Entanglement generation outside a Schwarzschild black hole and the Hawking effect

Jiawei Hu[a] **and Hongwei Yu**[a,b,1]

[a]*Institute of Physics and Key Laboratory of Low Dimensional Quantum Structures and Quantum Control of Ministry of Education, Hunan Normal University, Changsha, Hunan 410081, China*

[b]*Center for Nonlinear Science and Department of Physics, Ningbo University, Ningbo, Zhejiang 315211, China*

E-mail: jwhu3.14@gmail.com, hwyu@hunnu.edu.cn

ABSTRACT: We examine the Hawking effect by studying the asymptotic entanglement of two mutually independent two-level atoms placed at a fixed radial distance outside a Schwarzschild black hole in the framework of open quantum systems. We treat the two-atom system as an open quantum system in a bath of fluctuating quantized massless scalar fields in vacuum and calculate the concurrence, a measurement of entanglement, of the equilibrium state of the system at large times, for the Unruh, Hartle-Hawking and Boulware vacua respectively. We find, for all three vacuum cases, that the atoms turn out to be entangled even if they are initially in a separable state as long as the system is not placed right at the even horizon. Remarkably, only in the Unruh vacuum, will the asymptotic entanglement be affected by the backscattering of the thermal radiation off the space-time curvature. The effect of the back scatterings on the asymptotic entanglement cancels in the Hartle-Hawking vacuum case.

---

[1]Corresponding author.

Contents

1 Introduction

Classically, black holes are described as massive objects with such a strong gravitational field that even light cannot escape from them. However, Hawking finds, in the framework of quantum field theory in curved spacetime, that a black hole is not completely black, but emits thermal radiation with a black body spectrum due to quantum effects [1, 2]. Ever since this surprising discovery, the Hawking effect has attracted widespread interest in the physics community and extensive works have been done trying to understand it in various different physical contexts (See for example, ref. [1–13]).

In this paper, we will try to examine the Hawking effect in terms of the entanglement generation in the framework of open quantum systems. The open system we are going to study consists of two mutually independent static two-level atoms subjected to a bath of fluctuating quantized massless scalar fields in vacuum outside a Schwarzschild black hole. We will analyze the time evolution of the density matrix describing the system using the well-known techniques in the theory of open quantum systems. Let us note that the reduced dynamics of a single static detector (a two-level atom) placed outside a Schwarzschild black hole interacting with quantized massless scalar fields in the Unruh, Hartle-Hawking and Boulware vacua has been recently investigated [13] and it has been found that in both the Unruh and Hartle-Hawking vacua, the detector will spontaneously excite with a nonvanishing probability as if there were thermal radiation at the Hawking temperature. An another way to look at the issue is to study the equilibrium state of the detector. In this regard, it has been shown that the detector is asymptotically driven to a thermal state at the Hawking temperature in the spatial asymptotic region, regardless of its initial state [14]. This approach has also been applied to reproduce both the Unruh effect and the Gibbons-Hawking effect, in ref. [15] and [16], respectively.

If the single atom is replaced by two independent two-level atoms, the situation becomes physically more interesting. A study of the reduced dynamics of the two-atom system

may reveal whether the asymptotic equilibrium state of the system will be entangled or not. It is known that an environment usually leads to decoherence and noise, which may cause entanglement that might have been created before to disappear. However, in certain circumstances, the environment may enhance entanglement rather than destroying it [17–22]. The reason is that an external environment can also provide an indirect interaction between otherwise totally uncoupled subsystems through correlations that exist. It is of interest to see whether a bath of fluctuating vacuum scalar fields outside a Schwarzschild black hole can provide such an indirect coupling to enhance the entanglement. This is what we are going to pursue in the present paper. We will, in the hope of gaining new understanding of the Hawking effect from a different perspective, examine the asymptotic entanglement of the two-atom system in a bath of massless scalar fields in the Unruh [23], Hartle-Hawking [24], and Boulware [25] vacuums outside a black hole. Let us note here that a similar issue related to the Unruh effect has already been worked out by Benatti and Floreanini, in which they have studied a uniformly accelerating two-atom system [15] and found that the two initially separable atoms will become entangled, just as if they were immersed in a thermal bath at the Unruh temperature.

2 The master equation

The system we study consists of two mutually independent static two-level atoms of vanishing separation in interaction with a bath of fluctuating quantum scalar fields in vacuum outside a Schwarzschild black hole which is described by the following metric

$$ds^2 = \left(1 - \frac{2M}{r}\right)dt^2 - \frac{dr^2}{1 - 2M/r} - r^2(d\theta^2 + \sin^2\theta d\phi^2) \,. \tag{2.1}$$

Although the atoms are mutually independent, the fluctuating vacuum fields with which the atoms are coupled may provide an indirect interaction and therefore a means to generate entanglement between them. In this paper, we are particularly concerned with the issue of whether the atoms can be entangled when system reaches an equilibrium state as time becomes large. Generically, the total Hamiltonian of the system takes the form

$$H = H_s + H_\phi + \lambda\, H' \,. \tag{2.2}$$

Here H_s is the Hamiltonian of the two atoms,

$$H_s = H_s^{(1)} + H_s^{(2)}, \quad H_s^{(\alpha)} = \frac{\omega}{2}\, n_i\, \sigma_i^{(\alpha)}, \quad (\alpha = 1, 2), \tag{2.3}$$

where $\sigma_i^{(1)} = \sigma_i \otimes \sigma_0$, $\sigma_i^{(2)} = \sigma_0 \otimes \sigma_i$, σ_i $(i = 1, 2, 3)$ are the Pauli matrices, σ_0 the 2×2 unit matrix, $\mathbf{n} = (n_1, n_2, n_3)$ a unit vector, and ω_0 the energy level spacing. H_ϕ is the Hamiltonian of free massless scalar fields. The Hamiltonian H' that describes the interaction between the two atoms with the external scalar fields is given by

$$H' = \sum_{\mu=0}^{3} \left[(\sigma_\mu \otimes \sigma_0)\Phi_\mu(t, \mathbf{x}_1) + (\sigma_0 \otimes \sigma_\mu)\Phi_\mu(t, \mathbf{x}_2)\right] \,. \tag{2.4}$$

We assume that the scalar fields can be expanded as

$$\Phi_\mu(x) = \sum_{a=1}^{N} [\chi_\mu^a \phi^{(-)}(x) + (\chi_\mu^a)^* \phi^{(+)}(x)] , \qquad (2.5)$$

where $\phi^{(\pm)}(x)$ are positive and negative energy field operators of the massless scalar field, and χ_μ^a are the corresponding complex coefficients.

At the beginning, the whole system is characterized by the total density matrix $\rho_{\text{tot}} = \rho(0) \otimes |0\rangle\langle 0|$, in which $\rho(0)$ is the initial reduced density matrix of the two-atom system, and $|0\rangle$ is the vacuum state of field $\Phi(x)$. In the frame of the two-atom system, the evolution in the proper time τ of the total density matrix ρ_{tot} satisfies

$$\frac{\partial \rho_{\text{tot}}(\tau)}{\partial \tau} = -i L_H[\rho_{\text{tot}}(\tau)] , \qquad (2.6)$$

where the symbol L_H represents the Liouville operator associated with H

$$L_H[S] \equiv [H, S] . \qquad (2.7)$$

We assume that the interaction between the atoms and the field is weak, i.e., the coupling constant λ in (2.2) is small. In the limit of weak coupling, the evolution of the reduced density matrix $\rho(\tau)$ can be written in the Kossakowski-Lindblad form [15, 21, 26–28]

$$\frac{\partial \rho(\tau)}{\partial \tau} = -i\big[H_{\text{eff}}, \rho(\tau)\big] + \mathcal{L}[\rho(\tau)], \qquad (2.8)$$

with

$$H_{\text{eff}} = H_s - \frac{i}{2} \sum_{\alpha,\beta=1}^{2} \sum_{i,j=1}^{3} H_{ij}\, \sigma_i^{(\alpha)} \sigma_j^{(\beta)} , \qquad (2.9)$$

and

$$\mathcal{L}[\rho] = \frac{1}{2} \sum_{\alpha,\beta=1}^{2} \sum_{i,j=1}^{3} C_{ij}\big[2\, \sigma_j^{(\beta)}\rho\, \sigma_i^{(\alpha)} - \sigma_i^{(\alpha)}\sigma_j^{(\beta)}\, \rho - \rho\, \sigma_i^{(\alpha)}\sigma_j^{(\beta)}\big] . \qquad (2.10)$$

The coefficients of the matrix C_{ij} and H_{ij} are determined by the Fourier and Hilbert transforms of the field correlation functions

$$G_{ij}(x-y) = \langle 0|\Phi_i(x)\Phi_j(y)|0\rangle , \qquad (2.11)$$

which are defined as follows

$$\mathcal{G}_{ij}(\lambda) = \int_{-\infty}^{\infty} d\tau\, e^{i\lambda\tau}\, G_{ij}(\tau) , \qquad (2.12)$$

$$\mathcal{K}_{ij}(\lambda) = \int_{-\infty}^{\infty} d\tau\, \text{sign}(\tau)\, e^{i\lambda\tau}\, G_{ij}(\tau) = \frac{P}{\pi i} \int_{-\infty}^{\infty} d\omega\, \frac{\mathcal{G}_{ij}(\omega)}{\omega - \lambda} , \qquad (2.13)$$

in which P denotes principal value. It can be shown that the Kossakowski matrix C_{ij} can be written explicitly as

$$C_{ij} = \sum_{\xi=+,-,0} \sum_{k,l=1}^{3} \mathcal{G}_{kl}(\xi\omega)\, \psi_{ki}^{(\xi)}\, \psi_{lj}^{(-\xi)} , \qquad (2.14)$$

where

$$\psi_{ij}^{(0)} = n_i\, n_j\,, \qquad \psi_{ij}^{(\pm)} = \frac{1}{2}\big(\delta_{ij} - n_i\, n_j \pm i\epsilon_{ijk}n_k\big)\,. \tag{2.15}$$

Similarly, the coefficients of H_{ij} can be obtained by replacing $\mathcal{G}_{kl}(\xi\omega)$ with $\mathcal{K}_{kl}(\xi\omega)$ in the above equations.

If we assume that the field components $\Phi_i(x)$ are independent, or the coefficients χ_μ^a satisfy

$$\sum_{a=1}^{N} \chi_\mu^a (\chi_\nu^a)^* = \delta_{\mu\nu}\,, \tag{2.16}$$

the field correlation functions in (2.11) are diagonal such that

$$G_{ij}(x-y) = \delta_{ij}G(x-y)\,, \tag{2.17}$$

in which $G(x-y)$ is the standard Wightman function, and the Kossakowski matrix C_{ij} can be written as [15]

$$C_{ij} = A\,\delta_{ij} - iB\,\epsilon_{ijk}n_k + C\,n_i n_j\,, \tag{2.18}$$

where

$$A = \frac{1}{2}[\mathcal{G}(\omega_0) + \mathcal{G}(-\omega_0)]\,, \quad B = \frac{1}{2}[\mathcal{G}(\omega_0) - \mathcal{G}(-\omega_0)]\,, \quad C = \mathcal{G}(0) - A\,. \tag{2.19}$$

3 The asymptotic entanglement

In order to figure out whether the entanglement will be generated between the atoms and whether it can persist asymptotically through the analysis of the master equation governing the density matrix, we need to calculate the coefficients C_{ij} which are determined by the field correlation function in the vacuum state. However, when a vacuum state is concerned in a curved spacetime, a delicate issue then arises as to how the vacuum state of the quantum fields is determined. Normally, a vacuum state is associated with non-occupation of positive frequency modes. However, the positive frequency of field modes is defined with respect to the time coordinate. Therefore, to define positive frequency, one has to first specify a definition of time. In a spherically symmetric black hole background, three different vacuum states, i.e., the Boulware [25], Unruh [23] and Hartle-Hawking [24] vacuum states, have been defined, each corresponding to a different choice of time coordinate. In the next section, we examine the asymptotic entanglement for the atoms in all vacuums.

3.1 The Unruh vacuum

Let us begin our discussion with the Unruh vacuum which is supposed to be the vacuum state best approximating the state following the gravitational collapse of a massive body to a black hole. In order to analyze the asymptotic state of the system, we will deal with the master equation (2.8) and see how the reduced density matrix of the two-atom system ρ evolves.

Generally, we can write the reduced density matrix of the two-atom system in the form of

$$\rho(\tau) = \frac{1}{4}\left[\sigma_0 \otimes \sigma_0 + \rho_{0i}(\tau)\,\sigma_0 \otimes \sigma_i + \rho_{i0}(\tau)\sigma_i \otimes \sigma_0 + \rho_{ij}(\tau)\sigma_i \otimes \sigma_j\right], \qquad (3.1)$$

which is normalized as $\mathrm{Tr}(\rho) = 1$ with $\det(\rho) \geq 0$. Here we are interested in the final equilibrium density matrix ρ^∞, which does not change with time, i.e., $\partial_\tau \rho^\infty = 0$. Let us note that the unitary term involving Hamiltonian H_{eff} in the master equation can be ignored since it does not give rise to the entanglement phenomena [15] and we only need to examine the effects produced by the dissipative term $\mathcal{L}[\rho]$ in (2.8). As a result, the equilibrium condition becomes $\mathcal{L}[\rho^\infty] = 0$. After some direct calculations, one obtains the components of ρ^∞ [15]

$$\rho_{0i}^\infty = \rho_{i0}^\infty = -\frac{R}{3 + R^2}(\tau_* + 3)n_i\,,$$
$$\rho_{ij}^\infty = \frac{1}{3 + R^2}[(\tau_* - R^2)\delta_{ij} + R^2(\tau_* + 3)n_i n_j]\,, \qquad (3.2)$$

where $R = B/A$, τ_* is the trace of the density matrix $\tau_* = \Sigma_{i=1}^3 \rho_{ii}(\tau)$, which is actually a constant of motion, and the positivity of $\rho(0)$ requires $-3 \leq \tau_* \leq 1$.

In order to determine whether the final equilibrium state is entangled or not, we take the concurrence as a measurement of the entanglement, which is defined as $\mathcal{C}[\rho] = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$, where λ_μ ($\mu = 1, 2, 3, 4$) are the square roots of the non-negative eigenvalues of the matrix $\rho(\sigma_2 \otimes \sigma_2)\rho^T(\sigma_2 \otimes \sigma_2)$ in decreasing order, and T stands for transposition. The value of $\mathcal{C}[\rho]$ ranges from 0, for separable states, to 1, for maximally entangled states. For the current case, the concurrence is

$$\mathcal{C}(\rho^\infty) = \max\left\{\frac{(3 - R^2)}{2(3 + R^2)}\left[\frac{5R^2 - 3}{3 - R^2} - \tau_*\right], 0\right\}, \qquad (3.3)$$

which is non-zero provided τ_* obeys

$$\tau_* < \frac{5R^2 - 3}{3 - R^2}\,. \qquad (3.4)$$

This result implies that as long as the condition (3.4) is satisfied, the equilibrium state will turn out to be entangled, even if the initial state is separable.

In order to make our discussion more concise, let us come to a simple example. The initial state of the system is taken to be a separable state provided by the direct product of two pure states:

$$\rho(0) = \rho_n \otimes \rho_m\,, \qquad \rho_n = \frac{1}{2}\left(1 + \vec{n} \cdot \vec{\sigma}\right), \quad \rho_m = \frac{1}{2}\left(1 + \vec{m} \cdot \vec{\sigma}\right), \qquad (3.5)$$

where $\vec{n}$ and $\vec{m}$ are two unit vectors. Here $\tau_* = \vec{n} \cdot \vec{m}$, so the asymptotic entanglement is maximized when $\vec{n} = -\vec{m}$:

$$\mathcal{C}[\rho^\infty] = \frac{2R^2}{3 + R^2}\,. \qquad (3.6)$$

Eq. (3.6) shows that, in our simple case, the concurrence increases with R monotonically when R ranges from 0 to 1, and it reaches its maximum $\mathcal{C}[\rho^\infty] = 1/2$ for $R = 1$. So, we need only evaluate R in order to analyze the entanglement of the system.

Now let us begin to compute R in the Unruh vacuum case. The Wightman function for massless scalar fields in the Unruh vacuum is given by [29–31]

$$G^+(x, x') = \sum_{ml} \int_{-\infty}^\infty \frac{e^{-i\omega\Delta t}}{4\pi\omega} |Y_{lm}(\theta, \phi)|^2 \left[\frac{|\overrightarrow{R}_l(\omega, r)|^2}{1 - e^{-2\pi\omega/\kappa}} + \theta(\omega)|\overleftarrow{R}_l(\omega, r)|^2 \right] d\omega , \qquad (3.7)$$

where $\kappa = 1/4M$ is the surface gravity of the black hole. Its Fourier transform is

$$\begin{aligned}
\mathcal{G}(\lambda) &= \int_{-\infty}^\infty e^{i\lambda\tau} G^+(x, x') d\tau \\
&= \frac{1}{8\pi\lambda} \sum_{l=0}^\infty \left[\theta(\lambda\sqrt{g_{00}})(1 + 2l)|\overleftarrow{R}_l(\lambda\sqrt{g_{00}}, r)|^2 + \frac{(1 + 2l)|\overrightarrow{R}_l(\lambda\sqrt{g_{00}}, r)|^2}{1 - e^{-2\pi\lambda\sqrt{g_{00}}/\kappa}} \right] , \quad (3.8)
\end{aligned}$$

where we have used the relation

$$\sum_{m=-l}^l |Y_{lm}(\theta, \phi)|^2 = \frac{2l + 1}{4\pi} , \qquad (3.9)$$

and here κ_r is defined as $\kappa/\sqrt{g_{00}}$. The above Fourier transform, which is needed in our computation of R, is hard to evaluate, since we do not know the exact form of the radial functions $\overrightarrow{R}_l(\omega, r)$ and $\overleftarrow{R}_l(\omega, r)$. Here, we choose to compute it both close to the event horizon and at infinity. For this purpose, let us recall that the radial functions have the following properties in asymptotic regions [31]:

$$\sum_{l=0}^\infty (2l + 1)|\overrightarrow{R}_l(\omega, r)|^2 \sim \begin{cases} \dfrac{4\omega^2}{1 - \frac{2M}{r}} , & r \to 2M , \\[3mm] \dfrac{1}{r^2} \displaystyle\sum_{l=0}^\infty (2l + 1)|B_l(\omega)|^2 , & r \to \infty , \end{cases} \qquad (3.10)$$

$$\sum_{l=0}^\infty (2l + 1)|\overleftarrow{R}_l(\omega, r)|^2 \sim \begin{cases} \dfrac{1}{4M^2} \displaystyle\sum_{l=0}^\infty (2l + 1)|B_l(\omega)|^2, & r \to 2M , \\[3mm] \dfrac{4\omega^2}{1 - \frac{2M}{r}}, & r \to \infty . \end{cases} \qquad (3.11)$$

Inserting eq. (3.8) into eq. (2.19), and using eq. (3.10) and (3.11), one finds that

$$r \to 2M : \begin{cases} A \approx \dfrac{\omega_0}{4\pi} \left[1 + g_{00}\, f(\omega_0\sqrt{g_{00}}, 2M) + \dfrac{2}{e^{2\pi\omega_0/\kappa_r} - 1} \right] , \\[3mm] B \approx \dfrac{\omega_0}{4\pi} \left[1 + g_{00}\, f(\omega_0\sqrt{g_{00}}, 2M) \right] , \end{cases} \qquad (3.12)$$

and

$$r \to \infty : \begin{cases} A \approx \dfrac{\omega_0}{4\pi} \left[1 + g_{00}\, f(\omega_0\sqrt{g_{00}}, r) + \dfrac{2}{e^{2\pi\omega_0/\kappa_r} - 1} g_{00} f(\omega_0\sqrt{g_{00}}, r) \right] , \\[3mm] B \approx \dfrac{\omega_0}{4\pi} \left[1 + g_{00}\, f(\omega_0\sqrt{g_{00}}, r) \right] , \end{cases} \qquad (3.13)$$

where $f(\omega, r)$ is defined as

$$f(\omega, r) = \frac{1}{4\, r^2 \omega^2} \sum_{l=0}^{\infty} (2l+1)\,|\,B_l\,(\omega)|^2 \ . \tag{3.14}$$

Straightforward calculations then yield in the asymptotic regions,

$$R = \frac{B}{A} = \begin{cases} \dfrac{1 + g_{00} f(\omega_0 \sqrt{g_{00}}, 2M)}{1 + g_{00} f(\omega_0 \sqrt{g_{00}}, 2M) + \frac{2}{e^{2\pi\omega_0/\kappa_r}-1}} \ , & r \to 2M \ , \\[4mm] \dfrac{1 + g_{00} f(\omega_0 \sqrt{g_{00}}, r)}{1 + g_{00} f(\omega_0 \sqrt{g_{00}}, r) + \frac{2}{e^{2\pi\omega_0/\kappa_r}-1} g_{00} f(\omega_0 \sqrt{g_{00}}, r)} \ , & r \to \infty \ . \end{cases} \tag{3.15}$$

We can see that, in the vicinity of the event horizon, the first two terms in the denominator are the same as their counter-parts in the numerator, and the third one is the standard Planckian factor. At infinity, the Planckian factor is modified by a grey-body factor $g_{00} f(\omega_0 \sqrt{g_{00}}, r)$ caused by the backscattering off the space-time curvature. This suggests that at the horizon, there is a thermal flux going outwards, which is weakened by the backscattering off the curvature on its way to infinity. Actually, it has been shown that, in the framework of open quantum system, the spontaneous excitation rate per unit time of a particle detector from the initial ground state i to the final excited state f is just [13, 15]

$$\Gamma_{i \to f} = 2(A - B) = 2\mathcal{G}(-\omega_0) \ . \tag{3.16}$$

This means that the difference between the denominator and the numerator of R is proportional to the spontaneous excitation rate per unit time of a particle detector, i.e., the strength of the thermal radiation. The larger the difference between A and B, the stronger the thermal radiation, and the less the two-atom system gets entangled.

Here we note that, using the geometrical optics approximation [29], the transmission amplitude $B_l(\omega)$ can be approximated as $B_l(\omega) \sim \theta(\sqrt{27}M\omega - l)$, where $\theta(x)$ is the standard step function, which gives 0 for $x < 0$ and 1 for $x > 0$. So $g_{00} f(\omega_0 \sqrt{g_{00}}, r)$ can be simplified as

$$g_{00} f(\omega_0 \sqrt{g_{00}}, r) \approx \frac{27M^2 g_{00}}{4r^2} = \frac{27M^2}{4r^2}\left(1 - \frac{2M}{r}\right) \equiv f(r) \ , \tag{3.17}$$

and in both the asymptotic regions, $g_{00} f(\omega_0 \sqrt{g_{00}}, r) \to 0$. Allowing for this, we have, in the vicinity of the event horizon,

$$R = \frac{e^{2\pi\omega_0/\kappa_r} - 1}{e^{2\pi\omega_0/\kappa_r} + 1} \ . \tag{3.18}$$

It is interesting to note that the R we obtain here is exactly the same as that in the case of a two-atom system immersed in a thermal bath at the temperature $T = \kappa_r/2\pi = T_H/\sqrt{g_{00}}$ in a flat space-time, where $T_H = \kappa/2\pi$ is the Hawking temperature [1, 2]. When $r \to 2M$, the temperature T is divergent, since this effective temperature is a result of both the thermal flux from the black hole and the Unruh effect due to that the system is accelerating with

respect to the local free-falling inertial frame so as to maintain at a fixed distance from the black hole, and the acceleration diverges at the horizon. In this case, the concurrence $\mathcal{C}[\rho^\infty]$ approaches zero, which means that final equilibrium state of the two atom system, which is very close to the event horizon, will not be entangled. As the atoms are placed farther, then the thermal radiation becomes weaker due to the back scattering off the spacetime curvature and the concurrence grows larger. At the infinity, the grey-body factor vanishes and R approaches 1, so that the concurrence $\mathcal{C}[\rho^\infty]$ tends to reach its maximum $1/2$. This result suggests that, in the Unruh vacuum, no thermal radiation is felt at the infinity due to the back scattering of the outgoing thermal radiation off the spacetime curvature. Here it is clear that in the vicinity of the horizon, the entanglement of the system is enhanced due to the back scattering, while in the infinity, the concurrence is smaller than its maximum if we allow for the thermal radiation emitted from the horizon although it is back scattered by the space-time curvature.

Compared with the case of a thermal bath in a flat space-time, we find that the parameters A and B are modified by the grey-body factor $f(\omega, r)$, which is a function of M and r for a given energy gap ω_0. A similar result is derived when studying the entanglement generation in atoms immersed in a thermal bath of external quantum scalar fields with a reflecting boundary [32, 33]. This result implies that the back scattering of vacuum field modes off the space-time curvature of the black hole manifests in much the same way as the reflection of the field modes at the reflecting boundary in a flat space-time.

3.2 The Hartle-Hawking vacuum

Now let us move on to the Hartle-Hawking vacuum. The Wightman function is now given by [29–31]

$$G^+(x, x') = \sum_{ml} \int_{-\infty}^{\infty} \frac{|Y_{lm}(\theta, \phi)|^2}{4\pi\omega} \left[\frac{e^{-i\omega\Delta t}}{1 - e^{-2\pi\omega/\kappa}} |\overrightarrow{R_l}(\omega, r)|^2 + \frac{e^{i\omega\Delta t}}{e^{2\pi\omega/\kappa} - 1} |\overleftarrow{R_l}(\omega, r)|^2 \right] d\omega \ ,$$

(3.19)

and its Fourier transform is

$$\begin{aligned}
\mathcal{G}(\lambda) &= \int_{-\infty}^{\infty} e^{i\lambda\tau} G^+(x, x') d\tau \\
&= \sum_{l=0}^{\infty} \frac{(1 + 2l)}{8\pi\lambda} \left[\frac{|\overrightarrow{R_l}(\lambda\sqrt{g_{00}}, r)|^2}{1 - e^{-2\pi\lambda/\kappa_r}} + \frac{|\overleftarrow{R_l}(-\lambda\sqrt{g_{00}}, r)|^2}{1 - e^{-2\pi\lambda/\kappa_r}} \right] \ ,
\end{aligned}$$

(3.20)

Similar calculations then lead to

$$r \to 2M : \begin{cases} A \approx \dfrac{\omega_0}{4\pi} \dfrac{e^{2\pi\omega_0/\kappa_r} + 1}{e^{2\pi\omega_0/\kappa_r} - 1} \left[1 + g_{00} \, f(\omega_0\sqrt{g_{00}}, 2M) \right] \ , \\ B \approx \dfrac{\omega_0}{4\pi} \left[1 + g_{00} \, f(\omega_0\sqrt{g_{00}}, 2M) \right] \ , \end{cases}$$

(3.21)

$$r \to \infty : \begin{cases} A \approx \dfrac{\omega_0}{4\pi} \dfrac{e^{2\pi\omega_0/\kappa_r} + 1}{e^{2\pi\omega_0/\kappa_r} - 1} \left[1 + g_{00} \, f(\omega_0\sqrt{g_{00}}, r) \right] \ , \\ B \approx \dfrac{\omega_0}{4\pi} \left[1 + g_{00} \, f(\omega_0\sqrt{g_{00}}, r) \right] \ , \end{cases}$$

(3.22)

and

$$R = \frac{B}{A} = \begin{cases} \dfrac{1 + g_{00}f(\omega_0\sqrt{g_{00}}, 2M)}{1 + g_{00}f(\omega_0\sqrt{g_{00}}, 2M) + \frac{2}{e^{2\pi\omega_0/\kappa_r}-1} + \frac{2}{e^{2\pi\omega_0/\kappa_r}-1}g_{00}f(\omega_0\sqrt{g_{00}}, 2M)} , & r \to 2M , \\[3ex] \dfrac{1 + g_{00}f(\omega_0\sqrt{g_{00}}, r)}{1 + g_{00}f(\omega_0\sqrt{g_{00}}, r) + \frac{2}{e^{2\pi\omega_0/\kappa_r}-1} + \frac{2}{e^{2\pi\omega_0/\kappa_r}-1}g_{00}f(\omega_0\sqrt{g_{00}}, r)} , & r \to \infty . \end{cases}$$

(3.23)

Unlike that in the Unruh vacuum case, here R is the same in both the asymptotic regions. We have two Planckian terms in the denominator. One is the standard one and the other is a Planckian factor modified by a grey-body factor caused by the backscattering off the space-time curvature. This suggests that there are thermal radiation outgoing from the horizon and that incoming from infinity, both of which are weakened by the backscattering off the curvature on their way. Therefore, the Hartle-Hawking vacuum is actually a state that describes a black hole in equilibrium with an infinite sea of blackbody radiation [31].

Here we notice that eq. (3.23) can be simplified as

$$R = \frac{B}{A} = \frac{e^{2\pi\omega_0/\kappa_r} - 1}{e^{2\pi\omega_0/\kappa_r} + 1} .$$

(3.24)

At close to the horizon, i.e., when $r \to 2M$, $R \to 0$, which means that final state of the two-atom system is not entangled. When $r \to \infty$, $R \to \frac{e^{2\pi\omega_0/\kappa}-1}{e^{2\pi\omega_0/\kappa}+1}$, which is the maximal value it can reach in this case and it is nonzero, so two atoms will be entangled even if they are separable initially. Here, although both A and B are modified by a grey-body factor $f(\omega, r)$, the modification cancels when we evaluate R. So, R is the same as what we get in the case of a thermal bath. This result shows that, both in the vicinity of the horizon and at infinity, the impact of the black hole to the equilibrium entanglement of the system is the same as that of a thermal bath at temperature $T = T_H/\sqrt{g_{00}}$ in a flat space-time. As $r \to \infty$, the effective temperature becomes the Hawking temperature, since the acceleration needed to maintain the two-atom system at a fixed distance vanishes, and the temperature is purely due to the thermal bath the black hole immersed in.

3.3 The Boulware vacuum

For the Boulware vacuum, the Wightman function is given by [29–31]

$$G^+(x, x') = \sum_{lm} \int_0^\infty \frac{e^{-i\omega\Delta t}}{4\pi\omega} |Y_{lm}(\theta, \phi)|^2 \big[|\overrightarrow{R_l}(\omega, r)|^2 + |\overleftarrow{R_l}(\omega, r)|^2 \big] d\omega .$$

(3.25)

The Fourier transform with respect to the proper time is

$$\mathcal{G}(\lambda) = \int_{-\infty}^{\infty} e^{i\lambda\tau} G^+[x(\tau)] d\tau = \sum_{ml} \frac{2l+1}{8\pi\lambda} \big[|\overrightarrow{R_l}(\lambda\sqrt{g_{00}}, r)|^2 + |\overleftarrow{R_l}(\lambda\sqrt{g_{00}}, r)|^2 \big] \theta(\lambda) ,$$

(3.26)

where $\theta(\lambda)$ is the step function. Plugging eq. (3.26) into Eq (2.19), we have

$$A = B = \sum_{l=0}^{\infty} \frac{2l+1}{16\pi\omega} \big[|\overrightarrow{R_l}(\omega, r)|^2 + |\overleftarrow{R_l}(\omega, r)|^2 \big] .$$

(3.27)

So

$$R = \frac{B}{A} = 1 \; , \tag{3.28}$$

everywhere outside the event horizon, and the concurrence of the equilibrium state is

$$\mathcal{C}[\rho^\infty] = \mathcal{C}[\rho^\infty]_{\max} = \frac{1}{2} \tag{3.29}$$

which is the same as that in a Minkowski vacuum. No thermal radiation is present.

4   Conclusion

In summary, we have examined the asymptotic entanglement between two mutually independent two-level atoms at a fixed radial distance outside a Schwarzschild black hole in the paradigm of open quantum systems. We treat the two-atom system as an open quantum system in a bath of fluctuating quantized massless scalar fields in vacuum and have studied the Boulware, Unruh, and Hartle-Hawking vacua respectively.

In the Hartle-Hawking and the Unruh vacuum cases, the concurrence attains a non-zero value less than $1/2$ as long as the two-atom system is not placed right at the event horizon, indicating that the two atoms will turn out to be entangled even if they are initially separable. For the Unruh vacuum case, the concurrence is the same as if there were an outgoing thermal flux of radiation from the event horizon, which is backscattered by the curvature of the space-time. For the Hartle-Hawking vacuum case, the concurrence behaves as if the atom were in a thermal bath of radiation at a proper temperature which reduces to the Hawking temperature in the spatial asymptotic region. Remarkably, only in the Unruh vacuum, will the asymptotic entanglement be affected by the backscattering of the thermal radiation off the space-time curvature. The effect of the back scatterings on the asymptotic entanglement cancels in the Hartle-Hawking vacuum case.

References

[1] S.W. Hawking, *Black hole explosions*, *Nature* **248** (1974) 30 [SPIRES].

[2] S.W. Hawking, *Particle creation by black holes*, *Commun. Math. Phys.* **43** (1975) 199 [*Erratum ibid.* **46** (1976) 206] [SPIRES].

[3] G.W. Gibbons and S.W. Hawking, *Action integrals and partition functions in quantum gravity*, *Phys. Rev.* **D 15** (1977) 2752 [SPIRES].

[4] M.K. Parikh and F. Wilczek, *Hawking radiation as tunneling*, *Phys. Rev. Lett.* **85** (2000) 5042 [hep-th/9907001] [SPIRES].

[5] A. Strominger and C. Vafa, *Microscopic origin of the Bekenstein-Hawking entropy*, *Phys. Lett.* **B 379** (1996) 99 [hep-th/9601029] [SPIRES].

[6] A.W. Peet, *TASI lectures on black holes in string theory*, hep-th/0008241 [SPIRES].

[7] S.P. Robinson and F. Wilczek, *A relationship between Hawking radiation and gravitational anomalies*, *Phys. Rev. Lett.* **95** (2005) 011303 [gr-qc/0502074] [SPIRES].

[8] S. Iso, H. Umetsu and F. Wilczek, *Hawking radiation from charged black holes via gauge and gravitational anomalies*, *Phys. Rev. Lett.* **96** (2006) 151302 [hep-th/0602146] [SPIRES].

[9] S. Deser and O. Levin, *Mapping Hawking into Unruh thermal properties*, *Phys. Rev.* **D 59** (1999) 064004 [hep-th/9809159] [SPIRES].

[10] H.W. Yu and W. Zhou, *Relationship between Hawking radiation from black holes and spontaneous excitation of atoms*, *Phys. Rev.* **D 76** (2007) 027503 [arXiv:0706.2207] [SPIRES].

[11] H.W. Yu and W. Zhou, *Do static atoms outside a Schwarzschild black hole spontaneously excite?*, *Phys. Rev.* **D 76** (2007) 044023 [arXiv:0707.2613] [SPIRES].

[12] W. Zhou and H.W. Yu, *Lamb shift for static atoms outside a Schwarzschild black hole*, *Phys. Rev.* **D 82** (2010) 104030 [arXiv:1011.1619] [SPIRES].

[13] H.W. Yu and J. Zhang, *Understanding Hawking radiation in the framework of open quantum systems*, *Phys. Rev.* **D 77** (2008) 024031 [*Erratum ibid.* **D 77** (2008) 029904] [arXiv:0806.3602] [SPIRES].

[14] H.W. Yu and J. Zhang, *Hawking radiation and thermalization phenomena in open quantum systems*, in *Proceedings of the Ninth Asia-Pacific International Conference on Gravitation and Astrophysics*, Wuhan, China, 2009, J. Luo et al. eds., World Scientific Publishing, Singapore (2010), pg. 319.

[15] F. Benatti and R. Floreanini, *Entanglement generation in uniformly accelerating atoms: reexamination of the Unruh effect*, *Phys. Rev.* **A 70** (2004) 012112.

[16] H.W. Yu, *Open quantum system approach to Gibbons-Hawking effect of de Sitter space-time*, *Phys. Rev. Lett.* **106** (2011) 061101 [arXiv:1101.5235] [SPIRES].

[17] D. Braun, *Creation of entanglement by interaction with a common heat bath*, *Phys. Rev. Lett.* **89** (2002) 277901 [SPIRES].

[18] M.S. Kim, J. Lee, D. Ahn and P.L. Knight, *Entanglement induced by a single-mode heat environment*, *Phys. Rev.* **A 65** (2002) 040101(R) [quant-ph/0109052].

[19] L. Jakóbczyk, *Entangling two qubits by dissipation*, *J. Phys.* **A 35** (2002) 6383.

[20] S. Schneider and G.J. Milburn, *Entanglement in the steady state of a collective-angular-momentum (Dicke) model*, *Phys. Rev.* **A 65** (2002) 042107.

[21] F. Benatti, R. Floreanini and M. Piani, *Environment induced entanglement in Markovian dissipative dynamics*, *Phys. Rev. Lett.* **91** (2003) 070402 [quant-ph/0307052].

[22] A.M. Basharov, *Decoherence and entanglement in radiative decay of a diatomic system*, *J. Exp. Theor. Phys.* **94** (2002) 1070 [*Zh. Eksp. Teor. Fiz.* **121** (2002) 1249].

[23] W.G. Unruh, *Notes on black hole evaporation*, *Phys. Rev.* **D 14** (1976) 870 [SPIRES].

[24] J.B. Hartle and S.W. Hawking, *Path integral derivation of black hole radiance*, *Phys. Rev.* **D 13** (1976) 2188 [SPIRES].

[25] D.G. Boulware, *Quantum Field Theory in Schwarzschild and Rindler spaces*, *Phys. Rev.* **D 11** (1975) 1404 [SPIRES].

[26] F. Benatti and R. Floreanini, *Controlling entanglement generation in external quantum fields*, *J. Opt.* **B 7** (2005) S429 [quant-ph/0508150].

[27] V. Gorini, A. Kossakowski and E.C.G. Sudarshan, *Completely positive dynamical semigroups of N level systems*, *J. Math. Phys.* **17** (1976) 821 [SPIRES].

[28] G. Lindblad, *On the generators of quantum dynamical semigroups*, *Commun. Math. Phys.* **48** (1976) 119 [SPIRES].

[29] B.S. DeWitt, *Quantum Field Theory in curved space-time*, *Phys. Rept.* **19** (1975) 295 [SPIRES].

[30] S.M. Christensen and S.A. Fulling, *Trace anomalies and the Hawking effect*, *Phys. Rev.* **D 15** (1977) 2088 [SPIRES].

[31] P. Candelas, *Vacuum polarization in Schwarzschild space-time*, *Phys. Rev.* **D 21** (1980) 2185 [SPIRES].

[32] J. Zhang and H.W. Yu, *Entanglement generation in atoms immersed in a thermal bath of external quantum scalar fields with a boundary*, *Phys. Rev.* **A 75** (2007) 012101 [quant-ph/0611170] [SPIRES].

[33] J. Zhang and H.W. Yu, *The Unruh effect and entanglement generation for accelerated atoms near a reflecting boundary*, *Phys. Rev.* **D 75** (2007) 104014 [arXiv:0705.1092] [SPIRES].