

Chern-Simons black holes: scalar perturbations, mass and area spectrum and greybody factors

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ABSTRACT: We study the Chern-Simons black holes in d -dimensions and we calculate analytically the quasi-normal modes of the scalar perturbations and we show that they depend on the highest power of curvature present in the Chern-Simons theory. We obtain the mass and area spectrum of these black holes and we show that they have a strong dependence on the topology of the transverse space and they are not evenly spaced. We also calculate analytically the reflection and transmission coefficients and the absorption cross section and we show that at low frequency limit there is a range of modes which contributes to the absorption cross section.

KEYWORDS: Black Holes, Classical Theories of Gravity

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1 Introduction

Chern-Simons black holes are special solutions of gravity theories in higher than four dimensions which contain higher powers of curvature. These theories are consistent Lanczos-Lovelock theories resulting in second order field equations for the metric with well defined AdS asymptotic solutions. For spherically symmetric topologies, these black holes are labelled by an integer k which specifies the higher order of curvature present in the Lanczos-Lovelock action and it is related to the dimensionality d of spacetime by the relation $d - 2k = 1$ [1]. These solutions were further generalized to flat and hyperbolic topologies [2].

For spherical topologies the Chern-Simons black holes have similar causal structure as the (2+1)-dimensional BTZ black hole [3], and they have positive specific heat and therefore thermodynamical stability. For hyperbolic topologies, the Chern-Simons black holes resemble to the topological black holes [4–8] in their zero mass limit, and their thermodynamic behaviour was studied in [2].

An illustrative example of Chern-Simons black holes is provided by the Gauss-Bonnet theory for $d = 5$ and $k = 2$. Static local solutions of this theory are well studied over the years [9–16]. This theory has two branches of solutions. If there is a fine tuning between k and the Gauss-Bonnet coupling constant α , the two solutions coincide to the Chern-Simons black hole solution which has maximum symmetry. This is known as the Chern-Simons limit (for a review see [17]). The stability of these solutions has also been studied [18–21]. It was found in [22] that one of these solutions suffers from ghost-like instability up to the strongly coupled Chern-Simons limit where linear perturbation theory breaks down. Therefore, to study the stability at the exact Chern-Simons limit we have to go beyond perturbation theory. It was speculated in [22] that Chern-Simons black holes could be a transitional configuration between the two branches of solutions.

In this work we will consider a matter distribution outside the horizon of a Chern-Simons black hole with hyperbolic topology parameterized by a scalar field. We will perturb the scalar field assuming that there is no back reaction on the metric. This will result in the calculation of the quasi-normal modes (QNMs) which are characterized by a spectrum that is independent of the initial conditions of the perturbation and depends only on the black hole parameters and on the fundamental constants of the system (for a recent review see [23]). A novel and interesting feature of our calculation is that the QNMs of the Chern-Simons black holes depend also on the curvature parameter k . As we will discuss, because of the presence of the curvature parameter k as we approach the Chern-Simons limit the curvature singularity becomes milder. This may give detectable signature through the QNMs in the gravitational waves experiments.

Another interesting feature of the QNMs is their connection to thermal conformal field theories. According to the AdS/CFT correspondence [24], classical gravity backgrounds in AdS space are dual to conformal field theories at the boundary (for a review see [25]). Using this principle it was established [26] that the relaxation time of a thermal state of the boundary thermal theory is proportional to the inverse of the imaginary part of the QNMs of the dual gravity background. Therefore, the knowledge of the QNMs spectrum determines how fast a thermal state in the boundary theory will reach thermal equilibrium. We will show that, the rate at which a scalar field in the background of a Chern-Simons black hole will decay or the rate the boundary thermal theory will reach thermal equilibrium, depends on the value of the curvature parameter k .

The QNMs were also studied in relation to the quantum area spectrum of the black hole horizon. Bekenstein [27] was the first who proposed the idea that in quantum gravity the area of black hole horizon is quantized leading to a discrete spectrum which is evenly spaced. An interesting proposal was made by Hod [28] who conjectured that the asymptotic QNM frequency is related to the quantized black hole area. The black hole spectrum can be obtained imposing the Bohr-Sommerfeld quantization condition to an adiabatic invariant quantity involving the energy E and the vibrational frequency $\omega(E)$ [29]. Identifying $\omega(E)$ with the real part ω_R of the QNMs, the Hod's conjecture leads to an expression of the quantized black hole area, which however is not universal for all black hole backgrounds. Furthermore it was argued [30], that in the large damping limit the identification of $\omega(E)$ with the imaginary part of the QNMs could lead to the Bekenstein universal bound [27].

Both approaches have been followed in the literature [31–38]. In this work we will calculate the area spectrum and the mass spectrum of a Chern-Simons black hole with hyperbolic topology. We will show that there is a strong dependence of the spectrums on the hyperbolic geometry and that they are not evenly spaced.

The knowledge of black holes perturbations is also useful in studying the Hawking radiation. The Hawking radiation is a semiclassical effect and it gives the thermal radiation emitted by a black hole. However, black holes do not radiate strictly blackbody type radiation due to the well known frequency dependent greybody factors. These factors arise from frequency-dependent potential barriers outside the horizon which filter the initially blackbody spectrum emanating from the horizon [39]. In asymptotically AdS spacetimes, the light rays can reach spatial infinity and return to the origin in finite time. Black holes

in this kind of spacetimes are in thermal equilibrium with their environment. So, the radiation produced at black hole horizon is all re-absorbed and the radiation which reaches spatial infinity is reflected back, with part of this radiation travelling all the way through to the black hole horizon, and the rest being reflected back to spatial infinity [40].

In the present work, the reflection and the transmission coefficients and the greybody factors of Chern-Simons black holes are computed analytically. We make a numerical analysis of their behaviour in five-dimensions and in the low frequency limit and we found that contrary to the Schwarzschild-AdS case there is a range of modes with high angular momentum which contributes to the absorption cross section.

The paper is organized as follows. In section II we give a brief review of the Chern-Simons theory. In section III we calculate the exact QNMs of the scalar perturbations of d -dimensional Chern-Simons black holes with hyperbolic topology. In section IV using the calculated QNMs, the mass and area spectrum of Chern-Simons black holes are presented. In section V we calculate the reflection and the transmission coefficients and the greybody factors of Chern-Simons black holes. Finally, our conclusions are in section VI.

2 The Chern-Simons theory

The Einstein tensor is the only symmetric and conserved tensor depending on the metric and its derivatives, which is linear in the second derivatives of the metric. The field equations arise from the Einstein-Hilbert action with a cosmological constant Λ . In higher dimensions, the potential problem is to find the most general action that gives rise to a set of second order field equations. The solution to this problems is the Lanczos-Lovelock (LL) action [41]. This action is non linear in the Riemann tensor and differs from the Einstein-Hilbert action only if the spacetime has more than 4 dimensions. Therefore, the Lanczos-Lovelock action is the most natural extension of general relativity in higher dimensional spacetimes. In d -dimensions it can be written as follow

$$I_k = \kappa \int \sum_{q=0}^k c_q^k L^q, \tag{2.1}$$

with

$$L^q = \epsilon_{\alpha_1 \dots \alpha_d} R^{\alpha_1 \alpha_2} \dots R^{\alpha_{2q-1} \alpha_{2q}} e^{\alpha_{2q+1}} \dots e^{\alpha_d}, \tag{2.2}$$

where e^α and $R^{\alpha\beta}$ stand for the vielbein and the curvature two-form respectively and κ and l are related to the gravitational constant G_k and the cosmological constant Λ through

$$\kappa = \frac{1}{2(d-2)! \Omega_{d-2} G_k}, \tag{2.3}$$

$$\Lambda = -\frac{(d-1)(d-2)}{2l^2}, \tag{2.4}$$

and $\alpha_q := c_q^k$ where $c_q^k = \frac{l^{2(q-k)}}{d-2q} \binom{k}{q}$ for $q \leq k$ and vanishes for $q > k$, with $1 \leq k \leq [\frac{d-1}{2}]$ ($[x]$ denotes the integer part of x) and Ω_{d-2} corresponds to the volume of a unit $(d-2)$ -dimensional sphere. Static black hole-like geometries with spherical topology were

found [1] possessing topologically nontrivial AdS asymptotics. These theories and their corresponding solutions were classified by an integer k , which corresponds to the highest power of curvature in the Lagrangian. If $d - 2k = 1$, the solutions are known as Chern-Simons black holes (for a review on the Chern-Simons theories see [42]). These solutions were further generalized to other topologies [2] and they can be described in general by a non-trivial transverse spatial section \sum_γ of $(d - 2)$ -dimensions labelled by the constant $\gamma = +1, -1, 0$ that represents the curvature of the transverse section, corresponding to a spherical, hyperbolic or plane section respectively. The solution describing a black hole in a free torsion theory can be written as [2]

$$ds^2 = -\left(\gamma + \frac{r^2}{l^2} - \alpha\left(\frac{2G_k\mu}{r^{d-2k-1}}\right)^{\frac{1}{k}}\right)dt^2 + \frac{dr^2}{\left(\gamma + \frac{r^2}{l^2} - \alpha\left(\frac{2G_k\mu}{r^{d-2k-1}}\right)^{\frac{1}{k}}\right)} + r^2 d\sigma_\gamma^2, \quad (2.5)$$

where $\alpha = (\pm 1)^{k+1}$ and the constant μ is related to the horizon r_+ through

$$\mu = \frac{r_+^{d-2k-1}}{2G_k} \left(\gamma + \frac{r_+^2}{l^2}\right)^k, \quad (2.6)$$

and to the mass M by

$$\mu = \frac{\Omega_{d-2}}{\Sigma_{d-2}} M + \frac{1}{2G_k} \delta_{d-2k,\gamma}, \quad (2.7)$$

here Σ_{d-2} denotes the volume of the transverse space. As can be seen in (2.5), if $d - 2k \neq 1$ the k root makes the curvature singularity milder than the corresponding black hole of the same mass. At the exact Chern-Simons limit $d - 2k = 1$, the solution has similar structure like the (2+1)-dimensional BTZ black hole with a string-like singularity.

We are merely interested here for the hyperbolic topology with $\gamma = -1$. The reason is that Chern-Simons theories with $\gamma = -1$ always possess a single horizon and have interesting thermodynamic behaviour for even and odd k [2]. In this case $d\sigma_{-1}^2$ in (2.5) is the line element of the $(d - 2)$ -dimensional manifold \sum_{-1} , which is locally isomorphic to the hyperbolic manifold H^{d-2} or pseudosphere, which is a non-compact $(d - 2)$ -dimensional space of constant negative curvature and of the form

$$\sum = H^{d-2}/\Gamma \quad , \quad \Gamma \subset O(d - 2, d - 1), \quad (2.8)$$

where Γ is a freely acting discrete subgroup (i.e., without fixed points) of isometries. This space becomes a compact space of constant negative curvature with genus $g \geq 2$ by identifying, according to the connection rules of the discrete subgroup Γ , the opposite edges of a $4g$ -sided polygon whose sides are geodesics and is centered at the origin of the pseudosphere [4, 43]. In the case of $d = 2$, an octagon is the simplest such polygon, yielding a compact surface of genus $g = 2$ under these identifications. Thus in general, the $(d - 2)$ -dimensional manifold \sum_{-1} is a compact Riemann $(d - 2)$ -surface of genus $g \geq 2$. The configuration (2.5) is an asymptotically locally AdS spacetime.

3 d -dimensional quasi-normal modes of scalar perturbations

To obtain an exact expression for the quasi-normal modes of scalar perturbations of a Chern-Simons black hole in d -dimensions we need to impose boundary conditions on asymptotically AdS spacetime. These are that at the horizon there exist only ingoing waves and the vanishing of the flux of the field at infinity. The metric of Chern-Simons theories is

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2d\sigma_\gamma^2, \quad (3.1)$$

where,

$$f(r) = \gamma + \frac{r^2}{l^2} - \alpha(2\mu G_k)^{\frac{1}{k}}, \quad (3.2)$$

and the horizon is located at

$$r_+ = l\sqrt{\alpha(2\mu G_k)^{\frac{1}{k}} - \gamma}. \quad (3.3)$$

Considering the horizon geometry with a negative curvature constant, $\gamma = -1$, the allowed range of μ for $r_+ \geq 0$ are: if k is odd, $\alpha = 1$, $\mu \geq \frac{-1}{2G_k}$; if k is even and $\alpha = 1$, $\mu \geq 0$ and if k is even and $\alpha = -1$, $\frac{1}{2G_k} \geq \mu \geq 0$ [2]. Performing the change of variables $z = p - l^2/r^2$, where $p = 1 + \alpha(2\mu G_k)^{\frac{1}{k}}$ and $t = lt$, the metric (3.1) can be written

$$ds^2 = -f(z)l^2dt^2 + \frac{l^2}{4f(z)(p-z)^3}dz^2 + \frac{l^2}{p-z}d\sigma_\gamma^2, \quad (3.4)$$

where

$$f(z) = \frac{1 - p^2(1 - \frac{z}{p})}{p(1 - \frac{z}{p})}. \quad (3.5)$$

The horizon (3.3) now is located at $r_+ = l\sqrt{p}$. With the definition $x = 1 - \frac{z}{p}$ the metric (3.4) becomes

$$ds^2 = -\frac{(1-p^2x)l^2}{px}dt^2 + \frac{l^2}{4(1-p^2x)x^2}dx^2 + \frac{l^2}{px}d\sigma_\gamma^2. \quad (3.6)$$

If we define $y = p^2x$ the metric (3.6) can be written as

$$ds^2 = -\frac{(1-y)l^2p}{y}dt^2 + \frac{l^2}{4(1-y)y^2}dy^2 + \frac{l^2p}{y}d\sigma_\gamma^2, \quad (3.7)$$

and introducing $v = 1 - y$ we finally obtain

$$ds^2 = -\frac{l^2pv}{(1-v)}dt^2 + \frac{l^2}{4v(1-v)^2}dv^2 + \frac{l^2p}{1-v}d\sigma_\gamma^2. \quad (3.8)$$

A minimally coupled scalar field to curvature in the background of a Chern-Simons black hole in d -dimensions is given by the Klein-Gordon equation

$$\nabla^2\varphi = m^2\varphi. \quad (3.9)$$

We adopt the ansatz $\varphi = R(v)Y(\Sigma)e^{-i\omega t}$, where Y is a normalizable harmonic function on Σ_{d-2} which satisfies $\nabla^2 Y = -QY$, with ∇^2 the Laplace operator on Σ_{d-2} . The eigenvalues for the hyperbolic manifold are

$$Q = \left(\frac{d-3}{2}\right)^2 + \xi^2. \quad (3.10)$$

Without any identifications of the pseudosphere the spectrum of the angular wave equation is continuous, thus ξ takes any real value $\xi \geq 0$. Since the $(d-2)$ -dimensional manifold Σ is a quotient space of the form H^{d-2}/Γ and it is a compact space of constant negative curvature, the spectrum of the angular wave equation is discretized and thus ξ takes discrete real values $\xi \geq 0$ [43].

Then the radial function $R(v)$ becomes

$$v(1-v)\partial_v^2 R(v) + \left[1 + \left(\frac{d-5}{2}\right)v\right]\partial_v R(v) + \left[\frac{\omega^2}{4pv} - \frac{Q}{4p} - \frac{m^2 l^2}{4(1-v)}\right]R(v) = 0. \quad (3.11)$$

Under the decomposition $R(v) = v^\alpha(1-v)^\beta K(v)$, eq. (3.11) can be written as a hypergeometric equation for K

$$v(1-v)K''(v) + [c - (1+a+b)v]K'(v) - abK(v) = 0. \quad (3.12)$$

Where the coefficients a , b and c are given by

$$a = -\left(\frac{d-3}{4}\right) + \alpha + \beta + \frac{i}{2}\sqrt{\frac{\xi^2}{p} + \left(\frac{d-3}{2}\right)^2\left(\frac{1}{p} - 1\right)}, \quad (3.13)$$

$$b = -\left(\frac{d-3}{4}\right) + \alpha + \beta - \frac{i}{2}\sqrt{\frac{\xi^2}{p} + \left(\frac{d-3}{2}\right)^2\left(\frac{1}{p} - 1\right)}, \quad (3.14)$$

$$c = 1 + 2\alpha, \quad (3.15)$$

where c cannot be an integer and the exponents α and β are

$$\alpha = \pm \frac{i\omega\sqrt{p}}{2p}, \quad (3.16)$$

$$\beta = \beta_\pm = \left(\frac{d-1}{4}\right) \pm \frac{1}{2}\sqrt{\left(\frac{d-1}{2}\right)^2 + m^2 l^2}. \quad (3.17)$$

Without loss of generality, we choose the negative signs for α . The general solution of eq. (3.12) takes the form

$$K = C_1 F_1(a, b, c; v) + C_2 v^{1-c} F_1(a-c+1, b-c+1, 2-c; v), \quad (3.18)$$

which has three regular singular point at $v = 0$, $v = 1$ and $v = \infty$. Here, $F_1(a, b, c; v)$ is a hypergeometric function and C_1, C_2 are constants. Then, the solution for the radial function $R(v)$ is

$$R(v) = C_1 v^\alpha (1-v)^\beta F_1(a, b, c; v) + C_2 v^{-\alpha} (1-v)^\beta F_1(a-c+1, b-c+1, 2-c; v). \quad (3.19)$$

According to our change of variables at the vicinity of the horizon when $r \rightarrow r_+$, then $z \rightarrow p - \frac{1}{p}$, $x \rightarrow \frac{1}{p^2}$, $y \rightarrow 1$, $v \rightarrow 0$ and at the infinity when $r \rightarrow \infty$, then $z \rightarrow p$, $x \rightarrow 0$, $y \rightarrow 0$, $v \rightarrow 1$. In the vicinity of the horizon, $v = 0$ and using the property $F(a, b, c, 0) = 1$, the function $R(v)$ behaves as

$$R(v) = C_1 e^{\alpha \ln v} + C_2 e^{-\alpha \ln v}, \quad (3.20)$$

and the scalar field φ can be written in the following way

$$\varphi \sim C_1 e^{-i\omega(t + \frac{\sqrt{p}}{2p} \ln v)} + C_2 e^{-i\omega(t - \frac{\sqrt{p}}{2p} \ln v)}, \quad (3.21)$$

in which the first term represents an ingoing wave and the second one an outgoing wave in the black hole. For computing the QNMs, we have to impose our boundary conditions on the horizon that there exist only ingoing waves. This fixes $C_2 = 0$. Then the radial solution becomes

$$R(v) = C_1 e^{\alpha \ln v} (1-v)^\beta F_1(a, b, c; v) = C_1 e^{-i\omega \frac{\sqrt{p}}{2p} \ln v} (1-v)^\beta F_1(a, b, c; v). \quad (3.22)$$

In order to implement boundary conditions at infinity ($v = 1$), we shall apply in eq. (3.22) the Kummer's formula for the hypergeometric function [44],

$$F_1(a, b, c; v) = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} F_1(a, b, a+b-c, 1-v) + (1-v)^{c-a-b} \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)} F_1(c-a, c-b, c-a-b+1, 1-v). \quad (3.23)$$

With this expression the radial function results in

$$R(v) = C_1 e^{-i\omega \frac{\sqrt{p}}{2p} \ln v} (1-v)^\beta \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} F_1(a, b, a+b-c, 1-v) + C_1 e^{-i\omega \frac{\sqrt{p}}{2p} \ln v} (1-v)^{c-a-b+\beta} \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)} F_1(c-a, c-b, c-a-b+1, 1-v). \quad (3.24)$$

The flux is given by

$$F = \frac{\sqrt{-g} g^{rr}}{2i} (\varphi^* \partial_r \varphi - \varphi \partial_r \varphi^*), \quad (3.25)$$

and demanding that it vanishes at infinity, if $m^2 l^2 \geq \frac{25-(d-1)^2}{4}$, results in a set of two divergent terms of order $(1-v)^{2\beta-d/2+3}$ and $(1-v)^{-2\beta+d/2+2}$, for β_- and β_+ , respectively. The condition on the mass of the scalar field agrees with the Breitenlohner-Freedman condition that any effective mass must satisfy in order to have a stable AdS asymptotics in d-dimensions, $m^2 l^2 \geq -\frac{(d-1)^2}{4}$ [45–47]. It is worth noting also that for $d > 6$, a negative mass squared for a scalar field is consistent. Then according to eq. (3.24), for β_- , each of these terms is proportional to

$$\left| \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} \right|^2. \quad (3.26)$$

Since the gamma function $\Gamma(x)$ has the poles at $x = -n$ for $n = 0, 1, 2, \dots$, the wave function satisfies the considered boundary condition only upon the following additional

restriction $(c - a)|_\alpha = -n$ or $(c - b)|_\alpha = -n$ and these conditions determine the form of the quasi-normal modes as

$$\omega = \mp \sqrt{\xi^2 + \left(\frac{d-3}{2}\right)^2 (1-p)} - i\sqrt{p} \left(2n + 1 + \sqrt{\left(\frac{d-1}{2}\right)^2 + m^2 l^2}\right). \quad (3.27)$$

For β_+ , the divergent term is proportional to

$$\left| \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)} \right|^2. \quad (3.28)$$

Since the gamma function $\Gamma(x)$ has the poles at $x = -n$ for $n = 0, 1, 2, \dots$, the wave function satisfies the considered boundary condition only upon the following additional restriction $(a)|_\alpha = -n$ or $(b)|_\alpha = -n$ and these conditions determine the form of the quasi-normal modes as¹

$$\omega = \mp \sqrt{\xi^2 + \left(\frac{d-3}{2}\right)^2 (1-p)} - i\sqrt{p} \left(2n + 1 - \sqrt{\left(\frac{d-1}{2}\right)^2 + m^2 l^2}\right). \quad (3.29)$$

We observe that if $p = 1$ we recover the QNMs of the massless topological black holes in $d = 4$ dimensions [49]. Actually, if $p = 1$ then $\mu = 0$ and the metric (2.5) coincides with the metric of a massless topological black hole. It is also interesting to observe that if $p \neq 1$ the QNMs (3.27) and (3.29) of scalar perturbations of Chern-Simons black holes have the imprint of the high curvature of the original theory. We expect this to be also true for the flat and spherical topologies. This result may have observational implications to the gravitational wave experiments.

According to the AdS/CFT correspondence the relaxation time τ for a thermal state to reach thermal equilibrium in the boundary conformal field theory is $\tau = 1/\omega_I$ where ω_I is the imaginary part of QNMs. As can be seen in relation (3.27) ω_I scales with \sqrt{p} . Depending then on the sign of μ , p can be larger or smaller than one. This means that the scalar field will decay faster or slower depending on the value of the curvature parameter k .

An interesting question is what happens if the scalar field backreacts on the metric.² In [22] linear perturbations were studied of Gauss-Bonnet maximally symmetric vacuum solutions. It was found that these solutions are perturbatively unstable near the Chern-Simons limit but when this limit is reached the theory becomes strongly coupled and the perturbation theory breaks down. If we had considered a coupled scalar-tensor system this would have resulted in a Chern-Simons black hole solution with scalar hair. Actually such

¹Recently, a preprint appeared in the archives [48] which is based on an unpublished preprint (CECS-PHY-06/13) by the authors, in which the QNMs of a special class of black holes were presented including the QNMs of Chern-Simons black holes. Their method was depending on a suitably rewriting the Chern-Simons metric as a metric of a massless topological black hole. However, in our calculation, due to strong coupling problems, we followed the more physically transparent method of solving the Klein-Gordon equation in the background of the Chern-Simons black hole.

²We thank the referee for asking this question.

solutions exist [50] if you think that Chern-Simons black holes (with $\gamma = -1$) are generalizations of topological black holes. In a series of papers [51–53] the scalar, electromagnetic and tensor perturbations were studied of these hairy black holes. It was found that there exist a critical temperature below which the vacuum black hole acquires hair. This happens when the mass μ goes to zero and the position of the horizon is at $r_+ = 1$ (with the AdS length scale $l = 1$). Similar behaviour we expect also for the Chern-Simons black holes. We speculate that the strong coupling limit is attained when $\mu = 0$ which from (3.3) is at $r_+ = 1$. Then it is interesting to connect this limit with the phase transition observed in [51–53]. On the other hand notice that in [22] it was found with non-perturbative methods that at the Chern-Simons limit the Einstein and Gauss-Bonnet vacuum solutions coexist which is a transient configuration characteristic of a phase transition. This issue is very interesting and merits further study, which however is beyond the scope of the present work.

4 Mass and area spectrum from quasi-normal modes

Consider a system with energy E and vibrational frequency ω

$$\omega = \omega_R = \sqrt{\xi^2 + \left(\frac{d-3}{2}\right)^2 (1-p)}. \quad (4.1)$$

Define

$$V = -\alpha \left(\frac{d-3}{2}\right)^2 \left(2 \frac{\Omega_{d-2}}{\Sigma_{d-2}} G_k\right)^{\frac{1}{k}}, \quad (4.2)$$

and then (4.1) becomes

$$\omega = \sqrt{\xi^2 + VM^{\frac{1}{k}}}. \quad (4.3)$$

Assuming that this frequency is a fundamental vibrational frequency for a black hole of energy $E = M$, the quantity

$$I_1 = \int \frac{dE}{\omega(E)}, \quad (4.4)$$

is an adiabatic invariant [29]. Then eq. (4.4), using (4.3), has the following solution

$$I_1 = \frac{M\sqrt{VM^{\frac{1}{k}} + \xi^2}}{\xi^2} F_1 \left[1, \frac{1}{2} + k, 1 + k, \frac{-VM^{\frac{1}{k}}}{\xi^2} \right]. \quad (4.5)$$

Following Hod’s proposal,³ the Bohr-Sommerfeld quantization condition in the semi-classical limit gives the spectrum

$$I_1 \approx n\hbar. \quad (4.6)$$

For the Gauss-Bonnet case with $d = 5$ and $k = 2$, the adiabatic invariant eq. (4.5) can be written as

$$I_1 = \frac{4 \left(B^2 M - B\sqrt{M}\xi^2 + 2 \left(-1 + \sqrt{1 + \frac{B\sqrt{M}}{\xi^2}} \right) \xi^4 \right)}{3B^2\sqrt{1 + B\sqrt{M}}}, \quad (4.7)$$

³The Hod’s proposal, at least in the Schwarzschild case, is valid at the large damping limit. Here, contrary to the Schwarzschild case, the real part of the QNMs of the Chern-Simons black holes is independent of the mode number n .

where

$$B = -\alpha \left(2 \frac{\Omega_3}{\Sigma_3} G_2 \right)^{\frac{1}{2}}. \quad (4.8)$$

To simplify the above expression, without loosing the generality, we choose $\xi = 0$. Then equating the above equation with eq. (4.6) we obtain the mass spectrum

$$M(n) = \frac{1}{4} \left(3n\hbar \sqrt{\frac{B}{2}} \right)^{\frac{4}{3}}. \quad (4.9)$$

It is worth noting that for $k = 2$, α can take the values ± 1 . To find the area spectrum, we use the horizon area of the black hole, that is given by

$$A_{r_+} = \Sigma_3 r_+^3, \quad (4.10)$$

where r_+ is given by eq. (3.3). Then using the mass spectrum (4.9), the area spectrum becomes

$$A_n = \Sigma_3 l^3 \left(1 - \frac{B}{2} \left(3n\hbar \sqrt{\frac{B}{2}} \right)^{\frac{2}{3}} \right)^{\frac{3}{2}}. \quad (4.11)$$

We observe that the mass and area spectrum given by (4.9) and (4.11) respectively, are not evenly spaced. The no equidistance of the spectrum was also found in other black hole cases. The (2+1)-dimensional BTZ black hole was studied in connection to the Hod's conjecture [54]. It was found that there is a connection between the quasi-normal modes and the quantum (2+1) black holes but it was not found a quantization of the horizon area. Also in [55] a quantization of the horizon area was found for a non-rotating BTZ black hole but it is not evenly spaced. In [56] the high curvature Lanczos-Lovelock theories were studied and it was found that the area spacing is not equidistant and it was claimed that the notion of quantum of entropy is more natural in these theories. Also, in acoustic (2+1)-dimensional black holes, the mass and area spectrum is not evenly spaced [57].

It is interesting to note that the complexity of the horizon of the hyperbolic geometry has an important effect on the mass and area spectrum of the Chern-Simons black holes. The B factor that appears in (4.9) and (4.11) is inverse proportional to the volume of the hyperbolic space Σ_3 . For high genus the volume Σ_3 can be arbitrary large [58–60] giving an irregular spacing, leading eventually to a breakdown of the quantization conditions (4.9) and (4.11).

The same behaviour also appears in the Maggiore's approach [30]. In this approach we identify $\omega \simeq \omega_I$ of QNMs in the high damping limit (large n limit) and evaluate the adiabatic expression

$$I_2 = \int \frac{dM}{\omega_t}, \quad (4.12)$$

where, the transition frequency ω_t is given by

$$\omega_t = |(\omega_I)_n| - |(\omega_I)_{n-1}| = 2\sqrt{p} = 2\sqrt{1 + CM^{\frac{1}{k}}}, \quad (4.13)$$

where

$$C = \alpha \left(2 \frac{\Omega_{d-2}}{\Sigma_{d-2}} G_k \right)^{\frac{1}{k}}, \quad (4.14)$$

and

$$\omega_I(n \rightarrow \infty) = -2n\sqrt{p}. \quad (4.15)$$

Thus, eq. (4.12) has the following solution

$$I_2 = \frac{M}{2} F_2 \left[\frac{1}{2}, k, 1+k, -CM^{\frac{1}{k}} \right]. \quad (4.16)$$

In the specific example of $d = 5$ and $k = 2$, imposing the Bohr-Sommerfeld quantization condition the adiabatic invariant eq. (4.16) becomes

$$I_2 = \frac{2 \left[2 - \sqrt{1 - B\sqrt{M}} (B\sqrt{M} + 2) \right]}{3B^2}, \quad (4.17)$$

where $B = -C$. The solution (4.17) gives a complicated mass and area spectrum that they are not evenly spaced and they depend also on the factor B .

Having evaluated the mass and area spectrum we can ask if this information can be used to evaluate the spectrum of the entropy of Chern-Simons black holes. In Einstein's theory this is possible using the Bekenstein's entropy formula. However, in Lovelock theories the entropy is not proportional to the horizon area. The reason is that the entropy has a leading Lovelock correction term which appears as an induced curvature term which is nothing else but the Euler density of the horizon surface [61]. In light of this, can the entropy spectrum of Chern-Simons black holes be evenly spaced in spite that their area spectrum is not? In [56] a general argument is given that in Lovelock theories the entropy should be quantized with an equally spaced spectrum and they demonstrated this argument in the five-dimensional Gauss-Bonnet theory using the asymptotic form of the QNMs. We believe that the effect of Lovelock gravity on the entropy spectrum is a pure geometrical effect and considering that the Chern-Simons black holes depend explicitly on the curvature parameter k , a careful analysis should be carried out in the lines of [61] to settle this issue.

5 Reflection, transmission coefficients and absorption cross sections

The reflection and the transmission coefficients are defined by

$$\mathfrak{R} := \left| \frac{F_{\text{asympt}}^{\text{out}}}{F_{\text{asympt}}^{\text{in}}} \right|, \quad \text{and} \quad \mathfrak{U} := \left| \frac{F_{\text{hor}}^{\text{in}}}{F_{\text{asympt}}^{\text{in}}} \right|, \quad (5.1)$$

where F is the flux given in eq. (3.25) adapted for the radial function $R(v)$

$$F = \frac{\sqrt{-g}g^{rr}}{2i} (R^* \partial_r R - R \partial_r R^*). \quad (5.2)$$

To calculate the above coefficients we need to know the behaviour of the radial function both at the horizon and at the asymptotic infinity. The behaviour at the horizon is given by eq. (3.22) and using eq. (5.2), we get the flux at the horizon up to an irrelevant factor from the angular part of the solution

$$F_{\text{hor}}^{\text{in}} = -|C_1|^2 \omega l^{d-3} p^{\frac{d-4}{2}}. \quad (5.3)$$

To obtain the asymptotic behaviour of the $R(v)$ we use $1 - v = \frac{l^2 p}{r^2}$ and taking into account the limit of $R(v)$, eq. (3.19), when $v \rightarrow 1$, we have

$$R(r) = C_1 \left(\frac{l\sqrt{p}}{r} \right)^{2\beta} \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} + C_1 \left(\frac{l\sqrt{p}}{r} \right)^{d-1-2\beta} \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)}. \quad (5.4)$$

On the other hand, when $r \rightarrow \infty$, eq. (3.9) approximates to

$$\partial_r^2 R(r) + \frac{d}{r} \partial_r R(r) + \left(\frac{\omega^2 l^2}{r^4} - \frac{Q l^2}{r^4} - \frac{m^2 l^2}{r^2} \right) R(r) = 0, \quad (5.5)$$

where, we have used the ansatz $\varphi = R(r)Y(\Sigma)e^{-i\omega t}$. The behaviour at the asymptotic region is the same as for the Topological massless black holes [62]. The solution of eq. (5.5) is a linear combination of the Bessel function [44] given by

$$R(r) = \left(\frac{\sqrt{A}}{2r} \right)^{\frac{d-1}{2}} \left[D_1 \Gamma(1-C) J_{-C} \left(\frac{\sqrt{A}}{r} \right) + D_2 \Gamma(1+C) J_C \left(\frac{\sqrt{A}}{r} \right) \right], \quad (5.6)$$

where

$$A = l^2(l^2\omega^2 - Q), \quad C = \frac{1}{2} \sqrt{(d-1)^2 + 4m^2 l^2}. \quad (5.7)$$

Now, using the expansion of the Bessel function [44]

$$J_n(x) = \frac{x^n}{2^n \Gamma(n+1)} \left\{ 1 - \frac{x^2}{2(2n+2)} + \dots \right\}, \quad (5.8)$$

for $x \ll 1$, we find the asymptotic solution in the polynomial form

$$R_{asympt.}(r) = D_1 \left(\frac{\sqrt{A}}{2r} \right)^{\frac{d-1}{2}-C} + D_2 \left(\frac{\sqrt{A}}{2r} \right)^{\frac{d-1}{2}+C}, \quad (5.9)$$

for $\frac{\sqrt{A}}{r} \ll 1$. Introducing,

$$\hat{D}_1 \equiv D_1 \left(\frac{\sqrt{A}}{2} \right)^{\frac{d-1}{2}-C}, \quad \hat{D}_2 \equiv D_2 \left(\frac{\sqrt{A}}{2} \right)^{\frac{d-1}{2}+C}, \quad (5.10)$$

we write eq. (5.9) as

$$R_{asympt.}(r) = \hat{D}_1 \left(\frac{1}{r} \right)^{\frac{d-1}{2}-C} + \hat{D}_2 \left(\frac{1}{r} \right)^{\frac{d-1}{2}+C}. \quad (5.11)$$

In ref. [63] it was discussed that a scalar field with asymptotic behaviour similar to that of eq. (5.11), generically may lead to an unstable state (the so-called, big crunch singularity) which is a clear indication of the appearance of a nonlinear instability. However, such an instability influences the boundary conditions that scalar fields must satisfy at infinity [47]. Although the modified boundary conditions preserve the full set of asymptotic AdS symmetries and allow for a finite conserved energy to be defined, this energy can be

negative. We notice, that the imposition of regularity condition on the radial function (5.11) at the infinity implies $\frac{d-1}{2} - C \geq 0$ or $-\frac{(d-1)^2}{4} \leq m^2 l^2 \leq 0$. This is in agreement with the condition for the mass in order to have a stable asymptotic AdS spacetime in d -dimensions, $m^2 l^2 \geq -\frac{(d-1)^2}{4}$ [47]. Besides, $a + b - c = -C$, for $\beta = \beta_-$, and $c - a - b = -C$, for $\beta = \beta_+$. For this reason C can not be an integer, because the gamma function is singular at that point and the regularity conditions are not satisfied.

We now take advantage of the inherent symmetry that the radial solution possesses in the asymptotic region. More specifically, we have the freedom to choose the form of the constant β since by changing β_+ to β_- this solution is unchanged. Comparison of eqs. (5.4) and (5.11), regarding $\beta = \beta_-$, allows us to immediately read off the coefficients \widehat{D}_1 and \widehat{D}_2

$$\widehat{D}_1 = C_1 (l\sqrt{p})^{2\beta_-} \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}, \quad (5.12)$$

$$\widehat{D}_2 = C_1 (l\sqrt{p})^{d-1-2\beta_-} \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)}. \quad (5.13)$$

Therefore, the behaviour at the asymptotic region is given by eq. (5.11) and using eq. (5.2), we get the flux up to an irrelevant factor from the angular part of the solution and it is given by

$$F_{asympt.} = -iC \left(\frac{1}{l^2} - \frac{p}{r^2} \right) \left(\widehat{D}_2^* \widehat{D}_1 - \widehat{D}_1^* \widehat{D}_2 \right). \quad (5.14)$$

We notice here that the distinction between the ingoing and outgoing fluxes at the asymptotic region is a non trivial task because the spacetime is asymptotically AdS. In order to characterize the fluxes is convenient to split up the coefficients \widehat{D}_1 and \widehat{D}_2 in terms of the incoming and outgoing coefficients, D_{in} and D_{out} , respectively. We define $\widehat{D}_1 = D_{in} + D_{out}$ and $\widehat{D}_2 = ih(D_{out} - D_{in})$ with h being a dimensionless constant which will be assumed to be independent of the energy ω [64–67]. In this way, we rewrite the asymptotic flux eq. (5.14) as

$$F_{asympt.} \approx \frac{2hC}{l^2} \left(|D_{in}|^2 - |D_{out}|^2 \right). \quad (5.15)$$

Therefore, the reflection and transmission coefficients are given by

$$\mathfrak{R} = \frac{|D_{out}|^2}{|D_{in}|^2}, \quad (5.16)$$

$$\mathfrak{U} = \frac{\omega l^{d-1} p^{\frac{d-4}{2}} |C_1|^2}{2|h|C|D_{in}|^2}, \quad (5.17)$$

and the absorption cross section, σ_{abs} , is given by

$$\sigma_{abs} = \frac{\mathfrak{U}}{\omega} = \frac{l^{d-1} p^{\frac{d-4}{2}} |C_1|^2}{2|h|C|D_{in}|^2}, \quad (5.18)$$

where, the coefficients D_{in} and D_{out} are given by

$$D_{in} = \frac{C_1}{2} \left[(l\sqrt{p})^{2\beta_-} \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} + \frac{i}{h} (l\sqrt{p})^{d-1-2\beta_-} \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)} \right], \quad (5.19)$$

$$D_{out} = \frac{C_1}{2} \left[(l\sqrt{p})^{2\beta_-} \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} - \frac{i}{h} (l\sqrt{p})^{d-1-2\beta_-} \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)} \right]. \quad (5.20)$$

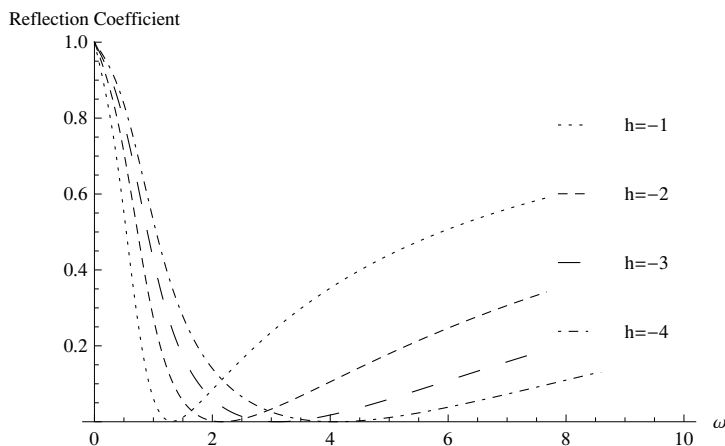


Figure 1. Reflection coefficient $v/s \omega$; $d = 5$, $m^2 l^2 = -15/4$, $l = 1$, $p = 0.5$ and $\xi = 0$.

We will carry out a numerical analysis of the reflection and transmission coefficients of eq. (5.16) and eq. (5.17) for a five-dimensional Chern-Simons black hole. For our numerics we will chose $p = 0.5, 2$, which indicates a black hole with $\alpha = \mp 1$, respectively. It is worth to note that this choice corresponds to different masses for the black holes. In the literature, there are various ways to chose the constant h . It can be chosen so that the absorption cross section can be expressed by the area of horizon in the zero-frequency limit [64, 68]. Also, it can be chosen to obtain the usual value of the Hawking temperature [65] or in such a way so that the sum of the reflection coefficient and the transmission coefficient be unity [69].

In view of these uncertainties, we leave h as a free parameter and our only requirement is that the reflection and transmission coefficients to be self-consistent so that the calculated greybody factors to have a physical meaning [66]. In consequence, for different values of h and for $m^2 l^2 = -15/4$, $l = 1$, $p = 0.5, 2$ and $\xi = 0$, we plot the reflection coefficient figure 1, 2, the transmission coefficient figure 3, 4 and the greybody factors, figure 5, 6. Essentially, we found the same general behaviour for the different values of h and the only difference is a shift in the location of the minimum or maximum of the reflection and transmission coefficients respectively. Also we observe that the parameter h must be less than zero and greater than some value such that the absorption cross section or the greybody factor be real in the zero-frequency limit, otherwise it is imaginary. However in the zero-frequency limit, the greybody factors depend on the value of h , so that the coefficient is increasing if the parameter h is increasing, as it can be seen in figure 5, 6. If we plot the combination $\mathfrak{R} + \mathfrak{U}$ for $m^2 l^2 = -15/4$, $l = 1$, $p = 0.5, 2$, $\xi = 0$ and $h = -1, -2, -3, -4$ using the figure 1, 3 and figure 2, 4 we get the $\mathfrak{R} + \mathfrak{U} = 1$ in accordance with [69]. As we discussed previously, our choice of $m^2 l^2$ is in agreement with the condition for the mass in order to have a stable asymptotic AdS spacetime in five dimensions.

We consider next without loss of generality, $m^2 l^2 = -15/4$, $l = 1$, $p = 0.5, 2$ and we fix $h = -1$, and then we analyze the behaviour of the coefficients in five dimensions for various values of ξ . Our results for $\xi = 0, 1, 2, 2.5$ are shown in figures 7, 8, 9, 10, and 11, 12, for the reflection, transmission coefficients and the greybody factors, respectively. We found,

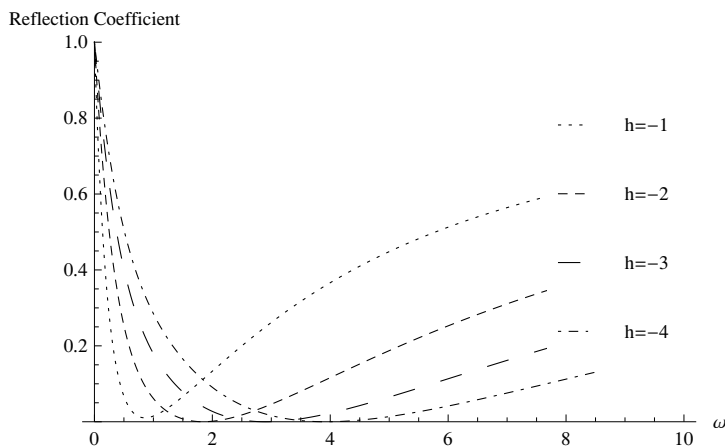


Figure 2. Reflection coefficient $v/s \omega$; $d = 5$, $m^2 l^2 = -15/4$, $l = 1$, $p = 2$ and $\xi = 0$.

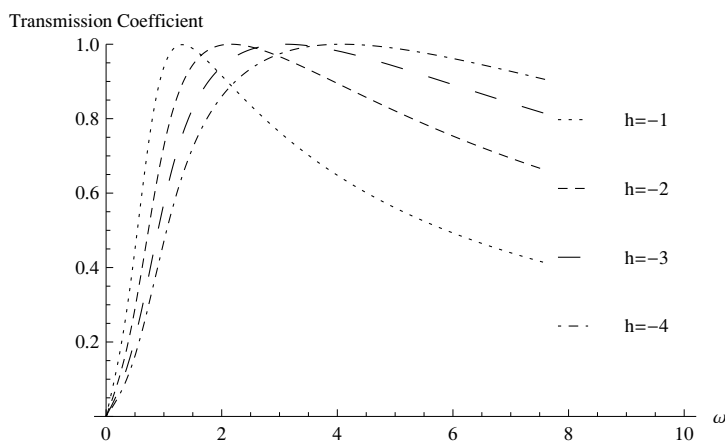


Figure 3. Transmission coefficient $v/s \omega$; $d = 5$, $m^2 l^2 = -15/4$, $l = 1$, $p = 0.5$ and $\xi = 0$.

in the zero-frequency limit that there is a range of values of ξ that contribute to the greybody factor, in contrast to the case analyzed by Das, Gibbons and Mathur [68], where in the zero-frequency limit, only the mode with lowest angular momentum contributes to the absorption cross section. Also we plot in figures 13, 14, 15, 16 and 17, 18, the mode with lowest angular momentum $\xi = 0$ for $d = 5$, $m^2 l^2 = -15/4, -7/4$, $p = 0.5, 2$, $l = 1$ and $h = -1$, the reflection the transmission coefficient and the greybody factor respectively.

We observed in the low frequency limit, that the reflection and transmission coefficients show a minimum and a maximum. Therefore, the coefficients have two branches in the reflection case, decreasing for low frequencies and then increasing. In the transmission case the behaviour is opposite, they are increasing and then decreasing, in such way as $\mathfrak{R} + \mathfrak{U} = 1$ in all cases. An interesting result is the existence of one optimal frequency to transfer energy out of the bulk.

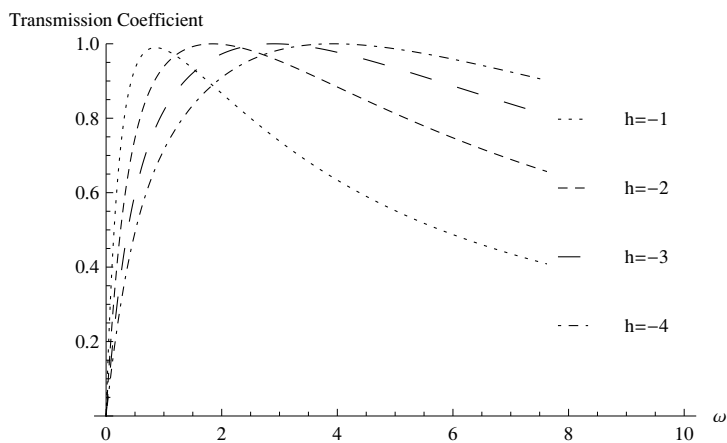


Figure 4. Transmission coefficient $v/s \omega$; $d = 5$, $m^2 l^2 = -15/4$, $l = 1$, $p = 2$ and $\xi = 0$.

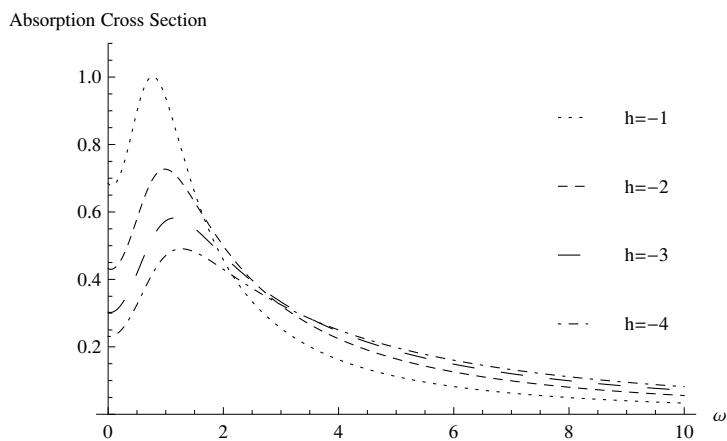


Figure 5. Absorption Cross section $v/s \omega$; $d = 5$, $m^2 l^2 = -15/4$, $l = 1$, $p = 0.5$ and $\xi = 0$.

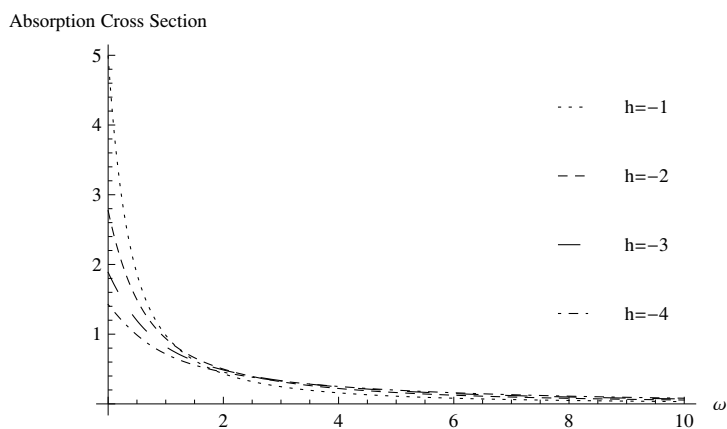


Figure 6. Absorption Cross section $v/s \omega$; $d = 5$, $m^2 l^2 = -15/4$, $l = 1$, $p = 2$ and $\xi = 0$.

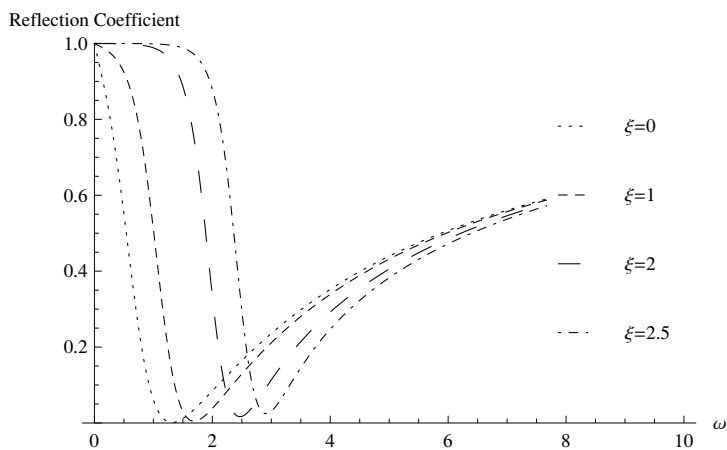


Figure 7. Reflection coefficient $v/s \omega$; $d = 5$, $m^2 l^2 = -15/4$, $l = 1$, $p = 0.5$ and $h = -1$.

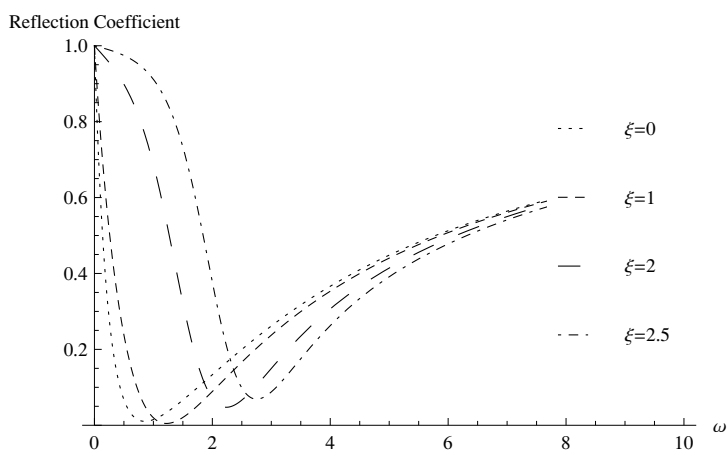


Figure 8. Reflection coefficient $v/s \omega$; $d = 5$, $m^2 l^2 = -15/4$, $l = 1$, $p = 2$ and $h = -1$.

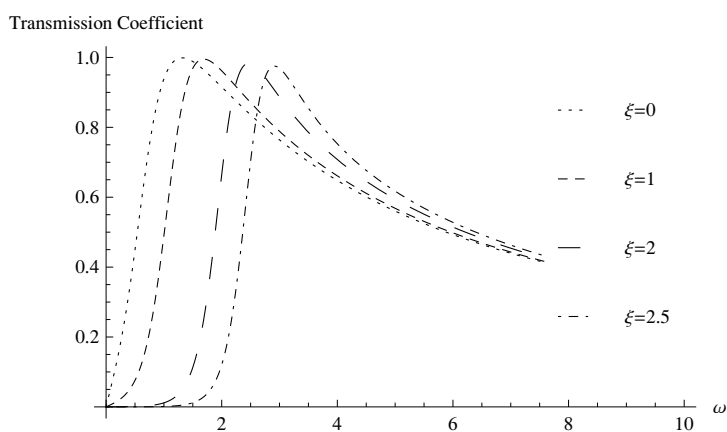


Figure 9. Transmission coefficient $v/s \omega$; $d = 5$, $m^2 l^2 = -15/4$, $l = 1$, $p = 0.5$ and $h = -1$.

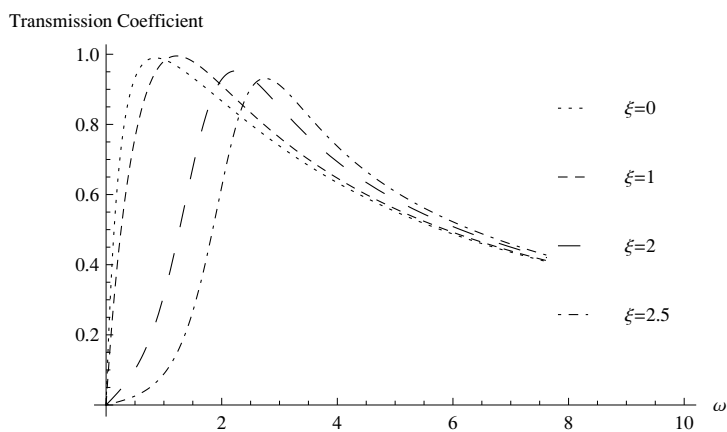


Figure 10. Transmission coefficient $v/s \omega$; $d = 5$, $m^2 l^2 = -15/4$, $l = 1$, $p = 2$ and $h = -1$.

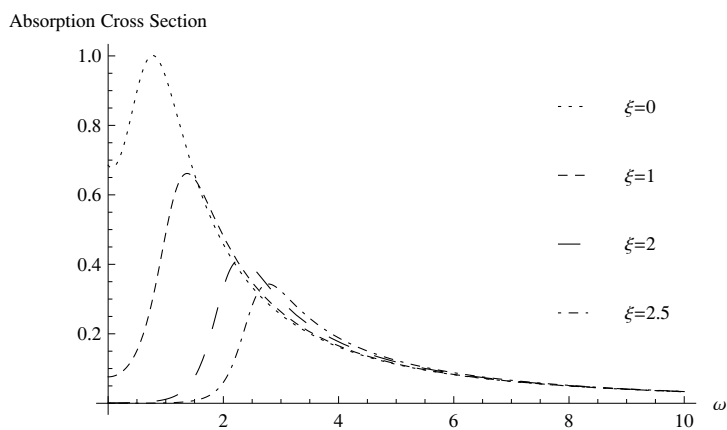


Figure 11. Absorption Cross section $v/s \omega$; $d = 5$, $m^2 l^2 = -15/4$, $l = 1$, $p = 0.5$ and $h = -1$.

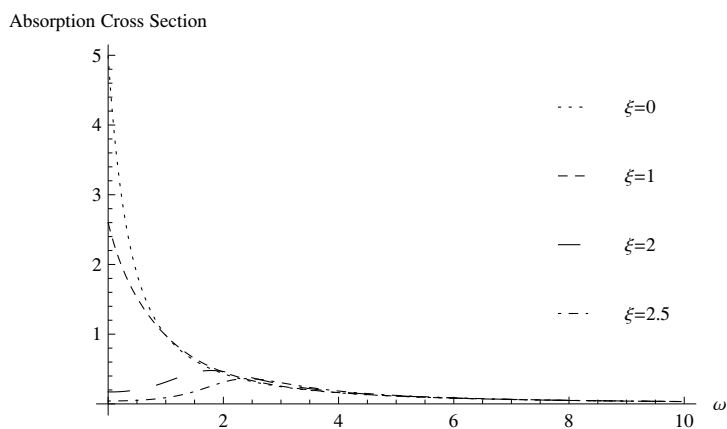


Figure 12. Absorption Cross section $v/s \omega$; $d = 5$, $m^2 l^2 = -15/4$, $l = 1$, $p = 2$ and $h = -1$.

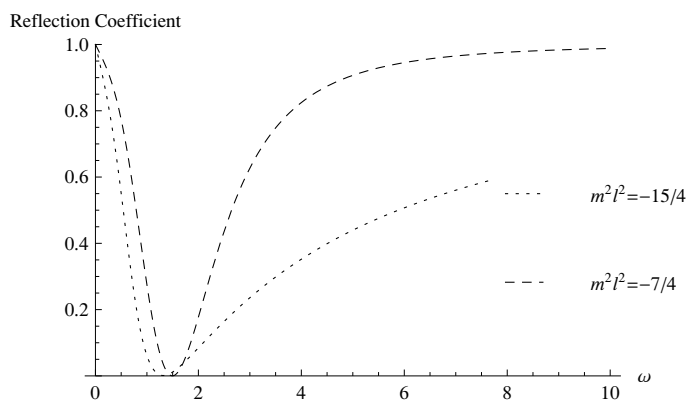


Figure 13. Reflection coefficient v/s ω ; $d = 5$, $m^2 l^2 = -15/4, -7/4$, $l = 1$, $h = -1$, $p = 0.5$ and $\xi = 0$.

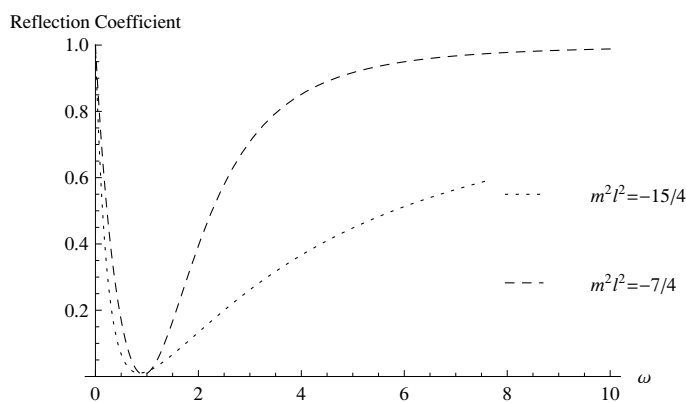


Figure 14. Reflection coefficient v/s ω ; $d = 5$, $m^2 l^2 = -15/4, -7/4$, $l = 1$, $h = -1$, $p = 2$ and $\xi = 0$.

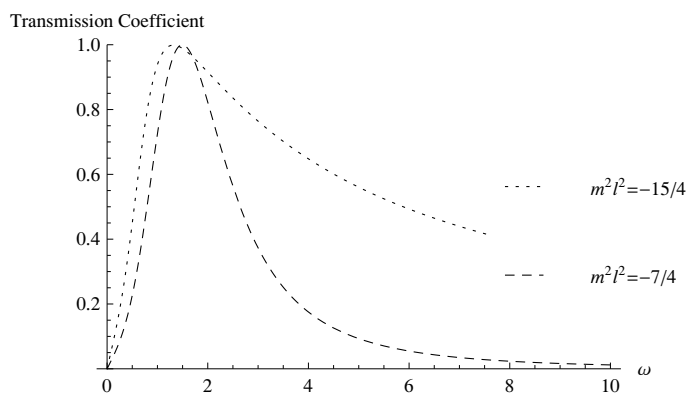


Figure 15. Transmission coefficient v/s ω ; $d = 5$, $m^2 l^2 = -15/4, -7/4$, $l = 1$, $h = -1$, $p = 0.5$ and $\xi = 0$.

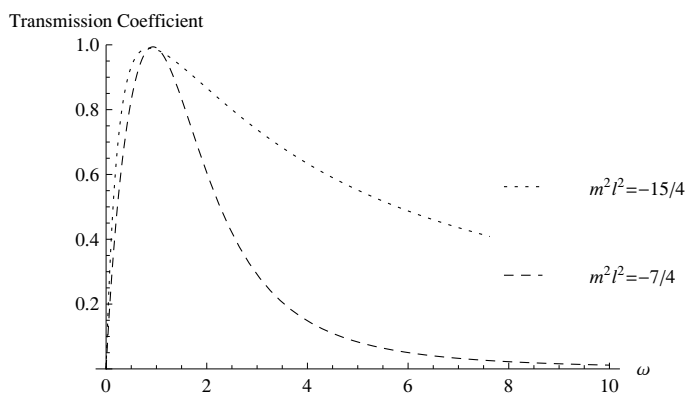


Figure 16. Transmission coefficient v/s ω ; $d = 5$, $m^2 l^2 = -15/4, -7/4$, $l = 1$, $h = -1$, $p = 2$ and $\xi = 0$.

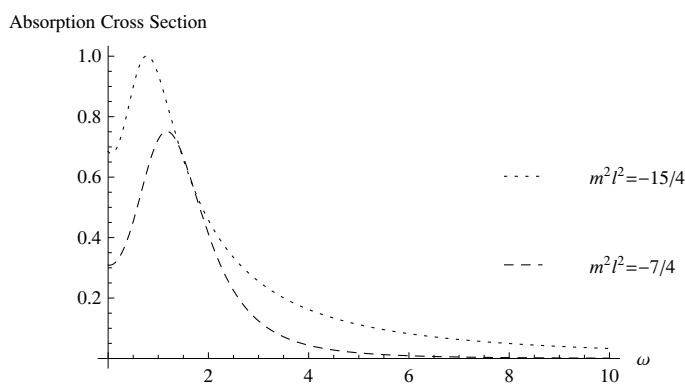


Figure 17. Absorption Cross section v/s ω ; $d = 5$, $m^2 l^2 = -15/4, -7/4$, $l = 1$, $h = -1$, $p = 0.5$ and $\xi = 0$.

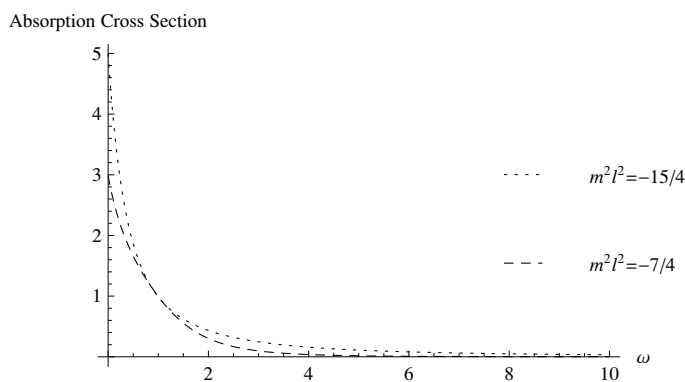


Figure 18. Absorption Cross section v/s ω ; $d = 5$, $m^2 l^2 = -15/4, -7/4$, $l = 1$, $h = -1$, $p = 2$ and $\xi = 0$.

6 Conclusions

Chern-Simons black holes are very interesting static solutions of Gravity theories which asymptotically approach spacetimes of constant negative curvature (AdS spacetimes). They can be considered as generalizations of the (2+1)-dimensional black holes in higher-dimensional Gravity theories containing higher powers of curvature. The Chern-Simons black holes of spherical topology have the same causal structure as the BTZ black holes and these solutions have a thermodynamical behavior which is unique among all possible black holes in competing Lanczos-Lovelock theories with the same asymptotics. The specific heat of these black holes is positive and therefore they can always reach thermal equilibrium with their surroundings and hence, are stable against thermal fluctuations.

These theories of high curvature also admit solutions which represent black objects of other topologies, whose singularity is surrounded by horizons of non-spherical topology. In the case of hyperbolic topology, introducing a discrete symmetry group and making the right identifications the resulting black holes resemble the topological black holes. The Chern-Simons black holes with hyperbolic topology have a single horizon, and the temperature is a linear function of the horizon. It is interesting one to study the possibility of the existence of a phase transition as it happens in the case of the topological black holes [51–53].

In this work we calculated the QNMs of scalar perturbations of the Chern-Simons black holes with hyperbolic topology. We found that the QNMs depend on the highest power of curvature present in the Lanczos-Lovelock theories. We also calculated the mass and area spectrum of these black holes. We found that there is a strong dependence of the quantization conditions on the underlying geometry resulting in making these conditions not evenly spaced.

We also computed analytically the greybody factors for Chern-Simons black holes in d -dimensions. We made a numerical analysis of the behaviour of the reflection and the transmission coefficients and the greybody factors in the low frequency limit and we found that there is a range of modes which contributes to the absorption cross section.

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