

$\mathcal{O}(\alpha_s^2)$ corrections to fermionic Higgs decays in the MSSM

L. Mihaila and C. Reiber

*Institut für Theoretische Teilchenphysik, Karlsruhe Institute of Technology (KIT),
76128 Karlsruhe, Germany*

E-mail: luminita@particle.uni-karlsruhe.de, reisser@kit.edu

ABSTRACT: We compute the two-loop corrections of $\mathcal{O}(\alpha_s^2)$ to the Yukawa couplings in the framework of the Minimal Supersymmetric Standard Model (MSSM). The calculation is performed using the effective Lagrangian approach under the approximation of neglecting the Higgs boson mass with respect to the top quark, gluino and all squark flavour masses. As an application we derive the $\mathcal{O}(\alpha_s^2)$ corrections to the partial decay width of the lightest Higgs boson to a bottom quark pair. We find that the two-loop corrections are sizable for large values of $\tan\beta$ and low CP-odd Higgs boson mass. With our calculation of the $\mathcal{O}(\alpha_s^2)$ corrections the remaining theoretical uncertainties reduce below a few percent.

KEYWORDS: Supersymmetry Phenomenology

Contents

1	Introduction	1
2	Notation and theoretical framework	2
2.1	Higgs decay width	5
3	Calculation of the coefficient function C_{2q} at next-to-next-to-leading order (NNLO)	7
3.1	Direct calculation of the coefficient function C_{2q}^0 at NNLO	8
3.2	LET derivation of the coefficient function C_{2q}^0 at NNLO	9
3.3	Regularization and renormalization scheme	10
4	Numerical results	11
5	Conclusions	19

1 Introduction

One of the main purposes of the CERN Large Hadron Collider (LHC) is the search for Higgs bosons. The discovery of a light Higgs boson is a decisive test for all models predicting supersymmetric (SUSY) particles at the TeV scale and in particular of the MSSM [1, 2]. A remarkable feature of the MSSM is the restricted Higgs sector. This allows Higgs searches without any assumption about the mechanism of SUSY breaking, but only constraints from the Higgs sector [3]. For the MSSM Higgs boson searches at the hadron colliders, two new complementary benchmark scenarios, the “small α_{eff} ” and the “gluophobic” scenarios, have been proposed in addition to those used at the CERN Large Electron-Positron Collider (LEP).

More precisely, in the “gluophobic” scenario the gluon fusion process is strongly suppressed due to cancellation between top quark and squark loop contributions. Nevertheless, the channel $t\bar{t} \rightarrow t\bar{t}h \rightarrow t\bar{t}b\bar{b}$ is enhanced as compared to the Standard Model (SM) case, so that this becomes the most promising detection mode. In the “small α_{eff} ” scenario the decay width for $h \rightarrow b\bar{b}$ is much smaller than its SM value. In this case the complementary channel $h \rightarrow \gamma\gamma$ is enhanced as compared to the SM and it becomes the preferred detection mode.

For a light Higgs boson ($m_h \leq 130$ GeV) the decay $h \rightarrow b\bar{b}$ is the dominant mode, but its detection at hadron colliders is difficult due to large QCD backgrounds for the b jets. However, at lepton colliders the Higgs boson search relies on b tagging that can be performed with high efficiency.

For the discovery of the Higgs bosons, the cross section of the main production channels, decay widths and branching ratios are necessary to be known with high accuracy. Within

the SM the radiative corrections to the fermionic Higgs decay were intensively studied in the literature. The QCD and EW corrections are known up to the three-loop order: $\mathcal{O}(\alpha_s^3)$ originating from the light degrees of freedom were first derived in ref. [4] and the top-induced $\mathcal{O}(\alpha_s^3)$ corrections in ref. [5]; the QCD-EW interference contributions of $\mathcal{O}(\alpha_s^2 x_t)$ ¹ can be found in ref. [6, 7]. In this paper we concentrate on the fermionic Higgs decay in the MSSM, for moderate Higgs boson masses $M_h \leq 130$ GeV. The processes $h \rightarrow b\bar{b}$ and $h \rightarrow \tau^+\tau^-$ (the second most important decay mode) are affected by large radiative corrections for scenarios with large values of $\tan\beta$ and moderate values of the neutral CP-odd Higgs boson mass M_A ($\tan\beta \geq 20, M_A \leq 250$ GeV) [8–10]. In this case, apart from pure QCD and EW corrections mentioned above, there are Higgs boson propagator corrections and vertex corrections due to SUSY particles. The first class of radiative corrections, can be taken into account by introducing the effective mixing angle α_{eff} that diagonalizes the neutral Higgs boson mass matrix [11]. Such type of corrections are known analytically up to two-loop order in supersymmetric QCD (SQCD) and the supersymmetric electroweak theory (SEW), see [12] and references therein. The second class of radiative corrections are especially important for large values of $\tan\beta$. Usually, they are derived using the effective Lagrangian approach [13]. The one-loop contributions are known since long [8–10, 14], while the two-loop corrections have been computed very recently [15, 16].

It is the aim of this paper to present the complete two-loop SQCD corrections to the decay width $h \rightarrow b\bar{b}$, taking into account the exact dependence on the supersymmetric particle masses and working in the full theory.

The paper is organized as follows: in the next section we introduce our framework and the quantities required for the computation of the decay width $\Gamma(h \rightarrow q\bar{q})$ through two loops in the MSSM. In section 3 we describe the actual two-loop calculation, pointing out the connection between the radiative corrections to the vertex $hq\bar{q}$ and those to the quark propagator through the low-energy theorem. The formalism discussed in this section is valid for a general quark flavour. However, we specify our calculation for the case of bottom quark which generates the dominant decay mode of a light Higgs boson. In section 4 we perform a numerical analysis and discuss the phenomenological implications of the two-loop SQCD vertex corrections to $\Gamma(h \rightarrow b\bar{b})$. We present the conclusions in section 5.

2 Notation and theoretical framework

The part of the MSSM Lagrangian describing the fermionic Higgs decay can be written in the following form

$$\mathcal{L} = \mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{SQCD}} + \sum_{i=1,2} \mathcal{L}_{q\phi_i} + \sum_{i=1,2} \mathcal{L}_{\tilde{q}\phi_i} \tag{2.1}$$

where

$$\mathcal{L}_{q\phi_i} = - \sum_{q=1}^6 \frac{m_q}{v} g_q^{\phi_i} \bar{q}q\phi_i \quad \text{and} \quad \mathcal{L}_{\tilde{q}\phi_i} = - \sum_{q=1}^6 \sum_{r,k=1,2} \frac{m_q}{v} g_{\tilde{q},kr}^{\phi_i} \tilde{q}_k^* \tilde{q}_r \phi_i. \tag{2.2}$$

¹For the definition of the parameter x_t see section 2.1.

f	$g_q^{\phi_1}$	$g_{\tilde{q};11}^{\phi_1}$	$g_{\tilde{q};12}^{\phi_1} = g_{\tilde{q};21}^{\phi_1}$	$g_{\tilde{q};22}^{\phi_1}$
up	0	$-\mu S_q/S_\beta$	$-\mu C_q/S_\beta$	$\mu S_q/S_\beta$
down	$1/C_\beta$	$(2m_q + A_q S_q)/C_\beta$	$A_q C_q/C_\beta$	$(2m_q - A_q S_q)/C_\beta$
f	$g_q^{\phi_2}$	$g_{\tilde{q};11}^{\phi_2}$	$g_{\tilde{q};12}^{\phi_2} = g_{\tilde{q};21}^{\phi_2}$	$g_{\tilde{q};22}^{\phi_2}$
up	$1/S_\beta$	$(2m_q + A_q S_q)/S_\beta$	$A_q C_q/S_\beta$	$(2m_q - A_q S_q)/S_\beta$
down	0	$-\mu S_q/C_\beta$	$-\mu C_q/C_\beta$	$\mu S_q/C_\beta$

Table 1. Yukawa coupling coefficients for up and down type quark and squark, where $S_q = \sin 2\theta_q$ and $C_q = \cos 2\theta_q$, and $S_\beta = \sin \beta$ and $C_\beta = \cos \beta$.

$\mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{SQCD}}$ denotes the supersymmetric extension of the full QCD Lagrangian with six quark flavours. The couplings $g_q^{\phi_i}$ and $g_{\tilde{q};kr}^{\phi_i}$ are defined in table 1, m_q denotes the mass of quark q , $v = \sqrt{v_1^2 + v_2^2}$ with $v_i, i = 1, 2$, the vacuum expectation values of the two Higgs doublets of the MSSM. The fields $\tilde{q}_i, i = 1, 2$, denote the squark mass eigenstates, while θ_q stands for the mixing angle defined through:

$$\sin 2\theta_q = \frac{2m_q X_q}{m_{\tilde{q}_1}^2 - m_{\tilde{q}_2}^2}, \quad X_q = A_q - \mu_{\text{SUSY}} \begin{cases} \tan \beta, & \text{for down-type quarks} \\ \cot \beta, & \text{for up-type quarks} \end{cases}, \quad (2.3)$$

where A_q is the trilinear coupling and μ_{SUSY} the Higgs-Higgsino bilinear coupling. The fields $\phi_i, i = 1, 2$, denote the neutral CP-even components of the MSSM Higgs doublets and they are related to the Higgs mass eigenstates through the orthogonal transformation

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}. \quad (2.4)$$

As usual, h stands for the lightest Higgs boson. The mixing angle α is determined at the leading order through

$$\tan 2\alpha = \tan 2\beta \frac{M_A^2 + M_Z^2}{M_A^2 - M_Z^2}; \quad -\frac{\pi}{2} < \alpha < 0, \quad (2.5)$$

where M_Z is the mass of the Z boson and $\tan \beta = v_2/v_1$.

In the following, we assume the mass of the lightest Higgs boson h to be much smaller than the mass of the top-quark and of the SUSY particles, as well as all the other Higgs bosons. In this case, the physical phenomena at low energies can be described with an effective theory containing five quark flavours and the light Higgs. At leading order in the heavy masses, the effective Lagrangian $\mathcal{L}_Y^{\text{eff}}$ can be written as a linear combination of three physical operators [17, 18] constructed from the light degrees of freedom

$$\mathcal{L} \longrightarrow \mathcal{L}_Y^{\text{eff}} + \mathcal{L}_{\text{QCD}}^{(5)}; \quad \mathcal{L}_Y^{\text{eff}} = -\frac{h^{(0)}}{v^{(0)}} \left[C_1^0 \mathcal{O}_1^0 + \sum_q (C_{2q}^0 \mathcal{O}_{2q}^0 + C_{3q}^0 \mathcal{O}_{3q}^0) \right], \quad (2.6)$$

where $\mathcal{L}_{\text{QCD}}^{(5)}$ denotes the Lagrangian of QCD with five active flavours and the coefficient functions $C_i, i = 1, 2q, 3q$, parametrize the effects of the heavy particles on the low-energy

phenomena. The superscript 0 labels bare quantities. The three operators are defined as

$$\begin{aligned}
 \mathcal{O}_1^0 &= (G_{\mu,\nu}^{0,\prime,a})^2, \\
 \mathcal{O}_{2q}^0 &= m_q^{0,\prime} \bar{q}^{0,\prime} q^{0,\prime}, \\
 \mathcal{O}_{3q}^0 &= \bar{q}^{0,\prime} (i\not{D}^{0,\prime} - m_q^{0,\prime}) q^{0,\prime},
 \end{aligned}
 \tag{2.7}$$

where $G_{\mu,\nu}^{0,\prime,a}$ and $D_\mu^{0,\prime}$ are the gluon field strength tensor and the covariant derivative, respectively, and the primes label the quantities in the effective theory. The operator \mathcal{O}_{3q} vanishes by the fermionic equation of motion and it will not contribute to physical observables. So, the last term in eq. (2.6) might be omitted, once the coefficients C_1^0, C_{2q}^0 are determined. The coefficient functions contain information about the heavy particles that were integrated out in the construction of the effective theory. On the contrary, as can be understood from eq. (2.7), the operators encounter only the effects of the light degrees of freedom.

The relations between the parameters and fields in the full and effective theories are given by

$$\begin{aligned}
 G_{\mu,\nu}^{0,\prime,a} &= (\zeta_3^{(0)})^{1/2} G_{\mu,\nu}^{0,a}, \\
 q^{0,\prime} &= (\zeta_2^{(0)})^{1/2} q^0, \\
 g_s^{0,\prime} &= \zeta_g^{(0)} g_s^0, \\
 m_q^{0,\prime} &= \zeta_m^{(0)} m_q^0,
 \end{aligned}
 \tag{2.8}$$

where $g_s = \sqrt{4\pi\alpha_s}$ is the strong coupling. The coefficients $\zeta_3^{(0)}, \zeta_2^{(0)}, \zeta_g^{(0)}, \zeta_m^{(0)}$ are the bare decoupling coefficients. They may be computed from the transverse part of the gluon polarization function and the vector and scalar part of the quark self-energy via [18]

$$\begin{aligned}
 \zeta_3^{(0)} &= 1 + \Pi^{0,h}(0), \\
 \zeta_2^{(0)} &= 1 + \Sigma_v^{0,h}(0), \\
 \zeta_m^{(0)} &= \frac{1 - \Sigma_s^{0,h}(0)}{1 + \Sigma_v^{0,h}(0)}.
 \end{aligned}
 \tag{2.9}$$

For the derivation of the coefficient $\zeta_g^{(0)}$ one has to consider in addition one vertex involving the strong coupling, for example $\bar{q}qg$ or $\bar{c}cg$, where c denotes the Faddeev-Popov ghost. The decoupling coefficients are independent of the momentum transfer, so that they can be evaluated at vanishing external momenta. The superscript h indicates that in the framework of Dimensional Regularization (DREG) or Dimensional Reduction (DRED) only diagrams containing at least one heavy particle inside the loops contribute and that only the hard regions in the asymptotic expansion of the diagrams are taken into account. They have been computed in QCD including corrections up to the four-loop order for the strong coupling [19, 20] and three-loop order for quark masses [18]. In the MSSM the two-loop SQCD [21–23] and SEW [24] expressions are known. Similar to the case of SM, the decoupling coefficients derived within the MSSM can be connected through the Low

Energy Theorem (LET) [25–31] with the coefficients C_1^0, C_{2q}^0 . We discuss in more detail the relation between the coefficients C_{2q}^0, C_{3q}^0 and $\zeta_m^{(0)}, \zeta_2^{(0)}$ in subsection 3.2.

The renormalization procedure of the dimension four operators in the Minimal Subtraction Scheme within DREG ($\overline{\text{MS}}$) [32] is known since long time [17]. The main aspect is that different operators in general mix under renormalization. For the convenience of the reader we reproduce the results for the renormalization constants of the operators \mathcal{O}_1^0 and \mathcal{O}_{2q}^0 that are of interest for the fermionic Higgs decays

$$\mathcal{O}_1 = Z_{11}\mathcal{O}_1^0 + Z_{12}\mathcal{O}_{2q}^0, \quad \mathcal{O}_2 = Z_{22}\mathcal{O}_{2q}^0, \quad \text{where}$$

$$Z_{11} = \left(1 - \frac{\pi}{\alpha'_s} \frac{\beta(\alpha'_s)}{\epsilon}\right)^{-1}, \quad Z_{12} = -\frac{4\gamma_m(\alpha'_s)}{\epsilon} \left(1 - \frac{\pi}{\alpha'_s} \frac{\beta(\alpha'_s)}{\epsilon}\right)^{-1}, \quad Z_{22} = 1, \quad (2.10)$$

$$C_1 = Z_{11}^{-1}C_1^0, \quad C_{2q} = C_{2q}^0 - \frac{Z_{12}}{Z_{11}}C_1^0. \quad (2.11)$$

The explicit expressions for the β -function and quark mass anomalous dimension γ_m of QCD with $n_l = 5$ active flavours at the one-loop order that are needed in the present paper, are given by

$$\beta(\alpha'_s) = -\left(\frac{\alpha'_s}{\pi}\right)^2 \beta_0 + \mathcal{O}((\alpha'_s)^3), \quad \beta_0 = \frac{11}{4} - \frac{n_l}{6},$$

$$\gamma_m(\alpha'_s) = -\frac{\alpha'_s}{\pi}\gamma_0 + \mathcal{O}((\alpha'_s)^2), \quad \gamma_0 = \frac{3}{4}C_F. \quad (2.12)$$

The bare coefficient functions $C_i^0, i = 1, 2q$, must be computed diagrammatically. For the calculation of the $\mathcal{O}(\alpha_s^2)$ corrections to the process $h \rightarrow q\bar{q}$, the knowledge of the coefficient functions C_1^0 and C_{2q}^0 is required at the one- and two-loop order, respectively.

The renormalized coefficient functions and operators are finite but not renormalization group (RG) invariant. In ref. [6, 7], a redefinition of the coefficient functions and operators was introduced so that they are separately RG invariant. This procedure allows us to choose independent renormalization scales for coefficient functions and operators. In practice, one makes a separation of scales: one chooses $\mu \approx M_h$ for the renormalization scale of the operators and $\mu \approx \tilde{M}$ (where \tilde{M} denotes an averaged mass for the heavy supersymmetric particles) for the coefficient functions. The new coefficient functions read [6, 7]

$$C_1(\tilde{M}, M_h) = \frac{\alpha'_s(\tilde{M})\beta^{(5)}(\alpha'_s(M_h))}{\alpha'_s(M_h)\beta^{(5)}(\alpha'_s(\tilde{M}))}C_1(\tilde{M}),$$

$$C_2(\tilde{M}, M_h) = \frac{4\alpha'_s(\tilde{M})}{\pi\beta^{(5)}(\alpha'_s(\tilde{M}))}[\gamma_m^{(5)}(\alpha'_s(\tilde{M})) - \gamma_m^{(5)}(\alpha'_s(M_h))]C_1(\tilde{M}) + C_{2q}(\tilde{M}), \quad (2.13)$$

where the superscript (5) marks that $n_l = 5$ in eq. (2.12). We employ this approach for the evaluation of the decay width $\Gamma(h \rightarrow \bar{q}q)$. More details about the practical calculation are discussed in the next section.

2.1 Higgs decay width

Once the renormalized coefficient functions C_1, C_2 are known, the decay width for the process $h \rightarrow \bar{q}q$ can be predicted. From eqs. (2.6) and (2.7) one can derive a general

formula for the inclusive $h \rightarrow \bar{q}q$ decay width [6, 7]

$$\Gamma(h \rightarrow \bar{q}q) = \Gamma^{(0)}(1 + \bar{\delta}_u)^2 \left[(1 + \Delta_q^{\text{QCD}})C_2^2 + \Xi_q^{\text{QCD}}C_1C_2 \right], \quad (2.14)$$

where $\Gamma^{(0)}$ represents the complete leading order (LO) result given by

$$\Gamma^{(0)} = \frac{N_c G_F M_h m_q^2}{4\pi\sqrt{2}} \left(1 - \frac{4m_q^2}{M_h^2} \right)^{3/2}. \quad (2.15)$$

As is well known, the large logarithms of the type $\ln(M_h^2/m_q^2)$ can be resummed by taking m_q in eq. (2.15) to be the $\overline{\text{MS}}$ mass $m_q^{\overline{\text{MS}}}(\mu)$ evaluated at the scale $\mu = M_h$. The QCD correction Δ_q^{QCD} is known since long time [33, 34],

$$\begin{aligned} \Delta_q^{\text{QCD}} &= \frac{\alpha'_s(\mu)}{\pi} \left(\frac{17}{3} + 2 \ln \frac{\mu^2}{M_h^2} \right) \\ &+ \left(\frac{\alpha'_s(\mu)}{\pi} \right)^2 \left[\frac{8851}{144} - \frac{47}{6} \zeta(2) - \frac{97}{6} \zeta(3) + \frac{263}{9} \ln \frac{\mu^2}{M_h^2} + \frac{47}{12} \ln^2 \frac{\mu^2}{M_h^2} \right], \end{aligned} \quad (2.16)$$

with $\zeta(x)$ being the Riemann's zeta function. The additional QCD correction generated through double-triangle topologies Ξ_q^{QCD} was first computed in ref. [6, 7],

$$\Xi_q^{\text{QCD}} = \frac{\alpha'_s(\mu)}{\pi} C_F \left(-19 + 6\zeta(2) - \ln^2 \frac{m_q^2}{M_h^2} - 6 \ln \frac{\mu^2}{M_h^2} \right). \quad (2.17)$$

The universal corrections $\bar{\delta}_u$ of $\mathcal{O}(\alpha_s^n x_t)$, where $x_t = (\alpha_t/4\pi)^2 = G_F M_t^2 / (8\pi^2 \sqrt{2})$, with α_t the top-Yukawa coupling, contain the contributions from the renormalization of the Higgs wave function and the vacuum expectation value [35],

$$\bar{\delta}_u = x_t \left[\frac{7}{2} + \frac{\alpha'_s(\mu)}{\pi} \left(\frac{19}{3} - 2\zeta(2) + 7 \ln \frac{\mu^2}{M_t^2} \right) + \mathcal{O}(\alpha_s^2) \right]. \quad (2.18)$$

The coefficient functions C_1, C_{2q} (and implicitly C_1, C_2) are known within SQCD at the one-loop order since quite some time [14, 36]. For completeness, we display them here providing also $\mathcal{O}(\epsilon)$ terms that are necessary for the two-loop calculation.

$$\begin{aligned} C_1 &= -\frac{\alpha'_s(\mu)}{12\pi} \left\{ -\frac{\sin \alpha}{\cos \beta} \left[\frac{M_t^2 \mu_{\text{SUSY}} X_t}{4m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2 \tan \beta} - \epsilon \frac{M_t \mu_{\text{SUSY}} \sin 2\theta_t}{8 \tan \beta} \left(\frac{L_{\tilde{t}_1}}{m_{\tilde{t}_1}^2} - \frac{L_{\tilde{t}_2}}{m_{\tilde{t}_2}^2} \right) \right] \right. \\ &+ \frac{\cos \alpha}{\sin \beta} \left[\frac{4m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2 + m_{\tilde{t}_1}^2 M_t^2 + m_{\tilde{t}_2}^2 M_t^2 - A_t M_t^2 X_t}{4m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2} \right. \\ &\left. \left. + \epsilon \frac{A_t M_t \sin 2\theta_t}{8} \left(\frac{L_{\tilde{t}_1}}{m_{\tilde{t}_1}^2} - \frac{L_{\tilde{t}_2}}{m_{\tilde{t}_2}^2} \right) + \epsilon \frac{M_t^2}{4} \left(\frac{4L_t}{M_t^2} + \frac{L_{\tilde{t}_1}}{m_{\tilde{t}_1}^2} + \frac{L_{\tilde{t}_2}}{m_{\tilde{t}_2}^2} \right) \right] \right\}, \end{aligned} \quad (2.19)$$

$$\begin{aligned}
C_{2q} = & -\frac{\sin \alpha}{\cos \beta} \frac{1 + \frac{\alpha'_s(\mu)}{2\pi} C_F A_b m_{\tilde{g}} \left[F_1(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, m_{\tilde{g}}^2) + \epsilon F_2(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, m_{\tilde{g}}^2) \right]}{1 + \frac{\alpha'_s(\mu)}{2\pi} C_F X_b m_{\tilde{g}} \left[F_1(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, m_{\tilde{g}}^2) + \epsilon F_2(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, m_{\tilde{g}}^2) \right]} \\
& + \frac{\cos \alpha}{\sin \beta} \frac{\frac{\alpha'_s(\mu)}{2\pi} C_F (-\mu_{\text{SUSY}} \tan \beta) m_{\tilde{g}} \left[F_1(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, m_{\tilde{g}}^2) + \epsilon F_2(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, m_{\tilde{g}}^2) \right]}{1 + \frac{\alpha'_s(\mu)}{2\pi} C_F X_b m_{\tilde{g}} \left[F_1(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, m_{\tilde{g}}^2) + \epsilon F_2(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, m_{\tilde{g}}^2) \right]}, \tag{2.20}
\end{aligned}$$

where $C_F = 4/3$, $L_x = \ln(\mu^2/m_x^2)$ and the functions F_1 and F_2 are defined through

$$\begin{aligned}
F_1(x, y, z) &= -\frac{xy \ln \frac{y}{x} + yz \ln \frac{z}{y} + zx \ln \frac{x}{z}}{(x-y)(y-z)(z-x)}, \\
F_2(x, y, z) &= -\frac{1}{(x-y)(y-z)(z-x)} \left[xy \ln \frac{y}{x} \left(1 + \ln \frac{\mu^2}{\sqrt{xy}} \right) \right. \\
&\quad \left. + yz \ln \frac{z}{y} \left(1 + \ln \frac{\mu^2}{\sqrt{yz}} \right) + zx \ln \frac{x}{z} \left(1 + \ln \frac{\mu^2}{\sqrt{xz}} \right) \right]. \tag{2.21}
\end{aligned}$$

In the above formulas, $\alpha'_s(\mu)$ denotes the strong coupling constant computed in the $\overline{\text{MS}}$ scheme and taking into account $n_l = 5$ active quark flavours.

The computation of the coefficient function C_{2q} through two loops in SQCD is discussed in some detail in the next section.

3 Calculation of the coefficient function C_{2q} at next-to-next-to-leading order (NNLO)

For the derivation of the coefficient functions one has to compute Green functions in the full and effective theory and make use of the decoupling relations eq. (2.8) to connect them [18]. In general, one Green function contains several coefficient functions. For example, the amputated Green function involving the $q\bar{q}$ pair and the zero-momentum insertion of the operator \mathcal{O}_h which mediates the couplings to the light Higgs boson h contains both coefficient functions C_{2q} and C_{3q} . Similarly, one possibility to compute the coefficient function C_1 involves the Green function formed by the coupling of the operators \mathcal{O}_h to two gluons.

In the following, we restrict the discussion to the computation of the coefficient function C_{2q} . Considering the appropriate one-particle-irreducible (1PI) Green function, we get

$$\Gamma_{\bar{q}q\mathcal{O}_h}^0(p, -p) = i^2 \int dx dy e^{ip(x-y)} \langle T q^0(x) \bar{q}^0(y) \mathcal{O}_h(0) \rangle^{1\text{PI}}, \tag{3.1}$$

where p is the outgoing momentum of q . In a next step we express the operator \mathcal{O}_h with the help of eqs. (2.6) and (2.7) and make use of the decoupling relations eq. (2.8). One can easily see that the above Green function will get contributions only from the operators \mathcal{O}_{2q}

and \mathcal{O}_{3q}

$$\begin{aligned}\Gamma_{\bar{q}q\mathcal{O}_h}^{0,h}(p, -p) &= -\zeta_2^{(0)} \int dx dy e^{ip(x-y)} \langle T q'^0(x) \bar{q}'^0(y) (C_{2q}\mathcal{O}_{2q} + C_{3q}\mathcal{O}_{3q}) \rangle^{\text{1PI}}, \\ &= \zeta_2^{(0)} \zeta_m^{(0)} (C_{2q}^0 - C_{3q}^0) m_b^0 + \zeta_2^{(0)} C_{3q}^0 \not{p}.\end{aligned}\tag{3.2}$$

In the last step we have used the Feynman rules for the scalar dimension four operators that can be found in ref. [37] and the fact that $\Gamma_{\bar{q}q\mathcal{O}_h}^{0,h}(p, -p)$ denotes an amputated Green function. Exploiting the fact that the coefficient functions do not depend on the momentum transfer, one can set also $p = 0$. In this case, on the l.h.s. of eq. (3.2) only the hard parts of the Green function survive, as the massless tadpoles are set to zero in DRED and DREG. As before, the superscript h stands for hard contributions.

The validity of the approximation $m_h^2 = p_h^2 \approx 0$ was extensively studied in the context of the SM and reconfirmed for the case of gluon fusion at two-loop order in SQCD in ref. [38]. We expect that this approximation holds also in the case of fermionic Higgs decays, due to the heavy supersymmetric mass spectrum.

There are two possibilities currently used in the literature for the derivation of $\Gamma_{\bar{q}q\mathcal{O}_h}^{0,h}(0, 0)$. The first one is the direct computation of the scalar and vector components of the vertex function making use of the appropriate projectors. The second one uses the LET, which relates the vertex corrections to the $hq\bar{q}$ coupling to the quark self-energy corrections via appropriate derivatives [25–31, 38].

3.1 Direct calculation of the coefficient function C_{2q}^0 at NNLO

Decomposing the Green function $\Gamma_{\bar{q}q\mathcal{O}_h}^{0,h}$ into its scalar and vector components $\Gamma_{\bar{q}q\mathcal{O}_h;s}^{0,h}$, $\Gamma_{\bar{q}q\mathcal{O}_h;v}^{0,h}$ one derives from eq. (3.2) two linearly independent relations for the coefficients C_{2q}^0 and C_{3q}^0

$$\begin{aligned}\Gamma_{\bar{q}q\mathcal{O}_h;s}^{0,h}(0, 0) &= \zeta_2^{(0)} \zeta_m^{(0)} (C_{2q}^0 - C_{3q}^0), \\ \Gamma_{\bar{q}q\mathcal{O}_h;v}^{0,h}(0, 0) &= \zeta_2^{(0)} C_{3q}^0.\end{aligned}\tag{3.3}$$

In SQCD at the two-loop order there is also an axial contribution to $\Gamma_{\bar{q}q\mathcal{O}_h}^{0,h}(0, 0)$, which arises from diagrams where a top quark-squark pair is exchanged from a gluino propagator. However, it generates only contributions $\mathcal{O}(\alpha_s^4)$ to $\Gamma(h \rightarrow q\bar{q})$ which are beyond the precision we are interested in this paper.

Finally, using eqs. (2.9) the expression for C_{2q}^0 through two loops reads

$$C_{2q}^0 = \frac{\Gamma_{\bar{q}q\mathcal{O}_h;s}^{0,h}(0, 0)}{1 - \Sigma_s^{0,h}(0)} + \frac{\Gamma_{\bar{q}q\mathcal{O}_h;v}^{0,h}(0, 0)}{1 + \Sigma_v^{0,h}(0)}.\tag{3.4}$$

One can either work in the (ϕ_1, ϕ_2) basis, i.e. generating two sets of vertex corrections for each Higgs field, or one derives the Feynman rules for the couplings of quarks and squarks to the light Higgs using the relation eq. (2.4).

For the computation of the vertex corrections up to two loops we have implemented two independent setups. In one of them, the Feynman diagrams are generate with the help

of the program `FeynArts` [39, 40], then its output is handled in a self-written `Mathematica` code which includes the two-loop tensor reduction and the mapping of the vertex topologies to the two-loop tadpole ones [41]. The last step is possible due to the fact, that we neglect the mass of the light Higgs boson and of the external light quarks. In the second setup, the Feynman diagrams are generated with the program `QGRAF` [42], and further processed with `q2e` and `exp` [43, 44]. The reduction of various vacuum integrals to the master integral was performed by a self-written `FORM` [45] routine.

3.2 LET derivation of the coefficient function C_{2q}^0 at NNLO

The connection between the coefficient functions C_1^0, C_{2q}^0 and the decoupling coefficients ζ_s^0, ζ_m^0 or equivalently $\Pi^{0,h}(0), \Sigma_s^{0,h}(0)$ and $\Sigma_v^{0,h}(0)$ was extensively studied in the context of the SM. The validity of the LET was verified up to three-loop order in QCD [18]. In the framework of the MSSM, however, the derivation of LET is much more involved due to the presence of the two Higgs doublets and of many massive particles and mixing angles. The applicability of LET in SQCD at two-loop order was verified only very recently. Namely, the relationship between the coefficient function C_1 and the hard part of the transverse gluon polarization function $\Pi^{0,h}(0)$ has been established in ref. [38]. Furthermore, the leading two-loop contributions to the effective bottom Yukawa couplings have been derived from the scalar part of the bottom quark self-energy in ref. [15, 16]. It is one of the aims of this paper to verify the relationship between the coefficient function C_{2q} and the hard part of the scalar and vector contributions to the quark self-energy.

For our calculation it is very convenient to work in (ϕ_1, ϕ_2) basis, which means that we have to decompose the Green functions according to eq. (2.4)

$$\Gamma_{\bar{q}q\mathcal{O}_h;a}^{0,h}(0,0) = -\sin\alpha \Gamma_{\bar{q}q\mathcal{O}_{\phi_1};a}^{0,h}(0,0) + \cos\alpha \Gamma_{\bar{q}q\mathcal{O}_{\phi_2};a}^{0,h}(0,0), \quad a = s, v. \quad (3.5)$$

Similarly, the coefficient function can be written as follows

$$C_{2q}^0 = -\sin\alpha C_{2q,\phi_1}^0 + \cos\alpha C_{2q,\phi_2}^0. \quad (3.6)$$

Applying the LET² to the individual components $\Gamma_{\bar{q}q\mathcal{O}_{\phi_i};a}$ we get

$$\begin{aligned} \Gamma_{\bar{q}q\mathcal{O}_{\phi_i};s}^{0,h}(0,0) &= \frac{1}{m_b} \frac{\partial}{\partial\phi_i} \left[m_b(1 - \Sigma_s^{0,h}(0)) \right] \Big|_{\phi_i=v_i} \equiv \frac{1}{m_b} \hat{D}_{q,\phi_i} \left[m_b(1 - \Sigma_s^{0,h}(0)) \right], \\ \Gamma_{\bar{q}q\mathcal{O}_{\phi_i};v}^{0,h}(0,0) &= \frac{\partial}{\partial\phi_i} \left[-\Sigma_v^{0,h}(0) \right] \Big|_{\phi_i=v_i} \equiv \hat{D}_{q,\phi_i} \left[-\Sigma_v^{0,h}(0) \right], \end{aligned} \quad (3.7)$$

with $i = 1, 2$ and $a = v, s$. As we are considering only the hard parts of the above Green functions no complication related to the occurrence of infrared divergences is encountered. In practice, it is convenient to express the operators \hat{D}_{q,ϕ_i} introduced in eq. (3.7) in terms of derivatives w.r.t. masses and mixing angles. This can be achieved using the field-dependent definition of the parameters, in our case quark and squark masses and squark mixing angles [47, 48].

²The gauge-fixing condition for SQCD is independent of the vacuum expectation values $v_{1,2}$, so that LET holds in its trivial form [46].

The formulas derived up to now are valid for a generic light quark flavour q . However, for phenomenological applications the decay channel $h \rightarrow b\bar{b}$ is the most important one. The explicit expressions for the operators \hat{D}_{b,ϕ_i} can be easily derived from the eqs. (11) and (12) in ref. [38]. We quote them here for completeness and to fix our normalization:

$$\begin{aligned}
 \hat{D}_{b,\phi_1} &= \frac{1}{\cos\beta}(m_b A_b \mathcal{F}_b + m_b \mathcal{G}_b) - \frac{1}{\sin\beta} m_t \mu_{\text{SUSY}} \sin 2\theta_t \mathcal{F}_t, \\
 \hat{D}_{b,\phi_2} &= \frac{1}{\cos\beta}(-m_b \mu_{\text{SUSY}} \mathcal{F}_b) + \frac{1}{\sin\beta}(m_t A_t \sin 2\theta_t \mathcal{F}_t + 2m_t^2 \mathcal{G}_t), \quad \text{with} \\
 \mathcal{F}_b &= \frac{2}{m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2}(1 - \sin^2 2\theta_b) \frac{\partial}{\partial \sin 2\theta_b}, \quad \mathcal{G}_b = \frac{\partial}{\partial m_b}, \\
 \mathcal{F}_t &= \frac{\partial}{\partial m_{\tilde{t}_1}^2} - \frac{\partial}{\partial m_{\tilde{t}_2}^2} + \frac{2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \frac{(1 - \sin^2 2\theta_t)}{\sin 2\theta_t} \frac{\partial}{\partial \sin 2\theta_t}, \\
 \mathcal{G}_t &= \frac{\partial}{\partial m_{\tilde{t}_1}^2} + \frac{\partial}{\partial m_{\tilde{t}_2}^2} + \frac{\partial}{\partial m_t^2}.
 \end{aligned} \tag{3.8}$$

In the above formulas we keep only the terms that do not vanish in the limit $m_b \rightarrow 0$.

As is well known, in eqs. (3.7) one has first to apply the derivative operators \hat{D}_{q,ϕ_i} and afterwards perform the renormalization. For simplicity of the notation we suppress the superscript (0), labeling bare quantities. We checked explicitly at the diagram level that eqs. (3.7) hold through two loops. The computation of the two-loop diagrams contributing to $\Sigma_a^{0,h}(0)$ goes along the same line as that for $\Gamma_{\bar{q}q\mathcal{O}_{\phi_i};a}^{0,h}(0,0)$. The exact results together with few expansions for special mass hierarchies can be found in ref. [22, 23].

Let us mention at this point that for large values of $\tan\beta$ the dominant contribution to the coefficient function C_{2q} is contained in the first term in eq. (3.4). The $\mu_{\text{SUSY}} \tan\beta$ -enhanced contributions are implicitly resummed in eq. (3.4), through the presence of the denominator $1 - \Sigma_s^{0,h}(0)$, which contains contribution of the form $\alpha_s^n \mu_{\text{SUSY}} \tan\beta$. In the framework of LET the $\mu_{\text{SUSY}} \tan\beta$ -enhanced contributions to $\Gamma_{\bar{q}q\mathcal{O}_{h;s}}$ are generated through the term proportional to the derivative \mathcal{F}_b in \hat{D}_{b,ϕ_2} . Taking into account the parametric dependence of $\Sigma_s^{0,h}$ on masses and mixing angles, one can easily derive these contributions from the terms proportional to $\sin 2\theta_b$ in $\Sigma_s^{0,h}$.³ Such contributions have also been derived in ref. [15, 16] using the effective Lagrangian approach. Indeed, after discarding the additional pieces comprised in our computation of the coefficient C_{2q} , namely the vector part and the rest of the derivative operators in eq. (3.8), we get good numerical agreement with the results of ref. [15, 16].

3.3 Regularization and renormalization scheme

It is well known that the appropriate regularization scheme for the computation of radiative corrections in supersymmetric theories is DRED. However, the most convenient regularization scheme for the handling of the dimension four operators at higher orders is DREG as discussed in section 2. For the renormalization we employed two different approaches. In one of them we computed the radiative corrections to the coefficient functions directly

³A similar observation was made in ref. [15, 16], too.

in DREG. This implies that some of the supersymmetric relations between couplings of quarks and squarks do not hold anymore. More precisely, one has to distinguish between the gluino-quark-squark coupling \hat{g}_s and the gauge coupling g_s , and between the Yukawa couplings of Higgs bosons to quarks $g_q^{\phi_i}$ and squarks $g_{\tilde{q},kr}^{\phi_i}$. The relationships between the different couplings are necessary only at the one-loop order and they are well known since long time [49].

In the second approach, we performed the two-loop computation of the coefficient functions in the DRED scheme and afterwards converted the results into the DREG scheme using the two-loop translation relations for the quark masses and strong couplings defined in the full [50] and effective theory [51], and eqs. (3.7). As a consistency check, we explicitly verified that the results obtained with the two methods agree.

For the renormalization of the divergent parameters we used the on-shell scheme for the gluino and bottom squark masses and mixing angle and the minimal subtraction scheme $\overline{\text{MS}}$ or $\overline{\text{DR}}$ for the strong coupling and the bottom quark mass. The renormalization of the trilinear coupling A_b was performed implicitly through the use of relation eq. (2.3). The explicit formulas for the one-loop counterterms are well-known in the literature (see for example ref. [52]).

The complete two-loop results for the SQCD corrections to the coefficient function C_{2q} discussed in this section are too lengthy to be given here. They are available in `MATHEMATICA` format upon request from the authors. For further applications, we also provide results for the case where all parameters are renormalized minimally.

In principle, the results obtained in this section can be easily generalized to the heavy Higgs decays taking into account the necessary changes in the Yukawa couplings as given in table 1. However, the application of effective theory formalism introduced in section 3 is not justified in this case, *i.e.* the condition $M_H \ll M_t, M_{\text{SUSY}}$ does not hold anymore.

4 Numerical results

In this section we study the phenomenological implications of the two-loop corrections to the coefficient function C_{2q} on the Higgs decay width $\Gamma(h \rightarrow b\bar{b})$. The SM input parameters are the strong coupling constant at the Z-boson mass scale $\alpha_s(M_Z) = 0.1184$ [53], the top quark pole mass $M_t = 173.1 \text{ GeV}$ [54] and the running bottom quark mass in the $\overline{\text{MS}}$ scheme $m_b(m_b) = 4.163 \text{ GeV}$ [55]. For the supersymmetric mass spectrum we adopted the corresponding values of the “small α_{eff} ” and “gluophobic” scenarios as defined in ref. [3].⁴

For the running of the strong coupling constant within QCD we use the `Mathematica` package `RunDec` [56]. For the evaluation of the strong coupling constant within the six-flavour SQCD and the $\overline{\text{DR}}$ scheme we follow ref. [23].⁵

An important ingredient for the computation of the decay width $\Gamma(h \rightarrow b\bar{b})$ is the effective mixing angle of the neutral Higgs sector α_{eff} [11] that takes into account the radiative corrections to the Higgs propagator. In practical applications one replaces the tree-level mixing angle α defined in eq. (2.5) with α_{eff} . This can be computed in perturbation theory

⁴We used the tree-level formulas to derive the mass eigenvalues for squark fields.

⁵For a more detailed discussion see ref. [23] and the references therein.

from the knowledge of the radiative corrections to the self-energy matrix of the neutral Higgs doublet $\hat{\Sigma}_{\phi_1}, \hat{\Sigma}_{\phi_2}, \hat{\Sigma}_{\phi_1\phi_2}$,

$$\tan \alpha_{\text{eff}} = \frac{-(M_A^2 + M_Z^2) \sin \beta \cos \beta - \hat{\Sigma}_{\phi_1\phi_2}}{M_Z^2 \cos^2 \beta + M_A^2 \sin^2 \beta - M_h^2 - \hat{\Sigma}_{\phi_1}}, \quad (4.1)$$

where M_h stands for the on-shell mass of the light Higgs boson. For the numerical analyses we implemented the exact two-loop results from ref. [57].⁶

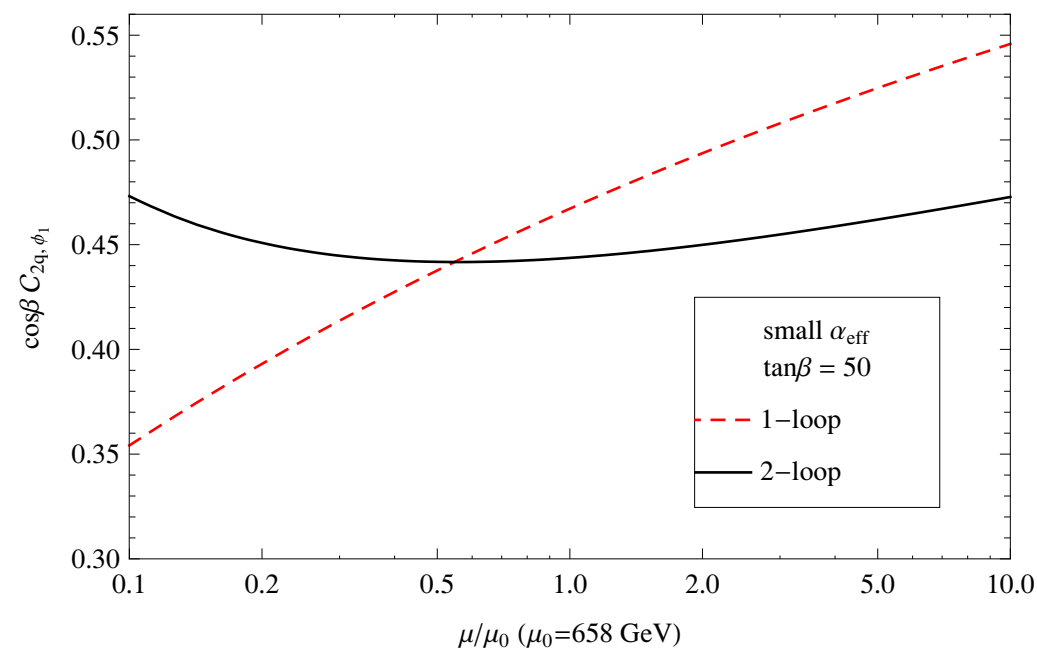
In figure 1 we show separately the renormalization scale dependence of the coefficient functions \mathcal{C}_{2q,ϕ_1} and \mathcal{C}_{2q,ϕ_2} , which describe the effective Yukawa couplings of the neutral Higgs fields ϕ_1 and ϕ_2 . We choose the “small α_{eff} ” scenario with $\tan \beta = 50$, $M_A = 300$ GeV and evaluate α_{eff} with the tree-level formula in order to avoid additional scale dependence. The dashed and solid lines correspond to the one- and two-loop results, respectively. As can be read from the figure, at the one-loop order the scale dependence amounts to about 50% when the renormalization scale is varied around the average value $\mu_0 = (m_{\bar{b}_1} + m_{\bar{b}_2} + m_{\bar{g}})/3 \simeq 658$ GeV by a factor 10. At the two-loop order the variation with the renormalization scale is significantly improved. The remaining scale dependence is below 6%. An interesting aspect is the opposite evolution of the two coefficient functions with the renormalization scale. This feature is reflected in a milder scale dependence of the full coefficient function \mathcal{C}_{2q} of about 4% and 0.5% at one- and two-loop order, respectively. However, for low A-boson masses $M_A \leq 120$ GeV this numbers change to 27% and 5%, respectively. As usual, we can interpret the last number as an estimation of the theoretical uncertainties due to unknown higher order corrections. So, the two-loop SQCD corrections are essential for the accurate prediction of the decay width $\Gamma(h \rightarrow b\bar{b})$.

In figure 2 we display the renormalization scale dependence for the “gluophobic” scenario, where we fixed the value of $\tan \beta$ to 20, $M_A = 300$ GeV and maintain the same convention for the lines. A similar behaviour as for the “small α_{eff} ” scenario is observed. However, in this case the coefficient function \mathcal{C}_{2q,ϕ_2} has a much stronger scale dependence at the one-loop order than the coefficient \mathcal{C}_{2q,ϕ_1} . The scale variation of the full coefficient function \mathcal{C}_{2q} sums up to 1.5% and 0.2% for $M_A = 300$ GeV at the one- and two-loop order, respectively. However for $M_A \leq 120$ GeV the scale variation amounts to 8% and 1.5% at one- and two-loop order, respectively, which shows the importance of the two-loop SQCD corrections for this region of the parameter space.

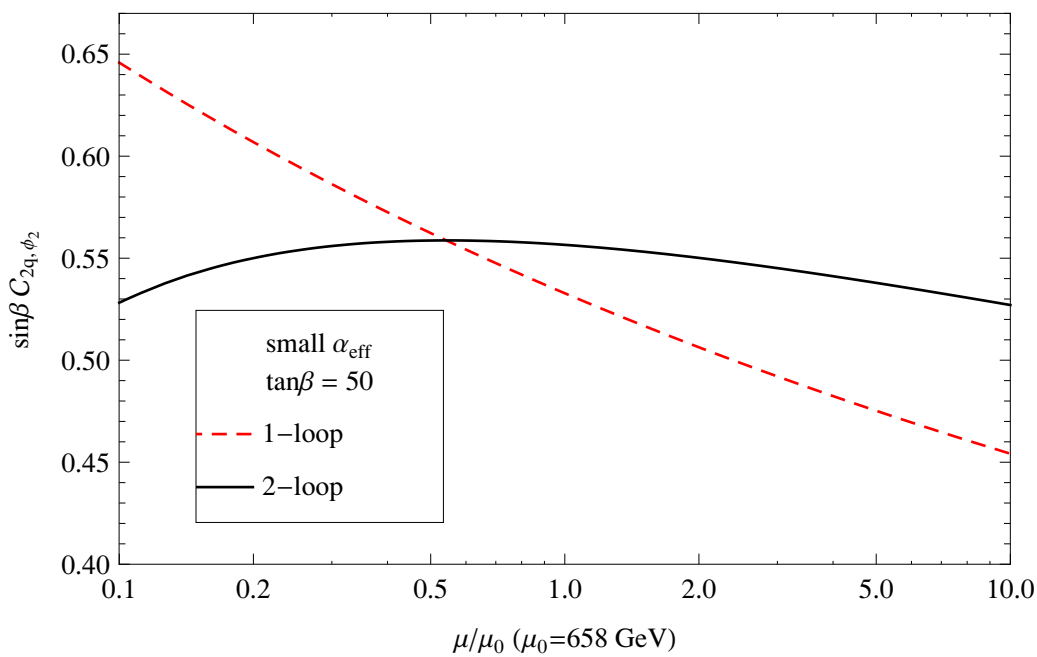
We can conclude that for phenomenological analyses the choice of the renormalization scale around μ_0 ensures small radiative corrections and a good convergence of the perturbative scale. In the following, we set $\mu \simeq \mu_0$ for the computation of the SQCD corrections to the coefficient functions \mathcal{C}_{2q} and the decay width $\Gamma(h \rightarrow b\bar{b})$.

In figure 3 we depict the dependence of the full coefficient function \mathcal{C}_{2q} on the A boson mass M_A (a) and the light Higgs boson mass M_h (b) for the “small α_{eff} ” scenario. We set $\tan \beta = 50$ and vary the mass of the A boson between $100 \text{ GeV} \leq M_A \leq 200 \text{ GeV}$. For the evaluation of the Higgs boson mass and the effective mixing angle α_{eff} we employed the

⁶Very recently the three-loop SQCD corrections to M_h have been computed [58]. However, they are valid only for specific mass hierarchies.

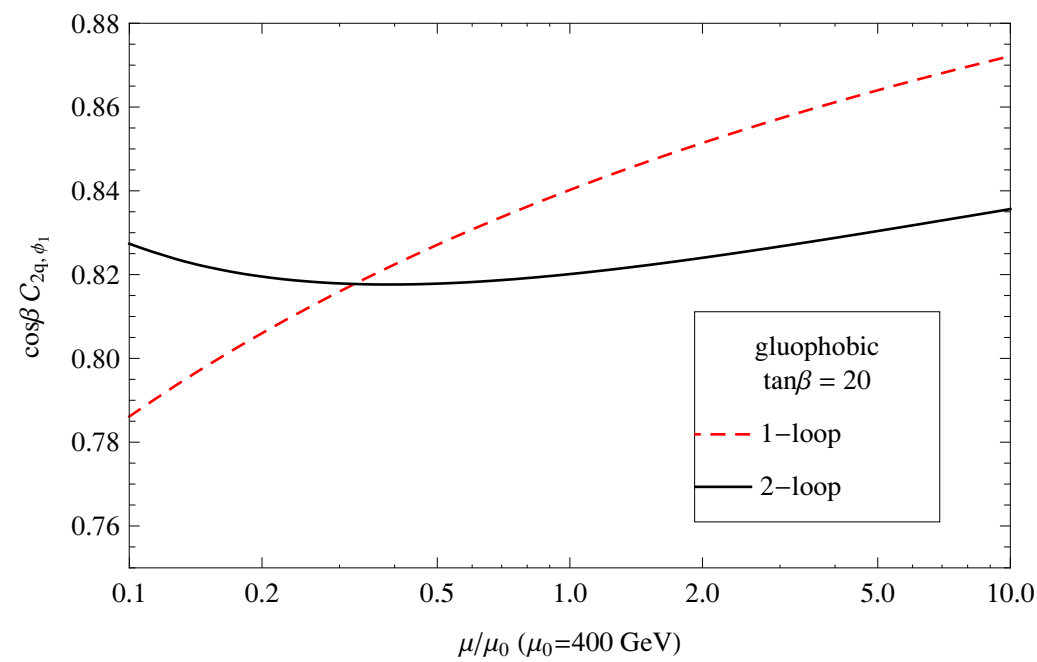


(a)

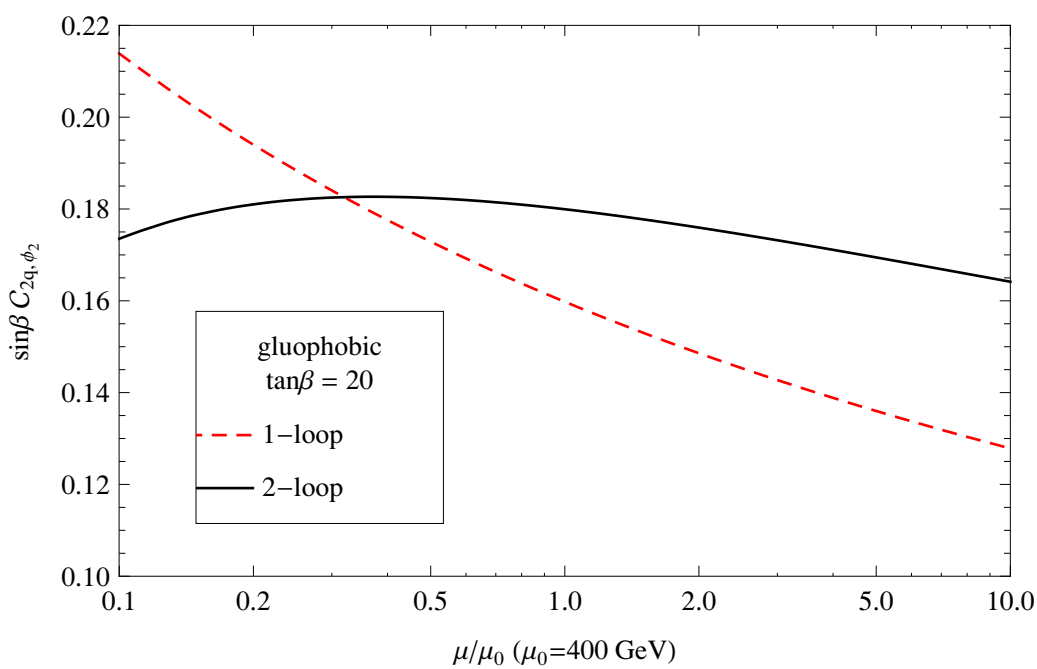


(b)

Figure 1. The renormalization scale dependence of the coefficient functions $C_{2q,\phi_1} \cos \beta$ (a) and $C_{2q,\phi_2} \sin \beta$ (b) is depicted at one- and two-loop order in the “small α_{eff} ” scenario.

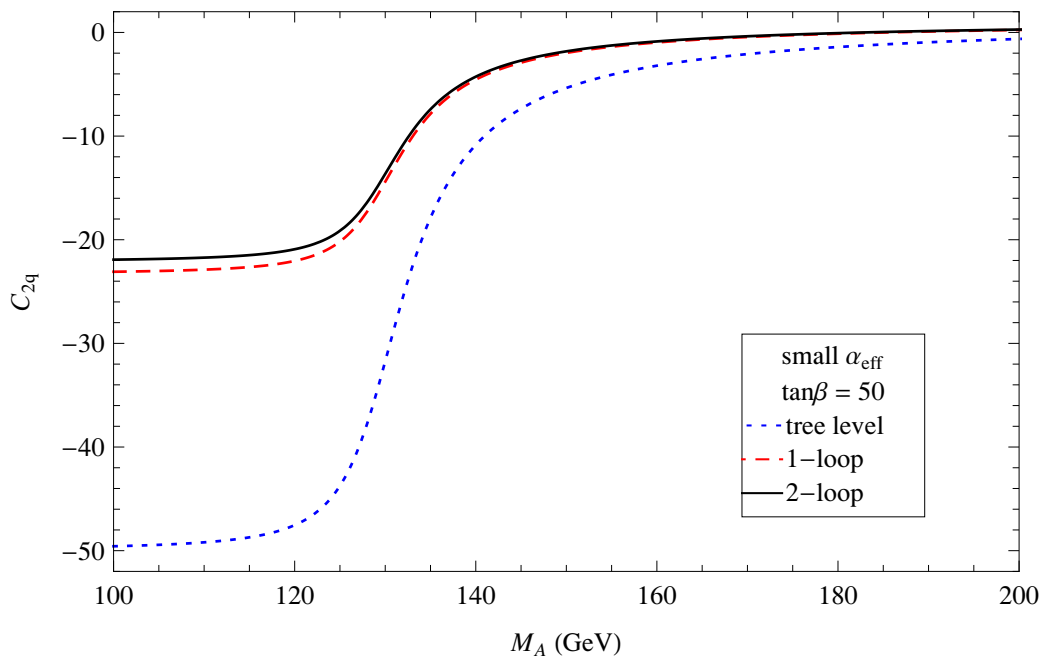


(a)

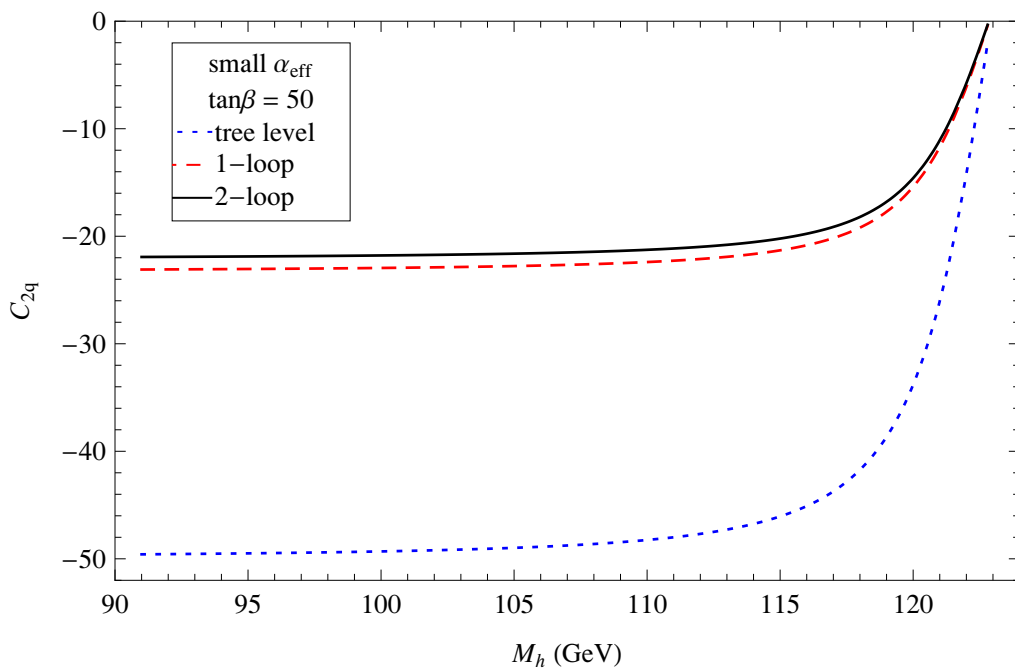


(b)

Figure 2. The renormalization scale dependence of the coefficient functions $C_{2q,\phi_1} \cos \beta$ (a) and $C_{2q,\phi_2} \sin \beta$ (b) is depicted at one- and two-loop order in the “gluophobic” scenario.



(a)



(b)

Figure 3. The coefficient function C_{2q} as a function of M_A (a) and M_h (b) is shown at one- and two-loop order in the “small α_{eff} ” scenario.

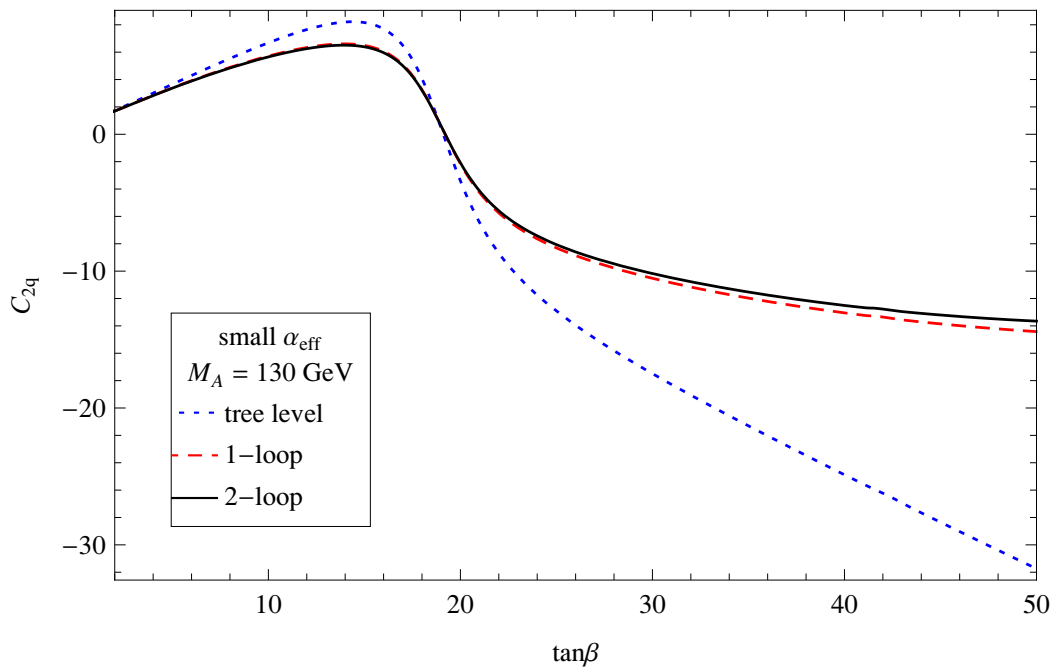
two-loop SQCD results [57]. As can be seen from figure 3(a) for low M_A values there are large one-loop corrections of about 60% of the tree-level values. They originate from the large corrections to the scalar part of the quark self-energy, that are actually resummed through the use of the formula given in the eq. (3.4). This feature is also reflected by the relatively small two-loop corrections of about 2% of the tree-level values. For M_A values larger than 130 GeV one observes a steep increase of the coefficient function C_{2q} (decrease of the absolute value) due to cancellation of eq. (4.1), that implies $\alpha_{\text{eff}} \rightarrow 0$. In this case, C_{2q} reaches its minimal absolute value. From panel (b) one notices a similar steep increase of the coefficient function C_{2q} when M_h reaches its maximal value of about 125 GeV. A similar behaviour is also observed for the “gluophobic” scenario. So, for both scenarios we expect a strong suppression of the decay width $\Gamma(h \rightarrow b\bar{b})$ for large A-boson masses or equivalently for M_h close to its maximal value for which $\alpha_{\text{eff}} \rightarrow 0$.

In figure 4 the dependence of the coefficient function C_{2q} on $\tan\beta$ is shown for the “small α_{eff} ” (a) and “gluophobic” (b) scenarios, where the A-boson mass was fixed to $M_A = 130$ GeV. As expected, we observe a significant increase of the magnitude of the radiative corrections with the increase of the $\tan\beta$ value. At the one-loop order the radiative corrections amount to about 70% (a) and to 33% (b) from the tree level values. At the two-loop order they sum up to 3% and 5%, respectively.

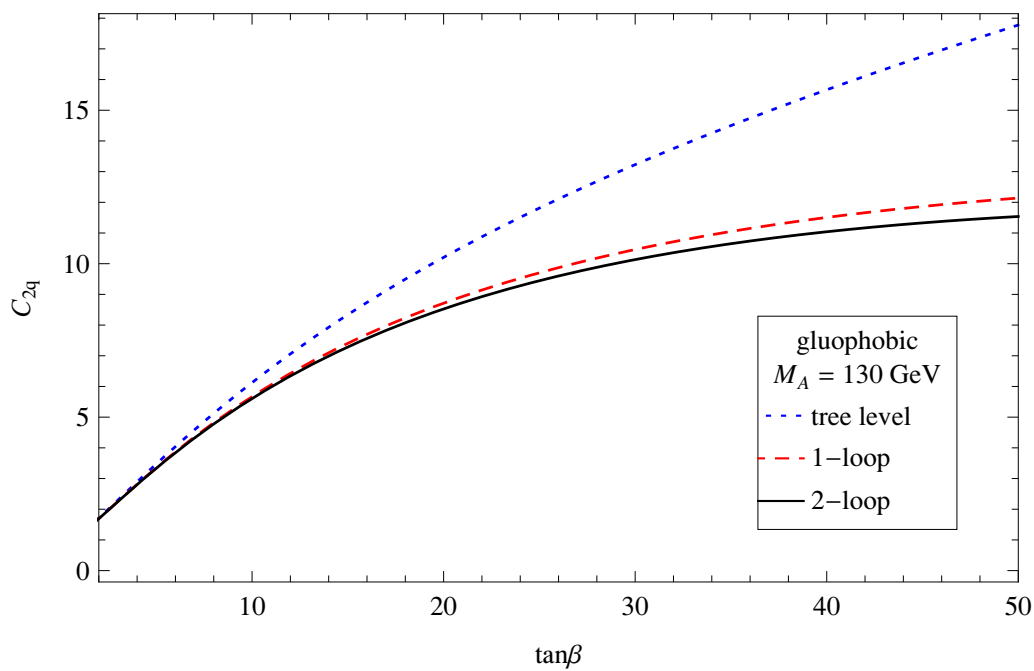
In figure 5 we display the decay width for $h \rightarrow b\bar{b}$ as a function of the Higgs boson mass M_h , considering the “small α_{eff} ” (a) and “gluophobic” (b) scenarios. We chose $\tan\beta = 50$ and $\tan\beta = 20$ for the case (a) and (b), respectively. The two-loop genuine QCD and EW corrections to the process $h \rightarrow b\bar{b}$, as well as the two-loop SQCD corrections to the Higgs boson propagator are depicted by the dotted lines. More precisely, they are derived from eq. (2.14), where the coefficient functions \mathcal{C}_1 and \mathcal{C}_2 are set to their tree-level values. The additional SQCD vertex corrections parametrized through the coefficient functions \mathcal{C}_1 and \mathcal{C}_2 are represented at the one- and two-loop order by the dashed and solid lines, respectively. We also take into account the one-loop SEW corrections to the coefficient function \mathcal{C}_2 and fix their renormalization scale at $\mu_{\text{SEW}} = (m_{\tilde{t}_1} + m_{\tilde{t}_2} + \mu_{\text{SUSY}})/15$, for which the two-loop SEW corrections become negligible [15, 16].

For a relatively light Higgs boson mass M_h , the large one-loop radiative corrections of about 70% (a) and 50% (b) are still amplified by mild two-loop corrections that can reach as much as about 8% from the decay width including QCD corrections even for the selected choice of the renormalization scale of SQCD corrections. The large SQCD radiative corrections to $\Gamma(h \rightarrow b\bar{b})$ have only a relatively small impact on the branching ratio $BR(h \rightarrow b\bar{b})$ but they can have a large impact on $BR(h \rightarrow \tau^+\tau^-)$. For sufficiently large $\tan\beta$ and μ_{SUSY} , the measurement of $BR(h \rightarrow \tau^+\tau^-)$ can provide information about the distinction between the SM and MSSM predictions.

For a large Higgs boson mass for which $\alpha_{\text{eff}} \rightarrow 0$, the partial decay widths for $h \rightarrow b\bar{b}$ and $h \rightarrow \tau^+\tau^-$ are significantly suppressed. In this case the radiative corrections (in particular the corrections to the Higgs boson propagator in eq. (4.1)) are essential for an accurate prediction of $\Gamma(h \rightarrow b\bar{b})$ and $\Gamma(h \rightarrow \tau^+\tau^-)$. Furthermore, the $BR(h \rightarrow \gamma\gamma)$ will be strongly enhanced, improving the LHC prospects of finding a light Higgs.

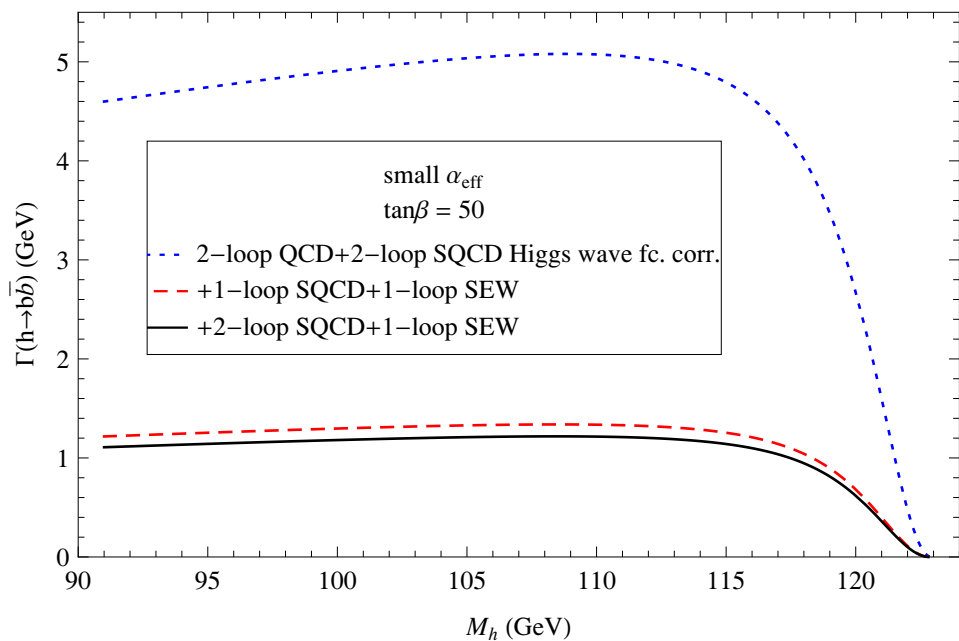


(a)

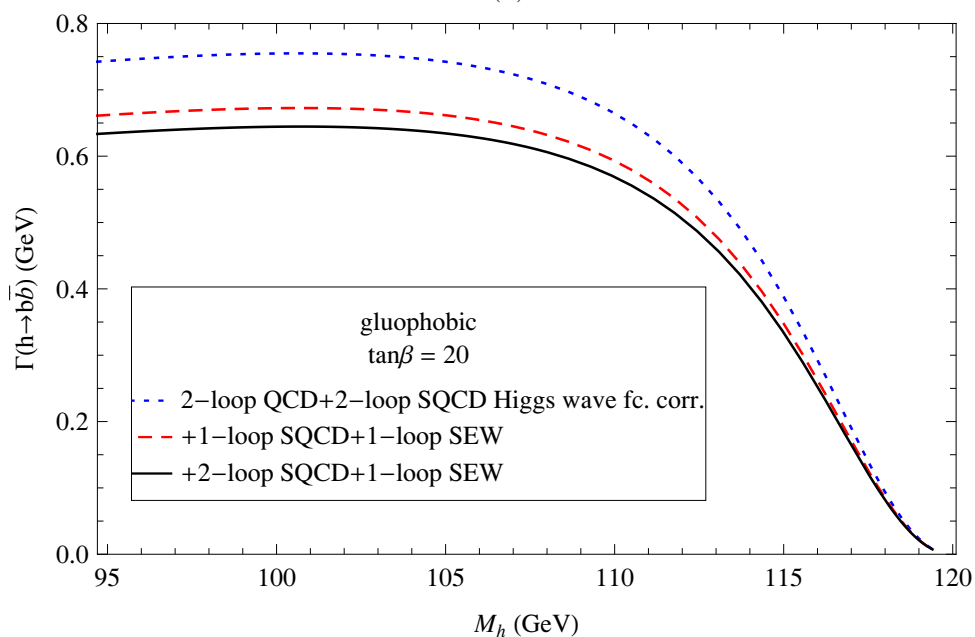


(b)

Figure 4. The coefficient function C_{2q} as a function of $\tan\beta$ for $M_A = 130$ GeV in (a) the “small α_{eff} ” and (b) the “gluophobic” scenario.



(a)



(b)

Figure 5. $\Gamma(h \rightarrow b\bar{b})$ for (a) the “small α_{eff} ” and (b) “gluophobic” scenario as a function of M_h . The dotted lines display the two-loop QCD and EW corrections together with two-loop corrections to the Higgs boson propagator. The dashed and solid lines depict in addition the one- and two-loop SUSY-QCD vertex corrections, respectively.

5 Conclusions

The knowledge of the Higgs boson couplings is essential for its searches at the present hadron colliders. In this paper we calculate the two-loop corrections of $\mathcal{O}(\alpha_s^2)$ to the Yukawa couplings within the MSSM. We employed the effective Lagrangian approach under the assumption of large top quark and supersymmetric particle masses. We calculate analytically the two-loop corrections to the coefficient function C_{2q} , taking into account the complete mass dependence. For large values of $\tan\beta$ the radiative corrections need to be resummed, which in our approach is performed through the use of the formula given in eq. (3.4). Furthermore, we verified at the diagram level the applicability of the low-energy theorem for Higgs interactions in the framework of the MSSM as stated in ref. [38].

From the phenomenological point of view, the two-loop corrections presented here reduce significantly the theoretical uncertainties, estimated through the variation with the renormalization scale, at the percent level. The two-loop SQCD corrections become sizable for $\tan\beta \geq 20$ and $M_A \leq 130$ GeV.

Acknowledgments

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