

Scale symmetry breaking and generation of mass at quantum critical points

Charlie Cresswell-Hogg  and Daniel F. Litim 

*Department of Physics and Astronomy, University of Sussex,
Falmer Campus, Brighton, BN1 9QH, U.K.*

E-mail: c.cresswell-hogg@sussex.ac.uk, d.litim@sussex.ac.uk

ABSTRACT: We study an asymptotically free theory of N relativistic Dirac fermions and a real scalar field coupled by Yukawa and scalar self-interactions in three dimensions using functional renormalisation. In the limit of many fermion flavours, the cubic scalar coupling becomes exactly marginal due to quantum fluctuations, leading to a line of strongly-coupled infrared fixed points. Fermion mass can be generated through a quantum phase transition even if chiral symmetry is absent. The line of fixed points terminates at a critical endpoint due to the loss of vacuum stability. Exactly at the endpoint, scale symmetry is broken spontaneously, leading to the generation of fermion mass. Intriguingly, the absence of chiral symmetry is a prerequisite for the spontaneous generation of fermion mass, and not a consequence thereof. We also highlight close similarities between Gross-Neveu and Gross-Neveu-Yukawa theories at and away from critical points, and establish the large- N equivalence of their functional RG flows and quantum effective actions. Further implications including for conformal field theories are indicated.

KEYWORDS: Field Theories in Lower Dimensions, Nonperturbative Effects, Renormalization Group, Scale and Conformal Symmetries

ARXIV EPRINT: [2311.16246](https://arxiv.org/abs/2311.16246)

Contents

1	Introduction	1
2	Gross-Neveu-Yukawa theories	2
3	Renormalisation group	3
4	Fixed points	5
5	Scaling dimensions	8
6	Phase diagram	9
7	Global fixed points	11
8	Spontaneously broken scale invariance and fermion mass	12
9	Duality with Gross-Neveu theory	14
10	Discussion and conclusions	18

1 Introduction

Fixed points of the renormalisation group (RG) play an important role in quantum and statistical field theory. Ultraviolet (UV) fixed points are key for a fundamental definition of quantum field theory, while infrared (IR) fixed points relate to continuous quantum phase transitions or chiral symmetry breaking. Moreover, scale symmetry at quantum critical points often entails full conformal invariance [1], and enforces that quantum critical theories are massless.

Interesting new effects occur if quantum scale symmetry at critical points is broken spontaneously. In particular, a dilaton arises alongside mass scales which are not determined by the fundamental parameters of the theory. In three dimensions, the classic example is scalar $(\phi^2)_{3d}^3$ theory at large N [2–5], where scale symmetry is broken spontaneously at the endpoint of a line of interacting ultraviolet (UV) fixed points owing to a non-analyticity of the quantum critical potential. Subsequently, scale symmetry breaking has been observed in more complex scalar [6], supersymmetric [7–9] and Chern-Simons theories coupled to matter [10, 11]. More recently, the phenomenon has been observed at interacting UV fixed points in perturbatively non-renormalisable fermionic $(\bar{\psi}\psi)_{3d}^3$ theories [12, 13]. An unexpected feature is that even though the dynamical breaking of chiral symmetry switches on a fermion mass and chirally odd interactions, the presence of chirally odd interactions alone does not entail fermion mass [12, 13]. Finally, in four dimensions, it has been speculated that spontaneous scale symmetry breaking may explain the Higgs as a “light dilaton” in extensions

of the Standard Model [14–16]. This type of mechanism may equally arise in UV conformal theories [17–21] which serve as templates for model building [22–25].

In this work, we investigate the phenomenon of spontaneous scale symmetry breaking and the generation of fermion mass in Gross-Neveu-Yukawa theories (GNY for short). They are asymptotically free and perturbatively renormalisable, and may develop critical points in the IR. In condensed matter physics, GNY models often arise as effective theories of massless Dirac fermion excitations in honeycomb lattice models [26–30] or topological insulators [28, 31], and are expected to belong to the same universality class as graphene [26, 27, 29, 30, 32–35]. Further, in the spirit of bosonisation, GNY theories are closely related to Gross-Neveu theories (GN for short) [36–45] and offer a testing ground for spontaneous scale symmetry breaking or the naturalness of fermion mass [12, 13].

With these considerations in mind, we identify the phase structure of GNY theories including all critical points, clarify how fermion mass is generated dynamically in the absence of chiral symmetry, and whether spontaneous scale symmetry breaking may occur at IR critical points. The main new addition of our study are chirally odd cubic scalar self-interactions, mimicking the role played by six-fermion interactions in GN theories. To achieve our goals, we employ functional renormalisation in the local potential approximation, which becomes exact in the limit of many fermion flavours. We also clarify in concrete terms the large- N duality between GN and GNY theories, both along their functional RG flows, and on the level of quantum effective actions.

The paper is organised as follows. After introducing GNY models and some of their key specifics (section 2), we provide their functional RG equations to be used throughout (section 3). We then identify interacting fixed points (section 4), scaling dimensions (section 5), and the phase diagram including UV-IR connecting trajectories (section 6). We also find the global fixed point potentials (section 7) to establish the spontaneous breaking of scale symmetry and the generation of fermion mass (section 8). We continue with an analysis of the large- N relationship between GNY and GN theories (section 9), and close with a discussion and conclusions (section 10).

2 Gross-Neveu-Yukawa theories

We are interested in 3d euclidean quantum field theories of the GNY type, featuring N flavours of four-component Dirac fermions ψ_a and a single real scalar field ϕ , with classical actions of the form

$$S_{\text{GNY}} = \int_x \left\{ \bar{\psi}_a \not{\partial} \psi_a + \frac{1}{2} (\partial\phi)^2 + H\phi \bar{\psi}_a \psi_a + U(\phi) \right\}. \quad (2.1)$$

Interactions in these theories are parametrised by the Yukawa coupling H and the scalar potential U . In addition to possessing a global $U(N)$ flavour symmetry,¹ the theories (2.1)

¹With four-component spinors, the flavour symmetry is actually $U(N) \times U(N)$ [46].

with four-component fermions may also be invariant under the discrete “chiral” symmetry

$$\begin{aligned}\psi_a &\mapsto \gamma^5 \psi_a, \\ \bar{\psi}_a &\mapsto -\bar{\psi}_a \gamma^5, \\ \phi &\mapsto -\phi,\end{aligned}\tag{2.2}$$

inherited from the four-component representation of the Clifford algebra, provided that the interaction potential $U(\phi)$ is an even function.

Incidentally, a similar role is played by parity transformations, $x = (x^0, x^1, x^2) \mapsto x' = (x^0, -x^1, x^2)$, with²

$$\begin{aligned}\psi_a(x) &\mapsto \gamma^1 \psi_a(x'), \\ \bar{\psi}_a(x) &\mapsto -\bar{\psi}_a(x') \gamma^1, \\ \phi(x) &\mapsto -\phi(x'),\end{aligned}\tag{2.3}$$

and ϕ a pseudoscalar. This symmetry also holds in the case of two-component fermions, where the (euclidean) gamma matrices can be taken to be the Pauli matrices [40, 47]. Imposing invariance under either chiral (2.2) or parity (2.3) transformations forbids any odd powers of ϕ in the potential as well as odd powers of $\bar{\psi}\psi$, such as a fermion mass term. In a slight abuse of language, we will use parity and chiral symmetry interchangeably.

The GNY theory (2.1) can also be viewed as a bosonised version of the GN theory with relevant four-fermion (4F) interactions $\sim \lambda_{4F}(\bar{\psi}_a \psi_a)^2$, whereby the scalar field $H\phi$ plays the role of $\bar{\psi}_a \psi_a$ [36–45]. We postpone a detailed analysis of their interrelations at large- N until section 9.

3 Renormalisation group

In this work, we are interested in theories (2.1) where chiral or parity symmetry is absent due to interactions. It is important to clarify how this impacts on the critical points, the phase structure of the theory, and the generation of fermion mass, which is no longer protected by symmetry. To that end, we investigate the theories (2.1) using functional renormalisation [48–51].

Briefly, the method proceeds by adding a Wilsonian cutoff term to the path integral representation of the theory, bilinear in the fields, which acts to integrate out successive momentum modes of the fields from the UV to the IR. By a Legendre transform, this defines a coarse-grained effective action Γ_k , dependent on the RG scale k . It interpolates between a classical action S at some UV scale $k = \Lambda$ and the full quantum effective action Γ , the generating functional of one-particle-irreducible correlation functions, obtained in the IR limit where all fluctuations are integrated out ($k \rightarrow 0$). The scale dependence of Γ_k is governed by an exact functional identity [49]

$$\partial_t \Gamma_k = \frac{1}{2} \text{STr} \left\{ [\Gamma_k^{(2)} + R_k]^{-1} \cdot \partial_t R_k \right\}\tag{3.1}$$

²Here we give the explicit form of the transformation in euclidean signature, where γ^1 is hermitian and squares to the identity. For lorentzian signature, the transformation rule for $\bar{\psi}$ has the opposite sign.

which derives from the regulated partition function, with $t = \ln(k/\Lambda)$. The right hand side of this equation features a functional trace in position or momentum space, as well as a trace over all internal indices. The quantity $\Gamma_k^{(2)} + R_k$ stands for the exact inverse propagator of the regulated theory and includes the cutoff function $R_k(q)$, which provides IR regularisation of the functional trace. UV regularisation is provided by the insertion of the scale derivative of the cutoff function, which vanishes rapidly for large momenta q . We use optimised cutoffs [52–55] for both the bosonic and fermionic regulators. Other choices of cutoff constitute different RG schemes which alters non-universal aspects of RG flows but has no impact on the physics.

Our study is based upon an ansatz for the scale dependent effective action $\Gamma_k = \Gamma_k[\phi, \psi, \bar{\psi}]$ of the form

$$\Gamma_k = \int_x \left\{ Z_\psi \bar{\psi}_a \not{\partial} \psi_a + \frac{1}{2} Z_\phi (\partial\phi)^2 + H_k \phi \bar{\psi}_a \psi_a + U_k(\phi) \right\}, \tag{3.2}$$

where $Z_{\psi,\phi}$ are scale dependent, but not field dependent, wavefunction renormalisation factors, H_k is the running Yukawa coupling, U_k the running scalar potential, and \int_x stands for the integral over 3d euclidean space. This ansatz with symmetric $U(\phi)$ has been widely studied in the literature [29, 44, 45, 56–61], as it contains all necessary ingredients to capture the physics of the theory’s interacting IR fixed point in the chirally symmetric setting, which is equivalent to the UV fixed point of the 3d GN model [39–42, 44, 45, 62, 63], and yields exact results for universal scaling dimensions in the large- N limit. As we will show, this equivalence also extends to the case of explicitly broken parity by including parity-odd interactions $\sim \phi^3$ in the scalar potential. Then, (3.2) accurately reproduces the line of fixed points found at large N in the GN model [12], including its endpoints where scale symmetry is spontaneously broken [13].

It is convenient to introduce dimensionless, renormalised fields and couplings scaled in units of the RG scale k as

$$\sigma = Z_\phi^{1/2} \phi / k^{1/2}, \tag{3.3}$$

$$h = (Z_\phi Z_\psi^2)^{-1/2} H_k / k^{1/2}, \tag{3.4}$$

$$u(\sigma) = U_k(\phi) / k^3. \tag{3.5}$$

We further scale the numerical factor $A = 3\pi^2/(2N)$ into the scalar field and the effective potential as $\sigma \rightarrow \sqrt{A}\sigma$ and $u \rightarrow Au$. This ensures that all scalar interactions are normalised in units of perturbative loop factors and powers in N suitable for a large- N limit.

In the infinite- N limit, the contributions of the scalar field fluctuations $\sim 1/N$ are suppressed, and the flow of the potential is driven entirely by the Yukawa interactions [45, 60]

$$\partial_t u = -3u + \frac{1}{2} (1 + \eta_\phi) \sigma \partial_\sigma u - \frac{1}{1 + (h\sigma)^2}. \tag{3.6}$$

While the anomalous dimension of the fermion field vanishes in the infinite- N limit, $\eta_\psi = 0$, the scalar field anomalous dimension, defined as $\eta_\phi = -\partial_t \ln Z_\phi|_{\sigma=0}$,³ can be non-trivial. Here, it takes the form

$$\eta_\phi = \frac{5}{2} h^2. \tag{3.7}$$

³We evaluate the scalar field anomalous dimension at the background value $\sigma = 0$, which is the location of the scalar potential’s minimum in all cases of interest.

In consequence, the Yukawa interaction evolves as

$$\partial_t h = -\frac{1}{2}(1 - \eta_\phi) h. \tag{3.8}$$

The RG flows (3.6) and (3.8) with (3.7) are the central equations controlling the model.

Let us briefly comment on the interplay of chiral symmetry and the generation of fermion mass $\sim M_F \bar{\psi}_a \psi_a$. In the presence of chiral symmetry (2.2), an explicit fermion mass term is forbidden. If chiral symmetry is absent, a bare fermion mass, or the generation of fermion mass by fluctuations, become a possibility. To check whether this can happen in practice, we project the functional flow (3.1) onto the fermion mass term. Expanding about vanishing scalar field, we find $\partial_t m_F = -m_F$, where $m_F = M_F/k$ is the dimensionless fermion mass. Evidently, fermion mass is a relevant perturbation. However, the large- N flow shows that the running is purely classical and fluctuations do not contribute to leading order in $1/N$. We may also expand the flow around the (dimensionless) minimum σ_0 of the potential where $u'(\sigma_0) = 0$, and $\lambda_2 = \partial_\sigma^2 u(\sigma_0)$ the scalar mass in units of k . This corresponds to the symmetric or symmetry broken phase if the dimensionful expectation value of the scalar field for $k \rightarrow 0$ takes a vanishing or finite value, respectively. We find

$$\partial_t m_F = -m_F \left(1 + \frac{2h^2}{(1 + m_F^2)^2 \lambda_2} \right). \tag{3.9}$$

We observe fluctuation-induced contributions to the flow, even though fermion mass remains natural in that the flow vanishes identically for vanishing mass. Most importantly, the results establish that fermion mass cannot be generated by fluctuations alone, even in the presence of chirally odd interactions. We conclude that full chiral or parity symmetry is not required to keep the fermions massless. Rather, the significantly milder constraint $m_F = 0$ is already sufficient.

4 Fixed points

In this section, we identify fixed points and scaling dimensions of the GNY theory in the presence of chirally odd interactions.

From the running Yukawa interactions (3.8), we note that fixed points are either free ($h_* = 0, \eta_\phi = 0$) or interacting ($h_* \neq 0$) with

$$\eta_\phi = 1. \tag{4.1}$$

The fixed point condition states that the scalar anomalous dimension must exactly cancel the canonical mass dimension of the Yukawa coupling. This entails that the mass dimension of the Yukawa coupling is transferred to the scalar field, which now scales anomalously with mass dimension unity rather than $\frac{1}{2}$. Together with (3.7), this also determines the critical Yukawa coupling as

$$h_*^2 = \frac{2}{5}. \tag{4.2}$$

Notice that the fixed point value depends on the RG scheme and the choice for the momentum cutoff function R_k , whereas the existence of the fixed point and its scaling dimension is

independent thereof. Also, the sign of h is physically irrelevant and we take $h \geq 0$ without loss of generality. Together with (3.7) and in terms of $h_\Lambda^2 = h^2(k = \Lambda)$, the Yukawa flow can be integrated explicitly to give

$$h^2(k) = h_\Lambda^2 \left[\frac{h_\Lambda^2}{h_*^2} + \frac{k}{\Lambda} \left(1 - \frac{h_\Lambda^2}{h_*^2} \right) \right]^{-1}. \quad (4.3)$$

The flow interpolates between the asymptotically free fixed point in the UV ($k \rightarrow \Lambda$) and the fixed point (4.2) in the IR ($k \rightarrow 0$). Provided that the initial Yukawa coupling takes values in the range

$$0 < h_\Lambda^2 < h_*^2, \quad (4.4)$$

the high scale can be removed ($\Lambda \rightarrow \infty$). The transition from the UV to the IR fixed point is characterised by the RG invariant cross-over scale

$$k_{\text{cr}} = \Lambda h_\Lambda^2 / h_*^2 \quad (4.5)$$

with $\Lambda \frac{d}{d\Lambda} k_{\text{cr}} = 0$, which arises from dimensional transmutation close to the UV fixed point ($h_\Lambda^2 \ll 1$). By extension, the anomalous dimension then interpolates between $\eta_\phi = 0$ in the UV and $\eta_\phi = 1$ in the IR.

In turn, provided the initial value of the Yukawa coupling h_Λ^2 exceeds its fixed point value h_*^2 , the running coupling invariably exhibits a Landau pole at the scale

$$k_L = \frac{\Lambda h_\Lambda^2}{h_\Lambda^2 - h_*^2}. \quad (4.6)$$

In this case, the flow is not connected to the free UV fixed point, the high scale Λ cannot be removed ($k_L \geq \Lambda$), and the theory is effective rather than fundamental.

Next, we turn our attention to the self-interactions of the scalar field. To this end, we expand the scalar potential in terms of n -scalar self-couplings λ_n at vanishing field

$$u(\sigma) = \sum_{n=0}^{\infty} \frac{1}{n!} \lambda_n \sigma^n, \quad (4.7)$$

allowing both even and odd interactions under the discrete symmetry (2.2). Inserting the series into the flow equation (3.6) yields RG equations for λ_n . For the dimensionless scalar mass term ($\lambda_2 \equiv m_s^2$) we find

$$\partial_t \lambda_2 = (-2 + \eta_\phi) \lambda_2 + 2h^2, \quad (4.8)$$

with the anomalous dimension η_ϕ given by (3.7). Notice that the scalar mass is not natural in that it will always be switched on by Yukawa interactions, regardless of its initial value. Integrating (4.8) together with (4.3), and using $\lambda_{2,\Lambda} = \lambda_2(k = \Lambda)$, we find

$$\lambda_2(k) = 2h^2(k) \left[1 + \left(\frac{\lambda_{2,\Lambda}}{2h_\Lambda^2} - 1 \right) \frac{\Lambda}{k} \right]. \quad (4.9)$$

Provided the UV initial conditions obey $\lambda_{2,\Lambda} > 2h_\Lambda^2$, the second term in (4.9) dominates in the IR and we conclude that the physical scalar mass $M_s^2 \equiv \lambda_2(k)k^{2-\eta_\phi}$ is positive and finite for $k \rightarrow 0$, giving a disordered phase with a vanishing vacuum expectation value. By the same token, $\lambda_{2,\Lambda} < 2h_\Lambda^2$ implies a massive, ordered phase with a non-vanishing vacuum expectation value. We conclude that the short-distance initial condition

$$\lambda_{2,\Lambda} = 2h_\Lambda^2 \tag{4.10}$$

defines the boundary between an ordered and disordered phase, which is also a necessary condition for a massless scalar field in the IR. We come back to this aspect in section 6.

Next, we turn to the chirally odd cubic scalar self-interaction λ_3 , whose flow is given by

$$\partial_t \lambda_3 = -\frac{3}{2}(1 - \eta_\phi)\lambda_3. \tag{4.11}$$

The flow for λ_3 is natural in that the coupling cannot be switched on if set to zero at any one scale. Further, the flow (4.11) vanishes identically as soon as $\eta_\phi = 1$, irrespective of the value for λ_3 . Stated differently, the Yukawa coupling taking an interacting fixed point, (4.1), invariably turns the cubic self-interaction λ_3 into an exactly marginal coupling. Thus, the canonical mass dimension $[\lambda_3] = \frac{3}{2}$ of the classically relevant interaction has been exactly marginalised quantum mechanically owing to strong fluctuations in the IR. This non-perturbative effect entails that the cubic coupling becomes a free parameter,

$$\lambda_3^* = \text{free parameter}, \tag{4.12}$$

characterising a line of interacting IR fixed points. Integrating the flow for the cubic coupling (4.11) and also using (4.3), we find

$$\lambda_3(k) = \lambda_{3,\Lambda} h^3(k)/h_\Lambda^3, \tag{4.13}$$

where $\lambda_{3,\Lambda} = \lambda_3(k = \Lambda)$. From (4.13), we observe that the exactly marginal parameter (4.12) at the IR fixed point can be traced back to initial conditions at the high scale as $\lambda_3^* = \lambda_{3,\Lambda} h_*^3/h_\Lambda^3$.

Turning to the quartic and higher order scalar self-interaction, we find

$$\partial_t \lambda_n = \left[-3 + \frac{n}{2}(1 + \eta_\phi)\right] \lambda_n - a_n h^n, \tag{4.14}$$

with coefficients $a_{n=\text{even}} = (-1)^{n/2} n!$ and $a_{n=\text{odd}} = 0$. It follows that all chirally odd self-interactions $\lambda_{n=\text{odd}}$ are natural and admit free fixed points $\lambda_{n=\text{odd}}^* = 0$, irrespective of what happens in the Yukawa sector, though with the exception of the cubic interaction which may become exactly marginal provided $h_* \neq 0$. On the other hand, all chirally even interactions $\lambda_{n=\text{even}}$ take free fixed points provided that $h_* = 0$, and non-trivial ones with

$$\lambda_n^* = \frac{a_n h_*^n}{n - 3}, \tag{4.15}$$

provided $h_* \neq 0$. Altogether, we conclude that the theory admits two types of fixed points, the Gaussian fixed point ($h_* = 0, \lambda_n^* = 0$) where all couplings vanish identically, and a line of interacting IR fixed points characterised by (4.2), (4.12) and (4.15). Notice also that only the cubic scalar self-interaction $\lambda_3^* \neq 0$ breaks the discrete symmetry (2.2) at the interacting fixed point.

5 Scaling dimensions

Next, we turn to the universal scaling dimensions. In the vicinity of a fixed point, the RG flow is characterised by a set of RG eigenoperators $\mathcal{O}_n \sim k^{3+\vartheta_n}$, which scale with universal exponents ϑ_n . Positive exponents are associated to irrelevant operators, whose contributions shrink towards the IR and do not affect the long distance physics, while negative exponents indicate relevant operators. At a UV fixed point, the latter are in one-to-one correspondence with the fundamentally free parameters of the theory.

At the asymptotically free UV fixed point, basic scaling operators are given by field monomials such as ϕ^n , $(\bar{\psi}\psi)^m$, and combinations $\phi^n(\bar{\psi}\psi)^m$ thereof. Scaling exponents are mostly determined by canonical power counting. Relevant perturbations are identified as the mass terms, the Yukawa interaction $\sim h\phi\bar{\psi}\psi$ with exponent

$$\vartheta_h^{\text{UV}} = -\frac{1}{2}, \tag{5.1}$$

and scalar interactions $\sim \lambda_n\phi^n$ with exponents

$$\vartheta_n^{\text{UV}} = -3 + \frac{n}{2}. \tag{5.2}$$

Of these, all interactions with $n \leq 5$ are relevant. The exponent $\vartheta_6 = 0$ relates to the classically marginal sextic interaction ϕ^6 . All higher order scalar self-interactions with $n > 6$ are irrelevant with $\vartheta_n^{\text{UV}} > 0$. Our results for UV scaling exponents of scalar field monomials are illustrated in figure 1 (magenta dots).

At the interacting IR fixed points, quantum fluctuations and strong coupling effects modify the scaling behaviour. Eigenoperators are found by linearising the flow around the fixed point. We find that the Yukawa interaction has become irrelevant with

$$\vartheta_h^{\text{IR}} = 1. \tag{5.3}$$

The scaling of the scalar self-interactions is modified by the anomalous dimension, giving

$$\vartheta_n^{\text{IR}} = -3 + n. \tag{5.4}$$

Results confirm that ϕ^3 interactions have become exactly marginal, (4.12), with λ_3 parametrising the line of IR fixed points. Overall, only the scalar and fermion mass terms remain relevant perturbations in the IR. Our results for IR scaling exponents of scalar field monomials are illustrated in figure 1 (red dots). We observe that quantum effects have induced large corrections to scaling exponents dictated by (4.1).

Switching on any of the relevant perturbations in the UV initiates a flow towards the IR. The set of RG trajectories running out of the UV fixed point is then characterised by small deviations $\delta\alpha(\Lambda) = \alpha(\Lambda) - \alpha_*$ from the (free) fixed point for any of the relevant or marginal couplings $\alpha \in \{m_F, \lambda_2 \equiv m_s^2, \lambda_3, \lambda_4, h, \lambda_5, \lambda_6\}$ at the high scale Λ . Below, we establish in (6.2) that a tuned combination of Yukawa and scalar mass perturbations guarantee that the interacting IR fixed points are reached.

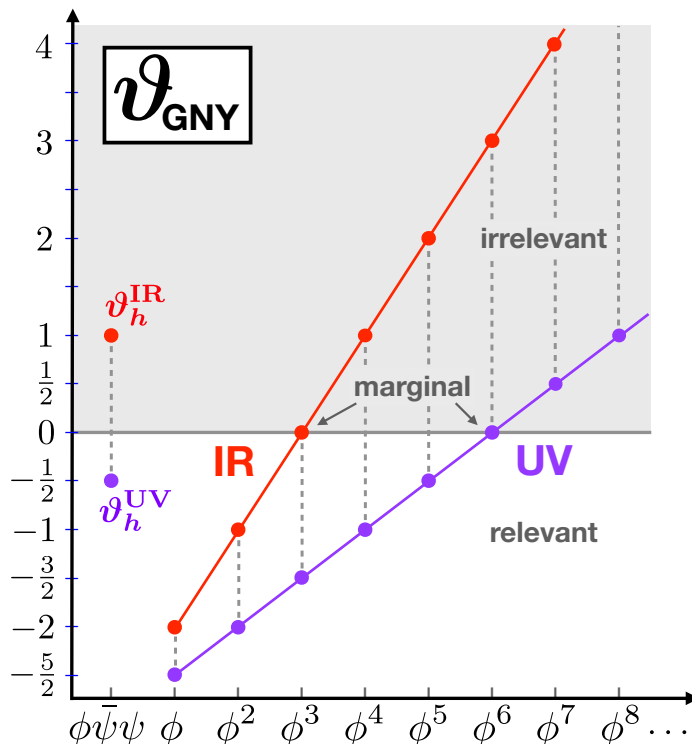


Figure 1. Shown are universal scaling exponents of the GNY theory at the UV and IR fixed points, and their association to field monomials (solid lines are added to guide the eye). Scaling exponents are shifted upwards in the IR due to quantum effects dictated by (4.1).

6 Phase diagram

In the UV, the theory displays an asymptotically free Gaussian fixed point where relevant perturbations trigger RG flows towards the IR. For these, specific choices need to be made in order to reach the line of interacting fixed points in the IR. A central role is played by the Yukawa interaction, whose RG flow is given by (4.3).

Next, we identify the UV-IR connecting separatrices. For that, the fermion mass does not play any role as it can safely be set to zero in the UV. However, since the scalar mass term $m_s^2 \equiv \lambda_2$ is relevant at the IR fixed points, its UV initial value must be tuned precisely to guarantee that the condition $\lambda_2^* = 2h_*^2$ is reached in the IR, see (4.9), (4.10). The condition for this to happen can also be understood by considering the ratio of couplings $X = (2h^2)/\lambda_2$. Using (3.8) and (4.8) we find

$$\partial_t X = X(1 - X). \tag{6.1}$$

The flow has a free IR fixed point $X_* = 0$ and an interacting UV fixed point $X_* = 1$, independent of η_ϕ . Therefore, the condition $\lambda_2 \equiv 2h^2$ can only be achieved in the IR limit provided that it already holds true for all scales. This demonstrates that the relation

$$\lambda_2(k) = 2h^2(k) \tag{6.2}$$

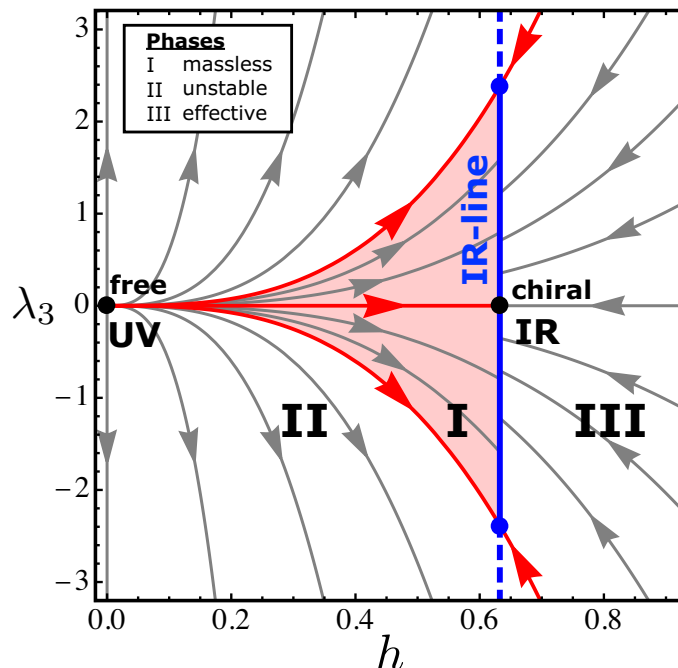


Figure 2. Phase diagram of the GNY theory in terms of the Yukawa h and cubic coupling λ_3 , and projected onto the hypersurface (6.2). We observe the free UV fixed point (black), a line of interacting IR fixed points with stable (blue), or unstable ground states (dashed blue), UV-IR connecting separatrices (red) and sample trajectories (gray). In region I (region III), we observe massless fundamental (effective) theories which (do not) originate out of an UV fixed point. In region II, theories are driven into vacuum instability; arrows point from the UV to the IR.

between the scalar mass and the Yukawa coupling identifies all separatrices connecting the UV and IR fixed points, for all scales $k \leq \Lambda$, in accord with (4.9), (4.10). Accordingly, the required initial condition for relevant perturbations in the UV read $\delta\lambda_2(\Lambda) = 2\delta h^2(\Lambda)$.

By the same token, UV initial conditions of the form $\delta\lambda_2(\Lambda) = 2\delta h^2(\Lambda) + \epsilon(\Lambda)$ which deviate from (6.2), cannot reach the line of massless IR fixed points. Instead, we find different types of massive theories in the IR, depending on the sign of ϵ . For $\epsilon(\Lambda) > 0$, the IR theory contains a massive scalar, whereas the fermions remain massless. For $\epsilon(\Lambda) < 0$, the scalar potential develops a nontrivial minimum, implying that the fermions also become massive due to Yukawa interactions. This pattern holds true regardless of whether or not chiral symmetry is explicitly broken by cubic scalar interactions.

We are now in a position to discuss the phase diagram of the theory with cubic interactions. The flow of the scalar mass does not influence this coupling, but we assume for argument's sake that m_s^2 has been tuned to lie on the separatrix trajectory. Figure 2 depicts the RG flow in the (h, λ_3) plane. Selected RG trajectories are shown, with arrows pointing from UV to IR. Highlighted on the left of the diagram is the UV free fixed point, which is connected via RG trajectories to the IR line of interacting fixed points, highlighted in blue on the right of the diagram. At the centre of the IR line, with $\lambda_3 = 0$, the chirally symmetric fixed point is marked “chiral”. Endpoints are marked on the line to show the range of cubic coupling values in the IR for which the scalar potential is bounded from below.

The phase diagram is divided into three regions. Region I consists of well-defined asymptotically free trajectories which flow towards interacting conformal theories with stable vacua in the IR. For these theories, all fields remain massless at all scales, despite the fact that chiral symmetry is explicitly broken by the cubic interaction. Region II also comprises asymptotically free trajectories, except that these are unphysical owing to an unstable vacuum in the IR limit. Region III consists of well-defined effective theories whose running couplings are not UV complete because they exhibit UV Landau poles. Outside the (h, λ_3) plane, mass can be generated either explicitly, or dynamically via a non-trivial expectation value for the scalar field, or spontaneously, via the breaking of quantum scale symmetry.

7 Global fixed points

In this section, we discuss the global form of the scalar potential $u_*(\sigma)$ at the IR fixed points of the flow equation (3.6) for all values of the field σ . The results we present in this section arise as a special case of those in appendix B of [60] and agree with [45] in the chiral limit. Setting the Yukawa coupling h to its IR fixed point value (4.2) where $\eta_\phi = 1$, and solving the fixed point condition $\partial_t u = 0$ for the entire scalar potential, we find

$$u_*(\sigma) = \frac{1}{6} \lambda_3 \sigma^3 + F(h_* \sigma), \tag{7.1}$$

with $F(x) = -\frac{1}{3} + x^2 + x^3 \arctan(x)$ an even function in x , and λ_3 the value of the exactly marginal cubic scalar coupling at the fixed point. Expanding (7.1) in powers of σ , we recover all couplings λ_n^* determined previously from (4.14), (4.15). Given that all parity-odd couplings bar λ_3 vanish at the fixed point, $\lambda_{n=\text{odd} \neq 3} = 0$, it follows that the non-perturbative function F resums all parity-even polynomial couplings.

Because F is an even function, and because the Yukawa interaction is invariant under (2.2), the fixed point theory represented by (7.1) with $\lambda_3 = 0$ is parity symmetric. For any $\lambda_3 \neq 0$, however, parity symmetry is explicitly broken by the appearance of a cubic term in the scalar potential. Note that the cubic coupling has mass dimension $\frac{3}{2}$ at the classical level. Its exact marginality in the IR originates from the anomalous scaling of the scalar field, which effectively acquires a mass dimension of unity at the interacting fixed points.

We emphasize that the cubic interaction term in the potential may destabilise the theory. This is so because the fluctuation-induced function $F(x)$ in (7.1) behaves asymptotically as $|x|^3$ for large arguments. As a result, the parity-odd and parity-even terms in (7.1) compete and the effective potential may become unbounded from below if the parameter λ_3 becomes large enough. More explicitly, for large fields, the potential behaves as

$$u_*(\sigma) \sim \frac{1}{6} [\lambda_3 + \lambda_{3,\text{crit}} \text{sgn}(\sigma)] \sigma^3, \tag{7.2}$$

where we have introduced the critical parameter

$$\lambda_{3,\text{crit}} \equiv 3\pi h_*^3. \tag{7.3}$$

Notice that the contribution $\sim \lambda_3 \sigma^3$ in (7.2) arises from the parity-odd cubic interaction whereas the contribution $\sim \lambda_{3,\text{crit}} \text{sgn}(\sigma) \sigma^3$ arises from the sum of all polynomial and parity-even scalar self interactions. The effective potential found in a mean field analysis of the

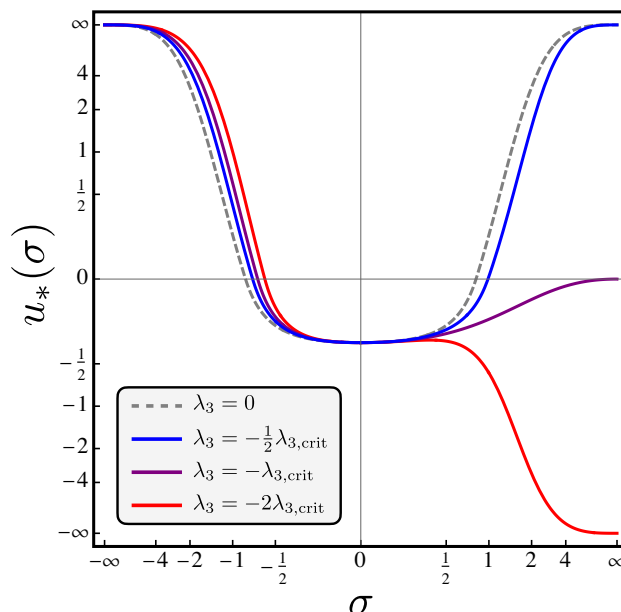


Figure 3. Shown are dimensionless global potentials at IR critical points (7.1) for different values of λ_3 , covering the parity-even potential ($\lambda_3 = 0$, dashed grey curve), and fixed points with $\lambda_3 < 0$ including a stable potential (solid blue), an asymptotically vanishing potential (purple), and an unbounded potential (red). Examples with $\lambda_3 \rightarrow -\lambda_3$ are obtained by reflection about the vertical axis. Axes are scaled as $X \rightarrow X/(1 + |X|)$ for better display.

chirally symmetric theory has such a form [58, 64, 65]. From the explicit expression (7.2), we conclude that the potential is bounded from below provided that

$$|\lambda_3| \leq \lambda_{3,\text{crit}} , \tag{7.4}$$

as indicated in figure 2, but otherwise unbounded for large positive ($\lambda_3 < -\lambda_{3,\text{crit}}$) or negative ($\lambda_3 > \lambda_{3,\text{crit}}$) fields, respectively. In the special case $|\lambda_3| = \lambda_{3,\text{crit}}$, parity-even and parity-odd contributions cancel out exactly for asymptotically large positive (negative) fields.

Results are illustrated in figure 3, showing the fixed point potential for the parity-symmetric theory ($\lambda_3 = 0$, dashed grey), as well as a (bounded) parity-asymmetric potential ($\lambda_3 = -\frac{1}{2}\lambda_{3,\text{crit}}$, blue), an unbounded potential ($\lambda_3 = -2\lambda_{3,\text{crit}}$, red), and an asymptotically flat potential ($\lambda_3 = -\lambda_{3,\text{crit}}$, purple). By symmetry, and given that F in (7.1) is even, it follows that critical potentials with parameter $\lambda_3 \leftrightarrow -\lambda_3$ relate to a reflection in field space $\sigma \leftrightarrow -\sigma$. Lastly, we emphasize that the loss of vacuum stability beyond the bound (7.3), (7.4) is a non-perturbative effect. It required the precise knowledge of the potential at asymptotically large fields, (7.2), for which any finite order polynomial approximation of (7.1) would not have been sufficient.

8 Spontaneously broken scale invariance and fermion mass

The critical theories discussed in the previous sections, characterised by the value of the exactly marginal cubic coupling, are all scale invariant at the quantum level. Provided that

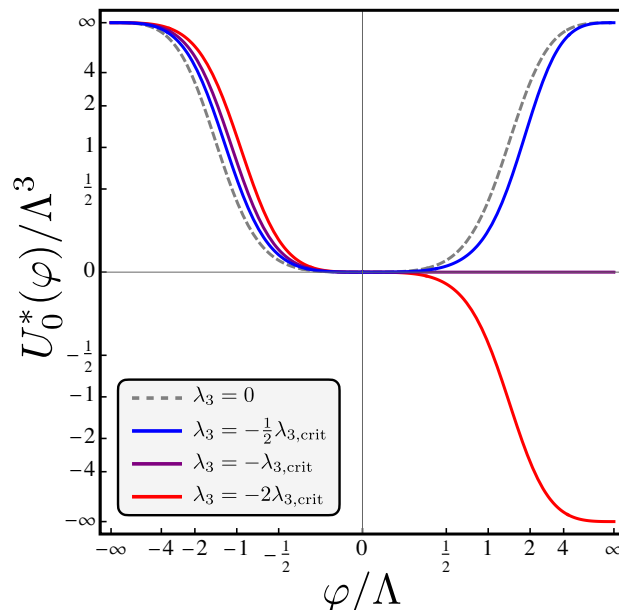


Figure 4. Shown are dimensionful fixed point potentials $U_*(\varphi)$ at $k = 0$ in units of the high scale Λ and for different λ_3 , covering the parity symmetric case ($\lambda_3 = 0$, dashed grey curve), and examples of fixed point potentials with $\lambda_3 < 0$ with a stable vacuum (solid blue), flat potential and spontaneously broken scale symmetry (purple), and an unbounded potential (red). Potentials are stable for $|\lambda_3| \leq \lambda_{3,\text{crit}}$; axes are scaled as in figure 3.

the cubic coupling is not too large in magnitude, $|\lambda_3| \leq \lambda_{3,\text{crit}}$, these theories also possess stable ground states. In this section, we discuss how fixed points at the borderline of vacuum stability ($|\lambda_3| = \lambda_{3,\text{crit}}$) give rise to the spontaneous breaking of quantum scale symmetry and the generation of fermion mass.

To make this explicit, we recast fixed point potentials (7.1) in terms of the dimensionful potential and fields. Recall that the scalar field scales anomalously at the IR fixed point, with $\eta_\phi|_* = -\partial_t \ln Z_\phi|_* = 1$. The anomalous scaling changes the mass dimension of the scalar field by $\frac{1}{2}$, corresponding to $Z_\phi|_* = k_0/k$ with k_0 a suitable reference scale. Therefore, and in view of (3.3), it is convenient to introduce a new field φ with canonical mass dimension one, $\varphi = \sqrt{k_0} \phi \equiv k \sigma$, and to express the dimensionful fixed point potentials as $U_k^*(\varphi) = k^3 u_*(\varphi/k)$. In the limit where all fluctuations are integrated out ($k \rightarrow 0$) we find the full quantum effective potential as

$$U_0^*(\varphi) = \frac{1}{6} [\lambda_3 + \lambda_{3,\text{crit}} \text{sgn}(\varphi)] \varphi^3. \quad (8.1)$$

We emphasize that the theory sits precisely at the IR fixed point. From the explicit result, it follows that the potential is globally unstable outside the range (7.4).

Figure 4 illustrates the fixed point potentials in (8.1) for several values of the free parameter λ_3 (corresponding to their dimensionless counterparts in figure 3). Most notably, spontaneous breaking of scale invariance is evidenced in the flatness of the critical potentials with $|\lambda_3| = \lambda_{3,\text{crit}}$. Here, the minima are degenerate, covering the entire positive (negative) field axis provided $\lambda_3 = -\lambda_{3,\text{crit}}$ ($\lambda_3 = \lambda_{3,\text{crit}}$). Using (8.1), the vacuum expectation value

is determined from the condition

$$[\lambda_3 + \lambda_{3,\text{crit}} \text{sgn}(\varphi_0)] \varphi_0^2 = 0, \tag{8.2}$$

valid in the range (7.4). It establishes that the vacuum expectation value of the scalar field either vanishes identically as long as $|\lambda_3| < \lambda_{3,\text{crit}}$, or, provided $|\lambda_3| = \lambda_{3,\text{crit}}$, turns into a free parameter of the theory,

$$|\varphi_0| = \text{free parameter} \tag{8.3}$$

up to its sign which is determined by the sign of λ_3 . Most importantly, a non-trivial vacuum expectation value (8.3) spontaneously introduces a mass scale despite of the fact that the theory is tuned to a quantum critical point. The fermions thereby acquire a mass

$$M_\psi = h_* \varphi_0 \tag{8.4}$$

owing to the Yukawa interactions, while the scalar remains strictly massless owing to the flatness of the critical potential, irrespective of φ_0 . Thus, quantum scale symmetry is broken spontaneously, leading to the generation of fermion mass. The breaking of scale symmetry also entails a dilaton, which is expected to be massless at large N .

As a final comment, we stress that for the above mechanism to be operative, the absence of parity symmetry is a prerequisite and not a consequence. Therefore, fermion mass is generated at a critical point, spontaneously, without the breaking of a symmetry other than scale symmetry itself.

9 Duality with Gross-Neveu theory

It is often argued that the IR fixed point of the chirally symmetric and perturbatively renormalisable GNY theory can be viewed as the UV completion of the perturbatively non-renormalisable GN theory with fundamental 4F interactions [36, 39–42, 44]. In this section, we highlight similarities between GNY (2.1) and GN theories [12, 13] beyond chiral symmetry, at and away from critical points, and establish the large- N equivalence of RG flows and quantum effective actions.

The action for the 3d GN theory of interacting fermions ψ_a can be written as

$$S_{\text{GN}} = \int_x \left\{ \bar{\psi}_a \not{\partial} \psi_a + V(\bar{\psi}_a \psi_a) \right\}. \tag{9.1}$$

The function $V(z)$ (with $z = \bar{\psi}_a \psi_a$) contains interaction monomials $\sim \lambda_{2n\text{F}} z^n$. The fermion anomalous dimension can be neglected in the large- N limit of many fermion flavours adopted here. The theory is chirally symmetric if all $\lambda_{2n\text{F}} = 0$ for $n = \text{odd}$.

The theory has been studied using functional renormalisation (3.1) in [12, 13, 62]. Even though the theory (9.1) is non-renormalisable in perturbation theory, it is non-perturbatively renormalisable and predictive up to highest energies, courtesy of an interacting UV fixed point [36, 39, 41, 45, 63, 64, 66–69]. Thereby, the classically irrelevant 4F coupling $\lambda_{4\text{F}}$ becomes relevant in the UV due to strong quantum effects. At the same time, the parity-odd $\lambda_{6\text{F}}$ becomes exactly marginal, while all other couplings $\lambda_{n\text{F}}$ with $n > 6$ remain irrelevant [12].

Consequently, the theory displays a line of UV fixed points characterised by the exactly marginal coupling λ_{6F} . The critical line is found to be finite,

$$|\lambda_{6F}| \leq \lambda_{6F,\text{crit}}, \tag{9.2}$$

because a well-defined quantum effective action ceases to exist for large sextic coupling [12, 13]. At the endpoint, quantum scale symmetry is broken spontaneously, leading to the generation of a fermion mass and a massless dilaton [13]. The critical 6F coupling reads

$$\lambda_{6F,\text{crit}} = 3\pi/8, \tag{9.3}$$

if we adopt the same RG scheme and fermionic regulator function as in this work [52, 53, 55].

In figure 5, we show the universal scaling dimensions associated to interaction monomials $\sim z^n$ at the interacting UV fixed point [12],

$$\vartheta_n^{\text{GN}} = -3 + n. \tag{9.4}$$

Notice that they differ substantially from the scaling dimensions $\vartheta_n^{\text{GN}}|_{\text{IR}} = -3 + 2n$ at the free IR fixed point. The infinite set of scaling dimensions of the GN fixed point (figure 5) agrees with those of scalar field monomials at the GNY fixed point (figure 1, IR line). The equivalence also persists in the presence of parity-odd interactions. We may associate the scaling dimensions (5.4) and the corresponding scaling fields in the GNY theory to the scaling dimensions (9.4) and the corresponding scaling operators in the GN theory.

Away from critical points, a common feature is that fermion mass is not generated explicitly by fluctuations, even though chiral symmetry is fundamentally absent. Instead, mass generation remains an exclusively dynamical mechanism, tied to the existence of quantum critical points, thus allowing the generation of mass without the (spontaneous) breaking of a symmetry.

Further, the role played by the exactly marginal 6F coupling in the GN theory, (9.2), is taken over by the exactly marginal cubic scalar coupling in the GNY theory, (7.4). In either case, the line of fixed points defined by the marginal coupling is finite, and characterised by a critical endpoint where quantum scale symmetry is broken spontaneously.

Given that the IR fixed points of the GNY theory (2.1) and the UV fixed points of the GN theory (9.1) display equivalent conformal scaling dimensions, it strengthens the view that the former can be interpreted as the UV completion of the latter even beyond the chirally symmetric setting. At infinite N , the GN theory is characterised by all polynomial interactions $\sim z^n$, while the GNY theory is characterised by all scalar field monomials $\sim \phi^n$, in either case scaling according to (9.4). However, in addition, the GNY theory also displays the Yukawa interaction, which, at the IR fixed point, is irrelevant with scaling exponent (5.3) (see figure 1). This scaling exponent is part of the conformal data of the critical GNY theory, yet it appears to have no direct counterpart in the critical GN theory.

This can be understood by noting that, although the Yukawa coupling is the crucial ingredient responsible for generating non-trivial IR scaling behaviour in the GNY model, any explicit dependence on H in the IR limit can actually be removed by rescaling the field $\phi \rightarrow H\phi$. From the point of view of bosonisation techniques, it is only the ratio h^2/λ_2 ,

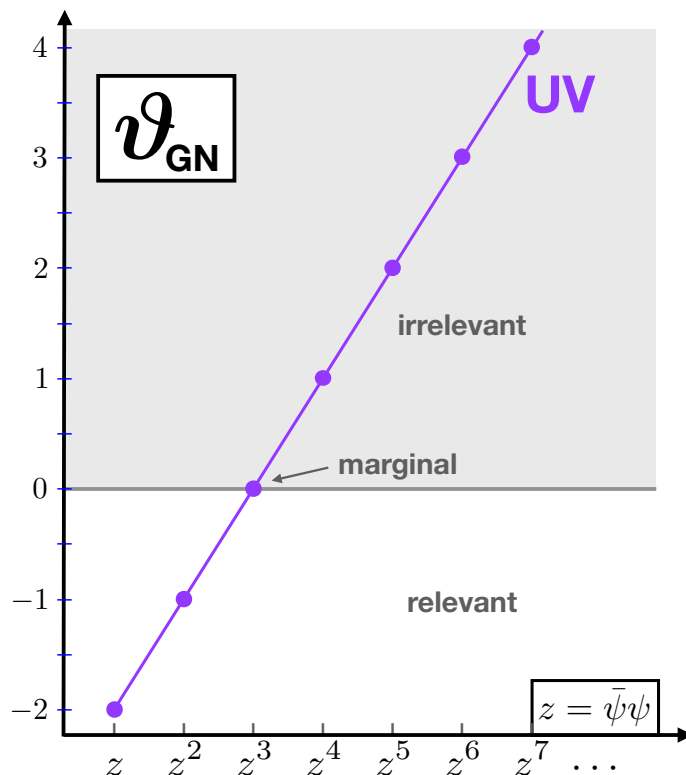


Figure 5. Shown are the universal scaling exponents at the non-chiral GN fixed points, and their association to field monomials (lines are added to guide the eye). Notice that the scaling dimensions agree with those at the interacting GNY fixed point (see figure 1).

which is proportional to the 4F coupling, that has physical meaning from the point of view of the GN model [36–45].

Next, we discuss the equivalence of GN and GNY theories on the level of effective actions. At large- N , it has been noted that they are related by a functional Legendre transformation [43], whereby the quantity $H\phi$ becomes canonically conjugate to the fermion bilinear $\bar{\psi}_a\psi_a$. Taking the constant-field limit of the map, we find that the fermionic potential V in (9.1) is related to the scalar potential U as⁴

$$V(\bar{\psi}_a\psi_a) = H\phi \bar{\psi}_a\psi_a + U(\phi), \tag{9.5}$$

where the scalar is considered as a function of the fermion bilinear via $\phi = [U']^{-1}(-H\bar{\psi}_a\psi_a)$ [43]. This entails a one-to-one correspondence between the scalar self-couplings λ_n of the GNY theory and the $2n$ -fermion couplings λ_{2nF} of the GN theory in which the Yukawa coupling becomes a spectator.

Expanding (9.5) around $U'(\phi) = 0$ which corresponds to $\bar{\psi}_a\psi_a = 0$, and using dimensionless variables, we find concrete bosonisation relations for the fermion mass and all $2nF$ fermion

⁴We assume that $U(\phi)$ is convex.

couplings in terms of the potential minimum σ_0 and all scalar self-couplings λ_n starting with

$$\begin{aligned}
 \lambda_{2\text{F}} &= h \sigma_0, \\
 \lambda_{4\text{F}} &= -h^2/\lambda_2, \\
 \lambda_{6\text{F}} &= -h^3\lambda_3/\lambda_2^3, \\
 \lambda_{8\text{F}} &= h^4 \left(\lambda_2\lambda_4 - 3\lambda_3^2 \right) / \lambda_2^5, \\
 &\vdots
 \end{aligned}
 \tag{9.6}$$

and similarly for the higher order couplings. A few comments are in order:

(i) *Equivalence along trajectories and fixed points*

Taking the scale derivative of the relations (9.6) on either side reproduces the GN flows given in [12] in terms of the GNY flows of this paper, and vice versa.⁵ The findings imply that the Legendre transform (9.5) can be lifted to an identity valid at all RG scales by promoting V , H and U to scale-dependent functions V_k , H_k , U_k under the functional RG flow (3.1). Notice that the same momentum cutoffs have been used in the GN and GNY versions of the functional RG flow to achieve the result. This also entails equivalence of universal scaling exponents at quantum critical points, independent of the momentum cutoff.

(ii) *Naturalness of fermion mass*

In the absence of explicit fermion mass terms, and irrespective of chiral symmetry, GNY fermions remain massless provided the expectation value $\langle H\phi \rangle$ of the scalar field vanishes. Correspondingly, since $H\phi$ is canonically conjugate to the fermion bilinear, the dual GN fermions remain massless as well. If $\langle H\phi \rangle \neq 0$, GNY fermions and their GN counterparts become massive. The flows of the dimensionless mass terms $\partial_t \lambda_{2\text{F}}$ in the GN theory [12] and $\partial_t(h\sigma_0)$ in the GNY theory (3.9) are identical after identifying couplings via the map (9.6). In either theory, fermion mass is found to be technically natural in the sense of ‘t Hooft, in that it cannot be switched on by fluctuations directly even if chiral symmetry is absent. Hence, the map on the level of mass terms now “explains” the technical naturalness of fermion mass in GN theories even if chiral symmetry is absent [12, 13] as a consequence of the very existence of a disordered phase $\langle H\phi \rangle = 0$ with massless fermions in the dual GNY theory, and vice versa.

(iii) *Critical endpoints and broken hyperscaling*

At the endpoints of the IR critical line (figure 2), the exactly marginal cubic couplings $\lambda_3 = \pm\lambda_{3,\text{crit}}$ (7.3) of the GNY theory are mapped by (9.6) onto their exactly marginal 6F counterparts $\lambda_{6\text{F}} = \mp\lambda_{6\text{F},\text{crit}}$ (9.3), which are the endpoints of the UV critical line of the GN theory. Further, along the entire critical lines of the GN and GNY theories bar their endpoints, the correlation length exponent ν and the specific heat exponent

⁵It was observed previously that the flow of the four fermion coupling $\lambda_{4\text{F}}$ is identical to the flow of $-h^2/\lambda_2$ in the GNY theory [45]. Our work establishes that this equivalence is embedded in the general map (9.5), (9.6) and its scale-dependent uplift, with similar relations for all couplings.

α are related by the hyperscaling relation

$$d\nu = 2 - \alpha, \tag{9.7}$$

with d the space-time dimensionality. At the endpoint of the UV line of the GN theory, however, it has been shown that the hyperscaling relation is violated as a consequence of scale symmetry breaking [13]. Given the map (9.5), (9.6), we conclude that the hyperscaling relation is equally broken at the endpoints of the IR line of the GNY theory studied in this paper (figure 2).

(iv) *Role of the Yukawa coupling*

As a final remark, we consider trajectories from the free UV fixed point to the line of interacting IR fixed points (figure 2). Here, we note that the 4F coupling in (9.6) relates to (6.1) as $X = \lambda_{4F}/\lambda_{4F}^*$. Since $X \equiv 1$ on UV-IR connecting separatrices (6.2), it follows that the corresponding 4F coupling does not run and sits at its interacting UV fixed point instead. The reason for this is that the canonical mass dimension of the Yukawa coupling in the UV is smoothly rolled-over to effectively become half the anomalous dimension of the scalar field in the IR, (3.8). In particular, this leaves ratios such as $\lambda_2(k)/h^2(k) = \lambda_{2,\Lambda}/h_\Lambda^2$ and $\lambda_3(k)/h^3(k) = \lambda_{3,\Lambda}/h_\Lambda^3$ invariant under the RG flow, (6.2), (4.13). For the higher order couplings with $n \geq 3$, and provided that $\lambda_{n,\Lambda}/(h_\Lambda)^n = \frac{a_n}{n-3}$, see (4.14), (4.15), the pattern percolates to the entire effective potential. Hence, even though the scalar potential $U_k(\phi)$ and the Yukawa coupling H_k run on UV-IR connecting separatrices, the corresponding fermionic potential $V_k(\bar{\psi}_\alpha\psi_\alpha)$ remains strictly at its UV fixed point, because the running is trivialised by the Legendre map. This result also makes the spectator role played by the Yukawa interaction explicit (see figure 1 vs. figure 5).

10 Discussion and conclusions

We have investigated 3d GNY theories (2.1) in the limit of many fermion flavours with the help of functional renormalisation. These theories are renormalisable in perturbation theory and asymptotically free in the UV. If chiral symmetry is manifest, the theory can develop an interacting fixed point in the IR. Relaxing chiral symmetry, which is the main novelty of this work, the classically relevant cubic scalar self-coupling with canonical mass dimension $\frac{3}{2}$ becomes exactly marginal in the IR due to strong quantum effects (4.11). This new effect has opened up an entire line of interacting and globally well-defined IR fixed points (figure 2), characterised by the value of the cubic scalar self interaction (4.12).

Moreover, we have found that the line of fixed points is finite rather than infinite. The reason for this is that the increase of the cubic coupling above a critical strength triggers the loss of vacuum stability, (7.4). Exactly at the endpoint of stability, the quantum effective potential becomes flat implying a degenerate ground state (figure 4), and scale symmetry is broken spontaneously. Most importantly, this leads to the spontaneous generation of a fermion mass at a quantum critical point while the scalar remains massless, and a massless dilaton. It will then be interesting to see if a similar mechanism can generate mass in weakly-coupled particle theories at the end of their conformal window [17–21].

Our results are also of interest from the viewpoint of symmetry. It is well known that fermion mass is protected in the presence of chiral (2.2) or parity symmetry (2.3). In our setting, however, chiral symmetry is absent due to scalar cubic interactions switched on at the high scale. Still, we find that a non-trivial fermion mass cannot be switched on by fluctuations alone, and chiral or parity symmetry are not required to protect fermion mass. Rather, the significantly milder constraint $m_F = 0$ at the high scale is already sufficient, e.g. (3.9).

From the viewpoint of mass generation, fermion mass only arises when the scalar field develops a non-trivial vacuum expectation value and transitions from a disordered phase into an ordered one. Along the IR line of critical points, and using the scalar mass parameter $\delta\lambda_2$ as a small perturbation away from the fixed point, we find a disordered phase with a vanishing vacuum expectation value and massless fermions for any $\delta\lambda_2 > 0$. On the other side, we find a massive and ordered phase with a non-trivial vacuum expectation value provided $\delta\lambda_2 < 0$. As such, the model provides an example where mass is generated through a quantum phase transition without breaking any symmetry.

We now discuss how our results for GNY theories (2.1) relate to GN theories (9.1). At their critical points, theories display equivalent lines of fixed points characterised by an exactly marginal coupling, and equivalent scaling exponents (see figure 1 and figure 5). Moreover, we have demonstrated that the functional map between the GN and GNY-type actions [43] remains valid along RG flows (3.1), resulting in exact relations between all running couplings, (9.6). This strengthens the view that the critical GNY theories can be interpreted as the UV completion of general GN theories. An intriguing implication of this Legendre map away from critical points is that the naturalness of fermion mass in GN theories [12, 13] even in the absence of chiral symmetry can now be understood as the consequence of naturalness of fermion mass in their dual GNY theories.

Finally, we comment on corrections beyond large N . What changes is that the cubic coupling λ_3 becomes weakly irrelevant in the critical region, the line of fixed points collapses to a single point, and subleading $1/N$ corrections modify the generation of mass. Still, the running of the λ_3 remains $1/N$ slow, and we expect to find a light rather than a massless dilaton as a remnant of broken scale symmetry. Further, chirally odd interactions generate a parametrically small fermion mass $\propto 1/N$, and mass generation in $\delta\lambda_2$ now takes the form of a crossover. With increasing N , the crossover becomes increasingly sharp, and practically indistinguishable from a second order quantum phase transition. We conclude that key features of theories at infinite N remain of significance at finite N . A more detailed study of finite N effects will be reported elsewhere [70].

Our results add to the extensive set of critical GNY theories studied in the context of conformal field theory (CFT), see e.g. [71–78]. Here, the powerful links between fixed points of the RG and CFTs [79] can be exploited to extract conformal data from our results. Further, large- N theories of critical scalars or critical fermions have been shown to be dual to versions of Vasiliev’s higher spin theories under the AdS/CFT conjecture [80–82]. As such, our study offers new large- N conformal theories involving bosons and fermions simultaneously, including in settings without parity symmetry. It would be intriguing to clarify whether critical GNY theories have interesting gravity duals, and how the spontaneous breaking of scale symmetry becomes visible in conformal correlators [83, 84].

Acknowledgments

This work is supported by the Science and Technology Facilities Council (STFC) under the Consolidated Grant T/X000796/1, and was performed in part during the 2023 Aspen Center for Physics workshop *Emergent Phenomena of Strongly-Interacting Conformal Field Theories and Beyond*, which is supported by National Science Foundation grant PHY-2210452.

Open Access. This article is distributed under the terms of the Creative Commons Attribution License ([CC-BY4.0](https://creativecommons.org/licenses/by/4.0/)), which permits any use, distribution and reproduction in any medium, provided the original author(s) and source are credited.

References

- [1] J. Polchinski, *Scale and Conformal Invariance in Quantum Field Theory*, *Nucl. Phys. B* **303** (1988) 226 [[INSPIRE](#)].
- [2] W.A. Bardeen, M. Moshe and M. Bander, *Spontaneous Breaking of Scale Invariance and the Ultraviolet Fixed Point in $O(n)$ Symmetric (ϕ^6 in Three-Dimensions) Theory*, *Phys. Rev. Lett.* **52** (1984) 1188 [[INSPIRE](#)].
- [3] F. David, D.A. Kessler and H. Neuberger, *The Bardeen-moshe-bander Fixed Point and the Ultraviolet Triviality of ϕ^6 in Three-dimensions*, *Phys. Rev. Lett.* **53** (1984) 2071 [[INSPIRE](#)].
- [4] D.F. Litim, E. Marchais and P. Mati, *Fixed points and the spontaneous breaking of scale invariance*, *Phys. Rev. D* **95** (2017) 125006 [[arXiv:1702.05749](#)] [[INSPIRE](#)].
- [5] D.F. Litim and M.J. Trott, *Asymptotic safety of scalar field theories*, *Phys. Rev. D* **98** (2018) 125006 [[arXiv:1810.01678](#)] [[INSPIRE](#)].
- [6] E. Rabinovici, B. Saering and W.A. Bardeen, *Critical Surfaces and Flat Directions in a Finite Theory*, *Phys. Rev. D* **36** (1987) 562 [[INSPIRE](#)].
- [7] O. Eyal and M. Moshe, *Flat Directions and Spontaneous Breaking of Scale Invariance in a Supersymmetric $O(N) \times O(N)$ Model*, *Phys. Lett. B* **178** (1986) 379 [[INSPIRE](#)].
- [8] D.F. Litim, M.C. Mastaler, F. Synatschke-Czerwonka and A. Wipf, *Critical behavior of supersymmetric $O(N)$ models in the large- N limit*, *Phys. Rev. D* **84** (2011) 125009 [[arXiv:1107.3011](#)] [[INSPIRE](#)].
- [9] M. Heilmann, D.F. Litim, F. Synatschke-Czerwonka and A. Wipf, *Phases of Supersymmetric $O(N)$ Theories*, *Phys. Rev. D* **86** (2012) 105006 [[arXiv:1208.5389](#)] [[INSPIRE](#)].
- [10] W.A. Bardeen and M. Moshe, *Spontaneous Breaking of Scale Invariance in a $D = 3$ $U(N)$ Model with Chern-Simons Gauge Fields*, *JHEP* **06** (2014) 113 [[arXiv:1402.4196](#)] [[INSPIRE](#)].
- [11] M. Moshe and J. Zinn-Justin, *3D Field Theories with Chern-Simons Term for Large N in the Weyl Gauge*, *JHEP* **01** (2015) 054 [[arXiv:1410.0558](#)] [[INSPIRE](#)].
- [12] C. Cresswell-Hogg and D.F. Litim, *Line of Fixed Points in Gross-Neveu Theories*, *Phys. Rev. Lett.* **130** (2023) 201602 [[arXiv:2207.10115](#)] [[INSPIRE](#)].
- [13] C. Cresswell-Hogg and D.F. Litim, *Critical fermions with spontaneously broken scale symmetry*, *Phys. Rev. D* **107** (2023) L101701 [[arXiv:2212.06815](#)] [[INSPIRE](#)].
- [14] W.D. Goldberger, B. Grinstein and W. Skiba, *Distinguishing the Higgs boson from the dilaton at the Large Hadron Collider*, *Phys. Rev. Lett.* **100** (2008) 111802 [[arXiv:0708.1463](#)] [[INSPIRE](#)].

- [15] B. Bellazzini et al., *A Higgslike Dilaton*, *Eur. Phys. J. C* **73** (2013) 2333 [[arXiv:1209.3299](#)] [[INSPIRE](#)].
- [16] C. Csaki, C. Grojean and J. Terning, *Alternatives to an Elementary Higgs*, *Rev. Mod. Phys.* **88** (2016) 045001 [[arXiv:1512.00468](#)] [[INSPIRE](#)].
- [17] D.F. Litim and F. Sannino, *Asymptotic safety guaranteed*, *JHEP* **12** (2014) 178 [[arXiv:1406.2337](#)] [[INSPIRE](#)].
- [18] A.D. Bond, D.F. Litim, G. Medina Vazquez and T. Steudtner, *UV conformal window for asymptotic safety*, *Phys. Rev. D* **97** (2018) 036019 [[arXiv:1710.07615](#)] [[INSPIRE](#)].
- [19] A.D. Bond, D.F. Litim and T. Steudtner, *Asymptotic safety with Majorana fermions and new large N equivalences*, *Phys. Rev. D* **101** (2020) 045006 [[arXiv:1911.11168](#)] [[INSPIRE](#)].
- [20] A.D. Bond, D.F. Litim and G.M. Vazquez, *Conformal windows beyond asymptotic freedom*, *Phys. Rev. D* **104** (2021) 105002 [[arXiv:2107.13020](#)] [[INSPIRE](#)].
- [21] D.F. Litim, N. Riyaz, E. Stamou and T. Steudtner, *Asymptotic safety guaranteed at four-loop order*, *Phys. Rev. D* **108** (2023) 076006 [[arXiv:2307.08747](#)] [[INSPIRE](#)].
- [22] A.D. Bond, G. Hiller, K. Kowalska and D.F. Litim, *Directions for model building from asymptotic safety*, *JHEP* **08** (2017) 004 [[arXiv:1702.01727](#)] [[INSPIRE](#)].
- [23] G. Hiller, C. Hormigos-Feliu, D.F. Litim and T. Steudtner, *Model Building from Asymptotic Safety with Higgs and Flavor Portals*, *Phys. Rev. D* **102** (2020) 095023 [[arXiv:2008.08606](#)] [[INSPIRE](#)].
- [24] R. Bause et al., *B -anomalies from flavorful $U(1)'$ extensions, safely*, *Eur. Phys. J. C* **82** (2022) 42 [[arXiv:2109.06201](#)] [[INSPIRE](#)].
- [25] G. Hiller, T. Höhne, D.F. Litim and T. Steudtner, *Portals into Higgs vacuum stability*, *Phys. Rev. D* **106** (2022) 115004 [[arXiv:2207.07737](#)] [[INSPIRE](#)].
- [26] I.F. Herbut, *Interactions and phase transitions on graphene's honeycomb lattice*, *Phys. Rev. Lett.* **97** (2006) 146401 [[cond-mat/0606195](#)] [[INSPIRE](#)].
- [27] I.F. Herbut, V. Juricic and O. Vafek, *Relativistic Mott criticality in graphene*, *Phys. Rev. B* **80** (2009) 075432 [[arXiv:0904.1019](#)] [[INSPIRE](#)].
- [28] O. Vafek and A. Vishwanath, *Dirac Fermions in Solids: From High-Tc cuprates and Graphene to Topological Insulators and Weyl Semimetals*, *Ann. Rev. Condensed Matter Phys.* **5** (2014) 83 [[arXiv:1306.2272](#)] [[INSPIRE](#)].
- [29] L. Janssen and I.F. Herbut, *Antiferromagnetic critical point on graphene's honeycomb lattice: A functional renormalization group approach*, *Phys. Rev. B* **89** (2014) 205403 [*Addendum ibid.* **102** (2020) 199902] [[arXiv:1402.6277](#)] [[INSPIRE](#)].
- [30] L. Classen, I.F. Herbut, L. Janssen and M.M. Scherer, *Mott multicriticality of Dirac electrons in graphene*, *Phys. Rev. B* **92** (2015) 035429 [[arXiv:1503.05002](#)] [[INSPIRE](#)].
- [31] S. Raghu, X.-L. Qi, C. Honerkamp and S.-C. Zhang, *Topological Mott Insulators*, *Phys. Rev. Lett.* **100** (2008) 156401 [[arXiv:0710.0030](#)] [[INSPIRE](#)].
- [32] S. Hands and C. Strouthos, *Quantum Critical Behaviour in a Graphene-like Model*, *Phys. Rev. B* **78** (2008) 165423 [[arXiv:0806.4877](#)] [[INSPIRE](#)].
- [33] N. Zerf et al., *Four-loop critical exponents for the Gross-Neveu-Yukawa models*, *Phys. Rev. D* **96** (2017) 096010 [[arXiv:1709.05057](#)] [[INSPIRE](#)].

- [34] J.A. Gracey, *Large N critical exponents for the chiral Heisenberg Gross-Neveu universality class*, *Phys. Rev. D* **97** (2018) 105009 [[arXiv:1801.01320](#)] [[INSPIRE](#)].
- [35] B. Ihrig, L.N. Mihaila and M.M. Scherer, *Critical behavior of Dirac fermions from perturbative renormalization*, *Phys. Rev. B* **98** (2018) 125109 [[arXiv:1806.04977](#)] [[INSPIRE](#)].
- [36] K.G. Wilson, *Quantum field theory models in less than four-dimensions*, *Phys. Rev. D* **7** (1973) 2911 [[INSPIRE](#)].
- [37] T. Eguchi, *A New Approach to Collective Phenomena in Superconductivity Models*, *Phys. Rev. D* **14** (1976) 2755 [[INSPIRE](#)].
- [38] A. Hasenfratz et al., *The equivalence of the top quark condensate and the elementary Higgs field*, *Nucl. Phys. B* **365** (1991) 79 [[INSPIRE](#)].
- [39] S. Hands, A. Kocic and J.B. Kogut, *Compositeness, anomalous dimensions and renormalizability in four Fermi theories*, *Phys. Lett. B* **273** (1991) 111 [[INSPIRE](#)].
- [40] J. Zinn-Justin, *Four fermion interaction near four-dimensions*, *Nucl. Phys. B* **367** (1991) 105 [[INSPIRE](#)].
- [41] S. Hands, A. Kocic and J.B. Kogut, *Four Fermi theories in fewer than four-dimensions*, *Annals Phys.* **224** (1993) 29 [[hep-lat/9208022](#)] [[INSPIRE](#)].
- [42] L. Karkkainen, R. Lacaze, P. Lacey and B. Petersson, *Critical behavior of the three-dimensional Gross-Neveu and Higgs-Yukawa models*, *Nucl. Phys. B* **415** (1994) 781 [[hep-lat/9310020](#)] [[INSPIRE](#)].
- [43] S. Weinberg, *Effective field theories in the large N limit*, *Phys. Rev. D* **56** (1997) 2303 [[hep-th/9706042](#)] [[INSPIRE](#)].
- [44] L. Rosa, P. Vitale and C. Wetterich, *Critical exponents of the Gross-Neveu model from the effective average action*, *Phys. Rev. Lett.* **86** (2001) 958 [[hep-th/0007093](#)] [[INSPIRE](#)].
- [45] J. Braun, H. Gies and D.D. Scherer, *Asymptotic safety: a simple example*, *Phys. Rev. D* **83** (2011) 085012 [[arXiv:1011.1456](#)] [[INSPIRE](#)].
- [46] F. Gehring, H. Gies and L. Janssen, *Fixed-point structure of low-dimensional relativistic fermion field theories: Universality classes and emergent symmetry*, *Phys. Rev. D* **92** (2015) 085046 [[arXiv:1506.07570](#)] [[INSPIRE](#)].
- [47] J. Zinn-Justin, *Quantum field theory and critical phenomena*, *Int. Ser. Monogr. Phys.* **113** (2002) 1 [[INSPIRE](#)].
- [48] J. Polchinski, *Renormalization and Effective Lagrangians*, *Nucl. Phys. B* **231** (1984) 269 [[INSPIRE](#)].
- [49] C. Wetterich, *Exact evolution equation for the effective potential*, *Phys. Lett. B* **301** (1993) 90 [[arXiv:1710.05815](#)] [[INSPIRE](#)].
- [50] U. Ellwanger, *Flow Equations for N Point Functions and Bound States*, *Z. Phys. C* **62** (1994) 503 [[hep-ph/9308260](#)] [[INSPIRE](#)].
- [51] T.R. Morris, *The exact renormalization group and approximate solutions*, *Int. J. Mod. Phys. A* **9** (1994) 2411 [[hep-ph/9308265](#)] [[INSPIRE](#)].
- [52] D.F. Litim, *Optimization of the exact renormalization group*, *Phys. Lett. B* **486** (2000) 92 [[hep-th/0005245](#)] [[INSPIRE](#)].
- [53] D.F. Litim, *Optimized renormalization group flows*, *Phys. Rev. D* **64** (2001) 105007 [[hep-th/0103195](#)] [[INSPIRE](#)].

- [54] D.F. Litim, *Mind the gap*, *Int. J. Mod. Phys. A* **16** (2001) 2081 [[hep-th/0104221](#)] [[INSPIRE](#)].
- [55] D.F. Litim, *Critical exponents from optimized renormalization group flows*, *Nucl. Phys. B* **631** (2002) 128 [[hep-th/0203006](#)] [[INSPIRE](#)].
- [56] F. Hoffing, C. Nowak and C. Wetterich, *Phase transition and critical behavior of the $D = 3$ Gross-Neveu model*, *Phys. Rev. B* **66** (2002) 205111 [[cond-mat/0203588](#)] [[INSPIRE](#)].
- [57] H. Gies and M.M. Scherer, *Asymptotic safety of simple Yukawa systems*, *Eur. Phys. J. C* **66** (2010) 387 [[arXiv:0901.2459](#)] [[INSPIRE](#)].
- [58] D.D. Scherer, J. Braun and H. Gies, *Many-flavor Phase Diagram of the $(2+1)d$ Gross-Neveu Model at Finite Temperature*, *J. Phys. A* **46** (2013) 285002 [[arXiv:1212.4624](#)] [[INSPIRE](#)].
- [59] J. Borchardt and B. Knorr, *Global solutions of functional fixed point equations via pseudospectral methods*, *Phys. Rev. D* **91** (2015) 105011 [Erratum *ibid.* **93** (2016) 089904] [[arXiv:1502.07511](#)] [[INSPIRE](#)].
- [60] G.P. Vacca and L. Zambelli, *Multimeson Yukawa interactions at criticality*, *Phys. Rev. D* **91** (2015) 125003 [[arXiv:1503.09136](#)] [[INSPIRE](#)].
- [61] B. Knorr, *Ising and Gross-Neveu model in next-to-leading order*, *Phys. Rev. B* **94** (2016) 245102 [[arXiv:1609.03824](#)] [[INSPIRE](#)].
- [62] A. Jakovác and A. Patkós, *Local potential approximation for the renormalization group flow of fermionic field theories*, *Phys. Rev. D* **88** (2013) 065008 [[arXiv:1306.2660](#)] [[INSPIRE](#)].
- [63] A. Jakovác, A. Patkós and P. Pósfay, *Non-Gaussian fixed points in fermionic field theories without auxiliary Bose-fields*, *Eur. Phys. J. C* **75** (2015) 2 [[arXiv:1406.3195](#)] [[INSPIRE](#)].
- [64] B. Rosenstein, B.J. Warr and S.H. Park, *The Four Fermi Theory Is Renormalizable in $(2+1)$ -Dimensions*, *Phys. Rev. Lett.* **62** (1989) 1433 [[INSPIRE](#)].
- [65] B. Rosenstein, B.J. Warr and S.H. Park, *Thermodynamics of $(2+1)$ -dimensional Four Fermi Models*, *Phys. Rev. D* **39** (1989) 3088 [[INSPIRE](#)].
- [66] K. Gawedzki and A. Kupiainen, *Renormalizing the Nonrenormalizable*, *Phys. Rev. Lett.* **55** (1985) 363 [[INSPIRE](#)].
- [67] K. Gawedzki and A. Kupiainen, *Renormalization of a Nonrenormalizable Quantum Field Theory*, *Nucl. Phys. B* **262** (1985) 33 [[INSPIRE](#)].
- [68] C. de Calan, P.A. Faria da Veiga, J. Magnen and R. Seneor, *Constructing the three-dimensional Gross-Neveu model with a large number of flavor components*, *Phys. Rev. Lett.* **66** (1991) 3233 [[INSPIRE](#)].
- [69] G. Gat, A. Kovner and B. Rosenstein, *Chiral phase transitions in $d = 3$ and renormalizability of four Fermi interactions*, *Nucl. Phys. B* **385** (1992) 76 [[INSPIRE](#)].
- [70] C. Cresswell-Hogg and D.F. Litim, *Fermion Mass Generation without Symmetry Breaking*, [arXiv:2406.00100](#) [[INSPIRE](#)].
- [71] L. Iliesiu et al., *Bootstrapping 3D Fermions*, *JHEP* **03** (2016) 120 [[arXiv:1508.00012](#)] [[INSPIRE](#)].
- [72] L. Fei, S. Giombi, I.R. Klebanov and G. Tarnopolsky, *Yukawa CFTs and Emergent Supersymmetry*, *PTEP* **2016** (2016) 12C105 [[arXiv:1607.05316](#)] [[INSPIRE](#)].
- [73] L. Iliesiu et al., *Bootstrapping 3D Fermions with Global Symmetries*, *JHEP* **01** (2018) 036 [[arXiv:1705.03484](#)] [[INSPIRE](#)].

- [74] M. Goykhman and R. Sinha, *CFT data in the Gross-Neveu model*, *Phys. Rev. D* **103** (2021) 125004 [[arXiv:2011.07768](#)] [[INSPIRE](#)].
- [75] S. Prakash, *Spectrum of a Gross-Neveu Yukawa model with flavor disorder in three dimensions*, *Phys. Rev. D* **107** (2023) 066025 [[arXiv:2207.13983](#)] [[INSPIRE](#)].
- [76] R.S. Erramilli et al., *The Gross-Neveu-Yukawa archipelago*, *JHEP* **02** (2023) 036 [[arXiv:2210.02492](#)] [[INSPIRE](#)].
- [77] C.P. Herzog and V. Schaub, *Fermions in Boundary Conformal Field Theory: Crossing Symmetry and ϵ -Expansion*, *JHEP* **02** (2023) 129 [[arXiv:2209.05511](#)] [[INSPIRE](#)].
- [78] S. Giombi, E. Helfenberger and H. Khanchandani, *Line defects in fermionic CFTs*, *JHEP* **08** (2023) 224 [[arXiv:2211.11073](#)] [[INSPIRE](#)].
- [79] J.L. Cardy, *Scaling and renormalization in statistical physics*, Cambridge University Press, Cambridge, U.K. (1996) [[INSPIRE](#)].
- [80] I.R. Klebanov and A.M. Polyakov, *AdS Dual of the Critical $O(N)$ Vector Model*, *Phys. Lett. B* **550** (2002) 213 [[hep-th/0210114](#)] [[INSPIRE](#)].
- [81] E. Sezgin and P. Sundell, *Holography in 4D (super) higher spin theories and a test via cubic scalar couplings*, *JHEP* **07** (2005) 044 [[hep-th/0305040](#)] [[INSPIRE](#)].
- [82] S. Giombi and X. Yin, *The Higher Spin/Vector Model Duality*, *J. Phys. A* **46** (2013) 214003 [[arXiv:1208.4036](#)] [[INSPIRE](#)].
- [83] J. Maldacena and A. Zhiboedov, *Constraining Conformal Field Theories with A Higher Spin Symmetry*, *J. Phys. A* **46** (2013) 214011 [[arXiv:1112.1016](#)] [[INSPIRE](#)].
- [84] J. Maldacena and A. Zhiboedov, *Constraining conformal field theories with a slightly broken higher spin symmetry*, *Class. Quant. Grav.* **30** (2013) 104003 [[arXiv:1204.3882](#)] [[INSPIRE](#)].