}\,(3\,Z\,\bar Z - Z_t\,\bar Z_{\bar t})}}{4\sqrt{3}\,Z\bar Z_{\bar t}} + \frac{Z_t}{\sqrt{3}\,Z}, \tag{4.16}$$

$$\begin{aligned}\lambda = {}&\left(\left(Z\bar Z - \frac{Z_t\bar Z_{\bar t}}{3}\right)^3 - I_4\,\left(Z\bar Z + Z_t\bar Z_{\bar t}\right) + \frac{4}{3\sqrt{3}}i\,(\bar Z Z_t^3 - Z\bar Z_{\bar t}^3)\,\sqrt{-I_4}\right)^{1/3} + \\ &+\left(\left(Z\bar Z - \frac{Z_t\bar Z_{\bar t}}{3}\right)^3 - I_4\,\left(Z\bar Z + Z_t\bar Z_{\bar t}\right) - \frac{4}{3\sqrt{3}}i(\bar Z Z_t^3 - Z\bar Z_{\bar t}^3)\sqrt{-I_4}\right)^{1/3},\end{aligned} \tag{4.17}$$

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where I_4 is quartic invariant. Plugging (4.16) back into multi-centered fake superpotential (4.4), the known expression for the non-BPS fake superpotential of the T^3 model [25] is retrieved:

$$W^2 = \frac{1}{4}\left(Z\bar{Z} + Z_t\bar{Z}_{\bar{t}}\right) + \frac{3}{8}\lambda. \tag{4.18}$$

Of course, one can also consider the single-centered limit after the second integration of electromagnetic potentials. In this case, one has to impose the condition $f_i = 0$, where f_i is defined by expression (4.13). By setting b to a constant, and integrating α_1 out, one achieves the following condition:

$$\left(x^2 + y^2\right)\left(-1 + \nu^2\right) - \sigma_-^2 + \nu^2\sigma_+^2 + 2be^{2u}y\nu(\sigma_- + \sigma_+) - 2x\left(\sigma_- + \nu^2\sigma_+\right) = 0, \tag{4.19}$$

where $x \equiv \mathrm{Re}\,(t)$ is the axion field, $y \equiv -\mathrm{Im}\,(t)$ is the dilaton field, and σ_+, σ_- and ν are defined in formula (3.19) of [25]. It is easy to check that solutions (4.19) of [25] indeed satisfy this condition. As for the BPS case, one obtains the condition (4.19) in order to have no rotation, whereas in the previous approach (namely, before integrating electromagnetic potential out) such condition had to be imposed by hand. Thus, one can conclude that the composite non-BPS class exhibits a smooth single-centered non-rotating non-BPS (extremal) limit.

2. The example concerns the generalization of Rasheed-Larsen black hole [27, 28], namely a non-BPS single-centered rotating black hole with all charges switched on. As we are considering a single-centered black hole, all harmonic functions sourced by the charges depend only on the radial coordinate r. Thus, condition (4.11) can be solved as in the case of a non-rotating single-center black hole. By computing α_1 and inserting it back into the expressions for the derivatives of warp factor and for scalar fields, one obtains the very same equations for the non-rotating single-centered black hole, and the very same solutions (see e.g. (4.19) of [25]), the unique difference being that, b now is not constant anymore, but instead it is a harmonic function which represents rotation:

$$b = b_0 + J\frac{\cos\theta}{r^2}, \tag{4.20}$$

where b_0, J are constants and J is the angular momentum. By setting all charges to zero but the $D0$ and $D6$ ones (which, in the notation of [25] amounts to setting $\nu \to 0$, $\sigma_- \to 0$ and $\nu\,\sigma_+ \to (q_0)^{1/3}/(p^0)^{1/3}$), and by imposing the additional condition $h_1 = h_0$ (corresponding to the vanishing of the axion), the Rasheed-Larsen solution is consistently reproduced.

3. As third example, we consider the (T^3-degeneration of the) composite multi-centered black holes investigated by Bossard and Ruef in [3, 4]. In the multi-centered case, the condition (4.11) can explicitly be solved only in some particular charge configurations. Namely, when all D6 charges or all D0 charges vanish on all centers

(respectively called magnetic and electric configurations). For the magnetic configuration, condition (4.11) yields $\alpha_1 = 0$. By solving all other equations, one gets the following solutions for warp factor, axion and dilaton, respectively:

$$U = -\frac{1}{4}\log\left[4\,A\,(-H_{p^1})^3 - b^2\right], \tag{4.21}$$

$$x = B + \frac{b}{2(H_{p^1})^2}, \tag{4.22}$$

$$y = \frac{e^{-2U}}{2(H_{p^1})^2}. \tag{4.23}$$

Here $H_{p^1} < 0$ denotes the harmonic function which corresponds to the magnetic charges p^1, while the harmonic function B satisfies

$$\partial_i B = \frac{6\,B\,\partial_i H_{p^1} - \partial_i H_{q_1}}{H_{p^1}}. \tag{4.24}$$

Furthermore, the non-harmonic function A is constrained to satisfy the equation

$$\partial_i A = \partial_i H_{q_0} + 3\,A\,\frac{\partial_i H_{p^1}}{H_{p^1}} + B\,\partial_i H_{q_1} - 3\,B^2\,\partial_i H_{p^1}.$$

It is here worth mentioning that one could have found this solution more easily by considering equations for the combinations $e^{-U}/\sqrt{y}$ and $x - y\,b\,e^{2U}$. As for f_i, it reads

$$f_i = \partial_i b + (H_{p^1})^2\,\partial_i B, \tag{4.25}$$

and b is such that implies f_i to be divergence-free. For the electric configuration, condition (4.11) yields $\alpha_1 = \mathrm{arccot}\,(x/y)$ and the following solutions can be obtained:

$$U = -\frac{1}{4}\log\left[\frac{4}{27}\,A\,(H_{q_1})^3 - b^2\right], \tag{4.26}$$

$$x = \frac{b - 2/9(H_{q_1})^2 B}{2\,(-bB + (((H_{q_1})^2B^2)/9 + (H_{q_1}A)/3)}, \tag{4.27}$$

$$y = \frac{e^{-2U}}{2\,(-bB + (((H_{q_1})^2B^2)/9 + (H_{q_1}A)/3)}, \tag{4.28}$$

where functions B (harmonic) and A (non-harmonic) satisfy:

$$\partial_i B = \frac{2\,\partial_i H_{q_1}\,B - 3\,\partial_i H_{p^1}}{2H_{q_1}}, \tag{4.29}$$

$$\partial_i A = \partial_i H_{p^0} + B\,(-3\,\partial_i H_{p^1} + B\partial_i H_{q_1}), \tag{4.30}$$

and f_i is given by

$$f_i = \partial_i b - \frac{2}{3}\,(H_{q_1})^2\,\partial_i B. \tag{4.31}$$

Since we are mostly interested in the construction of flow equations, we leave the analysis of the physical properties of these solutions for further future investigation.

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5 Almost BPS flow

5.1 General analysis

Finally, let us investigate the almost BPS class.

This class can be obtained from the BPS one by exploiting the very same procedure considered for the composite non-BPS class, the unique difference being the fact that in this case only the sign of the electric graviphoton charge is flipped: $q_0 \to -q_0$. This yields the following first order system:

$$\partial_i U = \frac{1}{4}\,\mathrm{Re}\left[\left(3\,e^{-i\,\alpha_0} - e^{3\,i\,\alpha_1}\right)\,\hat{Z}_i - \sqrt{3}\,e^{-i\,(\alpha_0+2\,\alpha_1)}\left(1 + e^{i\,(\alpha_0+3\,\alpha_1)}\right)\,\hat{Z}_{t\,i}\right] e^U, \quad (5.1)$$

$$\partial_i t = -\frac{1}{4}\,E^t\,\left(\sqrt{3}e^{i\,\alpha_1}\hat{Z}_i + \sqrt{3}e^{i\,(\alpha_0-2\alpha_1)}\hat{\bar{Z}}_i + 3e^{-i\,\alpha_1}\hat{Z}_{t\,i} - e^{i\,\alpha_0}\hat{\bar{Z}}_{\bar{t}\,i}\right)\,e^U. \quad (5.2)$$

By inserting (5.1) and (5.2) into the second order equations (2.6), a system of equations for the phases α_0 and α_1 and for f_i is obtained:

$$f_i = \frac{1}{2}\,\mathrm{Im}\left[\left(3\,e^{-i\,\alpha_0} + e^{3\,i\,\alpha_1}\right)\,\hat{Z}_i - \sqrt{3}\,e^{-i\,(\alpha_0+2\,\alpha_1)}\left(1 - e^{i\,(\alpha_0+3\,\alpha_1)}\right)\,\hat{Z}_{t\,i}\right] e^{-U}, \quad (5.3)$$

$$\begin{aligned}\partial_i\alpha_0 &= \frac{\sqrt{3}}{2}\,e^U\,\mathrm{Im}\left[e^{-2i\,\alpha_1}\left(-1 + 2e^{2i\,\alpha_1} + e^{i(\alpha_0+3\,\alpha_1)}\right)\hat{\bar{\Pi}}_i\right] + \\ &\quad + \frac{1}{2}\,e^U\,\sin\,(\alpha_0+3\alpha_1)\,\hat{W}_i + \frac{1}{2}\,e^{2U}\,\cos^2\frac{(\alpha_0+3\alpha_1)}{2}\,f_i, \end{aligned} \quad (5.4)$$

$$\begin{aligned}\partial_i\alpha_1 &= \frac{1}{2\sqrt{3}}\,e^U\,\mathrm{Im}\left[e^{-2i\,\alpha_1}\left(3 - 2e^{2i\,\alpha_1} + e^{i(\,\alpha_0+3\,\alpha_1)}\right)\hat{\bar{\Pi}}_i\right] - \\ &\quad - \frac{1}{2}\,e^U\,\sin\,(\alpha_0+3\alpha_1)\,\hat{W}_i - \frac{1}{2}\,e^{2U}\,\cos^2\frac{(\alpha_0+3\alpha_1)}{2}\,f_i. \end{aligned} \quad (5.5)$$

As in all other cases, f_i is the imaginary part of the quantity whose real part is W_i, and it enters both sides of (5.3).

Now, let us try to integrate out electromagnetic potential completely, as achieved in the composite non-BPS class. By starting from the Ansatz

$$\begin{aligned} b^\beta &= \frac{1}{2}\,\mathrm{Re}\left[\left(3\,e^{-i\,\alpha_0} - e^{3\,i\,\alpha_1}\right)\,v^\beta - \sqrt{3}\,e^{-i\,(\alpha_0+2\,\alpha_1)}\left(1 + e^{i\,(\alpha_0+3\,\alpha_1)}\right)\,v_t^\beta\right] e^U - \\ &\quad - b\,e^{3U}\frac{1}{2}\,\mathrm{Im}\left[\left(3\,e^{-i\,\alpha_0} + e^{3\,i\,\alpha_1}\right)\,v_i^\beta - \sqrt{3}\,e^{-i\,(\alpha_0+2\,\alpha_1)}\left(1 - e^{i\,(\alpha_0+3\,\alpha_1)}\right)\,v_t^\beta\right], \end{aligned} \quad (5.6)$$

and inserting it into equation (5.3), one achieved the expression

$$\mathrm{Im}\left[\left(3\,e^{-i\,\alpha_0} + e^{3\,i\,\alpha_1}\right)\,Z_i - \sqrt{3}\,e^{-i\,(\alpha_0+2\,\alpha_1)}\left(1 - e^{i\,(\alpha_0+3\,\alpha_1)}\right)\,Z_{t\,i}\right] = 0. \quad (5.7)$$

The equations for electromagnetic potential can be solved simultaneously *iff* one of these two conditions is fulfilled:

$$\begin{aligned} I &: \alpha_0 + 3\,\alpha_1 = \pi, \\ II &: \alpha_0 + 3\,\alpha_1 = 2\arctan(b\,e^{2U}). \end{aligned} \quad (5.8)$$

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Correspondingly, two sub-classes I and II of full-fledged almost BPS first order equations exist. As we will see in the treatment below, sub-class I corresponds to BPS-like configurations with a positive quartic invariant of the sum of all charges pertaining to each center (namely $I_4\left(\sum_a Q_a\right) > 0$: BPS black hole in the single-centered limit), whereas sub-class II corresponds to non-BPS composite-like configurations with a negative quartic invariant of the sum of all charges pertaining to each center (namely $I_4\left(\sum_a Q_a\right) < 0$: non-BPS black hole in the single-centered limit).

The system of flow equations pertaining to sub-class I reads:

$$
\begin{aligned}
\partial_i U &= \mathrm{Re}\left[e^{-i\alpha_0}\, Z_i\right] e^U + \frac{1}{2}\, b\, f_i\, e^{4U},\\
\partial_i t &= \frac{\sqrt{3}}{2}\, e^{2i\,\frac{\alpha_0-\pi}{3}} \left(e^U \left(i + b e^{2U}\right) f_i + 2\, i\, \mathrm{Im}\left[e^{-i\alpha_0} Z_i\right]\right) e^U\, E^t + e^{i\,\alpha_0}\, \bar{Z}_{\bar{t}\,i}\, e^U\, E^t,\\
f_i &= \partial_i b - 2\, e^{-U} \mathrm{Im}\left[e^{-i\,\alpha_0} Z_i\right] + \sqrt{3}\, b\, e^U\, \mathrm{Re}\left[\sqrt{3} e^{-i\,\alpha_0} Z_i + i\, e^{-i\,\frac{\alpha_0-\pi}{3}}\, Z_{t\,i}\right],
\end{aligned} \tag{5.9}
$$

where the phase α_0 satisfies the condition (already implemented in the system (5.9))

$$
\mathrm{Im}\left[e^{-i\,\alpha_0} Z_i + i\,\sqrt{3}\, e^{-i\,\frac{\alpha_0-\pi}{3}}\, Z_{t\,i}\right] = 0. \tag{5.10}
$$

By setting $b = 0$ in the system (5.9) (and using the condition (5.10)), one obtains the BPS first order system (3.10) constrained by (5.10). This leads us to identify this limit of sub-class I with BPS-like configurations. This conclusion is also confirmed by the asymptotical analysis, in which the rotation vanishes $f_i = 0$, and b becomes zero (more rapidly than any other functions, due to nature of rotation); by taking (5.10) into account, one then gets a correspondingly constrained BPS system (3.10), consistent with $I_4\left(\sum_a Q_a\right) > 0$ in the single-centered interpretation; this will be considered in one of the examples of the next section.

On the other hand, the system of flow equations pertaining to the sub-class II reads:

$$
\begin{aligned}
\partial_i U &= \frac{1}{2} e^U \mathrm{Re}\left[e^{3i\alpha_1} Z_i - \sqrt{3} e^{i\alpha_1} Z_{t\,i}\right] + \frac{1}{2} b\, e^{3U} \mathrm{Im}\left[e^{3i\alpha_1} Z_i - \sqrt{3} e^{i\alpha_1} Z_{t\,i}\right] + \frac{1}{2}\, b\, f_i\, e^{4U},\\
\partial_i t &= -\frac{1}{2} e^U E^t \left(\sqrt{3} e^{U-2i\,\alpha_1} \left(i - b e^{2U}\right) f_i + \sqrt{3} e^{i\,\alpha_1} Z_i + e^{-i\,\alpha_1} Z_{t\,i}\right) +\\
&\quad + \frac{1}{2} e^U E^t \left(\frac{1}{2} \mathrm{Re}\left[\sqrt{3}\, e^{3i\alpha_1} Z_i - e^{i\alpha_1} Z_{t\,i}\right] - \frac{i\, b\, e^{2U-2\,i\,\alpha}}{1 + b^2\, e^{4U}} \left(\sqrt{3}\, e^{-3\,i\,\alpha} \bar{Z}_i - e^{-i\alpha_1} \bar{Z}_{\bar{t}\,i}\right)\right),\\
f_i &= \partial_i b + \sqrt{3}\, e^{-U} \left(\mathrm{Im}\left[\sqrt{3} e^{3i\alpha_1} Z_i - e^{i\alpha_1} Z_{t\,i}\right] - b\, e^{2\,U} \mathrm{Re}\left[\sqrt{3}\, e^{3i\alpha_1} Z_i - e^{i\alpha_1} Z_{t\,i}\right]\right),
\end{aligned} \tag{5.11}
$$

where the phase α_1 satisfies the condition (already implemented in the system (5.11))

$$
\mathrm{Im}\left[e^{3i\alpha_1} Z_i\right] = \frac{\sqrt{3}}{2} \frac{b\, e^{2\,U}}{1 + b^2\, e^{4\,U}} \left(\mathrm{Re}\left[\sqrt{3}\, e^{3i\alpha_1} Z_i - e^{i\alpha_1} Z_{t\,i}\right] + b\, e^{2\,U} \mathrm{Im}\left[\sqrt{3}\, e^{3i\alpha_1} Z_i - e^{i\alpha_1} Z_{t\,i}\right]\right). \tag{5.12}
$$

At spatial infinity, the sub-class II first order system (5.11)–(5.12) reduces to the non-BPS composite one (4.14)–(4.15), when $f_i = 0$, b =constant and taking (5.12) into account. For this reason, as mentioned above we identify this sub-class as a composite non-BPS like one.

We leave the detailed analysis of the solutions (and related constraints) of the general almost BPS sub-classes determined above to future investigation. The next section will be devoted to some illustrative examples.

5.2 Some examples

1. As a first example, we consider the single-centered black hole before integrating the electromagnetic potential out. It should be pointed out that the almost BPS class admits two possible single-centered limits (namely, the BPS and non-BPS one), in contrast to the BPS and composite non-BPS classes. By setting $f_i = 0$ in (5.1) and in (5.3), and replacing the generalized central charge and its derivative by the standard ones, one obtains

$$e^{i\alpha_0} = e^{i\alpha_1}\,\frac{-\sqrt{3}e^{2i\alpha_1}Z + Z_t}{-\sqrt{3}\,\bar{Z} + e^{2i\alpha_1}\bar{Z}_{\bar{t}}} \tag{5.13}$$

for a non-BPS single-centered black hole, and

$$e^{i\alpha_1}\frac{\left(e^{2i\alpha_1}Z + \sqrt{3}Z_t\right)}{\bar{Z} + \sqrt{3}e^{2i\alpha_1}\bar{Z}_{\bar{t}}} = -e^{i\alpha_0} \tag{5.14}$$

for a BPS single-centered black hole. In particular, (5.14) is cubic in α_1; its solution is straightforward but cumbersome, and we will not present it here.

On the other hand, after integrating the electromagnetic fields out, the single-centered limit yields the following results. I) For case I of (5.8), the single-centered non-BPS limit yields a "small" black hole. Indeed, from I of (5.8), (5.13), (5.10) and from condition $f_i = 0$ (where f_i is defined by (5.9)), the following restrictions on special geometry invariants [29] follow:

$$i_3 = \frac{i}{3\sqrt{3}}\left(Z\bar{Z}_{\bar{t}}^3 - \bar{Z}Z_t^3\right) = 0, \tag{5.15}$$

$$i_2 = Z_t\bar{Z}_{\bar{t}} = 3\,i_1 = 3\,Z\bar{Z}, \tag{5.16}$$

which in turn imply the quartic invariant [29]

$$I_4 = (i_1 - i_2)^2 - 4i_2^2 + 4i_4 \tag{5.17}$$

to vanish. On the other hand, the single-centered BPS limit yields a "large" solution; by combining I of (5.8), (5.14), (5.10), conditions $f_i = 0$ and $b = 0$, one can indeed check that a "large" BPS black hole solution, constrained by the additional condition (5.15) is obtained.

II) The same procedure applied to case II of (5.8) yields no "large" BPS black holes (all special geometry invariants are zero), whereas the non-BPS single-centered limit yields the same solutions as the composite non-BPS class, with no additional constraints.

2. As a second and final example, we consider the well known almost BPS solution [5–7]. This is obtained by constraining the phase α_1 in the almost BPS sub-class II (5.11) as follows:
$$\alpha_1 = -\text{arccot}\left[b\, e^{2U}\right]. \tag{5.18}$$
The origin of such a constraint can be better understood by investigating the differential equations for α_0 and α_1, and noticing that $\alpha_0 + \alpha_1 = \pi$ is a particular solution. Combining this with the second of (5.8), one gets (5.18). By inserting this value of α_1 into (5.11) and considering equations for the combinations $e^{-U}/\sqrt{y}$ and $x + b\, y\, e^U$, the following solutions are obtained:
$$x = K_1 - \frac{b}{2\, H_{p^0}\, Z_1}, \quad y = \frac{e^{-2U}}{2\, H_{p^0}\, Z_1}, \tag{5.19}$$
$$e^{-4\,U} = 4\, H_{p^0}\, Z_1^3 - b^2, \tag{5.20}$$
in which the functions K_1 (harmonic) and Z_1 (non-harmonic) respectively satisfy the equations
$$\partial_i K_1 = \frac{\partial_i H_{p^0}\, K_1 - \partial_i H_{p^1}}{H_{p^0}}, \tag{5.21}$$
$$\partial_i Z_1 = \frac{1}{3}\partial_i H_{q_1} + 2\partial_i K_1 H_{p^0} K_1 - \partial_i H_{p^0} K_1^2. \tag{5.22}$$
As for f_i, it reads
$$f_i = \partial_i b - 6\, H_{p^0}\, Z_1\, \partial_i K_1, \tag{5.23}$$
where the harmonic functions are constrained by
$$\partial_i H_{q_0} = 3\, \partial_i K_1\, Z_1 - 3\, \partial_i Z_1\, K_1 + 3\, K_1^2\, \partial_i K_1\, H_{p^0} - \partial_i H_{p^0}\, K_1^3, \tag{5.24}$$
yielded by (5.12).

We leave the analysis of the physical properties of these solutions, along with an investigation of new classes of solutions, for future work.

6 Conclusion

In the present paper, the first order formalism for multi-centered and/or rotating black holes was constructed and investigated in full generality for the so-called T^3 model of $\mathcal{N} = 2$, $D = 4$ ungauged Maxwell-Einstein supergravity. The exploited procedure sets non-BPS classes on the same footing as the well known BPS one.

In order to highlight the various steps of our approach to the construction of the first order flow equations, we first considered the BPS class [16].

Similarly to the BPS class, the composite non-BPS class [3] is endowed with only one auxiliary phase field, which generally satisfies some algebraic constraint. For a single-centered static black hole, this phase can be integrated out without putting any restriction on the charges, and the well known non-BPS fake superpotential for the T^3 model is retrieved. It is worth stressing that a smooth single-centered limit is obtained also after integrating electromagnetic fields out. For what concerns rotating single-centered solutions, we recovered a generalization of the Rasheed-Larsen black hole [27, 28], in which all electric and magnetic charges are switched on. As for multi-centered solutions, we also have retrieved all known solutions.

Consistent with its nilpotent orbit characterization [3], the almost BPS class exhibits the most involved structure, in which two phase fields are present. We have shown that two general sub-classes (I and II) exist, depending on two possible constraints satisfied by the two phases. Also, the almost BPS class exhibits both a BPS and a non-BPS single-centered (extremal) black hole limit. In particular, the BPS single-centered limit yields some restrictions on the special geometry invariants (explicitly derived in the discussion of some examples). This is to be contrasted with the BPS and composite non-BPS classes, which admit only one single-centered (extremal) black hole limit (BPS and non-BPS, respectively), but without any restriction on the special geometry invariants. Furthermore, the classes I and II of almost BPS flow respectively correspond to a positive and negative quartic U-duality invariant polynomial I_4, evaluated on the total charge vector of the system. The well known almost BPS multi-centered solution was also recovered, and it was shown that it is nothing but a particular solution of sub-class II.

Having derived the most general sets of first order flow equations (consistent with the flatness of the three dimensional base-space), we leave the detailed study of their various new classes of solutions, along with their physical properties, to forthcoming works. The T^3 model provides a good framework, due to the presence of only one modulus and the absence of any flat direction. Furthermore, we plan to extend all such results to the STU model, which instead exhibits flat directions [30].

Note added. The new paper [31] on this subject appeared on arXiv after the present work had been completed.

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