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Spin 3 cubic vertices in a frame-like formalism

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Abstract: Till now most of the results on interaction vertices for massless higher spin fields were obtained in a metric-like formalism using completely symmetric (spin-)tensors. In this, the Lagrangians turn out to be very complicated and the main reason is that the higher the spin one want to consider the more derivatives one has to introduce. In this paper we show that such investigations can be greatly simplified if one works in a frame-like formalism. As an illustration we consider massless spin 3 particle and reconstruct a number of vertices describing its interactions with lower spin 2, 1 and 0 ones. In all cases considered we give explicit expressions for the Lagrangians and gauge transformations and check that the algebra of gauge transformations is indeed closed.

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Contents

1 Introduction

It has been known since a long time that it is not possible to construct standard gravitational interaction for massless higher spin $s \geq 5/2$ particles in flat Minkowski space [1–3]. At the same time, it has been shown [4, 5] that this task indeed has a solution in $(A)dS$ space with non-zero cosmological term. The reason is that gauge invariance, that turns out to be broken when one replaces ordinary partial derivatives by the gravitational covariant ones, could be restored with the introduction of higher derivative corrections containing gauge invariant Riemann tensor. These corrections have coefficients proportional to inverse powers of cosmological constant so that such theories do not have naive flat limit. However it is perfectly possible, for cubic vertices, to have a limit where both cosmological term and gravitational coupling constant simultaneously go to zero in such a way that only interactions with highest number of derivatives survive [6, 7]. Besides all, it means that the procedure can be reversed. Namely, one can start with the massless particle in flat Minkowski space and search for non-trivial (i.e. with non-trivial corrections to gauge transformations) higher derivatives cubic $s - s - 2$ vertex containing linearized Riemann tensor. Then, considering smooth deformation into $(A)dS$ space, one can try to reproduce standard minimal gravitational interaction as a by product of such deformation. Recently we have shown that such procedure is indeed possible on the example of massless spin 3 particle [6] using cubic four derivatives $3 - 3 - 2$ vertex constructed in [7, 8].

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Besides gravitational interaction one more classical and important test for any higher spin theory is electromagnetic interaction. The problem of switching on such interaction for massless higher spin particles looks very similar to the problem with gravitational interactions. Namely, if one replaces ordinary partial derivatives by the gauge covariant ones the resulting Lagrangian loses its gauge invariance and this non-invariance (arising due to non-commutativity of covariant derivatives) is proportional to field strength of vector field. In this, for the massless fields with $s \geq 3/2$ in flat Minkowski space there is no possibility to restore gauge invariance by adding non-minimal terms to Lagrangian and/or modifying gauge transformations. But such restoration becomes possible if one goes to $(A)dS$ space with non-zero cosmological constant. By the same reason, as in the gravitational case, such theories do not have naive flat limit, but it is possible to consider a limit where both cosmological constant and electric charge simultaneously go to zero so that only highest derivative non-minimal terms survive. Again it should be possible to reproduce standard minimal e/m interaction starting with some non-trivial cubic higher derivatives $s-s-1$ vertex containing e/m field strength and considering its smooth deformation into $(A)dS$ space. An example of such procedure for massless spin 2 particle has been given recently in [9], while candidate for appropriate $s-s-1$ vertex was given in [7].

It is natural to suggest that in any realistic higher spin theory (like in superstring) most of higher spin particles must be massive and their gauge symmetries spontaneously broken. As is well known, for massive higher spin particles any attempt to switch on standard minimal gravitational or electromagnetic interactions spoils a consistency of the theory leading first of all to appearance of non-physical degrees of freedom and/or non-causality. But having in our disposal mass m as a dimensionfull parameter even in a flat Minkowski space we can try to restore consistency of the theory by adding to Lagrangian non-minimal terms containing the linearized Riemann tensor (e/m field strength). Naturally such terms will have coefficients proportional to inverse powers of mass m so that the theory will not have naive massless limit. However, it is natural to suggest that there exists a limit where both mass and gravitational coupling constant (electric charge) simultaneously go to zero so that only some interactions containing Riemann tensor (e/m field strength) survive. Again it suggests that the procedure can be reversed. Namely, one can try to reproduce minimal gravitational (e/m) interactions starting with appropriate higher derivative non-minimal interactions for massless particle and performing smooth deformation into massive case. The first step towards such construction of gravitational interactions for massive spin 3 particles was performed in [6], while electromagnetic interactions for massive spin 2 particles where considered in [10].

In both cases it is crucial to have non-minimal higher derivative cubic vertices for massless particles in a flat Minkowski space (some recent reviews on higher spin interactions see [11–14]). Last years there appeared a number of important and interesting results in this direction both in a light cone [15, 16] and a Lorentz covariant [7, 8, 17–22] approaches as well as in attempts to extract useful information from strings [23–26]. One of the important general facts on these vertices is that the higher spins one tries to consider the more derivatives one has to introduce. It seems that there is a general agreement [15, 16, 21, 22] that the minimal number of derivatives necessary to construct non-trivial cubic vertex for massless particles with spins s_1, s_2 and s_3 such that $s_1 \geq s_2 \geq s_3$ is equal to:

$$n = s_1 + s_2 - s_3$$

Till now most of the results on such vertices where obtained in a metric-like formalism where for the description of massless spin s ($s+\frac{1}{2}$) particle one uses completely symmetric (spin-) tensor of rank s. In this, the Lagrangians for these vertices turn out to be very complicated. Moreover, higher derivatives in the field equations and especially higher derivatives of gauge parameters in gauge transformations make the consistency check in such theories to be highly non-trivial. The aim of this paper is to show that such investigations can be greatly simplified if one uses a frame-like formalism [27–29] (see also [30, 31]). In this, as it will be shown, higher derivatives of physical fields are replaced by so called auxiliary and extra fields, while higher derivatives of main gauge parameters are replaced by additional gauge parameters that are present in a frame like-formalism. As an illustration we choose massless spin 3 particle and try to reconstruct a number of cubic vertices describing interactions of this particle with lower spins 2, 1 and 0 ones.

The plan of the paper is simple. In section 2 we give all necessary information on the frame-like description of massless spin 3 particle, including Lagrangian, gauge transformations, expressions for auxiliary and extra fields in terms of derivatives of physical ones and a number of identities that will be heavily used in what follows. For completeness and to fix notations we also give relevant formulas for lower spins 2, 1 and 0 as well.

In section 3 we systematically reproduce a number of cubic vertices for the spin 3 particles interacting with the lower spin ones in such frame-like formalism. Almost all these vertices (except the $3-2-1$ one as far as we know) where known previously in a metric-like formalism. Note also that all vertices have minimal number of derivatives possible in agreement with the formula given above. In all cases we give Lagrangian and gauge transformations as well as check the closure of the algebra of gauge transformations.

Notations and conventions. We work in a flat Minkowski space with $d \geq 4$ dimensions. We use Greek letters for world indices and Latin letters for local ones. Surely, in a flat space one can freely convert world indices into local ones and vice-versa and we indeed will use such conversion whenever convenient. But separation of world and local indices plays very important role in a frame-like formalism. In particular, for all vertices we consider the Lagrangians can be written as a product of forms, i.e. as expressions completely antisymmetric on world indices and this property greatly simplifies all calculations.

2 Kinematics

In a frame-like formalism free Lagrangian for massless particle contains two main objects [27–29]: physical field (analogue of frame $e_\mu{}^a$) and auxiliary field (analogue of Lorentz connection $\omega_\mu{}^{ab}$). In this, equations for auxiliary field turn out to be algebraic and their solution allows one to express this field in terms of first derivatives of physical one. Besides, frame-like formalism contains a number of so called extra fields which do not enter free Lagrangian but play an important role for the description of interactions (as it will be seen in particular from the results of this paper). These extra fields also can be expressed in terms of higher derivatives of physical field. As it will be explained in the next section a modified 1 and $\frac{1}{2}$ order formalism we will use requires such explicit solutions for auxiliary and extra fields. Moreover, a number of identities that holds on the solutions only will be heavily used in what follows.

In this section we will give all necessary information on kinematics of massless spin 3 particle in flat Minkowski space including expressions for auxiliary and extra fields and corresponding identities. For completeness and to fix notations we also give relevant formulas for lower spin fields 2, 1 and 0.

2.1 Spin 3

Frame-like description of massless spin 3 particle in flat Minkowski space requires two main objects [27–29]: physical one form $\Phi_\mu{}^{ab}$ which is symmetric and traceless on local indices and auxiliary one form $\Omega_\mu{}^{ab,c}$ which is symmetric on first two indices, completely traceless on all local indices and satisfies a condition $\Omega_\mu{}^{(ab,c)} = 0$, where round brackets denote symmetrization. Corresponding free Lagrangian can be written as follows:

$$\mathcal{L}_0 = -\frac{1}{6}\left\{ {}^{\mu\nu}_{ab} \right\} [2\Omega_\mu{}^{ac,d}\Omega_\nu{}^{bc,d} + \Omega_\mu{}^{cd,a}\Omega_\nu{}^{cd,b}] - \frac{2}{3}\left\{ {}^{\mu\nu\alpha}_{abc} \right\} \Omega_\mu{}^{ad,b}\partial_\nu\Phi_\alpha{}^{cd} \tag{2.1}$$

where $\left\{ {}^{\mu\nu}_{ab} \right\} = e^\mu{}_a e^\nu{}_b - e^\nu{}_a e^\mu{}_b$ and so on. This Lagrangian is invariant under the following gauge transformations:

$$\delta\Phi_\mu{}^{ab} = \partial_\mu\xi^{ab} + \eta^{ab}{}_\mu, \qquad \delta\Omega_\mu{}^{ab,c} = \partial_\mu\eta^{ab,c} + \zeta^{ab,c}{}_\mu \tag{2.2}$$

where parameter ξ^{ab} is symmetric and traceless, $\eta^{ab,c}$ has the same properties on its local indices as $\Omega_\mu{}^{ab,c}$, while parameter $\zeta^{ab,cd}$ is symmetric on first as well as second pair of indices, completely traceless and satisfies a condition $\zeta^{(ab,c)d} = 0$.

As can be easily seen from the Lagrangian, the equation for Ω field is algebraic and allows one to express this field in terms of first derivatives of physical field Φ. To obtain explicit expression let us first of all introduce a "torsion" two form $T_{\mu\nu}{}^{ab}$ which is invariant under ξ^{ab} transformations (but not under the $\eta^{ab,c}$ ones):

$$T_{\mu\nu}{}^{ab} = \partial_\mu\Phi_\nu{}^{ab} - \partial_\nu\Phi_\mu{}^{ab} = \partial_{[\mu}\Phi_{\nu]}{}^{ab}, \qquad T_\mu{}^a = T_{\mu\nu}{}^{a\nu}, \qquad T_\mu{}^\mu = 0$$

By construction this two form satisfies the following identities:

$$\partial_{[\mu}T_{\nu\alpha]}{}^{ab} = 0, \qquad \partial_\mu T_{\nu\alpha}{}^{\mu b} = \partial_{[\nu}T_{\alpha]}{}^b \qquad \partial_\nu T_\mu{}^\nu = 0$$

Using this two form the explicit expression for Ω field can be written as follows:

$$\begin{aligned}\Omega_{\mu,\alpha\beta,\nu} = & \frac{1}{4}[2T_{\mu\nu,\alpha\beta} - T_{\mu\alpha,\nu\beta} - T_{\mu\beta,\nu\alpha} - T_{\nu\alpha,\mu\beta} - T_{\nu\beta,\mu\alpha}] - \\ & -\frac{1}{4(d-2)}[g_{\nu\alpha}T_{(\mu\beta)} + g_{\nu\beta}T_{(\mu\alpha)} + g_{\mu\alpha}T_{(\nu\beta)} + g_{\mu\beta}T_{(\nu\alpha)} - \\ & -2g_{\alpha\beta}T_{(\mu\nu)} - 2g_{\mu\nu}T_{(\alpha\beta)}]\end{aligned} \tag{2.3}$$

By straightforward calculations one can check that under $\delta\Phi_\mu{}^{ab} = \eta^{ab}{}_\mu$ such $\Omega_\mu{}^{ab,c}$ indeed transforms as $\delta\Omega_\mu{}^{ab,c} = \partial_\mu \eta^{ab,c} + \zeta^{ab,c}{}_\mu$ where

$$\begin{aligned}\zeta_{\mu\nu,\alpha\beta} = & \frac{1}{4}[\partial_\mu \eta_{\alpha\beta,\nu} + \partial_\nu \eta_{\alpha\beta,\mu} + \partial_\alpha \eta_{\mu\nu,\beta} + \partial_\beta \eta_{\mu\nu,\alpha}] + \\ & + \frac{1}{4(d-2)}[g_{\nu\alpha}(\partial\eta)_{\mu\beta} + g_{\nu\beta}(\partial\eta)_{\mu\alpha} + g_{\mu\alpha}(\partial\eta)_{\nu\beta} + g_{\mu\beta}(\partial\eta)_{\nu\alpha} - \\ & \qquad - 2g_{\alpha\beta}(\partial\eta)_{\mu\nu} - 2g_{\mu\nu}(\partial\eta)_{\alpha\beta}]\end{aligned}$$

Here $(\partial\eta)_{\alpha\beta} = \partial^\mu \eta_{\alpha\beta,\mu}$. Moreover, the following useful identity holds:

$$\Omega_{[\mu}{}^{ab}{}_{\nu]} = T_{\mu\nu}{}^{ab} \tag{2.4}$$

Now we introduce a curvature tensor for Ω field:

$$R_{\mu\nu}{}^{ab,c} = \partial_{[\mu}\Omega_{\nu]}{}^{ab,c}, \qquad R_\mu{}^{a,b} = R_{\mu\nu}{}^{a\nu,b}, \quad R^a = R_{\mu\nu}{}^{a\mu,\nu} = -R_\mu{}^{a,\mu}, \quad R_\mu{}^{\mu,a} = 0 \tag{2.5}$$

By construction it satisfies usual differential identities:

$$\partial_{[\mu} R_{\nu\alpha]}{}^{ab,c} = 0 \quad \Longrightarrow \quad \partial_\mu R_{\nu\alpha}{}^{\mu b,c} = -\partial_{[\nu} R_{\alpha]}{}^{b,c}, \quad 2\partial_\mu R_\nu{}^{a,\mu} + \partial_\mu R_\nu{}^{\mu,a} = -\partial_\nu R^a \tag{2.6}$$

Also, as a consequence of $\Omega_{[\mu}{}^{ab}{}_{\nu]} = T_{\mu\nu}{}^{ab}$, we obtain:

$$R_{[\mu\nu}{}^{ab}{}_{\alpha]} = \partial_{[\mu}\Omega_\nu{}^{ab}{}_{\alpha]} = \partial_{[\mu} T_{\nu\alpha]}{}^{ab} = 0 \quad \Longrightarrow \quad R_{[\mu}{}^a{}_{\nu]} = 0 \tag{2.7}$$

As can be easily seen from the Lagrangian, dynamical equations (i.e. equations for physical field $\Phi_\mu{}^{ab}$) can be written in terms of this curvature tensor. Direct calculations give us:

$$E_{\mu,ab} = \frac{\delta \mathcal{L}_0}{\delta \Phi_\mu{}^{ab}} = -\frac{2}{3}[R_{a,b,\mu} + R_{b,a,\mu} + R_{a,\mu,b}] - \frac{1}{3}[g_{\mu a} R_b + g_{\mu b} R_a] \tag{2.8}$$

The invariance of these equations under the $\delta\Phi_\mu{}^{ab} = \partial_\mu \xi^{ab} + \eta^{ab}{}_\mu$ gauge transformations is related with appropriate identities:

$$\partial^\mu E_\mu{}^{ab} = 0, \qquad 2E_{a,bc} - E_{(b,c)a} + \frac{1}{(d-1)}[2g_{bc}E_a - g_{a(b}E_{c)}] = 0 \tag{2.9}$$

where $E^a = E_\mu{}^{\mu a}$.

Curvature $R_{\mu\nu}{}^{ab,c}$ is invariant under ξ^{ab} and $\eta^{ab,c}$ transformations, but not under the $\zeta^{ab,cd}$ ones. So we proceed by introducing a so called extra field $\Sigma_\mu{}^{ab,cd}$ which has the same properties on local indices as parameter $\zeta^{ab,cd}$ and will play a role of gauge field for this transformations:

$$\delta\Sigma_\mu{}^{ab,cd} = \partial_\mu \zeta^{ab,cd} \tag{2.10}$$

Besides, we will require that the following identity holds:

$$\Sigma_{[\mu}{}^{ab,c}{}_{\nu]} \approx R_{\mu\nu}{}^{ab,c}$$

where "≈" means "on-shell". This requirement together with symmetry properties and the form of gauge transformations completely and unambiguously fix the solution for $\Sigma_\mu{}^{ab,cd}$ in terms of $R_{\mu\nu}{}^{ab,c}$. By straightforward but rather lengthy calculations we obtain:

$$\begin{aligned}
\Sigma_\rho{}^{ab,cd} &= \frac{1}{4}[R_\rho{}^{a,cd,b} + R_\rho{}^{b,cd,a} + R_\rho{}^{c,ab,d} + R_\rho{}^{d,ab,c}] + \\
&+ \frac{1}{12}[R^{ac}{}_\rho{}^{[b,d]} + R^{bc}{}_\rho{}^{[a,d]} + R^{ad}{}_\rho{}^{[b,c]} + R^{bd}{}_\rho{}^{[a,c]}] + \\
&- \frac{1}{2(d-2)}[2g^{ab}E_\rho{}^{cd} + 2g^{cd}E_\rho{}^{ab} - g^{ac}E_\rho{}^{bd} - g^{ad}E_\rho{}^{bc} - g^{bc}E_\rho{}^{ad} - g^{bd}E_\rho{}^{ac}] - \\
&- \frac{1}{(d-1)^2(d-2)}[2g^{ab}g^{cd} - g^{ac}g^{bd} - g^{ad}g^{bc}]E_\rho + \\
&+ \frac{1}{2(d-1)(d-2)}[(2g^{ab}e_\rho{}^{(c} - e_\rho{}^b g^{a(c} - e_\rho{}^a g^{b(c})E^{d)} + (ab \leftrightarrow cd)] \qquad (2.11)
\end{aligned}$$

In this, the exact form of algebraic identity (that will be heavily used in what follows) looks as follows:

$$\begin{aligned}
\Sigma_{[\mu}{}^{ab,c}{}_{\nu]} &= R_{\mu\nu}{}^{ab,c} + \frac{1}{2(d-2)}\Big[2e_{[\mu}{}^c E_{\nu]}{}^{ab} - e_{[\mu}{}^{(a}E_{\nu]}{}^{b)c} + \qquad (2.12) \\
&+ \frac{2}{(d-1)^2}[2g^{ab}e_{[\mu}{}^c E_{\nu]} - e_{[\mu}{}^{(a}g^{b)c}E_{\nu]}] - \frac{3}{(d-1)}e_{[\mu}{}^c e_{\nu]}{}^{(a}E^{b)}\Big]
\end{aligned}$$

At last we introduce a truly gauge invariant tensor — curvature for the Σ field:

$$\mathcal{R}_{\mu\nu}{}^{ab,cd} = \partial_{[\mu}\Sigma_{\nu]}{}^{ab,cd} \qquad (2.13)$$

Apart from being invariant under all ξ^{ab}, $\eta^{ab,c}$ and $\zeta^{ab,cd}$ gauge transformations, this tensor has one more very important property. Namely, its contraction vanish on-shell and can be expressed through the first derivatives of dynamical equations. By straightforward calculations (where all identities given above were heavily used) we obtain:

$$\begin{aligned}
\mathcal{R}_{\mu\nu}{}^{ab,c\nu} &= -\frac{(d-3)}{2(d-2)}\Big[2\partial^c E_\mu{}^{ab} - \partial^{(a}E_\mu{}^{b)c} + \frac{1}{(d-1)}(2g^{ab}(\partial E)_\mu{}^c - g^{c(a}(\partial E)_\mu{}^{b)}) - \\
&- \frac{1}{(d-1)^2}(2g^{ab}\partial_\mu E^c - g^{c(a}\partial_\mu E^{b)}) + \frac{2}{(d-1)^2}(2g^{ab}\partial^c E_\mu - g^{c(a}\partial^{b)}E_\mu) + \\
&+ \frac{1}{(d-1)}(e_\mu{}^c\partial^{(a}E^{b)} - 2e_\mu{}^{(a}\partial^c E^{b)} + e_\mu{}^{(a}\partial^{b)}E^c) - \\
&- \frac{1}{(d-1)^2}(2e_\mu{}^c g^{ab} - e_\mu{}^{(a}g^{b)c})(\partial E)\Big] \qquad (2.14)
\end{aligned}$$

where $(\partial E)_\mu{}^a = \partial_b E_\mu{}^{ab}$.

2.2 Spin 2

Frame-like description of massless spin 2 particle is very well known. We need main physical one form $h_\mu{}^a$ as well as auxiliary one form $\omega_\mu{}^{ab}$ antisymmetric on its local indices. In a flat Minkowski space the free Lagrangian can be written as follows:

$$\mathcal{L}_0 = \frac{1}{2}\left\{ {}^{\mu\nu}_{ab} \right\} \omega_\mu{}^{ac}\omega_\nu{}^{bc} - \frac{1}{2}\left\{ {}^{\mu\nu\alpha}_{abc} \right\} \omega_\mu{}^{ab}\partial_\nu h_\alpha{}^c \qquad (2.15)$$

This Lagrangian is invariant under the following gauge transformations:

$$\delta h_\mu{}^a = \partial_\mu \xi^a + \eta_\mu{}^a, \qquad \delta\omega_\mu{}^{ab} = \partial_\mu \eta^{ab} \tag{2.16}$$

In what follows we will need a solution for the algebraic equation for the ω field. It can be easily found to be:

$$\omega_{a,bc} = \frac{1}{2}[T_{ab,c} - T_{ac,b} - T_{bc,a}] \quad \Longrightarrow \quad \omega_{[\mu,\nu]}{}^a = T_{\mu\nu}{}^a \tag{2.17}$$

where we have introduced torsion two form $T_{\mu\nu}{}^a = \partial_\mu h_\nu{}^a - \partial_\nu h_\mu{}^a$, which is invariant under the ξ^a transformations (but not under the η^{ab} ones).

Then we introduce curvature tensor for the ω field

$$R_{\mu\nu}{}^{ab} = \partial_\mu \omega_\nu{}^{ab} - \partial_\nu \omega_\mu{}^{ab}, \qquad R_{a,b} = R_{ac,b}{}^c \tag{2.18}$$

which is invariant both under ξ^a and η^{ab} transformations. By construction it satisfies usual differential identity:

$$\partial_{[\mu} R_{\nu\alpha]}{}^{ab} = 0 \quad \Longrightarrow \quad \partial_\alpha R_{\mu\nu}{}^{\alpha a} = \partial_{[\mu} R_{\nu]}{}^a \tag{2.19}$$

Besides, as a consequence of $\omega_{[\mu,\nu]}{}^a = T_{\mu\nu}{}^a$ we have algebraic identity:

$$R_{[\mu\nu,\alpha]}{}^a = \partial_{[\mu} \omega_{\nu,\alpha]}{}^a = \partial_{[\mu} T_{\nu\alpha]}{}^a = 0 \quad \Longrightarrow \quad R_{[\mu,\nu]} = 0 \tag{2.20}$$

2.3 Spin 1

For the description of spin 1 particle we will also use frame-like (i.e. first order) formalism. We introduce main physical one form A_μ and auxiliary antisymmetric second rank tensor F^{ab}. The free Lagrangian then has the form:

$$\mathcal{L}_0 = \frac{1}{8} F_{ab}{}^2 - \frac{1}{4} \left\{ {}^{\mu\nu}_{ab} \right\} F^{ab} \partial_\mu A_\nu \tag{2.21}$$

Solution of algebraic equations for F^{ab} field gives us:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad \Longrightarrow \quad \partial_{[\mu} F_{\nu\alpha]} = 0 \tag{2.22}$$

2.4 Spin 0

Similarly, for the description of spin 0 particle we introduce physical scalar φ and auxiliary vector π^a. The free Lagrangian looks like:

$$\mathcal{L}_0 = -\frac{1}{2} \pi_a{}^2 + \left\{ {}^{\mu}_{a} \right\} \pi^a \partial_\mu \varphi \tag{2.23}$$

and by solving algebraic equations for the π^a we obtain:

$$\pi_\mu = \partial_\mu \varphi \quad \Longrightarrow \quad \partial_{[\mu} \pi_{\nu]} = 0 \tag{2.24}$$

3 Cubic vertices

In all investigations of massless particles interactions gauge invariance plays a crucial role. Not only it determines a kinematic structure of free theory and guarantees a right number of physical degrees of freedom, but also to a large extent it fixes all possible interactions of such particles. This leads, in particular, to formulation of so-called constructive approach to investigation of massless particles models [3, 7, 32–40]. In this approach one starts with free Lagrangian for the collection of massless fields with appropriate gauge transformations and tries to construct interacting Lagrangian and modified gauge transformations iteratively by the number of fields so that:

$$\mathcal{L} \sim \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \dots, \qquad \delta \sim \delta_0 + \delta_1 + \delta_2 + \dots$$

where $\mathcal{L}_1$ — cubic vertex, $\mathcal{L}_2$ — quartic one and so on, while δ_1 — corrections to gauge transformations linear in fields, δ_2 — quadratic in fields and so on.

In a frame-like formalism it means that one starts with the free Lagrangian $\mathcal{L}_0$ containing physical Φ and auxiliary Ω fields and their initial gauge transformations $\delta_0\Phi$ and $\delta_0\Omega$ such that:

$$\frac{\delta\mathcal{L}_0}{\delta\Phi}\delta_0\Phi + \frac{\delta\mathcal{L}_0}{\delta\Omega}\delta_0\Omega = 0$$

Then in the first non-trivial approximation one has to achieve:

$$\frac{\delta\mathcal{L}_1}{\delta\Phi}\delta_0\Phi + \frac{\delta\mathcal{L}_1}{\delta\Omega}\delta_0\Omega + \frac{\delta\mathcal{L}_0}{\delta\Phi}\delta_1\Phi + \frac{\delta\mathcal{L}_0}{\delta\Omega}\delta_1\Omega = 0$$

From one hand one can use honest first order formalism here treating both Φ and Ω as independent fields. But this requires a lot of calculations including corrections to gauge transformations of auxiliary field Ω which often turn out to be the most complicated ones. At the other hand in frame-like formulation of gravity and supergravity there is a well known 1 and $\frac{1}{2}$ order formalism. Here one takes into account variations of physical field Φ only but all calculations are made on the solutions of complete algebraic equations for auxiliary field Ω:

$$\left[\frac{\delta\mathcal{L}_1}{\delta\Phi}\delta_0\Phi + \frac{\delta\mathcal{L}_0}{\delta\Phi}\delta_1\Phi\right]_{\frac{\delta(\mathcal{L}_0+\mathcal{L}_1)}{\delta\Omega}=0} = 0$$

Thus there is no need to consider corrections to Ω field gauge transformations but one has to solve non-linear equations for this field and this can be a non-trivial task. In this paper we will use modified 1 and $\frac{1}{2}$ order formalism very well suited namely for investigations of cubic vertices:

$$\left[\frac{\delta\mathcal{L}_1}{\delta\Phi}\delta_0\Phi + \frac{\delta\mathcal{L}_1}{\delta\Omega}\delta_0\Omega + \frac{\delta\mathcal{L}_0}{\delta\Phi}\delta_1\Phi\right]_{\frac{\delta\mathcal{L}_0}{\delta\Omega}=0} = 0$$

Here also there is no need to consider corrections to Ω field gauge transformations but we have to make all calculations on the solutions of free Ω field equations only. And the very same solutions of free Ω field equations will be used in investigations of different cubic vertices. Note at last that we will use the same strategy for the extra field Σ as well.

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3.1 Vertex 3-0-0

One of the simplest examples of cubic vertices for spin 3 particle is a three derivatives 3-0-0 vertex [41] (see also [14, 18, 40, 42, 43]). As is known, to construct such a vertex one needs at least two different spin 0 fields, the vertex being antisymmetric on them.

Let us first consider this vertex in a metric-like formalism. We introduce completely symmetric third rank tensor $\Phi_{\mu\nu\alpha}$ with gauge transformations:

$$\delta\Phi_{\mu\nu\alpha} = \partial_{(\mu}\xi_{\nu\alpha)}, \qquad \xi_{\mu\nu} = \xi_{\nu\mu}, \qquad \xi_{\mu\mu} = 0$$

and a pair of scalars φ^i, $i = 1, 2$. Then the most general ansatz for the vertex can be written as follows:

$$\begin{aligned} \mathcal{L}_1 &= \varepsilon^{ij}\Phi^{\mu\nu\alpha}[a_1\partial_{\mu\nu\alpha}\varphi^i\varphi^j + a_2\partial_{\mu\nu}\varphi^i\partial_\alpha\varphi^j] + \\ &+\varepsilon^{ij}\tilde\Phi^\mu[a_3\partial^2\partial_\mu\varphi^i\varphi^j + a_4\partial^2\varphi^i\partial_\mu\varphi^j + a_5\partial_{\mu\beta}\varphi^i\partial_\beta\varphi^j] \end{aligned} \tag{3.1}$$

where $\tilde\Phi_\mu = \Phi_{\mu\nu}{}^\nu$, $\partial_{\mu\nu} = \partial_\mu\partial_\nu$ and so on. To compensate a non-invariance of this vertex under the $\xi_{\mu\nu}$ gauge transformations we have to consider all possible transformations for scalar fields with two derivatives. The most general ansatz looks like:

$$\delta\varphi^i = \varepsilon^{ij}[\alpha_1\xi^{\mu\nu}\partial_{\mu\nu}\varphi^j + \alpha_2(\partial\xi)^\mu\partial_\mu\varphi^j + \alpha_3(\partial\partial\xi)\varphi^j] \tag{3.2}$$

Recall that in any case where the number of derivatives in the interaction Lagrangian is greater or equal to that in a free Lagrangian one always has a possibility to make field redefinitions. In this, all interacting Lagrangians related by such redefinitions are physically equivalent, so one can freely use this freedom to simplify Lagrangian and/or gauge transformations. In the case at hands such redefinitions have the following form:

$$\Phi_{\mu\nu\alpha} \Longrightarrow \Phi_{\mu\nu\alpha} + \kappa_1\varepsilon^{ij}g_{(\mu\nu}\varphi^i\partial_{\alpha)}\varphi^j, \qquad \varphi^i \Longrightarrow \varphi^i + \varepsilon^{ij}[\kappa_2\tilde\Phi^\mu\partial_\mu\varphi^j + \kappa_3(\partial\tilde\Phi)\varphi^j]$$

We use this redefinitions to set $a_1 = 0$, $\alpha_2 = \alpha_3 = 0$. Then the requirement that the Lagrangian be invariant under the gauge transformations (in the linear approximation) gives us:

$$\begin{aligned} \mathcal{L}_1 &= \frac{\alpha_0}{6}\varepsilon^{ij}[-2\Phi^{\mu\nu\alpha}\partial_{\mu\nu}\varphi^i\partial_\alpha\varphi^j + \tilde\Phi^\mu(2\partial^2\varphi^i\partial_\mu\varphi^j + \partial_{\mu\alpha}\varphi^i\partial_\alpha\varphi^j)] \\ \delta\varphi^i &= \alpha_0\varepsilon^{ij}\xi^{\mu\nu}\partial_{\mu\nu}\varphi^j \end{aligned} \tag{3.3}$$

Now let us reconstruct this vertex in a frame-like formalism. In this case the ansatz for interacting Lagrangian can be written as follows:

$$\mathcal{L}_1 = \varepsilon^{ij}\left\{ {}^{\mu\nu}_{ab} \right\}\Phi_\mu{}^{ac}(a_1\partial_\nu\pi^{b,i}\pi^{c,j} + a_2\partial_\nu\pi^{c,i}\pi^{b,j}) \tag{3.4}$$

But now we have to take care on two gauge transformations $\delta\Phi_\mu{}^{ab} = \partial_\mu\xi^{ab} + \eta^{ab}{}_\mu$. It is easy to check that this Lagrangian is invariant under $\eta^{ab,c}$ transformations provided $a_1 = 2a_2$. Then the non-invariance of the Lagrangian under the ξ^{ab} transformations can be compensated by appropriate transformations of scalar fields:

$$\delta\varphi^i = -3a_2\varepsilon^{ij}\xi^{ab}\partial_a\pi_b{}^j \tag{3.5}$$

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3.2 Vertex 3-1-1

Similarly to the previous case to construct such vertex [14, 41] we need three derivatives and at least two different spin 1 particles, the vertex being antisymmetric on them.

Let us consider metric-like formalism first. In this case the most general ansatz for the Lagrangian and gauge transformations turns out to be rather complicated. At the same time there exists a lot of possible field redefinitions. We have explicitly checked that by using these redefinitions one can bring the Lagrangian into the form which is trivially invariant under the vector field gauge transformations $\delta A_\mu{}^i = \partial_\mu \lambda^i$, so that vector fields enter the Lagrangian and gauge transformations through gauge invariant field strengths $A_{\mu\nu}{}^i = \partial_{[\mu} A_{\nu]}{}^i$ only. In this case the most general such Lagrangian and gauge transformations can be written in the following form:

$$\mathcal{L}_1 = \varepsilon^{ij}[a_1 \Phi^{\mu\nu\alpha}\partial_\mu A_{\nu\beta}{}^i A_{\alpha\beta}{}^j + a_2 \tilde{\Phi}^\mu \partial_\mu A_{\alpha\beta}{}^i A_{\alpha\beta}{}^j + a_3 \tilde{\Phi}^\mu (\partial A)_\beta{}^i A_{\mu\beta}{}^j] \tag{3.6}$$

$$\delta A_\mu{}^i = \varepsilon^{ij}[\alpha_1 \xi^{\alpha\beta}\partial_\alpha A_{\beta\mu}{}^j + \alpha_2 \xi_{\mu\alpha}(\partial A)_\alpha{}^j + \alpha_3 \partial_\alpha \xi_{\beta\mu} A_{\alpha\beta}{}^j + \alpha_4 (\partial\xi)^\alpha A_{\alpha\mu}{}^j] \tag{3.7}$$

Note that the transformation with parameter α_2 is a so called trivial symmetry, i.e. just a symmetry of free Lagrangian not related with any non-trivial interactions. Note also that we still have one possible field redefinition of the form:

$$A_\mu{}^i \Longrightarrow A_\mu{}^i + \kappa\varepsilon^{ij}\tilde{\Phi}^\alpha A_{\alpha\mu}{}^j$$

We use this freedom to set $\alpha_4 = 0$. Then the requirement that the Lagrangian be invariant under the gauge transformations (in linear approximation) leads to the following result:

$$\begin{aligned} \mathcal{L}_1 &= \frac{a_0}{4}\varepsilon^{ij}[-2\Phi^{\mu\nu\alpha}\partial_\mu A_{\nu\beta}{}^i A_{\alpha\beta}{}^j + \tilde{\Phi}^\mu \partial_\mu A_{\alpha\beta}{}^i A_{\alpha\beta}{}^j + 4\tilde{\Phi}^\mu (\partial A)_\beta{}^i A_{\mu\beta}{}^j] \\ \delta A_\mu{}^i &= a_0 \varepsilon^{ij}[3\xi^{\alpha\beta}\partial_\alpha A_{\beta\mu}{}^j + \partial_\alpha \xi_{\beta\mu} A_{\alpha\beta}{}^j] \end{aligned} \tag{3.8}$$

Now let us reconstruct this vertex in a frame-like formalism. In this case the most general ansatz has the form:

$$\mathcal{L}_1 = \varepsilon^{ij} \left\{ {}^{\mu\nu}_{ab} \right\} [a_1 \Phi_\mu{}^{cd}\partial_\nu F^{ac,i} F^{bd,j} + a_2 \Phi_\mu{}^{ac}\partial_\nu F^{bd,i} F^{cd,j} + a_3 \Phi_\mu{}^{ac}\partial_\nu F^{cd,i} F^{bd,j}] \tag{3.9}$$

Again we have to take care on two transformations with parameters ξ^{ab} and $\eta^{ab,c}$. By straightforward calculations it easy to check that if we set $a_2 = 2a_1$ and $a_3 = a_1$ then the non-invariance of the Lagrangian can be compensated by the following transformations for vector fields:

$$\delta A_\mu{}^i = a_1 \varepsilon^{ij}[3\xi^{ab}\partial_a F_{b\mu}{}^j - \eta_{\mu a,b} F^{ab,j}] \tag{3.10}$$

3.3 Vertex 3-2-2

In a metric-like formalism such cubic 3-2-2 vertex with three derivatives has been constructed in [8]. As in both previous cases its construction requires at least two different spin 2 particles. In metric-like formalism such vertex turns out to be very complicated, so we will not reproduce these results here.

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Let us try to reconstruct this vertex in a frame-like formalism. Results of metric like formalism, obtained in [8], suggests the following form of the Lagrangian and gauge transformations:

$$\mathcal{L}_1 \sim \Phi R\omega \oplus \Omega\omega\omega, \qquad \delta h \sim R\xi \oplus \omega\eta_3 \oplus \Omega\eta_2, \qquad \delta\Phi \sim \omega\eta_2$$

Here Φ and Ω — physical and auxiliary fields for spin 3 particle, ω and R — Lorentz connection and curvatute tensor for spin 2 particle, while η_3 and η_2 — $\eta^{ab,c}$ and η^{ab} correspondingly.

Let us consider $\Phi R\omega$ terms. The most general ansatz appears to be very simple:

$$\mathcal{L}_1 = \varepsilon^{ij} \left\{ {}^{\mu\nu\alpha\beta}_{abcd} \right\} \Phi_\mu{}^{ae}[a_1 R_{\nu\alpha}{}^{be,i}\omega_\beta{}^{cd,j} + a_2 R_{\nu\alpha}{}^{bc,i}\omega_\beta{}^{de,j}] \tag{3.11}$$

Due to well known identity $\partial_{[\mu}R_{\nu\alpha]}{}^{ab} = 0$ variation of this Lagrangian under $\delta\Phi_\mu{}^{ab} = \partial_\mu\xi^{ab}$ gauge transformations gives us terms of the form ξRR only:

$$\delta_\xi \mathcal{L}_1 = (a_2 - a_1)\varepsilon^{ij} \left\{ {}^{\mu\nu\alpha\beta}_{abcd} \right\} \xi^{ae} R_{\mu\nu}{}^{be,i} R_{\alpha\beta}{}^{cd,j} = 8(a_1 - a_2)\varepsilon^{ij}[R_{ab}{}^i - \frac{1}{2}g_{ab}R^i]R_{ac,bd}{}^j\xi^{cd}$$

where the last form was obtained using $R_{[\mu\nu,\alpha]}{}^a = 0$ and such terms can be compensated by $\delta h \sim R\xi$ transformations (see below).

Now we introduce all possible terms of the form $\Omega\omega\omega$:

$$\mathcal{L}_2 = \varepsilon^{ij} \left\{ {}^{\mu\nu\alpha}_{abc} \right\} [b_1 \Omega_\mu{}^{ad,b}\omega_\nu{}^{ce,i}\omega_\alpha{}^{de,j} + b_2\Omega_\mu{}^{ad,e}\omega_\nu{}^{bc,i}\omega_\alpha{}^{de,j} + b_3\Omega_\mu{}^{ad,e}\omega_\nu{}^{bd,i}\omega_\alpha{}^{ce,j}] \tag{3.12}$$

First of all it is easy to check that at $b_3 = -2b_2$ such Lagrangian is invariant under the $\delta\Omega_\mu{}^{ab,c} = \zeta^{ab,c}{}_\mu$ transformations. So we proceed and consider $\delta\Phi_\mu{}^{ab} = \eta^{ab}{}_\mu$, $\delta\Omega_\mu{}^{ab,c} = \partial_\mu\eta^{ab,c}$ transformations. Both Lagrangians give contributions of the form $\eta_3 R\omega$. Moreover, if we set

$$b_1 = 2a_2, \qquad b_2 = -a_2, \qquad a_1 = -2a_2$$

then these variations are reduced to the form:

$$\delta_{\eta_3}(\mathcal{L}_1 + \mathcal{L}_2) = -8a_2\varepsilon^{ij}[R_{ab}{}^i - \frac{1}{2}g_{ab}R^i]\omega_a{}^{cd,j}\eta^{bc,d}$$

and can be compensated by $\delta h \sim \omega\eta_3$ transformations.

We have no free parameters left but we still have to take care on $\delta\omega_\mu{}^{ab} = \partial_\mu\eta^{ab}$ transformations which give us terms of two types. The first ones — $R\Omega\eta_2$ happily combine into:

$$8a_2\varepsilon^{ij}[R_{ab}{}^i - \frac{1}{2}g_{ab}R^i]\Omega_a{}^{bc,d}\eta^{cd,j}$$

and can be compensated by $\delta h \sim \Omega\eta_2$ transformations. At the same time variations of the second type $\partial\Omega\omega\eta_2$ can be compensated by corrections to $\Phi_\mu{}^{ab}$ transformations (recall that dynamical equations for Φ field are related with curvature tensor for Ω field):

$$\delta\Phi_\mu{}^{ab} = 6a_2\varepsilon^{ij}[\omega_\mu{}^{c(a,i}\eta^{b)c,j} - Tr]$$

Collecting all pieces together we obtain finally the Lagrangian:

$$\begin{aligned}\mathcal{L} &= a_0\varepsilon^{ij}\left\{ \begin{smallmatrix}\mu\nu\alpha\beta \\ abcd\end{smallmatrix}\right\} \Phi_\mu{}^{ae}[-2R_{\nu\alpha}{}^{be,i}\omega_\beta{}^{cd,j} + R_{\nu\alpha}{}^{bc,i}\omega_\beta{}^{de,j}] + \\ &+a_0\varepsilon^{ij}\left\{ \begin{smallmatrix}\mu\nu\alpha \\ abc\end{smallmatrix}\right\} [2\Omega_\mu{}^{ad,b}\omega_\nu{}^{ce,i}\omega_\alpha{}^{de,j} - \Omega_\mu{}^{ad,e}\omega_\nu{}^{bc,i}\omega_\alpha{}^{de,j} + 2\Omega_\mu{}^{ad,e}\omega_\nu{}^{bd,i}\omega_\alpha{}^{ce,j}]\end{aligned} \tag{3.13}$$

as well as corresponding corrections to gauge transformations:

$$\begin{aligned}\delta h_{\mu b}{}^i &= 8a_0\varepsilon^{ij}[3R_{\mu c,bd}{}^j\xi^{cd} + \omega_\mu{}^{cd,j}\eta^{bc,d} - \Omega_\mu{}^{bc,d}\eta^{cd,j}] \\ \delta\Phi_\mu{}^{ab} &= 6a_0\varepsilon^{ij}[\omega_\mu{}^{c(a,i}\eta^{b)c,j} - Tr]\end{aligned} \tag{3.14}$$

One more important requirement for the consistency of this vertex is that the algebra of gauge transformations has to be closed. Due to simple structure of results obtained it is an easy task to check that in this case algebra is indeed closed (in the lowest order):

$$\begin{aligned}&[\delta_1,\delta_2]h_\mu{}^{a,i} = \partial_\mu\tilde{\xi}^{a,i} + \tilde{\eta}_\mu{}^{a,i}, \qquad \tilde{\xi}^{a,i} = 8a_0\varepsilon^{ij}\eta^{bc,j}\eta^{ab,c}, \qquad \tilde{\eta}^{ab,i} = -8a_0\varepsilon^{ij}\zeta^{ac,bd}\eta^{cd,j} \\ &[\delta_1,\delta_2]\Phi_\mu{}^{ab} = \partial_\mu\tilde{\xi}^{ab}, \qquad \tilde{\xi}^{ab} = 6a_0\varepsilon^{ij}\eta_1{}^{ac,i}\eta_2{}^{bc,j} - Tr - (1\leftrightarrow 2)\end{aligned}$$

3.4 Vertex 3-3-2

In a metric-like formalism a cubic vertex 3-3-2 with four derivatives has been constructed in [8] (see also [6, 7]). Again the results in a metric like formalism appear to be very complicated so we will not reproduce them here.

Let us try to reconstruct this vertex in a frame-like formalism. The structure of results obtained suggests the following general structure for the Lagrangian and gauge transformations:

$$\mathcal{L} \sim \Omega\Omega R, \qquad \delta h \sim \Sigma\eta \oplus \Omega\zeta, \qquad \delta\Phi \sim R\eta$$

Note that the spin 2 field enter through the curvature tensor only so the Lagrangian is trivially invariant under its gauge transformations. The most general ansatz for such vertex has the following form:

$$\begin{aligned}\mathcal{L}_1 = \left\{ \begin{smallmatrix}\mu\nu\alpha\beta \\ abcd\end{smallmatrix}\right\} [&a_1\Omega_\mu{}^{ae,f}\Omega_\nu{}^{be,f}R_{\alpha\beta}{}^{cd} + a_2\Omega_\mu{}^{ef,a}\Omega_\nu{}^{ef,b}R_{\alpha\beta}{}^{cd} + a_3\Omega_\mu{}^{ae,b}\Omega_\nu{}^{cf,d}R_{\alpha\beta}{}^{ef} + \\ &+a_4\Omega_\mu{}^{ae,b}\Omega_\nu{}^{ce,f}R_{\alpha\beta}{}^{df} + a_5\Omega_\mu{}^{ae,b}\Omega_\nu{}^{ef,c}R_{\alpha\beta}{}^{df}]\end{aligned} \tag{3.15}$$

By construction this Lagrangian is invariant under the ξ^{ab} transformations so we have to take care on $\eta^{ab,c}$ and $\zeta^{ab,cd}$ transformations only. Let us begin with the $\delta\Omega_\mu{}^{ab,c} = \zeta^{ab,c}{}_\mu$ transformations. By straightforward calculations one can show that at:

$$a_1 = 2a_0, \qquad a_2 = -5a_0, \qquad a_3 = -4a_0, \qquad a_4 = 16a_0, \qquad a_5 = 8a_0$$

corresponding variations of the Lagrangian are reduced to a simple form:

$$\delta_\zeta\mathcal{L}_1 = -48a_0\left[R_{ab} - \frac{1}{2}g_{ab}R\right]\Omega_a{}^{cd,e}\zeta^{cd,eb}$$

and can be compensated by $\delta h \sim \Omega\zeta$ transformations (see below).

Let us turn to the $\delta\Omega_\mu{}^{ab,c} = \partial_\mu \eta^{ab,c}$ transformations. We have no free parameters left, nevertheless by rather lengthy calculations we can show that all such variations can be compensated by $\delta\Phi \sim R\eta$ and $\delta h \sim \Sigma\eta$ transformations. Thus we obtain:

$$\begin{aligned}
\delta\Phi_\mu{}^{ab} &= -72a_0 \Big[R_{\mu\nu}{}^{ac}\eta^{\nu c,b} + \frac{1}{6(d-1)} e_\mu{}^{(a} R^{b)c,de}\eta^{cd,e} + \\
&\qquad + \frac{1}{(d-2)} R_\mu{}^c \eta^{ab,c} + \frac{1}{2(d-2)^2} R_{cd}\eta^{cd,(a} e_\mu{}^{b)} \Big] \qquad (3.16) \\
\delta h_\mu{}^a &= 48a_0 [\Omega_\mu{}^{cd,b}\zeta^{ab,cd} - \Sigma_\mu{}^{ab,cd}\eta^{cd,b}]
\end{aligned}$$

Again due to a simple structure of gauge transformations it is an easy task to see that the algebra of gauge transformations is closed:

$$[\delta_1, \delta_2] h_\mu{}^a = \partial_\mu \tilde{\xi}^a + \tilde{\eta}_\mu{}^a, \qquad \tilde{\xi}^a = 48a_0\zeta^{ab,cd}\eta^{cd,b}, \qquad \tilde{\eta}^{ab} = 48a_0\zeta_1^{ac,de}\zeta_2^{bc,de} - (1 \leftrightarrow 2)$$

3.5 Vertex 3-2-1

As far as we know cubic vertex 3-2-1 with four derivatives has not been considered earlier.[1] Our analysis of this vertex in a metric-like formalism (which we will not reproduce here due to its complexity) showed that by using possible field redefinitions one can always bring this vertex into the form that is trivially invariant under the spin 2 and spin 1 gauge transformations so that these fields enter the Lagrangian and gauge transformations through curvature tensor and field strength correspondingly. This in turn suggests the following general structure for the Lagrangian and gauge transformations in a frame like formalism:

$$\mathcal{L} \sim \Omega R F, \qquad \delta h \sim \partial F \eta, \qquad \delta A \sim R\eta$$

The most general ansatz for this vertex can be written as follows:

$$\begin{aligned}
\mathcal{L} = \left\{ {}^{\mu\nu\alpha}_{abc} \right\} [& a_1\Omega_\mu{}^{da,b}R_{\nu\alpha}{}^{ce}F^{de} + a_2\Omega_\mu{}^{da,b}R_{\nu\alpha}{}^{de}F^{ce} + a_3\Omega_\mu{}^{ad,e}R_{\nu\alpha}{}^{bc}F^{de} + \\
&+ a_4\Omega_\mu{}^{ad,e}R_{\nu\alpha}{}^{de}F^{bc} + a_5\Omega_\mu{}^{ad,e}R_{\nu\alpha}{}^{bd}F^{ce} + a_6\Omega_\mu{}^{ae,d}R_{\nu\alpha}{}^{bd}F^{ce}
\end{aligned} \qquad (3.17)$$

But due to identity $R_{[\mu\nu,\alpha]}{}^a = 0$ (which holds on the solutions of algebraic equation for the $\omega_\mu{}^{ab}$ field) not all these terms are independent. Namely, there exist combinations of parameters a_1, a_5, a_6 and a_2, a_4 which turn out to be proportional to this identity. In what follows we choose $a_1 = 0$, $a_2 = 0$. Moreover, there exists one possible field redefinition:

$$h_\mu{}^a \Longrightarrow h_\mu{}^a + \kappa\Omega_\mu{}^{ab,c}F^{bc}$$

and we use this freedom to set $a_3 = -a_4/2$.

By construction such vertex is invariant under the $\delta\Phi_\mu{}^{ab} = \partial_\mu\xi^{ab}$ transformations, so we have to take care on $\eta^{ab,c}$ and $\zeta^{ab,cd}$ transformations only. Direct calculations show that

[1]Partial results on this vertex were obtained and used in [6] where gravitational interactions for massive spin 3 particle were investigated.

the vertex will be invariant under the $\delta\Omega_\mu{}^{ab,c} = \zeta^{ab,c}{}_\mu$ transformations provided $a_5 = -2a_4$. At the same time, if we set $a_6 = 2a_5$ then variations of the vertex under the $\delta\Omega_\mu{}^{ab,c} = \partial_\mu\eta^{ab,c}$ transformations can be compensated by appropriate corrections for $h_\mu{}^a$ and A_μ fields. We obtain:

$$\begin{aligned} \mathcal{L} &= \frac{a_0}{2}\left\{ {}^{\mu\nu\alpha}_{abc} \right\} [-\Omega_\mu{}^{ad,e}R_{\nu\alpha}{}^{bc}F^{de} + 2\Omega_\mu{}^{ad,e}R_{\nu\alpha}{}^{de}F^{bc} \\ &\qquad -4\Omega_\mu{}^{ad,e}R_{\nu\alpha}{}^{bd}F^{ce} - 8\Omega_\mu{}^{ae,d}R_{\nu\alpha}{}^{bd}F^{ce}] \end{aligned} \tag{3.18}$$

$$\begin{aligned} \delta A_\mu &= 4a_0 R_{\mu a,bc}\eta^{ab,c}, \\ \delta h_\mu{}^a &= 6a_0\left[\partial_b F_{c\mu}\eta^{bc,a} - \frac{2}{3(d-2)}e_\mu{}^a\partial_b F_{cd}\eta^{bc,d}\right] \end{aligned} \tag{3.19}$$

3.6 Vertex 3-3-1

In a metric-like formalism cubic vertex 2-2-1 with three derivatives of the form $\partial h\partial hF$ as well as its generalization on arbitrary integer spin of the form $\partial^{s-1}\Phi\partial^{s-1}\Phi F$ have been constructed in [7]. In [9, 10] frame-like version of 2-2-1 vertex has been constructed and used in the investigations of electromagnetic interactions for massless and massive spin 2 particles. This vertex has the form:

$$\mathcal{L} = -\frac{a_0}{4}\varepsilon^{ij}\left\{ {}^{\mu\nu}_{ab} \right\} [\omega_\mu{}^{i,cd}\omega_\nu{}^{j,cd}F^{ab} - 2\omega_\mu{}^{i,ab}\omega_\nu{}^{j,cd}F^{cd} + 4\omega_\mu{}^{i,ac}\omega_\nu{}^{j,bd}F^{cd}] \tag{3.20}$$

By construction its invariant under the $\delta h_\mu{}^a = \partial_\mu\xi^a$ transformations, while invariance under the η^{ab} transformations requires appropriate corrections to gauge transformations:

$$\begin{aligned} \delta A_\mu &= a_0\varepsilon^{ij}\omega_\mu{}^{i,ab}\eta^{j,ab} \\ \delta h_\mu{}^{i,a} &= 2a_0\varepsilon^{ij}\left[2F_\mu{}^b\eta^{j,ab} - \frac{1}{(d-2)}e_\mu{}^a(F\eta)^j\right] \end{aligned} \tag{3.21}$$

Results obtained in a metric-like formalism [7] suggest the following general structure of the Lagrangian and gauge transformations in a frame-like version for the case of spin 3 particle:

$$\mathcal{L} \sim \Sigma\Sigma F, \qquad \delta A \sim \Sigma\zeta, \qquad \delta\Phi \sim \partial(F\zeta)$$

Moreover, if we introduce a notation:

$$\hat{\Sigma}_\mu{}^{ab,cd} = \Sigma_\mu{}^{ab,cd} - \Sigma_\mu{}^{cb,ad}$$

then corresponding cubic vertex with five derivatives can be written exactly in the same form as in the case of spin 2:

$$\mathcal{L} = -\frac{a_0}{4}\varepsilon^{ij}\left\{ {}^{\mu\nu}_{ab} \right\} [\hat{\Sigma}_\mu{}^{i,ce,df}\hat{\Sigma}_\nu{}^{j,ce,df}F^{ab} - 2\hat{\Sigma}_\mu{}^{i,ae,bf}\hat{\Sigma}_\nu{}^{j,ce,df}F^{cd} + 4\hat{\Sigma}_\mu{}^{i,ae,cf}\hat{\Sigma}_\nu{}^{j,be,df}F^{cd}] \tag{3.22}$$

By construction such Lagrangian is invariant both under ξ^{ab} and $\eta^{ab,c}$ transformations, while invariance under the $\delta\Sigma_\mu{}^{ab,cd} = \partial_\mu\zeta^{ab,cd}$ transformations requires corresponding corrections:

$$\begin{aligned} \delta A_\mu &= 3a_0\varepsilon^{ij}\Sigma_\mu{}^{i,ab,cd}\zeta^{j,ab,cd} \\ \delta\Phi_\mu{}^{i,ab} &= 18a_0\varepsilon^{ij}\partial^c[F_\mu{}^d\zeta^{j,ab,cd} - Tr] \end{aligned} \tag{3.23}$$

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Again it is trivial to see that the algebra of gauge transformations is closed:

$$[\delta_1, \delta_2] A_\mu = \partial_\mu \lambda, \qquad \lambda = 3a_0 \varepsilon^{ij} {\zeta_1}^{i,ab,cd} {\zeta_2}^{j,ab,cd} - (1 \leftrightarrow 2)$$

4 Conclusion

Thus, we have seen that in a frame-like formalism the Lagrangians for higher derivative non-minimal vertices indeed become much simpler. In this, an important role here plays the fact that such Lagrangians can be written as a product of forms. It is this (almost) coordinate independence that greatly simplifies calculations and, in principle, allows straightforward deformation into $(A)dS$ spaces. Also the structure of gauge transformations turns out to be rather simple and it is almost trivial task to check that the algebra of gauge transformations is closed. At last but not least, in many cases the very structure of the vertex suggests natural generalization on arbitrary spins.

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