

A variant of 3-3-1 model for the generation of the SM fermion mass and mixing pattern

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ABSTRACT: We propose an extension of the 3-3-1 model with an additional symmetry group $Z_2 \times Z_4 \times U(1)_{L_g}$ and an extended scalar sector. To our best knowledge this is the first example of a renormalizable 3-3-1 model, which allows explanation the SM fermion mass hierarchy by a sequential loop suppression: tree-level top and exotic fermion masses, 1-loop bottom, charm, tau and muon masses; 2-loop masses for the light up, down, strange quarks as well as for the electron. The light active neutrino masses are generated from a combination of linear and inverse seesaw mechanisms at two loop level. The model also has viable fermionic and scalar dark matter candidates.

KEYWORDS: Beyond Standard Model, Discrete Symmetries, Gauge Symmetry, Quark Masses and SM Parameters

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1 Introduction

Despite the great consistency of the Standard Model (SM) with experimental data, recently confirmed by the LHC discovery of the 126 GeV Higgs boson [1, 2], it has several unexplained issues [3]. Among the most pressing ones are the smallness of neutrino masses, the fermion mass and mixing hierarchy, and the existence of three fermion families.

In the SM the flavor structure of the Yukawa interactions is not restricted by gauge invariance. Consequently, fermion masses and mixings are left unfixed, and the SM does not provide an explanation for their large hierarchy, which spreads over a range of five orders of magnitude in the quark sector, and a dramatically broader range of about 11 orders of magnitude, if we include the neutrinos. Even though in the SM these parameters appear only through Yukawa interaction terms and not in explicit mass terms, this mechanism does not provide an explanation for their values, but only translates the problem to fitting different Yukawa couplings, one for each mass and with disparate values for some of them. The origin of quark mixing and the size of CP violation in this sector is also a related issue. A fundamental theory is expected to provide a dynamical explanation for the masses and mixings.

While the mixing angles in the quark sector are very small, in the lepton sector two of the mixing angles are large, and one mixing angle is small. This suggests a different kind of New Physics for the neutrino sector from the one present in the quark mass and mixing pattern. Experiments with solar, atmospheric and reactor neutrinos have brought clear evidence of neutrino oscillations from the measured non vanishing neutrino mass

squared splittings. This brings compelling and indubitable evidence that at least two of the neutrinos have non vanishing masses, much smaller, by many orders of magnitude, than the SM charged fermion masses, and that the three neutrino flavors mix with each other.

The flavor puzzle of the SM indicates that New Physics has to be advocated in order to explain the prevailing pattern of fermion masses and mixings. To tackle the limitations of the SM, various extensions, including larger scalar and/or fermion sectors, as well as extended gauge groups with additional flavor symmetries, have been proposed in the literature [4–56]. Recent reviews on flavor symmetries are provided in refs. [57–62]. Another approach to describe the fermion mass and mixing pattern consists in postulating particular mass matrix textures (see refs. [63–93] for works which consider textures). In addition, the hierarchy of SM charged fermion masses can also be explained by considering the charged fermion Yukawa matrices as products of a few random matrices, which typically feature strong hierarchies in their eigenvalue spectrum, even though the individual entries are of order unity, as was recently observed in ref. [94].

Concerning models with an extended gauge symmetry, those based on the gauge symmetry $SU(3)_c \times SU(3)_L \times U(1)_X$, also called 3-3-1 models, which introduce a family non-universal $U(1)_X$ symmetry [95–106], can provide an explanation for the origin of the family structure of the fermions. These models have the following phenomenological advantages: (i) The three family structure in the fermion sector can be understood in the 3-3-1 models from the cancellation of chiral anomalies and asymptotic freedom in QCD. (ii) The fact that the third family is treated under a different representation can explain the large mass difference between the heaviest quark family and the two lighter ones. (iii) The 3-3-1 models allow for the quantization of electric charge [107, 108]. (iv) These models have several sources of CP violation [109, 110]. (v) These models explain why the Weinberg mixing angle satisfies $\sin^2 \theta_W < \frac{1}{4}$. (vi) These models contain a natural Peccei-Quinn symmetry, which solves the strong-CP problem [111–114]. (vii) The 3-3-1 models with heavy sterile neutrinos include cold dark matter candidates as weakly interacting massive particles (WIMPs) [115–118]. A concise review of WIMPs in 3-3-1 Electroweak Gauge Models is provided in ref. [119].

In most versions of 3-3-1 models, one heavy triplet field with a Vacuum Expectation Value (VEV) at a high energy scale breaks the symmetry $SU(3)_L \times U(1)_X$ into the SM electroweak group $SU(2)_L \times U(1)_Y$, thus generating masses for the non SM fermions and non SM gauge bosons, while other two lighter triplets with VEVs at the electroweak scale, trigger the Electroweak Symmetry Breaking [80] and provide the masses for the SM particles. To provide an explanation for the observed pattern of SM fermion masses and mixings, various 3-3-1 models with flavor symmetries [33–45, 120–123] and radiative seesaw mechanisms [80, 120, 124–133] have been proposed in the literature. However, some of them involve non renormalizable interactions [39, 40, 42, 43, 45], others are renormalizable but do not address the observed pattern of fermion masses and mixings due to the unexplained huge hierarchy among the Yukawa couplings [33, 35–38, 122, 123, 134] and others are only focused either in the quark mass hierarchy [34, 128, 130], or in the study of the neutrino sector [120, 124–127, 129, 132, 133, 135, 136], or only include the description of SM fermion mass hierarchy, without addressing the mixings in the fermion sector [131]. It is interesting

to find an alternative explanation for the observed SM fermion mass and mixing pattern, in the framework of 3-3-1 models, by considering that it arises by a sequential loop suppression, so that the masses are generated according to: three level top quark mass, one loop level bottom, charm, tau and muon masses and two loop level masses for the light up, down and strange quarks as well as for the electron and neutrinos. This way of generating the SM fermion mass hierarchy was proposed for the first time in ref. [137]. However, the proposed model includes non-renormalizable Yukawa terms with a quite low cutoff scale. In this paper we propose the first renormalizable extension of the 3-3-1 model with the electric charge constructed from the $SU(3)_L$ generators as $Q = T_3 + \beta T_8 + XI$ with $\beta = -\frac{1}{\sqrt{3}}$. The model explains the SM fermion mass and mixing pattern by a sequential loop suppression mechanism.

The paper is organized as follows. In section 2 we present the theoretical setup of the proposed model. In section 3 we discuss the quark masses and mixings within the model, while the discussion of the lepton masses and mixings is given in section 4.

2 The model

The $SU(3)_C \times SU(3)_L \times U(1)_X$ model (3-3-1 model) with $\beta = -\frac{1}{\sqrt{3}}$ and right-handed Majorana neutrinos in the $SU(3)_L$ lepton triplet was proposed for the first time in [138]. However, the observed pattern of fermion masses and mixings was not addressed at that time due to the unexplained huge hierarchy among the Yukawa couplings [122, 129, 134]. Here we propose the first renormalizable extension of the 3-3-1 model with the parameter $\beta = -\frac{1}{\sqrt{3}}$, which includes a loop suppression mechanism to generate the observed pattern of the SM fermion masses and mixings. In our model only the top quark and the charged exotic fermions acquire tree level masses, whereas the remaining SM fermions get their masses via radiative corrections: 1 loop bottom, charm, tau and muon masses; 2-loop masses for the light up, down, strange quarks as well as for the electron. Light active neutrinos acquire their masses from a combination of linear and inverse seesaw mechanisms at two loop level, and the quark mixings arise from a combination of one and two loop level effects.

In order to realize this scenario we extend the $SU(3)_C \times SU(3)_L \times U(1)_X$ group with an extra $Z_4 \times Z_2$ discrete group, where the Z_4 symmetry is softly broken and the remaining Z_2 symmetry is broken both spontaneously and softly. We also introduce a global $U(1)_{L_g}$ of the generalized lepton number L_g [127], which is spontaneously broken down to a residual discrete $Z_2^{(L_g)}$ lepton number symmetry by a VEV of a gauge-singlet scalar ξ^0 to be introduced below. The corresponding massless Goldstone boson, Majoron, is phenomenologically harmless being a gauge-singlet. The full symmetry \mathcal{G} of the model experiences a two-step spontaneous breaking, as follows:

$$\begin{aligned}
 \mathcal{G} &= SU(3)_C \times SU(3)_L \times U(1)_X \times Z_4 \times Z_2 \times U(1)_{L_g} \\
 &\xrightarrow{v_\chi, v_\xi} SU(3)_C \times SU(2)_L \times U(1)_Y \times Z_4 \times Z_2^{(L_g)} \\
 &\xrightarrow{v_\eta} SU(3)_C \times U(1)_{em} \times Z_4 \times Z_2^{(L_g)},
 \end{aligned}
 \tag{2.1}$$

where the different symmetry breaking scales satisfy the following hierarchy

$$v_\eta = v = 246\text{GeV} \ll v_\chi \sim v_\xi \sim \mathcal{O}(10)\text{ TeV}, \quad (2.2)$$

which corresponds in our model to the VEVs of the scalar fields to be introduced below. In the 3-3-1 model under consideration, the electric charge is defined as [96, 122, 138]:

$$Q = T_3 + \beta T_8 + XI = T_3 - \frac{1}{\sqrt{3}}T_8 + XI, \quad (2.3)$$

where T_3 and T_8 are the $SU(3)_L$ diagonal generators, I is the 3×3 identity matrix and X is the $U(1)_X$ charge.

Different versions of the 3-3-1 models are determined by the choice of the β parameter, which is related to the different possible fermion assignments. The most studied versions of 3-3-1 models have $\beta = \pm\frac{1}{\sqrt{3}}$ [95, 138] and $\beta = \pm\sqrt{3}$ [97, 99, 139], and if we want to avoid exotic charges we are led to only two different models: $\beta = \pm\frac{1}{\sqrt{3}}$. Those having $\beta = \pm\frac{1}{\sqrt{3}}$ contain non SM fermions with non-exotic electric charges, i.e., equal to the electric charge of some SM fermions [140–142]. Those with $\beta = \pm\sqrt{3}$ have non SM fermions with large exotic electric charges and require a departure from the perturbative regime at a scale of several TeV, in order to successfully account for the measured value of the weak mixing angle at low energies, as shown in detail in ref. [143]. Other versions of 3-3-1 models have $\beta = 0, \pm\frac{2}{\sqrt{3}}$ and contain non SM particles with fractional electric charges [144]. For instance, 3-3-1 models with $\beta = 0$ contains exotic quarks and exotic charged leptons with electric charges $\frac{1}{6}$ and $-\frac{1}{2}$, respectively [144]. Since electric charge conservation implies that the lightest exotic particles of the 3-3-1 models with $\beta = 0, \pm 2/\sqrt{3}$ should be stable, the phenomenological viability of such models requires a detailed analysis of the abundance of such stable exotic charged particles in cosmology.

For these reasons, 3-3-1 models with $\beta = -\frac{1}{\sqrt{3}}$ have advantages over those with $\beta = 0, \pm\frac{2}{\sqrt{3}}, \pm\sqrt{3}$. In addition, choosing $\beta = -\frac{1}{\sqrt{3}}$ implies that the third component of the weak lepton triplet is a neutral field ν_R^C , which allows building the Dirac matrix with the usual field ν_L of the weak doublet. If one introduces a sterile neutrino N_R in the model, the light neutrino masses can be generated via low scale seesaw mechanisms, which could be inverse or linear. The 3-3-1 models with $\beta = -\frac{1}{\sqrt{3}}$ can also provide an alternative framework to generate neutrino masses, where the neutrino spectrum includes the light active sub-eV scale neutrinos, as well as sterile neutrinos, which could be dark matter candidates, if they are light enough, or candidates for detection at the LHC, if their masses are at the TeV scale. Therefore, pair production of TeV scale sterile neutrinos via the Drell-Yan mechanism at the LHC could be a signal supporting models with extended gauge symmetries such as the 3-3-1 models. In addition, Drell-Yan heavy vector pair production processes at the LHC may help to distinguish the 3-3-1 models from other models with extended gauge symmetry. With respect to the quark spectrum, we assign each of the first two families of quarks to an $SU(3)_L$ antitriplet 3^* , whereas the third family is assigned to a $SU(3)_L$ triplet 3 , as required by the $SU(3)_L$ anomaly cancellation condition. Therefore, considering that there are 3 quark colors, we have six 3^* irreducible

representations. In addition, there are six $SU(3)_L$ triplets 3 of fermionic fields, considering the three lepton families. Thus, the $SU(3)_L$ representations are vector like and anomaly free. The quantum numbers for the fermion families are assigned in such a way that the combination of the $U(1)_X$ representations with other gauge sectors is anomaly free. As a consequence, one finds that the number of chiral fermion generations is an integer multiple of the number of colors, which provides an explanation for the existence of three generations of quarks and leptons in terms of the 3 colors. The $U(1)_X$ -charge assignments of the fermionic fields are obtained from eq. (2.3) and the requirement of reproducing the electric charges of the SM quarks and leptons. Then the $U(1)_X$ charge of the first two families of quark antitriplets is $X_{Q_nL} = \frac{1}{6} + \frac{\beta}{2\sqrt{3}}$ ($n = 1, 2$), whereas for the third family of quark triplet is $X_{Q_{3L}} = \frac{1}{6} - \frac{\beta}{2\sqrt{3}}$, and the corresponding $U(1)_X$ -charges of the right handed quarks are equal to their electric charges, given by $X_{u_{jR}, d_{jR}, J_{nR}} = \frac{2}{3}, -\frac{1}{3}, \frac{1}{6} + \frac{\sqrt{3}}{2}\beta$, ($j = 1, 2, 3$ and $n = 1, 2$). The third generation non SM right handed quark T_R has a $U(1)_X$ -charge given by $X_{T_R} = \frac{1}{6} - \frac{\sqrt{3}}{2}\beta$. The three left-handed lepton families are grouped into $SU(3)_L$ triplets with $X_{L_{jL}} = -\frac{1}{2} - \frac{\beta}{2\sqrt{3}}$ ($j = 1, 2, 3$), while the right-handed leptons are assigned as $SU(3)_L$ singlets with $U(1)_X$ -charges equal to their electric charges, given by $X_{e_{iR}, \tilde{e}_{iR}} = -1, -\frac{1}{2} - \frac{\sqrt{3}}{2}\beta$, where \tilde{e}_{iR} are the right handed exotic leptons. These exotic fermions reside in vector-like representations of the SM gauge group and are singlets under the $SU(2)_L$. Since we are considering a 3-3-1 model with $\beta = -\frac{1}{\sqrt{3}}$, the cancellation of chiral anomalies implies that quarks are unified in the following $SU(3)_C \times SU(3)_L \times U(1)_X$ left- and right-handed representations [96, 101, 145, 146]:

$$\begin{aligned}
 Q_{nL} &= \begin{pmatrix} D_n \\ -U_n \\ J_n \end{pmatrix}_L \sim (3, 3^*, 0), & Q_{3L} &= \begin{pmatrix} U_3 \\ D_3 \\ T \end{pmatrix}_L \sim \left(3, 3, \frac{1}{3}\right), & n &= 1, 2, \\
 D_{iR} &\sim \left(3, 1, -\frac{1}{3}\right), & U_{iR} &\sim \left(3, 1, \frac{2}{3}\right), & i &= 1, 2, 3, \\
 J_{nR} &\sim \left(3, 1, -\frac{1}{3}\right), & T_R &\sim \left(3, 1, \frac{2}{3}\right), & &
 \end{aligned} \tag{2.4}$$

where U_{iL} and D_{iL} ($i = 1, 2, 3$) are the left handed up and down type quarks fields in the flavor basis, respectively. The right handed SM quarks, i.e., U_{iR} and D_{iR} ($i = 1, 2, 3$) and right handed exotic quarks, i.e., T_R and J_{nR} ($n = 1, 2$) are assigned to be $SU(3)_L$ singlets with $U(1)_X$ quantum numbers equal to their electric charges.

Furthermore, the requirement of chiral anomaly cancellation constrains the leptons to the following $SU(3)_C \times SU(3)_L \times U(1)_X$ left- and right-handed representations [96, 101, 145]:

$$L_{iL} = \begin{pmatrix} \nu_i \\ e_i \\ \nu_i^c \end{pmatrix}_L \sim \left(1, 3, -\frac{1}{3}\right), \quad e_{iR} \sim (1, 1, -1), \quad i = 1, 2, 3, \tag{2.5}$$

where $\nu_{iL}, \nu^c \equiv \nu_R^c$ and e_{iL} (e_L, μ_L, τ_L) are the neutral and charged lepton families, respectively. Let us note that we assign the right-handed leptons to $SU(3)_L$ singlets, which implies that their $U(1)_X$ quantum numbers correspond to their electric charges.

	Q_{1L}	Q_{2L}	Q_{3L}	U_{1R}	U_{2R}	U_{3R}	T_R	D_{1R}	D_{2R}	D_{3R}	J_{1R}	J_{2R}	\tilde{T}_{1L}	\tilde{T}_{1R}	\tilde{T}_{2L}	\tilde{T}_{2R}	B_L	B_R
L_g	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{2}{3}$	0	0	0	-2	0	0	0	2	2	0	0	0	0	0	0
Z_4	-1	-1	1	1	-i	1	1	1	1	1	-1	-1	i	1	i	1	-1	-1
Z_2	1	1	1	1	1	-1	-1	1	1	1	-1	-1	1	1	1	1	1	1

Table 1. Quark assignments under $Z_4 \times Z_2$ and the values of generalized Lepton Number L_g .

	L_{1L}	L_{2L}	L_{3L}	e_{1R}	e_{2R}	e_{3R}	E_{1L}	E_{2L}	E_{3L}	E_{1R}	E_{2R}	E_{3R}	N_{1R}	N_{2R}	N_{3R}	Ψ_R
L_g	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	1	1	1	1	1	1	1	1	1	-1	-1	-1	1
Z_4	i	i	i	-i	-i	-i	1	i	i	-i	-i	-i	i	i	i	1
Z_2	-1	1	1	-1	1	1	-1	1	1	-1	1	1	-1	-1	-1	-1

Table 2. Lepton assignments under $Z_4 \times Z_2$ and the values of generalized Lepton Number L_g .

To implement the radiative seesaw mechanisms that generate the observed hierarchy of the SM charged fermion masses and mixing angles by a sequential loop suppression and the light active neutrino masses from a combination of linear and inverse seesaw mechanisms at two loop level, we extend both the fermion and the scalar sectors of the 3-3-1 models with $\beta = -\frac{1}{\sqrt{3}}$ previously considered in the literature. We introduce $SU(3)_L$ singlet exotic up type quarks $\tilde{T}_{L,R}$, down type quarks $B_{L,R}$ and charged leptons $E_{L,R}$ as well as four gauge group eq. (2.1) singlet leptons N_R, Ψ_R . Their complete $SU(3)_C \times SU(3)_L \times U(1)_X$ assignments are:

$$\begin{aligned} \tilde{T}_{1L} &\sim (3, 1, 2/3), & \tilde{T}_{1R} &\sim (3, 1, 2/3), & \tilde{T}_{2L} &\sim (3, 1, 2/3), & \tilde{T}_{2R} &\sim (3, 1, 2/3), \\ B_L &\sim (3, 1, -1/3), & B_R &\sim (3, 1, -1/3), \end{aligned} \quad (2.6)$$

$$\begin{aligned} E_{1L} &\sim (1, 1, -1), & E_{2L} &\sim (1, 1, -1), & E_{3L} &\sim (1, 1, -1), \\ E_{1R} &\sim (1, 1, -1), & E_{2R} &\sim (1, 1, -1), & E_{3R} &\sim (1, 1, -1), \\ N_{1R} &\sim (1, 1, 0), & N_{2R} &\sim (1, 1, 0), & N_{3R} &\sim (1, 1, 0), & \Psi_R &\sim (1, 1, 0). \end{aligned} \quad (2.7)$$

The $U(1)_{L_g} \times Z_4 \times Z_2$ assignments for all the fermions of the model are shown in tables 1, 2.

Compared to the simplified versions of 3-3-1 models with the scalar sector composed only of three $SU(3)_L$ scalar triplets — χ , η and ρ — we introduce seven $SU(3)_L$ singlets $\varphi_1^0, \varphi_2^0, \xi^0, \phi_1^+, \phi_2^+, \phi_3^+$ and ϕ_4^+ . All these scalars are assigned in our model to the following representations of $SU(3)_C \times SU(3)_L \times U(1)_X$:

$$\begin{aligned} \chi &= \begin{pmatrix} \chi_1^0 \\ \chi_2^- \\ \frac{1}{\sqrt{2}}(v_\chi + \xi_\chi \pm i\zeta_\chi) \end{pmatrix} \sim \left(1, 3, -\frac{1}{3}\right), & \rho &= \begin{pmatrix} \rho_1^+ \\ \frac{1}{\sqrt{2}}(\xi_\rho \pm i\zeta_\rho) \\ \rho_3^+ \end{pmatrix} \sim \left(1, 3, \frac{2}{3}\right), \\ \eta &= \begin{pmatrix} \frac{1}{\sqrt{2}}(v_\eta + \xi_\eta \pm i\zeta_\eta) \\ \eta_2^- \\ \eta_3^0 \end{pmatrix} \sim \left(1, 3, -\frac{1}{3}\right), & \varphi_1^0 &\sim (1, 1, 0), & \varphi_2^0 &\sim (1, 1, 0), \\ \phi_1^+ &\sim (1, 1, 1), & \phi_2^+ &\sim (1, 1, 1), & \phi_3^+ &\sim (1, 1, 1), & \phi_4^+ &\sim (1, 1, 1), & \xi^0 &\sim (1, 1, 0), \end{aligned} \quad (2.8)$$

Their $U(1)_{L_g} \times Z_4 \times Z_2$ assignments are shown in table 3.

	χ	η	ρ	φ_1^0	φ_2^0	ϕ_1^+	ϕ_2^+	ϕ_3^+	ϕ_4^+	ξ^0
L_g	$\frac{4}{3}$	$-\frac{2}{3}$	$-\frac{2}{3}$	0	0	0	-2	-2	-2	-2
Z_4	1	1	-1	-1	i	i	-1	-1	1	1
Z_2	-1	-1	1	1	1	1	1	-1	-1	1

Table 3. Scalar assignments under $Z_4 \times Z_2$ and the values of generalized Lepton Number L_g .

The spontaneous symmetry breaking (2.1) in our model is triggered by the VEVs (2.2) of the scalar fields χ , η and ξ^0 , neutral under the Z_4 discrete symmetry. As seen from (2.1), the first stage of the breaking is done by a TeV scale VEV v_χ of an $SU(3)_L$ triplet χ handing masses to the non-SM fermions and gauge bosons as well as by the TeV scale VEV v_ξ of the gauge-singlet scalar ξ^0 , which spontaneously breaks the generalized lepton number symmetry $U(1)_{L_g}$. The corresponding Majoron is a gauge-singlet and, therefore, unobservable. Note that Lepton Number (LN) is broken together with Generalized Lepton Number (GLN) by the VEV of ξ^0 , which has both LN and GLN equal to -2 . Since the gauge singlet scalar ξ^0 breaks $U(1)_{L_g}$ in a way that respects the condition $|\Delta L_g| = |\Delta L| = 2$, there survives a residual discrete $Z_2^{(L_g)}$ lepton number symmetry under which the leptons are charged and the other particles are neutral. This means that in any reaction leptons can appear only in pair, thus, forbidding proton decay. The TeV scale VEVs v_χ of the $SU(3)_L$ triplet χ also breaks Z_2 symmetry. Another $SU(3)_L$ triplet η with a Fermi scale VEV v_η is responsible for the electroweak symmetry breaking and the masses of the SM fermions and W, Z -bosons.

Let us explain the VEV pattern of the $SU(3)_L$ scalar triplets χ and η . Since the χ triggers the $SU(3)_L \times U(1)_X \rightarrow SU(2)_L \times U(1)_Y$ breaking, the following conditions have to be fulfilled:

$$T_1 \langle \chi \rangle = T_2 \langle \chi \rangle = T_3 \langle \chi \rangle = (\beta T_8 + XI) \langle \chi \rangle = 0. \quad (2.9)$$

whereas the remaining generators do not leave the vacuum $\langle \chi \rangle$ invariant. From the first three conditions for $\langle \chi \rangle$ given in eq. (2.9), it follows that:

$$\langle \chi \rangle = \begin{pmatrix} 0 \\ 0 \\ \frac{v_\chi}{\sqrt{2}} \end{pmatrix} \quad (2.10)$$

The last condition in eq. (2.9) for $\langle \chi \rangle$, i.e. $(\beta T_8 + XI) \langle \chi \rangle = 0$, yields the following relation between the $U(1)_X$ charge of the $SU(3)_L$ scalar triplet χ and the β parameter:

$$X_\chi = \frac{\beta}{\sqrt{3}}, \quad (2.11)$$

which for $\beta = -\frac{1}{\sqrt{3}}$, results in $X_\chi = -\frac{1}{3}$, as indicated by eq. (2.8).

The electroweak symmetry breaking $SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$ in our model is realized by the VEV v_η of the $SU(3)_L$ scalar triplet η . Requiring that all the $SU(3)_L$ generators are broken, with the exception of the electric charge generator Q , we arrive at the following VEV pattern

$$\langle \eta \rangle = \begin{pmatrix} \frac{v_\eta}{\sqrt{2}} \\ 0 \\ 0 \end{pmatrix}. \tag{2.12}$$

From the requirement of the $U(1)_{EM}$ invariance we have

$$Q \langle \eta \rangle = (T_3 + \beta T_8 + XI) \langle \eta \rangle = 0, \tag{2.13}$$

thus producing the following relation for the $U(1)_X$ charge X_η of the η field:

$$X_\eta = -\frac{1}{2} - \frac{\beta}{2\sqrt{3}}, \tag{2.14}$$

which for $\beta = -\frac{1}{\sqrt{3}}$ results in $X_\eta = -\frac{1}{3}$ as indicated by eq. (2.8).

Note that, the difference between the η and χ Higgs triplets can be explained using the generalized lepton number L_g , discussed in appendix A. Its values for the fields of the model are specified in tables 1–3.

The choice of the VEV structure in (2.10) and (2.12) shows that only the neutral Higgs field without lepton number is allowed to have the VEV. In addition, the patterns of the $SU(3)_L$ scalar triplets χ and η shown in eq. (2.10) and (2.12) are consistent with a global minimum of the scalar potential of our model for all the region of parameter space. We adopt $X_\rho = 2/3$ for another $SU(3)_L$ scalar triplet ρ in eq. (2.8) from the simplified versions of the 3-3-1 model [101, 102, 138], where both η and ρ scalars participate in the electroweak symmetry breaking. The extra $SU(3)_L$ scalar triplet ρ is introduced in simplified versions of the 3-3-1 models to give masses to charged leptons, as well as to the bottom, up and charm quarks. In our model the $SU(3)_L$ scalar triplet ρ is crucial to give one loop level masses for the bottom and charm quarks, to the tau and muon leptons as well as two loop level masses for the up, down and strange quarks as well to the electron, as shown in figures 1, 2. The $SU(3)_L$ scalar triplet ρ also contributes to some entries of the neutrino mass matrix as indicated in figure 3. On the other hand, the conditions similar to (2.12), (2.13) are applied to $\langle \rho \rangle$ as well and lead to $X_\rho = 2/3$. In our model we have $\langle \rho \rangle = 0$ due to the Z_4 conservation (2.1), and the above symmetry breaking conditions do not restrict X_ρ . We choose $X_\rho = 2/3$ in order to maintain resemblance with the previous versions of the 3-3-1 model. Another motivation for the choice $X_\rho = 2/3$ is the $U(1)_X$ invariance of the $SU(3)_L$ invariant trilinear scalar interaction $\chi\eta\rho$. Let us note that our choice $\beta = -\frac{1}{\sqrt{3}}$ yields $X_\eta = X_\chi = -\frac{1}{3}$, which in turn leads to $X_\rho = 2/3$.

With the above particle content, the relevant quark and lepton Yukawa terms invariant under the symmetry group (2.1) of our model take the form:

$$\begin{aligned}
 -L_{gY}^{(q)} &= h_\chi^{(T)} \bar{Q}_{3L} \chi T_R + h_\eta^{(U)} \bar{Q}_{3L} \eta U_{3R} \\
 &+ \sum_{n=1}^2 \sum_{m=1}^2 h_{\rho nm}^{(\tilde{T})} \bar{Q}_{nL} \rho^* \tilde{T}_{mR} + \sum_{n=1}^2 h_{\varphi_1^0 n 2}^{(U)} \bar{\tilde{T}}_{nL} \varphi_1^0 U_{2R} + \sum_{n=1}^2 h_{\varphi_2^0 n 1}^{(U)} \bar{\tilde{T}}_{nL} \varphi_2^0 U_{1R} \\
 &+ \sum_{n=1}^2 \sum_{m=1}^2 h_{\chi nm}^{(J)} \bar{Q}_{nL} \chi^* J_{mR} + h_\rho^{(B)} \bar{Q}_{3L} \rho B_R + \sum_{j=1}^3 h_{\varphi_1^0 j}^{(D)} \bar{B}_L \varphi_1^0 D_{jR} \\
 &+ \sum_{n=1}^2 \sum_{j=1}^3 h_{\phi_1^+ n j}^{(D)} \bar{\tilde{T}}_{nL} \phi_1^+ D_{jR} + \sum_{n=1}^2 \sum_{m=1}^2 h_{\varphi_2^0 nm}^{(\tilde{T})} \bar{\tilde{T}}_{nL} \varphi_2^0 \tilde{T}_{mR} + m_B \bar{B}_L B_R + h.c.,
 \end{aligned} \tag{2.15}$$

$$\begin{aligned}
 -L_{gY}^{(l)} &= h_\rho^{(E)} \bar{L}_{1L} \rho E_{1R} + h_{\varphi_2^0}^{(E)} \bar{E}_{1L} \varphi_2^0 E_{1R} + h_{\varphi_2^0}^{(e)} \bar{E}_{1L} \varphi_2^0 e_{1R} + \sum_{n=2}^3 \sum_{m=2}^3 h_{\rho nm}^{(E)} \bar{L}_{nL} \rho E_{mR} \\
 &+ h_\rho^{(e)} \bar{L}_{1L} \rho e_{1R} + \sum_{n=2}^3 \sum_{m=2}^3 h_{\rho nm}^{(e)} \bar{L}_{nL} \rho e_{mR} + \sum_{n=2}^3 \sum_{m=2}^3 h_{\varphi_1^0 nm}^{(E)} \bar{E}_{nL} \varphi_1^0 E_{mR} \\
 &+ \sum_{n=2}^3 \sum_{m=2}^3 h_{\varphi_1^0 nm}^{(e)} \bar{E}_{nL} \varphi_1^0 e_{mR} + \sum_{n=2}^3 \sum_{j=1}^3 h_{\chi nj}^{(L)} \bar{L}_{nL} \chi N_{jR} \\
 &+ \sum_{j=1}^3 \sum_{n=2}^3 h_{\phi_4^- nj}^{(e)} \bar{E}_{nL} \phi_4^- N_{jR} + \sum_{j=1}^3 h_{\varphi_2^0}^{(N)} \bar{\Psi}_R^c (\varphi_2^0)^* N_{jR} + y_\Psi \bar{\Psi}_R^c \Psi_R \xi^0 \\
 &+ h_{\rho 11}^{(L)} \varepsilon_{abc} \bar{L}_{1L}^a (L_{1L}^C)^b (\rho^*)^c + \sum_{n=2}^3 \sum_{m=2}^3 h_{\rho nm}^{(L)} \varepsilon_{abc} \bar{L}_{nL}^a (L_{mL}^C)^b (\rho^*)^c + h.c.
 \end{aligned} \tag{2.16}$$

where the dimensionless parameters in eqs. (2.16) and (2.16) are $\mathcal{O}(1)$ dimensionless couplings. From the quark Yukawa terms it follows that the top quark mass mainly arises from the interaction with the $SU(3)_L$ scalar triplet η , which breaks the $SU(2)_L \times U(1)_Y$ gauge group. Consequently, the dominant contribution to the SM-like 126 GeV Higgs boson arises mainly from the CP even neutral component ξ_η of the $SU(3)_L$ scalar triplet η . The terms of the scalar potential relevant for the implementation of the radiative seesaw mechanisms that generate the observed hierarchy of the SM charged fermion masses and mixing angles by a sequential loop suppression are:

$$\begin{aligned}
 V \supset & \lambda_1 \eta \chi \rho \varphi_1^0 + \lambda_2 \eta \chi \rho (\varphi_1^0)^* + \lambda_3 \phi_3^- \rho \eta^\dagger \xi^0 + \lambda_4 \phi_1^- \phi_2^+ (\varphi_2^0)^* (\xi^0)^* \\
 & + w_1 (\varphi_2^0)^2 \varphi_1^0 + w_2 \phi_3^- \rho \chi^\dagger + h.c..
 \end{aligned} \tag{2.17}$$

After the spontaneous breaking of the electroweak symmetry, the above-given Yukawa interactions generate the observed hierarchy of SM fermion masses and mixing angles by a sequential loop suppression, provided that one introduces the $Z_4 \times Z_2$ soft breaking mass

terms for the electroweak singlet fermions:

$$\begin{aligned}
 L_{gsoft}^F = & \sum_{n=1}^2 \sum_{m=1}^2 (m_{\tilde{T}})_{nm} \tilde{T}_{nL} \tilde{T}_{mR} + m_{E_1} \bar{E}_{1L} E_{1R} + \sum_{n=2}^3 \sum_{m=2}^3 (m_E)_{nm} \bar{E}_{nL} E_{mR} \\
 & + \sum_{n=2}^3 (m_E)_{n1} \bar{E}_{nL} E_{1R} + h.c., \tag{2.18}
 \end{aligned}$$

as well as soft $Z_4 \times Z_2$ breaking in the electroweak singlet scalar sector:

$$L_{gsoft}^{scalars} = \mu_1^2 (\varphi_2^0)^2 + \mu_2^2 \phi_2^- \phi_3^+ + \mu_3^2 \phi_4^- \phi_3^+ + h.c.. \tag{2.19}$$

Let us note that in the simplified version of the 3-3-1 model with $\beta = -\frac{1}{\sqrt{3}}$, whose scalar sector contains three $SU(3)_L$ scalar triplets, the flavor constraints can be fulfilled by considering the scale of breaking of the $SU(3)_L \times U(1)_X$ gauge symmetry much larger than the electroweak symmetry breaking scale $v = 246$ GeV, which corresponds to the alignment limit of the mass matrix for the CP-even Higgs bosons [147]. Our model has a more extended scalar sector since it is composed of three $SU(3)_L$ scalar triplets (from which one is inert $SU(3)_L$ triplet) and six $SU(3)_L$ scalar singlets. Consequently, following ref. [147], we expect that the FCNC effects as well as the constraints arising from $K^0 - \bar{K}^0$, $B^0 - \bar{B}^0$ and $D^0 - \bar{D}^0$ mixings will be fulfilled in our model, by considering the scale of breaking of the $SU(3)_L \times U(1)_X$ gauge symmetry much larger than the scale of breaking of the electroweak symmetry. The scalar sector of our model is not predictive as its corresponding scalar potential has many free uncorrelated parameters that can be adjusted to get the required pattern of scalar masses. Therefore, the loop effects of the heavy scalars contributing to certain observables can be suppressed by the appropriate choice of the free parameters in the scalar potential. Fortunately, all these adjustments do not affect the charged fermion and neutrino sector, which is completely controlled by the fermion-Higgs Yukawa couplings.

Despite the fact that the scalar and fermion sectors of our model are considerably larger than the corresponding to the simplified version of the 3-3-1 model with $\beta = -\frac{1}{\sqrt{3}}$, and the fields assignments under the discrete group $Z_4 \times Z_2$ look rather sophisticated, each introduced element plays its own role in the implementation of the radiative seesaw mechanisms that allow us to explain the SM fermion mass hierarchy by a sequential loop suppression. In what follows we provide a justification and summary of our above presented model setup:

1. The spontaneously and softly broken Z_2 symmetry is crucial to generate a two loop level electron mass as it distinguishes the first generation left leptonic triplet, i.e., L_{1L} , neutral under Z_2 from the second and third generation ones i.e., L_{2L} and L_{3L} which are Z_2 even. This symmetry also separates the SM right handed charged leptonic field, i.e, e_{1R} , which is Z_2 odd from the remaining SM right handed charged leptonic fields, i.e, e_{2R} and e_{3R} , neutral under the Z_2 symmetry. This results in one loop level tau and muon lepton masses and a two loop level mass for the electron.
2. The softly broken Z_4 symmetry separates the third generation left handed quark fields from the first and second generation ones, giving rise to a tree level top and

exotic quark masses and to radiatively generated masses for the remaining quarks. Besides that, the Z_4 symmetry differentiates the second generation right handed SM up quark fields, i.e., U_{2R} , charged under this symmetry, from the first generation SM one, i.e., U_{1R} , which is Z_4 neutral, thus giving rise to a one loop level charm quark mass and two loop level up quark mass.

3. The scalar sector of our model is composed of three $SU(3)_L$ scalar triplets, i.e., χ , η and ρ , seven $SU(3)_L$ scalar singlets, from which three are electrically neutral, i.e., φ_1^0 , φ_2^0 and ξ^0 and four electrically charged, i.e., ϕ_1^+ , ϕ_2^+ , ϕ_3^+ and ϕ_4^+ . The inclusion of the spontaneously and softly broken Z_2 symmetry requires the introduction of a $SU(3)_L$ scalar singlet ϕ_3^+ , which is odd under this symmetry. The presence of the $SU(3)_L$ scalar singlet ϕ_3^+ , is needed in order to build the Z_2 invariant trilinear scalar interactions required to generate two loop level down and strange quark masses, as shown in figure 1. Besides that, in order to implement a two loop level radiative seesaw mechanism for the generation of the up, down and strange quark masses as well as the electron mass, the Z_4 charged $SU(3)_L$ scalar singlets φ_1^0 , φ_2^0 , ϕ_1^+ , ϕ_2^+ (which do not acquire a vacuum expectation value) are also required in the scalar sector. The Z_4 charged $SU(3)_L$ scalar singlet φ_1^0 is also needed for the implementation of the one loop level radiative seesaw mechanism that generates the charm, the bottom quark masses as well as the tau and muon lepton masses, as shown in figure 2. The Z_4 charged $SU(3)_L$ scalar singlets φ_2^0 and ϕ_3^+ as well as the $SU(3)_L$ scalar singlet ϕ_4^+ , neutral under Z_4 are also crucial for the implementation of two loop level linear and inverse seesaw mechanisms that give rise to the light active neutrino masses. The $SU(3)_L$ scalar singlet ξ^0 is introduced to spontaneously break the $U(1)_{L_g}$ generalized lepton number symmetry and thus giving rise to a tree-level mass for the right handed Majorana neutrino Ψ_R . Lets us note that ξ^0 is the only electrically neutral $SU(3)_L$ scalar singlet that has a non-vanishing generalized Lepton Number L_g . It is crucial for generating two loop-level masses for the down and strange quarks. This is due to the fact that the electrically charged $SU(3)_L$ scalar singlets ϕ_2^+ and ϕ_3^+ appearing in the two loop level diagrams that give rise to the down and strange quark masses carry non-vanishing generalized Lepton Numbers thus implying that the quartic scalar interaction $\lambda_3 \phi_3^- \rho \eta^\dagger \xi^0$ is crucial to generate the masses for the down and strange quarks, as shown in figure 1. Note that we assign non-vanishing generalized Lepton Numbers for ϕ_2^+ and ϕ_3^+ because ϕ_3^+ mix with ϕ_4^+ as well as with ϕ_2^+ via the soft breaking mass terms of eq. (2.19) and ϕ_4^+ carry a non-vanishing generalized Lepton Number as required from the invariance of the lepton Yukawa interaction $\bar{E}_{nL} \phi_4^- N_{jR}$ under the $U(1)_{L_g}$ symmetry.
4. The fermion sector of the 3-3-1 model, with right-handed neutrinos ν_R^c in the $SU(3)_L$ lepton triplet, is extended by introducing two $SU(3)_L$ singlet exotic up type quarks, i.e. \tilde{T}_1 and \tilde{T}_2 , a $SU(3)_L$ singlet exotic down type quark, i.e., B , three $SU(3)_L$ singlet exotic charged leptons, i.e., E_j ($j = 1, 2, 3$) and four right handed Majorana neutrinos N_{jR} ($j = 1, 2, 3$), Ψ_R . The $SU(3)_L$ singlet exotic down type quarks, i.e. B , is crucial for the implementation of the one loop level radiative seesaw mechanism that generate

the bottom quark mass. The $SU(3)_L$ singlet exotic up type quarks, i.e., \tilde{T}_1 and \tilde{T}_2 , are needed to generate a one loop level charm quark mass as well as two loop level down and strange quark masses. The three $SU(3)_L$ singlet exotic charged leptons, i.e., E_j ($j = 1, 2, 3$), are required in order to provide the radiative seesaw mechanisms that generate one loop level tau and muon masses and two loop level electron mass. The four right handed Majorana neutrinos, i.e., N_{jR} ($j = 1, 2, 3$), Ψ_R , are crucial for the implementation of the two loop level linear and inverse seesaw mechanisms that give rise to the light active neutrino masses. It is worth mentioning that out of these four right handed Majorana neutrinos, only Ψ_R acquires a tree level mass, whereas the three remaining right handed Majorana neutrinos, i.e., N_{jR} ($j = 1, 2, 3$), get their masses via a one loop level radiative seesaw mechanism mediated by Ψ_R and φ_2^0 , as shown in figure 3.

In what follows we briefly comment on some phenomenological aspects of our model concerning LHC signals of non-SM fermions. From the quark Yukawa interactions it follows that the heavy exotic $SU(3)_L$ singlet down (up) type quark(s), i.e., B (\tilde{T}_n ($n = 1, 2$)) will decay predominantly into a SM down (up) type quark and the $Re\varphi_1^0$ or $Im\varphi_1^0$ neutral scalar, which is identified as missing energy, due to the preserved Z_4 symmetry. Furthermore, from the lepton Yukawa interactions it follows that the heavy $SU(3)_L$ singlet exotic charged leptons, i.e. E_j ($j = 1, 2, 3$), will have a dominant decay mode into a SM charged lepton and a neutral CP even ξ_ρ or CP odd ζ_ρ scalar state, which can also be identified as missing energy, due to the preserved Z_4 symmetry. The exotic $SU(3)_L$ singlet up type quarks, i.e. \tilde{T}_1 and \tilde{T}_2 and down type quark, i.e., B , are produced in pairs at the LHC via gluon fusion and the Drell-Yan mechanism, and the charged exotic leptons E_j ($j = 1, 2, 3$) are also produced in pairs but only via the Drell-Yan mechanism. Thus, observing an excess of events with respect to the SM background in the dijet and opposite sign dileptons final states at the LHC, can be a signal in support of this model. With respect to the exotic T , J^1 or J^2 quarks, they mainly decay into a top quark and either neutral or charged scalar. The precise signature of the decays of the exotic quarks depends on details of the spectrum and other parameters of the model. The present lower limits on the Z' gauge boson mass in 3-3-1 models arising from LHC searches, reach around 2.5 TeV [148]. These bounds can be translated into limits of about 6.3 TeV on the $SU(3)_C \times SU(3)_L \times U(1)_X$ gauge symmetry breaking scale v_χ . Furthermore, electroweak data from the decays $B_{s,d} \rightarrow \mu^+\mu^-$ and $B_d \rightarrow K^*(K)\mu^+\mu^-$ set lower bounds on the Z' gauge boson mass ranging from 1 TeV up to 3 TeV [146, 149–152]. The exotic quarks can be pair produced at the LHC via Drell-Yan and gluon fusion processes mediated by charged gauge bosons and gluons, respectively. A detailed study of the exotic quark production at the LHC and the exotic quark decay modes is beyond the scope of this work and is deferred for a future publication.

Furthermore, from the quark Yukawa terms of eq. (2.16), it follows that the flavor changing top quark decays $t \rightarrow hc$, $t \rightarrow hu$ and $t \rightarrow cZ$ are absent in our model. Besides that, the decays of charged Higgses into a SM up-type and SM down-type quarks, namely, $H_1^+ \rightarrow u_i \bar{d}_j$, $H_1^- \rightarrow d_i \bar{u}_j$, $H_2^+ \rightarrow u_i \bar{d}_j$, $H_2^- \rightarrow d_i \bar{u}_j$, ($i, j = 1, 2, 3$) with $H_1^\pm = -\rho_1^\pm$ and $H_2^\pm = -\rho_3^\pm$, $(u_1, u_2, u_3) = (u, c, t)$ and $(d_1, d_2, d_3) = (d, s, b)$, are forbidden at tree level in

our model. Out of the charged Higgs decays into SM quarks, only the decays $H_1^+ \rightarrow u_1 \bar{d}_n$, $H_1^- \rightarrow \bar{u}_1 d_n$ ($n = 1, 2$) appear at one loop level whereas the decays $H_1^+ \rightarrow u_2 \bar{d}_n$, $H_1^- \rightarrow \bar{u}_2 d_n$ ($n = 1, 2$) are allowed at two loop level. In addition, the dominant SM leptonic decay modes of the charged Higgses H_1^\pm and H_2^\pm only appear at one loop level and correspond to the processes $H_1^\pm \rightarrow \nu_1 e^\pm$ and $H_2^\pm \rightarrow \nu_1^c e^\pm$. The remaining decay modes $H_1^\pm \rightarrow \nu_1 \mu^\pm$, $H_2^\pm \rightarrow \nu_1^c \mu^\pm$, $H_1^\pm \rightarrow \nu_1 \tau^\pm$, $H_2^\pm \rightarrow \nu_1^c \tau^\pm$ are very tiny with respect to the decays $H_1^\pm \rightarrow \nu_1 e^\pm$ and $H_2^\pm \rightarrow \nu_1^c e^\pm$, due to the very small mixing angles in the rotation matrix that connects the SM right handed charged leptonic fields in the interaction eigenstates with the physical SM right handed charged leptonic fields. Consequently, a measurement of the branching fraction for the $t \rightarrow hc$, $t \rightarrow hu$, $t \rightarrow cZ$, $H_1^+ \rightarrow t \bar{d}_j$, $H_1^- \rightarrow d_i \bar{t}$, $H_2^+ \rightarrow u_i \bar{d}_j$, $H_2^- \rightarrow d_i \bar{u}_j$ ($i, j = 1, 2, 3$), $H_1^\pm \rightarrow \nu_1 \mu^\pm$, $H_2^\pm \rightarrow \nu_1^c \mu^\pm$, $H_1^\pm \rightarrow \nu_1 \tau^\pm$, $H_2^\pm \rightarrow \nu_1^c \tau^\pm$ decays at the LHC will be crucial for ruling out this model.

2.1 Tadpole cancellation mechanisms

Notice that after $Z_2 \times Z_4$ is softly broken, the terms $\bar{E}_{nL} \varphi_1^0 E_{mR}$ and $(m_E)_{nm} \bar{E}_{nL} E_{mR}$ ($m, n = 2, 3$) will generate a tadpole for φ_1^0 . Since this contribution is known to give an infinite value, in order to make the theory renormalizable without giving a VEV to the φ_1^0 , one has to consider also the contribution to the φ_1^0 tadpole arising from the scalar interaction $\omega_1 \varphi_1^0 (\varphi_2^0)^2$ with the virtual φ_2^0 in the loop. We require that these two tadpoles cancel so that $\langle \varphi_1^0 \rangle = 0$ be guaranteed at one-loop level. This requirement of tadpole cancellation is an *ad hoc* condition of viability of our model. It implies fine-tuning of the model parameters $(m_E)_{nm}$ and $\varphi_1^0 (\varphi_2^0)^2$, which is unstable under the renormalization flow. In our model we do not have a symmetry to stabilize the required tadpole cancellation. Moreover, it is not possible to introduce such a symmetry without a radical modification of the model structure with all its nice features. The solution to this problem can be expected from the appropriate imbedding of our model into a more fundamental setup with additional symmetries protecting the tadpole cancellation. Given that this condition relates the parameters of the fermionic and scalar sector one may think of imbedding our model into a supersymmetric or warped five-dimensional framework (see refs. [153, 154] for recent reviews on extra-dimensions). Thinking of a supersymmetric (SUSY) version of our model (for some examples of SUSY 3-3-1 models see refs. [121, 131, 155–170]) we hope that even in the case of softly-broken SUSY the tadpole cancellation would be technically natural. More conservatively we may expect a violation of this cancellation not stronger than logarithms of the high-scale cutoff. In this case $\langle \varphi_1^0 \rangle \neq 0$, but due to the logarithmic sensitivity to the cutoff, it would be around the electroweak scale. This is phenomenologically safe, giving rise to tree level mixing $\bar{F}_L f_R$ between an exotic, F , and a SM, f , charged fermions. Despite the presence of this mixing terms, the first and second rows of the up type quark mass matrix as well as the first three rows of the down type quark and charged lepton mass matrices will still be vanishing at tree level, which is a consequence of the symmetries of the model as well as from the fact that the $SU(3)_L$ scalar triplet ρ is inert. This implies that only the top quark and exotic fermions do acquire tree level masses, whereas the remaining SM fermions will be massless at tree level. The

masses for the remaining SM fermions will still appear via the radiative seesaw mechanisms described in the previous subsection.

The implementation of supersymmetry or embedding our model in a warped extra-dimensional setup, requires careful studies, which are beyond the scope of the present paper and will be addressed elsewhere.

3 Quark masses and mixings

From the quark Yukawa interactions (2.16) it follows that the SM quark mass matrices are given by:

$$M_U = \begin{pmatrix} \tilde{\varepsilon}_{11}^{(u)} & \varepsilon_{12}^{(u)} & 0 \\ \tilde{\varepsilon}_{21}^{(u)} & \varepsilon_{22}^{(u)} & 0 \\ 0 & 0 & y \end{pmatrix} \frac{v}{\sqrt{2}}, \quad M_D = \begin{pmatrix} \tilde{\varepsilon}_{11}^{(d)} & \tilde{\varepsilon}_{12}^{(d)} & \tilde{\varepsilon}_{13}^{(d)} \\ \tilde{\varepsilon}_{21}^{(d)} & \tilde{\varepsilon}_{22}^{(d)} & \tilde{\varepsilon}_{23}^{(d)} \\ \varepsilon_{31}^{(d)} & \varepsilon_{32}^{(d)} & \varepsilon_{33}^{(d)} \end{pmatrix} \frac{v}{\sqrt{2}} \quad (3.1)$$

where $y \simeq 1$ is generated at tree level from the renormalizable Yukawa interaction $\bar{Q}_{3L}\eta U_{3R}$, thus giving rise to a tree level top quark mass. Furthermore, $\varepsilon_{n2}^{(u)}$ ($n = 1, 2$) and $\varepsilon_{3j}^{(d)}$ ($j = 1, 2, 3$) are dimensionless parameters generated at one loop level, whereas the dimensionless parameters $\tilde{\varepsilon}_{n1}^{(u)}$ and $\tilde{\varepsilon}_{nj}^{(d)}$ arise at two loop level. The corresponding Feynman diagrams are shown in figure 1.

In what follows we will show that the SM quark mass matrices given above are consistent with the low energy quark flavor data. To this end, and considering that the $\varepsilon_{n2}^{(u)}$ ($n = 1, 2$) and $\varepsilon_{3j}^{(d)}$ ($j = 1, 2, 3$) dimensionless parameters are generated at one loop level, whereas the dimensionless parameters $\tilde{\varepsilon}_{n1}^{(u)}$ and $\tilde{\varepsilon}_{nj}^{(d)}$ arise at two loop level, we choose a benchmark scenario where we set:

$$\begin{aligned} \varepsilon_{n2}^{(u)} &= a_{n2}^{(u)} l, & \tilde{\varepsilon}_{n1}^{(u)} &= b_{n1}^{(u)} l^2, \\ \varepsilon_{3j}^{(d)} &= a_{3j}^{(d)} l, & \tilde{\varepsilon}_{nj}^{(d)} &= b_{nj}^{(d)} l^2, \end{aligned} \quad n = 1, 2, \quad j = 1, 2, 3, \quad (3.2)$$

where $l \approx (1/4\pi)^2 \approx 2.0 \times \lambda^4$ is the loop suppression factor and $\lambda = 0.225$ is the Wolfenstein parameter. Then we expect in the model that $a_{n2}^{(u)}$, $b_{n1}^{(u)}$, $a_{3j}^{(d)}$, $b_{nj}^{(d)}$ ($n, m = 1, 2$ and $j = 1, 2, 3$) be $\mathcal{O}(1)$ parameters.

Let us note that the large amount of independent model parameters in the fermion and scalar sectors of our model, entering in the Feynman diagrams contributing to the entries of the SM fermion mass matrices, can be absorbed in the effective parameters $\varepsilon_{n2}^{(u)}$, $\tilde{\varepsilon}_{n1}^{(u)}$, $\varepsilon_{3j}^{(d)}$, $\tilde{\varepsilon}_{nj}^{(d)}$ ($n, m = 1, 2$ and $j = 1, 2, 3$) given by eq. (3.2). They amount to 26 real free parameters, which is a large number compared with the number of quark sector observables with the experimental values¹

$$\begin{aligned} m_u(MeV) &= 1.45_{-0.45}^{+0.56}, & m_d(MeV) &= 2.9_{-0.4}^{+0.5}, & m_s(MeV) &= 57.7_{-15.7}^{+16.8}, & (3.3) \\ m_c(MeV) &= 635 \pm 86, & m_t(GeV) &= 172.1 \pm 0.6 \pm 0.9, & m_b(GeV) &= 2.82_{-0.04}^{+0.09}, \\ \sin \theta_{12} &= 0.2254, & \sin \theta_{23} &= 0.0414, & \sin \theta_{13} &= 0.00355, & J &= 2.96_{-0.16}^{+0.20} \times 10^{-5}. \end{aligned}$$

¹We use the experimental values of the quark masses at the M_Z scale, from ref. [171], which are similar to those in [172]. The experimental values of the CKM parameters are taken from ref. [173].

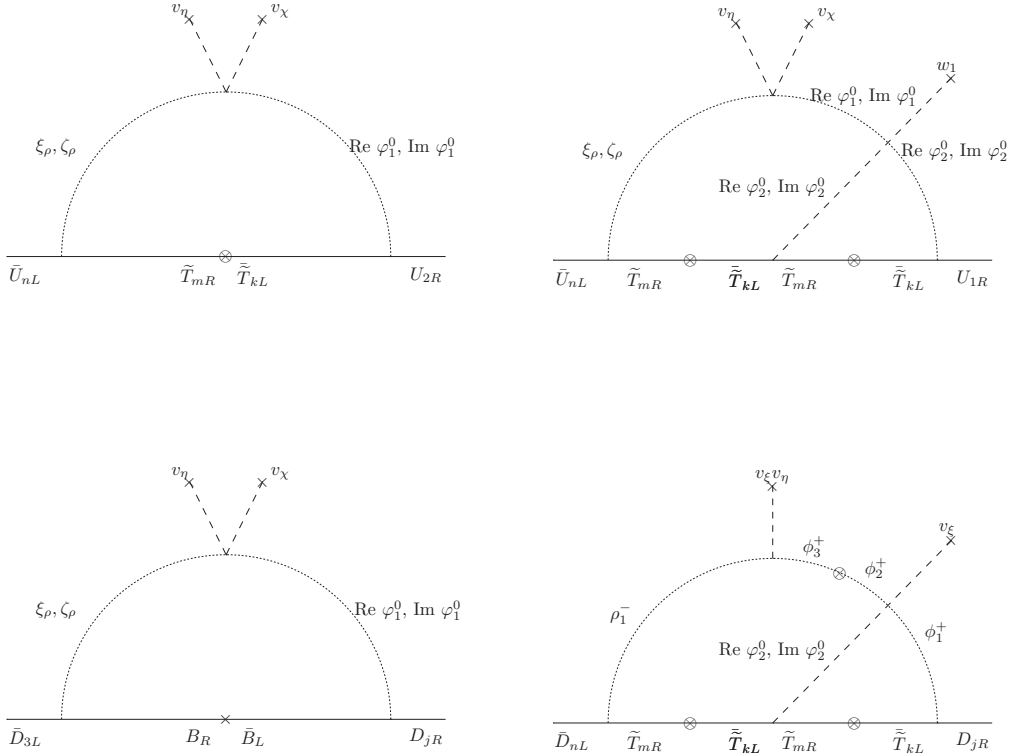


Figure 1. Loop Feynman diagrams contributing to the entries of the SM quark mass matrices. Here $m, n, k = 1, 2$ and $j = 1, 2, 3$, whereas w_1 corresponds to the mass dimension coefficient of the trilinear scalar coupling $\left((\varphi_2^0)^*\right)^2 (\varphi_1^0)^*$. The cross marks \times and \otimes in the internal lines correspond to the symmetry preserving and softly breaking mass insertions, respectively.

being t, u, c, d, s, b quark masses, $\theta_{12}, \theta_{23}, \theta_{13}$ mixing angles and the Jarlskog parameter. Therefore, the model in its present form does not predict these observables. However, as we already commented, we only pretend to reproduce the hierarchy of the quark masses via the loop suppression predicted by the model and expressed by eq. (3.2). To wit, we consider the mass matrices (3.1) with the hierarchical matrix elements (3.2) predicted by the model. For these matrices we look for the eigenvalue problem solutions reproducing the values in eq. (3.3), under the condition that $a^{(u,d)}, b^{(u,d)}$ be most close to $\mathcal{O}(1)$. Applying the standard procedure we find a solution

$$\begin{aligned}
 a_{12}^{(u)} &\simeq a_{22}^{(u)} \approx 0.5, & b_{11}^{(u)} &\approx 0.25, & b_{21}^{(u)} &\approx 0.7, & (3.4) \\
 a_{31}^{(d)} &\simeq -1.6, & a_{32}^{(d)} &\simeq -2.2, & a_{33}^{(d)} &\simeq 1.6, \\
 b_{11}^{(d)} &\simeq -12.3 - 0.7i, & b_{12}^{(d)} &\simeq -7.6 - 1.0i, & b_{13}^{(d)} &\simeq 11.6 + 0.7i, \\
 b_{21}^{(d)} &\simeq -14.3 + 0.7i, & b_{22}^{(d)} &\simeq -5.6 + 1.0i, & b_{23}^{(d)} &\simeq 14.8 - 0.7i.
 \end{aligned}$$

The above values reproduce exactly the central values in (3.3). The absolute values of the parameters in the first two rows are $\mathcal{O}(1)$. The values of the remaining two-loop level parameters are around ~ 10 , but this is still within the ballpark of the same loop level,

since the loop-suppression factor is $l \sim 10^{-2}$ (see eq. (3.2)). Thus the model reproduces fairly well the hierarchical structure of the observable quark mass spectrum as a result of the sequential loop suppression mechanism, where the top quark mass is generated at tree level, the masses for the bottom and charm quarks arise at one loop level and the light up, down and strange quarks get their masses at two loop level.

4 Lepton masses and mixings

The charged lepton masses are generated by the charged lepton Yukawa terms in eq. (2.16) via the loop diagrams shown in figure 2. The corresponding charged lepton mass matrix takes the form:

$$M_l = \begin{pmatrix} \tilde{\varepsilon}_{11}^{(l)} & \varepsilon_{12}^{(l)} & \varepsilon_{13}^{(l)} \\ 0 & \varepsilon_{22}^{(l)} & \varepsilon_{23}^{(l)} \\ 0 & \varepsilon_{32}^{(l)} & \varepsilon_{33}^{(l)} \end{pmatrix} \frac{v}{\sqrt{2}}, \quad (4.1)$$

where $\varepsilon_{jn}^{(l)}$ ($n = 2, 3$ and $j = 1, 2, 3$) are dimensionless parameters generated at one loop level, whereas the dimensionless parameter $\tilde{\varepsilon}_{11}^{(l)}$ arises at two loop level. In order to express the loop order suppression explicitly we define new parameters

$$\varepsilon_{j3}^{(l)} = a_{j3}^{(l)} \cdot l, \quad \varepsilon_{j2}^{(l)} = a_{j2}^{(l)} \cdot l, \quad \tilde{\varepsilon}_{11}^{(l)} = a_{11}^{(l)} \cdot l^2, \quad j = 1, 2, 3. \quad (4.2)$$

Here $l = (1/4\pi)^2 \approx 2.0 \times \lambda^4$ is the loop suppression factor introduced after eq. (3.2). Having 7 complex parameters, the model does not pretend to predict the charged lepton masses, but only reproduce the observed mass hierarchy. Therefore, again, as in the quark sector, we are looking for values of the $a^{(l)}$ parameters so that on one hand they reproduce the observable central values of the charged lepton masses $m_e = 0.487\text{MeV}$, $m_\mu = 102.8\text{MeV}$, $m_\tau = 1.75\text{GeV}$ and on the other hand the condition $|a^{(l)}| \sim \mathcal{O}(1)$ is achieved as close as possible. A benchmark point in the model parameter space of this kind is

$$a_{11}^{(l)} \approx a_{22}^{(l)} \approx -0.2, \quad a_{12}^{(l)} \approx a_{32}^{(l)} \approx -0.14, \quad a_{13}^{(l)} \approx a_{23}^{(l)} \approx a_{33}^{(l)} \approx 1.1. \quad (4.3)$$

All the values are not unnaturally small compared to the loop hierarchy (2-loop level)/(1-loop level) $\sim l \approx 6.3 \times 10^{-3}$. Thus, as in the quark sector, the model proves to be able to reproduce the observed charged lepton mass hierarchy by a sequential loop suppression.

From the neutral lepton Yukawa interactions in eq. (2.16) we find the neutral lepton mass terms:

$$-L_{gmass}^{(\nu)} = \frac{1}{2} \begin{pmatrix} \overline{\nu}_L^C & \overline{\nu}_R & \overline{N}_R \end{pmatrix} M_\nu \begin{pmatrix} \nu_L \\ \nu_R^C \\ N_R^C \end{pmatrix} + m_\Psi \overline{\Psi}_R^c \Psi_R + h.c., \quad (4.4)$$

where the neutrino mass matrix M_ν

$$M_\nu = \begin{pmatrix} M_1 & 0_{3 \times 3} & M_3 \\ 0_{3 \times 3} & M_2 & M_4 \\ M_3 & M_4 & \mathcal{M} \end{pmatrix}, \quad (4.5)$$

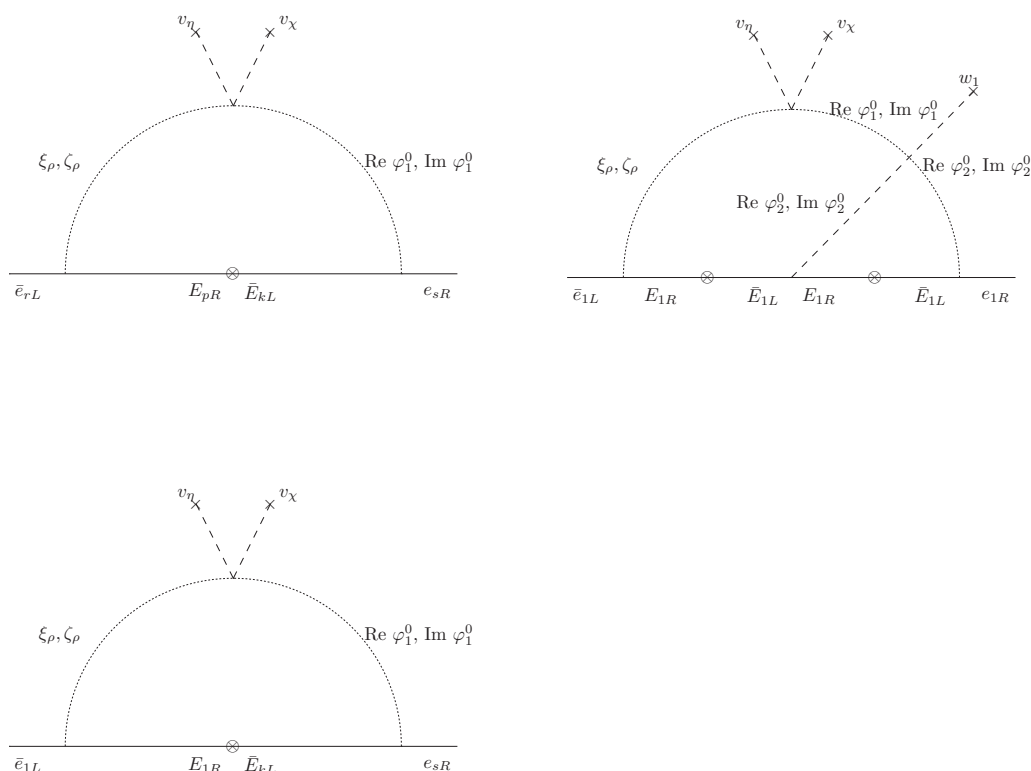


Figure 2. Loop Feynman diagrams contributing to the entries of the charged lepton mass matrices. Here $k, p, r, s = 2, 3$ and w_1 corresponds to the mass dimension coefficients of the trilinear scalar coupling $\left((\varphi_2^0)^*\right)^2 (\varphi_1^0)^*$. The cross marks in the internal lines denote the m_E mass insertions from eq. (2.18).

is generated by the loop diagrams shown in figure 3. The sub-matrices M_1, M_2, M_3, M_4 and \mathcal{M} are given by

$$\begin{aligned}
 M_1 &= \begin{pmatrix} 0 & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{pmatrix}, & M_2 &= \begin{pmatrix} 0 & b_{12} & b_{13} \\ b_{12} & b_{22} & b_{23} \\ b_{13} & b_{23} & b_{33} \end{pmatrix}, & (4.6) \\
 M_3 &= \begin{pmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{pmatrix} \frac{v}{\sqrt{2}}, & M_4 &= \begin{pmatrix} \epsilon_1 & \epsilon_2 & \epsilon_3 \\ d_1 & d_2 & d_3 \\ d_4 & d_5 & d_6 \end{pmatrix} \frac{v_\chi}{\sqrt{2}}, & \mathcal{M} &= \begin{pmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} & \mathcal{M}_{13} \\ \mathcal{M}_{12} & \mathcal{M}_{22} & \mathcal{M}_{23} \\ \mathcal{M}_{13} & \mathcal{M}_{23} & \mathcal{M}_{33} \end{pmatrix},
 \end{aligned}$$

where the matrix elements d_l ($l = 1, 2, \dots, 6$) arise at tree level, $\epsilon_{ij}, \epsilon_j$ and \mathcal{M}_{ij} ($i, j = 1, 2, 3$) at one loop level, whereas a_{nm}, a_{1n}, b_{1n} and b_{nm} ($n, m = 2, 3$) arise at two loop level. Let us note that a_{1n} and b_{1n} are generated by the $h_{\rho 11}^{(L)}$ and $h_{\rho nm}^{(L)}$ terms in eq. (2.16). These terms give rise to the four Feynman diagrams shown in the last two lines of figure 3. Consequently, the light active neutrino masses are generated by a combination of linear and inverse seesaw mechanisms at two loop level.

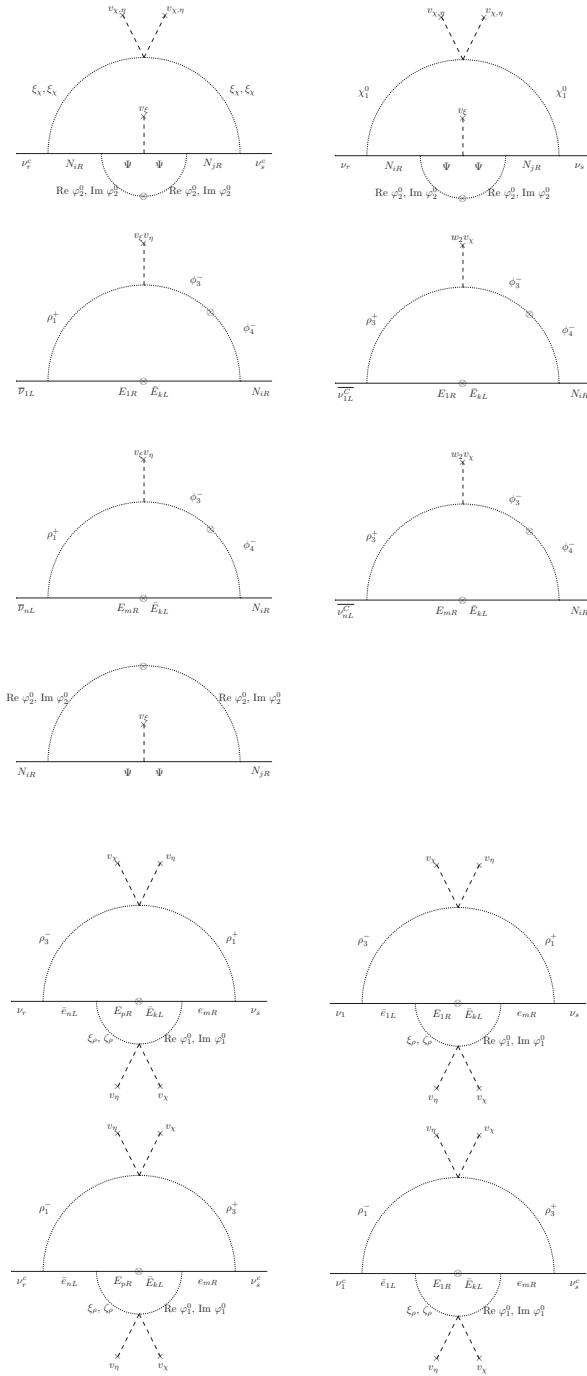


Figure 3. Loop Feynman diagrams contributing to the entries of the neutrino mass matrix. Here $n, m, k, p, r, s = 2, 3$ and $i, j = 1, 2, 3$, whereas w_2 corresponds to the mass dimension coefficient of the trilinear scalar coupling $\phi_3^+ \rho^\dagger \chi$. The cross mark \otimes in the internal lines correspond to the softly breaking mass insertions.

By performing the perturbative block diagonalization of the 9×9 neutrino mass matrix M_ν of eq. (4.5), which is shown in appendix B, we find that the light active neutrino mass matrix has the form:

$$M_{1\nu} = M_1 + \frac{1}{16} M_3 (M_4)^{-2} M_3^T M_3 (M_4)^{-1} M_3^T M_3 (M_4)^{-2} M_3^T + \frac{1}{8} M_3 (M_4)^{-1} \left(\mathcal{M} - M_3^T M_3 (M_4)^{-1} \right) (M_4)^{-1} \left(\mathcal{M} - (M_4)^{-1} M_3^T M_3 \right) (M_4)^{-1} M_3^T \quad (4.7)$$

whereas the sterile neutrino mass matrices are given by:

$$M_{2\nu} = -M_4 \\ M_{3\nu} = M_4 + \frac{1}{\sqrt{2}} \left(M_3^T M_3 (M_4)^{-1} + (M_4)^{-1} M_3^T M_3 \right). \quad (4.8)$$

Let us analyze eq. (4.8) and see what are the typical mass scales of the model, which allow us to reproduce the neutrino mass scale of $m_\nu \sim 50$ meV. The non-zero matrix elements of M_1 are determined by the 2-loop diagrams in figure 3. For the benchmark region where $m_\Psi = y_\Psi v_\xi \gg m_{\phi_2^0}, \mu_1$ their contribution is

$$a_{ij} \sim \alpha_1 \left(\frac{1}{4\pi} \right)^2 \left(\frac{v_\chi}{v} \right)^2 \frac{\mu_1^2}{m_\Psi} \log \left(m_\Psi / m_{\phi_2^0} \right). \quad (4.9)$$

This 2-loop-level contribution has typical inverse or linear seesaw structure proportional to the soft symmetry breaking parameter μ_1^2 . This parameter is stable against radiative corrections due to the model symmetries and, therefore, any of its possible values is technically natural. Then we can choose it arbitrarily small to adjust the observable neutrino mass scale $m_\nu \sim 50$ meV. Note that $a_{ij} \rightarrow 0$ in the limit $m_\Psi \rightarrow 0$, since m_Ψ is the only Lepton Number Violating parameter in our model.

The second term in eq. (4.8) is of the order

$$(\text{2nd term in eq. (4.8)}) \sim \left(\frac{v}{v_\chi} \right)^5 v. \quad (4.10)$$

From the condition $(M_{1\nu})_{ij} \lesssim m_\nu \sim 50$ meV ($i, j = 1, 2, 3$), we find that the scale of the first stage of symmetry breaking (2.1) is limited to

$$v_\chi \gtrsim 90 \text{ TeV} \quad (4.11)$$

It is worth mentioning that, as follows from eqs. (4.7) and (4.8), the physical neutrino eigenstates include three active neutrinos and six exotic neutrinos. Due to the structure of $M_{3,4}$ in eq. (4.6) and using eq. (4.8), it is shown in appendix C that the second and third generation of exotic neutrinos arising from $M_{2\nu}$ and $M_{3\nu}$ have $\mathcal{O}(10)$ TeV scale masses, whereas the first generation ones from $M_{2\nu}$ and $M_{3\nu}$ have masses at the electroweak symmetry breaking scale. The $\mathcal{O}(10)$ TeV scale exotic neutrinos of $M_{2\nu}$ have a small splitting of $\sim \frac{v^2}{v_\chi}$ with respect to the ones of $M_{3\nu}$, as indicated by eq. (4.8). These heavy quasi Dirac neutrinos can be produced in pairs at the LHC, via a Drell-Yan mechanism, mediated by a heavy non Standard Model neutral gauge boson Z' . The heavy quasi Dirac neutrinos

can decay into a Standard Model charged lepton and W gauge boson, due to their mixings with the light active neutrinos. Thus, the observation of an excess of events in the dilepton final states with respect to the SM background at the LHC would be a signal supporting this model. Studies of inverse seesaw neutrino signatures at the LHC and ILC as well as the production of Heavy neutrinos at the LHC are performed in refs. [174, 175]. A detailed study of the collider phenomenology of this model is beyond the scope of the present paper and is left for future studies.

5 Discussions and conclusions

We have built the first renormalizable extension of the 3-3-1 model with $\beta = -\frac{1}{\sqrt{3}}$, which explains the SM fermion mass hierarchy by a sequential loop suppression. Our model, based on the 3-3-1 symmetry extended with the $U(1)_{L_g} \times Z_4 \times Z_2$ group is consistent with the low energy fermion flavor data. In the model only the top quark and the charged exotic fermions acquire tree level masses, whereas the remaining SM fermions get their masses via radiative corrections: 1 loop bottom, charm, tau and muon masses; 2-loop masses for the light up, down, strange quarks as well as for the electron. Furthermore, the light active neutrinos acquire their masses from a combination of linear and inverse seesaw mechanisms at two loop level. In our model the quark and lepton mixings arise from radiative effects. At tree level there is no quark mixing, the mixing angles in the quark sector are generated from a combination of one and two loop level radiative seesaw mechanisms. In the lepton sector, the contribution to the leptonic mixing angles coming from the charged leptons arise at one loop level, whereas the mixings in the light active neutrino sector are generated from a two loop level radiative seesaw mechanism.

Furthermore our model predicts the absence of the decays $t \rightarrow hc$, $t \rightarrow hu$, $t \rightarrow cZ$, $H_1^+ \rightarrow t\bar{d}_j$, $H_1^- \rightarrow d_i\bar{t}$, $H_2^+ \rightarrow u_i\bar{d}_j$, $H_2^- \rightarrow d_i\bar{u}_j$ ($i, j = 1, 2, 3$), $H_1^\pm \rightarrow \nu_1\mu^\pm$, $H_2^\pm \rightarrow \nu_1^c\mu^\pm$, $H_1^\pm \rightarrow \nu_1\tau^\pm$, $H_2^\pm \rightarrow \nu_1^c\tau^\pm$, which implies that a measurement of the branching fraction for these decays at the LHC will be crucial for ruling out the model. Consequently, charged Higgses can be searched at the LHC through their decay into a SM up-type (down-type) and a exotic SM down-type B (\tilde{T}_k ($k = 1, 2$) up-type) quarks, as well as into a exotic charged lepton and neutrino. Since the heavy exotic $SU(3)_L$ singlet down (up) type quark(s), i.e., B (\tilde{T}_n ($n = 1, 2$)) will decay predominantly into a SM down (up) type quark and the $Re\varphi_1^0$ or $Im\varphi_1^0$ neutral scalar (which is identified as missing energy, due to the preserved Z_4 symmetry), it follows that the observation of an excess of events with respect to the SM background in the dijet final states at the LHC can be a signal of charged Higgs decays of this model. Finally, it is worth mentioning that since charged exotic fermions are produced in pairs, and they predominantly decay into a SM charged fermion and an electrically neutral Z_4 charged scalar (identified as a missing energy), observing an excess of events with respect to the SM background in the dijet and opposite sign dilepton final states at the LHC can be a signal in support of this model. A detailed study of the exotic charged fermion production at the LHC and the exotic charged fermion decay modes is beyond the scope of this work and is deferred for a future publication.

The final remark deals with the possible DM candidates in our model, which could be either the right handed Majorana neutrinos N_{iR} ($i = 1, 2, 3$), Ψ_R , or the lightest scalars $\varphi_1^R \equiv \text{Re}\varphi_1^0$, $\varphi_1^I \equiv \text{Im}\varphi_1^0$ as well as $\varphi_2^R \equiv \text{Re}\varphi_2^0$, $\varphi_2^I \equiv \text{Im}\varphi_2^0$. Let us note that the masses $m_1^{R,I}, m_2^R$ of the scalars $\varphi_1^{R,I}$, φ_2^R and the fermion Ψ_R mass m_Ψ are arbitrary parameters, since the corresponding mass terms are compatible with all the symmetries of the model, while φ_2^I squared mass is $(m_2^R)^2 - 4\mu_1^2$. The mass splitting parameter μ_1^2 is the soft Z_4 breaking mass (2.19), which already showed up in the light neutrino sector (4.9). Since the light neutrino mass scale should be small, then the $\varphi_2^R - \varphi_2^I$ mass splitting should not be very large. A superficial survey shows that N_{iR} could be a DM candidate only in a rather restricted domain of the model parameter space, due to the presence of $N_R - \nu_L$ -mixing $U_{\nu N}$ at least at one-loop level (fifth diagram in figure 3). As a result, there is the SM charged current decay $N_R \rightarrow e_L^- \nu_1 e_R^+$. The requirement that the DM lifetime be greater than the universe lifetime sets stringent constraints on $U_{\nu N}$. We do not analyze the impact of this constraint on the model parameter space and the possible correlations of the DM and the light neutrino sectors. Instead we consider the other more viable DM candidates. The gauge group singlet Ψ_R is one of them, if its mass satisfies the condition $m_\Psi < m_2^R$, and then, as follows from eq. (2.16), it does not decay at tree level. Assuming that our model be valid only up to some high-energy scale $\Lambda \gg v_\chi$, we have to consider the possible non-renormalizable operators induced by the physics beyond this scale. It is easy to check that all such operators compatible with the symmetry group \mathcal{G} of our model involve the exotic scalar φ_2^0 . The lowest dimensional operator is

$$\frac{1}{\Lambda^3} (\bar{L}\Psi_R) (\bar{e}_R L) \varphi_2^0 \quad (5.1)$$

Therefore, with the condition $m_\Psi < m_2^\Psi$ the non-renormalizable operators do not lead to kinematically allowed decays of Φ_R , making it a stable DM particle. There is also a viable scalar DM candidate φ^0 in our model. This is the lightest of the exotic scalars $\text{Re}\varphi_{1,2}^0$, $\text{Im}\varphi_{1,2}^0$, which is also lighter than the exotic charged fermions, as well as lighter than Ψ_R , and then, as follows from eqs. (2.16), (2.16), its tree-level decays are kinematically forbidden. However, as before, we check the possible non-renormalizable operators originating from the scales Λ , above the theoretical validity of our model. In the case of the DM candidate $\varphi^0 \equiv \text{Re}\varphi_1^0$ or $\text{Im}\varphi_1^0$, we find the dominant operator

$$\frac{1}{\Lambda^2} \epsilon_{abc} (\eta^\dagger)^a (\chi^\dagger)^b \varphi_1^0 \bar{L}_1^c e_{kR} \quad \text{for } k = 2, 3 \quad (5.2)$$

compatible with all the symmetries of our model. This operator induces the decays $\varphi^0 \rightarrow e_1^+ e_{2,3}^- \xi_\eta$, $\varphi^0 \rightarrow e_1^+ e_{2,3}^- \zeta_\eta$, $\varphi^0 \rightarrow e_1^+ e_{2,3}^- \xi_\chi$, $\varphi^0 \rightarrow e_1^+ e_{2,3}^- \zeta_\chi$, $\varphi^0 \rightarrow e_1^+ e_{2,3}^-$, respectively. Here $\xi_\eta = \cos \alpha h^0 + \sin \alpha H_1^0$, $\zeta_\eta \simeq G_1^0$, with $\tan \theta \sim \mathcal{O}(\frac{v}{v_\chi})$ and α a mixing angle, which depends on the scalar potential parameters. Furthermore, h is the 126 GeV SM Higgs boson, H_1^0 is one of the physical heavy neutral Higgses, whereas G_1^0 is the Goldstone boson associated with the longitudinal component of the Z gauge boson. For the scenario where the scalar φ^0 is heavier than the 126 GeV Higgs, the partial decay rates of the kinematically allowed

processes can be estimated as

$$\begin{aligned} \Gamma(\varphi^0 \rightarrow Z e_1^+ e_{2,3}^-) &\simeq \Gamma(\varphi^0 \rightarrow \zeta_\eta e_1^+ e_{2,3}^-) \sim \Gamma(\varphi^0 \rightarrow h e_1^+ e_{2,3}^-) \sim m_{\varphi^0}^3 \frac{v_\chi^2}{\Lambda^4}, \\ \Gamma(\varphi^0 \rightarrow e_1^+ e_{2,3}^-) &\sim m_{\varphi^0} \left(\frac{v_\chi v_\eta}{\Lambda^2} \right)^2. \end{aligned} \quad (5.3)$$

Requiring that the DM candidate φ^0 lifetime be greater than the universe lifetime $\tau_u \approx 13.8$ Gyr, taking into account the limit (4.11) and assuming $m_{\varphi^0} \sim 1$ TeV, we estimate the cutoff scale of our model

$$\Lambda > 3 \times 10^{10} \text{ GeV}. \quad (5.4)$$

Thus we conclude that under the above specified conditions the model contains viable fermionic Ψ_R and scalar φ^0 DM candidates. A detailed study of the dark matter constraints in our model is beyond the scope of the present paper and will be considered elsewhere.

Acknowledgments

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A Generalized lepton number

Since the lepton and anti-lepton lie in the triplet, the lepton number operator L does not commute with the $SU(3)_C \times SU(3)_L \times U(1)_X$ gauge symmetry and has the form [127]

$$L = \frac{4}{\sqrt{3}} T_8 + L_g = \begin{pmatrix} \pm \frac{2}{3} + L_g & 0 & 0 \\ 0 & \pm \frac{2}{3} + L_g & 0 \\ 0 & 0 & \mp \frac{4}{3} + L_g \end{pmatrix}, \quad (A.1)$$

where the upper and lower signs correspond to triplet and antitriplet of $SU(3)_L$, respectively. Here L_g is a conserved charge corresponding to the $U(1)_{L_g}$ global symmetry, which commutes with the gauge symmetry. According to the analysis done in ref. [127], the $SU(3)_L$ Higgs triplets χ and η have different L_g charges, which are given by:

$$L_g(\chi) = \frac{4}{3}, \quad L_g(\eta) = -\frac{2}{3}. \quad (A.2)$$

From the application of (A.1) to (2.8), it follows that the top component of χ and the bottom one of η carry lepton number $L(\chi_1^0) = -L(\eta_3^0) = 2$ whereas the other components do not $L(\chi_3^0) = L(\eta_1^0) = 0$.

B Perturbative diagonalization of the neutrino mass matrix

In this appendix we show explicitly the perturbative diagonalization of the 9×9 neutrino mass matrix M_ν of our model, which is given by eqs. (4.4)–(4.6). The elements of the submatrices $M_{1,2,3,4}$ obey the following hierarchy:

$$(M_1)_{ij} \sim (M_2)_{ij} \ll \mathcal{M}_{ij} \ll (M_3)_{ij} \ll (M_4)_{ij} \quad (\text{B.1})$$

with $i, j = 1, 2, 3$.

We first apply the following orthogonal transformation to the matrix M_ν :

$$S_\nu^T M_\nu S_\nu \simeq \begin{pmatrix} M_1 & \frac{1}{\sqrt{2}} M_3 & \frac{1}{\sqrt{2}} M_3 \\ \frac{1}{\sqrt{2}} M_3^T & M_4 & \frac{1}{2} \mathcal{M} \\ \frac{1}{\sqrt{2}} M_3^T & \frac{1}{2} \mathcal{M} & -M_4 \end{pmatrix}, \quad S_\nu = \begin{pmatrix} I & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} I & \frac{1}{\sqrt{2}} I \\ 0 & -\frac{1}{\sqrt{2}} I & \frac{1}{\sqrt{2}} I \end{pmatrix} \quad (\text{B.2})$$

where I is the 3×3 identity matrix.

Then, a second orthogonal transformation is applied under the matrix M_ν , as follows:

$$R_{1\nu}^T S_\nu^T M_\nu S_\nu R_{1\nu} \simeq \begin{pmatrix} M_1 + \frac{1}{\sqrt{2}} M_3 B_1^T + \frac{1}{\sqrt{2}} B_1 M_3^T - B_1 M_4 B_1^T & \frac{1}{\sqrt{2}} M_3 + \frac{1}{2} B_1 \mathcal{M} & \frac{1}{\sqrt{2}} M_3 - B_1 M_4 \\ \frac{1}{\sqrt{2}} M_3^T + \frac{1}{2} \mathcal{M} B_1^T & M_4 & \frac{1}{2} \mathcal{M} - \frac{1}{\sqrt{2}} M_3^T B_1 \\ \frac{1}{\sqrt{2}} M_3^T - M_4 B_1^T & \frac{1}{2} \mathcal{M} - \frac{1}{\sqrt{2}} B_1^T M_3 & -M_4 - \frac{1}{\sqrt{2}} M_3^T B_1 - \frac{1}{\sqrt{2}} B_1^T M_3 + B_1^T M_1 B_1 \end{pmatrix} \quad (\text{B.3})$$

where the rotation matrix $R_{1\nu}$ is given by:

$$R_{1\nu} = \begin{pmatrix} 1 - \frac{1}{2} B_1 B_1^T & 0 & -B_1 \\ 0 & 1 & 0 \\ B_1^T & 0 & 1 - \frac{1}{2} B_1 B_1^T \end{pmatrix} \quad (\text{B.4})$$

The partial diagonalization condition:

$$(R_{1\nu}^T S_\nu^T M_\nu S_\nu R_{1\nu})_{nm} = (R_{1\nu}^T S_\nu^T M_\nu S_\nu R_{1\nu})_{mn} = 0, \quad n = 1, 2, 3 \quad m = 7, 8, 9. \quad (\text{B.5})$$

yields the following relation:

$$B_1 \simeq \frac{1}{\sqrt{2}} M_3 (M_4)^{-1} \quad (\text{B.6})$$

Thus, eq. (B.3) takes the form:

$$R_{1\nu}^T S_\nu^T M_\nu S_\nu R_{1\nu} \simeq \begin{pmatrix} M_1 + \frac{1}{2} M_3 (M_4)^{-1} M_3^T & \frac{1}{\sqrt{2}} M_3 & 0 \\ \frac{1}{\sqrt{2}} M_3^T & M_4 & \frac{1}{2} \mathcal{M} - \frac{1}{2} M_3^T M_3 (M_4)^{-1} \\ 0 & \frac{1}{2} \mathcal{M} - \frac{1}{2} (M_4)^{-1} M_3^T M_3 & -M_4 \end{pmatrix} \quad (\text{B.7})$$

Now, a third orthogonal transformation is applied under the matrix M_ν :

$$R_{2\nu}^T R_{1\nu}^T S_\nu^T M_\nu S_\nu R_{1\nu} R_{2\nu} \simeq \begin{pmatrix} f_c (M_1 + \frac{1}{2} M_3 (M_4)^{-1} M_3^T) f_c + \frac{1}{\sqrt{2}} f_c M_3 B_2^T + \frac{1}{\sqrt{2}} B_2 M_3^T f_c + B_2 M_4 B_2^T & \frac{1}{\sqrt{2}} M_3 + B_2 M_4 & \frac{1}{2} B_2 \mathcal{M} - \frac{1}{2} B_2 M_3^T M_3 (M_4)^{-1} \\ \frac{1}{\sqrt{2}} M_3^T + M_4 B_2^T & M_4 - \frac{1}{\sqrt{2}} (M_3^T B_2 + B_2^T M_3) & \frac{1}{2} \mathcal{M} - \frac{1}{2} M_3^T M_3 (M_4)^{-1} \\ \frac{1}{2} \mathcal{M} B_2^T - \frac{1}{2} (M_4)^{-1} M_3^T M_3 B_2^T & \frac{1}{2} \mathcal{M} - \frac{1}{2} (M_4)^{-1} M_3^T M_3 & -M_4 \end{pmatrix} \quad (\text{B.8})$$

where

$$f_s = B_2, \quad f_c = 1 - \frac{1}{2}B_2B_2^T, \quad (\text{B.9})$$

and the rotation matrix $R_{2\nu}$ is given by:

$$R_{2\nu} = \begin{pmatrix} 1 - \frac{1}{2}B_2B_2^T & 0 & -B_2 \\ 0 & 1 & 0 \\ B_2^T & 0 & 1 - \frac{1}{2}B_2B_2^T \end{pmatrix} \quad (\text{B.10})$$

The resulting partial diagonalization condition:

$$(R_{2\nu}^T R_{1\nu}^T S_\nu^T M_\nu S_\nu R_{1\nu} R_{2\nu})_{nm} = (R_{2\nu}^T R_{1\nu}^T S_\nu^T M_\nu S_\nu R_{1\nu} R_{2\nu})_{mn} = 0, \quad n = 1, 2, 3 \quad m = 4, 5, 6 \quad (\text{B.11})$$

yields the following relation:

$$B_2 \simeq -\frac{1}{\sqrt{2}}M_3(M_4)^{-1} \quad (\text{B.12})$$

Consequently, eq. (B.8) takes the form:

$$\begin{aligned} & R_{2\nu}^T R_{1\nu}^T S_\nu^T M_\nu S_\nu R_{1\nu} R_{2\nu} \simeq \\ & \begin{pmatrix} \widetilde{M}_1 & 0 & -\frac{1}{\sqrt{2}}M_3(M_4)^{-1} \left(\frac{1}{2}\mathcal{M} - \frac{1}{2}M_3^T M_3 (M_4)^{-1} \right) \\ 0 & \widetilde{M}_4 & \frac{1}{2}\mathcal{M} - \frac{1}{2}M_3^T M_3 (M_4)^{-1} \\ -\frac{1}{\sqrt{2}} \left(\frac{1}{2}\mathcal{M} - \frac{1}{2}(M_4)^{-1} M_3^T M_3 \right) (M_4)^{-1} M_3^T & \frac{1}{2}\mathcal{M} - \frac{1}{2}(M_4)^{-1} M_3^T M_3 & -M_4 \end{pmatrix} \\ & = \begin{pmatrix} W & 0 & X \\ 0 & Z & Y \\ X^T & Y^T & -M_4 \end{pmatrix} \end{aligned} \quad (\text{B.13})$$

where

$$\begin{aligned} W &= \widetilde{M}_1 = M_1 + \frac{1}{16}M_3(M_4)^{-2}M_3^T M_3(M_4)^{-1}M_3^T M_3(M_4)^{-2}M_3^T \\ Z &= \widetilde{M}_4 = M_4 + \frac{1}{\sqrt{2}} \left(M_3^T M_3 (M_4)^{-1} + (M_4)^{-1} M_3^T M_3 \right) \\ X &= -\frac{1}{\sqrt{2}}M_3(M_4)^{-1} \left(\frac{1}{2}\mathcal{M} - \frac{1}{2}M_3^T M_3 (M_4)^{-1} \right) \\ Y &= \frac{1}{2}\mathcal{M} - \frac{1}{2}(M_4)^{-1} M_3^T M_3 \end{aligned} \quad (\text{B.14})$$

Then we apply a fourth orthogonal transformation under the matrix M_ν , as follows:

$$\begin{aligned} & R_{3\nu}^T R_{2\nu}^T R_{1\nu}^T S_\nu^T M_\nu S_\nu R_{1\nu} R_{2\nu} R_{3\nu} \\ & \simeq \begin{pmatrix} W + X(M_4)^{-1}X^T & 0 & 0 \\ 0 & Z & Y \\ 0 & Y^T - M_4 - X^T B_3 - B_3^T X + B_3^T W B_3 \end{pmatrix} \end{aligned} \quad (\text{B.15})$$

where the rotation matrix $R_{3\nu}$ is given by:

$$R_{3\nu} = \begin{pmatrix} 1 - \frac{1}{2}B_3B_3^T & 0 & -B_3 \\ 0 & 1 & 0 \\ B_3^T & 0 & 1 - \frac{1}{2}B_3B_3^T \end{pmatrix}, \quad B_3 \simeq X(M_4)^{-1} \quad (\text{B.16})$$

Consequently, the light active neutrino mass matrix takes the form:

$$\begin{aligned} M_{1\nu} &= W + X(M_4)^{-1}X^T \\ &= M_1 + \frac{1}{16}M_3(M_4)^{-2}M_3^T M_3(M_4)^{-1}M_3^T M_3(M_4)^{-2}M_3^T \\ &\quad + \frac{1}{8}M_3(M_4)^{-1} \left(\mathcal{M} - M_3^T M_3(M_4)^{-1} \right) (M_4)^{-1} \left(\mathcal{M} - (M_4)^{-1} M_3^T M_3 \right) (M_4)^{-1} M_3^T \end{aligned} \quad (\text{B.17})$$

On the other hand, from eq. (B.14) and considering the hierarchy given by eq. (B.1), it follows that the matrix of eq. (B.15) is nearly block diagonal, which implies that the sterile neutrino mass matrices are given by:

$$\begin{aligned} M_{2\nu} &= -M_4 \\ M_{3\nu} &= M_4 + \frac{1}{\sqrt{2}} \left(M_3^T M_3(M_4)^{-1} + (M_4)^{-1} M_3^T M_3 \right) = M_4 + \Delta. \end{aligned} \quad (\text{B.18})$$

C Sterile neutrino mass spectrum

In this appendix we compute the sterile neutrino mass spectrum. Our starting point is the fact that the sterile neutrino mass matrices $M_{2\nu} = -M_4$ and $M_{3\nu} = M_4 + \Delta$ satisfy the relation:

$$M_4 M_4^T = \begin{pmatrix} \varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2 & \varepsilon_1 d_1 + \varepsilon_2 d_2 + \varepsilon_3 d_3 & \varepsilon_1 d_4 + \varepsilon_2 d_5 + \varepsilon_3 d_6 \\ \varepsilon_1 d_1 + \varepsilon_2 d_2 + \varepsilon_3 d_3 & d_1^2 + d_2^2 + d_3^2 & d_1 d_4 + d_2 d_5 + d_3 d_6 \\ \varepsilon_1 d_4 + \varepsilon_2 d_5 + \varepsilon_3 d_6 & d_1 d_4 + d_2 d_5 + d_3 d_6 & d_4^2 + d_5^2 + d_6^2 \end{pmatrix} \frac{v_\chi^2}{2}, \quad (\text{C.1})$$

where the subleading $\mathcal{O}\left(\frac{v^2}{v_\chi}\right)$ corrections presented in $M_{3\nu}$ have been neglected. Then, from the previous expression, it follows that:

$$\det(M_4 M_4^T) = (\varepsilon_1 d_2 d_6 - \varepsilon_1 d_3 d_5 - \varepsilon_2 d_1 d_6 + \varepsilon_2 d_3 d_4 + \varepsilon_3 d_1 d_5 - \varepsilon_3 d_2 d_4)^2 \frac{v_\chi^6}{8}, \quad (\text{C.2})$$

Since $\varepsilon_i \ll d_k$ ($i = 1, 2, 3$ and $k = 1, 2, \dots, 6$), the mixing angles between the first generation sterile neutrinos and the second and third generation ones can be neglected, being of the order of $\frac{\varepsilon_i}{d_k}$. Consequently, the masses for the first, second and third generation sterile neutrinos are respectively given by:

$$\begin{aligned} m_1^{(1,2)} &= \frac{4(\varepsilon_1 d_2 d_6 - \varepsilon_1 d_3 d_5 - \varepsilon_2 d_1 d_6 + \varepsilon_2 d_3 d_4 + \varepsilon_3 d_1 d_5 - \varepsilon_3 d_2 d_4)}{(r^2 - s)} \frac{v_\chi}{\sqrt{2}}, \\ m_2^{(1,2)} &= \frac{1}{2}(r - \sqrt{s}) \frac{v_\chi}{\sqrt{2}}, \quad m_3^{(1,2)} = \frac{1}{2}(r + \sqrt{s}) \frac{v_\chi}{\sqrt{2}}, \end{aligned} \quad (\text{C.3})$$

where the superscripts 1 and 2 in eq. (C.3) correspond to the physical neutrino states arising from $M_{2\nu}$ and $M_{3\nu}$, respectively. Furthermore, r and s are given by:

$$\begin{aligned}
 r &= d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2 + d_6^2, \\
 s &= d_1^4 + 2d_1^2d_2^2 + 2d_1^2d_3^2 + 2d_1^2d_4^2 - 2d_1^2d_5^2 - 2d_1^2d_6^2 + 8d_1d_2d_4d_5 + 8d_1d_3d_4d_6 + d_2^4 + 2d_2^2d_3^2 \\
 &\quad - 2d_2^2d_4^2 + 2d_2^2d_5^2 - 2d_2^2d_6^2 + 8d_2d_3d_5d_6 + d_3^4 - 2d_3^2d_4^2 - 2d_3^2d_5^2 + 2d_3^2d_6^2 + d_4^4 + 2d_4^2d_5^2 \\
 &\quad + 2d_4^2d_6^2 + d_5^4 + 2d_5^2d_6^2 + d_6^4
 \end{aligned} \tag{C.4}$$

Since $v_\chi \sim \mathcal{O}(10)\text{TeV}$, $d_k \sim \mathcal{O}(1)$ and $\varepsilon_i \sim \mathcal{O}(10^{-2})$, we find $m_1^{(1,2)} \sim \mathcal{O}(100)\text{GeV}$ and $m_{2,3}^{(1,2)} \sim \mathcal{O}(10)\text{TeV}$. This shows that the second and third generation of exotic neutrinos arising from $M_{2\nu}$ and $M_{3\nu}$ have $\mathcal{O}(10)\text{TeV}$ scale masses, whereas the first generation ones from $M_{2\nu}$ and $M_{3\nu}$ have masses at the electroweak symmetry breaking scale. The $\mathcal{O}(10)\text{TeV}$ scale exotic neutrinos of $M_{2\nu}$ have a small splitting of $\sim \frac{v^2}{v_\chi}$ with the ones of $M_{3\nu}$, as indicated by eq. (B.18).

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