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The point of E_8 in F-theory GUTs

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Abstract: We show that in F-theory GUTs, a natural explanation of flavor hierarchies in the quark and lepton sector requires a single point of E_8 enhancement in the internal geometry, from which all Yukawa couplings originate. The monodromy group acting on the seven-brane configuration plays a key role in this analysis. Moreover, the E_8 structure automatically leads to the existence of the additional fields and interactions needed for minimal gauge mediated supersymmetry breaking, *and almost nothing else*. Surprisingly, we find that in all but one Dirac neutrino scenario the messenger fields in the gauge mediated supersymmetry breaking sector transform as vector-like pairs in the $10 \oplus \overline{10}$ of SU(5). We also classify dark matter candidates available from this enhancement point, and rule out both annihilating and decaying dark matter scenarios as explanations for the recent experiments PAMELA, ATIC and FERMI. In F-theory GUT models, a $10-100$ MeV mass gravitino remains as the prime candidate for dark matter, thus suggesting an astrophysical origin for recent experimental signals.

Keywords: Cosmology of Theories beyond the SM, F-Theory, Intersecting branes models, GUT

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Contents

1 Introduction

The paradigm of unification provides a compelling and predictive framework for high energy physics. In the context of string theory, this might at first suggest that gauge and gravitational degrees of freedom should also unify. Nevertheless, in constructions of gauge theories within string theory, there is often a limit where gravity decouples. This is because gauge theory degrees of freedom can localize on subspaces of the internal geometry. Moreover, the fact that $M_{\text{GUT}}/M_{\text{Planck}} \ll 1$ suggests that the existence of a limit where gravity decouples may also be relevant for the gauge theory defined by the Standard Model.

Flexibility in potential model building applications suggests branes with maximal dimension such that a consistent decoupling limit exists. Such considerations point to seven-branes, and thus type IIB string theory. On the other hand, the requisite elements of Grand Unified Theories (GUTs), such as the $5_H \times 10_M \times 10_M$ interaction require E-type gauge theory structures, which is incompatible with perturbative IIB string theory. Importantly, however, the strong coupling limit of IIB strings, namely F-theory is flexible enough to accommodate such elements. Recent work on F-theory GUTs in [1–9] (see also [10–29]) has shown that many realistic features of particle phenomenology naturally emerge, with potentially observable consequences for the LHC [4, 7] and upcoming neutrino experiments [8].

Low energy constraints impose important conditions that the internal geometry must satisfy, and point towards the especially important role of exceptional groups in F-theory GUTs. For example, the existence of an order one top quark Yukawa requires a point where the singularity type of the geometry enhances to E_6 [1] (see also [30]). The existence of

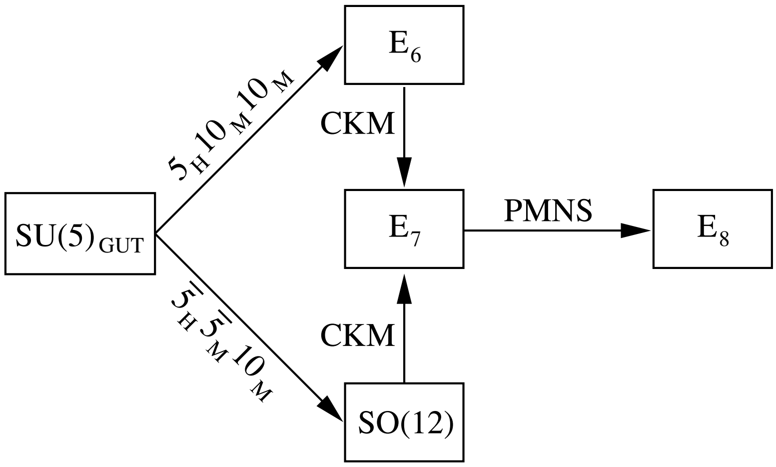

Figure 1. Starting from a GUT seven-brane with SU(5) symmetry, each additional phenomenological condition leads to a further jump in the rank at a point of the geometry. Including the $5_H 10_M 10_M$ interaction requires an E_6 point, and the $\overline{5}_H \overline{5}_M 10_M$ interaction requires an SO(12) point. A hierarchical CKM matrix then suggests that these points should also unify to an E_7 point of enhancement. Incorporating leptonic mixing structure pushes this all the way up to E_8.

higher unification structures is also important in the context of flavor physics. For example, the CKM matrix exhibits a hierarchical structure provided the up and down type Yukawas localize at points which are sufficiently close [5], which is suggestive of a single point of at least E_7 enhancement. As we show in this paper, incorporating a minimal neutrino sector with a mildly hierarchical lepton mixing matrix (PMNS matrix) pushes this all the way up to E_8. It is in principle possible to construct models where a globally well-defined E_8 structure plays no special role, and in which flavor hierarchies are solved through fine tuning. Even so, the most natural option, and the option which flavor hierarchies dictate is the existence of a *single* E_8 *enhanced symmetry point in the internal geometry from which all the interactions descend*. In this paper we will assume this is indeed the case. In fact in this paper we classify all the minimal F-theory GUT scenarios which descend from a single E_8 point, which turn out to be interestingly predictive. See figure 1 for a depiction of how various enhancement points each demand a higher unification structure.

Cosmological considerations provide another window into the physics of F-theory GUTs. The cosmology of such models neatly avoids many of the problems which sometimes afflict supersymmetric models [6]. For example, assuming the supersymmetry breaking scenario of [3], the mass of the gravitino is $10 - 100\,\mathrm{MeV}$.[1] It is known that in other contexts this typically leads to over-production of such particles. This issue is bypassed in F-theory GUTs because the saxion comes to dominate the energy density of the Universe. The subsequent decay of the saxion dilutes this relic abundance, leaving gravitinos instead as a prominent component of dark matter. In a certain regime of parameters, axionic dark

[1]As in early work on local F-theory GUTs such as [3], we assume that the dominant contribution to supersymmetry breaking originates from gauge mediated supersymmetry, and ingredients present in the local model.

matter is also possible. Both the gravitino and axion interact very weakly with the Standard Model, and are therefore unlikely to be detected. On the other hand, recent results from experiments such as PAMELA [31, 32], ATIC [33], PPB-BETS [34], HESS [35] and FERMI [36] can potentially be explained by weak to TeV scale dark matter which interacts more strongly with the Standard Model. One of the aims of this paper is to address whether there are any additional dark matter candidates in minimal F-theory GUT models besides the gravitino and axion.

The matter content of F-theory GUTs roughly divides into the degrees of freedom on the GUT seven-brane, nearby seven-branes which share a mutual intersection with the GUT seven-brane, and branes which are far away in the sense that they do not directly couple to the GUT sector. In addition, there are also degrees of freedom which do not localize on a brane but instead propagate in the bulk of the geometry. Since the dynamics of supersymmetry breaking takes place due to the dynamics of a seven-brane with Peccei-Quinn (PQ) gauge symmetry, weak to TeV scale dark matter candidates must also be relatively nearby. See figure 2 for a depiction of the possible ingredients which can in principle participate in this construction.

Assuming a single E_8 enhancement point in our local patch, we classify all visible and dark matter which can descend from the adjoint of E_8. Phenomenological requirements then lead to a rich interplay between group theoretic and geometric conditions considerations. In particular, having available suitable curves which accommodate the Higgs and the matter fields severely restricts the possibilities. Moreover, even though E_8 contains the maximal subgroup $\mathrm{SU}(5)_{\mathrm{GUT}} \times \mathrm{SU}(5)_{\perp}$, the presence of non-trivial monodromies in the seven-brane configuration required by phenomenology cuts down $\mathrm{SU}(5)_{\perp}$ to either $\mathrm{U}(1)_{\mathrm{PQ}} \times \mathrm{U}(1)_{\chi}$, as in Dirac neutrino scenarios, or $\mathrm{U}(1)_{\mathrm{PQ}}$ in Majorana scenarios. As the notation suggests, in both cases, one of the U(1) factors can be identified with a Peccei-Quinn symmetry. Here, $\mathrm{U}(1)_{\chi}$ of the Dirac scenario corresponds to a non-anomalous symmetry and is a linear combination of $\mathrm{U}(1)_Y$ and $\mathrm{U}(1)_{B-L}$. We find that only a few possible monodromy groups lead to consistent flavor physics. The full list of possible monodromy groups are $\mathbb{Z}_2, \mathbb{Z}_2 \times \mathbb{Z}_2, \mathbb{Z}_3, S_3$ and Dih_4 (the symmetry group of the square). A quite surprising outcome of the classification of visible matter fields is that all the fields needed for a successful implementation of gauge mediated supersymmetry breaking automatically follows from the existence of this E_8 point. Moreover, we find that in all but one Dirac neutrino scenario, the messenger fields of the minimal gauge mediated supersymmetry breaking (mGMSB) sector are *forced* to transform as vector-like pairs in the $10 \oplus \overline{10}$ of SU(5). In fact, in two of the Majorana neutrino scenarios all the charged matter that descends from the E_8 point are necessary and sufficient for mGMSB and the interactions of the MSSM! This rigid structure also extends to the list of available dark matter candidates, providing only a few options with very specific $\mathrm{U}(1)_{\mathrm{PQ}}$ (as well as $\mathrm{U}(1)_{\chi}$ for Dirac scenarios) charge assignments.

Having classified extra matter fields available from the E_8 point, we next turn to whether any of these can serve as dark matter candidates for the purpose of explaining the PAMELA, ATIC and FERMI experiments. Many of these experiments require some additional component of electrons and positrons generated by either dark matter physics

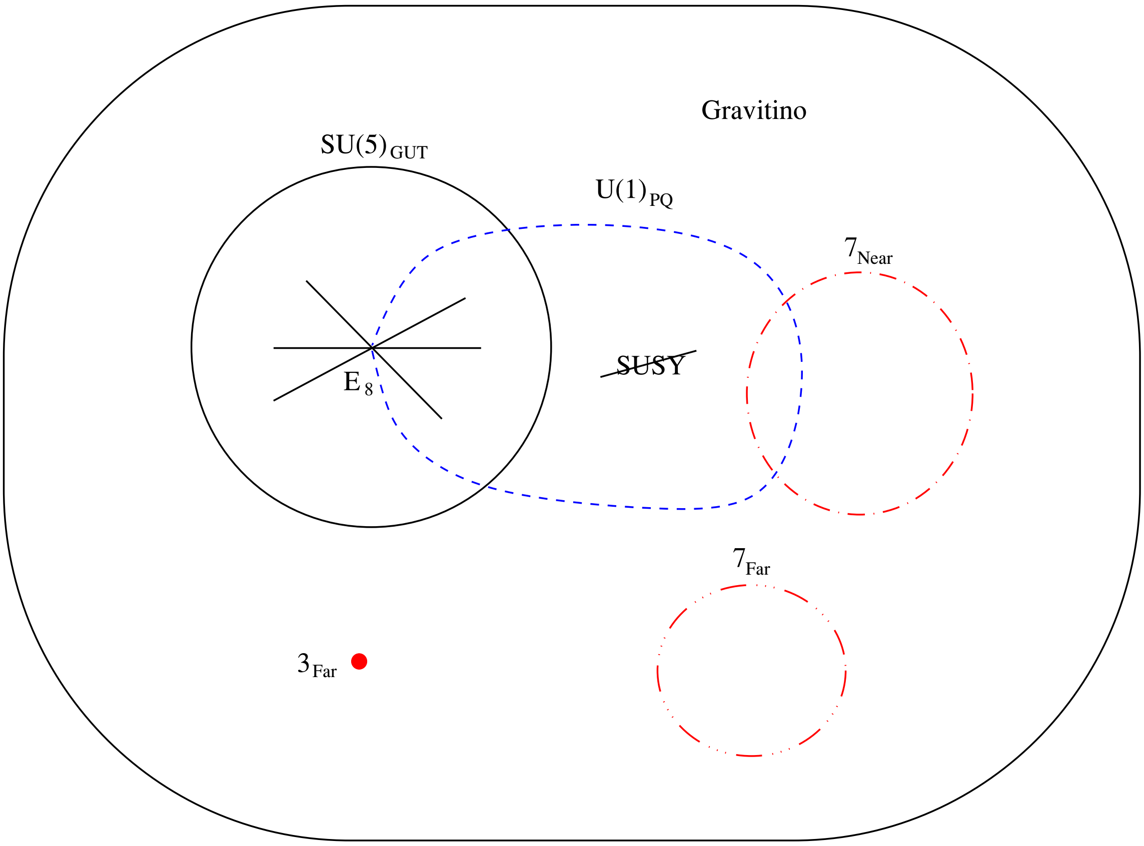

Figure 2. Depiction of the $SU(5)_{GUT}$ seven-brane, the $U(1)_{PQ}$ seven-brane, and other possible branes and bulk modes which can in principle appear as "dark objects" in F-theory GUTs. Here, the 7_{Near}-branes share a mutual intersection with the $SU(5)_{GUT}$ seven-brane along the PQ seven-brane. As in [3], supersymmetry breaking occurs due to dynamics localized on the PQ seven-brane. In addition, we have also included the possibility of additional three-branes and seven-branes, 3_{Far} and 7_{Far}, as well as bulk modes, such as the gravitino. These latter possibilities are less directly connected to the supersymmetry breaking sector, and so typically do not contain weak to TeV scale dark matter candidates.

or astrophysics, and our focus will be on whether the former possibility can be realized in F-theory GUTs. In the context of dark matter scenarios there are two main types of models based on either annihilating or decaying dark matter. In annihilating dark matter scenarios, dark matter states annihilate and produce electrons and positrons. In the case of decaying dark matter, GUT scale suppressed higher dimension operators trigger a decay of the dark matter into electrons and positions. The decaying scenario is more in line with the idea of F-theory GUTs, where unification plays a key role in low energy phenomenology due to GUT scale suppressed operators. Nevertheless, we find that none of the available fields provide a viable dark matter candidate for either decaying, or annihilating scenarios. There are multiple obstacles (though fewer obstacles in the decaying scenario). The overarching problem is that in both scenarios the available TeV scale dark matter candidates decay too rapidly.

JHEP08(2010)040

There are also other issues, even if one imposes extra structures to avoid these decays. For example, there is a cosmological issue: In scenarios where the dark matter relic abundance is generated non-thermally from the decay of the saxion, the saxion mass needs to be bigger than 1 TeV and this is in conflict with the fact that the saxion mass cannot be that large without significant fine tuning to avoid the stau mass from becoming tachyonic.[2] In addition, the saxion decay also overproduces dark matter candidates in non-thermal scenarios. In scenarios where the dark matter is generated thermally, the decay of the saxion overdilutes the relic abundance. This issue can be overcome in decaying scenarios by a mild fine tuning, but is especially problematic in annihilating scenarios. Also in the annihilating scenario one needs a light field to communicate between the visible and the hidden sector. One can rule out a gauge boson playing this role, thanks to the classification of allowed gauge factors. Light scalar mediators from inside the E_8 do exist that can in principle do the job, but even in this case one needs to assume many additional ingredients for this to work.

One could ask if there are other possible charged matter which can communicate with our sector by gauge interactions. For example if we consider matter which is charged under our E_8 as well as some other group G, this could in principle provide another class of dark matter candidates. In this regard, E-type singularities resist the intuition derived from quiver diagrams. Indeed, matter charged under $E_8 \times G$ is not allowed in string theory! There are two ways to state why this is not possible. One way of arguing for the absence of such particle states is that matter on colliding branes can be explained by locally Higgsing a higher singularity of a simple group [37], so that E_8 places an upper cap on the allowed singularity type. More directly, it is also known that the collision of E-type branes with one another in higher dimensions lead to tensionless strings! The four-dimensional reflection of such tensionless strings are conformal theories with E-type symmetry. If we are to avoid a tower of nearly massless particles, we can consider a sector with badly broken conformal symmetry. Even though this is logically allowed we find it to be somewhat exotic from the perspective of F-theory GUT constructions. Of course one can also speculate about the potential implications of having a nearly conformal sector with an E-type symmetry, and we offer some speculations along this line later in the paper.

It is in principle possible to also consider dark matter candidates which originate from bulk gravitational modes, or modes associated with other branes of the compactification. In this case, the main issue is that generating a weak scale mass for the candidate involves a non-trivial extension of the model, which appears quite ad hoc. All of this reinforces the idea that if one takes seriously the notion of unifying geometric structures in F-theory GUTs, the gravitino remains as the main candidate for dark matter. This also points to an astrophysical origin for the signals observed in the PAMELA, ATIC and FERMI experiments.

The rest of this paper is organized as follows. We first review the main building blocks of F-theory GUTs in section 2, and in particular provide in crude terms a characterization

[2]This is especially problematic when the messengers must transform in the $10 \oplus \overline{10}$, as we have found to be the generic case with a single E_8 point of enhancement.

JHEP08(2010)040

of the expected mass scales for matter within, close to, and disjoint from the GUT seven-brane. In section 3 we review the fact that in F-theory GUTs, gravitinos provide a natural dark matter candidate. In section 4 we demonstrate that hierarchical structures in the CKM and PMNS matrix require a single point of E_8 enhancement. This is followed in section 5 by a classification of F-theory GUTs which respect this property, as well as other mild phenomenological conditions. In sections 6 and 7, we respectively consider matter which intersects a part of E_8, and matter disjoint from E_8. Having catalogued potential dark matter candidates, in section 8 we review the main features of current experiments and their potential dark matter and astrophysical explanations. Section 9 contains our analysis of annihilating and decaying scenarios, where we find obstructions in all cases to realizing a viable interacting dark matter scenario. In section 10 we present our conclusions and potential directions for future investigation. Appendix A contains the classification of viable monodromy groups associated with a single E_8 interaction point.

2 Building blocks and mass scales of F-theory GUTs

Before proceeding to a specific dark matter candidate, in this section we briefly review the main ingredients and mass scales which enter into F-theory GUTs. The discussion roughly separates into those objects which are part of the GUT seven-brane, those which communicate directly with the supersymmetry breaking sector localized in the seven-brane with Peccei-Quinn (PQ) gauge symmetry, and those degrees of freedom which are not directly connected with the PQ seven-brane. We argue that weak to TeV scale matter must either possess a non-trivial PQ charge, or interact closely with such a field. Fields cutoff from the dynamics of the PQ seven-brane have far lower mass, on the order of the gravitino mass $m_{3/2} \sim 10 - 100\,\mathrm{MeV}$.

In F-theory GUTs, the relevant degrees of freedom of the GUT model localize on matter curves or propagate in the complex surface wrapped by the GUT seven-brane. All of the matter content of F-theory GUTs is controlled by the local enhancement in the singularity type of the geometry. For example, the bulk seven-brane corresponds to a locus where an SU(5) singularity is present. Matter trapped on Riemann surfaces or "curves" corresponds to places where the local singularity type of the geometry enhances further. This matter can then be described in terms of a local Higgsing operation. At points of the geometry, a further enhancement is possible, and this is where matter curves meet, and Yukawa couplings localize.

Many of the necessary ingredients localize within the GUT seven-brane. On the other hand, the existence of matter curves indicates the presence of other seven-branes which intersect the GUT brane. The gauge symmetries of these additional branes provide additional nearly exact global symmetries for the low energy theory. In principle, extra matter fields which are GUT singlets can also localize on curves inside of such "$7_\perp$-branes". Such matter fields will typically interact with the charged matter of the GUT model. For example, the field responsible for supersymmetry breaking, X is neutral under the GUT seven-brane, but is nevertheless charged under a U(1) Peccei-Quinn seven-brane. This field develops a vev of the form:

$$\langle X \rangle = x + \theta^2 F_X, \tag{2.1}$$

where:

$$x \sim 10^{12}\ \text{GeV}, \tag{2.2}$$

$$F_X \sim 10^{17}\ \text{GeV}^2. \tag{2.3}$$

Within this framework, it is possible to accommodate a μ term correlated with supersymmetry breaking through the higher-dimension operator $X^\dagger H_u H_d/\Lambda_{\text{UV}}$, as well as a minimal gauge mediated supersymmetry breaking sector through the F-term XYY', where the Y's denote messenger fields [3]. See [1–9] for other recent work on the phenomenology of F-theory GUTs. In principle, there can be additional contributions to supersymmetry breaking effects in passing from local to global compactifications. In this paper we assume that the dominant source of supersymmetry breaking effects in the visible sector descends from a gauge mediation scenario, with degrees of freedom present in the local model.

So far, our discussion has been purely local in the sense that we have decoupled the effects of gravity, and have focussed on degrees of freedom which are geometrically nearby the GUT seven-brane. In a globally consistent model, additional degrees of freedom will necessarily be present. Tadpole cancellation conditions will likely require the presence of additional seven-branes and three-branes. Moreover, bulk gravitational modes which propagate in the threefold base could also be present.

In this paper we shall assume that the masses of all degrees of freedom are either specified by high scale supersymmetric dynamics, or are instead correlated with the effects of supersymmetry breaking. Since supersymmetry breaking originates from the X field, it follows that proximity to the PQ seven-brane strongly influences the mass scale for the corresponding degree of freedom. Very massive modes can always be integrated out, so we shall focus on the mass scales of objects with mass correlated with either the Higgs or X field vev. Generating masses for scalars is typically more straightforward than for fermions, and so we shall focus on the ways in which fermions can develop mass.

First consider objects which are charged under the GUT seven-brane. Such matter fields can develop a mass by coupling to the Higgs fields. Once the Higgs develops a suitable vacuum expectation value, this induces a weak scale mass for the corresponding particles. Here, we are using a loose notion of "weak scale" so that for example the mass of the electron is effectively controlled by the same dynamics. In gauge mediation scenarios, the soft scalars and gauginos all develop mass through loop suppressed interactions with the messenger fields. Communicating supersymmetry breaking to the MSSM then leads to a sparticle spectrum with masses in the range of $\sim 100 - 1000\,\text{GeV}$.

The situation is far different for matter fields which are not charged under the GUT group. Such matter fields subdivide into those which are charged under $U(1)_{\text{PQ}}$, and those which are uncharged. Assuming that the relevant degrees of freedom are relatively "nearby" the GUT seven-brane, the suppression scale Λ_{UV} for the relevant higher dimension operators will be close to the GUT scale. In this case, the relevant F-terms and D-terms which can generate masses for all components of chiral superfields Φ_1 and Φ_2 with bosonic and fermionic components ϕ_1, ϕ_2 and ψ_1, ψ_2 are:[3]

$$\int d^4\theta \frac{X^\dagger \Phi_1 \Phi_2}{\Lambda_{\rm UV}} \rightarrow \mu \int d^2\theta \Phi_1 \Phi_2 \tag{2.4}$$

$$\int d^4\theta \frac{X^\dagger X \Phi_1 \Phi_2}{\Lambda_{\rm UV}^2} \rightarrow |\mu|^2 \phi_1 \phi_2 + \frac{\mu \cdot x}{\Lambda_{\rm UV}} \psi_1 \psi_2 + \cdots \tag{2.5}$$

$$\int d^2\theta X \Phi_1 \Phi_2 \rightarrow F_X \phi_1 \phi_2 - |x|^2 |\phi_i|^2 + x \psi_1 \psi_2, \tag{2.6}$$

where in the first two lines we have used the rough relation [3]:

$$\mu \sim \frac{\overline{F_X}}{\Lambda_{\rm UV}}. \tag{2.7}$$

Note that the first and third lines both require non-trivial PQ charge for at least one of the chiral superfields Φ_i. In particular, we conclude that PQ charged objects, or direct interactions with PQ objects lead to weak scale masses, while uncharged chiral multiples have lower fermionic masses.

Integrating out the heavy PQ gauge boson generates operators of the form:

$$4\pi\alpha_{\rm PQ} e_X e_\Phi \int d^4\theta \frac{X^\dagger X \Phi^\dagger \Phi}{M_{{\rm U}(1)_{\rm PQ}}^2} \rightarrow 4\pi\alpha_{\rm PQ} e_X e_\Phi \left| \frac{F_X}{M_{{\rm U}(1)_{\rm PQ}}} \right|^2 |\phi|^2 , \tag{2.8}$$

where $\alpha_{\rm PQ}$ denotes the fine structure constant of the PQ gauge theory, and e_X and e_Φ denote the charges of X and Φ under $\mathrm{U}(1)_{\rm PQ}$. This term generates a contribution to the mass squared of the scalar component, but does not produce a fermionic mass. An interesting feature of this "PQ deformation":

$$\Delta_{\rm PQ} \propto \left| \frac{F_X}{M_{{\rm U}(1)_{\rm PQ}}} \right| \tag{2.9}$$

is that depending on the relative charges of X and Φ, this can either produce a positive or tachyonic contribution to the overall mass squared.

The presence of the PQ deformation actually provides another potential means by which fields could develop weak scale masses. For appropriate PQ charges, the PQ deformation induces a tachyonic mass squared, so that the corresponding field can develop a non-zero vev on the order of the size of the PQ deformation. Couplings between this singlet and other fields can then generate weak scale masses, either through cubic superpotential terms, or through couplings to vector multiplets.

The fermions of vector multiplets can also develop mass through couplings to X:

$$\int d^2\theta \log X \cdot Tr\left(W^\alpha W_\alpha\right) \rightarrow \frac{F_X}{x} \lambda^\alpha \lambda_\alpha \tag{2.10}$$

$$\int d^4\theta \frac{X^\dagger X}{\Lambda_{\rm UV}^2} \cdot Tr\left(W^\alpha W_\alpha\right) \rightarrow \frac{\overline{\mu} \cdot \overline{x}}{\Lambda_{\rm UV}} \cdot \lambda^\alpha \lambda_\alpha. \tag{2.11}$$

[3] While it is tempting to include contributions of the form $\int d^4\theta \log |X|^2 \Phi^\dagger \Phi$, as would be present in gauge mediated supersymmetry breaking, note that this generates mass for the scalars of the chiral multiplet, but not the fermions.

Let us comment on the two mass scales present in lines (2.10) and (2.11). The first case is the characteristic value expected in gauge mediated supersymmetry breaking, and is due to integrating out heavy messenger fields. This type of coupling requires, however, that the corresponding vector multiplet couple to fields which are charged under $U(1)_{PQ}$. Indeed, in the absence of such fields, line (2.11) establishes a far lower mass for the corresponding gaugino, which is in line (up to factors of the gauge coupling) with the estimate of line (2.5).

Returning to the explicit values of x and F_X given in equations (2.2) and (2.3), it follows that the F-term coupling of line (2.6) produces masses for the scalars and bosons far above the weak scale. On the other hand, we see that generic couplings to the X field always generate weak scale to TeV scale masses for the bosons. Note, however, that while the operators $X^\dagger \Phi_1 \Phi_2 / \Lambda_{\rm UV}$ and $\log X \cdot Tr\left(W^\alpha W_\alpha\right)$ leads to fermion masses in the same range, in the case of the operators $X^\dagger X \Phi_1 \Phi_2 / \Lambda_{\rm UV}^2$ and $X^\dagger X Tr\left(W^\alpha W_\alpha\right) / \Lambda_{\rm UV}^2$, the corresponding operators have far lower mass. Indeed, in this case, the fermion masses are:

$$\left| m_{\rm fermion}^{(\Lambda_{\rm UV})} \right| \sim \left| \frac{\mu \cdot x}{\Lambda_{\rm UV}} \right| \sim |\mu| \cdot 10^{-3} \sim 100 - 1000 \text{ MeV}. \tag{2.12}$$

In all cases, the essential point is that generating a weak or TeV scale mass for the fermions requires the degree of freedom to closely interact with PQ charged fields.

Next consider degrees of freedom which propagate in the bulk of the threefold base, or which are completely sequestered from the PQ seven-brane. In such cases, the relevant suppression scale for all higher dimension operators is more on the order of the reduced Planck scale $M_{\rm PL} \sim 2.4 \times 10^{18}$ GeV rather than the GUT scale. The absence of PQ charged objects significantly limits the available couplings of X between vector and chiral multiplets. Here, the expected mass scales are:

$$\int d^4\theta \frac{X^\dagger X \Phi_1 \Phi_2}{M_{\rm PL}^2} \rightarrow \frac{|\mu|^2 \Lambda_{\rm UV}^2}{M_{\rm PL}^2} \phi_1 \phi_2 + \frac{\mu \cdot x}{\Lambda_{\rm UV}} \frac{\Lambda_{\rm UV}}{M_{\rm PL}} \psi_1 \psi_2 \tag{2.13}$$

$$\int d^4\theta \frac{X^\dagger X}{\Lambda_{\rm UV}^2} \cdot Tr\left(W^\alpha W_\alpha\right) \rightarrow \frac{\Lambda_{\rm UV}}{M_{\rm PL}} \frac{\overline{\mu} \cdot \overline{x}}{\Lambda_{\rm UV}} \cdot \lambda^\alpha \lambda_\alpha . \tag{2.14}$$

In all cases then, the relevant mass scale is related to that of equation (2.12) as:

$$\left| m_{\rm fermion}^{(M_{\rm pl})} \right| \sim \frac{\Lambda_{\rm UV}}{M_{\rm PL}} \left| m_{\rm fermion}^{(\Lambda_{\rm UV})} \right| \sim 1 - 100 \text{ keV}. \tag{2.15}$$

Bulk gravitational degrees of freedom have similar mass to that of the gravitino. In the context of F-theory GUTs, the mass of the gravitino is:

$$m_{3/2} \sim \frac{F_X}{M_{\rm PL}} \sim 10 - 100 \text{ MeV}. \tag{2.16}$$

To summarize, weak to TeV scale degrees of freedom must interact with the PQ seven-brane. This implies that the most natural TeV scale dark matter candidates are effectively "nearby" the other ingredients of the local F-theory GUT.

3 Gravitino dark matter and the cosmology of F-theory GUTs

In this section we review some of the main features of the cosmology of F-theory GUTs, and in particular, the fact that the gravitino already provides a very natural dark matter candidate [6]. In certain scenarios, the axion can also contribute towards the dark matter relic abundance. As shown in [6], this is due to a rich interplay between the cosmology of various components of the axion supermultiplet in F-theory GUTs.[4] We now review the primary features of the analysis in [6] which naturally evades some of the typical problems present in the cosmology of gauge mediated supersymmetry breaking scenarios. As in [6], we shall focus on the cosmology of F-theory GUTs, treated as a model of particle physics at temperatures $T < T_{\mathrm{RH}}^{0}$, where T_{RH}^{0} denotes the "initial reheating temperature" of the Universe, which corresponds to the temperature at which the Universe transitions to an era of radiation domination.

Since F-theory GUTs correspond to a deformation away from the minimal gauge mediation scenario, the gravitino corresponds to the lightest supersymmetric partner with mass:

$$m_{3/2} \sim \frac{F_X}{M_{\mathrm{PL}}} \sim 10 - 100 \text{ MeV} \tag{3.1}$$

where $\sqrt{F_X}$ denotes the scale of supersymmetry breaking. In the context of supersymmetric models with a stable gravitino in this mass range, there is a strong tendency to over-produce gravitinos because the gravitino decouples from the thermal bath quite early on in the thermal history of the Universe. Indeed, in the context of F-theory GUTs, the freeze out temperature for gravitinos is:

$$T_{3/2}^{f} \sim 10^{10} \text{ GeV}. \tag{3.2}$$

One common way to solve the "gravitino problem" is to lower the reheating temperature T_{RH}^{0} to the point where the relic abundance of gravitinos is truncated to a sufficiently low level. For example, in the parameter range preferred by a $10-100$ MeV mass gravitino, this translates into the upper bound $T_{\mathrm{RH}}^{0} < 10^{6}$ GeV [40]. This is problematic for generating a suitable baryon asymmetry because aside from the Affleck-Dine mechanism, mechanisms such as leptogenesis and GUT scale baryogenesis typically require thermal processes at much higher temperatures between $10^{12}-10^{16}$ GeV to be available. As we now explain, the cosmology of the saxion in F-theory GUTs plays a crucial role in bypassing this constraint.

The scalar part of the axion supermultiplet corresponds to two real degrees of freedom, given by the axion, and the saxion. The mass of the saxion is sensitive to a stringy effect which also shifts the soft scalar masses of the MSSM. With respect to this parameter Δ_{PQ}, the corresponding mass of the saxion is:

$$m_{\mathrm{sax}} \propto \Delta_{\mathrm{PQ}}, \tag{3.3}$$

where the actual constant of proportionality depends on details of the saxion potential, and how $\mathrm{SU}(5) \times \mathrm{U}(1)_{\mathrm{PQ}}$ embeds in E_8 [3, 8].

[4]Although ultimately different, see also [38, 39] for features of gravitino dark matter in the context of the "sweet spot" model of supersymmetry breaking.

The cosmological history of the saxion leads to a significant shift in the expected relic abundance of the gravitino. Below the initial reheating temperature $T^0_{\rm RH}$, the saxion field will typically be displaced from its origin by some initial amplitude. At a temperature $T^{\rm sax}_{\rm osc}$, this field begins to oscillate. An interesting numerical coincidence is that in F-theory GUTs:

$$T^{\rm sax}_{\rm osc} \sim T^f_{3/2} \sim 10^{10}~{\rm GeV}. \tag{3.4}$$

At somewhat lower temperatures, the energy stored in the oscillation of the saxion comes to dominate the energy density of the Universe.

The era of saxion domination terminates when the saxion decays. This typically occurs above the starting temperature for BBN, $T_{\rm BBN} \sim 1 - 10\,{\rm MeV}$. Efficient reheating of the Universe from saxion decay further requires that the saxion decay primarily to Higgs fields, which imposes a lower bound on its mass. Remarkably, this translates into a lower bound on the size of the PQ deformation parameter on the order of 50 GeV.

The decay of the saxion significantly dilutes the relic abundance of all thermally produced relics. The relation between the relic abundances before and after the decay of the saxion are:

$$\Omega_{\rm after} = D_{\rm sax} \Omega_{\rm before}, \tag{3.5}$$

where in the context of F-theory GUTs the saxion dilution factor is roughly:

$$D_{\rm sax} \sim \frac{M^2_{\rm PL}}{s^2_0} \frac{T^{\rm sax}_{\rm RH}}{\min(T^{\rm sax}_{\rm osc}, T^0_{\rm RH})}, \tag{3.6}$$

with $T^{\rm sax}_{\rm osc}$ the temperature of saxion oscillation, and $T^{\rm sax}_{\rm RH}$ is the temperature at which the saxion reheats the Universe. The initial amplitude s_0 is naturally in the range:

$$s_0 \sim \Lambda_{\rm UV} \sim 10^{15.5}~{\rm GeV}, \tag{3.7}$$

while the saxion reheating temperature is set by the total saxion decay rate:

$$T^{\rm sax}_{\rm RH} \sim 0.5\sqrt{M_{\rm PL}\Gamma_{\rm sax}} \sim 0.1 - 10~{\rm GeV}. \tag{3.8}$$

The typical size of the dilution factor is roughly [6]:

$$D_{\rm sax} \sim 10^{-4} \tag{3.9}$$

in the range of maximal interest when $T^0_{\rm RH} > T^{\rm sax}_{\rm osc}$.

The decay of the saxion dilutes the thermally produced gravitinos, while introducing additional gravitinos as decay products. The confluence of temperatures in equation (3.4) actually causes the resulting relic abundance of thermally produced gravitinos to remain independent of $T^0_{\rm RH}$, provided there is still an era where the oscillations of the saxion dominates the energy density of the Universe. As shown in [6], the resulting relic abundance for thermally produced gravitinos is roughly given as:

$$\Omega^{\rm TP}_{3/2} h^2 \sim 0.01 - 0.1, \tag{3.10}$$

which is in the required range for gravitinos to constitute a substantial component of dark matter. In addition, gravitinos produced through the decay of the saxion can, in certain parameter regimes can account for at most 10% of the gravitino relic abundance. In this regime, the gravitino relic abundance is independent of the reheating temperature T^0_{RH}. This reintroduces possible high temperature mechanisms for generating a suitable baryon asymmetry, such as leptogenesis, or GUT scale baryogenesis.

At lower temperature scales where the reheating temperature T^0_{RH} is below the required temperature for saxion domination, it is also in principle possible for oscillations of the axion to constitute a component of the dark matter relic abundance. In F-theory GUTs, the axion decay constant is roughly $f_a \sim 10^{12}$ GeV, which is the requisite range for the axion to play a prominent role. Here it is important to note that the oscillations of the saxion can sometimes disrupt coherent oscillations of the axion.

As we have already mentioned, the confluence of various temperature scales suggests gravitinos as a natural dark matter component which can in suitable circumstances be supplemented by an axionic dark matter component. On the other hand, both the gravitino and axion only interact with the matter content of the Standard Model through higher derivative terms. This in particular means that such candidates are unlikely to be observed either by direct, or indirect dark matter detection experiments. Given the recent influx of tantalizing hints at dark matter detection, in this paper we focus on whether there are any TeV scale dark matter candidates available in minimal F-theory GUT models which could potentially interact more directly with the Standard Model.

4 Flavor implies E_8

Up to this point, we have simply reviewed many of the ingredients which have figured in previous F-theory GUT constructions. This leaves open the question, however, as to how many of these ingredients are required, and how many are additional inputs necessary to solve a particular phenomenological problem. Here we show that assuming only that there is a flavor hierarchy in the CKM and PMNS matrix *requires* an E_8 point of enhancement! In fact, many of the extra fields left over can then play a specific role in the phenomonology of F-theory GUT scenarios, and in section 5 we classify these options.

The only assumptions we shall make in this section are that the following interaction terms:

$$\int d^2\theta \; 5_H \times 10_M \times 10_M + \int d^2\theta \; \overline{5}_H \times \overline{5}_M \times 10_M + \text{Neutrinos} \tag{4.1}$$

be present, where we shall assume that the neutrino sector can correspond to either a Dirac [8], or Majorana scenario of the form:

$$\text{Dirac:} \quad \int d^4\theta \; \frac{H_d^\dagger L N_R}{\Lambda_{\mathrm{UV}}} \tag{4.2}$$

$$\text{Majorana:} \quad \int d^2\theta \; H_u L N_R + M_{\mathrm{maj}} N_R N_R. \tag{4.3}$$

In addition, we assume that all of the matter fields 5_H, $\overline{5}_H$, 10_M and N_R localize on curves of the geometry. We will not even assume the mechanism of doublet triplet splitting via fluxes proposed in [2], but will instead simply require a much milder genericity statement from index theory that if a four-dimensional zero mode localizes on a curve, then the conjugate representation cannot have any zero modes localized on the same curve.[5]

Flavor hierarchy imposes a number of significant restrictions on the available enhancements in the singularity type. First, the presence of the interaction term $5_H \times 10_M \times 10_M$ requires a point where SU(5) enhances to at least E_6. Moreover, the presence of the interaction term $\overline{5}_H \times \overline{5}_M \times 10_M$ requires an enhancement to at least SO(12). As found in [5], the CKM matrix will exhibit a hierarchical structure provided these points of enhancement are close together, so that there is at least an E_7 point of enhancement.

In this section we show that generating a mildly hierarchical PMNS matrix forces this enhancement up to E_8. Our strategy for obtaining this result will be to ask whether an E_7 point of enhancement is sufficient for realizing all of the required interaction terms. The obstructions we encounter will imply that only E_8 is available. Since E_7 is a subgroup of E_8, we can phrase our analysis in terms of interaction terms inside of E_8. What we will effectively show is that enough of E_8 is used that it cannot all be fit inside of E_7.

The maximal subgroup of E_8 which contains the GUT group SU(5) is $SU(5)_{GUT} \times SU(5)_\perp$. The adjoint representation of E_8 decomposes into irreducible representations of $SU(5)_{GUT} \times SU(5)_\perp$ as:

$$E_8 \supset SU(5)_{GUT} \times SU(5)_\perp \tag{4.4}$$
$$248 \to (1,24) + (24,1) + (5,\overline{10}) + (\overline{5},10) + (10,5) + (\overline{10},\overline{5}). \tag{4.5}$$

Although this would appear to provide a large number of additional ingredients for model building, consistency with qualitative phenomenological features imposes a number of identifications in the low energy effective field theory. For example, the interaction term $5_H \times 10_M^{(1)} \times 10_M^{(2)}$ would at first appear to involve three matter curves. This is problematic, because if the 10_M's localize on distinct curves, then the resulting mass matrix will without fine tuning lead to at least two massive generations [2]. At the level of the effective field theory, achieving one heavy generation then requires the existence of an interchange symmetry:

$$10_M^{(1)} \leftrightarrow 10_M^{(2)}. \tag{4.6}$$

The presence of such symmetries may appear as an extra level of fine tuning. In fact, as noted in [20], such identifications will generically occur. This is essentially because the positions of the seven-branes are dictated by the locations of singularities in the fibration structure of the geometry, which are in turn controlled by polynomial equations in several

[5]After this paper appeared, there have been various claims made in the literature, starting with [41] that activating GUT breaking by a hyperflux is incompatible with doublet triplet splitting, in the sense that it induces exotics in the low energy spectrum. This analysis applies to a very limited class of local models which can be treated using the spectral cover description. It is still an open problem to realize a consistent local model with no charged exotics, but we stress that the obstruction [41] only rules out a small fraction of possible ways of building a local model. See section 5 for further discussion on this point.

JHEP08(2010)040

variables. For example, polynomials with rational numbers for coefficients will often contain non-trivial symmetry groups which interchange the various roots. A similar phenomenon is present in the factorization of polynomials of more than one variable. This means in particular that rather than being a special feature of the geometry, it is to be expected on quite general grounds that such identifications will occur.

In the context of $SU(5)_\perp$, this discrete symmetry group is maximally given by its Weyl group S_5, the symmetric group on five letters. The group S_5 acts by identifying directions in the 5 of $SU(5)_\perp$. From the perspective of the effective field theory, this occurs because inside of $SU(5)_\perp$, there are additional discrete symmetries which act as identifications on the representations of $SU(5)_\perp$. Thus, there will generically be identifications of some of the 5's and 10's. In the physical "quotient theory" where all identifications have taken place, the actual number of distinct matter curves will be greatly reduced compared to the "covering theory". For example, there will be five distinct 10_M curves in the covering theory, since the 10_M transforms as a 5 of $SU(5)_\perp$. Acting with the discrete symmetry group will then identify some of these fields.

As we show in section 5, this analysis leaves us with just a few additional ingredients which can also interact at the same point of enhancement. Quite remarkably, these also play a significant role: They are the fields of the supersymmetry breaking sector utilized in [3]! Thus, simply achieving the correct flavor structure will automatically include the necessary ingredients for supersymmetry breaking.

The rest of this section is organized as follows. Before proceeding to the main result of this section, we first setup notation, and introduce the features of monodromy groups which will be important for our analysis. Using some minimal facts about such monodromy groups coupled with the existence of all interactions in line (4.1) will then allow us to deduce that flavor implies E_8.

4.1 Monodromy groups: generalities

Since the action of monodromy groups play such a crucial role in the analysis to follow, here we explain in more precise terms how this group acts on the available matter curves. Geometrically, matter fields localize along loci where elements in the Cartan of $SU(5)_\perp$ combine with $SU(5)_{\mathrm{GUT}}$ so that the singularity type enhances. The precise location of each matter curve can be analyzed by introducing a weight space decomposition of the various representations. The Cartan of $SU(5)_\perp$ can be parameterized by the coordinates $t_1, \ldots, t_5$ subject to the constraint:

$$t_1 + \ldots + t_5 = 0. \tag{4.7}$$

The weights of the $5_\perp$, $10_\perp$ and $24_\perp$ of $SU(5)_\perp$ are then given as:

$$5_\perp : t_i \tag{4.8}$$

$$10_\perp : t_i + t_j \tag{4.9}$$

$$24_\perp : \pm (t_i - t_j) + 4 \times (0 \text{ weights}) \tag{4.10}$$

for $1 \leq i, j \leq 5$ such that $i \neq j$. Using the decomposition of the adjoint of E_8 into irreducible representations of $SU(5)_{GUT} \times SU(5)_{\perp}$, the matter fields with appropriate $SU(5)_{GUT}$ representation content localize along the following curves:

$$5_{\mathrm{GUT}} : -t_i - t_j = 0 \tag{4.11}$$

$$10_{\mathrm{GUT}} : t_i = 0 \tag{4.12}$$

$$1_{\mathrm{GUT}} : t_i - t_j = 0, \tag{4.13}$$

so that the vanishing loci then correspond to local enhancements in the singularity type of the compactification. A matter field with a given weight will necessarily also be charged under a U(1) subgroup of $SU(5)_{\perp}$, dictated by its weight.

Geometric considerations and minimal requirements from phenomenology impose significant constraints. For example, the deformation of a singularity will generically contain monodromies in the seven-brane configuration whereby some of the matter curves will in fact combine to form a single irreducible matter curve. Group theoretically, the deformations of a singularity are parameterized by the Cartan modulo the Weyl subgroup of the singularity. In the present context, the Weyl group of $SU(5)_{\perp}$ is S_5, the symmetric group on five letters. This discrete group acts as permutations on the five t_i's. The most generic geometry will involve identifications by the full S_5 monodromy group. In particular to have any smaller monodromy we need to assume certain factorization properties of the unfolding singularity. But this generic choice is already too restrictive: For example it will force the $\overline{5}_M$,H_d, H_u to all come from a single curve, which would be in conflict with the resolution of the doublet-triplet splitting problem found in [2], and would also not allow a consistent identification of matter parity. Thus to fit with phenomenological constraints we need to assume a less generic monodromy group than S_5. The bigger the monodromy group, the more generic the geometry. In this sense, the most generic monodromies we find are for two Majorana neutrino scenarios discussed in section 5, where the monodromy group is the order 8 symmetry group of the square, Dih_4. Quite remarkably in these cases, all the available orbits which are charged under the standard model gauge group are utilized in the minimal GUT model, including the messenger fields!

Note that the monodromy group cannot be trivial. Indeed, the appearance of monodromies is quite important for achieving a single massive generation in the up-type quark sector. For example, as found in [20], a rank one $5_H \times 10_M \times 10_M$ is quite natural once monodromies in the seven-brane configuration are taken into account. In the covering theory, there are then at least two 10_M curves, which are exchanged under monodromy.

Consistency with all necessary interaction terms then implies that the monodromy group may act non-trivially on the other covering theory matter fields of F-theory GUTs. Each matter field in the covering theory fills out an orbit under the monodromy group, G, so that if w denotes the corresponding weight, then elements of the form:

$$Orb(w) = \{\sigma(w) | \sigma \in G\} \tag{4.14}$$

are all identified under the action of the monodromy group. We shall often refer to the

JHEP08(2010)040

"length" of the orbit as the number of elements so that:

$$\text{Length}(Orb(w)) = \#Orb(w). \tag{4.15}$$

As a final piece of notation, we will often denote the action of a permutation group element using cycle notation. For example, (123) acts on the weights $t_1, \ldots, t_5$ as:

$$(t_1, t_2, t_3, t_4, t_5) \overset{(123)}{\longrightarrow} (t_2, t_3, t_1, t_4, t_5). \tag{4.16}$$

The element (12)(34) instead acts on the weights as:

$$(t_1, t_2, t_3, t_4, t_5) \overset{(12)(34)}{\longrightarrow} (t_2, t_1, t_4, t_3, t_5). \tag{4.17}$$

In our conventions, multiplication of two elements proceeds as in the composition of two functions, so that $(123) \cdot (12) = (13)$.

The monodromy group will also sometimes identify continuous global symmetries. The effect of this can lead to the presence of additional discrete symmetries in the low energy theory.

Returning to the interaction term $5_H \times 10_M \times 10_M$, the weight assignments in $\text{SU}(5)_\perp$ for the 5_H and 10_M's are of the form $-t_i - t_j$ and t_k, respectively. If these weight assignments form an invariant interaction term, then they must satisfy the constraint:

$$(-t_i - t_j) + (t_k) + (t_l) = 0, \tag{4.18}$$

where we have grouped the weight assignments for each field by parentheses. In this case, it follows that the weights for the 10_M's must correspond to t_i and t_j. The existence of a single orbit for the 10_M's implies that there must exist an element of the monodromy group which sends t_i to t_j. Note, however, that a given element in the orbit of the 5_H need not form an interaction term with all of the weights in the orbit of the 10_M's. Indeed, the only condition is that there is some weight in the orbit of the 10_M which can form an appropriate interaction.

4.2 E_7 does not suffice

We now proceed to show that quark and lepton flavor hierarchies are incompatible with a single point of E_7 enhancement. To start our analysis, recall that we require the following interaction terms:

$$\int d^2\theta \; 5_H \times 10_M \times 10_M + \int d^2\theta \; \overline{5}_H \times \overline{5}_M \times 10_M + \text{Neutrinos}. \tag{4.19}$$

Suppose to the contrary that an E_7 point of enhancement did suffice for all required interaction terms. In this case, the resulting breaking pattern would fit as $E_8 \supset E_7 \times \text{SU}(2)_\perp$. In particular, of the five t_i's present in $\text{SU}(5)_\perp$, the monodromy group inside of E_7 would have to leave the generators t_4 and t_5 invariant. Indeed, this $\text{SU}(2)_\perp$ is one factor in the "Standard Model subgroup" $\text{SU}(3)_\perp \times \text{SU}(2)_\perp \times \text{U}(1)_\perp \subset \text{SU}(5)_\perp$. Our strategy will be to show that the orbits non-trivially involve the weights from the $\text{SU}(2)_\perp$ factor. Thus, we will have established that the only available enhancement point must be E_8.

A necessary condition in this regard is that all available weights must be uncharged under the Cartan generator of this $SU(2)_\perp$ factor. Viewing this generator as an element in the dual space, this direction can be written as:

$$t^*_{SU(2)_\perp} \equiv t^*_4 - t^*_5, \tag{4.20}$$

so that:

$$t^*_{SU(2)_\perp}(t_4) = +1 \tag{4.21}$$

$$t^*_{SU(2)_\perp}(t_5) = -1 \tag{4.22}$$

and for $i = 1, 2, 3$,

$$t^*_{SU(2)_\perp}(t_i) = 0. \tag{4.23}$$

The available monodromy groups which act on t_1, t_2 and t_3 are given by permutations on these letters. The subgroups of the symmetric group on three letters are isomorphic to S_3, $\mathbb{Z}_3$, $\mathbb{Z}_2$ and the trivial group. Since the $5_H \times 10_M \times 10_M$ interaction requires at least two weights in the orbit of the 10_M, it follows that the monodromy group must be non-trivial. We now show that in all cases, the available monodromy group orbits are inconsistent with a neutrino sector.

4.2.1 $G_{\text{mono}} \simeq \mathbb{Z}_3$ or S_3

First suppose that the monodromy group is either S_3 or $\mathbb{Z}_3$. In this case, there exists an order three element which acts on the weights, which without loss of generality we take as:

$$(123) \in G_{\text{mono}} \tag{4.24}$$

Since the orbit for the 10_M is non-trivial, it follows that this orbit must in fact involve t_1, t_2 and t_3. In particular, compatibility with the interaction term $5_H \times 10_M \times 10_M$ now implies that the orbit for the 5_H must involve $-t_1 - t_2$, $-t_2 - t_3$, $-t_1 - t_3$. Since index theory considerations require the $\overline{5}$'s to localize on a curve distinct from 5_H, there are precisely three orbits available, which we label by subscripts:

$$Orb(\overline{5}_{(1)}) = t_1 + t_4, t_2 + t_4, t_3 + t_4 \tag{4.25}$$

$$Orb(\overline{5}_{(2)}) = t_1 + t_5, t_2 + t_5, t_3 + t_5 \tag{4.26}$$

$$Orb(\overline{5}_{(3)}) = t_4 + t_5. \tag{4.27}$$

The last option given by $Orb(\overline{5}_{(3)}) = t_4 + t_5$ is incompatible with the interaction term $\overline{5}_H \times \overline{5}_M \times 10_M$. Indeed, this interaction term requires the weights for the various fields to satisfy:

$$(t_i + t_j) + (t_k + t_l) + t_m = 0, \tag{4.28}$$

Where $t_m = t_1, t_2$ or t_3. In all available cases where $t_4 + t_5$ appears as a weight, either t_4 or t_5 will appear twice, and the sum will not vanish.

We therefore conclude that the only candidate orbits for the $\overline{5}_H$ and $\overline{5}_M$ involve $t_i + t_4$ or $t_i + t_5$ for $i = 1, 2, 3$. Since nothing distinguishes t_4 or t_5 in our analysis so far, we can now fix the orbits of $\overline{5}_M$ to be $t_i + t_4$. Note that compatibility with the interaction term $\overline{5}_H \times \overline{5}_M \times 10_M$ now forces the orbit $\overline{5}_H$ to be $t_i + t_5$. Hence, the available orbits are:

$$Orb(10_M) = t_1, t_2, t_3 \tag{4.29}$$

$$Orb(\overline{5}_M) = t_1 + t_4, t_2 + t_4, t_3 + t_4 \tag{4.30}$$

$$Orb(\overline{5}_H) = t_1 + t_5, t_2 + t_5, t_3 + t_5 \tag{4.31}$$

$$Orb(5_H) = -t_1 - t_2, -t_1 - t_3, -t_2 - t_3. \tag{4.32}$$

It is now straightforward to eliminate both neutrino scenarios. For example, in the Dirac neutrino scenario, the interaction term $H_d^\dagger L N_R / \Lambda_{\text{UV}}$ requires the weight assignments to obey the constraint:

$$(-t_i - t_5) + (t_j + t_4) + (t_m - t_n) = 0. \tag{4.33}$$

In particular, this implies that the weight of N_R is given by $t_4 - t_5$. This, however, requires a non-trivial participation from the roots of SU(2). In other words, the right-handed neutrino will only touch the E_7 point of enhancement provided it is actually an E_8 point of enhancement!

Next consider Majorana neutrino scenarios. In this case, the interaction term $H_u L N_R$ leads to the constraint on the weights:

$$(-t_i - t_j) + (t_k + t_4) + (t_m - t_n) = 0. \tag{4.34}$$

By inspection of the 5_H orbit, it now follows that t_4 can only cancel out provided $t_n = t_4$.

On the other hand, the Majorana mass term $N_R N_R$ requires $t_n - t_m$ must also be present. Thus, there exists another set of weights such that:

$$(-t_{i'} - t_{j'}) + (t_{k'} + t_4) + (t_n - t_m) = 0. \tag{4.35}$$

But this implies that $t_m = t_4$. In other words, the right-handed neutrino is given by the weight $t_n - t_m = t_4 - t_4$, which is a contradiction!

To summarize, in the case where the monodromy group is $\mathbb{Z}_3$ or S_3, the Dirac scenario is consistent with requirements from flavor, but requires an E_8 enhancement point, and in the case of Majorana neutrinos, we do not find a consistent scenario.

4.2.2 $G_{\text{mono}} \simeq \mathbb{Z}_2$

Next suppose that the monodromy group is given by $\mathbb{Z}_2$, so that it acts by interchanging t_1 and t_2:

$$G_{\text{mono}} = \langle (12) \rangle \simeq \mathbb{Z}_2 \tag{4.36}$$

This last conclusion is a consequence of the fact that there are only three t_i's available which can participate in the monodromy group action. In this case, the orbits for the 10_M and 5_H are completely fixed to be:

$$Orb(10_M) = t_1, t_2 \tag{4.37}$$

$$Orb(5_H) = -t_1 - t_2. \tag{4.38}$$

JHEP08(2010)040

In this case, the available orbits for the $\overline{5}$'s are:

$$Orb(\overline{5}_{(1)}) = t_1 + t_3, t_2 + t_3 \tag{4.39}$$
$$Orb(\overline{5}_{(2)}) = t_1 + t_4, t_2 + t_4 \tag{4.40}$$
$$Orb(\overline{5}_{(3)}) = t_1 + t_5, t_2 + t_5 \tag{4.41}$$
$$Orb(\overline{5}_{(4)}) = t_3 + t_4 \tag{4.42}$$
$$Orb(\overline{5}_{(5)}) = t_3 + t_5 \tag{4.43}$$
$$Orb(\overline{5}_{(6)}) = t_4 + t_5. \tag{4.44}$$

Fixing the weight of 10_M as t_1, this imposes the weight constraint:

$$(t_i + t_j) + (t_k + t_l) + t_1 = 0. \tag{4.45}$$

The available pairs of orbits which can satisfy this constraint are then:

$$\text{Option 1: } Orb(\overline{5}_{(1)}) = t_1 + t_3, t_2 + t_3 \text{ and } Orb(\overline{5}_{(6)}) = t_4 + t_5 \tag{4.46}$$
$$\text{Option 2: } Orb(\overline{5}_{(2)}) = t_1 + t_4, t_2 + t_4 \text{ and } Orb(\overline{5}_{(5)}) = t_3 + t_5 \tag{4.47}$$
$$\text{Option 3: } Orb(\overline{5}_{(3)}) = t_1 + t_5, t_2 + t_5 \text{ and } Orb(\overline{5}_{(4)}) = t_3 + t_4. \tag{4.48}$$

It is now immediate that all Dirac scenarios are ruled out. Indeed, the presence of the operator $H_d^\dagger L N_R / \Lambda_{\text{UV}}$ requires the weight constraint:

$$\text{Option 1: } (-t_i - t_3) + (t_4 + t_5) + (t_m - t_n) = 0 \tag{4.49}$$
$$\text{Option 2: } (-t_i - t_4) + (t_3 + t_5) + (t_m - t_n) = 0 \tag{4.50}$$
$$\text{Option 3: } (-t_i - t_5) + (t_3 + t_4) + (t_m - t_n) = 0. \tag{4.51}$$

Since $t_i = t_1$ or t_2, it follows that there does not exist a solution for any t_m and t_n. Thus, we conclude that in all Dirac scenarios, flavor considerations require E_8.

Next consider Majorana scenarios. In this case, the presence of the operator $H_u L N_R$ imposes the weight constraint:

$$(-t_1 - t_2) + (t_j + t_k) + (t_m - t_n) = 0. \tag{4.52}$$

We therefore conclude that $t_m = t_1$ or t_2. In fact, since both $t_m - t_n$ and $t_n - t_m$ must be present in the same orbit, it now follows that the orbit for N_R is precisely given as:

$$Orb(N_R) = t_1 - t_2, t_2 - t_1. \tag{4.53}$$

Returning to equation (4.52), it now follows that the weight $t_j + t_k$ must satisfy the constraint:

$$(-t_1 - t_2) + (t_j + t_k) + (t_1 - t_2) = 0, \tag{4.54}$$

which is not possible, because t_2 appears twice in the constraint. Hence, in all cases we find that an E_7 point of enhancement cannot accommodate either a Dirac, or a Majorana scenario.

It is quite remarkable that flavor considerations from quarks and leptons push the enhancment point all the way up to E_8. We now turn to a classification of possible mondromy group orbits compatible with the other elements of F-theory GUTs.

JHEP08(2010)040

5 Matter and monodromy in E_8

Taking seriously the idea of unification naturally suggests combining the various ingredients of F-theory GUTs in the minimal number of geometric ingredients necessary. Indeed, as we have seen in section 4, the presence of an E_8 point is not so much an aesthetic criterion, as a *necessary* one in order to generate hierarchical CKM and PMNS matrices. The aim of this section is to classify F-theory GUT models consistent with this E_8 point of enhancement. The only assumptions we shall make are that the following interaction terms be present in the low energy theory:

$$\int d^2\theta\; 5_H \times 10_M \times 10_M + \int d^2\theta\; \overline{5}_H \times \overline{5}_M \times 10_M + \text{Neutrinos} + \int d^4\theta \frac{X^\dagger H_u H_d}{\Lambda_{\text{UV}}}, \quad (5.1)$$

where here, X is a GUT singlet localized on a curve. In fact, the result we obtain will not strictly require $X^\dagger H_u H_d/\Lambda_{\text{UV}}$, but will also apply to μ-terms generated through the vev of a GUT singlet S through the F-term:[6]

$$\int d^2\theta\; S H_u H_d. \quad (5.2)$$

In order for a GUT scale μ-term to not be present, we shall also require the presence of a continuous global Peccei-Quinn symmetry. In the context of Majorana neutrino scenarios, there is a unique PQ symmetry available, with charge assignments [8]:

	$\overline{5}_M$	10_M	5_H	$\overline{5}_H$	$X^\dagger$	N_R
Majorana $U(1)_{PQ}$	$+2$	$+1$	-2	-3	$+5$	0

. (5.3)

In the case of Dirac neutrino scenarios, there is a certain degree of flexibility. For simplicity, we shall take the same convention for PQ charge assignments used in [3, 8]:

	$\overline{5}_M$	10_M	5_H	$\overline{5}_H$	$X^\dagger$	N_R
Dirac $U(1)_{PQ}$	$+1$	$+1$	-2	-2	$+4$	-3

. (5.4)

Summarizing, we shall classify all available monodromy group actions in F-theory GUTs consistent with the assumptions:

- Hierarchical CKM and PMNS matrices
- μ-term from the vev of a GUT singlet
- A PQ symmetry of the type given by line (5.3) for Majorana scenarios and (5.4) for Dirac scenarios

A remarkable byproduct of this analysis is that after performing this classification, there is typically just enough room for a messenger sector of a minimal gauge mediated supersymmetry breaking scenario where the field X couples to a vector-like pair of messengers Y and Y' through the F-term:

$$L \supset \int d^2\theta\; XYY'. \quad (5.5)$$

[6]Note that the field S whose vev gives rise to the μ-term is related to our field X by $S = \overline{D}^2 X^\dagger/\Lambda_{\text{UV}}$.

In fact, one of the messengers localizes on the same curve as a matter field. More surprisingly, in all but one Dirac scenario, the actual representation content of the messengers will uniquely be fixed to transform in the $10 \oplus \overline{10}$ of $SU(5)_{GUT}$! This has distinctive consequences for phenomenology, which we shall comment on later in this section. See figure 3 for a depiction of the relevant geometry.

Quotienting by a discrete subgroup of S_5 can also generate highly non-trivial symmetries in the effective field theory. For example, the identification of two U(1) factors in $U(1) \times U(1)$ can lead to discrete $\mathbb{Z}_2$ subgroup factors.

The action of the mondromy group in Majorana scenario eliminates essentially all other gauge degrees of freedom other than $U(1)_{PQ}$. In the case of the Dirac scenarios, we find that there is one additional gauge boson $U(1)_\chi$ which is a linear combination of $U(1)_Y$ and $U(1)_{B-L}$. In addition, we also classify the available chiral multiplets which can localize on matter curves normal to the GUT seven-brane. These can then constitute potential dark matter candidates, although we shall return to issues connected with dark matter later in section 9.

The rest of this section is organized as follows. We first explain in greater detail the main conditions which we shall demand of the monodromy group action at the E_8 enhancement point. Next, we proceed to review the available monodromy groups in Dirac and Majorana neutrino scenarios. See appendix A for further details of this classification. A remarkable feature of the classification is that in all but one scenario, it forces the messengers to transform in the $10 \oplus \overline{10}$ of $SU(5)_{GUT}$. After commenting on some of the implications of this for phenomenology, we next discuss potential "semi-visible" dark matter candidates corresponding to electrically neutral components of non-trivial $SU(5)_{GUT}$ multiplets.

5.1 Flux and monodromy

Before reviewing the main elements of the classification, we first discuss some of the necessary conditions on matter curves and fluxes which a monodromy group action must respect in order to remain consistent with the assumptions spelled out at the beginning of this section.

Recall that the chiral matter is determined by the choice of background fluxes through the matter curves of the geometry. In particular, we must require that if a zero mode in a representation localizes on a curve, then the conjugate representation cannot appear. In this context, keeping the 5_H, $\overline{5}_H$ and $\overline{5}_M$ localized on distinct curves imposes the following condition on the orbits of these fields:

$$Orb(\overline{5}_M),\ Orb(\overline{5}_H),\ \overline{Orb(5_H)} \text{ all distinct.} \tag{5.6}$$

It is important to note that the flux passing through a matter curve need not give precisely three generations. For example, a 10_M curve could in principle contain another GUT multiplet, and another curve could contain an extra $\overline{10}_M$. Indeed, as noted in [8], because both sets of fields fill out full GUT multiplets, it is in principle possible to allow some of the messengers and matter fields to localize on the same curve. This will prove quite important when we discuss conditions necessary for unifying a messenger sector with the other matter content of F-theory GUTs.

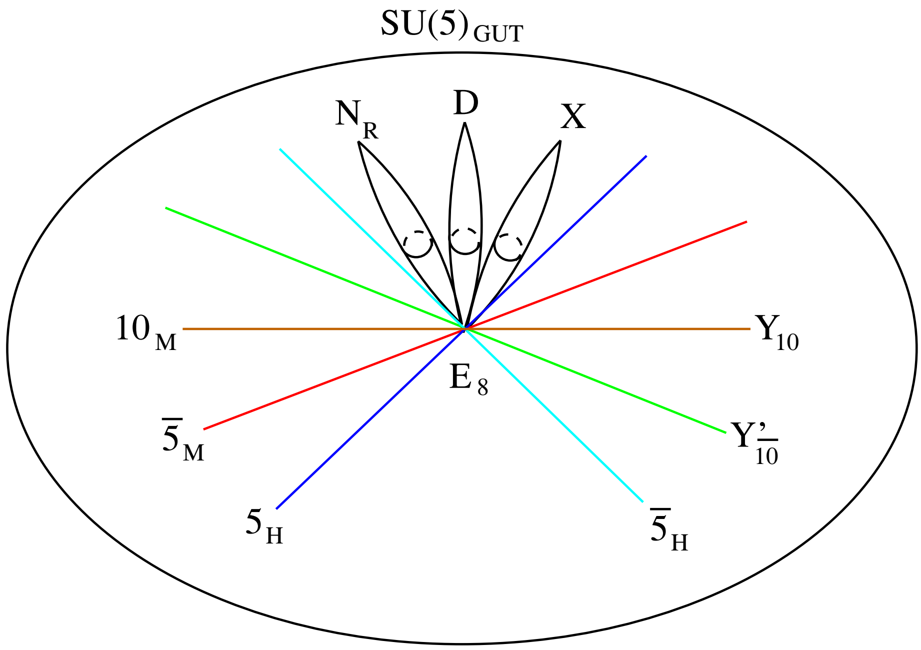

Figure 3. Depiction of an F-theory GUT in which all of the necessary interaction terms descend from a single point of E_8 enhancement. In all but one Dirac neutrino scenario, accommodating messenger fields in the gauge mediated supersymmetry breaking sector turns out to force the messengers (Y_{10} and $Y'_{\overline{10}}$) to transform in the $10 \oplus \overline{10}$ of $SU(5)_{GUT}$. GUT singlets such as the right-handed neutrinos N_R and X field localize on curves normal to the GUT seven-brane. Here we have also included a dark matter candidate D which is localized on a curve.

Before closing this subsection, let us also comment on the sense in which we demand an E_8 structure. The primary condition we consider in this paper is that in a sufficiently small patch of the Kähler surface wrapped by the GUT seven-brane, there is a point of enhancement to E_8. This does *not* mean that the geometries we consider must descend from a single globally well-defined E_8 singularity over the entire surface, as in [23]. Indeed, it is a very strong assumption on the class of compactifications to literally import all of the structures of the perturbative heterotic string to the F-theory setting. To give just one example of how this can fail, consider the heterotic $E_8 \times E_8$ theory compactified on a T^2 with radii chosen so that the full gauge group is $E_8 \times E_8 \times SU(2)$. This latter factor is simply missing from the spectral cover description, and points to a significant limitation on the class of compactifications covered by such global unfoldings. Along these lines, let us also note that though there is a Kodaira classification of degenerations of an elliptic fiber for K3 surfaces, no such classification is known for codimension three singularities, of the type considered here.[7] In particular, though we can find a *local patch* which describes the singular fibers in terms of enhancement to E_8, extending this over the surface S may include

[7]As a related point, let us note that orbifold singularities given by quotienting $\mathbb{C}^2$ by a discrete subgroup of SU(2) admit an ADE classification, though there is no similar ADE classification for orbifolds of $\mathbb{C}^3$ and $\mathbb{C}^4$, in part because now collapsing Kähler surfaces may be present at these more general orbifold singularities.

additional seven-branes which intersect the GUT stack. These additional six-dimensional matter fields cannot be embedded inside a single E_8 factor, but nevertheless are expected to be present in a general compactification.

The reason that it is important to point out these limitations is that certain statements have appeared in the literature, which appear to have propagated from [41] that there is an obstruction to activating a GUT breaking flux without inducing charged exotics. Under the strong assumptions that the geometry over S is described by the unfolding of a *single* globally defined E_8, and moreover, that no additional factorization occurs in the discriminant locus, this can be established [41]. However, this result crucially relies on knowing that everything unfolds from a single global E_8. It is through this assumption that one determines more information about the homology classes of the matter curves. Since the index theory is controlled by the cohomology class of the gauge field flux and the homology classes of the matter curves, this can in principle produce a non-trivial constraint on the low energy content.

But in the case of a single E_8 unfolding at a point, we in general expect to have some flexibility in the homology class of the corresponding matter fields. Indeed, it is only after compactifying all curves by specifying the content of the unfolding over all of S that we can hope to read off the homology classes, and thus determine by index theory considerations the chiral matter content on a curve. To summarize and repeat: In general local models based on a compact Kähler surface S with a non-compact normal direction, *no obstruction to achieving doublet triplet splitting with hyperflux has been proven in general*, and we shall assume in this paper that this condition can be satisfied.

5.2 Dirac scenarios

We now discuss the various Dirac neutrino scenarios. We refer to appendix A for a derivation of the orbit classification. As found in appendix A, there are essentially three distinct orbits available for matter fields in Dirac neutrino scenarios. In all three cases, we find that the monodromy group preserves two U(1) factors in $SU(5)_\perp$. Labeling these U(1) factors as $U(1)_{PQ}$ and $U(1)_\chi$ the charge assignments we find for the visible matter are:

Minimal Matter	$10_M, Y_{10}$	$\overline{5}_M, Y'_{\overline{5}}$	$Y'_{\overline{10}}$	Y_5	5_H	$\overline{5}_H$	$X^\dagger$	N_R
$U(1)_{PQ}$	$+1$	$+1$	$+3$	$+3$	-2	-2	$+4$	-3
$U(1)_\chi$	-1	$+3$	$+1$	-3	$+2$	-2	0	-5

. (5.7)

where the Y's denote messenger fields, and the subscript indicates the representation content. An interesting feature of this analysis is that it suggests a potentially more general choice for the $U(1)_{PQ}$ charges. The symmetry $U(1)_\chi$ is a linear combination of hypercharge and $U(1)_{B-L}$. In an integral normalization of $U(1)_Y$ so that the lepton doublet has $U(1)_Y$ charge -3, the $B-L$ generator is given as:

$$U(1)_{B-L} = -\frac{1}{5}U(1)_\chi + \frac{2}{15}U(1)_Y. \tag{5.8}$$

Note that the charge assignments in the messenger sector have non-trivial $U(1)_{B-L}$ charge. As one final comment, although our conventions for the $U(1)_{PQ}$ symmetry are chosen so as to coincide with those taken in [3, 8], there is a more general possibility for the "PQ deformation" given by taking an arbitrary linear combination of $U(1)_{PQ}$ and $U(1)_\chi$.

The actual choice of the $U(1)_{PQ}$ generator naturally splits into two distinct scenarios, depending on the action of the monodromy group. We find that the monodromy group is isomorphic to $\mathbb{Z}_2$, $\mathbb{Z}_2 \times \mathbb{Z}_2$, $\mathbb{Z}_3$ or S_3. Invariance under the corresponding subgroup largely fixes the direction of the Cartan. Viewing $U(1)_{PQ}$ and $U(1)_\chi$ as elements in the vector space dual to the weights spanned by the t_i's, we find that when the monodromy group is isomorphic to either $\mathbb{Z}_2$ or $\mathbb{Z}_2 \times \mathbb{Z}_2$,

$$G^{\text{Dirac}}_{\text{mono}} \simeq \mathbb{Z}_2 \text{ or } \mathbb{Z}_2 \times \mathbb{Z}_2: \tag{5.9}$$
$$t^*_{PQ} = t^*_1 + t^*_2 - 3t^*_3 - 3t^*_4 + 4t^*_5 \tag{5.10}$$
$$t^*_\chi = -(t^*_1 + t^*_2 + t^*_3 + t^*_4) + 4t^*_5, \tag{5.11}$$

where the t^*_i correspond to weights in the dual space such that:

$$t^*_i(t_j) = \delta_{\text{ij}}. \tag{5.12}$$

The monodromy group action leaves these generators invariant. With respect to this convention, $\mathbb{Z}_2$ is generated by the permutation group element (12)(34), which acts in the obvious way on the t_i's. The $\mathbb{Z}_2 \times \mathbb{Z}_2$ group is generated by (12) and (34). In the case where the monodromy group is isomorphic to $\mathbb{Z}_3$ or S_3, the generators for $U(1)_{PQ}$ are instead given as:

$$G^{\text{Dirac}}_{\text{mono}} \simeq \mathbb{Z}_3 \text{ or } S_3: \tag{5.13}$$
$$t^*_{PQ} = t^*_1 + t^*_2 + t^*_3 - 3t^*_4 \tag{5.14}$$
$$t^*_\chi = -(t^*_1 + t^*_2 + t^*_3 + t^*_4) + 4t^*_5. \tag{5.15}$$

Without loss of generality, the $\mathbb{Z}_3$ group can be taken to be generated by the three cycle (123). Having specified the underlying symmetries of each Dirac scenario, we now turn to the explicit orbits in each case.

5.2.1 $\mathbb{Z}_2$ and $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbits

In this subsection we summarize the classification of the orbits in the cases where the monodromy group is isomorphic to either $\mathbb{Z}_2$ or $\mathbb{Z}_2 \times \mathbb{Z}_2$. In this case, the generators for $U(1)_{PQ}$ and $U(1)_\chi$ lie in the directions:

$$G \simeq \mathbb{Z}_2 \text{ or } \mathbb{Z}_2 \times \mathbb{Z}_2: \tag{5.16}$$
$$t^*_{PQ} = t^*_1 + t^*_2 - 3t^*_3 - 3t^*_4 + 4t^*_5 \tag{5.17}$$
$$t^*_\chi = -(t^*_1 + t^*_2 + t^*_3 + t^*_4) + 4t^*_5. \tag{5.18}$$

JHEP08(2010)040

In the case where the monodromy group is generated by the order two element (12)(34), the orbits of the monodromy group are:

$$G_{\text{mono}}^{\text{Dirac}} = \langle (12)(34) \rangle \simeq \mathbb{Z}_2\text{:} \tag{5.19}$$

$$\text{Minimal Matter} \tag{5.20}$$

$$Orb(10_M, Y_{10}) = t_1, t_2 \tag{5.21}$$

$$Orb(Y'_{10}) = -t_3, -t_4 \tag{5.22}$$

$$Orb(\overline{5}_M) = t_4 + t_5, t_3 + t_5 \tag{5.23}$$

$$Orb(5_H) = -t_1 - t_2 \tag{5.24}$$

$$Orb(\overline{5}_H) = t_1 + t_4, t_2 + t_3 \tag{5.25}$$

$$Orb(X^\dagger) = t_2 - t_4, t_1 - t_3 \tag{5.26}$$

$$Orb(N_R) = t_1 - t_5, t_2 - t_5 \tag{5.27}$$

for minimal matter. There are in principle additional charged matter fields, which lie in the additional orbits:

$$G_{\text{mono}}^{\text{Dirac}} = \langle (12)(34) \rangle \simeq \mathbb{Z}_2\text{:} \tag{5.28}$$

$$\text{Extra Charged Matter} \tag{5.29}$$

$$Orb(10_{(1)}) = t_5 \tag{5.30}$$

$$Orb(\overline{5}_{(1)}) = t_1 + t_3, t_2 + t_4 \tag{5.31}$$

$$Orb(\overline{5}_{(2)}) = t_1 + t_5, t_2 + t_5 \tag{5.32}$$

$$Orb(\overline{5}_{(3)}) = t_3 + t_4, \tag{5.33}$$

as well as additional matter curves supporting neutral fields:

$$G_{\text{mono}}^{\text{Dirac}} = \langle (12)(34) \rangle \simeq \mathbb{Z}_2\text{:} \tag{5.34}$$

$$\text{Extra Neutral} \tag{5.35}$$

$$Orb(D_{(1)}) = t_1 - t_2, t_2 - t_1 \tag{5.36}$$

$$Orb(D_{(2)}) = t_1 - t_4, t_2 - t_3 \tag{5.37}$$

$$Orb(D_{(3)}) = t_3 - t_4, t_4 - t_3 \tag{5.38}$$

$$Orb(D_{(4)}) = t_3 - t_5, t_4 - t_5. \tag{5.39}$$

Note that here, and in what follows, by 'Extra Neutral' we mean extra fields which are not charged under the standard model gauge group.

Each such orbit defines a matter curve, so that in addition to the dark chiral matter listed, we also have the conjugate representations as well. Besides the chiral matter localized on curves, there are also two zero weights, Z_{PQ} and Z_χ which descend from the adjoint of $\text{SU}(5)_\perp$. These can be identified with bulk modes which are roughly the center of mass motion degrees of freedom for the seven-branes. The $\text{U}(1)_{\text{PQ}}$ and $\text{U}(1)_\chi$ charges for all of

JHEP08(2010)040

the fields listed above are:

Minimal	$10_M, Y_{10}$	$\overline{5}_M$	$Y'_{\overline{10}}$	5_H	$\overline{5}_H$	$X^\dagger$	N_R
$\mathrm{U}(1)_{\mathrm{PQ}}$	$+1$	$+1$	$+3$	-2	-2	$+4$	-3
$\mathrm{U}(1)_\chi$	-1	$+3$	$+1$	$+2$	-2	0	-5

(5.40)

Extra Charged	$10_{(1)}$	$\overline{5}_{(1)}$	$\overline{5}_{(2)}$	$\overline{5}_{(3)}$
$\mathrm{U}(1)_{\mathrm{PQ}}$	$+4$	-2	$+5$	-6
$\mathrm{U}(1)_\chi$	$+4$	-2	$+3$	-2

(5.41)

Extra Neutral	$D_{(1)}$	$D_{(2)}$	$D_{(3)}$	$D_{(4)}$	Z_{PQ}	Z_χ
$\mathrm{U}(1)_{\mathrm{PQ}}$	0	$+4$	0	-7	0	0
$\mathrm{U}(1)_\chi$	0	0	0	-5	0	0

. (5.42)

In the Dirac scenario where the monodromy group is enlarged to $\mathbb{Z}_2 \times \mathbb{Z}_2 = \langle (12), (34) \rangle$, the only difference is that the size of the orbits are enlarged. The orbits of the monodromy group in this case are:

$$G_{\mathrm{mono}}^{\mathrm{Dirac}} = \langle (12), (34) \rangle \simeq \mathbb{Z}_2 \times \mathbb{Z}_2 \text{:} \tag{5.43}$$

$$\text{Minimal Matter} \tag{5.44}$$

$$Orb(10_M, Y_{10}) = t_1, t_2 \tag{5.45}$$

$$Orb(Y'_{\overline{10}}) = -t_3, -t_4 \tag{5.46}$$

$$Orb(\overline{5}_M) = t_4 + t_5, t_3 + t_5 \tag{5.47}$$

$$Orb(5_H) = -t_1 - t_2 \tag{5.48}$$

$$Orb(\overline{5}_H) = t_1 + t_4, t_2 + t_3, t_2 + t_4, t_1 + t_3 \tag{5.49}$$

$$Orb(X^\dagger) = t_2 - t_4, t_1 - t_3, t_1 - t_4, t_2 - t_3 \tag{5.50}$$

$$Orb(N_R) = t_1 - t_5, t_2 - t_5, \tag{5.51}$$

while the extra charged matter is:

$$G_{\mathrm{mono}}^{\mathrm{Dirac}} = \langle (12), (34) \rangle \simeq \mathbb{Z}_2 \times \mathbb{Z}_2 \text{:} \tag{5.52}$$

$$\text{Extra Charged Matter} \tag{5.53}$$

$$Orb(10_{(1)}) = t_5 \tag{5.54}$$

$$Orb(\overline{5}_{(2)}) = t_1 + t_5, t_2 + t_5 \tag{5.55}$$

$$Orb(\overline{5}_{(3)}) = t_3 + t_4, \tag{5.56}$$

and the extra neutral states available are:

$$G_{\mathrm{mono}}^{\mathrm{Dirac}} = \langle (12), (34) \rangle \simeq \mathbb{Z}_2 \times \mathbb{Z}_2 \text{:} \tag{5.57}$$

$$\text{Extra Neutral} \tag{5.58}$$

$$Orb(D_{(1)}) = t_1 - t_2, t_2 - t_1 \tag{5.59}$$

$$Orb(D_{(3)}) = t_3 - t_4, t_4 - t_3 \tag{5.60}$$

$$Orb(D_{(4)}) = t_3 - t_5, t_4 - t_5. \tag{5.61}$$

JHEP08(2010)040

5.2.2 $\mathbb{Z}_3$ and S_3 orbits

We now turn to Dirac scenarios where the monodromy group is isomorphic to either $\mathbb{Z}_3$ or S_3. In this case, the two surviving U(1) directions in the Cartan of $\mathrm{SU}(5)_\perp$ are:

$$G_{\rm mono}^{\rm Dirac} \simeq \mathbb{Z}_3 \text{ or } S_3: \tag{5.62}$$

$$t_{\rm PQ}^* = t_1^* + t_2^* + t_3^* - 3t_4^* \tag{5.63}$$

$$t_\chi^* = -(t_1^* + t_2^* + t_3^* + t_4^*) + 4t_5^*. \tag{5.64}$$

With respect to this convention, the possible monodromy groups are generated by either (123), (132), or the entire symmetric group on three letters, S_3. In all cases, the orbits for the matter fields are the same. Using the results of appendix A, the resulting orbits are:

$$G_{\rm mono}^{\rm Dirac} \simeq \mathbb{Z}_3 \text{ or } S_3 \tag{5.65}$$

$$\text{Minimal Matter} \tag{5.66}$$

$$Orb(10_M, Y_{10}) = t_1, t_2, t_3 \tag{5.67}$$

$$Orb(Y'_{\overline{10}}) = -t_4 \tag{5.68}$$

$$Orb(\overline{5}_M, Y'_{\overline{5}}) = t_1 + t_5, t_2 + t_5, t_3 + t_5 \tag{5.69}$$

$$Orb(Y_5) = -t_4 - t_5 \tag{5.70}$$

$$Orb(5_H) = -t_1 - t_2, -t_2 - t_3, -t_3 - t_1 \tag{5.71}$$

$$Orb(\overline{5}_H) = t_1 + t_4, t_2 + t_4, t_3 + t_4 \tag{5.72}$$

$$Orb(X^\dagger) = t_1 - t_4, t_2 - t_4, t_3 - t_4 \tag{5.73}$$

$$Orb(N_R) = t_4 - t_5. \tag{5.74}$$

In this case, the available extra matter fields are more limited, in comparison to the other Dirac scenarios. The orbits of potentially extra charged matter are:

$$G_{\rm mono}^{\rm Dirac} \simeq \mathbb{Z}_3 \text{ or } S_3 \text{ Orbits} \tag{5.75}$$

$$\text{Extra Charged Matter} \tag{5.76}$$

$$Orb(10_{(1)}) = t_5, \tag{5.77}$$

Next consider extra GUT singlets which descend from the adjoint of $\mathrm{SU}(5)_\perp$. Due to the action of the monodromy group, there are only two zero weights, which we denote as $Z_{\rm PQ}$ and Z_χ. The remaining weights are of the form $t_m - t_n$ and localize on curves in the geometry. The available orbits can all be obtained by acting with a $\mathbb{Z}_3$ subgroup of the monodromy group. Up to complex conjugation, the orbits are:

$$G_{\rm mono}^{\rm Dirac} \simeq \mathbb{Z}_3 \text{ or } S_3 \text{ Orbits} \tag{5.78}$$

$$\text{Extra Singlets} \tag{5.79}$$

$$Orb(D_{(1)}) = t_1 - t_5, t_2 - t_5, t_3 - t_5 \tag{5.80}$$

$$Orb(D_{(2)}) = t_2 - t_3, t_3 - t_1, t_1 - t_2. \tag{5.81}$$

In addition, there are again two zero weights, $Z_{\rm PQ}$ and Z_χ given by the two unidentified directions $\mathrm{U}(1)_{\rm PQ}$ and $\mathrm{U}(1)_\chi$. These correspond to bulk modes which do not localize on curves. This is also the only scenario in which messengers in the $10 \oplus \overline{10}$ and $5 \oplus \overline{5}$ are both in principle possible.

Finally, the charges of the various fields under these U(1) symmetries are:

Minimal	$10_M, Y_{10}$	$\overline{5}_M, Y'_{\overline{5}}$	Y_5	$Y'_{\overline{10}}$	5_H	$\overline{5}_H$	$X^\dagger$	N_R
$\mathrm{U}(1)_{\mathrm{PQ}}$	$+1$	$+1$	$+3$	$+3$	-2	-2	$+4$	-3
$\mathrm{U}(1)_\chi$	-1	$+3$	-3	$+1$	$+2$	-2	0	-5

(5.82)

Extra Charged	$10_{(1)}$
$\mathrm{U}(1)_{\mathrm{PQ}}$	0
$\mathrm{U}(1)_\chi$	$+4$

(5.83)

Extra Neutral	$D_{(1)}$	$D_{(2)}$	Z_{PQ}	Z_χ
$\mathrm{U}(1)_{\mathrm{PQ}}$	$+1$	0	0	0
$\mathrm{U}(1)_\chi$	-5	0	0	0

. (5.84)

5.3 Majorana scenarios: $\mathbb{Z}_2 \times \mathbb{Z}_2$ and Dih_4 orbits

In this subsection we discuss minimal Majorana neutrino scenarios. As in [8], we shall assume that the right-handed neutrino states localize on matter curves. In this case, the covering theory must contain interaction terms which are schematically of the form:

$$L \supset \int d^2\theta \; H_u L N_R + H'_u L' N'_R + M_{\mathrm{maj}} N_R N'_R, \tag{5.85}$$

where the primes denote fields in the same orbit. The actual list of interaction terms will typically be much larger. This can partially be traced to the presence of a Majorana mass term, which couples states with conjugate quantum numbers. This has the effect of also eliminating candidate global U(1) symmetries in the quotient theory which would be explicitly broken by the Majorana mass term.

Majorana scenarios admit three consistent choices of orbits. Before proceeding to explicit examples, we first focus on the common aspects of all such scenarios, and defer a more complete characterization to appendix A. In all cases, the monodromy group action identifies all four U(1) factors of $\mathrm{SU}(5)_\perp$. Adopting the same convention used in appendix A, this leads to a unique direction in the Cartan for $\mathrm{U}(1)_{\mathrm{PQ}}$:

$$t^*_{\mathrm{PQ}} = t^*_1 + t^*_2 + t^*_3 - 4t^*_4 + t^*_5. \tag{5.86}$$

By inspection, this choice of $\mathrm{U}(1)_{\mathrm{PQ}}$ is consistent with permutations which leave t_4 invariant. Thus, the monodromy group must be a subgroup of S_4, the symmetric group on the remaining t_i's. The $\mathrm{U}(1)_{\mathrm{PQ}}$ charges of all visible matter are:

Visible Matter	$10_M, Y_{10}$	$Y'_{\overline{10}}$	$\overline{5}_M$	5_H	$\overline{5}_H$	$X^\dagger$	N_R
$\mathrm{U}(1)_{\mathrm{PQ}}$	$+1$	$+4$	$+2$	-2	-3	$+5$	0

. (5.87)

An interesting feature of all Majorana scenarios is that the only messenger fields which can be accommodated transform in the $10 \oplus \overline{10}$ of SU(5). In addition, note that in this case, the right-handed neutrinos are neutral under $\mathrm{U}(1)_{\mathrm{PQ}}$.

An additional feature of such scenarios is that because the X field carries odd PQ charge, its vev will not retain a $\mathbb{Z}_2$ subgroup, so we cannot embed matter parity inside of $\mathrm{U}(1)_{\mathrm{PQ}}$. Rather, we shall assume that there is an additional source of $\mathbb{Z}_2$ matter parity under which X and the Higgs fields have charge $+1$, while the MSSM chiral matter supefields have charge -1. Such symmetries can in principle originate from the geometry of an F-theory compactification, as discussed for example in [2].

We now proceed to list the consistent Majorana orbits. In the case where the mondromy group is $\langle(12)(35),(13)(25)\rangle \simeq \mathbb{Z}_2 \times \mathbb{Z}_2$, the corresponding orbits are:

$$G^{\mathrm{Maj}}_{\mathrm{mono}} = \langle(12)(35),(13)(25)\rangle \simeq \mathbb{Z}_2 \times \mathbb{Z}_2 \tag{5.88}$$
$$\text{Minimal Matter} \tag{5.89}$$
$$Orb(10_M, Y_{10}) = t_1, t_2, t_3, t_5 \tag{5.90}$$
$$Orb(Y'_{\overline{10}}) = -t_4 \tag{5.91}$$
$$Orb(\overline{5}_M) = t_2 + t_3, t_5 + t_1 \tag{5.92}$$
$$Orb(5_H) = -t_1 - t_2, -t_5 - t_3 \tag{5.93}$$
$$Orb(\overline{5}_H) = t_1 + t_4, t_2 + t_4, t_5 + t_4, t_3 + t_4 \tag{5.94}$$
$$Orb(X^\dagger) = t_2 - t_4, t_1 - t_4, t_5 - t_4, t_3 - t_4 \tag{5.95}$$
$$Orb(N_R) = \pm\left(t_1 - t_3\right), \pm\left(t_5 - t_2\right) \tag{5.96}$$

Besides fields with non-trivial weights, there is also a single zero weight Z_{PQ}, which lies in the same direction as the $\mathrm{U}(1)_{\mathrm{PQ}}$ generator. For small monodromy group orbits, there is one possible extra charged matter curve given as:

$$G^{\mathrm{Maj}}_{\mathrm{mono}} = \langle(12)(35),(13)(25)\rangle \simeq \mathbb{Z}_2 \times \mathbb{Z}_2 \tag{5.97}$$
$$\text{Extra Charged} \tag{5.98}$$
$$Orb(\overline{5}_{(1)}) = t_1 + t_3, t_2 + t_5. \tag{5.99}$$

Next consider extra GUT singlets of the theory. Since there is a single available $\mathrm{U}(1)$ in $\mathrm{SU}(5)_\perp$, only one zero weight denoted as Z_{PQ} can descend from the adjoint of $\mathrm{SU}(5)_\perp$. The remaining weights of the adjoint are of the form $t_m - t_n$. Under the provided monodromy group action, these separate into the following orbits:

$$G^{\mathrm{Maj}}_{\mathrm{mono}} = \langle(12)(35),(13)(25)\rangle \simeq \mathbb{Z}_2 \times \mathbb{Z}_2 \tag{5.100}$$
$$\text{Extra Neutral} \tag{5.101}$$
$$Orb(D_{(1)}) = \pm(t_1 - t_2), \pm(t_3 - t_5) \tag{5.102}$$
$$Orb(D_{(2)}) = \pm(t_1 - t_5), \pm(t_2 - t_3). \tag{5.103}$$

The actual charge assignments in the case where the monodromy group is $\mathbb{Z}_2 \times \mathbb{Z}_2$ are given as:

Visible Matter	$10_M, Y_{10}$	$Y'_{\overline{10}}$	$\overline{5}_M$	5_H	$\overline{5}_H$	$X^\dagger$	N_R
$\mathrm{U}(1)_{\mathrm{PQ}}$	$+1$	$+4$	$+2$	-2	-3	$+5$	0

(5.104)

Extra Charged	$\overline{5}_{(1)}$
$\mathrm{U}(1)_{\mathrm{PQ}}$	$+2$

(5.105)

Extra Neutral	$D_{(1)}$	$D_{(2)}$	Z_{PQ}
$\mathrm{U}(1)_{\mathrm{PQ}}$	0	0	0

. (5.106)

Thus, in this case, there is exactly one additional unused orbit, corresponding to the $\overline{5}_{(1)}$.

The other Majorana scenarios simply correspond to enlarging the monodromy group by allowing more elements from S_4 to participate in the various orbits. There are two remaining monodromy groups of order eight, which are both isomorphic to the dihedral group Dih_4 given by rotations and reflections of the square. The explicit generators for these two monodromy groups are:

$$G_8^{Maj(2)} \simeq \langle (13)(25), (1253) \rangle \simeq Dih_4 \simeq \mathbb{Z}_2 \ltimes \mathbb{Z}_4 \tag{5.107}$$

$$G_8^{Maj(3)} = \langle (13)(25), (1325) \rangle \simeq Dih_4 \simeq \mathbb{Z}_2 \ltimes \mathbb{Z}_4, \tag{5.108}$$

where the $\mathbb{Z}_2$ acts by inversion on the $\mathbb{Z}_4$ factor of the semi-direct product. This also enlarges the orbits of some of the visible matter field sectors, and identifies some of the orbits for the dark matter fields. We refer the interested reader to appendix A for a full classification of all possible orbits. Quite remarkably, in the two cases of the maximal monodromy group where $G_{\mathrm{mono}}^{\mathrm{Maj}} \simeq Dih_4$, *all* of the orbits in the visible sector are utilized except for one 10 curve. But this is precisely where the messenger $\overline{10}$ can localize! Thus, the E_8 enhancement point is just flexible to accommodate all of the minimal matter, but nothing more! In this case, the matter content and charge assignments are:

Visible Matter	$10_M, Y_{10}$	$Y'_{\overline{10}}$	$\overline{5}_M$	5_H	$\overline{5}_H$	$X^\dagger$	N_R
$\mathrm{U}(1)_{\mathrm{PQ}}$	$+1$	$+4$	$+2$	-2	-3	$+5$	0

(5.109)

Extra Neutral	$D_{(1)}$	Z_{PQ}
$\mathrm{U}(1)_{\mathrm{PQ}}$	0	0

. (5.110)

5.4 E_8 and the absence of exotica

The presence of a single E_8 interaction point also addresses a puzzle encountered in the context of the gauge mediated supersymmetry breaking sector introduced in [3]. While the PQ charge assignments for matter fields suggestively hinted at higher unification structures such as E_6, E_7 and E_8, in [3], it was assumed that this required full multiplets such as the 27 of E_6 to localize on matter curves in the SU(5) GUT seven-brane. This would appear to require a local enhancement from SU(5) to at least E_7. In analyzing the matter trapped along the corresponding curve, however, it was often difficult to remove additional exotic states from the low energy theory. For example, the adjoint 133 of E_7 breaks to $E_6 \times \mathrm{U}(1)$

as:

$$E_7 \supset E_6 \times \mathrm{U}(1) \tag{5.111}$$

$$133 \rightarrow 78_0 + 27_2 + \overline{27}_{-2} + 1_0. \tag{5.112}$$

Breaking further to SU(5), it seemed in [3] that in addition to the states in the 27, the matter curve would also contain irreducible components descending from the 78 of E_6. Here, we have seen that rank one enhancements along matter curves are sufficient in realizing these higher unification structures. Indeed, there are no exotic fields in this case, and the presence of the E-type singularity is instead concentrated at a point of the geometry.

5.5 Monodromy and messengers

An intriguing byproduct of the classification of monodromy group orbits is that while it is indeed possible to accommodate a messenger sector, some of the messengers must localize on the same curve as the matter fields of the MSSM. Moreover, we have also seen that aside from the $\mathbb{Z}_3$ or S_3 Dirac scenarios, in all scenarios the only available messengers transform in the $10 \oplus \overline{10}$ of $\mathrm{SU}(5)_{\mathrm{GUT}}$. It is important to note that the presence of additional matter fields does not alter the main features of the flavor hierarchy found in [5] and [8] which was obtained by estimating the overlap of wave functions. A crucial ingredient in the flavor hierarchy story is that the internal wave function for the massive generation must not vanish near the interaction point. Indeed, letting $\psi^{(1)}, \ldots, \psi^{(N)}$ denote the internal wave functions for the matter fields localized on a curve, a generic linear combination of these wave functions corresponds to the matter field which actually couples to X in the messenger sector. Thus, provided there is another interaction point where the messengers and X fields meet, the other linear combinations for the matter fields will not vanish at the interaction point, so that the analysis of flavor hierarchies found in [5] and [8] will carry over to this case as well.

Another consequence of localizing the messenger field Y_{10} on the same curve as the 10_M is that these messengers will now couple to MSSM fields through the coupling:

$$5_H Y_{10} 10_M. \tag{5.113}$$

This provides a rapid channel of decay for messengers to MSSM fields. Similar considerations hold for the $\mathbb{Z}_3$ or S_3 Dirac scenarios with messengers in the $5 \oplus \overline{5}$. In particular, this means that the messengers will not persist as thermal relics. This is a welcome feature of this model, because stable thermal relics of this type would lead to far greater matter density than is observed. To a certain extent, this is to be expected because the classification of monodromy orbits performed earlier eliminates all U(1) symmetries other than $\mathrm{U}(1)_{\mathrm{PQ}}$, and in the case of Dirac neutrino scenarios, also $\mathrm{U}(1)_\chi$. Since there is no conserved messenger number, it is natural to expect nothing to protect the messengers from exhibiting such decays.

We have also seen that in all but one scenario, the messenger fields organize into vector-like pairs in the $10 \oplus \overline{10}$ of SU(5), rather than the $5 \oplus \overline{5}$. Recall that in minimal gauge mediation, the soft mass terms are controlled by $C(R)$ of the messenger fields, where:

$$T^A_{\mathrm{ij}} T^B_{\mathrm{ji}} = C(R)\delta^{\mathrm{AB}} \tag{5.114}$$

for matrices T^A_{ij} in the representation R. Since $C(10)/C(5) = 3$, it follows that each 10 effectively counts as three 5's.[8] The presence of 10's also scales the soft scalar masses relative to their gaugino counterparts. Indeed, letting $m^{\mathrm{scalar}}_{(1)}$ and $m^{\mathrm{gaugino}}_{(1)}$ denote the soft masses in the case of a minimal gauge mediation scenario with a single vector-like pair in the $5 \oplus \overline{5}$, a minimal gauge mediation model with N_5 vector-like pairs in the $5 \oplus \overline{5}$ has soft masses:

$$m^{\mathrm{scalar}} = \sqrt{N_5} \cdot m^{\mathrm{scalar}}_{(1)} \tag{5.115}$$

$$m^{\mathrm{gaugino}} = N_5 \cdot m^{\mathrm{gaugino}}_{(1)}. \tag{5.116}$$

Thus, the soft scalar masses decrease relative to the gaugino masses.

For the generic case of interest where N_5 is a multiple of three, this causes the lightest stau to become comparable in mass to the bino. In the presence of the PQ deformation, this can lead to a further decrease in the lightest stau mass which quite suggestively hints at scenarios with a stau NLSP. Furthermore, this also lowers the upper bound on Δ_{PQ}. On the other hand, the size of the PQ deformation is bounded below by the requirement that the saxion have available a decay channel to MSSM particles [6]. This then leads to an interesting constraint on the viable window of PQ deformations.

5.6 Semi-visible TeV scale dark matter candidates

Strictly speaking, a viable dark matter candidate need only be neutral under $U(1)_{\mathrm{EM}}$ and not under all of $SU(5)_{\mathrm{GUT}}$. Indeed, bino LSP scenarios provide an explicit example of precisely this type. With this in mind, it is therefore important to examine potential TeV scale dark matter candidates which are electrically neutral, but not necessarily fully neutral under $SU(5)_{\mathrm{GUT}}$. The 5 and 10 of $SU(5)_{\mathrm{GUT}}$ decompose into irreducible representations of the Standard Model gauge group as:

$$SU(5)_{\mathrm{GUT}} \supset SU(3)_C \times SU(2)_L \times U(1)_Y \tag{5.117}$$

$$5 \to (3,1)_{-2} + (1,2)_3 \tag{5.118}$$

$$10 \to (1,1)_6 + (\overline{3},1)_{-4} + (3,2)_1. \tag{5.119}$$

Once $SU(2)_L \times U(1)_Y$ breaks to $U(1)_{\mathrm{EM}}$, it follows that only the 5 and $\overline{5}$ possess electrically neutral candidates, which can be thought of as the "neutrino component" of the fundamental and anti-fundamental. Note further that in order to retain successful gauge coupling unification, a full GUT multiplet must persist below the GUT scale.[9]

[8] More generally, recall that for the group $SU(N)$, $C(N) = 1/2$ and for the two index anti-symmetric representation of dimension $N(N-1)/2$, $C(N(N-1)/2) = (N-2)/2$.

[9] In principle, one could consider scenarios where a piece of the messenger is retained, and another piece of the dark matter GUT multiplet is also kept, retaining unification. This appears to be a somewhat artificial means by which to preserve unification in this context, so we shall not pursue this possibility here. In the context of F-theory GUTs, more motivated choices for splitting up GUT multiplets through appropriate fluxes can also be arranged, for example as in [16].

The available orbits for additional 5's are typically quite limited. As we now explain, minimal realizations of TeV scale dark matter are problematic in this context, and can only be accommodated in a single Dirac neutrino scenario.[10] Since interaction terms with the Higgs fields both involve couplings to a 10 of $SU(5)_{GUT}$, we shall assume that the same mechanism responsible for generating the μ-term is also responsible for generating mass for the dark matter candidate so that:

$$\int d^4\theta \frac{X^\dagger D_5 D'_{\overline{5}}}{\Lambda_{UV}}. \tag{5.120}$$

Thus, when X develops a supersymmetry breaking vev, it will induce a mass of $\mu \sim 100-1000\,\text{GeV}$ for the bosons and fermions of the dark matter multiplet D_5 and $D'_{\overline{5}}$.

The interaction term of line (5.120) is difficult to incorporate at a single point of E_8 unification. The reason for this obstruction is quite similar to the one already encountered for messengers. In all scenarios considered in this section, the orbit of $X^\dagger$ contains a weight of the form:

$$Orb(X^\dagger) \ni t_2 - t_4. \tag{5.121}$$

Thus, the weights for D_5 and $D_{\overline{5}}$ must satisfy the constraint:

$$(t_2 - t_4) + (-t_i - t_j) + (t_k + t_l) = 0, \tag{5.122}$$

so that the weights for D_5 and $D_{\overline{5}}$ are respectively of the form $-t_2 - t_i$ and $t_4 + t_i$. The options for semi-visible dark matter are therefore quite limited. Indeed, there are essentially three possible options for all of the dark matter candidates:

$$\text{Option 1: } Orb(D_5) \ni -t_2 - t_1,\ Orb(D_{\overline{5}}) \ni t_4 + t_1 \tag{5.123}$$

$$\text{Option 2: } Orb(D_5) \ni -t_2 - t_3,\ Orb(D_{\overline{5}}) \ni t_4 + t_3 \tag{5.124}$$

$$\text{Option 3: } Orb(D_5) \ni -t_2 - t_5,\ Orb(D_{\overline{5}}) \ni t_4 + t_5. \tag{5.125}$$

Returning to the discussion of viable orbits in Dirac and Majorana scenarios, the analysis of subsections 5.2 and 5.3 establishes that typically, the length of the orbits other than $\overline{5}_M$ are too large to admit additional 5's in full GUT multiplets. Rather, we find that only in the $\mathbb{Z}_2$ and $\mathbb{Z}_2 \times \mathbb{Z}_2$ Dirac neutrino scenarios where the orbit $\overline{5}_M$ is:

$$Orb(\overline{5}_M) = t_4 + t_5, t_3 + t_5 \tag{5.126}$$

is it even possible to accommodate additional dark matter in the $5 \oplus \overline{5}$. The corresponding orbits for these fields are then:

$$Orb(\overline{5}_M, D_{\overline{5}}) = t_4 + t_5, t_3 + t_5 \tag{5.127}$$

$$Orb(D_5) = -t_1 - t_5, -t_2 - t_5, \tag{5.128}$$

and theire charges under $U(1)_{PQ}$ and $U(1)_\chi$ are:

Semi-Dark	$\overline{5}_M, D_{\overline{5}}$	D_5
$U(1)_{PQ}$	$+1$	-5
$U(1)_\chi$	$+3$	-3

(5.129)

[10] We thank T. Hartman for a related discussion which prompted this analysis.

Even so, this by itself is insufficient for specifying a dark matter candidate. We will encounter some obstructions to realizing such a candidate in section 9.

6 Bifundamentals and E-type singularities

In the previous section we focussed primarily on the constraints derived from the condition that all of the matter of the MSSM consistently embed inside the unfolding of a single E_8 singularity. Indeed, the E-type structure is what leads to an appealing unification structure as well as geometric rigidity in the construction. In the context of a more complete geometry, tadpole cancellation considerations generically requires the presence of additional seven-branes, as well three-branes. These additional branes will introduce additional gauge group factors, which can in principle contribute to the low energy phenomenology of the theory.

Given the presence of such objects, it is natural to study whether additional bifundamental matter could localize at a pairwise intersection of an E-type seven-brane with another seven-brane. Even though this would at first appear to represent a mild generalization of the quiver-type constructions which are quite common in perturbatively realized intersecting seven-brane configurations, the presence of an exceptional gauge symmetry significantly shields the E-type seven-brane from massless bifundamentals charged under gauge groups of the form $E_8 \times G$. As we now explain, states localized at such pairwise intersections correspond to more exotic objects which resist an interpretation in terms of conventional particles.

To see the source of this restriction, we first review how bifundamental matter charged under distinct seven-branes comes about in F-theory constructions. Letting G_S and $G_{S'}$ denote the gauge groups of the corresponding seven-branes wrapping surfaces S and S', suppose that S and S' intersect over a matter curve Σ. This curve corresponds to the locus of the elliptic fibration where the singularity $G_S \times G_{S'}$ enhances to $G_\Sigma \supset G_S \times G_{S'}$. "Bifundamental" matter trapped on the curve can be analyzed in terms of a gauge theory with gauge group G_Σ which locally Higgses to G_S along S and $G_{S'}$ along S' [37]. As an example, note that this covers perturbatively realized intersecting seven-brane configurations, where $G_S = \mathrm{SU}(N)$, $G_{S'} = \mathrm{SU}(M)$ and $G_\Sigma = \mathrm{SU}(N+M)$.

While this clearly generalizes (at least in a non-compact model) to arbitrary N and M, exceptional singularities impose far more significant restrictions. Indeed, the gauge theory description just presented requires that G_Σ admits an interpretation as a gauge theory which is locally Higgsed. On the other hand, if $G_S = E_8$, then there is no compact simple gauge group such that $G \supset E_8 \times G_{S'}$.[11] As a consequence, there is little sense in which "bifundamental" particle states localize along the intersection locus. Note that this does not require the presence of a single, globally defined E_8 singularity which unfolds over an isolated surface. Indeed, it is in principle possible to consider geometries where the singularity enhancements embed in different E_8 gauge groups.

[11] Although there is no sense in which a compact simple gauge group contains $E_8 \times G_{S'}$, affine extensions such as $E_9 \supset E_8$ provide one possible gauge theory interpretation. In fact this seems compatible with the interpretation of tensionless strings carrying an E_8 current algebra as the analog of "bifundamental matter".

This begs the question, however, as to what occurs when an E_8 singularity collides with another stack of seven-branes with gauge group $G_{S'}$. These types of colliding singularities can occur geometrically and in fact have been studied in [42]. In six dimensional theories, this phenomenon leads to tensionless strings. In the context of F-theory compactified on a Calabi-Yau threefold, this corresponds to the presence of a vanishing $\mathbb{P}^1$ in the base wrapped by a D3-brane. Compactifying the six-dimensional theory on a T^2, a T-dual version of this intersecting brane configuration maps to zero size E_8 instantons. For example, a seven-brane which carries E_8 gauge symmetry can be Higgsed by turning on an instanton. A zero size instanton of the seven-brane theory carries non-zero D3-brane charge. Thus, we can view a zero size E_8 instanton as an intersection of a single D3-brane with the E_8 seven-brane. As argued in [43], the small instanton limit includes tensionless strings. The presence of such tensionless strings can be deduced as follows. One can lift this configuration to an M-theory configuration with an M5-brane near the boundary of the space which carries an E_8 symmetry. If the M5-brane is brought to the boundary, it corresponds to a zero size E_8 instanton. On the other hand an M2-brane stretched between the M5-brane and the boundary leads to a tensionless string as the M5-brane approaches the boundary. This can be further analyzed in F-theory, as in [44–46], and a great deal of information can be extracted from various duality chains. In particular one can construct the elliptic genus of such E-strings [47].

Note, however, that although tensionless strings can be BPS objects in 5 and 6 dimensions, this is not the case in four dimensions. The remnants of such tensionless strings in four dimensions are conformal fixed points. Thus, the exotic possibility of a brane intersecting an E-brane can be phrased in four dimensional language as the study of a conformal theory with an E-type global symmetry, as studied for example in [48, 49]. Some examples of precisely this type have recently been studied in [50], and F-theory based constructions involving D3-brane probes of seven-branes with E-type gauge symmetries have been analyzed in [51], which can be viewed roughly as T-dual to the theories we are considering.

In the present context, the E_8 symmetry need only be restored at a single point of the GUT seven-brane. In other words, one can view our background as a theory near an E_8 type seven-brane. Thus, if any other brane intersects the GUT or PQ seven-branes, we have the remnant of a "nearly" conformal theory with an approximate E-type global symmetry.

We are thus led to potentially consider the possibility of an additional sector which is nearly conformal with an approximate E-type symmetry. In other words we could contemplate having an "unparticle sector", with terminology as in [52]. As the qualification of the previous paragraph suggests, it is first important to settle how badly the conformal symmetry of the theory is broken. There are at least two reasons to expect that the conformal symmetry is badly broken at scales below $M_{\rm GUT}$ or M_*, the string scale. The first reason has to do with the fact that the scalar vevs in the geometry which break the E_8 symmetry to SU(5) are typically of the GUT scale or higher (as they determine the background geometry), and thus a typical brane will not be particularly close to the E_8 point. Thus, the would be "tensionless string" will have a tension on the order of $T \sim M_*^3 \cdot R_{\rm GUT} \sim (10^{15}\,{\rm GeV})^2$. Hence, the mass scale for the operator responsible for breaking conformal invariance is relatively large.

Another reason why the conformal symmetry should be badly broken is the assumption of naturalness: If the mass scale for breaking of conformal symmetry is small, we will have a tower of particles charged under the Standard Model group which will dramatically accelerate the running of the coupling constants, and thus make the unification scale much smaller than 10^{16} GeV. This could lead to a certain amount of tension with current constraints on the lifetime of the proton, since lowering the GUT scale will also decrease the suppression scale of dimension six operators. Moreover, lowering the GUT seven-brane far below 10^{16} GeV may be difficult to arrange in practice without significant fine tuning.

Thus, even if there are such sectors we are naturally led to consider the corresponding conformal symmetry to be badly broken at energies near the weak scale. Without knowing more about the nature of such conformal theories and their breaking, it is difficult to say much about the light spectrum of particles associated with this conformal system, or whether any such particles will even survive to low energies. Although perhaps somewhat exotic, this possibility might produce novel phenomenological signatures at low energies, and would be interesting to study further.

7 The sequestered sector

The progression of ideas so far has been to examine potential dark objects, first "inside" of the E_8 singularity. Next, we examined possible dark objects which may only "touch" some part of the E_8 singularity. As the next logical step, in this section we consider objects which are "fully dark" in the sense that they do not directly intersect any seven-brane connected with the E_8 enhancement point.

Such dark objects can either correspond to degrees of freedom localized on other branes, or fields which propagate in the threefold base of the compactification. In the context of a local model, these degrees of freedom will decouple in the same limit which turns off the effects of gravity. If the corresponding degrees of freedom do not develop high scale supersymmetric masses, the corresponding masses of such particles are then cut off from the PQ seven-brane, and will not lead to TeV scale objects. For example, the gravitino is of this type, and has a mass in the far lower range of $10 - 100$ MeV. Other bulk gravitational modes have similar masses, but will also interact quite weakly with the visible sector.

Next consider modes localized on three-branes and seven-branes, which will typically be present in order to satisfy tadpole cancellation conditions. Here there is again a problem with generating a TeV scale dark matter candidate because the associated degrees of freedom for these branes only indirectly communicate with the PQ seven-brane. Although somewhat ad hoc, we can still consider scenarios where such objects develop a TeV scale mass through some other mechanism. Even though these degrees of freedom are geometrically "sequestered" from the GUT seven-brane, they can still interact non-trivially with the visible sector. In particular, abelian gauge fields for these branes can mix non-trivially with the $U(1)_Y$ gauge boson of the Standard Model. As we now explain, this type of effect is difficult to avoid, and suggests that in fact all of the abelian gauge bosons will mix to a certain extent. This would suggest that "dark gauge bosons" and "dark gauginos" cannot be dark matter candidates, as they will decay too rapidly.

7.1 Ubiquitous kinetic mixing

In this subsection we show that dark gauge bosons associated with "sequestered branes" generically mix with $U(1)_Y$ because massive states charged under both groups induce one loop mixing effects. This kinetic mixing then provides a source by which other aspects of dark physics can potentially communicate with the Standard Model. To illustrate the main features of kinetic mixing, we first review this phenomenon in the context of a $U(1)_1 \times U(1)_2$ gauge theory with bifundamental matter. Next, we explain how in the context of string based constructions, such kinetic mixing will generically arise from massive modes charged under both group factors.

We now review the primary features of kinetic mixing [53] in the context of a $U(1)\times U(1)$ field theory with Lagrangian:

$$L \supset \mathrm{Im}\,\tau_1 \int d^2\theta W^{\alpha}_{(1)} W^{(1)}_{\alpha} + \mathrm{Im}\,\tau_2 \int d^2\theta W^{\alpha}_{(2)} W^{(2)}_{\alpha} + \mathrm{Im}\,\varepsilon \int d^2\theta W^{\alpha}_{(1)} W^{(2)}_{\alpha}. \tag{7.1}$$

The final term proportional to ε denotes the effects of kinetic mixing. For further discussion on kinetic mixing see for example [53–57]. If matter is charged under both U(1) factors, then loop suppressed contributions can convert a $U(1)_1$ gauge field into a $U(1)_2$ gauge field. This introduces a kinetic mixing term which in a holomorphic basis leads to the coupling:[12]

$$\varepsilon(\mu) = \varepsilon(\mu_0) + \frac{e_1 e_2}{16\pi^2} \log \frac{\mu_0^2}{\mu^2}, \tag{7.2}$$

where the e_i denote the charges of the bifundamental under the two gauge groups and μ_0 denotes the scale of the UV boundary conditions for the field theory. The crucial point is that even if the mass of the bifundamental is quite large, it can still contribute to kinetic mixing between the U(1) gauge fields. In particular, even very heavy states can also contribute to kinetic mixing.

Returning to more string based constructions, there will generically be massive modes charged as bifundamentals under the gauge groups of the compactification. Because kinetic mixing does not decouple, all of these massive states will induce small effects which mix the various U(1) factors. This can occur even when considering mixing with $U(1)_Y$ embedded inside of SU(5) due to flux breaking of the GUT group. Note, however, that for non-abelian hidden sector gauge groups, gauge invariance forbids such kinetic mixing terms. In particular, this implies that although sequestered, kinetic mixing between abelian gauge fields will generically occur in string based models, and is in fact difficult to turn off. Moreover, this also implies that it is in fact quite difficult to completely sequester the effects of a "dark" abelian gauge boson which is light compared to the GUT scale. While a dark matter scenario involving kinetic mixing is therefore still a possibility, such scenarios constitute a significant departure from the spirit of minimality advocated here, and so we shall not pursue this option further in this paper.

[12]We thank D.E. Morrissey for discussion on this point.

8 Recent dark matter experiments and theoretical explanations

In the previous sections we have presented a classification of possible dark matter candidates in the context of F-theory GUTs. In this section we provide a very brief overview of current experiments which provide tantalizing hints at the observation of dark matter. After reviewing the primary signals of each such experiment, we turn to possible theoretical explanations for these results. Aside from astrophysical sources, such experiments may point towards the observation of dark matter. In this vein, we discuss possible dark matter explanations for such experiments. These fall into two main categories, corresponding to dark matter which annihilates, and dark matter which decays. As a disclaimer, our aim here is not to provide an exhaustive overview of experimental constraints and theoretical models, but rather, to provide a minimal characterization of dark matter explanations of these experiments.

8.1 Experiments

There are currently many different astrophysically based experiments which are potentially capable of detecting dark matter. Recently, the PAMELA satellite experiment [31, 32] has observed an excess in the positron fraction $e^+/(e^+ + e^-)$ in comparison to the cosmic ray background in the range of 10 GeV to 100 GeV. In particular, as opposed to the expected power law falloff, this experiment instead observes an increase in the observed positron fraction as a function of energy in this range. Moreover, PAMELA does not detect an excess in the analogous anti-proton fraction. If one accepts a dark matter explanation for this experiment, this suggests the presence of a dark particle with mass at least 100 GeV which preferentially produces positrons, rather than anti-protons.

Corroborating evidence for heavy dark matter with mass near the weak scale has also emerged from various recent experiments which measure the total $e^+ + e^-$ flux in different energy ranges. Experiments such as ATIC [33], PPB-BETS [34] and HESS [35] all appear to be consistent with the PAMELA experiment. Moreover, ATIC appears to detect an increase in flux up to around 700 GeV, with a steep falloff at higher energies. Again assuming a dark matter explanation for such experiments, this motivates an even heavier source of dark matter.

Most recently, however, the FERMI satellite experiment [36], which is also sensitive to the electron energy spectrum from 20 GeV to 1 TeV detects a milder increase in signal at energies above 300 GeV, when compared with the results of ATIC. At lower energy scales FERMI appears to be consistent with ATIC. This leads to a certain amount of tension with the results of the other experiments. Moreover, the fact that there is no steep falloff in the signal casts some doubt on dark matter explanations for the other signals. Nevertheless, FERMI points towards a lower bound on the mass of the dark matter around 1 TeV [58]. Presented with this at least suggestive evidence for the presence of dark matter, we now turn to some of the broad features of such scenarios.

8.2 Theoretical scenarios

The suggestive energy range of excess positrons and electrons found in recent experiments begs for a theoretical explanation of some sort. Either some astrophysical explanation, or some new production mechanism beyond that found in the Standard Model must be introduced. In this subsection, we discuss these two possibilities, focussing first on the exciting prospect that such experiments may require a novel extension of the Standard Model. After reviewing some of the features of annihilating and decaying dark matter scenarios, we next discuss potential astrophysical origins for the observed experimental results.

8.2.1 Annihilating dark matter

Assuming that the current experiments possess a particle physics origin, this suggests an exciting window into the physics of dark objects. Generating an appropriate excess of electrons can potentially be generated provided dark matter particles preferentially annihilate into leptonic final states. Modulo effects related to the propagation of dark matter across the galaxy, the annihilation cross section for dark matter must obey the relation:

$$\Phi_{e^+}^{\mathrm{ann}} \propto \left(\frac{\rho_{\mathrm{DM}}^{\mathrm{local}}}{m_{\mathrm{DM}}}\right)^2 \cdot \langle\sigma_{\mathrm{ann}} v\rangle_{\mathrm{present}}^{\mathrm{DM}} \tag{8.1}$$

where in the above, Φ_{e^+} denotes the positron flux, $\rho_{\mathrm{DM}}^{\mathrm{local}}$ denotes the local energy density of dark matter, and $\langle\sigma_{\mathrm{ann}} v\rangle_{\mathrm{present}}^{\mathrm{DM}}$ denotes the thermally averaged annihilation cross section for dark matter. Here, the local density for dark matter is known to be greater than the overall cold dark matter density as:

$$\rho_{\mathrm{DM}}^{\mathrm{local}} = A \cdot \rho_{\mathrm{DM}}, \tag{8.2}$$

where A is the "accumulation factor" which is on the order of 10^2 for weak scale dark matter.

To get a sense of the required cross section necessary to explain the results of the PAMELA experiment, we note that if the corresponding dark matter candidate comprises all of the dark matter relic abundance, then as explained for example in [59]:

$$\langle\sigma_{\mathrm{ann}} v\rangle_{\mathrm{present}}^{\mathrm{DM}} \sim 10^{-24} - 10^{-23}\ \mathrm{cm}^{-3}\ \mathrm{s} \sim 10^{-7} - 10^{-6}\ \mathrm{GeV}^{-2}, \tag{8.3}$$

will generate a sufficiently large signal to explain current data available from PAMELA.

On the other hand, if the dark matter is given by a WIMP thermal relic, then (ignoring saxion dilution effects), generating the required relic abundance leads to $\langle\sigma_{\mathrm{ann}} v\rangle_{\mathrm{WIMP}} \sim 10^{-9} - 10^{-8}\ \mathrm{GeV}^{-2}$. Generating a sufficient signal from this value of the cross section would then require an enhancement in the overall size of the cross section, by a factor of 10^2 to 10^3. Indeed, we can write:

$$\Phi_{e^+}^{\mathrm{PAMELA}} = \Phi_{e^+}^{\mathrm{WIMP}} \cdot B_{\mathrm{WIMP}}, \tag{8.4}$$

where

$$B_{\mathrm{WIMP}} \sim 10^2 - 10^3 \tag{8.5}$$

is some enhancement factor. This enhancement factor can either be argued for by some astrophysical reason, or through an infrared enhancement in the cross section. In the latter scenario, an infrared enhancement is in principle possible if dark matter interacts with a light particle. Summing the corresponding ladder diagrams associated with this annihilation can lead to a "Sommerfeld enhancement" in the annihilation cross section as discussed for example in [60–63]. An important ingredient for this infrared enhancement is the presence of a light bosonic mediator between the dark and visible sectors, with a mass around a few GeV.

It is important to note that because the flux depends on the square of the number density, this leads to the expectation that a large number of gamma rays should be generated near the center of the galaxy. This leads to a certain amount of tension with present observation [64]. In addition, there are potential issues with constraints from BBN [65].

8.2.2 Decaying dark matter

It is also possible to consider scenarios where a sufficiently long-lived dark matter particle decays preferentially to leptonic final states. In this case, the corresponding flux is dictated by the local dark matter energy density as:

$$\Phi_{e^+}^{\mathrm{dec}} \propto \left(\frac{\rho_{\mathrm{DM}}^{\mathrm{local}}}{m_{\mathrm{DM}}}\right) \cdot \Gamma_{\mathrm{DM}}, \tag{8.6}$$

where Γ_{DM} denotes the decay rate to positron final states. In order to accommodate PAMELA, for example, this requires the corresponding lifetime to be on the order of [66]:

$$\Gamma_{\mathrm{DM}} \sim 10^{-51}\ \mathrm{GeV} \sim \left(10^{26}\ \mathrm{sec}\right)^{-1}. \tag{8.7}$$

The numerology connected with this lifetime leads to a quite suggestive link with dimension six operators suppressed by the GUT scale [66]. For example, the decay of a TeV scale dark matter particle via a "dark analogue" of the operator responsible for proton decay in ordinary GUT models naturally lead to the requisite decay rate of the form:

$$\Gamma_{\mathrm{DM}} \sim \frac{m_{\mathrm{DM}}^5}{M_{\mathrm{GUT}}^4} \sim 6 \times 10^{-51}\ \mathrm{GeV} \cdot \left(\frac{m_{\mathrm{DM}}}{\mathrm{TeV}}\right)^5 \left(\frac{2 \times 10^{16}\ \mathrm{GeV}}{M_{\mathrm{GUT}}}\right)^4. \tag{8.8}$$

Moreover, because the flux scales only linearly with the number density, there is considerably less tension with current constraints on gamma rays from the galactic center, in comparison with annihilating dark matter scenarios. It is important to stress, however, that in order for a decaying dark matter candidate to generate a sufficiently large signal, it cannot contain any rapid decay channels.

8.2.3 Astrophysical explanations

Although less exciting from the perspective of particle physics, astrophysical sources based on pulsars provides a source of distortion in the energy dependence in the flux measured by various experiments. Pulsar winds appear to be capable of generating up to 1000 TeV boosts in the energy of electrons (see for example [67]). Contributions from both nearby

JHEP08(2010)040

and distant pulsars can in principle produce signals which can explain the types of signals found in the current slew of experiments. As shown for example in [68–71] pulsars appear to provide an explanation for the PAMELA excess. Most conservatively then, pulsars appear to provide a promising explanation using established physics. This is certainly quite economical, and the obstructions we shall encounter later in realizing dark matter capable of generating a sufficient flux in F-theory GUTs will point back to pulsars as an attractive option.

Upcoming experiments such as FERMI will study anisotropies in the flux background [72]. This should provide a means by which to distinguish between astrophysical compact sources and dark matter scenarios based on a larger halo.

9 Exceptional obstructions to annihilating and decaying scenarios

Having discussed potential dark matter candidates in F-theory GUTs, we now proceed to analyze whether any of these options provide an adequate explanation for recent dark matter experiments. In all cases we encounter significant obstructions to the existence of a weak to TeV scale dark matter candidate which can account for these experiments. *This leads us to the conclusion that at least with the minimal geometric ingredients necessary for other aspects of F-theory GUTs, recent dark matter experiments have an astrophysical explanation, and moreover, that the gravitino remains as the primary dark matter candidate in F-theory GUTs.*

In principle, there could be other candidate dark matter candidates in the context of less minimal F-theory GUT models. This would, however, lead to a less motivated as well as less predictive framework. Given the tight interrelations between various phenomenological ingredients already found in previous work on F-theory GUTs, in this paper we shall exclusively focus on those elements which are compatible with minimal considerations.

Even though one might initially think that global considerations could provide many dark matter candidates invisible to the local model, such candidates do not provide weak to TeV scale dark matter candidates. Indeed, recall that the available dark matter candidates roughly split into those which are "inside" the minimal E_8 of the local GUT model, possible candidates "nearby" which interact with at least some seven-brane factor of this E_8, and candidates which are fully sequestered. Of these possibilities, note that since the weak scale is controlled by the scale of supersymmetry breaking, which is in turn controlled by the vev of the chiral superfield X, weak or TeV scale dark matter must interact closely with X, or at least with other degrees of freedom which interact closely with X. As shown in section 2, degrees of freedom which are completely sequestered from the PQ seven-brane either have very large masses set by high scale supersymmetric dynamics, or have lower masses on the order of $10-100\,\text{MeV}$ as for the gravitino, while for some fermions the mass can as be low as $10-100$ keV.

Weak to TeV scale dark matter candidates must interact with degrees of freedom more closely linked to the supersymmetry breaking sector, and thus, to the PQ seven-brane. In principle, viable candidates could descend either from the minimal E_8 enhancement point, or from "nearby branes" which intersect the PQ seven-brane, but do not embed in this minimal E_8. This latter possibility is somewhat exotic, as well as non-minimal as discussed in section 6, so we shall not dwell on this possibility here. Rather, in this section we shall instead focus on those candidates which the presence of the E_8 enhancement point allows. Whether or not such candidates are present depends on the specific choice of fluxes. In principle, there could be many candidates, or none at all. Some of our discussion will be more general and will also apply to any PQ charged object.

Restricting further to minimal dark matter which descends from a local Higgsing of the E_8 enhancement point, we have seen in section 5 that there are essentially two types of possible dark matter candidates, corresponding to electrically neutral matter which can descend either from the 5 or $\overline{5}$ of SU(5), or as a GUT singlet. This first possibility is quite limited, and meets with significant constraints.

Having dispensed with this option, we are then left with dark matter candidates which transform as singlets under the GUT group. In the covering theory, such matter fields descend from the adjoint representation of $SU(5)_\perp$. Our aim in this section will be to show that these singlets also meet with little success because all available candidates decay faster than cosmological timescales. This again points to the natural role of gravitino dark matter in F-theory GUTs.

The rest of this section is organized as follows. We first illustrate that already at the level of cosmological considerations, the decay of the saxion presents significant obstructions to realizing a dark matter scenario capable of generating large positron fluxes, as required to provide a dark matter explanation of experiments such as PAMELA. Indeed, this decay dilutes the abundance of thermally produced relics, and, in certain instances can overproduce relics through non-thermal processes. After explaining some of the issues with such scenarios, we next turn to a list of potential dark matter candidates. First, we explain in greater detail why current experimental limits strongly disfavor electrically neutral components in extra $5 \oplus \overline{5}$'s. Next, we study the available GUT singlets which can descend from the local Higgsing of an E_8 point of enhancement. We find that in all cases, dark matter candidates which develop a suitable mass are also unstable against rapid decay to MSSM fields, and briefly comment on some non-minimal possibilities. Finally, we discuss other aspects of annihilating dark matter scenarios, such as the absence of light GeV scale gauge bosons inside of E_8, and constraints on light scalars.

9.1 Saxion decay and dark matter production in F-theory

In order to generate a sufficiently large signal capable of providing an explanation for recent dark matter experiments, the relic abundance of the corresponding dark matter particles must comprise at least a portion of the total dark matter relic abundance. Note that as the fraction of dark matter given by a candidate decreases, the resulting strength of the signal generated by this candidate must increase. For example, the flux generated in annihilating and decaying dark matter scenarios respectively scale as:

$$\Phi_{e^+}^{\rm ann} \propto \left(\frac{\rho_{\rm DM}^{\rm local}}{m_{\rm DM}}\right)^2 \cdot \langle\sigma_{\rm ann} v\rangle_{\rm present}^{\rm DM} \tag{9.1}$$

$$\Phi_{e^+}^{\rm dec} \propto \left(\frac{\rho_{\rm DM}^{\rm local}}{m_{\rm DM}}\right) \cdot \Gamma_{\rm DM}, \tag{9.2}$$

with notation as in equations (8.1) and (8.6). Thus, decreasing the fraction of the dark matter relic abundance requires a corresponding increasing in either the annihilation cross section, or decay rate.

Generally speaking, the relic abundance for dark matter can be generated either from the thermal bath of the early Universe, or at a later stage due to a late decaying particle. This distinction is especially important in F-theory GUTs, because in the most common cosmological scenario, oscillations of the saxion will dominate the energy density of the Universe prior to its decay.[13] Indeed, once the saxion decays, it will release a significant amount of entropy, diluting the relic abundance of all previously generated thermal relics through the relation:

$$\Omega_{\text{after}} = D_{\text{sax}} \Omega_{\text{before}}, \tag{9.3}$$

where as reviewed in section 3, the typical size of the dilution factor is:

$$D_{\text{sax}} \sim 10^{-4}. \tag{9.4}$$

On the other hand, the decay of the saxion can also generate new sources of dark matter relics. In the following subsections we analyze both possibilities.

9.1.1 Thermal production

We now consider the expected flux from annihilating and decaying scenarios, under the assumption that a species of dark matter is produced thermally from the primordial bath, in which it freezes out at a temperature $T^f_{(i)}$. The decay of the saxion imposes quite severe conditions on annihilating scenarios, but somewhat milder conditions on decaying dark matter scenarios.

First consider annihilating scenarios. In the case of an annihilating scenario, the flux is:

$$\Phi^{(i)}_{e^+} \propto \left(\frac{r_i \rho^{\text{local}}_{\text{DM}}}{m_i} \right)^2 \cdot \left\langle \sigma_{(i)} v \right\rangle_{\text{present}}, \tag{9.5}$$

where r_i denotes the actual fraction of dark matter of the i^{th} species participating in annihilation processes so that:

$$r_i \equiv \frac{\Omega_i}{\Omega^{\text{total}}_{\text{DM}}} \leq 1. \tag{9.6}$$

Since we are assuming the relic abundance is generated thermally, we have the relation:

$$\Omega_i h^2 \propto D_{\text{sax}} \cdot \frac{1}{\left\langle \sigma_{(i)} v \right\rangle_{T=T^f_{(i)}}}, \tag{9.7}$$

where here, we have taken into account the effects of saxion dilution.

To compare this with the expected signal from PAMELA, we first compare the expected signal of this scenario with dilution, to a WIMP scenario with no dilution. Recall that (in

[13] It is in principle possible to decrease the initial reheating temperature T^0_{RH} to such a low value that the oscillation of the saxion never comes to dominate the energy density of the Universe. In keeping with the broad outlines of the cosmological scenario found in [6], we shall typically take the initial reheating temperature higher than this low value.

JHEP08(2010)040

the absence of saxion dilution), WIMPs naturally produce the correct relic abundance to account for all of the dark matter. In particular, we have:

$$\Omega_{\rm DM}^{\rm total} h^2 = \Omega_{\rm WIMP} h^2 \propto \frac{1}{\langle \sigma v \rangle_{\rm WIMP}}, \tag{9.8}$$

where $\langle \sigma v \rangle_{\rm WIMP}$ is the annihilation cross section for WIMPs and the constant of proportionality is roughly the same in lines (9.7) and (9.8). Taking the ratio of $\Omega_i h^2$ to $\Omega_{\rm DM}^{\rm total} h^2$, we therefore obtain:

$$\frac{\Omega_i h^2}{\Omega_{\rm DM}^{\rm total} h^2} = D_{\rm sax} \cdot \frac{\langle \sigma v \rangle_{\rm WIMP}}{\langle \sigma_{(i)} v \rangle_{T=T^f_{(i)}}}, \tag{9.9}$$

In other words, returning to equation (9.6), we obtain:

$$\langle \sigma_{(i)} v \rangle_{T=T^f_{(i)}} = D_{\rm sax} \cdot \frac{\langle \sigma v \rangle_{\rm WIMP}}{r_i}. \tag{9.10}$$

Assuming that the cross section at freeze out is roughly the same as at present, we can now express the flux as:

$$\Phi_{e^+}^{(i)} \propto \left(\frac{r_i \rho_{\rm DM}^{\rm local}}{m_{\rm WIMP}} \right)^2 \cdot D_{\rm sax} \cdot \frac{\langle \sigma v \rangle_{\rm WIMP}}{r_i}. \tag{9.11}$$

In other words, in comparison with a WIMP scenario, the expected flux from the annihilating scenario is:

$$\Phi_{e^+}^{(i)} = r_i D_{\rm sax} \Phi_{e^+}^{\rm WIMP} = \frac{r_i D_{\rm sax}}{B_{\rm WIMP}} \Phi_{e^+}^{\rm PAMELA} \lesssim 10^{-7} \cdot \Phi_{e^+}^{\rm PAMELA}, \tag{9.12}$$

where in the second relation we have used equation (8.4), and in the last inequality, we have used the fact that r_i, $D_{\rm sax} \sim 10^{-4}$ and $B_{\rm WIMP}^{-1} \sim 10^{-2} - 10^{-3}$ are all factors less than one. Thus, saxion dilution exacerbates the problems with generating a sufficiently large signal already present in WIMP scenarios! It is therefore necessary to boost the cross section by a factor of at least 10^7, which is unappealing. To bypass this constraint, it seems necessary to consider cosmological scenarios where the saxion never comes to dominate the energy density of the Universe. This requires a significant decrease in the initial reheating temperature $T_{\rm RH}^0$, to values in the range of $10^4 - 10^6$ GeV [6]. In principle, this is a logical possibility, but runs counter to the idea that gravity should in principle decouple from gauge theory considerations. Indeed, lowering the initial reheating temperature so much is in conflict with this parameter range.

The consequences of saxion dilution in decaying dark matter scenarios are somewhat milder. Assuming that the decay rate for weak scale decaying dark matter is controlled by a dimension six operator with suppression scale $\Lambda_{\rm UV}$, the relevant decay rate scales much as in equation (8.8):

$$\Gamma_{\rm DM} \sim \frac{m_{\rm DM}^5}{\Lambda_{\rm UV}^4} \sim 6 \times 10^{-51} \text{ GeV} \cdot \left(\frac{m_{\rm DM}}{\rm TeV} \right)^5 \left(\frac{2 \times 10^{16} \text{ GeV}}{\Lambda_{\rm UV}} \right)^4. \tag{9.13}$$

Thus, while dilution introduces a decrease in the expected flux by a factor of 10^{-4}, note that a factor of 10 decrease in $\Lambda_{\rm UV}$ below the GUT scale easily makes up this difference. In the context of F-theory GUTs, such suppression scales are quite commonplace, either as the mass scale of a normal curve, or from enhancements in the effective scale due to the local behavior of Green's functions. In other words, the effects of saxion dilution are far milder in decaying scenarios.

9.1.2 Non-thermal production

Even though the decay of the saxion dilutes the thermally produced relics, its decays can generate a new source for such particles. This type of production mechanism requires the mass of the saxion to be greater than the decay products. When this is not the case, the relevant decay channel is closed, and some other means must be found to generate a sufficient relic abundance. Indeed, we find that if saxion decays can generate the relevant dark matter, then they will be overproduced.

Kinematic considerations require that the saxion mass be at least as heavy as the dark matter candidate. Otherwise, the needed decay channel will be unavailable. Assuming a TeV scale dark matter candidate, this is already quite problematic, because the mass of the saxion is more typically in the range of 500 GeV, than a TeV. The reason for this upper bound comes about because of the interplay between the mass of the saxion and the PQ deformation through the relation [6]:

$$m_{\rm sax} \propto \Delta_{\rm PQ}, \tag{9.14}$$

where the constant of proportionality depends on the embedding of $SU(5) \times U(1)_{\rm PQ}$ in E_8, and details of the saxion potential. For illustrative purposes, we use the mass scale induced by the PQ deformation for X as a rough estimate that this constant is on the order of four. On the other hand, the PQ deformation also induces a tachyonic contribution to the mass squared of many of the scalars, including the stau [3]:

$$m^2 = m_0^2 - q\Delta_{\rm PQ}^2, \tag{9.15}$$

where m_0 denotes the mass in the absence of the PQ deformation. This leads to an upper bound on the size of the PQ deformation. To overcome this obstacle, it is necessary to increase m_0, which will increase the amount of fine-tuning present in the gauge mediation sector.

To take an explicit example, in a Dirac neutrino scenario with zero PQ deformation, the mass of the lightest stau with minimal fine tuning is on the order of 200 GeV. With a single pair of $10 \oplus \overline{10}$ messengers with minimal fine tuning, the maximum allowed $\Delta_{\rm PQ}$ is on the order of 150 GeV. This would lead to a saxion mass on the order of 600 GeV, which is significantly less than 1 TeV. To generate a TeV scale mass would require a factor of two increase in the soft scalar masses generated by gauge mediation effects. This would also increase the mass of the squarks, and would exacerbate the fine-tuning already present.

Leaving aside this potential worry, we now show that there are further issues with generating the required relic abundance from saxion decays. It is important to note that the decay to a dark matter candidate produced in this way need not be the dominant decay channel. For example, in F-theory GUTs, the dominant decay channel for the saxion is to a pair of Higgs fields. The branching fraction to gravitinos is significantly smaller, as the relevant decay amplitude derives from a Planck suppressed higher-dimension operator. Even so, because the gravitino is stable, the resulting relic abundance of gravitinos produced in this way can still comprise up to ten percent of the relic abundance [6]. Thus, even if a dark matter candidate couples only very weakly to the saxion through a higher dimension operator, the branching fraction can still generate a sizable dark matter relic abundance.

We now compute the relic abundance generated by the decay of the saxion:

$$\Omega_{\phi}^{\mathrm{NT}} h^2 = \left(\frac{s_0}{\rho_{c,0}}\right) m_{\phi} Y_{\phi}^{\mathrm{NT}}, \tag{9.16}$$

where s_0 and $\rho_{c,0}$ respectively denote the present entropy and critical density, and Y_{ϕ}^{NT} denotes the yield non-thermally generated by the decay of the saxion:

$$Y_{\phi}^{\mathrm{NT}} \sim \frac{n_{\phi,after}^{\mathrm{NT}}}{s_{\mathrm{after}}}, \tag{9.17}$$

where $n_{\phi,after}^{\mathrm{NT}}$ denotes the number density of the dark matter generated non-thermally, and s_{after} denotes the entropy density after the decay of the saxion. As reviewed for example in [6], this can be related to the reheating temperature for the saxion, and the branching fraction of the saxion to dark matter so that:

$$Y_{\phi}^{\mathrm{NT}} \sim \frac{3}{2} B_{\phi} \frac{T_{\mathrm{RH}}^{\mathrm{sax}}}{m_{\mathrm{sax}}}, \tag{9.18}$$

where B_{ϕ} denotes the saxion branching fraction to the dark matter species ϕ. The resulting relic abundance is then given by:

$$\Omega_{\phi}^{\mathrm{NT}} h^2 = \left(\frac{s_0}{\rho_{c,0}} h^2\right) \cdot \frac{3}{2} m_{\phi} B_{\phi} \frac{T_{\mathrm{RH}}^{\mathrm{sax}}}{m_{\mathrm{sax}}}. \tag{9.19}$$

Using $s_0/\rho_{c,0} \sim 3\times 10^8\ (h^2\,\mathrm{GeV})^{-1}$, and the rough estimate $m_{\phi} \sim m_{\mathrm{sax}}/2$, we therefore find:

$$\Omega_{\phi}^{\mathrm{NT}} h^2 \sim 2 \times 10^8\ \mathrm{GeV}^{-1} \cdot B_{\phi} \cdot T_{\mathrm{RH}}^{\mathrm{sax}}. \tag{9.20}$$

To estimate the resulting relic abundance, we now determine the branching fraction to the dark matter candidate ϕ. Assuming the mass of dark matter chiral superfields Φ_1 and Φ_2 is generated through the higher dimension operator:

$$L \supset \gamma \int d^4\theta \frac{X^{\dagger} \Phi_1 \Phi_2}{\Lambda_{\mathrm{UV}}}, \tag{9.21}$$

we now show that the coupling to the saxion leads to an overproduction of dark matter particles.

For the purposes of this discussion we may take the bosonic component of the chiral superfield to be the quasi-stable dark matter candidate. If the fermionic component is lighter, then the bosonic component will eventually decay to its fermionic partner, also emitting a gravitino. Letting x, ϕ_1 and ϕ_2 denote the scalar components of these chiral superfields, taking derivatives of the Kähler potential shows that the bosonic component fields couple as:

$$L \supset \gamma \frac{\phi_2}{\Lambda_{\rm UV}} \partial^\mu \overline{x} \partial_\mu \phi_1 + \gamma \frac{\phi_1}{\Lambda_{\rm UV}} \partial^\mu \overline{x} \partial_\mu \phi_2 + h.c. \tag{9.22}$$

Here we shall assume that the bosons are stable against further decays, and then compute the branching fraction. Writing $x = \langle x \rangle \exp(ia + s)$, with s the saxion, it follows that the saxion indeed couples to the ϕ_i's. Provided the mass of the saxion is larger than the combined mass of the two decay products, we obtain a rough estimate for the decay rate:

$$\Gamma_{x \to \phi_1 \phi_2} \sim \frac{\gamma^2}{32\pi} \frac{m_{\rm sax}^3}{\Lambda_{\rm UV}^2}. \tag{9.23}$$

With the decay rate in hand, we now compute the expected branching fraction. Returning to equation (3.8), the total saxion decay rate is related to the axion branching fraction as:

$$\Gamma_{\rm sax} \sim \frac{1}{B_a} \Gamma_a \sim \frac{1}{B_a} \frac{1}{64\pi} \frac{m_{\rm sax}^3}{f_a^2}, \tag{9.24}$$

where B_a is the branching fraction to axions and f_a is the axion decay constant. Hence, the branching fraction to dark matter is:

$$B_\phi = \frac{\Gamma_{x \to \phi_1 \phi_2}}{\Gamma_{\rm sax}} \sim B_a \frac{m_\phi^2}{\Lambda_{\rm GMSB}^2} \sim 1 \times 10^{-4} \cdot B_a \left(\frac{m_\phi}{1 \text{ TeV}} \right)^2 \left(\frac{10^5 \text{ GeV}}{\Lambda_{\rm GMSB}} \right)^2, \tag{9.25}$$

where we have used the fact that in F-theory GUTs, the scale of supersymmetry breaking and axion decay constant are related to the characteristic scale of gauge mediation as $F_X / f_a = \Lambda_{\rm GMSB} \sim 10^5$ GeV [3]. Further, we have taken the natural value $m_\phi \sim$TeV for the mass of the dark matter candidate. The saxion reheating temperature is:

$$T_{\rm RH}^{\rm sax} \sim 0.5 \sqrt{\Gamma_{\rm sax} M_{\rm PL}} \sim 10 \text{ GeV} \cdot \frac{1}{B_a^{1/2}} \left(\frac{m_\phi}{1 \text{ TeV}} \right)^{3/2}. \tag{9.26}$$

Returning to equation (9.20), the resulting thermal relic abundance is then:

$$\Omega_\phi^{\rm NT} h^2 \sim 2 \times 10^5 \cdot B_a^{1/2} \left(\frac{m_\phi}{1 \text{ TeV}} \right)^{7/2} \left(\frac{10^5 \text{ GeV}}{\Lambda_{\rm GMSB}} \right)^2. \tag{9.27}$$

The actual branching fraction to axions depends on the mass of the saxion. Indeed, as the saxion becomes more massive than the corresponding states in the MSSM, additional channels will begin to open up. As a consequence, the value of B_a can decrease, even though the decay rate to axions scales with $m_{\rm sax}^3$. Note, though, that to generate a suitable relic abundance would require $B_a \sim 10^{-10}$, which is far smaller than the value of $B_a \sim 10^{-3} - 10^{-1}$ found in [6]. Hence, we deduce that the decay of the saxion actually overproduces the corresponding relic. In the above we also assumed that the bosonic component was the dark matter. If this is not the case, note that it will decay, generating a further source of gravitino relics, which appears to be far greater than can otherwise be allowed.

JHEP08(2010)040

Summarizing, the above analysis demonstrates that the decay of the saxion overproduces TeV scale dark matter candidates, when such a decay channel is available. Turning this discussion around, we can view this either as a constraint on how the saxion couples to the dark matter, or as an upper bound on the mass of the saxion, so that kinematics can forbid such decays. In the former case, this leaves open exactly how the dark matter develops a weak scale mass. This can in principle be accommodated if another GUT singlet develops a weak scale, but this is somewhat less minimal. Assuming that the saxion can decay to the dark matter candidate, this imposes an intriguing cosmological upper bound on the mass, of around 1 TeV. Combined with the condition that the saxion must be allowed to decay to an MSSM scalar such as the Higgs, this leads to the mass range:

$$230\ \text{GeV} \lesssim m_{\text{sax}} \lesssim 1\ \text{TeV}. \tag{9.28}$$

We have also seen that in the case of thermally produced annihilating dark matter, saxion dilution tends to eliminate any candidate signatures. On the other hand, thermally produced decaying dark matter can still generate a sizable signal capable of explaining excess fluxes. Even so, it is possible that including additional non-minimal ingredients could produce an appropriate relic abundance. Assuming this is the case, we now turn to other obstructions with realizing such scenarios in minimal F-theory GUT models.

9.2 Eliminating semi-visible candidates

In this section we discuss obstructions in F-theory associated with realizing dark matter from the electrically neutral component of the $5 \oplus \overline{5}$ of SU(5). In subsection 5.6 we found that in the case of a Dirac scenario with $\mathbb{Z}_2$ monodromy group, it is indeed possible to consider vector-like pairs in the $5 \oplus \overline{5}$ which develop a mass through the Giudice-Masiero operator:

$$\int d^4\theta \frac{X^\dagger D_5 D'_{\overline{5}}}{\Lambda_{\text{UV}}} \to \int d^2\theta\ \mu_D D_5 D'_{\overline{5}}. \tag{9.29}$$

This induces a mass μ_D on the order of the μ parameter. The electrically neutral components of the 5 and $\overline{5}$ then provide potential dark matter candidates. A quite attractive feature of such semi-visible dark matter candidates is that although electrically neutral, this component can still couple to the Standard Model through SU(2) interactions.

Current experimental limits impose strong restrictions on the existence of such electrically neutral objects. First note that in order not to spoil the unification of the coupling constants, all of the components of the GUT multiplet must have comparable mass. Thus, we can expect extra states with non-trivial $SU(3)_C$ and $U(1)_{\text{EM}}$ charges. Direct searches at the Tevatron only rule out extra colored states with mass less than about 300 GeV [73], which is well below the expected mass of D_5 and $D'_{\overline{5}}$.

A more stringent constraint comes from the SU(2) doublet components of the dark matter candidate.[14] Experimental bounds on dark matter scattering off of nuclei in experiments such as CDMS are sensitive to a convolution between the total flux from dark matter and the effective cross section for scattering with nuclei from Z^0 gauge boson exchange. In

[14] We thank D.E. Morrissey for discussion on this point.

this context, the corresponding effective cross section for matter far above the GeV scale is [74]:[15]

$$\sigma_{DM,nuc} \sim 7.5 \times 10^{-39}\ \mathrm{cm}^2. \tag{9.30}$$

This is to be contrasted with current experimental constraints from CDMS. Assuming that all of the dark matter relic abundance participates in nuclei scattering, and moreover, that the local dark matter density is roughly $0.3\,\mathrm{GeV/cm}^3$, this leads to a lower bound on the size of the allowed cross section [76]:

$$\sigma_{\mathrm{CDMS}} \lesssim 10^{-43}\ \mathrm{cm}^2. \tag{9.31}$$

It is important to note that this result scales linearly with the overall dark matter number density, so in principle, lowering the local density by roughly a factor of 10^{-4} can provide one way for a dark matter candidate to evade such a bound. Note, however, that this will also lower the expected positron flux which experiments such as PAMELA can in principle produce.

An alternative way to evade such a bound is in principle possible in inelastic dark matter (iDM) scenarios, in which a small Majorana mass is also included in the effective Lagrangian [77]. This induces a mass splitting between the two Dirac mass states. In particular, when the lighter dark matter state interacts with the nuclei, it can "up-scatter" to the heavier state. Kinematic considerations now imply that the resulting phase space available for the convolution between dark matter flux and the total cross section is somewhat lower. In order for this mechanism to lead to a sufficient suppression in the reaction rate expected from CDMS, this requires a Majorana mass term of at least 100 keV.

Generating such a large Majorana mass term is quite problematic in minimal F-theory GUT setups. Indeed, this requires the presence of a higher dimension operator such as:

$$\int d^2\theta \frac{\left(H_u L_{D'_{\overline{5}}}\right)^2}{\Lambda_{\mathrm{UV}}}, \tag{9.32}$$

where $L_{D_{\overline{5}}}$ denotes the "lepton doublet" part of $D_{\overline{5}}$. On the other hand, similar operators in the Majorana neutrino scenario of [8] generate masses closer to 10 meV. Without adding a significantly more elaborate sector with which D_5 and $D'_{\overline{5}}$ interact, this effectively rules out such semi-visible dark matter candidates.

9.3 Unstable GUT singlets

In the previous subsections we found that the decay of the saxion in F-theory GUTs dilutes thermal relics, and can also overproduce dark matter candidates. Leaving aside these potential concerns, in this section we show that all of the available dark matter candidates decay far too rapidly. This then eliminates all annihilating and decaying dark matter scenarios based on particles embedded in E_8.

Any dark matter candidate must be sufficiently long-lived. Dimension five decay operators pose significant problems for the lifetime of a dark matter candidate. Indeed, the

[15] See for example [75] for a brief discussion of how to extract the value of this cross section from [74].

expected decay rate in this case is of the form:

$$\Gamma_{\rm DM} \sim \frac{m_{\rm DM}^3}{M_{\rm GUT}^2} \sim 10^{-24}\ {\rm GeV} \cdot \left(\frac{m_{\rm DM}}{1\ {\rm TeV}}\right)^3 \left(\frac{3\times 10^{16}\ {\rm GeV}}{M_{\rm GUT}}\right)^2 \tag{9.33}$$

$$\sim 10\ {\rm sec}^{-1} \cdot \left(\frac{m_{\rm DM}}{1\ {\rm TeV}}\right)^3 \left(\frac{3\times 10^{16}\ {\rm GeV}}{M_{\rm GUT}}\right)^2, \tag{9.34}$$

which is far shorter than any cosmological timescale. In other words, if a dark matter candidate can couple to MSSM fields through a dimension five operator, and if the decay is kinematically allowed, the corresponding field will be unstable on cosmological timescales.

The aim of this section is to show that all of the GUT singlets which descend from the local Higgsing of E_8 decay far too rapidly to constitute dark matter candidates. This is a general statement, and does not require delving into the particular details of a specific annihilating or decaying scenario. The relevant higher dimension operators can all be generated much as in [2, 3, 8] by integrating out heavy modes localized on curves.

Available GUT singlets which can constitute dark matter descend from the adjoint of $SU(5)_\perp$, so in the languague of section 5 are either given as non-trivial weights of the form $t_m - t_n$, or as zero weights. Of the available states which descend from the adjoint of $SU(5)_\perp$, note that the zero weights correspond to bulk modes of other seven-branes. Since no symmetry forbids F-terms involving one zero weight and three MSSM chiral superfields, we conclude that such modes cannot constitute dark matter candidates. This leaves only the weights of the form $t_m - t_n$ as possibilities.

With notation as in section 5, the available GUT singlets are given as either $D_{(i)}$, the conjugates $D^c_{(i)}$ which localize on the same curve, or as GUT singlets which localize on either the X curve or the N_R curve. Index theory considerations exclude the possibility of fields conjugate to X or N_R, but in principle, extra "generations" on these curves could be present, which we denote as $\widehat{X}$ and $\widehat{N}_R$. Using the classification of section 5, the available dark matter candidates, and their respective charges under the available symmetries in the various neutrino scenarios are:

$G^{\rm Dirac}_{\rm mono} \simeq \mathbb{Z}_2$ or $\mathbb{Z}_2 \times \mathbb{Z}_2$	$\widehat{X}$	$\widehat{N}_R$	$D_{(1)}, D^c_{(1)}$	$D_{(2)}$	$D^c_{(2)}$	$D_{(3)}, D^c_{(3)}$	$D_{(4)}$	$D^c_{(4)}$
$U(1)_{\rm PQ}$	-4	-3	0	$+4$	-4	0	-7	$+7$
$U(1)_\chi$	0	-5	0	0	0	0	-5	$+5$

(9.35)

$G^{\rm Dirac}_{\rm mono} \simeq \mathbb{Z}_3$ or S_3	$\widehat{X}$	$\widehat{N}_R$	$D_{(1)}$	$D^c_{(1)}$	$D_{(2)}, D^c_{(2)}$
$U(1)_{\rm PQ}$	-4	-3	$+1$	-1	0
$U(1)_\chi$	0	-5	-5	$+5$	0

(9.36)

$G^{\rm Maj}_{\rm mono}$	$\widehat{X}$	$\widehat{N}_R$	$D_{(1)}, D^c_{(1)}$	$D_{(2)}, D^c_{(2)}$
$U(1)_{\rm PQ}$	-5	0	0	0

. (9.37)

Of these candidates, it is necessary to state how each available dark matter candidate can develop a weak to TeV scale mass. The most minimal option is to correlate the mass through the same type of mechanism which generates the μ term, through a Giudice-Masiero operator. It is in principle possible to also consider cubic superpotential couplings between three GUT singlets where one field develops a weak scale vev. Although less minimal, this possibility can occur in F-theory GUTs provided the field which develops a vev has a suitable PQ charge. Indeed, the PQ deformation can induce a tachyonic contribution to the mass squared for the bosonic component of this field. Letting $\Phi_{(i)}$ denote candidate dark matter chiral multiplets, this leaves the following possibilities for how the dark fields can develop a suitable mass:

$$\int d^4\theta \frac{X^\dagger \Phi_{(1)}\Phi_{(2)}}{\Lambda_{\text{UV}}} \text{ or } \int d^2\theta\ \Phi_{(1)}\Phi_{(2)}\Phi_{(3)}, \tag{9.38}$$

where in the second possibility, at least one of the $\Phi_{(i)}$'s develops a weak scale vev.

Our strategy to deduce the absence of any quasi-stable dark matter candidates is as follows. Given a pair of chiral superfields $\Phi_{(i)}$ and $\Phi_{(j)}$which develop a weak to TeV scale mass through the coupling:,

$$\mu_{\text{ij}} \int d^2\theta\ \Phi_{(i)}\Phi_{(j)}, \tag{9.39}$$

then either both fields are quasi-stable, or neither is. Indeed, if the components of $\Phi_{(i)}$ decay rapidly, then mixing through the mass term will also induce a decay for $\Phi_{(j)}$. It is therefore enough to show that for any available pair of chiral superfields, at least one member of the pair decays rapidly to MSSM fields.

We now demonstrate that in all cases, a TeV scale GUT singlet possesses a rapid decay channel. Since the actual weight assignments are somewhat specific to the details of a given monodromy group, we discuss these possibilities separately.

9.3.1 Dirac scenarios: $G^{\text{Dirac}}_{\text{mono}} \simeq \mathbb{Z}_2$ or $\mathbb{Z}_2 \times \mathbb{Z}_2$

First consider monodromy scenarios where $G^{\text{Dirac}}_{\text{mono}} \simeq \mathbb{Z}_2$ or $\mathbb{Z}_2 \times \mathbb{Z}_2$. Consistency with $\text{U}(1)_{\text{PQ}}$ and $\text{U}(1)_\chi$ leads to the following options for generating weak to TeV scale masses in this Dirac scenario:

$$G^{\text{Dirac}}_{\text{mono}} \simeq \mathbb{Z}_2 \text{ or } \mathbb{Z}_2 \times \mathbb{Z}_2 \tag{9.40}$$

$$\int d^4\theta \frac{X^\dagger \Phi_{(1)}\Phi_{(2)}}{\Lambda_{\text{UV}}} \Longrightarrow \Phi_{(1)} = D^c_{(2)},\ \Phi_{(2)} = D_{(1)}, D^c_{(1)}, D_{(3)}, D^c_{(3)} \tag{9.41}$$

$$\int d^2\theta\ \Phi_{(1)}\Phi_{(2)}\Phi_{(3)} \Longrightarrow \Phi_{(1)} = \widehat{X},\ \Phi_{(2)} = D_{(2)},\ \Phi_{(3)} = D_{(1)}, D^c_{(1)}, D_{(3)}, D^c_{(3)} \tag{9.42}$$

$$\int d^2\theta\ \Phi_{(1)}\Phi_{(2)}\Phi_{(3)} \Longrightarrow \Phi_{(1)} = \widehat{X},\ \Phi_{(2)} = \widehat{N}_R,\ \Phi_{(3)} = D^c_{(4)} \tag{9.43}$$

$$\int d^2\theta\ \Phi_{(1)}\Phi_{(2)}\Phi_{(3)} \Longrightarrow \Phi_{(1)} = \widehat{N}_R,\ \Phi_{(2)} = D^c_{(2)},\ \Phi_{(3)} = D^c_{(4)}. \tag{9.44}$$

As we now explain, all available dark matter candidates are unstable on cosmological timescales. To establish this, recall that the available orbits for minimal matter and possible dark matter candidates are:

$$G_{\text{mono}}^{\text{Dirac}} = \langle (12)(34) \rangle \simeq \mathbb{Z}_2 \colon \tag{9.45}$$

$$Orb(10_M, Y_{10}) = t_1, t_2 \tag{9.46}$$

$$Orb(Y'_{10}) = -t_3, -t_4 \tag{9.47}$$

$$Orb(\overline{5}_M) = t_4 + t_5, t_3 + t_5 \tag{9.48}$$

$$Orb(5_H) = -t_1 - t_2 \tag{9.49}$$

$$Orb(\overline{5}_H) = t_1 + t_4, t_2 + t_3 \tag{9.50}$$

$$Orb(X^\dagger) = t_2 - t_4, t_1 - t_3 \tag{9.51}$$

$$Orb(\widehat{X}) = t_4 - t_2, t_3 - t_1 \tag{9.52}$$

$$Orb(N_R, \widehat{N}_R) = t_1 - t_5, t_2 - t_5 \tag{9.53}$$

$$Orb(D_{(1)}, D^c_{(1)}) = t_1 - t_2, t_2 - t_1 \tag{9.54}$$

$$Orb(D_{(2)}) = t_1 - t_4, t_2 - t_3 \tag{9.55}$$

$$Orb(D^c_{(2)}) = t_4 - t_1, t_3 - t_2 \tag{9.56}$$

$$Orb(D_{(3)}, D^c_{(3)}) = t_3 - t_4, t_4 - t_3 \tag{9.57}$$

$$Orb(D^c_{(4)}) = t_5 - t_3, t_5 - t_4. \tag{9.58}$$

By inspection, $D_{(1)}$ and $D^c_{(1)}$ are in the same orbit, and similar considerations apply for $D_{(3)}$ and $D^c_{(3)}$. Hence, nothing forbids the mass terms:

$$\int d^2\theta \; D_{(1)} D_{(1)} + \int d^2\theta \; D_{(3)} D_{(3)}, \tag{9.59}$$

which will generically be at the GUT scale. Thus, these fields are unsuitable as dark matter candidates.[16] In particular, this means that potential couplings involving these operators are now excluded as dark matter candidates, since as in the usual seesaw mechanism, the mass of the heavy state is near the GUT scale, and the light state will have mass closer to μ^2 / M_{GUT}. This eliminates lines (9.41) and (9.42) as viable options.

The remaining dark matter candidate fields are then given by either the bosonic or fermionic component of the chiral superfields $\widehat{N}_R$, $\widehat{X}$, $D^c_{(2)}$, or $D^c_{(4)}$. As we now argue, $\widehat{N}_R$ must constitute a TeV scale dark matter candidate. Note that of the two remaining options given by lines (9.43) and (9.44), both involve the field $\widehat{N}_R$. Either $\widehat{N}_R$ corresponds to a dark matter candidate, or it generates a weak to TeV scale vev, inducing a mass term for the other dark matter candidates.

In the context of F-theory GUTs, however, $\widehat{N}_R$ will typically not acquire a vev due to the PQ deformation. Indeed, since it has the same charge as the X field, the PQ deformation will induce a positive mass squared contribution. Moreover, if $\widehat{N}_R$ did develop a vev, it would induce matter parity couplings in the low energy theory:

$$\int d^4\theta \frac{H_d^\dagger L N_R}{\Lambda_{\text{UV}}} \to \int d^4\theta \frac{H_d^\dagger L \langle N_R \rangle}{\Lambda_{\text{UV}}}. \tag{9.60}$$

[16] It is also possible to show that these candidates also couple to MSSM fields through dimension five operators, so that they will also decay rapidly.

This is a somewhat more exotic possibility in the context of F-theory GUTs, and so we will not dwell on this in any great detail. Another fine-tuned possibility would be to require that the mass of $\widehat{N}_R$ is quite low. In this case, kinematic considerations could exclude a potential decay channel. This is again a somewhat exotic possibility in the context of F-theory GUTs. Indeed, if anything, the PQ deformation tends to further increase the mass of $\widehat{N}_R$. Thus, in all cases we conclude that in any candidate dark matter scenario, there must exist a quasi-stable $\widehat{N}_R$ field of TeV scale mass which is localized on the same curve as the other right-handed neutrinos.

As we now explain, the candidate field $\widehat{N}_R$ always couples to operators which induce rapid decays of the field. Since $\widehat{N}_R$ couples through a mass term to the remaining dark matter candidates, it follows that all remaining states will be unstable against rapid decay. In this regard, it is tempting to simply end the discussion here by invoking the presence of operators of the form:

$$\int d^4\theta \frac{H_d^\dagger L \widehat{N}_R}{\Lambda_{\rm UV}}. \tag{9.61}$$

Note, however, that this is the same type of term which induces a Dirac neutrino mass [8]. Since there are only three generations of L's, one linear combination of the modes localized on the N_R curve will not couple to $H_d^\dagger L$. Thus, this coupling is by itself not enough to guarantee that $\widehat{N}_R$ decays rapidly.

Besides $H_d^\dagger L \widehat{N}_R/\Lambda_{\rm UV}$, $\widehat{N}_R$ also couples to MSSM fields through the dimension five operator:

$$\int d^2\theta \frac{\widehat{N}_R \overline{5}_M \overline{5}_M 10_M}{\Lambda_{\rm UV}} : \widehat{N}_R = t_1 - t_5,\, \overline{5}_M = t_4 + t_5,\, \overline{5}_M = t_3 + t_5,\, 10_M = t_2, \tag{9.62}$$

where we have also listed the corresponding weight assignments. Both the bosonic and fermionic components of the $\widehat{N}_R$ chiral multiplet can now decay rapidly.

Some examples of the available decay channels for the bosonic and fermionic components of $\widehat{N}_R$ are respectively:

$$\phi_{\widehat{N}_R} \to \widetilde{\tau}_1^{\pm} \tau^{\mp} \nu_\tau. \tag{9.63}$$

$$\psi_{\widehat{N}_R} \to \widetilde{\tau}_1^{\pm} \widetilde{\tau}_1^{\mp} \nu_\tau, \tag{9.64}$$

where $\widetilde{\tau}_1^{\pm}$ denotes the lightest stau. In F-theory GUTs, the mass of the lightest stau is typically in the range of $100 - 300\,\text{GeV}$. Since the components of $\widehat{N}_R$ are required to be in the TeV range just to provide a possible dark matter candidate, it follows that this dark matter candidate is unstable against rapid decay. It now follows that since all remaining candidates mix by a mass term with the components of $\widehat{N}_R$, all available dark matter candidates are unstable against rapid decay.

9.3.2 Dirac scenarios: $G_{\rm mono}^{\rm Dirac} \simeq \mathbb{Z}_3$ or S_3

Next consider monodromy scenarios where $G_{\rm mono}^{\rm Dirac} \simeq \mathbb{Z}_3$ or S_3. Returning to our general discussion of available GUT singlets, compatibility with $\mathrm{U}(1)_{\rm PQ}$ and $\mathrm{U}(1)_\chi$ leads to the following options:

$$G_{\text{mono}}^{\text{Dirac}} \simeq \mathbb{Z}_3 \text{ or } S_3 \tag{9.65}$$

$$\int d^4\theta \frac{X^\dagger \Phi_{(1)}\Phi_{(2)}}{\Lambda_{\text{UV}}} \Longrightarrow \Phi_{(1)} = \widehat{N}_R,\, \Phi_{(2)} = D^c_{(1)} \tag{9.66}$$

$$\int d^4\theta \frac{X^\dagger \Phi_{(1)}\Phi_{(2)}}{\Lambda_{\text{UV}}} \Longrightarrow \Phi_{(1)} = \widehat{X},\, \Phi_{(2)} = D_{(2)}, D^c_{(2)} \tag{9.67}$$

$$\int d^2\theta\ \Phi_{(1)}\Phi_{(2)}\Phi_{(3)} \Longrightarrow\ \Phi_{(1)} = D_{(1)},\, \Phi_{(2)} = D^c_{(1)},\, \Phi_{(3)} = D_{(2)}, D^c_{(2)}. \tag{9.68}$$

Of these options, we can immediately eliminate the last possibility, because it involves a vector-like pair of fields localized on the same curve. Such modes will generically lift from the low energy theory, and so this coupling is ruled out. This leaves the first two options, and $\widehat{N}_R$, $D^c_{(1)}$, $\widehat{X}$, $D_{(2)}$ and $D^c_{(2)}$ as potential dark matter candidates. To establish the absence of a stable dark matter candidate, it is enough to show that the candidates $\widehat{N}_R$, $D_{(2)}$ and $D^c_{(2)}$ are all unstable on cosmological timescales. To this end, recall that the orbits for the minimal matter and relevant dark matter candidates are as obtained in section 5:

$$G_{\text{mono}}^{\text{Dirac}} \simeq \mathbb{Z}_3 \text{ or } S_3 \tag{9.69}$$

$$Orb(10_M, Y_{10}) = t_1, t_2, t_3 \tag{9.70}$$

$$Orb(Y'_{10}) = -t_4 \tag{9.71}$$

$$Orb(\overline{5}_M) = t_1 + t_5, t_2 + t_5, t_3 + t_5 \tag{9.72}$$

$$Orb(5_H) = -t_1 - t_2, -t_2 - t_3, -t_3 - t_1 \tag{9.73}$$

$$Orb(\overline{5}_H) = t_1 + t_4, t_2 + t_4, t_3 + t_4 \tag{9.74}$$

$$Orb(X^\dagger) = t_1 - t_4, t_2 - t_4, t_3 - t_4 \tag{9.75}$$

$$Orb(N_R, \widehat{N}_R) = t_4 - t_5. \tag{9.76}$$

$$Orb(D^c_{(1)}) = t_5 - t_1, t_5 - t_2, t_5 - t_3 \tag{9.77}$$

$$Orb(D_{(2)}) = t_2 - t_3, t_3 - t_1, t_1 - t_2 \tag{9.78}$$

$$Orb(D^c_{(2)}) = t_3 - t_2, t_1 - t_3, t_2 - t_1. \tag{9.79}$$

We first show that the candidate dark matter field $\widehat{N}_R$ again decays too rapidly. Indeed, this field participates in the dimension five operator:

$$\int d^2\theta \frac{\widehat{N}_R \overline{5}_M \overline{5}_M 10_M}{\Lambda_{\text{UV}}} : \widehat{N}_R = t_4 - t_5,\, \overline{5}_M = t_1 + t_5,\, \overline{5}_M = t_2 + t_5,\, 10_M = t_3, \tag{9.80}$$

with the weights of all fields indicated. Thus, just as in subsection 9.3.1, we conclude that both the bosonic and fermionic components of $\widehat{N}_R$ decay too rapidly to constitute viable dark matter candidates.

Next consider the dark matter candidates $D_{(2)}$ and $D^c_{(2)}$. These candidates also decay quite rapidly due to the higher dimension operators:

$$\int d^2\theta \frac{D_{(2)} 5_H 10_M 10_M}{\Lambda_{\rm UV}} : D_{(2)} = t_1 - t_2, 5_H = -t_3 - t_1, 10_M = t_2, 10_M = t_3 \tag{9.81}$$

$$\int d^2\theta \frac{D^c_{(2)} \overline{5}_H \overline{5}_M 10_M}{\Lambda_{\rm UV}} : D^c_{(2)} = t_2 - t_1, \overline{5}_H = t_1 + t_4, \overline{5}_M = t_1 + t_5, 10_M = t_3, \tag{9.82}$$

where in the above, we have also indicated the corresponding weights of the fields.

Since the Higgs fields develop a weak scale vev anyway, it is enough to consider some of the induced cubic couplings of lines (9.81) and (9.82) with non-zero Higgs vevs. These operators have the form:

$$\frac{M_{\rm weak}}{\Lambda_{\rm UV}} \int d^2\theta \; D E_L E_R, \tag{9.83}$$

where E_L is the charged part of the lepton doublet. Thus, both the scalar and fermionic components of D can decay to leptons quite rapidly. For example, the fermionic component of the D's can decay via:

$$\psi_D \rightarrow \widetilde{\tau}_1^{\pm} \tau^{\mp}. \tag{9.84}$$

Since the lightest stau typically has mass in the range of $100 - 300\,\text{GeV}$, nothing kinematically forbids the decay of a TeV scale D.

Thus, we conclude that in all Dirac scenarios, generating a TeV scale mass for the dark matter candidate is incompatible with it being quasi-stable.

9.3.3 Majorana scenarios

The Majorana scenarios are significantly more constrained than their Dirac counterparts because the sizes of the orbits are that much longer. This also means that the number of available GUT singlets will also be significantly reduced. Returning to line (9.37), the available GUT singlets are $\widehat{X}$, $\widehat{N}_R$, $D_{(1)}$, $D^c_{(1)}$, $D_{(2)}$ and $D^c_{(2)}$. Of these possibilities, note that only $\widehat{X}$ has non-trivial PQ charge. Indeed, the analysis of appendix A establishes that in all Majorana scenarios, the weights for the orbit of $D_{(i)}$ and $\widehat{N}_R$ contains both $t_m - t_n$ as well as $t_n - t_m$. Hence, there is no symmetry in the covering theory which forbids the dark matter candidates $\Phi_{(i)}$ from developing a large mass through the terms:

$$M_{\rm GUT} \int d^2\theta \; \Phi_{(i)} \Phi_{(i)} : \Phi_{(i)} = t_m - t_n, \Phi_{(i)} = t_n - t_m. \tag{9.85}$$

In other words, the dark matter candidate lifts from the low energy theory.[17] This also means that no light states are available with which $\widehat{X}$ can pair up to develop a weak to TeV scale mass. Thus, in all Majorana neutrino scenarios, there are simply no candidates available.

Combining this with the analysis of the Dirac scenarios, we conclude that in no case do we have a TeV scale quasi-stable dark matter candidate.

[17]Note that such heavy fields are not cosmologically problematic because they can also decay due to couplings of the form $\Phi_{(i)} \overline{5}_H \overline{5}_M 10_M / \Lambda_{\rm UV}$.

9.3.4 Non-minimal scenarios

Although beyond the scope of this paper, we note that less minimal models in which nearby seven-branes intersecting the PQ seven-brane also participate can potentially lead to additional possibilities, such as when an E_8-type singularity nearly collides with other singularities.

At least at the level of effective field theory considerations, it is conceivable that a dark matter candidate could have PQ charges and additional discrete symmetries as necessary to exclude operators which induce rapid decay. For example, in the context of a Dirac neutrino scenario, we can introduce dark matter chiral multiplets $\Phi_{(1)}$ and $\Phi_{(2)}$, with PQ charges:

	$\overline{5}_M$	10_M	5_H	$\overline{5}_H$	$X^\dagger$	N_R	$\Phi_{(1)}$	$\Phi_{(2)}$
Dirac $U(1)_{PQ}$	$+1$	$+1$	-2	-2	$+4$	-3	q_1	$q_2 = -q_1 - 4$

. (9.86)

Provided q_1 is at least an order ten number, it is now immediate that an odd number of $\Phi_{(i)}$'s cannot appear in an operator. It is therefore enough to consider operators which are quadratic in the $\Phi_{(i)}$'s appearing in the combinations $\Phi_{(1)}\Phi_{(2)}$ or $\Phi^\dagger_{(i)}\Phi_{(i)}$. To induce a decay, one of the $\Phi_{(i)}$'s must then develop a weak scale vev. Note that although very large PQ charges constitute a significant departure from the minimal scenarios considered in this paper, PQ charges as large as ± 7 naturally appeared in the monodromy orbit classification reviewed in section 5.

We now rule out potentially problematic dimension five or lower operators which could destabilize the dark matter candidate. The available operators are of the form $\Phi_{(1)}\Phi_{(2)}\mathcal{O}_{12}$ or $\Phi^\dagger_{(i)}\Phi_{(i)}\mathcal{O}_{\bar{i}i}$, where $\mathcal{O}_{12}$ and $\mathcal{O}_{\bar{i}i}$ are operators constructed from the minimal matter of F-theory GUTs with respective PQ charges +4 and 0. In the former case, note that by inspection of the available minimal matter and their charges, no dimension four or five F-terms can be generated, and as for D-terms, only $\mathcal{O}_{12} = X^\dagger$ is available. By gauge invariance, the combination $\Phi^\dagger_{(i)}\Phi_{(i)}$ must involve at least two minimal fields. We therefore conclude that in all cases, no dangerous operator is available.[18]

On the other hand, symmetry considerations are compatible with operators of the form:

$$L \supset \int d^4\theta \frac{\Phi^\dagger_{(i)}\Phi_{(j)}\Sigma^\dagger_{\rm MSSM}\Sigma_{\rm MSSM}}{\Lambda^2_{\rm UV}}, \tag{9.87}$$

where $\Sigma_{\rm MSSM}$ denotes a chiral superfield of the MSSM. See [66] for further discussion on this and other operators which are suitable for decaying dark matter scenarios. Letting the same variables denote the bosonic components of Σ, and ϕ and ψ the respective bosonic and fermionic components of Φ, the Lagrangian therefore contains the terms:

$$L \supset \frac{\phi_{(j)}\Sigma^\dagger_{\rm MSSM}}{\Lambda^2_{\rm UV}}\partial^\mu\phi^\dagger_{(i)}\partial_\mu\Sigma_{\rm MSSM} + \frac{\phi_{(j)}\Sigma^\dagger_{\rm MSSM}}{\Lambda^2_{\rm UV}}\overline{\psi}_{(i)}i\overline{\sigma}^\mu\partial_\mu\psi_\Sigma. \tag{9.88}$$

[18] Since we have already excluded all combinations involving X and $X^\dagger$, this also rules out instanton generated operators. Indeed, an instanton suppressed contribution can effectively be viewed as multiplication by a field with the same charge as $X^\dagger$ since instantons also induce the Polonyi term $q_{\rm inst}X$, as in the gauge mediated supersymmetry breaking scenario in [3] (see also [13]).

In other words, provided one of the $\phi_{(i)}$'s develops a weak scale vev, this leads to the required decay rate associated with a dimension six operator.

As a cautionary note, this is by itself still insufficient to demonstrate that such possibilities can be realized in F-theory. A complete realization would require introducing a somewhat exotic sector of the type discussed in section 6.

9.4 Other aspects of annihilating scenarios

In the previous subsections we have encountered significant obstructions to realizing other dark matter candidates besides the gravitino in F-theory GUTs. These obstructions are especially severe in the case of annihilating scenarios, where we have seen that the dilution of thermally produced dark matter requires an even bigger boost in the annihilation cross section, as compared with WIMP scenarios. Moreover, when the saxion decay generates such candidates non-thermally, we have also found that the relic abundance is too large by several orders of magnitude! In addition, all of the available GUT singlets from E_8 which could in principle develop a weak to TeV scale mass are unstable on cosmological timescales, and are therefore unsuitable as dark matter candidates.

In this subsection we comment on some additional aspects of annihilating scenarios where the candidate dark sector embeds inside of E_8. The first point is that a common ingredient in many annihilating scenarios, namely the existence of an additional light "dark" gauge boson is simply unavailable within E_8. On the other hand, we show that with sufficient fine tuning of the parameters of F-theory GUTs, a light scalar could potentially communicate between the dark and visible sectors. This by itself is interesting, although it is not completely clear even in this case how to overcome all of the other obstacles found earlier in this section.

As reviewed in section 8, there is already some tension in annihilating dark matter scenarios because the expected annihilation cross section from thermally produced WIMPs leads to a signal which is too small to explain the results of PAMELA by a factor of $10^{-2} - 10^{-3}$. One way that the annihilation cross section can be boosted (as required by PAMELA), is through an infrared enhancement due to the exchange of a light boson with mass far below the weak scale, and closer to a few GeV. See for example [60–63] for discussion on this possibility. It is therefore natural to ask whether such degrees of freedom are available in F-theory GUTs.

The classification of monodromy orbits reviewed in section 5 already shows that no spin one mediators are available inside of a minimal E_8 F-theory GUT. Indeed, in the Dirac neutrino scenarios, there are only two U(1)'s, $\mathrm{U}(1)_{\mathrm{PQ}}$ and $\mathrm{U}(1)_\chi$, and in the Majorana neutrino scenarios, only $\mathrm{U}(1)_{\mathrm{PQ}}$ is available. Since $\mathrm{U}(1)_{\mathrm{PQ}}$ is anomalous, the generalized Green-Schwarz mechanism cancels the corresponding anomaly, but also cause this gauge boson to develop a large mass via the Stückelberg mechanism. This leaves only $\mathrm{U}(1)_\chi$ in the Dirac neutrino scenario as even a candidate for consideration. As reviewed in section 5, $\mathrm{U}(1)_\chi$ is a very specific linear combination of $\mathrm{U}(1)_Y$ and $\mathrm{U}(1)_{B-L}$ dictated by the embedding $\mathrm{SO}(10) \supset \mathrm{SU}(5)_{\mathrm{GUT}} \times \mathrm{U}(1)_\chi$. The experimental bounds on the mass of this

JHEP08(2010)040

type of gauge boson are quite stringent, and well above 100 GeV.[19] We therefore conclude that there are no available gauge bosons in an E_8 F-theory GUT which can provide the required Sommerfeld enhancement.

This leaves open the possibility, however, that a light scalar could play the role of the required mediator. Indeed, in the case of Dirac neutrino scenarios there are typically several extra GUT singlet curves available. Rather than present an exhaustive list of possible scenarios involving such models, here we simply comment on the fact that at least some of the ingredients of such scenarios can indeed be accommodated within E_8. Even so, this by itself does not overcome all of the obstacles reviewed at the beginning of this subsection, and we will encounter additional problems in the specific example considered.

To illustrate some of the issues involved, consider the model of [79]. This model includes a $U(1)_{B-L}$ gauge boson, and a chiral superfield S which couples to another GUT singlet S' through a "dark μ-term", so that the following terms are present in the effective Lagrangian:

$$L \supset \int d^4\theta S^\dagger e^{V_{B-L}} S + \int d^4\theta S'^\dagger e^{-V_{B-L}} S' + \mu_{\rm dark} \int d^2\theta \; SS'. \tag{9.89}$$

When the S's develop TeV scale vevs, this will Higgs the $U(1)_{B-L}$ symmetry. Letting $\tan\beta_{\rm dark}$ denote the ratio:

$$\tan\beta_{\rm dark} = \frac{\langle S\rangle}{\langle S'\rangle}, \tag{9.90}$$

when $\tan\beta_{\rm dark}$ is sufficiently close to one (which requires fine tuning in the parameters of the F-theory GUT), the physical mass spectrum will consist of a triplet of TeV scale "neutralinos", given by the $U(1)_{B-L}$ gaugino and the two S and S' "Higgsinos", and a scalar with mass on the order of a few GeV [79]. Our aim in this section will not be to realize every detail of the construction in [79], but rather to show that the elements in the Lagrangian of line (9.89) can indeed be accommodated in some Dirac neutrino scenarios.

To present one example of this type, recall that when the monodromy group of a Dirac scenario is either $\mathbb{Z}_3$ or S_3, we have available the following GUT singlets as dark matter candidates:

$G^{\rm Dirac}_{\rm mono} \simeq \mathbb{Z}_3$ or S_3	$\widehat{X}$	$\widehat{N}_R$	$D_{(1)}$	$D^c_{(1)}$	$D_{(2)}$	$D^c_{(2)}$	$Z_{\rm PQ}$	Z_χ
$U(1)_{\rm PQ}$	-4	-3	$+1$	-1	0	0	0	0
$U(1)_\chi$	0	-5	-5	$+5$	0	0	0	0

. (9.91)

At the level of symmetries then, we can consider an extra right-handed neutrino "generation" $\widehat{N}_R$, and pair this with $D^c_{(1)}$. Making the identification:

$$S = \widehat{N}_R \tag{9.92}$$

$$S' = D^c_{(1)}, \tag{9.93}$$

note that the effective field theory does not forbid the operator $X^\dagger SS'/\Lambda_{\rm UV}$, which induces a "$\mu_{\rm dark}$-term" once X develops a supersymmetry breaking vev.

[19] See for example [78] for further discussion on the physics of extra U(1) gauge bosons.

It is important to check that $\mathrm{U}(1)_\chi$ (and hence $\mathrm{U}(1)_{B-L}$) is non-anomalous with respect to this choice of matter assignments. Note that in addition to the MSSM matter fields, as in [8], there are three generations of right-handed neutrinos, and, by inspection of the $\mathrm{U}(1)_\chi$ charge assignments of the messengers, $\widehat{N}_R$ and $D^c_{(1)}$, all of these additional contributions appear in vector-like pairs with respect to $\mathrm{U}(1)_\chi$. Thus, there is at least the possibility that the mass of the $\mathrm{U}(1)_{B-L}$ gauge boson could remain close to the TeV scale.

In fact, it is also straightforward to check that the operator $X^\dagger SS'/\Lambda_{\mathrm{UV}}$ is compatible with the orbits of the monodromy group. Indeed, using the orbits reviewed in section 5, we can take the following weights for $X^\dagger$, $S = \widehat{N}_R$, and $S' = D^c_{(1)}$:

$$Orb(X^\dagger) \ni t_1 - t_4 \tag{9.94}$$

$$Orb(\widehat{N}_R) \ni t_4 - t_5 \tag{9.95}$$

$$Orb(D^c_{(1)}) \ni t_5 - t_1. \tag{9.96}$$

Thus, it is therefore in principle possible to reproduce the Lagrangian of line (9.89).

Note, however, that as argued in subsection 9.3, it is unappealing to allow $\widehat{N}_R$ to develop a vev, as this breaks matter parity in the model. Moreover, it is unclear in the present context how to arrange for the vevs of $\widehat{N}_R$ and $D^c_{(1)}$ to be very close to each other so that $\tan\beta_{\mathrm{dark}} \simeq 1$. Indeed, the PQ charges of these two fields are different, and will therefore experience different PQ deformations. Thus, without significant fine tuning of the model, it appears difficult to generate the required scalar with mass on the order of a few GeV.

Although all of the ingredients of the construction presented above fall inside of E_8, making this model fully realistic still appears to be quite problematic. Indeed, the required elements include a TeV scale $\mathrm{U}(1)_\chi$ gauge boson, an extra generation on the right-handed neutrino curve, and an additional GUT singlet matter curve. This constitutes a significant departure from the spirit of minimality adopted in this paper. Coupled with the fact that we have also encountered significant phenomenological problems with saxion dilution, and the instability of all available dark matter candidates, we again end with the same conclusion that TeV scale dark matter in both annihilating and decaying scenarios is unfeasible in minimal F-theory GUTs.

10 Conclusions

F-theory GUTs provide a quite predictive framework for making contact between string theory and phenomenology. In particular, the rich interplay between geometric and group theoretic ingredients leads to significant constraints which appear unmotivated from the perspective of the four-dimensional effective field theory. In this paper we have exploited the rigid structure present in this unified approach to study whether additional charged and uncharged matter of such models can provide potential candidates for dark matter beyond the gravitino. Assuming that all of the interaction terms of the F-theory GUT descend from a single point of E_8 enhancement, we have found a remarkably rigid structure which does not admit many options for additional matter. A surprising outcome of this analysis

JHEP08(2010)040

is that in all but one Dirac neutrino scenario, incorporating a minimal gauge mediated supersymmetry breaking sector forces the messengers to transform in vector-like pairs in the $10 \oplus \overline{10}$ of SU(5). Moreover, embedding in E_8 also significantly limits the available dark matter candidates. Indeed, since supersymmetry breaking is correlated with the dynamics of a seven-brane which is present in the local model, TeV scale dark matter candidates are forced to also interact closely with the GUT seven-brane. Expanding out from the E_8 point of enhancement, we have studied other potential dark objects which either intersect some locus connected with the E_8 point of enhancement, or which are fully cut off from the visible sector. In all cases, we have encountered obstructions which render all available candidates as unsuitable explanations for recent experimental results connected with the detection of dark matter. *This again reinforces the point that in F-theory GUTs, gravitinos remain as the prime candidate for dark matter.* In the remainder of this section we speculate on possible directions of future investigation.

While aesthetically quite pleasing, the existence of a single point of E_8 enhancement leads to surprisingly strong restrictions on the form of both the visible and dark sectors of F-theory GUTs. In particular, contrary to a perhaps naive notion that a single vector-like pair of messengers in the $5 \oplus \overline{5}$ is "minimal", here we have seen that geometric minimality instead selects as the more natural option messengers organized into multiples of $10 \oplus \overline{10}$ (where each such pair effectively plays the role of *three* $5 \oplus \overline{5}$ pairs). This leads to a class of signatures which are distinct from the single $5 \oplus \overline{5}$ messenger case, and it would be interesting to study the phenomenology of such models in greater detail. In this vein, we have seen that in the Dirac scenario, there is naturally another candidate U(1) symmetry which can combine with the $U(1)_{PQ}$ symmetry. This defines a whole family of PQ deformations, with different consequences for phenomenology.

More broadly speaking, however, less minimal implementations of F-theory GUTs need not contain a single point of E_8 enhancement. In this case, some of the constraints will nevertheless survive provided there is still a sense in which all matter descends from a single well-defined E_8 singularity. On the other hand, more general F-theory compactifications provide another element of flexibility. Our goal here has to been to study a particularly motivated choice which is compatible with the requirements of unification. It would be interesting to determine what can be said about dark matter in this more general case.

The primary restrictions we have encountered have centered on obstructions encountered in trying to fit a TeV scale dark matter candidate with recent experimental results. We have also seen that F-theory GUTs can quite naturally accommodate additional fields with masses in the range of 10 MeV to a few GeV. It would be interesting to study the consequences of such objects for other dark matter experiments, such as DAMA.

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A Monodromy orbit classification

In this appendix we present a full classification of possible monodromy groups such that all of the interaction terms of an F-theory GUT consistently embed at a single unified E_8 interaction point. With notation as in section 5, we consider the decomposition of the adjoint representation of E_8 into irreducible representations of $\mathrm{SU}(5)_{\mathrm{GUT}} \times \mathrm{SU}(5)_{\perp}$:

$$E_8 \supset \mathrm{SU}(5)_{\mathrm{GUT}} \times \mathrm{SU}(5)_{\perp} \tag{A.1}$$

$$248 \to (1,24) + (24,1) + (5,\overline{10}) + (\overline{5},10) + (10,5) + (\overline{10},\overline{5}). \tag{A.2}$$

The dark objects transform as singlets under $\mathrm{SU}(5)_{\mathrm{GUT}}$ and must therefore descend from the 24 of $\mathrm{SU}(5)_{\perp}$. The Cartan of $\mathrm{SU}(5)_{\perp}$ can be parameterized by the coordinates $t_1, \ldots, t_5$ subject to the constraint:

$$t_1 + \ldots + t_5 = 0. \tag{A.3}$$

The weights of the $5_{\perp}$, $10_{\perp}$ and $24_{\perp}$ of $\mathrm{SU}(5)_{\perp}$ are then given as:

$$5_{\perp} : t_i \tag{A.4}$$

$$10_{\perp} : t_i + t_j \tag{A.5}$$

$$24_{\perp} : \pm(t_i - t_j) + 4 \times (0 \text{ weights}) \tag{A.6}$$

for $1 \leq i, j \leq 5$ such that $i \neq j$. It thus follows that matter fields with appropriate $\mathrm{SU}(5)_{\mathrm{GUT}}$ representation content localize along the following curves:

$$5_{\mathrm{GUT}} : -t_i - t_j = 0 \tag{A.7}$$

$$10_{\mathrm{GUT}} : t_i = 0 \tag{A.8}$$

$$1_{\mathrm{GUT}} : t_i - t_j = 0, \tag{A.9}$$

so that the vanishing loci then correspond to local enhancements in the singularity type of the compactification. A matter field with a given weight will necessarily also be charged under a U(1) subgroup of $\mathrm{SU}(5)_{\perp}$, dictated by its weight.

In this appendix we classify possible monodromy orbits consistent with the interaction terms:

$$\int d^4\theta \frac{X^\dagger H_u H_d}{\Lambda_{\mathrm{UV}}} + \int d^2\theta \; 5_H \times 10_M \times 10_M + \int d^2\theta \; \overline{5}_H \times \overline{5}_M \times 10_M + \text{Neutrinos}, \tag{A.10}$$

with the neutrino sector as specified in section 5. More precisely, we shall also demand that that all three generations in the 10_M localize on one curve, and further all three generations in the $\overline{5}_M$ localize on one curve.

The rest of this appendix is organized as follows. First, we discuss constraints on the orbit imposed by the interaction terms of the MSSM. Next, we separate our classification of orbits under the monodromy group into Dirac and Majorana neutrino scenarios.

A.1 Constraints from the MSSM

We now proceed to classify the possible orbits of the monodromy group inherited from $SU(5)_\perp$ which are compatible with the MSSM interactions of line (A.10). We will return to constraints imposed by the neutrino sector in later subsections.

Although we do not know the full orbit of the monodromy group G, we do know that it must be compatible with the presence of the interaction term $5_H \times 10_M \times 10_M$. In particular, without loss of generality, we may fix the weight assignment of the 10_M orbit to include the $SU(5)_\perp$ weights t_1 and t_2. Hence, we also conclude that the orbit of the 5_H matter field must also contain a weight of the form $-t_1 - t_2$. Note that there can in principle be additional weights in each orbit. Nevertheless, we can now conclude that the orbits of 5_H and 10_M minimally contain:

$$Orb(10_M) = t_1, t_2, \ldots \tag{A.11}$$

$$Orb(5_H) = -t_1 - t_2, \ldots . \tag{A.12}$$

Next consider the presence of the interaction term $X^\dagger H_u H_d/\Lambda_{\rm UV}$. In principle, various components of each orbit in the cover can participate in such an interaction term. Thus, the H_u field which participates in a given covering theory interaction term need not correspond to the weight $-t_1 - t_2$ but might instead correspond to a weight of the form $-t_i - t_j$. Note, however, that since $-t_i - t_j$ and $-t_1 - t_2$ both lie in the same orbit, there exists an element of the monodromy group which maps $-t_i - t_j$ to $-t_1 - t_2$. This group action will also map the weights for $X^\dagger$ and H_d to other elements of their respective orbits. Since $X^\dagger$ is a singlet under $SU(5)_{\rm GUT}$ which is charged under $U(1)_{\rm PQ}$, it must correspond to a weight of the form $t_k - t_l$. In addition, H_d must correspond to a weight of the form $t_m + t_n$. The presence of the interaction term $X^\dagger H_u H_d/\Lambda_{\rm UV}$ then imposes the constraint:

$$(t_k - t_l) + (-t_1 - t_2) + (t_m + t_n) = 0, \tag{A.13}$$

which implies one of the two sets of conditions must be met:

$$t_m + t_n = t_1 + t_l \text{ and } t_k - t_l = t_2 - t_l \tag{A.14}$$

$$t_m + t_n = t_2 + t_l \text{ and } t_k - t_l = t_1 - t_l. \tag{A.15}$$

In fact, since line (A.11) already requires the monodromy group to identify t_1 and t_2 in the same orbit, the distinction between these two options is ambiguous. Since nothing in our discussion fixes t_l, without loss of generality, we may take one element of the orbit for $X^\dagger$ to be $t_2 - t_4$, with one weight in the orbit of H_d then fixed to be $t_1 + t_4$. We therefore conclude that in addition to the 10_M and 5_H orbits, we must minimally also include the following terms in the orbit for H_d and $X^\dagger$:

$$Orb(10_M) = t_1, t_2, \ldots \tag{A.16}$$

$$Orb(5_H) = -t_1 - t_2, \ldots \tag{A.17}$$

$$Orb(\overline{5}_H) = t_1 + t_4, \ldots \tag{A.18}$$

$$Orb(X^\dagger) = t_2 - t_4, \ldots . \tag{A.19}$$

The final non-neutrino constraint which we can obtain derives from the presence of the interaction term $\overline{5}_H \times \overline{5}_M \times 10_M$. Again, by a suitable action of the monodromy group, we know that there will be an interaction in the cover which involves the $\overline{5}_H$ with weight $t_1 + t_4$. The weight of $\overline{5}_M$ is then of the form $t_i + t_j$, and the weight for 10_M is t_k. The presence of the interaction term then requires:

$$(t_1 + t_4) + (t_i + t_j) + t_k = 0. \tag{A.20}$$

Hence, t_i, t_j and t_k must all be distinct, and all different from t_1 and t_4.

We now further fix the weight assignments for the $\overline{5}_M$ field. Recall that the orbit of this field contains a weight of the form $t_i + t_j$ with t_i and t_j distinct from t_1 and t_2. There are in principle three possibilities available, once we fix the weight of the 10_M:

$$\text{Option 1: } Orb(\overline{5}_M) \ni t_3 + t_5,\ Orb(10_M) \ni t_2 \tag{A.21}$$

$$\text{Option 2: } Orb(\overline{5}_M) \ni t_2 + t_5,\ Orb(10_M) \ni t_3 \tag{A.22}$$

$$\text{Option 3: } Orb(\overline{5}_M) \ni t_2 + t_3,\ Orb(10_M) \ni t_5. \tag{A.23}$$

Options two and three are in some sense the same because up to this point our discussion has not distinguished between t_3 and t_5. We may therefore focus without loss of generality on options one and two.

To proceed further, we next turn to constraints derived from the specific content of the Dirac and Majorana neutrino scenarios. After fixing a convention for the direction of the $U(1)_{PQ}$ symmetry, we return to the three options listed in lines (A.21)–(A.23).

A.2 Dirac neutrino scenarios

First consider Dirac neutrino scenarios. Our strategy will be to first constrain the form of $U(1)_{PQ}$, and to then use this information and compatibility with the remaining interaction terms to fix further properties of the weights.

Our first constraint stems from a proper identification of the $U(1)_{PQ}$ symmetry. This can be viewed as vector in the space dual to the weight space, such that for each weight t_i, we assign a definite charge. The general form of $U(1)_{PQ}$ is:

$$t^*_{PQ} = a_1 t^*_1 + a_2 t^*_2 + a_3 t^*_3 + a_4 t^*_4 + a_5 t^*_5, \tag{A.24}$$

where:

$$t^*_i(t_j) = \delta_{ij}. \tag{A.25}$$

In order for an element of the Cartan to remain intact under monodromy, it must be invariant under the action of the monodromy group.

To fix conventions, as in [3, 8] we use the same PQ charge assignments associated with the embedding $E_6 \supset SO(10) \times U(1)_{PQ}$:[20]

	$\overline{5}_M$	10_M	5_H	$\overline{5}_H$	$X^\dagger$	N_R
Dirac $U(1)_{PQ}$	$+1$	$+1$	-2	-2	$+4$	-3

(A.26)

[20] We will find below that there is in fact another U(1) in Dirac scenarios, so that the actual "PQ deformation" can correspond to a linear combination of this U(1) and the other U(1) we find. Note, however, that since all of the interaction terms we find will be invariant under both U(1)'s, there is no loss of generality in remaining with this convention.

With this convention, we now fix some of the values of the a_i. The orbit of the 10_M contains the weights t_1 and t_2. Since the 10_M has PQ charge $+1$, we conclude:

$$a_1 = a_2 = 1. \tag{A.27}$$

Note that this also fixes the PQ charge of the 5_H weight $-t_1 - t_2$. Next consider the PQ charge for the $\overline{5}_H$, which is -2. The orbit of this field contains the weight $t_1 + t_4$, which implies:

$$a_1 + a_4 = -2. \tag{A.28}$$

Hence, we also deduce that $a_4 = -3$.

A.2.1 First Dirac scenario

Returning to lines (A.21)–(A.23), we now show that both physically distinct options yield an acceptable monodromy group action. First consider the possibility that option 1 of line (A.21) is realized so that:

$$\text{Option 1: } Orb(\overline{5}_M) \ni t_3 + t_5,\ Orb(10_M) \ni t_2. \tag{A.29}$$

Since the PQ charges of $\overline{5}_M$ and 10_M are both $+1$, this imposes the constraint:

$$a_3 + a_5 = 1. \tag{A.30}$$

Combining all of the constraints derived previously, the form of the $U(1)_{PQ}$ generator in the dual to the weight space is fixed to be of the form:

$$t^*_{PQ} = t^*_1 + t^*_2 + a_3 t^*_3 - 3t^*_4 + (1 - a_3)t^*_5. \tag{A.31}$$

We now determine the weight assignments for the neutrino sector by requiring the presence of the interaction term $H^\dagger_d L N_R/\Lambda_{UV}$. Fixing the weight assignment for $H^\dagger_d$, L and N_R as $-t_1 - t_4$, $t_i + t_j$ and $t_m - t_n$, it follows that these weights obey the constraint:

$$(-t_1 - t_4) + (t_i + t_j) + (t_m - t_n) = 0. \tag{A.32}$$

There are two possible solutions, corresponding to the weight assignments:

$$\text{Option 1a: } Orb(\overline{5}_M) \ni t_1 + t_n,\ Orb(N_R) \ni t_4 - t_n \tag{A.33}$$

$$\text{Option 1b: } Orb(\overline{5}_M) \ni t_4 + t_n,\ Orb(N_R) \ni t_1 - t_n. \tag{A.34}$$

Since the PQ charge of $\overline{5}_M$ and N_R are respectively $+1$ and -3, this leads to the constraints:

$$\text{Option 1a: } 1 + a_n = 1,\ \ -3 - a_n = 0 \tag{A.35}$$

$$\text{Option 1b: } -3 + a_n = 1,\ 1 - a_n = -3. \tag{A.36}$$

Option 1a does not possess a consistent solution. We therefore conclude that $a_n = 4$ is the unique possibility. This implies that either $(a_3, a_5) = (4, -3)$ or $(a_3, a_5) = (-3, 4)$. Since none of our analysis has so far distinguished t_3 or t_5, without loss of generality, we may take $a_3 = -3$. Hence, the $\mathrm{U}(1)_{\mathrm{PQ}}$ direction is fixed to be:

$$t^*_{\mathrm{PQ}} = t^*_1 + t^*_2 - 3t^*_3 - 3t^*_4 + 4t^*_5. \tag{A.37}$$

Returning to the weight assignment for the right-handed neutrino, this also fixes the orbit of N_R to contain the weight $t_1 - t_5$, and the orbit $\overline{5}_M$ to contain the weight $t_4 + t_5$. We therefore conclude that the orbits for each field are of the form:

$$Orb(10_M) = t_1, t_2, \ldots \tag{A.38}$$
$$Orb(\overline{5}_M) = t_4 + t_5, \ldots \tag{A.39}$$
$$Orb(5_H) = -t_1 - t_2, \ldots \tag{A.40}$$
$$Orb(\overline{5}_H) = t_1 + t_4, \ldots \tag{A.41}$$
$$Orb(X^\dagger) = t_2 - t_4, \ldots \tag{A.42}$$
$$Orb(N_R) = t_1 - t_5, \ldots. \tag{A.43}$$

The form of the $\mathrm{U}(1)_{\mathrm{PQ}}$ generator t_{PQ} also allows us to deduce the allowed monodromy groups. The essential point is that a permutation on the t_i's must leave t^*_{PQ} fixed. In particular, this means that the only possible generators are, in terms of cycles (12), (34), and (12)(34). Thus, the only available monodromy groups are generated by such elements, and are therefore isomorphic to either $\mathbb{Z}_2$ or $\mathbb{Z}_2 \times \mathbb{Z}_2$.

In fact, it is possible to go further and deduce that the monodromy group must act non-trivially on t_3 and t_4. Suppose to the contrary that t_4 is invariant under the monodromy group. Returning to the weight assignments of lines (A.38)–(A.43), the interaction $\overline{5}_H \times \overline{5}_M \times 10_M$ would then be of the form $2t_4 + \ldots$. Since t_4 appears twice, this interaction term would then be forbidden. This in particular implies that either the orbit of $\overline{5}_M$, or $\overline{5}_H$ must include a contribution from t_3. Hence, the monodromy group acts non-trivially on t_3 and t_4. In this case, there are therefore precisely two monodromy groups:

$$G^{Dir(1)}_{(2)} = \langle (12)(34) \rangle \simeq \mathbb{Z}_2 \tag{A.44}$$
$$G^{Dir(1)}_{(4)} = \langle (12), (34) \rangle \simeq \mathbb{Z}_2 \times \mathbb{Z}_2, \tag{A.45}$$

where the subscript indicates the order of the group. It is therefore possible to write out the full orbits in this case. In the case where the monodromy group is $G^{Dir(1)}_{(2)}$, we find:

$$G^{Dir(1)}_{(2)} = \langle (12)(34) \rangle \simeq \mathbb{Z}_2 \text{ Orbits:} \tag{A.46}$$
$$Orb(10_M) = t_1, t_2 \tag{A.47}$$
$$Orb(\overline{5}_M) = t_4 + t_5, t_3 + t_5 \tag{A.48}$$
$$Orb(5_H) = -t_1 - t_2 \tag{A.49}$$
$$Orb(\overline{5}_H) = t_1 + t_4, t_2 + t_3 \tag{A.50}$$
$$Orb(X^\dagger) = t_2 - t_4, t_1 - t_3 \tag{A.51}$$
$$Orb(N_R) = t_1 - t_5, t_2 - t_5. \tag{A.52}$$

Note that the orbit for the 10_M, for example truncates at exactly two weights. Adding additional weights would lead to distinct orbits, which is counter the condition that there is a single 10_M curve. Similar considerations apply for the other length two orbits. In the case of $G_{(4)}^{Dir(1)}$ we have:

$$G_{(4)}^{Dir(1)} = \langle(12),(34)\rangle \simeq \mathbb{Z}_2 \times \mathbb{Z}_2 \text{ Orbits:} \tag{A.53}$$

$$Orb(10_M) = t_1, t_2 \tag{A.54}$$

$$Orb(\overline{5}_M) = t_4 + t_5, t_3 + t_5 \tag{A.55}$$

$$Orb(5_H) = -t_1 - t_2 \tag{A.56}$$

$$Orb(\overline{5}_H) = t_1 + t_4, t_2 + t_3, t_2 + t_4, t_1 + t_3 \tag{A.57}$$

$$Orb(X^\dagger) = t_2 - t_4, t_1 - t_3, t_1 - t_4, t_2 - t_3 \tag{A.58}$$

$$Orb(N_R) = t_1 - t_5, t_2 - t_5. \tag{A.59}$$

It is very tempting to include in this list the messenger fields Y and Y' reviewed in section 5 required for gauge mediated supersymmetry breaking [3]. A priori, such messenger fields can either localize on curves distinct from the matter curves, or as noted in [8] can potentially reside on the same curve as other matter fields. For example, the $\overline{5}_M$ curve could in principle support another generation in the $\overline{5}$ provided another curve is free to support an entire GUT multiplet in the 5.

To classify possible matter curve assignments for messenger fields, we demand that the interaction terms XYY' be present. First consider the case of messengers in the $10 \oplus \overline{10}$. Fixing the weight of the X field as $t_4 - t_2$, the constraint on the weights now reads:

$$(t_4 - t_2) + t_i - t_j = 0. \tag{A.60}$$

Hence, one of the messenger fields must localize on t_2, as an additional 10 in the orbit of the 10_M, and the other messenger in the $\overline{10}$ has weight assignment $-t_4$:

$$Orb(10_M, Y_{10}) = t_1, t_2 \tag{A.61}$$

$$Orb(Y'_{\overline{10}}) = -t_3, -t_4. \tag{A.62}$$

We therefore conclude that messengers in the $10 \oplus \overline{10}$ are indeed possible, but that Y_{10} must localize on the same curve as the 10_M.

Next consider messengers in the $5 \oplus \overline{5}$ of SU(5). Again fixing the weight of X as $t_4 - t_2$, this imposes the constraint:

$$(t_4 - t_2) - (t_i + t_j) + (t_k + t_l) = 0. \tag{A.63}$$

Hence, the $Y'_{\overline{5}}$ must be of the form $t_2 + t_i$, and the weight of the Y_5 is given by $-t_4 - t_i$. There are in principle three possible options for the weight assignments of the messengers:

$$\text{Option 1: } Orb(Y_5) \ni -t_4 - t_1, Orb(Y'_{\overline{5}}) \ni t_2 + t_1 \tag{A.64}$$

$$\text{Option 2: } Orb(Y_5) \ni -t_4 - t_3, Orb(Y'_{\overline{5}}) \ni t_2 + t_3 \tag{A.65}$$

$$\text{Option 3: } Orb(Y_5) \ni -t_4 - t_5, Orb(Y'_{\overline{5}}) \ni t_2 + t_5. \tag{A.66}$$

JHEP08(2010)040

Returning to the list of already specified orbits, we see that in the case of option 1, $Y'_{\overline{5}}$ would localize on the 5_H curve. Similarly, for option 2 $Y'_{\overline{5}}$ would localize on the $\overline{5}_H$ curve. Since a non-trivial hyperflux pierces both curves, a full GUT multiplet of messengers cannot localize on these curves. This leaves only option 3. While this is in principle possible, note that this requires Y_5 to localize on the same matter curve as the $\overline{5}_M$. Index theory considerations then imply that while it is perhaps possible to arrange for three $\overline{5}_M$'s and one Y_5 to localize on the *same* curve, this is quite unnatural. Hence, the messengers cannot transform in the $5 \oplus \overline{5}$.

The orbits for all of the visible matter fields are then:

$$G_{(2)}^{Dir(1)} = \langle (12)(34) \rangle \simeq \mathbb{Z}_2 \text{ Orbits:} \tag{A.67}$$

$$Orb(10_M, Y_{10}) = t_1, t_2 \tag{A.68}$$

$$Orb(Y'_{\overline{10}}) = -t_3, -t_4 \tag{A.69}$$

$$Orb(\overline{5}_M) = t_4 + t_5, t_3 + t_5 \tag{A.70}$$

$$Orb(5_H) = -t_1 - t_2 \tag{A.71}$$

$$Orb(\overline{5}_H) = t_1 + t_4, t_2 + t_3 \tag{A.72}$$

$$Orb(X^\dagger) = t_2 - t_4, t_1 - t_3 \tag{A.73}$$

$$Orb(N_R) = t_1 - t_5, t_2 - t_5. \tag{A.74}$$

Similarly, the existence of messengers in the $\overline{5}$ is not possible in the case of the larger monodromy group $G_{(4)}^{Dir(1)} \simeq \mathbb{Z}_2 \times \mathbb{Z}_2$. It thus follows that in the case of $G_{(4)}^{Dir(1)}$, only 10's can correspond to messenger fields. In this case, the full orbits including the messengers are:

$$G_{(4)}^{Dir(1)} = \langle (12), (34) \rangle \simeq \mathbb{Z}_2 \times \mathbb{Z}_2 \text{ Orbits:} \tag{A.75}$$

$$\text{Minimal Matter} \tag{A.76}$$

$$Orb(10_M, Y_{10}) = t_1, t_2 \tag{A.77}$$

$$Orb(Y'_{\overline{10}}) = -t_3, -t_4 \tag{A.78}$$

$$Orb(\overline{5}_M) = t_4 + t_5, t_3 + t_5 \tag{A.79}$$

$$Orb(5_H) = -t_1 - t_2 \tag{A.80}$$

$$Orb(\overline{5}_H) = t_1 + t_4, t_2 + t_3, t_2 + t_4, t_1 + t_3 \tag{A.81}$$

$$Orb(X^\dagger) = t_2 - t_4, t_1 - t_3, t_1 - t_4, t_2 - t_3 \tag{A.82}$$

$$Orb(N_R) = t_1 - t_5, t_2 - t_5. \tag{A.83}$$

Having specified the content of the visible matter, we now turn to extra charged and neutral fields which can fit inside of E_8. An interesting consequence of the classification already performed is that it eliminates half of the possible U(1) dark gauge bosons. Indeed, since the 1 and 2 directions, and the 3 and 4 directions are also identified, it follows that of the original $\mathrm{U}(1)^4 \subset \mathrm{SU}(5)_\perp$ abelian gauge bosons, only two gauge bosons remain. We have identified one such gauge boson with $\mathrm{U}(1)_{\mathrm{PQ}}$. The other invariant can be identified as:

$$t^*_{PQ'} = a\,(t^*_1 + t^*_2) + b(t^*_3 + t^*_4) + ct^*_5. \tag{A.84}$$

In keeping with conventions associated with the embedding $E_6 \supset \mathrm{SO}(10) \times \mathrm{U}(1)_{\mathrm{PQ}} \supset \mathrm{SU}(5) \times \mathrm{U}(1)_\chi \times \mathrm{U}(1)_{\mathrm{PQ}}$ used in [3, 8], we demand that the pattern of charges consistent with the decomposition of the spinor 16 of SO(10) appear as:

$$\mathrm{SO}(10) \supset \mathrm{SU}(5) \times \mathrm{U}(1)_\chi \tag{A.85}$$
$$16 \to 1_{-5} + \overline{5}_{+3} + 10_{-1}, \tag{A.86}$$

which will occur provided $a = -1$, $b = +3$ and $c = 0$. Summarizing, the two U(1) generators are given as:

$$t^*_{\mathrm{PQ}} = (t^*_1 + t^*_2) - 3(t^*_3 + t^*_4) + 4t^*_5 \tag{A.87}$$
$$t^*_\chi = -(t^*_1 + t^*_2 + t^*_3 + t^*_4) + 4t^*_5. \tag{A.88}$$

Under these two U(1)'s the minimal matter fields of the F-theory GUT have charges:

Minimal	$10_M, Y_{10}$	$\overline{5}_M$	$Y'_{\overline{10}}$	5_H	$\overline{5}_H$	$X^\dagger$	N_R
$\mathrm{U}(1)_{\mathrm{PQ}}$	+1	+1	+3	−2	−2	+4	−3
$\mathrm{U}(1)_\chi$	−1	+3	+1	+2	−2	0	−5

. (A.89)

Besides the minimal matter required for realizing an F-theory GUT, there could in principle be additional matter either charged or uncharged under the GUT group. Under the monodromy group actions just encountered, there are a few additional matter curves on which a 5 or a 10 could localize. The corresponding orbits for extra 5's and 10's not already used are:

$$G^{Dir(1)}_{(2)} = \langle (12)(34) \rangle \simeq \mathbb{Z}_2 \text{ Orbits:} \tag{A.90}$$
$$\text{Extra Charged} \tag{A.91}$$
$$Orb(10_{(1)}) = t_5 \tag{A.92}$$
$$Orb(\overline{5}_{(1)}) = t_1 + t_3, t_2 + t_4 \tag{A.93}$$
$$Orb(\overline{5}_{(2)}) = t_1 + t_5, t_2 + t_5 \tag{A.94}$$
$$Orb(\overline{5}_{(3)}) = t_3 + t_4, \tag{A.95}$$

while in the case of the larger monodromy group $G^{Dir(1)}_{(4)}$, $5_{(1)}$ combines to form a larger orbit in the minimal matter sector, so that we now have:

$$G^{Dir(1)}_{(4)} = \langle (12), (34) \rangle \simeq \mathbb{Z}_2 \times \mathbb{Z}_2 \text{ Orbits:} \tag{A.96}$$
$$\text{Extra Charged} \tag{A.97}$$
$$Orb(10_{(1)}) = t_5 \tag{A.98}$$
$$Orb(\overline{5}_{(2)}) = t_1 + t_5, t_2 + t_5 \tag{A.99}$$
$$Orb(\overline{5}_{(3)}) = t_3 + t_4. \tag{A.100}$$

Besides these possibilities, there can also be extra matter fields neutral under $\mathrm{SU}(5)_{\mathrm{GUT}}$ which transform in the adjoint of $\mathrm{SU}(5)_\perp$. These correspond either to the zero weights, which we can identify with directions in the Cartan, as well as weights of the form $t_m - t_n$. Of the zero weights, only two survive, because of the action of the monodromy group, which we denote as $Z_{\rm PQ}$ and Z_χ. Next consider the non-zero weights. Some of these have already been encountered in terms of the X and N_R fields. Besides these possibilities, the full list of singlets (omitting conjugate weights unless in the same orbit) then fill out the following distinct orbits:

$$G_{(2)}^{Dir(1)} = \langle (12)(34) \rangle \simeq \mathbb{Z}_2 \text{ Orbits:} \tag{A.101}$$

$$\text{Extra Singlets} \tag{A.102}$$

$$Orb(D_{(1)}) = t_1 - t_2, t_2 - t_1 \tag{A.103}$$

$$Orb(D_{(2)}) = t_1 - t_4, t_2 - t_3 \tag{A.104}$$

$$Orb(D_{(3)}) = t_3 - t_4, t_4 - t_3 \tag{A.105}$$

$$Orb(D_{(4)}) = t_3 - t_5, t_4 - t_5. \tag{A.106}$$

In the case of the larger monodromy group, $D_{(2)}$ joins the orbit of $X^\dagger$, and the orbits are instead:

$$G_{(4)}^{Dir(1)} = \langle (12), (34) \rangle \simeq \mathbb{Z}_2 \times \mathbb{Z}_2 \text{ Orbits:} \tag{A.107}$$

$$\text{Extra Singlets} \tag{A.108}$$

$$Orb(D_{(1)}) = t_1 - t_2, t_2 - t_1 \tag{A.109}$$

$$Orb(D_{(3)}) = t_3 - t_4, t_4 - t_3 \tag{A.110}$$

$$Orb(D_{(4)}) = t_3 - t_5, t_4 - t_5. \tag{A.111}$$

Under the two surviving U(1)'s, the charges of the various extra objects (omitting complex conjugates) are then:

Extra Charged	$10_{(1)}$	$\overline{5}_{(1)}$	$\overline{5}_{(2)}$	$\overline{5}_{(3)}$
$\text{U}(1)_{\text{PQ}}$	$+4$	-2	$+5$	-6
$\text{U}(1)_\chi$	$+4$	-2	$+3$	-2

(A.112)

Extra Neutral	$D_{(1)}$	$D_{(2)}$	$D_{(3)}$	$D_{(4)}$	Z_{PQ}	Z_χ
$\text{U}(1)_{\text{PQ}}$	0	$+4$	0	-7	0	0
$\text{U}(1)_\chi$	0	0	0	-5	0	0

. (A.113)

A.2.2 Second Dirac scenario

In the previous section we presented a complete classification of possible orbits and monodromy groups under the assumption that option 1 of line (A.21) is realized in a Dirac scenario. Next consider the possibility that option 2 of line (A.21) is realized so that:

$$\text{Option 2: } Orb(\overline{5}_M) \ni t_2 + t_5,\ Orb(10_M) \ni t_3 \tag{A.114}$$

Since the PQ charges of $\overline{5}_M$ and 10_M are both $+1$, this imposes the constraint:

$$a_2 + a_5 = 1 \tag{A.115}$$

$$a_3 = 1. \tag{A.116}$$

Combining all of the constraints derived previously, the form of the $\mathrm{U}(1)_{\mathrm{PQ}}$ generator in the dual to the weight space is fixed to be of the form:

$$t_{\mathrm{PQ}}^{*} = t_1^{*} + t_2^{*} + t_3^{*} - 3t_4^{*}. \tag{A.117}$$

In order for this choice to be invariant under the action of the monodromy group, it follows that the monodromy group can only act non-trivially on the directions t_1^*, t_2^* and t_3^*. Hence, the monodromy group must be a subgroup of the permutation group S_3 acting on these three letters. We now deduce the remaining orbits and weight assignments by demanding consistency with the interaction term $\overline{5}_H \times \overline{5}_M \times 10_M$ and the presence of the Dirac interaction term $H_d^{\dagger} L N_R / \Lambda_{\mathrm{UV}}$.

Returning to the interaction term $\overline{5}_H \times \overline{5}_M \times 10_M$, fix the weight appearing in the orbit of $\overline{5}_H$ as $t_1 + t_4$. The existence of this interaction term then requires:

$$(t_1 + t_4) + (t_i + t_j) + t_k = 0. \tag{A.118}$$

Since 10_M already involves t_1 and t_2 it cannot involve t_5, as this would lead to two distinct orbits for 10_M. Thus, $\overline{5}_M$ must be of the form $t_i + t_5$ for some $i = 2, 3$. Now, although we do not know the full action of the monodromy group, we do know that t_4 is also invariant. As a consequence, since 10_M has in its orbit t_1 and t_2, it follows that the orbit of $\overline{5}_H$ must include $t_1 + t_4$, as well as its image, $t_2 + t_4$. Repeating the same argument as before, it now follows that $\overline{5}_M$ must be of the form $t_i + t_5$ for $i = 1$ or 3. Summarizing, we now learn that the orbits of the various matter fields contains the following weights:

$$Orb(10_M) = t_1, t_2, \ldots \tag{A.119}$$

$$Orb(\overline{5}_M) = t_i + t_5, \ldots \tag{A.120}$$

$$Orb(5_H) = -t_1 - t_2, \ldots \tag{A.121}$$

$$Orb(\overline{5}_H) = t_1 + t_4, t_2 + t_4, \ldots \tag{A.122}$$

$$Orb(X^{\dagger}) = t_1 - t_4, t_2 - t_4, \ldots \tag{A.123}$$

for some t_i given by either t_1, t_2 or t_3.

To fully specify the weight assignment for the $\overline{5}_M$, next consider the interaction term $H_d^{\dagger} L N_R / \Lambda_{\mathrm{UV}}$. Fixing the weight of $H_d^{\dagger}$ as $-t_1 - t_4$, this interaction term requires:

$$(-t_1 - t_4) + (t_i + t_j) + (t_m - t_n) = 0. \tag{A.124}$$

Thus, we deduce that the orbits for $\overline{5}_M$ and N_R must satisfy one of two options:

$$\text{Option 1: } Orb(\overline{5}_M) \ni t_1 + t_j,\ Orb(N_R) \ni t_4 - t_j \tag{A.125}$$

$$\text{Option 2: } Orb(\overline{5}_M) \ni t_4 + t_j,\ Orb(N_R) \ni t_1 - t_j. \tag{A.126}$$

In the latter case, it is note possible for $\overline{5}_M$ to attain PQ charge +1 with the PQ direction of equation (A.117). Hence, only option 1 is available. Since N_R must have PQ charge -3,

t_j must correspond to t_5. Indeed, for all other choices of t_j, $t_4 - t_j$ has PQ charge -4. We can therefore further fix the orbit of $\overline{5}_M$ and N_R to include:

$$Orb(10_M) = t_1, t_2, \ldots \tag{A.127}$$
$$Orb(\overline{5}_M) = t_1 + t_5, t_2 + t_5, \ldots \tag{A.128}$$
$$Orb(5_H) = -t_1 - t_2, \ldots \tag{A.129}$$
$$Orb(\overline{5}_H) = t_1 + t_4, t_2 + t_4, \ldots \tag{A.130}$$
$$Orb(X^\dagger) = t_1 - t_4, t_2 - t_4, \ldots \tag{A.131}$$
$$Orb(N_R) = t_4 - t_5. \tag{A.132}$$

In fact, the listed weights do not fill out a complete orbit. Indeed, returning to the $\overline{5}_H \times \overline{5}_M \times 10_M$ interaction, using the weights listed above, t_1 or t_2 always appears twice.

Fixing the weight assignment for the $\overline{5}_M$ as $t_1 + t_5$, note that the remaining weights for the 10_M and $\overline{5}_H$ must satisfy the relation:

$$(t_1 + t_5) + (t_i + t_j) + t_k = 0. \tag{A.133}$$

Thus, either t_i, t_j or t_k must involve t_3. Since t_4 and t_5 are fixed under monodromy, the orbits for 10_M and $\overline{5}_H$ respectively include t_1 and $t_1 + t_5$, this implies that the monodromy group contains an element which sends t_1 to t_3. Hence, the orbit of 10_M must include t_3, the orbit of $\overline{5}_M$ must include $t_3 + t_5$, and the orbit of $\overline{5}_H$ must include $t_3 + t_4$. Similar considerations apply to the orbit of $X^\dagger$ so that the orbits include $t_1 - t_4, t_2 - t_4, t_3 - t_4$. Hence, the orbits include:

$$Orb(10_M) = t_1, t_2, t_3 \tag{A.134}$$
$$Orb(\overline{5}_M) = t_1 + t_5, t_2 + t_5, t_3 + t_5 \tag{A.135}$$
$$Orb(5_H) = -t_1 - t_2, \ldots \tag{A.136}$$
$$Orb(\overline{5}_H) = t_1 + t_4, t_2 + t_4, t_3 + t_4 \tag{A.137}$$
$$Orb(X^\dagger) = t_1 - t_4, t_2 - t_4, t_3 - t_4 \tag{A.138}$$
$$Orb(N_R) = t_4 - t_5. \tag{A.139}$$

To fix the remaining orbits, we now appeal to some facts from group theory. As we have already noted, the monodromy group is a subgroup of S_3. Recall that the elements of S_3 are given by the following list of cycles:

$$S_3 = \{id,\ (12),\ (13),\ (23),\ (123),\ (132)\}\,. \tag{A.140}$$

On the other hand, since the orbit for 10_M contains three distinct weights, the monodromy group must have order at least three. Now, the order of any subgroup of S_3 must divide that of the subgroup, so $G_{\text{mono}} \simeq \mathbb{Z}_3$ or S_3. In either case, it follows that the monodromy group contains a $\mathbb{Z}_3$ subgroup, and thus a three cycle. There are only two available three cycles, namely (123) and (132), and both generate the same orbits. We thus conclude that the full set of orbits is:

$$G_{\rm mono}^{Dir(2)} \simeq \mathbb{Z}_3 \text{ or } S_3 \tag{A.141}$$
$$Orb(10_M) = t_1, t_2, t_3 \tag{A.142}$$
$$Orb(\overline{5}_M) = t_1 + t_5, t_2 + t_5, t_3 + t_5 \tag{A.143}$$
$$Orb(5_H) = -t_1 - t_2, -t_2 - t_3, -t_3 - t_1 \tag{A.144}$$
$$Orb(\overline{5}_H) = t_1 + t_4, t_2 + t_4, t_3 + t_4 \tag{A.145}$$
$$Orb(X^\dagger) = t_1 - t_4, t_2 - t_4, t_3 - t_4 \tag{A.146}$$
$$Orb(N_R) = t_4 - t_5. \tag{A.147}$$

As in subsection A.2.1, we now address whether it is possible to combine this monodromy group action with a messenger sector. First consider messenger fields in the $5 \oplus \overline{5}$ of SU(5). Fixing the weight of X as $t_4 - t_1$, the interaction term $XY_5Y'_{\overline{5}}$ imposes the weight constraint:

$$(t_4 - t_1) + (-t_i - t_j) + (t_k + t_l) = 0. \tag{A.148}$$

Thus, the orbit of the Y_5 contains $-t_4 - t_i$ and the orbit of $Y'_{\overline{5}}$ contains $t_1 + t_i$. Compatibility with the other orbits then fixes this weight to be $t_1 + t_5$, which is in the same orbit as the $\overline{5}_M$ field.

Next consider messengers in the $10 \oplus \overline{10}$ of SU(5). Fixing the weight of X as $t_4 - t_1$, the interaction term $XY_{10}Y'_{\overline{10}}$ imposes the constraint:

$$(t_4 - t_1) + t_i - t_j = 0. \tag{A.149}$$

Thus, the orbit of Y_{10} contains the weight t_1, and the orbit of $Y'_{\overline{10}}$ contains the weight $-t_4$. Acting with the three cycle present in the monodromy group it follows that Y_{10} lies in the same orbit as the 10_M, while the orbit of $Y'_{\overline{10}}$ is given by t_4. Summarizing, the weight assignments for all of the visible matter are:

$$G_{\rm mono}^{Dir(2)} \simeq \mathbb{Z}_3 \text{ or } S_3 \tag{A.150}$$
$$Orb(10_M, Y_{10}) = t_1, t_2, t_3 \tag{A.151}$$
$$Orb(Y'_{\overline{10}}) = -t_4 \tag{A.152}$$
$$Orb(\overline{5}_M, Y'_{\overline{5}}) = t_1 + t_5, t_2 + t_5, t_3 + t_5 \tag{A.153}$$
$$Orb(Y_5) = -t_4 - t_5 \tag{A.154}$$
$$Orb(5_H) = -t_1 - t_2, -t_2 - t_3, -t_3 - t_1 \tag{A.155}$$
$$Orb(\overline{5}_H) = t_1 + t_4, t_2 + t_4, t_3 + t_4 \tag{A.156}$$
$$Orb(X^\dagger) = t_1 - t_4, t_2 - t_4, t_3 - t_4 \tag{A.157}$$
$$Orb(N_R) = t_4 - t_5. \tag{A.158}$$

The action of the monodromy group manifestly identifies three directions in the Cartan. Hence, of the four possible U(1) gauge bosons in $SU(5)_\perp$, only two remain. We have already identified one such direction with $\mathrm{U}(1)_{\mathrm{PQ}}$. With notation as in subsection A.2.1, the other remaining direction is given as $\mathrm{U}(1)_\chi$ which is now specified by the direction:

$$t_\chi^* = -(t_1^* + t_2^* + t_3^* + t_4^*) + 4t_5^*. \tag{A.159}$$

The charge assignments for each matter field are then:

Minimal Matter	$10_M, Y_{10}$	Y'_{10}	$\overline{5}_M, Y'_{\overline{5}}$	Y_5	5_H	$\overline{5}_H$	$X^\dagger$	N_R
$\mathrm{U}(1)_{\mathrm{PQ}}$	$+1$	$+3$	$+1$	$+3$	-2	-2	$+4$	-3
$\mathrm{U}(1)_\chi$	-1	$+1$	$+3$	-3	$+2$	-2	0	-5

. (A.160)

As in the case of the first Dirac scenario, it is of interest to classify extra matter fields which live in unidentified orbits. In this case, the available orbits for extra 10 curves are:

$$G_{\mathrm{mono}}^{Dir(2)} \simeq \mathbb{Z}_3 \text{ or } S_3 \text{ Orbits} \tag{A.161}$$

$$\text{Extra Charged} \tag{A.162}$$

$$Orb(10_{(1)}) = t_5 \tag{A.163}$$

Next consider extra GUT singlets which descend from the adjoint of $\mathrm{SU}(5)_\perp$. In this case, we have only two possible gauge bosons corresponding to $\mathrm{U}(1)_{\mathrm{PQ}}$ and $\mathrm{U}(1)_\chi$. Moreover, due to the action of the monodromy group, there are only two zero weights, which we denote as Z_{PQ} and Z_χ. The remaining weights are of the form $t_m - t_n$ and localize on curves in the geometry. The available orbits can all be obtained by acting with a $\mathbb{Z}_3$ subgroup of the monodromy group. Thus, (omitting complex conjugate weights) we have the orbits:

$$G_{\mathrm{mono}}^{Dir(2)} \simeq \mathbb{Z}_3 \text{ or } S_3 \text{ Orbits} \tag{A.164}$$

$$\text{Extra Singlets} \tag{A.165}$$

$$Orb(D_{(1)}) = t_1 - t_5, t_2 - t_5, t_3 - t_5 \tag{A.166}$$

$$Orb(D_{(2)}) = t_2 - t_3, t_3 - t_1, t_1 - t_2. \tag{A.167}$$

Up to complex conjugates, the corresponding charges of the extra matter are then:

Extra Charged	$10_{(1)}$
$\mathrm{U}(1)_{\mathrm{PQ}}$	0
$\mathrm{U}(1)_\chi$	$+4$

(A.168)

Extra Neutral	$D_{(1)}$	$D_{(2)}$	Z_{PQ}	Z_χ
$\mathrm{U}(1)_{\mathrm{PQ}}$	$+1$	0	0	0
$\mathrm{U}(1)_\chi$	-5	0	0	0

. (A.169)

A.3 Majorana neutrino scenarios

In the previous subsections we classified all possible monodromy group interactions consistent with a Dirac neutrino scenario. In this subsection we return to the case of a minimal

Majorana neutrino scenario. Returning to our general discussion, the appropriate MSSM interaction terms required the following weights to be present in the orbit of each field:

$$Orb(10_M) = t_1, t_2, \ldots \tag{A.170}$$
$$Orb(5_H) = -t_1 - t_2, \ldots \tag{A.171}$$
$$Orb(\overline{5}_H) = t_1 + t_4, \ldots \tag{A.172}$$
$$Orb(X^\dagger) = t_2 - t_4, \ldots . \tag{A.173}$$

In the case of a minimal Majorana neutrino scenario, the action of the monodromy group is more involved, as it involves a more non-trivial identification of matter fields. We will be interested in covering theories which contain the F-term:

$$\int d^2\theta \; H_u L N_R, \tag{A.174}$$

where the N_R's develop a suitable Majorana mass. In fact, as explained in [8], the existence of the higher-dimension operator:

$$\int d^2\theta \; \frac{(H_u L)^2}{\Lambda_{\mathrm{UV}}} \tag{A.175}$$

requires the PQ charge assignments:

	$\overline{5}_M$	10_M	5_H	$\overline{5}_H$	$X^\dagger$	N_R
Majorana $\mathrm{U}(1)_{\mathrm{PQ}}$	$+2$	$+1$	-2	-3	$+5$	0

. (A.176)

Letting t^*_{PQ} correspond to the direction corresponding to the $\mathrm{U}(1)_{\mathrm{PQ}}$ generator in the Cartan, the general form of t^*_{PQ} is:

$$t^*_{\mathrm{PQ}} = a_1 t^*_1 + a_2 t^*_2 + a_3 t^*_3 + a_4 t^*_4 + a_5 t^*_5. \tag{A.177}$$

Since the charge of 10_M is $+1$, we conclude that $a_1 = a_2 = 1$. Moreover, since the PQ charge of $\overline{5}_H$ is -3, we further conclude that $a_4 = -4$. Thus, we can already fix part of t^*_{PQ} to be of the form:

$$t^*_{\mathrm{PQ}} = t^*_1 + t^*_2 + a_3 t^*_3 - 4t^*_4 + a_5 t^*_5. \tag{A.178}$$

We now use the presence of the F-term $H_u L N_R$ to further constrain the orbits of the various fields. Since N_R is assumed to localize on a curve, it has weight $t_m - t_n$. Fixing the weight of H_u as $-t_1 - t_2$, it follows that since the weight for L must be of the form $t_i + t_j$, we obtain the constraint:

$$(-t_1 - t_2) + (t_i + t_j) + (t_m - t_n). \tag{A.179}$$

Thus, N_R must involve a weight of the form $t_1 - t_n$ as well as (under the action of the monodromy group sending t_1 to t_2) $t_2 - t_{n'}$. The weight t_n must be distinct from t_1 and t_2 since otherwise L would localize on the same curve as H_u. Nothing in our discussion so far has distinguished t_3 and t_5, so without loss of generality, we may further take the N_R orbit to contain the weight $t_1 - t_3$. This also implies that the orbit of the $\overline{5}_M$ contains the weight $t_2 + t_3$. Since N_R has PQ charge 0 in the minimal Majorana scenario, it follows that we can also fix $a_3 = 1$. Thus, the form of $t^*_{\rm PQ}$ is constrained to be of the form:

$$t^*_{\rm PQ} = t^*_1 + t^*_2 + t^*_3 - 4t^*_4 + a_5 t^*_5. \tag{A.180}$$

Note that invariance of this generator under the monodromy group implies that t_4 cannot be mapped to any of t_1, t_2 or t_3. To summarize, the orbits then must contain the terms:

$$Orb(10_M) = t_1, t_2, \ldots \tag{A.181}$$
$$Orb(\overline{5}_M) = t_2 + t_3, \ldots \tag{A.182}$$
$$Orb(5_H) = -t_1 - t_2, \ldots \tag{A.183}$$
$$Orb(\overline{5}_H) = t_1 + t_4, \ldots \tag{A.184}$$
$$Orb(X^\dagger) = t_2 - t_4, \ldots \tag{A.185}$$
$$Orb(N_R) = t_1 - t_3, \ldots. \tag{A.186}$$

To further fix the form of the monodromy group, note that the listed weights are incompatible with the interaction term $\overline{5}_H \times \overline{5}_M \times 10_M$. Returning to lines (A.21)–(A.23), recall that the orbits of the $\overline{5}_M$ and $\overline{5}_H$ contain one of the two following possibilities:

$$\text{Option 1: } Orb(\overline{5}_M) \ni t_3 + t_5,\ Orb(10_M) \ni t_2 \tag{A.187}$$
$$\text{Option 2: } Orb(\overline{5}_M) \ni t_2 + t_5,\ Orb(10_M) \ni t_3 \tag{A.188}$$
$$\text{Option 3: } Orb(\overline{5}_M) \ni t_2 + t_3,\ Orb(10_M) \ni t_5. \tag{A.189}$$

Since the PQ charges for the $\overline{5}_M$ and 10_M are respectively +2 and +1, it follows that the condition on a_5 in the three cases is:

$$\text{Option 1: } 1 + a_5 = 2 \tag{A.190}$$
$$\text{Option 2: } 1 + a_5 = 2 \tag{A.191}$$
$$\text{Option 3: } a_5 = 1. \tag{A.192}$$

Thus, in all three cases, we find the same requirement that $a_5 = 1$. Hence, the form of $t^*_{\rm PQ}$ is uniquely fixed to be of the form:

$$t^*_{\rm PQ} = t^*_1 + t^*_2 + t^*_3 - 4t^*_4 + t^*_5. \tag{A.193}$$

By inspection of $t^*_{\rm PQ}$, t_4 is fixed by the monodromy group, and so the monodromy group must be a subgroup of S_4, the permutation group on the letters t_1, t_2, t_3 and t_5. Since $t_1 + t_4$ is in the orbit of the monodromy group, and moreover, since there exists an element of the monodromy group which sends t_1 to t_2 with t_4 fixed, we also deduce that the orbits for $\overline{5}_H$ must include both $t_1 + t_4$ and $t_2 + t_4$. Similar reasoning implies that the orbit for $X^\dagger$ must include both $t_2 - t_4$ and $t_1 - t_4$. Hence, the orbits under the monodromy

JHEP08(2010)040

group must be enlarged so that:

$$Orb(10_M) = t_1, t_2, \dots \tag{A.194}$$
$$Orb(\overline{5}_M) = t_2 + t_3, \dots \tag{A.195}$$
$$Orb(5_H) = -t_1 - t_2, \dots \tag{A.196}$$
$$Orb(\overline{5}_H) = t_1 + t_4, t_2 + t_4 \dots \tag{A.197}$$
$$Orb(X^\dagger) = t_2 - t_4, t_1 - t_4, \dots \tag{A.198}$$
$$Orb(N_R) = t_1 - t_3, \dots . \tag{A.199}$$

The interaction term $\overline{5}_H \times \overline{5}_M \times 10_M$ requires a further enlargement in the orbits. Indeed, fixing the weight of $\overline{5}_M$ as $t_2 + t_3$, we obtain the constraint:

$$(t_i + t_j) + (t_2 + t_3) + t_k = 0. \tag{A.200}$$

Since t_4 is not in the orbit of 10_M, the only available weights for 10_M are then t_1 and t_5. The available orbits consistent with this constraint are:

$$\text{Option 1: } Orb(\overline{5}_H) \ni t_4 + t_5,\ Orb(10_M) \ni t_1 \tag{A.201}$$
$$\text{Option 2: } Orb(\overline{5}_H) \ni t_1 + t_4,\ Orb(10_M) \ni t_5. \tag{A.202}$$

Note that in either case, the fact that t_4 is fixed under the monodromy group then forces the existence of an element in the monodromy group which sends t_1 to t_5. In particular, it follows that *both* weights must be included in a consistent orbit. Enlarging the orbits further to include maps from t_1 to t_5, t_1 to t_2 and t_2 to t_5 (consistent with the orbit of the 10_M) we have:

$$Orb(10_M) = t_1, t_2, t_5, \dots \tag{A.203}$$
$$Orb(\overline{5}_M) = t_2 + t_3, t_5 + t_i, t_1 + t_j \dots \tag{A.204}$$
$$Orb(5_H) = -t_1 - t_2, -t_5 - t_k, \dots \tag{A.205}$$
$$Orb(\overline{5}_H) = t_1 + t_4, t_2 + t_4, t_5 + t_4, \dots \tag{A.206}$$
$$Orb(X^\dagger) = t_2 - t_4, t_1 - t_4, t_5 - t_4, \dots \tag{A.207}$$
$$Orb(N_R) = t_1 - t_3, t_5 - t_l, t_2 - t_m, \dots, \tag{A.208}$$

for some t_i, t_j and t_k.

By far the most significant constraint stems from the requirement that the Majorana mass term $N_R N_R^c$ be present. Since N_R and N_R^c must localize on the same curve, and must also lie in the same orbit, it follows that the orbit for N_R must be enlarged to include the complex conjugate weights. Thus, we deduce that the orbits are minimally:

$$Orb(10_M) = t_1, t_2, t_5, \dots \tag{A.209}$$
$$Orb(\overline{5}_M) = t_2 + t_3, t_5 + t_i, t_1 + t_j \dots \tag{A.210}$$
$$Orb(5_H) = -t_1 - t_2, -t_5 - t_k, \dots \tag{A.211}$$
$$Orb(\overline{5}_H) = t_1 + t_4, t_2 + t_4, t_5 + t_4, \dots \tag{A.212}$$
$$Orb(X^\dagger) = t_2 - t_4, t_1 - t_4, t_5 - t_4, \dots \tag{A.213}$$
$$Orb(N_R) = \pm(t_1 - t_3), \pm(t_5 - t_l), \pm(t_2 - t_m), \dots . \tag{A.214}$$

In particular, this implies that there exists an element of the monodromy group which acts by interchanging $t_1 - t_3$ with $t_3 - t_1$. Hence, there exists an element of the monodromy group which interchanges t_1 with t_3. It follows that the orbit of 10_M includes t_1, t_2, t_3, t_5 so that the length of the 10_M orbit is precisely four (since t_4 is invariant under the monodromy group). Thus, there exist elements of the monodromy group such that t_1 maps to any of t_2, t_3 or t_5. Using the fact that t_4 remains fixed by the monodromy group, we can thus further enlarge the orbits to:

$$Orb(10_M) = t_1, t_2, t_3, t_5, \ldots \tag{A.215}$$
$$Orb(\overline{5}_M) = t_2 + t_3, t_5 + t_i, t_1 + t_j \ldots \tag{A.216}$$
$$Orb(5_H) = -t_1 - t_2, -t_5 - t_l, -t_3 - t_m \ldots \tag{A.217}$$
$$Orb(\overline{5}_H) = t_1 + t_4, t_2 + t_4, t_5 + t_4, t_3 + t_4 \tag{A.218}$$
$$Orb(X^\dagger) = t_2 - t_4, t_1 - t_4, t_5 - t_4, t_3 - t_4 \tag{A.219}$$
$$Orb(N_R) = \pm (t_1 - t_3), \pm (t_5 - t_n), \pm (t_2 - t_p), \ldots. \tag{A.220}$$

In particular, it follows that the length of the $\overline{5}_H$ orbit, and the $X^\dagger$ orbit are both four.

We now deduce further properties of the monodromy group action. Since $Orb(N_R)$ contains both $t_1 - t_3$ as well as $t_3 - t_1$, there exists a monodromy group element which permutes t_1 and t_3. Note that this does not fix the action on the rest of the t's. Indeed, the action of the two cycle (13) maps the weight $t_2 + t_3$ of $\overline{5}_M$ to $t_2 + t_1$, which is in the conjugate orbit of 5_H. Since the $\overline{5}_M$ and 5_H must localize on distinct matter curves, this implies that the group element which interchanges t_1 and t_3 also acts on t_2. Now, the only other available t_i is t_5, since t_4 is invariant under the monodromy group. It thus follows that the monodromy group contains the element $\sigma = (13)(25)$. Note that this also excludes the two cycle (25) since $(25) \cdot (13)(25) = (13)$. To summarize, we therefore deduce that the monodromy contains neither (13) nor (25), but does contain (13)(25):

$$(13), (25) \notin G^{\text{Maj}}_{\text{mono}} \tag{A.221}$$
$$(13)(25) \in G^{\text{Maj}}_{\text{mono}}. \tag{A.222}$$

Acting on the available weights with $\sigma = (13)(25)$ on the listed weights allows us to further enlarge the available orbits. For example, acting on the weight $t_2 + t_3$ in the orbit of $\overline{5}_M$ with σ, it follows that $Orb(\overline{5}_M)$ also contains $t_5 + t_1$. Moreover, acting on the weight $-t_1 - t_2$ in the orbit of 5_H with σ, it follows that $Orb(5_H)$ also contains the weight $-t_3 - t_5$. Hence, we can further fix the orbits under the monodromy group as:

$$Orb(10_M) = t_1, t_2, t_3, t_5 \tag{A.223}$$
$$Orb(\overline{5}_M) = t_2 + t_3, t_5 + t_1, \ldots \tag{A.224}$$
$$Orb(5_H) = -t_1 - t_2, -t_5 - t_3, \ldots \tag{A.225}$$
$$Orb(\overline{5}_H) = t_1 + t_4, t_2 + t_4, t_5 + t_4, t_3 + t_4 \tag{A.226}$$
$$Orb(X^\dagger) = t_2 - t_4, t_1 - t_4, t_5 - t_4, t_3 - t_4 \tag{A.227}$$
$$Orb(N_R) = \pm (t_1 - t_3), \pm (t_5 - t_n), \pm (t_2 - t_p), \ldots. \tag{A.228}$$

Using the listed weights, we now show that the monodromy group does not contain any three cycles. The essential point is that the existence of any three cycle would lead to an identification of the $\overline{5}_M$ and 5_H orbits. The three cycles in S_4 are:

$$\text{Three Cycles in } S_4 = \{(123), (125), (132), (135), (152), (153), (235), (253)\}\,. \tag{A.229}$$

In each case, there exists a weight of one orbit of either $\overline{5}_M$ or 5_H which gets mapped to a weight in the other conjugate orbit. As explicit examples, we have three cycles which map elements of $\overline{Orb(5_H)}$ to $Orb(\overline{5}_M)$:

$$\overline{Orb(5_H)} \to Orb(\overline{5}_M) \tag{A.230}$$
$$(123) : t_1 + t_2 \to t_2 + t_3 \tag{A.231}$$
$$(135) : t_5 + t_3 \to t_1 + t_5 \tag{A.232}$$
$$(152) : t_1 + t_2 \to t_5 + t_1 \tag{A.233}$$
$$(253) : t_1 + t_2 \to t_1 + t_5 \tag{A.234}$$

The remaining three cycles all map an element of $Orb(\overline{5}_M)$ to $\overline{Orb(5_H)}$:

$$Orb(\overline{5}_M) \to \overline{Orb(5_H)} \tag{A.235}$$
$$(125) : t_5 + t_1 \to t_1 + t_2 \tag{A.236}$$
$$(132) : t_2 + t_3 \to t_1 + t_2 \tag{A.237}$$
$$(153) : t_2 + t_3 \to t_2 + t_1 \tag{A.238}$$
$$(235) : t_5 + t_1 \to t_2 + t_1. \tag{A.239}$$

It therefore follows that the monodromy group contains no three cycles. In particular, we therefore conclude that the monodromy group does not contain any order three subgroups. By the Sylow theorem, it thus follows that the order of the monodromy group must not be divisible by 3 so that:

$$3 \nmid \#G^{\text{Maj}}_{\text{mono}}. \tag{A.240}$$

On the other hand, the available subgroups of S_4 have order 24, 12, 8, 6, 4, 3, 2 and 1. This excludes order 24, 12, 6 and 3 monodromy groups as possibilities. Moreover, because the orbit of the 10_M is length 4, the monodromy group has order at least 4. We therefore deduce that the monodromy group has order 4 or 8:

$$\#G^{\text{Maj}}_{\text{mono}} = 4 \text{ or } \#G^{\text{Maj}}_{\text{mono}} = 8. \tag{A.241}$$

To proceed further, we examine each possibility in turn.

A.3.1 First Majorana scenario: $\#G^{\text{Maj}}_{\text{mono}} = 4$

First consider scenarios where the monodromy group is order four, which we denote as $G_4^{Maj(1)}$:

$$\#G^{\text{Maj}}_{\text{mono}} = 4. \tag{A.242}$$

In this case, the maximal orbit length is 4. Returning to the parts of the orbits fixed by prior considerations, recall that we have:

$$Orb(10_M) = t_1, t_2, t_3, t_5 \tag{A.243}$$
$$Orb(\overline{5}_M) = t_2 + t_3, t_5 + t_1, \ldots \tag{A.244}$$
$$Orb(5_H) = -t_1 - t_2, -t_5 - t_3, \ldots \tag{A.245}$$
$$Orb(\overline{5}_H) = t_1 + t_4, t_2 + t_4, t_5 + t_4, t_3 + t_4 \tag{A.246}$$
$$Orb(X^\dagger) = t_2 - t_4, t_1 - t_4, t_5 - t_4, t_3 - t_4 \tag{A.247}$$
$$Orb(N_R) = \pm(t_1 - t_3), \pm(t_5 - t_n), \pm(t_2 - t_p), \ldots. \tag{A.248}$$

Since the orbit of N_R has length four, it follows that this orbit truncates to $\pm(t_1 - t_3)$ and $\pm(t_5 - t_2)$ so that:

$$Orb(10_M) = t_1, t_2, t_3, t_5 \tag{A.249}$$
$$Orb(\overline{5}_M) = t_2 + t_3, t_5 + t_1, \ldots \tag{A.250}$$
$$Orb(5_H) = -t_1 - t_2, -t_5 - t_3, \ldots \tag{A.251}$$
$$Orb(\overline{5}_H) = t_1 + t_4, t_2 + t_4, t_5 + t_4, t_3 + t_4 \tag{A.252}$$
$$Orb(X^\dagger) = t_2 - t_4, t_1 - t_4, t_5 - t_4, t_3 - t_4 \tag{A.253}$$
$$Orb(N_R) = \pm(t_1 - t_3), \pm(t_5 - t_2). \tag{A.254}$$

As we now explain, the monodromy group does not contain any two cycles either. The full list of two cycles in S_4 are:

$$\text{Two Cycles in } S_4 = \{(12), (13), (15), (23), (25), (35)\}. \tag{A.255}$$

By inspection of the action of these group elements on weights in the orbit of N_R, the following two cycles cannot be included in $G_4^{Maj(1)}$:

$$(12), (15), (23), (35) \notin G_4^{Maj(1)}. \tag{A.256}$$

On the other hand, we have already seen that the action of (13) and (25) would identify the orbits for $\overline{5}_M$ and 5_H. Thus, the monodromy group does not contain any two cycles:

$$(12), (15), (23), (35), (13), (25) \notin G_4^{Maj(1)}. \tag{A.257}$$

The absence of two cycles now allows us to completely fix the form of the monodromy group. Indeed, since there exists an element which maps $t_1 - t_3$ to $t_2 - t_5$, the monodromy group either contains either (12)(35) or (1235). Note, however, that (1235) sends the $\overline{5}_M$ weight $t_2 + t_3$ to $t_3 + t_5$. Since $t_3 + t_5$ lies in the orbit conjugate to 5_H, we therefore conclude that (12)(35) is an element of the monodromy group. Since we have already argued that the monodromy group contains (13)(25), we deduce that:

$$(12)(35), (13)(25) \in G_4^{Maj(1)}. \tag{A.258}$$

Taking the product of these two elements generates a third non-trivial element:

$$(13)(25) \cdot (12)(35) = (12)(35) \cdot (13)(25) = (15)(23). \tag{A.259}$$

Taking all products of group elements, it is now immediate that $G_4^{Maj(1)}$ is an abelian group. Since $G_4^{Maj(1)}$ is an abelian order four group with an order two element, it follows from the Chinese remainder theorem that $G_4^{Maj(1)}$ is isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_2$. Having specified explicitly the group elements of $G_4^{Maj(1)}$, we can now fix all of the orbits as:

$$G_4^{Maj(1)} = \langle (12)(35), (13)(25) \rangle \simeq \mathbb{Z}_2 \times \mathbb{Z}_2 \tag{A.260}$$

$$Orb(10_M) = t_1, t_2, t_3, t_5 \tag{A.261}$$

$$Orb(\overline{5}_M) = t_2 + t_3, t_5 + t_1 \tag{A.262}$$

$$Orb(5_H) = -t_1 - t_2, -t_5 - t_3 \tag{A.263}$$

$$Orb(\overline{5}_H) = t_1 + t_4, t_2 + t_4, t_5 + t_4, t_3 + t_4 \tag{A.264}$$

$$Orb(X^\dagger) = t_2 - t_4, t_1 - t_4, t_5 - t_4, t_3 - t_4 \tag{A.265}$$

$$Orb(N_R) = \pm (t_1 - t_3), \pm (t_5 - t_2). \tag{A.266}$$

We will return to the case where the monodromy group contains more generators in the following subsection.

Next consider the messenger sector of the theory. First consider messengers in the $5 \oplus \overline{5}$ of SU(5). Fixing the weight assignment of X as $t_4 - t_2$, the presence of the interaction term $XY_5Y'_{\overline{5}}$ requires the weights for the messengers to satisfy the constraint:

$$(t_4 - t_2) + (-t_i - t_j) + (t_k + t_l) = 0. \tag{A.267}$$

Hence, the orbits of the Y_5 and $Y'_{\overline{5}}$ respectively include $-t_4 - t_i$ and $t_2 + t_i$. Note, however, that the conjugate to the weight for Y_5 already falls in the orbit for the 5_H. Since full GUT multiplets do not localize on this curve, it follows that this model does not admit messengers in the $5 \oplus \overline{5}$. Next consider messenger fields in the $10 \oplus \overline{10}$. In this case, the constraint on the weights requires:

$$(t_4 - t_2) + t_i + -t_k = 0. \tag{A.268}$$

Thus, the orbit for the Y_{10} contains the weight t_2 and the orbit for the $Y'_{\overline{10}}$ contains the weight $-t_4$. It thus follows that just as for the Dirac scenario, one of the messenger fields fits inside of the same orbit as the 10_M. Summarizing, the visible sector fields consists of the following orbits:

$$G_4^{Maj(1)} = \langle (12)(35), (13)(25) \rangle \simeq \mathbb{Z}_2 \times \mathbb{Z}_2 \tag{A.269}$$

$$Orb(10_M, Y_{10}) = t_1, t_2, t_3, t_5 \tag{A.270}$$

$$Orb(Y'_{\overline{10}}) = -t_4 \tag{A.271}$$

$$Orb(\overline{5}_M) = t_2 + t_3, t_5 + t_1 \tag{A.272}$$

$$Orb(5_H) = -t_1 - t_2, -t_5 - t_3 \tag{A.273}$$

$$Orb(\overline{5}_H) = t_1 + t_4, t_2 + t_4, t_5 + t_4, t_3 + t_4 \tag{A.274}$$

$$Orb(X^\dagger) = t_2 - t_4, t_1 - t_4, t_5 - t_4, t_3 - t_4 \tag{A.275}$$

$$Orb(N_R) = \pm (t_1 - t_3), \pm (t_5 - t_2). \tag{A.276}$$

JHEP08(2010)040

The action of the monodromy group manifestly identifies four directions in the Cartan. Hence, of the four possible U(1) gauge bosons in $SU(5)_\perp$, only $U(1)_{PQ}$ remains. The PQ charge assignments for the matter fields are:

Minimal	$10_M, Y_{10}$	$Y'_{\overline{10}}$	$\overline{5}_M$	5_H	$\overline{5}_H$	$X^\dagger$	N_R
$U(1)_{PQ}$	$+1$	$+4$	$+2$	-2	-3	$+5$	0

(A.277)

In this case, all of the orbits of the 10 are already exhausted. Moreover, the available orbits for additional 5's are quite limited:

$$G_4^{Maj(1)} = \langle (12)(35), (13)(25) \rangle \simeq \mathbb{Z}_2 \times \mathbb{Z}_2 \tag{A.278}$$

$$\text{Extra Charged} \tag{A.279}$$

$$Orb(\overline{5}_{(1)}) = t_1 + t_3, t_2 + t_5. \tag{A.280}$$

Next consider extra GUT singlets of the theory. Since there is a single available U(1) in $SU(5)_\perp$, only one zero weight denoted as Z_{PQ} can descend from the adjoint of $SU(5)_\perp$. The remaining weights of the adjoint are of the form $t_m - t_n$. Under the provided monodromy group action, these separate into the following orbits:

$$G_4^{Maj(1)} = \langle (12)(35), (13)(25) \rangle \simeq \mathbb{Z}_2 \times \mathbb{Z}_2 \tag{A.281}$$

$$Orb(D_{(1)}) = \pm(t_1 - t_2), \pm(t_3 - t_5) \tag{A.282}$$

$$Orb(D_{(2)}) = \pm(t_1 - t_5), \pm(t_2 - t_3) \tag{A.283}$$

The corresponding PQ charge of these fields is:

Extra Charged	$\overline{5}_{(1)}$
$U(1)_{PQ}$	$+2$

(A.284)

Extra Neutral	$D_{(1)}$	$D_{(2)}$	Z_{PQ}
$U(1)_{PQ}$	0	0	0

(A.285)

A.3.2 Second and third Majorana scenarios: $\#G_{\text{mono}}^{\text{Maj}} = 8$

In the previous section we completely classified the available monodromy groups when the order of the monodromy group is four. We now turn to monodromy groups with eight elements. Even without specifying the explicit action of the monodromy group, it is already possible to deduce that the monodromy group must be isomorphic to the dihedral group Dih_4, namely the group generated by rotations and reflections of the square. At the level of abstract groups, we have:

$$Dih_4 \simeq \mathbb{Z}_2 \ltimes \mathbb{Z}_4, \tag{A.286}$$

where the $\mathbb{Z}_2$ acts by inversion on the $\mathbb{Z}_4$ elements.

To show that the monodromy group must be isomorphic to Dih_4, recall that $G_{\text{mono}}^{\text{Maj}}$ is a subgroup of S_4 of order $8 = 2^3$. It now follows from the Sylow theorems that all groups of this order are isomorphic. Further note that S_4 is the group of symmetries of the cube. Since Dih_4 is a symmetry which acts on one of the faces of the cube, we can already deduce

JHEP08(2010)040

that all of the order eight monodromy groups are isomorphic to Dih_4. It is important to note that specifying the abstract group is not enough to fix the actual orbits.

Returning to the parts of the orbit already specified before subsection (A.3.1), we have:

$$Orb(10_M) = t_1, t_2, t_3, t_5 \tag{A.287}$$
$$Orb(\overline{5}_M) = t_2 + t_3, t_5 + t_1, \ldots \tag{A.288}$$
$$Orb(5_H) = -t_1 - t_2, -t_5 - t_3, \ldots \tag{A.289}$$
$$Orb(\overline{5}_H) = t_1 + t_4, t_2 + t_4, t_5 + t_4, t_3 + t_4 \tag{A.290}$$
$$Orb(X^{\dagger}) = t_2 - t_4, t_1 - t_4, t_5 - t_4, t_3 - t_4 \tag{A.291}$$
$$Orb(N_R) = \pm(t_1 - t_3), \pm(t_5 - t_n), \pm(t_2 - t_p), \ldots. \tag{A.292}$$

To this end, we now proceed to classify the possible options for the corresponding orbits. The analysis we consider splits to two cases, namely the case where the orbit of $\overline{5}_M$ has length two, and the case where the orbit of the $\overline{5}_M$ is bigger. In the following subsections we consider these two cases separately.

A.3.3 Length two $\overline{5}_M$ orbit

First suppose that the orbit for the $\overline{5}_M$ truncates at exactly two weights so that:

$$Orb(\overline{5}_M) = t_2 + t_3, t_5 + t_1. \tag{A.293}$$

On the other hand, we have also seen that the monodromy group must contain a four cycle. There are six candidate four cycles of S_4, given as:

$$\text{Four Cycles in } S_4 = \{(1235), (1253), (1325), (1352), (1523), (1532)\}. \tag{A.294}$$

Of these possibilities, it is enough to focus on the cases $(1235), (1253), (1325)$ since the remaining possibilities are inverse elements. Note, however, that neither (1235) nor (1325) preserves $Orb(\overline{5}_M)$. It therefore follows that the four cycle of $G_{\text{mono}}^{\text{Maj}}$ must contain (1253). Combined with the analysis leading to line (A.222) showing that the group element $(13)(25)$ must always be present, we conclude that:

$$(13)(25), (1253) \in G_8^{Maj(2)}. \tag{A.295}$$

In fact, these elements generate an order eight group since $(1253) \cdot (1253) = (15)(23) \neq (13)(25)$. Taking all available products between powers of these two generators, the explicit elements are:

$$G_8^{Maj(2)} = \{id, (1253), (15)(23), (1352), (13)(25), (23), (12)(35), (15)\}. \tag{A.296}$$

The corresponding orbits are then:

$$G_8^{Maj(2)} \simeq \langle (13)(25), (1253) \rangle \simeq Dih_4 \tag{A.297}$$
$$Orb(10_M) = t_1, t_2, t_3, t_5 \tag{A.298}$$
$$Orb(\overline{5}_M) = t_2 + t_3, t_5 + t_1 \tag{A.299}$$
$$Orb(5_H) = -t_1 - t_2, -t_5 - t_3, -t_1 - t_3, -t_5 - t_2 \tag{A.300}$$
$$Orb(\overline{5}_H) = t_1 + t_4, t_2 + t_4, t_5 + t_4, t_3 + t_4 \tag{A.301}$$
$$Orb(X^\dagger) = t_2 - t_4, t_1 - t_4, t_5 - t_4, t_3 - t_4 \tag{A.302}$$
$$Orb(N_R) = \pm(t_1 - t_3), \pm(t_5 - t_2), \pm(t_5 - t_3), \pm(t_1 - t_2). \tag{A.303}$$

As in subsection A.3.1, the only available messengers can fit into vector-like pairs in the $10 \oplus \overline{10}$ of SU(5). This can be traced back to the fact that the $X^\dagger$ field always contains a contribution from t_4. The full list of orbits is therefore given by:

$$G_8^{Maj(2)} \simeq \langle (13)(25), (1253) \rangle \simeq Dih_4 \tag{A.304}$$
$$Orb(10_M, Y_{10}) = t_1, t_2, t_3, t_5 \tag{A.305}$$
$$Orb(Y'_{\overline{10}}) = -t_4 \tag{A.306}$$
$$Orb(\overline{5}_M) = t_2 + t_3, t_5 + t_1 \tag{A.307}$$
$$Orb(5_H) = -t_1 - t_2, -t_5 - t_3, -t_1 - t_3, -t_5 - t_2 \tag{A.308}$$
$$Orb(\overline{5}_H) = t_1 + t_4, t_2 + t_4, t_5 + t_4, t_3 + t_4 \tag{A.309}$$
$$Orb(X^\dagger) = t_2 - t_4, t_1 - t_4, t_5 - t_4, t_3 - t_4 \tag{A.310}$$
$$Orb(N_R) = \pm(t_1 - t_3), \pm(t_5 - t_2), \pm(t_5 - t_3), \pm(t_1 - t_2). \tag{A.311}$$

In this case, all available charged GUT matter is already part of the minimal matter sector. Next consider extra GUT singlets. There is again a single gauge boson, and the adjoints are either given by the one remaining zero weight $Z_{\rm PQ}$, or by charged weights in the adjoint. Tracing through all orbits, we now have:

$$G_8^{Maj(2)} \simeq \langle (13)(25), (1253) \rangle \simeq Dih_4 \tag{A.312}$$
$$\text{Extra Singlets} \tag{A.313}$$
$$Orb(D_{(1)}) = \pm(t_1 - t_5), \pm(t_2 - t_3). \tag{A.314}$$

The corresponding charges under $U(1)_{\rm PQ}$ are then:

Extra Neutral	$D_{(1)}$	$Z_{\rm PQ}$
$U(1)_{\rm PQ}$	0	0

. (A.315)

A.3.4 Length four $\overline{5}_M$ orbit

In the previous subsection we deduced the form of the monodromy group in the case where the length of the $\overline{5}_M$ orbit is two. Next suppose that the length of the $\overline{5}_M$ orbit is greater than two. In fact, the orbit for $\overline{5}_M$ cannot have length three because of the orbit-stabilizer theorem. The stabilizer of a weight w is given as the set of elements of the monodromy group leaving it invariant:

Returning to the available list of orbits:

$$Stab_G(w) = \{\sigma \in G | \sigma(w) = w\} . \tag{A.316}$$

The orbit-stabilizer theorem establishes that the product of the length of the orbit and the order of the stabilizer are equal to the order of the group:

$$\#Orb(w) \cdot \#Stab_G(w) = \#G. \tag{A.317}$$

Since $\#G$ is not divisible by 3, we conclude that the length of the $\overline{5}_M$ orbit must be precisely 4. On the other hand, returning to the list of orbits already specified, we have:

$$Orb(10_M) = t_1, t_2, t_3, t_5 \tag{A.318}$$

$$Orb(\overline{5}_M) = t_2 + t_3, t_5 + t_1, \ldots \tag{A.319}$$

$$Orb(5_H) = -t_1 - t_2, -t_5 - t_3, \ldots \tag{A.320}$$

$$Orb(\overline{5}_H) = t_1 + t_4, t_2 + t_4, t_5 + t_4, t_3 + t_4 \tag{A.321}$$

$$Orb(X^\dagger) = t_2 - t_4, t_1 - t_4, t_5 - t_4, t_3 - t_4 \tag{A.322}$$

$$Orb(N_R) = \pm (t_1 - t_3), \pm (t_5 - t_n), \pm (t_2 - t_p), \ldots . \tag{A.323}$$

Prior considerations have already fixed eight out of the ten weights available for orbits of the 5, leaving only $t_1 + t_3$ and $t_2 + t_5$ as options. We can therefore uniquely fix the orbits for the 5's as:

$$Orb(\overline{5}_M) = t_2 + t_3, t_5 + t_1, t_1 + t_3, t_2 + t_5 \tag{A.324}$$

$$Orb(5_H) = -t_1 - t_2, -t_5 - t_3 \tag{A.325}$$

$$Orb(\overline{5}_H) = t_1 + t_4, t_2 + t_4, t_5 + t_4, t_3 + t_4. \tag{A.326}$$

Note that the orbit for the 5_H is now of length two. Just as in the previous subsection, we now ask which four cycles of S_4 leave this orbit fixed. Of the available candidates:

$$\text{Four Cycles in } S_4 = \{(1235), (1253), (1325), (1352), (1523), (1532)\} , \tag{A.327}$$

only (1325) and (1523) preserve this orbit. Combined with the analysis leading to line (A.222) showing that the group element (13)(25) must always be present, we conclude that:

$$(13)(25), (1325) \in G_8^{Maj(3)}. \tag{A.328}$$

In fact, these elements generate an order eight group since $(1325) \cdot (1325) = (12)(35) \neq (13)(25)$. Taking all available products between powers of these two generators, the explicit elements are:

$$G_8^{Maj(3)} = \{id, (1325), (12)(35), (1523), (13)(25), (12), (15)(23), (35)\} . \tag{A.329}$$

Just as in the other Majorana scenarios, the messenger fields can only consistently embed in the $10 \oplus \overline{10}$ of SU(5). This can again be traced to the larger size of orbit lengths in the

Majorana scenario. It therefore follows that in this case as well, the full list of orbits are:

$$G_8^{Maj(3)} = \langle(13)(25), (1325)\rangle \simeq Dih_4 \tag{A.330}$$
$$\text{Minimal Matter} \tag{A.331}$$
$$Orb(10_M, Y_{10}) = t_1, t_2, t_3, t_5 \tag{A.332}$$
$$Orb(\overline{Y}'_{10}) = -t_4 \tag{A.333}$$
$$Orb(\overline{5}_M) = t_2 + t_3, t_5 + t_1, t_1 + t_3, t_2 + t_5 \tag{A.334}$$
$$Orb(5_H) = -t_1 - t_2, -t_5 - t_3 \tag{A.335}$$
$$Orb(\overline{5}_H) = t_1 + t_4, t_2 + t_4, t_5 + t_4, t_3 + t_4 \tag{A.336}$$
$$Orb(X^\dagger) = t_2 - t_4, t_1 - t_4, t_5 - t_4, t_3 - t_4 \tag{A.337}$$
$$Orb(N_R) = \pm(t_1 - t_3), \pm(t_5 - t_2), \pm(t_5 - t_1), \pm(t_2 - t_3). \tag{A.338}$$

In this case, note that there are no additional orbits for additional matter charged under $SU(5)_{GUT}$ to localize.

Next consider extra matter corresponding to GUT singlets. As for all of the Majorana scenarios, there is a single U(1) invariant under the action of the monodromy group. The dark chiral matter is given by the zero weight of the adjoint of $SU(5)_\perp$ denoted by Z_{PQ}, and weights of the form $t_m - t_n$. The extra matter candidates fill out the following orbit:

$$G_8^{Maj(3)} = \langle(13)(25), (1325)\rangle \simeq Dih_4$$
$$\text{Extra Singlet Orbits}$$
$$Orb(D_{(1)}) = \pm(t_1 - t_2), \pm(t_3 - t_5).$$

The PQ charge assignments are then:

Extra Neutral	$D_{(1)}$	Z_{PQ}
$U(1)_{PQ}$	0	0

. (A.339)

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