

The swampland conjecture and F-term axion monodromy inflation

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ABSTRACT: We continue the investigation of F-term axion monodromy inflation in string theory, while seriously taking the issue of moduli stabilization into account. For a number of closed and open string models, we show that they suffer from serious control issues once one is trying to realize trans-Planckian field excursions. More precisely, the flux tuning required to delay the logarithmic scaling of the field distance to a trans-Planckian value cannot be done without leaving the regime where the employed effective supergravity theory is under control. Our findings are consistent with the axionic extension of the Refined Swampland Conjecture, stating that in quantum gravity the effective theory breaks down for a field excursion beyond the Planck scale. Our analysis suggests that models of F-term axion monodromy inflation with a tensor-to-scalar ratio $r \geq O(10^{-3})$ cannot be parametrically controlled.

KEYWORDS: Cosmology of Theories beyond the SM, Flux compactifications, Superstring Vacua

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1 Introduction

Even though meanwhile dismissed, the 2014 BICEP2 announcement of a detection of primordial B-modes with a large tensor-to-scalar ratio of $r \sim 0.2$, triggered much research in string cosmology. Indeed, the main model building challenge is that for a ratio of $r > 0.01$ the Lyth bound [1] implies that the inflaton has to roll over trans-Planckian field

distances, hence making the process highly UV sensitive. Therefore, string theory as a UV complete quantum theory of gravity provides a well defined framework to discuss high scale inflation. Interestingly, there are some hints supporting the existence of an underlying quantum gravity constraint that forbids trans-Planckian excursions. Further investigation in this direction is therefore, not only phenomenological, but also conceptually interesting.

To forbid higher order Planck suppressed operators in the inflaton action, one can employ a pseudo-scalar field with a continuous shift symmetry, called an axion. There are essentially two mostly followed approaches towards realizing axionic inflation in string theory. The first employs the periodic cosine potential [2] generically generated by instantons, possibly with more than one axion to enlarge the field range [3, 4]. For the simplest model of natural inflation, string theory requires to work outside the regime of a controlled low-energy effective action [5]. It was realized [6–9] that this behavior is precisely reflected in the Weak Gravity Conjecture (WGC) [10] extended from point particles to instantons.

The second approach is to impose a controlled spontaneous breaking of the axionic shift symmetry [11] by adding branes or fluxes, inducing a potential energy that increases by a certain amount over every period the inflaton transverses. This ansatz is called axion monodromy inflation and was introduced in the stringy context in [12]. One mechanism to generate a polynomial potential for axion monodromy inflation is to turn on background fluxes generating a tree-level F-term scalar potential [13–15], see also [16–25] and for reviews [26, 27]. For other attempts to realize axion monodromy inflation in string theory see e.g. [28–30].

Turning on fluxes has the advantage that the same mechanism generating the axion potential also stabilizes the other moduli and breaks supersymmetry. Therefore, the question arose whether one can control the trans-Planckian regime for the axion in a consistent scheme of moduli stabilization. This was analyzed in a series of papers [31–34] in the framework of orientifolded Calabi-Yau compactification of the 10-dimensional type IIA or type IIB theory giving rise to a four dimensional $N = 1$ supergravity theory with usually plenty of massless scalar fields and axions. This geometry is then perturbed by turning on geometric and non-geometric background fluxes leading to a gauged supergravity theory, that can be deduced via dimensional reduction of double field theory [35].

A detectable tensor-to-scalar ratio of $r > 0.01$ and the so far not detected non-Gaussianities favor single large-field inflation. In this case, the potential energy during inflation is $M_{\text{inf}} \sim 10^{16}$ GeV, the Hubble-scale during inflation is $H_{\text{inf}} \sim 10^{14}$ GeV and the inflaton mass is $M_\theta \sim 10^{13}$ GeV. In order to use an effective supergravity approach, the string scale M_s and the Kaluza-Klein scale M_{KK} must lie above all these scales. Moreover, the other moduli masses should lie above the Hubble scale to guarantee a model of single field inflation. Therefore, altogether we have the ordered hierarchy of mass scales

$$M_{\text{Pl}} > M_s > M_{\text{KK}} > M_{\text{mod}} > H_{\text{inf}} > M_\theta, \quad (1.1)$$

where neighboring scales can differ only by a factor of $\mathcal{O}(10)$. This is obviously a major challenge for concrete string model building.

Since for single field inflation, the inflaton should be the lightest scalar field, all other moduli should better acquire their masses already at tree-level. In the type IIB setting this

implies that the universal axio-dilaton requires an NS-NS three-form flux and the overall volume a non-geometric Q -flux to be turned on. Closed string moduli stabilization with solely fluxes was discussed in [32, 33] (see also [28]). There it was found that control over the trans-Planckian regime in all examples required to violate at least one of the required hierarchies in (1.1). Moreover, the backreaction of the rolling axion onto the other moduli was substantial and led to a flattening of the potential [36] and in the extreme case to a potential of plateau(Starobinsky)-type. The reason behind this is that for large field excursions of an axion θ , the backreacted proper field distance showed a logarithmic behavior $\Theta \sim \lambda^{-1} \log \theta$. Here λ^{-1} can be considered as the scale in field distance where the backreaction becomes substantial.

It was realized in [37, 38] that this logarithmic scaling of the proper field distance is very generic and that it precisely reflects the conjectured behavior by Ooguri/Vafa [39] to distinguish effective field theory models that can be realized in string theory (the landscape) from those that cannot be coupled in a UV complete way to gravity (the swampland) [40]. This, later called, swampland conjecture [38] says that if one moves over very large distances in the moduli space of an effective quantum gravity theory, there appears an infinite tower of states whose mass scales as $m \sim m_0 \exp(-\lambda \Delta \Theta)$. This means that for $\Delta \Theta > \lambda^{-1}$ the effective theory breaks down. The prototype example of this appears for string theory compactified on a circle, where it is the Kaluza-Klein tower that shows this behavior in terms of the proper field distance.

The string theory models discussed in [37] always had $\lambda = O(1)$, i.e. the cut-off in the field distance where one could trust the effective description was close to the Planck-scale. This led Kläwer and Palti in [38], to formulate the *Refined Swampland Conjecture* (RSC), extending the former one by the statement that $\lambda = O(1)$, i.e. one cannot push λ^{-1} to values parametrically larger than one. Furthermore, the RSC applies to any scalar field, including axions, unlike the original conjecture from [39] which only applies to the geometric moduli space.

It was motivated in [41], though, that one should aim for engineering models with a flux dependent λ in such a way that the backreaction can in principle be delayed in field distance. The authors of [42] analyzed inflationary models with an open string modulus, namely the deformation modulus of a $D7$ -brane [15, 18, 19, 22, 43, 44], playing the role of the inflaton. These models looked a priori promising to admit a parametrically large value of λ^{-1} . However, the *Refined Swampland Conjecture* implies that also F-term axion monodromy inflation cannot be realized in a parametrically controlled way in string theory. Let us mention that an argument based on entropy of de-Sitter space has led J. Conlon to the same general conclusion [45] (see also [46]).

It is the purpose of this paper to challenge or find further evidence for this intricate relation between F-term axion monodromy inflation and the *Refined Swampland Conjecture*. Despite the danger of repeating parts of this introduction, in section 2 we review former attempts to build string models of large field inflation, discuss the challenges one faces when combining this with full moduli stabilization and also present the swampland conjecture and its refinement. In section 3, we will revisit a simple purely closed string model from [32, 33] and demonstrate how it fits nicely into this picture. Moreover, we

will show that also the proposed backreacted plateau-like model [32] is not parametrically under control.

In section 4, we extend and further examine the open string models discussed in [42]. We indeed find that the backreacted proper field distance always exhibit the predicted logarithmic scaling at large field. Our aim is, though, to identify and analyze in detail models where λ^{-1} is flux-dependent and can in principle be tuned parametrically large to delay the backreaction. We find that also these models require $\lambda \approx 1$ in order to have parametric control over the effective field theories. For concreteness, we consider models in which all scalars are fixed at tree level by fluxes. This requires the addition of geometric fluxes in IIA, which become non-geometric in IIB. We identify two simple representative models of having a tunable λ , and show that the necessary flux tuning would imply that the scale of moduli masses becomes larger than the Kaluza-Klein scale. The (quantum gravity) ingredients in the string effective action that are responsible for this behavior can be identified as:

- The leading order Kähler potential always shows a logarithmic dependence on the saxions.
- The specific form of the superpotential appearing in string theory.
- The moduli dependence of the various mass scales, like string, Kaluza-Klein and moduli mass, resulting from dimensional reduction and moduli stabilization.
- The fact that fluxes are quantized.

These observations lead us to a change of perspective. Instead of trying to make the models more baroque and to find loop-holes, maybe one should better believe in the *Refined Swampland Conjecture* and figure out where these control issues were hidden or ignored in the previous attempts that (naively) looked successful to realize large field inflation. We also critically revisit attempts to build axion monodromy models where the Kähler moduli were stabilized via non-perturbative effects, like in KKLT and the Large Volume Scenario. We notice that the required flux tuning gets into conflict with the original assumptions of small W_0 and large volume, respectively. Our conclusions in section 5 will also discuss possible loopholes and future directions to continue investigating the realization of axion monodromy inflation and its relation with the Swampland Conjecture.

2 F-term axion monodromy inflation

In this section we review former attempts to realize large field inflation in string theory and challenges one faces, when combining this with the issue of moduli stabilization. We also review the Swampland Conjecture [39] as formulated by Ooguri/Vafa and following [38] how it is related to large field inflation.

2.1 Large field inflation

The large number of difficulties encountered when embedding large field inflation in a controlled string theory framework gave rise to the suspicion that a fundamental reason

might underly the obstruction of getting trans-Planckian field ranges in a consistent theory of quantum gravity. The search of this fundamental reason has triggered plenty of recent work aiming to identify the constraints that quantum gravity imposes over an, a priori, consistent quantum field theory.

The obstruction of getting a trans-Planckian decay constant to realize natural inflation can be related, for instance, to the Weak Gravity Conjecture (WGC) [10]. This conjecture generalized to axions reads

$$f S_{\text{inst}} \leq 1, \tag{2.1}$$

where f is the axion decay constant and S_{inst} the instanton action. Thus, it states that for any axion with a trans-Planckian decay constant there must exist an instanton, electrically coupled to the axion, with an action at most of order one. Therefore, the potential for the axion will generically receive non-suppressed instantonic corrections which signal the breakdown of the effective theory and will reduce the effective field range to a sub-Planckian value [6–9]. Attempts to engineer trans-Planckian flat directions by using multiple fields are also highly constrained by strong versions of the Weak Gravity Conjecture [47].

As outlined in the introduction, a promising alternative is F-term axion monodromy inflation [13]. The basic idea is to induce a non-periodic potential for the axion while leaving the discrete shift symmetry unbroken. This leads to the familiar multi-branched structure which allows for a non-compact field range for the axion. By rolling down one of the branches a trans-Planckian excursion can be achieved even if the axionic decay constant f (and therefore the underlying periodicity of the system) is sub-Planckian. This implies that the above constraints coming from the WGC do not apply in this case. Furthermore, the discrete shift of the axion if combined with a shift of the integer labeling the different branches is still a symmetry of the theory. This protects the effective theory from dangerous UV corrections coming from states above the cut-off scale.

The realization in four dimensions is given by coupling the axion ϕ to a 3-form gauge field $F_4 = dC_3$ as follows,

$$\mathcal{L} = -f^2(d\phi)^2 - F_4 \wedge *F_4 + 2mF_4 \phi. \tag{2.2}$$

This description was first analyzed in detail by Dvali [48, 49] and applied to inflation by Kaloper and Sorbo [11, 50–52]. The gauge field has no dynamics in four dimensions but its field strength can have a non-vanishing (quantized) value f_0 in the vacuum. Upon integrating out the 3-form field,

$$*F_4 = f_0 + m\phi \rightarrow V = (f_0 + m\phi)^2 \tag{2.3}$$

one recovers the scalar potential for the axion with multiple branches labeled by f_0 . Notice that this is not a particular model of F-term axion monodromy, but a dual formulation in four dimensions, since for any massive axion one can always define an effective 3-form field generating the corresponding scalar potential. This formulation makes the underlying symmetries of the system manifest. In particular, the combined discrete shift

$$f_0 \rightarrow f_0 + c, \quad \phi \rightarrow \phi - c/m \tag{2.4}$$

is still a symmetry of the system, and for $c/m = 2\pi f$ this transformation identifies gauge equivalent branches.

Furthermore, transitions between different branches are mediated by nucleation of membranes electrically charged under the 3-form gauge field. By crossing a membrane, f_0 shifts by an integer times the charge of the membrane. The tunneling rate is exponentially suppressed, and can indeed be estimated by applying the WGC to the 3-form gauge field. However, recent results show that the tunneling rate is not fast enough to constrain large field inflation [53–55].

Remarkably, this is also the mechanism underlying flux stabilization of axions in string theory, since the discrete axionic shift symmetry is indeed a gauge identification and cannot be explicitly broken. As explained, this does not prevent the axions to become massive in a consistent way with the discrete shift symmetry. Thus, all axions arising in string compactifications which are stabilized by internal fluxes are examples of the aforementioned multi-branched structure and candidates for F-term axion monodromy. In those cases, the 3-form fields come from dimensionally reducing higher NS-NS and R-R p -form fields and are dual to the internal fluxes [25, 56].

Despite all these appealing features, including the apparent robustness against the WGC, we think that there does not exist any completely successful and convincing string realization of F-term axion monodromy inflation, yet. The difficulties are related to moduli stabilization and backreaction effects from the other scalars of the compactification.¹ When taking the backreaction into account, the physical field range of the inflaton might be drastically reduced, as we proceed to explain in section 2.2. More than a technical issue, these difficulties might again point towards a fundamental obstruction of any consistent theory of quantum gravity. As noticed in [37, 38], in this case these control issues can be related to the Swampland Conjecture. We will review and extend this relation in section 2.3.

2.2 Challenge with moduli stabilization

Any attempt to construct a realistic inflationary model in string theory has to deal with the issue of moduli stabilization. The strong experimental bounds on non-Gaussianities and isocurvature perturbations favor a scheme of single field inflation or, at most, moderate multi-field inflation involving a few weakly-coupled scalars. To guarantee the consistency of the effective field theory approach as well as to realize a model of single field inflation, one has to stabilize the moduli such that the following hierarchy of mass scales is realized

$$M_{\text{Pl}} > M_{\text{s}} > M_{\text{KK}} > M_{\text{mod}} > H_{\text{inf}} > M_{\theta}, \tag{2.5}$$

where H_{inf} is the Hubble scale during inflation and M_{θ} the inflaton mass. These scales are constrained by the amplitude of scalar density perturbations and the value of the tensor-to-scalar ratio. For chaotic inflation, $M_{\theta} \sim 10^{13}$ GeV and $H_{\text{inf}} \sim 10^{14}$ GeV. Therefore there is not much room left to stabilize the rest of the moduli (M_{mod}) above the inflaton mass

¹From this perspective, inflationary string model building attempts that did not consider these issues are not yet complete and need to be reevaluated.

and below the Kaluza-Klein scale (which is also usually of order $M_{\text{KK}} \sim 10^{16} - 10^{17}$ GeV in perturbative string theory). To achieve this hierarchy of scales at the minimum of the potential is already a challenge for many flux compactifications (see [28, 31] for some no-go theorems for the complex structure moduli space of a Calabi-Yau three-fold). But to guarantee the stabilization of these scales during the whole inflationary trajectory is an even bigger challenge (see also [23, 28, 57, 58]).

Let us assume a pseudo-scalar θ parametrizing the inflationary trajectory. When θ is displaced from its minimum, generically the minima of the other scalars will also change,

$$s(\theta) = s_0 + \delta s(\theta) \tag{2.6}$$

where s_0 denotes the vacuum expectation value of the scalar s at the minimum of the potential, i.e. when θ is also at its minimum. We will use the word *saxions* to refer to all non-periodic (non-axionic) scalars. By plugging this back into the effective theory, the scalar potential and the kinetic term for the inflaton can be substantially modified. In other words, the inflationary trajectory is no longer only along θ but corresponds to a combination of θ and s . This backreaction leads to a flattening of the inflaton potential [36].

Note that the above simple procedure of freezing s and plugging (2.6) back into the effective theory is an approximation that relies on neglecting the variation of the kinetic energy of the saxion with respect to the potential energy, so it is valid only as long as there is a mass hierarchy between θ and s . Otherwise, a multifield analysis is required to consider simultaneously the dynamics of both fields.

In the Kaloper-Sorbo formulation of the axion coupled to the 3-form gauge field, these corrections do not appear from higher dimensional operators breaking the shift symmetry. They arise from the fact that the kinetic metric of the 3-form gauge fields is also field dependent (in particular, it depends on the saxions) [41]. When integrating out the 3-form gauge field, the shape of the branches becomes field dependent and can be substantially modified when displacing the inflaton away from the minimum (in a shift invariant way, but potentially dangerous for inflation anyway).

In [33, 37] it was pointed out that the displacement of the saxions will generically backreact on the kinetic metric of the inflaton leading at best to a logarithmic behavior of the proper field distance at large field. More concretely,

$$\Theta = \int \sqrt{K_{\theta\theta}(s)} d\theta \sim \int \frac{1}{s(\theta)} \sim \frac{1}{\lambda} \log(\theta) \tag{2.7}$$

where we have assumed that $K = -\log(s)$ with s being the saxionic partner of the inflaton, and that for large field excursions $\delta s(\theta) \simeq \lambda\theta$. In (2.7), Θ is the canonically normalized inflaton field. This implies that parametrically large displacements are strongly disfavored in string theory, but in principle trans-Planckian field ranges are still possible if $\lambda \ll 1$, so that backreaction effects can be delayed far out in field space. In other words, the field range available before backreaction effects become important and the logarithmic scaling takes place, is given by

$$\Theta_c = \int^{\theta_c} \sqrt{K_{\theta\theta}(s)} d\theta \sim \frac{\theta_c}{s_0} \sim \frac{1}{\lambda} \tag{2.8}$$

in Planck units. Here θ_c is the critical value before backreaction effects dominate, which occurs when $\delta s(\theta_c) \simeq s_0$ implying² $\theta_c \simeq s_0/\lambda$. In [37, 38] it was claimed that λ is a flux independent parameter of order one, implying that the backreaction effects are therefore tied to the Planck mass. If this is true in general, it is a very powerful statement which indicates a clear obstruction for having trans-Planckian field ranges.

However, the flux independence of λ was only proved [37] in type IIA flux compactifications where the inflaton belonged to the closed string sector. In [41] a possible loophole involving the open string sector was pointed out (and examined in more detail in [42]). There, the parameter λ is not flux-independent anymore but indeed proportional to the mass hierarchy $M_\Theta/M_{\text{heavy}}$. Therefore a mass hierarchy between the inflaton and the saxions can help to delay the backreaction effects which are not anymore tied to the Planck mass. However, the incorporation of more ingredients to the compactification makes the model more difficult to control, and it is not clear if such a hierarchy can be really achieved in a fully reliable global compactification.

It is the purpose of this paper to continue the investigation of these models and similar ones, in which λ can depend on the above mass hierarchy. We will see that in some representative models, by setting λ small, we are inevitably also decreasing the Kaluza-Klein scale compared to the moduli mass scale, signaling the breakdown of the effective theory. But before turning to our results, let us discuss in more detail the relation between the logarithmic scaling of the field distance, the breakdown of the effective theory and the Swampland Conjecture.

2.3 The swampland conjecture

It is clear that not all effective quantum field theories can be obtained as effective theories from string theory. As made more precise in [40], besides the string landscape there exist a vast swampland of such theories that cannot be consistently coupled to quantum gravity. In [39] Ouguri and Vafa formulated this in a more concise manner. They provided a couple of conjectured criteria that an effective theory in the landscape necessarily should satisfy. The most quantitative criterium was termed the Swampland Conjecture in [38] and it says:

Swampland conjecture: for any point p_0 in the continuous scalar moduli space of a consistent quantum gravity theory (the landscape), there exist other points p at arbitrarily large distance. As the distance $d(p_0, p)$ diverges, an infinite tower of states exponentially light in the distance appears, meaning that the mass scale of the tower varies as

$$M \sim M_0 e^{-\alpha d(p_0, p)}. \tag{2.9}$$

Thus, the number of states in the tower which are below any finite mass scale diverges as $d \rightarrow \infty$.

Here, the distance is measured with the metric on the moduli space. Moreover, α is a still undetermined parameter that specifies when this behavior sets in, namely beyond

²If the Kähler metric for the inflaton depends on more than one saxion, one can extract the value of λ from $K_{\theta\theta}^{-1/2}(s^i) \simeq K_{\theta\theta}^{-1/2}(s_0^i) + \delta K_{\theta\theta}^{-1/2}(s^i(\theta))$ with $\delta K_{\theta\theta}^{-1/2}(s^i(\theta)) \simeq \lambda\theta$ at large field, and all previous formulae apply.

$d(p_0, p) \sim \alpha^{-1}$ the exponential drop-off becomes essential. Infinitely many states becoming light beyond a certain distance in field space indicates that the quantum gravity theory valid at the point p_0 only has a finite range d_c of validity in the scalar moduli space. As a consequence any physics that we might derive for larger values $d > d_c$ cannot be trusted.

In this formulation, the flat axion moduli space is assumed to be compact and the logarithmic behavior is expected to hold rather for the saxions. Therefore, it is not immediately clear how this conjecture is related to the question of realizing large field inflation in string theory. How this proceeds has been suggested in [37, 38] and will also be demonstrated in the very explicit prototype models to be discussed in sections 3 and 4. Let us already sketch here, how this works.

Say one has managed to stabilize the moduli such that there is only a single light axion Θ with mass M_Θ and a set of heavy other moduli stabilized at M_{heavy} . Then, after integrating out the heavy moduli one can derive an effective polynomial potential $V_{\text{eff}}(\theta)$ for the light axion, potentially supporting large field inflation. However, this picture is a bit too naive as we are interested in field excursion of θ that are trans-Planckian. As explained in the previous section, for very large θ one has to take the backreaction of the rolling inflaton onto the other moduli into account. The critical value in proper field space where this behavior becomes essential is $\Theta_c \sim 1/\lambda$ (see eq. (2.8)). As discussed above, for field excursions beyond this value, the backreaction causes the following relation between the proper field distance and θ

$$\Theta = \frac{1}{\lambda} \log(\theta) . \tag{2.10}$$

Therefore, e.g. KK-modes whose mass scales like $M_{\text{KK}} \sim s(\theta)^{-n} \sim \theta^{-n}$ have the scaling $M_{\text{KK}} \sim \exp(-n\lambda\Theta)$ with respect to the proper field distance. This is precisely the behavior stated in the Swampland Conjecture after identifying

$$\alpha \sim \lambda . \tag{2.11}$$

Thus, it seems that the original version of the swampland conjecture can be extended to axion directions upon taking into account backreaction effects. It is this generalization that we consider in this paper. Notice that this formulation of the conjecture not only implies a constraint on the field metrics but also on the shape of the scalar potentials coming from string theory, since the backreaction on the saxions is essential to obtain such a logarithmic behaviour at large field.

The essential question now is about the value of λ . The original swampland conjecture leaves this open.³ The set of examples studied in [37] led the authors to define the so-called *Refined Swampland Conjecture*, that in addition to the contents of the swampland conjecture above states $\alpha = O(1)$. We will see that those examples are only particular

³For an axion, the WGC implies $f S_{\text{inst}} \leq 1$ which can be rewritten in the presence of supersymmetry in terms of the saxionic partner φ as $\sqrt{g_{\varphi\varphi}} \varphi \leq 1$ [37]. After integration one gets

$$\int \sqrt{g_{\varphi\varphi}} d\varphi \leq \int \frac{1}{\varphi} d\varphi \iff \phi \leq \log \varphi , \tag{2.12}$$

i.e. the proper field distance grows at best logarithmically as $\phi_c \log \varphi$ with $\phi_c = O(1)$.

cases and that in general one can have

$$\Theta_c \sim \frac{1}{\lambda} \sim \left(\frac{M_{\text{heavy}}}{M_{\Theta}} \right)^p \tag{2.13}$$

where $p = 0, 1$ depending on the model under consideration. In particular, the models in [37] satisfy $p = 0$, while $p = 1$ corresponds to the loopholes in [41, 42]. For the latter class of models, if one can manage to dynamically freeze the moduli such that $\lambda < O(1/10)$, then one has control over the effective theory for the required $N_e = 60$ e-foldings. However we will see that for $\lambda \ll 1$ there are other reasons beyond the exponential drop-off, why the effective theory fails.

3 Closed string models

In this section, we revisit a simple prototype model [32, 33] of closed string moduli stabilization and analyze its relation to the Swampland Conjecture and how this restricts the potential to provide a controllable string (inspired) model of F-term axion monodromy inflation.

In [33] it was found that the considered single field inflationary models with a parametrically light axion fail to also preserve the required hierarchy of mass scales, thus spoiling parametric control over the employed effective action. This perfectly matches with the results found in [37, 41] for their IIA counterparts. Within the closed string sector of IIA flux compactifications with RR and NS fluxes, it is not possible to get the mass hierarchy required to suppress backreaction, implying that one always get a flux-independent $\lambda \sim O(1)$. Therefore, we do not expect these closed string IIB models to work either. However, they are a perfect playground to exemplify the backreaction problems and the relation to the Swampland Conjecture. Therefore, instead of analyzing an exhaustive list of elaborated models, we will choose the simplest one and discuss the problems arising when trying to drive inflation in the regime $\Theta > \Theta_c$.

Let us emphasize that, in this paper, our focus is on analytically solvable models, where in order to be able to compute also the string and KK-scales, all relevant moduli are included. It is clear that e.g. the string and the KK-scales are only dynamically fixed when we include the axio-dilaton as well as the Kähler moduli as dynamical fields.

For the presented representative examples, we focus on the parametric dependence of certain relevant quantities in terms of the background fluxes. Our philosophy is that parametric control is essential to claim that certain mass hierarchies can be naturally achieved. Just an accidental, model dependent numerical factor of e.g. order $O(1) - O(10^2)$ is not sufficient and is certainly not related to general arguments from quantum gravity.

3.1 Moduli stabilization and non-geometric fluxes

Before analyzing concrete models for axion monodromy inflation in detail, let us briefly review the necessary concepts of closed string moduli stabilization with various fluxes in type IIB orientifold compactifications. Later we will not just consider moduli coming from the closed string sector, but are furthermore taking open string moduli into account as

number	modulus	name
1	$S = g_s^{-1} - iC_0$	axio-dilaton
$h_-^{2,1}(\mathcal{M})$	$U^i = u^i + iv^i$	complex structure
$h_+^{1,1}(\mathcal{M})$	$T_\alpha = \tau_\alpha + i\rho_\alpha + \dots$	Kähler
$h_-^{1,1}(\mathcal{M})$	$G^a = Sb^a + ic^a$	axionic odd

Table 1. Closed string moduli in type IIB orientifold compactifications.

they might provide an independent source for inflation. Let us postpone the discussion of open string moduli stabilization to section 4.1.

We start with compactifying type IIB string theory on orientifolds of Calabi-Yau threefolds \mathcal{M} , which are equipped with a holomorphic three-form Ω_3 . The orientifold projection $\Omega_P(-1)^{F_L}\sigma$ contains, besides the world-sheet parity operator Ω_P and the left-moving fermion number F_L , a holomorphic involution $\sigma : \mathcal{M} \rightarrow \mathcal{M}$. We choose the latter to act on the Kähler form J and the holomorphic (3, 0)-form Ω_3 of the Calabi-Yau three-fold \mathcal{M} as

$$\sigma^* : J \rightarrow +J, \quad \sigma^* : \Omega_3 \rightarrow -\Omega_3. \tag{3.1}$$

The fixed loci of this involution correspond to O7- and O3-planes, which in general require the presence of D7- and D3-branes to satisfy the tadpole cancellation conditions. The holomorphic involution σ of the orientifold projection splits the cohomology into even and odd parts

$$H^{p,q}(\mathcal{M}) = H_+^{p,q}(\mathcal{M}) \oplus H_-^{p,q}(\mathcal{M}), \quad h^{p,q} = h_+^{p,q} + h_-^{p,q}. \tag{3.2}$$

Reducing the ten-dimensional bosonic field content of type IIB string theory on the Calabi-Yau threefold \mathcal{M} and taking the orientifold projection into account leads to numerous massless moduli in the effective four-dimensional supergravity theory.

The closed string moduli relevant for later constructions are summarized in table 1, where the convention was chosen such that the imaginary parts of the moduli correspond to axions.⁴

Note that in the following we have redefined the axio-dilaton as $S = s + ic$. Moduli are stabilized by turning on non-trivial background fluxes generating a scalar potential for the moduli, see for instance the review [59]. Here we will not just focus on R-R and NS-NS three-form fluxes, but supplementary make use of geometric and non-geometric fluxes. For more details we refer to [32] as well as references therein.

As already mentioned in the introduction, for single field inflation one needs to achieve a considerable mass hierarchy between the inflaton and the other moduli. The KKLT and

⁴The full definition of the Kähler moduli T_α is given by

$$T_\alpha = \frac{1}{2} \kappa_{\alpha\beta\gamma} t^\beta t^\gamma + i \left(\rho_\alpha - \frac{1}{2} \kappa_{\alpha ab} c^a b^b \right) - \frac{1}{4} e^\phi \kappa_{\alpha ab} G^a (G + \bar{G})^b, \tag{3.3}$$

where $\kappa_{\alpha\beta\gamma}$ denote the triple intersection numbers.

Large Volume Scenarios (LVS) [60, 61] incorporate small non-perturbative effects to fix certain saxionic Kähler moduli, which makes it unnatural to obtain a mass hierarchy with the axionic inflaton stabilized at tree-level. Therefore, it is more natural to fix all moduli already at tree-level by employing geometric and non-geometric fluxes for the stabilization of the Kähler moduli. Such fluxes appear in the context of $\mathcal{N} = 2$ gauged supergravity and double field theory. However, for completeness, we will also analyze models within the framework of KKLT and LVS without non-geometric fluxes in section 4.4.

In addition to the usual R-R and NS-NS three-form fluxes $\mathfrak{F} = \langle dC_2 \rangle$ and $H = \langle dB_2 \rangle$ there are the geometric flux F^I_{JK} and the non-geometric fluxes Q_I^{JK} and R^{IJK} . Including these new fluxes, the Gukov-Vafa-Witten superpotential [62] can be extended in the following compact way [63, 64]

$$W = \int_{\mathcal{M}} \left[\mathfrak{F} + \mathcal{D} \Phi_c^{\text{ev}} \right]_3 \wedge \Omega_3, \tag{3.4}$$

with the complex multi-form $\Phi_c^{\text{ev}} = iS - iG^a \omega_a - iT_\alpha \tilde{\omega}^\alpha$ and the cohomology bases $\{\omega_a\} \in H^{1,1}(\mathcal{M})$ and $\{\tilde{\omega}^\alpha\} \in H^{2,2}(\mathcal{M})$. The twisted differential \mathcal{D} is defined by

$$\mathcal{D} = d - H \wedge - F \circ - Q \bullet - R_{\perp}, \tag{3.5}$$

where the operators appearing in (3.5) implement the mapping

$$\begin{aligned} H \wedge : & \quad p\text{-form} \rightarrow (p+3)\text{-form}, & F \circ : & \quad p\text{-form} \rightarrow (p+1)\text{-form}, \\ Q \bullet : & \quad p\text{-form} \rightarrow (p-1)\text{-form}, & R_{\perp} : & \quad p\text{-form} \rightarrow (p-3)\text{-form}. \end{aligned} \tag{3.6}$$

One can be more specific about the action of \mathcal{D} after introducing a symplectic basis for the third cohomology $H^3(\mathcal{M})$ of the Calabi-Yau threefold. Eventually the non-vanishing flux components⁵ can be summarized by:

\mathfrak{F}	H	F	Q
$\{f_\lambda, \tilde{f}^\lambda\}$	$\{h_\lambda, \tilde{h}^\lambda\}$	$\{f_{\lambda a}, \tilde{f}^\lambda_a\}$	$\{q_\lambda^\alpha, \tilde{q}^{\lambda\alpha}\}$

(3.7)

where $\lambda = 0, \dots, h_-^{2,1}$ and the indices a, α label the moduli G^a, T_α , respectively. Let us stress that all these fluxes, coupling to moduli of the closed string sector, are quantized and may only take integer values.

Introducing the periods X^λ and F_λ of the holomorphic three-form Ω_3 , the complex structure moduli are determined by $U^i = -iX^i/X^0$. In terms of the periods, the superpotential (3.4) simplifies to

$$\begin{aligned} W = & - (f_\lambda X^\lambda - \tilde{f}^\lambda F_\lambda) + iS (h_\lambda X^\lambda - \tilde{h}^\lambda F_\lambda) \\ & + iG^a (f_{\lambda a} X^\lambda - \tilde{f}^\lambda_a F_\lambda) - iT_\alpha (q_\lambda^\alpha X^\lambda - \tilde{q}^{\lambda\alpha} F_\lambda). \end{aligned} \tag{3.8}$$

Apparently the superpotential depends only linearly on the moduli S, G^a, T_α and in particular the Kähler moduli couple to non-geometric fluxes at tree-level. Together with

⁵It turns out that the purely non-geometric R^{IJK} flux does not appear in the superpotential.

the perturbative Kähler potential at large volume and small string coupling

$$K = -\log\left(-i\int_{\mathcal{M}}\Omega_3\wedge\bar{\Omega}_3\right) - \log(S+\bar{S}) - 2\log\mathcal{V}, \quad (3.9)$$

where \mathcal{V} denotes the overall volume of the Calabi-Yau threefold \mathcal{M} in Einstein frame, the flux-induced F-term scalar potential of the moduli in the four-dimensional supergravity theory is given by

$$V_F = e^K\left(K^{I\bar{J}}D_I W D_{\bar{J}}\bar{W} - 3|W|^2\right), \quad (3.10)$$

with Kähler metric $K_{I\bar{J}} = \partial_I\partial_{\bar{J}}K$ and Kähler-covariant derivative $D_I W = \partial_I W + (\partial_I K)W$. In general, this scalar potential stabilizes all the moduli and generates flux-dependent mass terms for them.

The NS-NS fluxes also give rise to generalized Bianchi identities and to Freed-Witten anomaly cancellation conditions. Let us remark that for the examples to be discussed in this paper, these will all be satisfied. Let us finally remark that most non-geometric type IIB fluxes considered in this paper would correspond to geometric fluxes in the T-dual IIA compactification.

3.2 Closed string model: C1

Let us revisit the most simple model of tree-level flux induced moduli stabilization, that only contains the two always present moduli, the axio-dilaton $S = s + ic$ and the overall volume modulus $T = \tau + i\rho$. This exactly solvable example already reveals the main problem with achieving large field inflation for F-term axion monodromy. It can be thought of as an isotropic T^6 with frozen complex structure modulus.

3.2.1 Moduli stabilization, masses and backreaction

At large values of the saxions (s, τ) , the Kähler potential at leading order is given by

$$K = -\log(S+\bar{S}) - 3\log(T+\bar{T}), \quad (3.11)$$

and the flux-induced superpotential is chosen to be

$$W = -if_0 + ihS + iqT. \quad (3.12)$$

The resulting scalar potential reads

$$V = \frac{(hs + f_0)^2}{16s\tau^3} - \frac{6hqs - 2qf_0}{16s\tau^2} - \frac{5q^2}{48s\tau} + \frac{\theta^2}{16s\tau^3} \quad (3.13)$$

with the linear combination $\theta = hc + q\rho$. This field will be our inflaton candidate. There exists a non-supersymmetric, tachyon-free AdS minimum at

$$\tau_0 = \frac{6f_0}{5q}, \quad s_0 = \frac{f_0}{h}, \quad \theta_0 = 0. \quad (3.14)$$

The masses for the canonically normalized fields are

$$M_{\text{mod},i}^2 = \nu_i \frac{hq^3}{f_0^2}, \quad (3.15)$$

with $\nu \in \{0, 0.43, 0.21, 0.78\}$. The cosmological constant in the minimum is $V_0 = -\frac{25}{216} \frac{hq^3}{f_0^2}$.

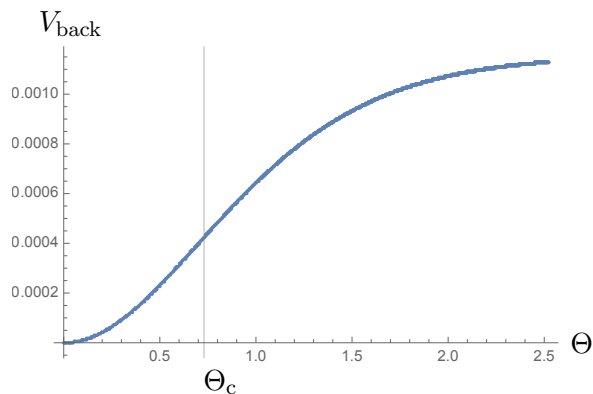


Figure 1. The backreacted potential $V_{\text{back}}(\Theta)$ (after adding a constant uplift) depending on the proper field distance.

Thus, the mass of the axion θ is parametrically of the same order as the masses of the two saxions. Comparing to section 2.3, this means that $\lambda = O(1)$ and the backreaction should set in right at the Planck-scale. Indeed, for field excursions in the direction θ , the backreaction on the saxions can be exactly solved and gives

$$\begin{aligned} \tau_0(\theta) &= \frac{3}{20q} \left(4f_0 + \sqrt{10\theta^2 + 16f_0^2} \right), \\ s_0(\theta) &= \frac{1}{4h} \sqrt{10\theta^2 + 16f_0^2}. \end{aligned} \tag{3.16}$$

Looking at the discriminant, it is clear that beyond the critical field-value $\theta_c = \sqrt{\frac{8}{5}}f_0$ the backreaction becomes substantial. The kinetic term for θ is

$$\mathcal{L}_{\text{kin}}^{\text{ax}} = \frac{3}{4(3h^2s^2 + q^2\tau^2)} \partial_\mu\theta\partial^\mu\theta, \tag{3.17}$$

implying that for $\theta < \theta_c$ the canonically normalized axion is $\Theta = \frac{5}{\sqrt{74}}\frac{\theta}{f_0}$. The critical proper field distance is flux independent $\Theta_c = \sqrt{\frac{20}{37}} \approx 0.73$, i.e. for the canonically normalized axion the backreaction becomes substantial right at the Planck-scale. The backreacted potential as a function of the proper field distance is shown in figure 1. Note that we added a constant uplift.

It is evident that beyond Θ_c the potential is not any more of quadratic form and therefore one cannot realize large field inflation. Indeed, in the trans-Planckian regime one finds

$$\mathcal{L}_{\text{kin}}^{\text{ax}} = \frac{2}{\gamma^2} \left(\frac{\partial\theta}{\theta} \right)^2, \tag{3.18}$$

with $\gamma = 2\sqrt{\frac{7}{5}}$. The canonically normalized field can be defined as

$$\Theta = \frac{2}{\gamma} \log \left(\frac{\theta}{2\theta_c} \right). \tag{3.19}$$

This is precisely the logarithmic behavior (2.10) satisfying $\lambda \sim \mathcal{O}(1)$ expected from the Refined Swampland Conjecture. After assuming a constant uplift by $|V_0|$, the scalar potential reads

$$V_{\text{back}}(\Theta) = |V_0| \left[1 - \left(\frac{2\theta_c}{\theta} \right)^2 \right] = |V_0| [1 - e^{-\gamma\Theta}]. \quad (3.20)$$

Like the Starobinsky model, V_{back} is a plateau potential for $\Theta > \Theta_c$.

Therefore, the strong backreaction led to a significant flattening of the potential, the initial quadratic potential of the axion became plateau-like. If $H_{\text{inf}} < M_{\text{mod}} < M_{\text{KK}}$ could be parametrically guaranteed, the potential (3.20) by itself could still support inflation with a resulting lower value of the tensor-to-scalar ratio

$$r = \frac{8}{(\gamma N_e)^2} \sim \mathcal{O}(10^{-3}). \quad (3.21)$$

This looks promising at a first glance, but as we work just at the limit of having control, there are three serious caveats:

- In the trans-Planckian regime, the KK-masses show the expected exponential drop-off

$$M_{\text{KK}} \sim \frac{1}{\tau} \sim \frac{q}{f_0} \exp\left(-\frac{\gamma}{2}\Theta\right), \quad (3.22)$$

while the inflationary mass scale $M_{\text{inf}} = |V_0|^{\frac{1}{4}}$ stays constant on the plateau. Using the relation $V_0 = 3M_{\text{pl}}^2 H_{\text{inf}}^2$, one finds for the ratio

$$\frac{M_{\text{KK}}}{H_{\text{inf}}} \sim \frac{1}{(qh)^{\frac{1}{2}}} \exp\left(-\frac{\gamma}{2}\Theta_*\right). \quad (3.23)$$

Thus we parametrically get $H_{\text{inf}} \stackrel{>}{\sim} M_{\text{KK}}$ so that we are outside the regime of controlling the effective action.

- We were assuming here a constant uplift potential, which is however not realistic, as in string theory all known potentials drop-off at infinity. The task then is to identify a realistic uplift term that still admits the plateau up to the pivot scale before it drops-off towards larger values for the inflaton. This issue will be addressed below in section 3.2.2.
- Since the mass of the inflaton candidate is of the same scale as the mass of the other moduli, the latter cannot really be integrated out and one has to treat the model in the framework of multifield inflation. This will affect the trajectory and the scalar potential along it.

Thus, this example confirms in an analytically deducible way the statement of the *Refined Swampland Conjecture* even for the case of axionic fields with a shift symmetry. It is the backreaction onto the saxionic fields that limits the parametrically controllable field range to be smaller than the Planck-scale. We have also identified a tower of Kaluza-Klein modes that become exponentially light in the trans-Planckian regime. Hence, even Starobinsky-like inflation on a sufficiently broad plateau is not under parametric control.

As we will explain next, to get such a plateau is also challenged from another perspective, namely by considering more realistic (non-constant) uplift terms. This latter point has also been observed in [23] for a class of models including instanton contributions, like for KKLT or the Large Volume Scenario.

3.2.2 A semi-realistic uplift

So far we were just assuming a constant uplift. Due to the backreaction this implied to a constant plateau for $\Theta \rightarrow \infty$. For models with a realistic uplift potential, like $\overline{D3}$ branes in a warped throat, such a behavior will not happen. Instead there will be another critical value Θ_{up} beyond which the uplift term dominates the backreaction.

For the simple closed string model from section 3.2, it is found that an uplift potential via $\overline{D3}$ branes in a warped throat

$$V_{\overline{D3}} = \frac{\epsilon}{\tau^2} \tag{3.24}$$

does not work as the full potential $V_F + V_{\overline{D3}}$ does not admit tachyon-free Minkowski-minima (after fine-tuning of the warp factor ϵ). In principle, an assumed uplift potential

$$V_{\text{up}} = \frac{\epsilon}{s} \tag{3.25}$$

works much better.⁶ Here, the full potential provides a tachyon-free Minkowski-minimum for the values

$$\tau_0 = \frac{3f_0}{2q}, \quad s_0 = \frac{7f_0}{2h}, \quad \theta_0 = 0, \quad \epsilon = \frac{2q^3}{9f_0}. \tag{3.26}$$

Note that in the perturbative regime ϵ becomes small. The masses for the canonically normalized fields scale in the same way as in the non-supersymmetric AdS minimum

$$M_{\text{mod},i}^2 = \nu_i \frac{h q^3}{f_0^2}, \tag{3.27}$$

with $\nu \in \{0, 0.55, 0.10, 0.87\}$.

When computing the backreaction of a large field excursion of θ onto the saxions, one finds that the scaling (3.16) only holds up to a threshold scale

$$\theta_{\text{up}} \approx 2f_0, \tag{3.28}$$

above which the uplift term becomes dominant. The consequence of this behavior is that for values $\theta > \theta_{\text{up}}$, the local minimum for the saxions is not present any more, i.e. the valley one is following up comes to an end at θ_{up} . This is shown for a concrete choice of fluxes in figure 2. In this example, the critical scale θ_{up} is between θ_c (the convex-concave turning scale of the potential) and the scale where one reaches the top of the plateau. Therefore, for this more realistic non-constant uplift potential, including the backreaction, one can never reach the top of the plateau. Of course, this is just a simple model but, together with the observations made in [23], we think that it exemplifies another generic obstacle

⁶We do not know which string theoretic, supersymmetry breaking object can lead to this functional form of an uplift potential.

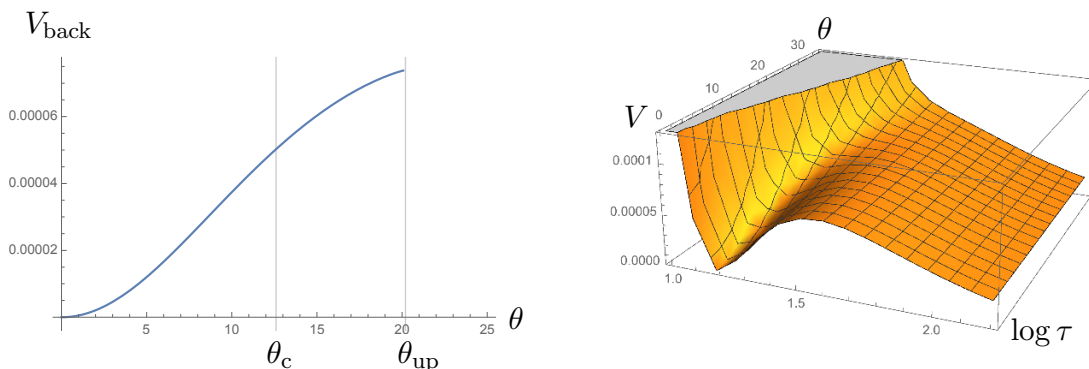


Figure 2. We plotted on the left the backreacted potential $V_{\text{back}}(\theta)$ including the uplift and on the right a slice of the potential $V(\theta, s_0(\tau), \tau)$. Both pictures show the destabilization of the inflationary valley.

to realize plateau-like large field inflation in string theory. We will come back to this point when we discuss large field inflation in KKLT and Large Volume Scenario in section 4.4.

Therefore, it seems clear that one cannot drive inflation in the regime $\Theta > \Theta_c$. After having familiarized ourselves with the relevant issues that appear when one wants to realize large field inflation in a controlled manner, let us now challenge the *Refined Swampland Conjecture* by trying to follow a recent idea on how one could achieve a trans-Planckian critical field value $\Theta_c \gg 1$ by introducing open string fields. Notice that we also found a closed string model showing this feature when incorporating an axionic odd G modulus. As it turned out, this model suffers, however, from the same issues which we will describe in the next section about open string moduli.

4 Open string models

The example in the previous section featured $\Theta_c = O(1)$, providing support for the *Refined Swampland Conjecture*. In this example, Θ_c was flux independent and we had no chance to tune it larger. The aim of this central section of this paper is to provide examples involving brane deformation moduli that admit an in principle tunable flux dependent Θ_c .

4.1 Stabilization of D7-brane moduli

Again, before starting a detailed analysis of models including open string moduli, let us briefly review the necessary conceptual ingredients.

4.1.1 D7-brane deformation moduli

Consider a space-time filling D7-brane with gauge group $U(1)$ wrapping a 4-cycle C_4 of the orientifolded Calabi-Yau threefold \mathcal{M} . The spectrum of the D7-brane leads to two different types of open string moduli in the 4d effective supergravity theory. On the one hand, there are moduli from deformations transverse to the D7-brane, i.e. D7-brane position moduli, and on the other hand we have Wilson lines of the $U(1)$ gauge field on the 4-cycle C_4 , see for instance [65]. As shown in [66], Wilson line moduli are not stabilized by fluxes which

makes them unattractive for our setup. For that reason we are exclusively focusing on D7-brane position moduli denoted by

$$\Phi^I = \varphi^I + i\theta^I \quad \text{with} \quad I = 1, \dots, h_-^{2,0}(C_4). \quad (4.1)$$

If the transverse space of the D7-brane supports 1-cycles, like in a toroidal compactification, the above real fields ϕ^I and θ^I enjoy a shift symmetry. For simplicity we restrict our analysis in the following to the case of a single D7-brane with one complex position modulus Φ .

It is well-known that open string moduli lead to a redefinition of the holomorphic chiral variables. Whereas Wilson line moduli change the Kähler moduli, the D7-brane position moduli we are employing here, modify the axio-dilaton S [65, 67, 68]. For a D7-brane wrapping a 4-cycle T^4 inside $T^6 = T^2 \times T^4$, the redefinition reads

$$S \longrightarrow S - \frac{1}{2} \Phi \frac{\Phi + \bar{\Phi}}{U + \bar{U}}, \quad (4.2)$$

with U being the complex structure modulus of the transverse T^2 . This can be used to determine the Kähler potential. In our prototype models we will compactify on an isotropic six-torus, whose closed string Kähler potential reads

$$K_{\text{cl}} = -3 \log(T + \bar{T}) - \log(S + \bar{S}) - 3 \log(U + \bar{U}). \quad (4.3)$$

Taking now also the open string modulus of the D7-brane into account, according to the redefinition of eq. (4.2), one arrives at the Kähler potential we will use for our prototype models [65]

$$K_{\text{op}} = -3 \log(T + \bar{T}) - 2 \log(U + \bar{U}) - \log \left[(S + \bar{S})(U + \bar{U}) - \frac{(\Phi + \bar{\Phi})^2}{2} \right]. \quad (4.4)$$

It is known that α' corrections from the Dirac-Born-Infeld action of the brane give rise to a non-canonical kinetic term for the inflaton which leads to an additional flattening of the effective scalar potential [22, 44]. These corrections will appear as higher derivative corrections to the above Kähler potential and can have implications in the determination of the critical value Θ_c . However, since we do not have control over all analogous α' corrections in the closed string sector, we will restrict our analysis to leading order in α' in both open and closed string sectors.

Let us finally specify the superpotential we are working with. It was argued in [19, 22] that D7-brane position moduli give rise to a superpotential of the form

$$W \supset \mu \Phi^2. \quad (4.5)$$

Its microscopical origin can be deduced from reducing the DBI and Chern-Simons actions of the D7-brane or from the T-dual type IIA description with D6-branes [42, 56]. Additional motivation of this superpotential arises from F-theory where complex structure and D7 position moduli are put on an equal footing. Let us elucidate this in more detail.

4.1.2 Superpotential for brane deformations

Recall that the D7-brane is wrapping a homological 4-cycle C_4 in a CY threefold ambient space \mathcal{M} and is embedded via a map $\iota : C_4 \rightarrow \mathcal{M}$. In the perturbative type IIB superstring theory the relevant F-term potential is (see e.g. [69])

$$W_o = \int_{\Gamma_5} \Omega_3 \wedge (\iota^* B + F) + \Delta W_o \tag{4.6}$$

where Γ denotes the 5-chain swept out by pulling the D7-brane off the orientifold $O(7)$ -plane. Moreover, $\iota^* B$ denotes the pull-back of the ambient NS-NS two-form B onto the world-volume of the D7-brane. The gauge field strength F on the brane can be expanded into a basis of $H^2(C_4, \mathbb{Z})$ and splits into two-cocycles that are pull-backs from two-cocycles on \mathcal{M} and those whose push-forward to \mathcal{M} is trivial, i.e. $F = F^{\mathcal{M}} + \tilde{F}$.

Clearly, Γ_5 depends on the deformation moduli $\Phi \in H^0(C_4, N_{C_4}) = H^{2,0}(C_4, \mathbb{Z})$ and the induced obstruction appears when by pulling off the brane from the $O7$ -plane a $(0, 2)$ -component of $\mathcal{F} = (\iota^* B + F)$ is generated. Since the CY ambient space itself does not have any closed $(0, 2)$ form, this can only happen if $dB = H \neq 0$ or for the flux components \tilde{F} that are cohomologically trivial on \mathcal{M} . In a toroidal set-up, the generation of such an obstruction via a non-trivial H -flux was demonstrated explicitly in [70]. The discussion of the \tilde{F} fluxes appeared in [67] and for toroidal configurations does not provide a contribution to W_o .

Note that in type IIB the co-chain $\iota^* B$ (for $H = dB$) is not necessarily quantized as an integer. It was argued in [19] that by taking the weak coupling limit of F-theory, an additional term

$$\Delta W_o = \frac{i}{2\pi} \int_{\mathcal{M}} H \wedge \log \left(\frac{P_{D7}}{P_{O7}} \right) \Omega_3 \tag{4.7}$$

appears. Here P_{D7} and P_{O7} are polynomials in the coordinates on the base that vanish at the location of the D7-branes and $O7$ -planes, respectively. In particular, they depend on the complex structure and brane moduli. They arise due to the fact that in F-theory the axio-dilaton is not constant but

$$\tau = \tau_0 + \frac{i}{2\pi} \log \left(\frac{P_{D7}}{P_{O7}} \right) \tag{4.8}$$

in the orientifold limit. In F-theory all fluxes reside in $G_4 \in H^4(Y, \mathbb{Z})$ and are quantized. Therefore, the extra term ΔW_o in the type IIB superpotential can be considered to be necessary for compensating the non-quantization of the term involving $\iota^* B$.

Thus, the naive type IIB superpotential (that treats the brane as a probe, thus ignoring backreaction effects) presumably admits non-quantized open string fluxes, whereas in the full F-theory treatment the quantization of all open and closed string fluxes is manifest.

Since the Kähler potential that we use is motivated by a single D7-brane wrapping the isotropic T^6 , let us lay out what the form of the superpotential could be.

4.1.3 Superpotential for D7-brane on a six-torus

Consider a $T^6 = (T^2)^3$ and on each T^2 we introduce a complex structure via $z_a = x_a + iU_a y_a$ with $a = 1, 2, 3$. Moreover, we introduce a D7-brane wrapping the first two T^2 factors. Since

this brane does not contain any 2-cycles that are trivial in the bulk T^6 , the only source for a brane superpotential is a non-vanishing H -flux. Such a flux will however generate both a bulk and a brane superpotential.

Using the conventions and techniques from [70], let us see what type of terms can in principle be generated. Turning on the general H_3 form flux

$$\begin{aligned}
 H = & h_0 dy_1 \wedge dy_2 \wedge dy_3 \\
 & + h_1 dx_1 \wedge dy_2 \wedge dy_3 + h_2 dy_1 \wedge dx_2 \wedge dy_3 + h_3 dy_1 \wedge dy_2 \wedge dx_3 \\
 & + \tilde{h}_1 dy_1 \wedge dx_2 \wedge dx_3 + \tilde{h}_2 dx_1 \wedge dy_2 \wedge dx_3 + \tilde{h}_3 dx_1 \wedge dx_2 \wedge dy_3 \\
 & + \tilde{h}_0 dx_1 \wedge dx_2 \wedge dx_3,
 \end{aligned} \tag{4.9}$$

introduces a bulk superpotential

$$\begin{aligned}
 W_b = & \left(h_0 - ih_1 U_1 - ih_2 U_2 - ih_3 U_3 - \tilde{h}_1 U_2 U_3 \right. \\
 & \left. - \tilde{h}_2 U_1 U_3 - \tilde{h}_3 U_1 U_2 + i\tilde{h}_0 U_1 U_2 U_3 \right) iS.
 \end{aligned} \tag{4.10}$$

Here all fluxes are integers and, since the H -fluxes do have one leg on each T^2 factor, the Freed-Witten anomaly cancellation condition $\int_{D7} H = 0$ is satisfied. In order to find the open string superpotential, we restrict the three-form onto the brane-worldvolume

$$B_{D7} = h_0 y_3 dy_1 \wedge dy_2 + \dots + \tilde{h}_0 x_3 dx_1 \wedge dx_2. \tag{4.11}$$

Now, we have to check whether this contains a $(0, 2)$ component. Indeed, we find

$$B_{D7}^{(0,2)} = \omega^{(0,2)} \left[\frac{\partial_S W_b}{2\text{Re}(U_3)} (\Phi - \bar{\Phi}) + \left(-h_3 + i\tilde{h}_1 U_2 + i\tilde{h}_2 U_1 + \tilde{h}_0 U_1 U_2 \right) \Phi \right] \tag{4.12}$$

where $\Phi = z_3$ and

$$\omega^{(0,2)} = \frac{d\bar{z}_1 d\bar{z}_2}{4\text{Re}(U_1)\text{Re}(U_2)} \tag{4.13}$$

denotes the $(0, 2)$ -form on the worldvolume of the D7-brane. On the supersymmetric locus $\partial_S W = 0$ the $(0, 2)$ component of B depends holomorphically on the brane position as

$$B_{D7}^{(0,2)} = \left(-h_3 + i\tilde{h}_1 U_2 + i\tilde{h}_2 U_1 + \tilde{h}_0 U_1 U_2 \right) \Phi \omega^{(0,2)}. \tag{4.14}$$

Therefore, the brane position is frozen at $\Phi = 0$. In the full F-theory picture, where the brane is not treated as a probe in a supersymmetric bulk, the bulk/brane superpotential is expected to read

$$\begin{aligned}
 W_{\text{tot}} = & ih_0 S + h_1 U_1 S + h_2 U_2 S + h_3 (U_3 S - \Phi^2) - i\tilde{h}_1 U_2 (U_3 S - \Phi^2) \\
 & - i\tilde{h}_2 U_1 (U_3 S - \Phi^2) - i\tilde{h}_3 U_1 U_2 S - \tilde{h}_0 U_1 U_2 (U_3 S - \Phi^2).
 \end{aligned} \tag{4.15}$$

As we want to deal with the most simple model, we restrict this to the isotropic torus. We do this in two steps. First we set all complex structures to be equal, $U_1 = U_2 = U_3 \equiv U$. Then (4.15) becomes

$$\begin{aligned}
 W_{\text{tot}} = & ih_0 S + (h_1 + h_2 + h_3) U S - h_3 \Phi^2 - i(\tilde{h}_1 + \tilde{h}_2 + \tilde{h}_3) U^2 S \\
 & + i(\tilde{h}_1 + \tilde{h}_2) U \Phi^2 - \tilde{h}_0 (U^3 S - U^2 \Phi^2).
 \end{aligned} \tag{4.16}$$

Still treating the various fluxes as independent parameters, the coefficients of e.g. the US -term and the C^2 -term could be disentangled. In the following, we will call this the *weakly isotropic* torus. In section 4.2, we will present an exactly solvable toy model of this type. Since it has the advantage of being exactly solvable, many of the issues about large field excursions can be seen very explicitly.

However, thinking of the isotropic torus as proper Calabi-Yau with only one complex structure modulus, one would not expect to have more components of the H -flux available than the number of three cycles, that would be $b_3 = 4$. This is the reason why for the *strongly isotropic* torus, we also restrict the fluxes to be symmetric, i.e. $h_1 = h_2 = h_3 \equiv \mu_1$, $\tilde{h}_1 = \tilde{h}_2 = \tilde{h}_3 \equiv \mu_2$ and $\tilde{h}_0 \equiv \mu_3$. In this case the superpotential (4.15) becomes

$$W_{\text{tot}} = ih_0 S + \mu_1(3US - \Phi^2) - i\mu_2(3U^2S - 2U\Phi^2) - \mu_3(U^3S - U^2\Phi^2) \quad (4.17)$$

and a $U^n\Phi^2$ term is always accompanied by a corresponding $U^{n+1}S$ term. We will also discuss examples of this more realistic type in section 4.3.

4.1.4 Criteria for models with tunable Θ_c

The purpose of introducing open string fields relies on extending our analysis to models with a tunable flux-dependent critical value Θ_c . Then, one might be able to delay the backreaction and the consequent exponential drop-off of the massive states to a trans-Planckian value for the inflaton $\Theta_c > 1$. As first remarked in [41], this requires the minimum of the potential to satisfy the following condition:

Θ_c will be tunable if one can set the inflaton mass to zero without destabilizing the other scalars.

In other words, one needs to engineer a flat direction which is stabilized by an additional subleading flux μ in a second step. The new minimum will correspond then to the old minimum (without the inflaton) corrected by a term proportional to μ . This is precisely the approach that was also followed in [31] and for the *flux scaling models* considered in [32]. It turns out that the backreacted minima for the saxions — once we move the inflaton away from its minimum — take the following schematic form,

$$s = s_0 + \delta s(\theta), \quad \delta s(\phi) \simeq \lambda \theta \quad (4.18)$$

with λ depending on the mass hierarchy as $\lambda \sim (M_\Theta/M_{\text{heavy}})^p$. In the closed string models of section 3 and those first analyzed in [37], the above condition is not satisfied since the value of s_0 blows up in the limit $\mu \rightarrow 0$. In those models, the critical canonical field distance before the logarithmic behavior dominates is inevitably fixed at $\Theta_c = \lambda^{-1} = O(1)$ in Planck units (or equivalently $p = 0$). The inclusion of open string fields allows us to engineer models with $p = 1$ that satisfy the previous condition.

Let us consider the flux superpotential (4.17) of the effective theory of a D7-brane living in a strongly isotropic torus derived in the previous section. Every term Φ^2 is accompanied by a bulk term SU . This implies that the only superpotential term for the dilaton which is independent of Φ is the linear term ih_0S . Therefore, we need to have $h_0 \neq 0$ in order

to stabilize the dilaton while keeping $\theta = \text{Im}(\Phi)$ massless. We also assume that there are some RR fluxes stabilizing the complex structure modulus U and a non-geometric flux stabilizing T via a superpotential term iqT . We are left then with two possibilities:

- $\mu_1 \neq 0$ and/or $\mu_3 \neq 0$.

As a consequence the superpotential mixes real and imaginary parts of the moduli differently (i.e. even and odd powers of the fields), e.g.

$$W = ihS + \mu_1(3US - \Phi^2) + \dots \tag{4.19}$$

The new minimum cannot be understood as a deformation of the old minimum proportional to μ_1 . In particular, the orthogonal direction to the axionic combination $\sigma_0 = hc_0 + q\rho_0$ remains unfixed in the old minimum and gets a vacuum expectation value in the new minimum proportional to μ_1^{-1} . This modifies the vevs of the saxions leading to the same parametric dependence on μ_1^{-1} , so that we do not recover the old minima when setting $\mu_1 = 0$. The strong backreaction then implies $\lambda \sim O(1)$ independently of the flux choice. A solution comes from adding a term q_1UT with the non-geometric flux satisfying $q_1 = q\mu_1/h$, which vanishes when μ_1 goes to zero. In this way, the problematic axionic direction remains unfixed and the new minimum is simply a deformation of the old minimum, giving rise to a good candidate for having a flux-tunable λ .

- $\mu_1 = \mu_3 = 0$.

The only possibility to stabilize the open string modulus is now to turn on μ_2 , hence

$$W = ihS + i\mu_2U(3US - 2\Phi^2) + \dots \tag{4.20}$$

This model enters within the class of *flux-scaling models* analyzed in [32]. The new minimum can be understood as a deformation of the old minimum which goes to zero when μ is vanishing. This model is thus a good candidate to obtain a λ depending on the flux-tunable mass hierarchy.

For later convenience, we dub the first model with $\mu_1, q_1 \neq 0$ as O2 and analyze it further in section 4.3. Let us remark, though, that we get the same conclusions from analyzing the model with $\mu_2 \neq 0$ and we do not include the explicit analysis simply to avoid cluttering and repetition of results. We will also analyze an extension of O2 by having both μ_1 and μ_3 non-vanishing. This allows us to discuss an example in which the μ -parameter entering on λ is not a flux integer but an effective parameter depending also on field vacuum expectation values. Notice that the other possibility, having both μ_1 and μ_2 non-vanishing, does not really lead to an effective parameter. This is due to the relative factor of $i = \sqrt{-1}$ in the superpotential.

In addition, one can also consider the weakly isotropic torus (4.16) which allows us to drop the condition of having the same flux parameter for the SU and Φ^2 terms. In this manner we can stabilize the dilaton independently of the inflaton, without the need of a linear term ihS . The new minimum will be a deformation of the old minimum, yielding

a good candidate for having again a tunable flux-dependent λ . Due to its computational simplicity, we will first analyze this model, dubbed as *O1*, in section 4.2, and leave the model *O2* for section 4.3.

Our analysis will show that, in spite of having in principle a tunable flux dependent λ , the flux choice required to delay backreaction cannot be done without losing parametric control of the effective theory. In particular, by requiring a mass hierarchy leading to $\lambda < 1$, the moduli masses become heavier than the Kaluza-Klein scale.

4.2 Open string model: O1

Consider now the so-called *STU*-model extended by a complex open string modulus Φ that parametrizes the transversal deformation of the D7-brane. Here the four complex moduli are

$$S = s + ic, \quad T = \tau + i\rho, \quad U = u + iv, \quad \Phi = \varphi + i\theta \quad (4.21)$$

where the imaginary parts are axion-like scalars. At large values of the saxions (s, τ, u) , the Kähler potential at leading order is given as

$$K = -3 \log(T + \bar{T}) - 2 \log(U + \bar{U}) - \log \left[(S + \bar{S})(U + \bar{U}) - \frac{1}{2}(\Phi + \bar{\Phi})^2 \right]. \quad (4.22)$$

As we have seen, the model could be realized as a D7-brane wrapping a four-cycle T^4 on an isotropic $T^6 = (T^2)^3$. Now we turn on fluxes to generate the superpotential

$$W = f_0 + 3f_2 U^2 - h S U - q T U - \mu \Phi^2. \quad (4.23)$$

Note that for the *strongly isotropic* torus, the fluxes h and μ would not be independent. Thus, this model only makes sense for the *weakly isotropic* torus and could therefore still be in the swampland. Nevertheless, as we will see, it reveals many interesting features and hence is a very good toy model to sharpen our tools. Furthermore, in a more complicated Calabi-Yau, one could aim to disentangle the h and μ fluxes via additional bilinear couplings of the dilaton to other complex structure moduli that contribute to the first but not to the second one. Therefore, it is a good candidate to exemplify the problems arising even if one manages to get $h \neq \mu$. Let us mention that this model is related via mirror-symmetry to a type IIA model with only geometric fluxes.⁷

⁷Applying three T-dualities in the three x -directions (of $(T^2)^3$), one gets a type IIA flux model, where the D7 becomes a D6-brane and the complex structure moduli get exchanged with the Kähler moduli. The Kähler potential reads

$$K = -3 \log(U + \bar{U}) - 2 \log(T + \bar{T}) - \log \left[(S + \bar{S})(T + \bar{T}) - \frac{1}{2}(\Phi + \bar{\Phi})^2 \right], \quad (4.24)$$

and the superpotential

$$W = f_6 + 3f_2 T^2 - f_0 S T - f_1 U T - \mu \Phi^2. \quad (4.25)$$

Here f_6 denotes a R-R six-form flux, f_2 a R-R two-form flux and f_i geometric fluxes.

4.2.1 Moduli stabilization and masses

This model admits an analytically solvable non-supersymmetric tachyon-free AdS minimum at

$$\begin{aligned}
 s_0 &= \frac{2^{\frac{7}{4}} \cdot 3^{\frac{1}{2}} (f_0 f_2)^{\frac{1}{2}}}{5^{\frac{1}{4}} h}, & \tau_0 &= \frac{5^{\frac{3}{4}} \cdot 3^{\frac{1}{2}} (f_0 f_2)^{\frac{1}{2}}}{2^{\frac{1}{4}} q} \\
 u_0 &= \frac{1}{10^{\frac{1}{4}} \cdot 3^{\frac{1}{2}}} \left(\frac{f_0}{f_2} \right)^{\frac{1}{2}}, & \varphi_0 &= 0 \\
 v_0 &= hc_0 + q\rho_0 = \theta_0 = 0,
 \end{aligned} \tag{4.26}$$

leaving one axionic direction unconstrained. The value of the scalar potential in the AdS minimum is

$$V_0 = -\frac{1}{120 \cdot 3^{\frac{1}{2}} \cdot 10^{\frac{1}{4}} f_0^{\frac{3}{2}} f_2^{\frac{1}{2}}} h q^3. \tag{4.27}$$

For the canonically normalized mass-matrix we obtain

$$M_{\text{closed}}^2 = \nu_i \frac{h q^3}{f_0^{\frac{3}{2}} f_2^{\frac{1}{2}}} \tag{4.28}$$

with $\nu \in \{0, 0.0001, 0.0019, 0.0029, 0.0117, 0.0162\}$ and

$$\begin{aligned}
 M_\phi^2 &= 0.0022 \left[1 + 14 \frac{\mu}{h} + 24 \left(\frac{\mu}{h} \right)^2 \right] \frac{h q^3}{f_0^{\frac{3}{2}} f_2^{\frac{1}{2}}} \simeq 0.0022 \frac{h q^3}{f_0^{\frac{3}{2}} f_2^{\frac{1}{2}}} \\
 M_\theta^2 &= 0.0065 \mu \frac{(3.1623 + 8 \frac{\mu}{h}) q^3}{f_0^{\frac{3}{2}} f_2^{\frac{1}{2}}} \simeq 0.0205 \frac{\mu q^3}{f_0^{\frac{3}{2}} f_2^{\frac{1}{2}}}
 \end{aligned} \tag{4.29}$$

where on the right hand side we assumed $\mu/h \ll 1$. Therefore, in this regime the open string axion θ is *parametrically* lighter than all the other massive moduli, indeed

$$\frac{M_{\text{heavy}}}{M_\Theta} \sim \sqrt{\frac{h}{\mu}} = \lambda^{-1}. \tag{4.30}$$

Comparing this to the relation (2.13) from the general discussion of the Swampland Conjecture, one expects that $\lambda = \sqrt{\mu/h}$ is the now flux dependent parameter that controls the backreaction of the inflaton onto the other moduli.

4.2.2 Backreaction

We can now analyze the model further, in particular to relate to the general swampland discussion in section 2.3.

Since this model features a parametrically light axion mass, we expect that the backreaction in the slow-roll regime is also under control. Let us analyze this in more detail under the assumption $\lambda \ll 1$. Up to subleading corrections of order $\mathcal{O}(\lambda^2)$, the conditions

for the backreacted minima can be solved

$$\begin{aligned}
 s_0(\theta) &\sim \frac{2^{\frac{7}{4}} 3^{\frac{1}{2}} (f_0 + \mu\theta^2)^{\frac{1}{2}} f_2^{\frac{1}{2}}}{5^{\frac{1}{4}} h}, & \tau_0(\theta) &\sim \frac{5^{\frac{3}{4}} 3^{\frac{1}{2}} (f_0 + \mu\theta^2)^{\frac{1}{2}} f_2^{\frac{1}{2}}}{2^{\frac{1}{4}} q} \\
 u_0(\theta) &\sim \frac{1}{10^{\frac{1}{4}} 3^{\frac{1}{2}}} \left(\frac{f_0 + \mu\theta^2}{f_2} \right)^{\frac{1}{2}}
 \end{aligned} \tag{4.31}$$

with all other fields sitting in their minimum at zero. Thus, the critical value of θ where the backreaction becomes significant is

$$\theta_c = \sqrt{\frac{f_0}{\mu}}. \tag{4.32}$$

The kinetic term for the inflaton becomes

$$\mathcal{L}_{\text{kin}}^{\text{ax}} = K_{\Phi\bar{\Phi}} \partial_\mu \theta \partial^\mu \theta = \frac{1}{8} \sqrt{\frac{5}{2}} \frac{h}{f_0 + \mu\theta^2} (\partial\theta)^2 \tag{4.33}$$

so that the critical value for the canonically normalized inflaton field Θ is

$$\Theta_c = \gamma \sqrt{\frac{h}{f_0}} \theta_c = \gamma \sqrt{\frac{h}{\mu}} = \gamma \lambda^{-1} \tag{4.34}$$

with $\gamma = \frac{1}{2} \left(\frac{5}{2}\right)^{\frac{1}{4}} = 0.63$. Therefore, from this perspective, for $\lambda \ll 1$ and $\Theta \ll \Theta_c$ the backreaction can be neglected and one gets the effective potential for the inflaton (after adding a constant uplift)

$$V_{\text{eff}} \simeq \frac{\mu h q^3}{f_0^{\frac{7}{2}} f_2^{\frac{1}{2}}} (2f_0\theta^2 + \mu\theta^4) \simeq \frac{\mu h q^3}{f_0^{\frac{5}{2}} f_2^{\frac{1}{2}}} \theta^2 \simeq \frac{\mu q^3}{f_0^{\frac{3}{2}} f_2^{\frac{1}{2}}} \Theta^2. \tag{4.35}$$

Note that the quartic term is parametrically suppressed by a factor θ^2/θ_c^2 relative to the quadratic one. Thus, it seems that by parametrically choosing $\Theta_c \sim \lambda^{-1} > 10$ one can achieve a stringy model featuring large field inflation with a quadratic potential. This is consistent with the observation already made in [42] for a more complicated, only numerically treatable open string model (without non-geometric fluxes).

Beyond the critical value, the kinetic term for the inflaton takes the form

$$\mathcal{L}_{\text{kin}}^{\text{ax}} = \frac{1}{8} \sqrt{\frac{5}{2}} \frac{h}{\mu} \left(\frac{\partial\theta}{\theta} \right)^2 \tag{4.36}$$

so that the canonically normalized inflaton shows the logarithmic behavior

$$\Theta = \Theta_c \log \left(\frac{\theta}{\theta_c} \right) \simeq \frac{1}{\lambda} \log \theta \simeq \frac{M_{\text{heavy}}}{M_\Theta} \log \theta. \tag{4.37}$$

Let us mention that, in this regime, the backreacted scalar potential (after constant uplift) becomes

$$V_{\text{back}} \simeq |V_0| \left[1 - \left(\frac{\theta_c}{\theta} \right)^3 \right] = |V_0| \left[1 - \exp \left(-3 \frac{\Theta}{\Theta_c} \right) \right]. \tag{4.38}$$

Thus, in this large field regime $\Theta \gg \Theta_c$ the backreacted potential is not polynomial but of Starobinsky-like type.

4.2.3 Mass scales and the swampland conjecture

From the previous section, the model seems promising to realize large field inflation with an effective quadratic potential once we are able to choose the fluxes such that $\Theta_c \sim \lambda^{-1} \gg 1$ and $\Theta < \Theta_c$. Thus we need $h/\mu = O(10^2)$. This could easily be achieved, if the flux μ could be tuned much smaller than one. However, the origin of this flux in F-theory suggests that also this open string flux is a quantized integer (see section 4.1.2). In this case, one can only introduce a large flux $h > O(10^2)$.

The question is whether such large fluxes are consistent with the use of the low-energy effective field theory that we employed for our analysis. To see what happens let us consider the various mass scales, like string scale, Kaluza-Klein scales, heavy moduli masses and the inflaton mass. As mentioned in the beginning of this section, we will not be concerned with model dependent numerical prefactors, but will focus on desired mass hierarchies that are guaranteed or spoiled parametrically.

Thus, up to numerical coefficients, the relevant masses scale in the following way with the fluxes (recall that we set $M_{\text{pl}} = 1$): the string scale is

$$M_s^2 \sim \frac{1}{\tau^{\frac{3}{2}} s^{\frac{1}{2}}} \sim \frac{h^{\frac{1}{2}} q^{\frac{3}{2}}}{f_0 f_2}. \tag{4.39}$$

Moreover, considering our model as being realized on the isotropic T^6 , we now have *two* Kaluza-Klein scales

$$M_{\text{KK}}^2 \sim \frac{1}{\tau^2} u^{\pm 1}, \tag{4.40}$$

for $u > 1$, yielding a *heavy* and a *light* Kaluza-Klein mass

$$M_{\text{KK,h}}^2 \sim \frac{q^2}{f_0^{\frac{1}{2}} f_2^{\frac{3}{2}}}, \quad M_{\text{KK,l}}^2 \sim \frac{q^2}{f_0^{\frac{3}{2}} f_2^{\frac{1}{2}}}. \tag{4.41}$$

Recall that the mass of the heavy moduli and the inflaton scaled as

$$M_{\text{mod}}^2 \sim \frac{h q^3}{f_0^{\frac{3}{2}} f_2^{\frac{1}{2}}}, \quad M_{\Theta}^2 \sim \frac{\mu q^3}{f_0^{\frac{3}{2}} f_2^{\frac{1}{2}}}. \tag{4.42}$$

Therefore, one gets

$$\frac{M_s^2}{M_{\text{KK,h}}^2} \sim \left(\frac{h f_2}{q f_0} \right)^{\frac{1}{2}}. \tag{4.43}$$

Thus, by choosing the fluxes $\{f_0, f_2, h, q\}$ all of the same size, parametrically one can still keep all moduli at the boundary of the perturbative regime and have the *heavy* KK-scale parametrically not bigger than the string scale, i.e. $M_s \stackrel{\simeq}{p} M_{\text{KK,h}}$.

To relate the mass structure of this model to the Swampland Conjecture, reviewed in section 2.3, we can also evaluate the various mass-scales in the large field regime. Due to (4.31), this means that we just have to change

$$f_0 \rightarrow f_0 \left(\frac{\theta}{\theta_c} \right)^2 \rightarrow f_0 \exp \left(2 \frac{\Theta}{\Theta_c} \right) \tag{4.44}$$

so that the string scale becomes

$$M_s^2 = M_s^2|_0 \exp\left(-2\frac{\Theta}{\Theta_c}\right). \quad (4.45)$$

Similarly, the KK-scales in the large field regime are

$$M_{\text{KK,h}}^2 = M_{\text{KK,h}}^2|_0 \exp\left(-\frac{\Theta}{\Theta_c}\right), \quad M_{\text{KK,l}}^2 = M_{\text{KK,l}}^2|_0 \exp\left(-3\frac{\Theta}{\Theta_c}\right) \quad (4.46)$$

and for the heavy moduli masses we obtain

$$M_{\text{mod}}^2 = M_{\text{mod}}^2|_0 \exp\left(-3\frac{\Theta}{\Theta_c}\right). \quad (4.47)$$

Therefore, all these mass scales show the expected exponential drop off (2.9) at large values in the field space. Thus, for very large values of Θ/Θ_c we have many exponentially light states that invalidate the use of the low-energy effective action. For still moderate values of Θ/Θ_c , one might argue that this by itself would not be disastrous, as long as the order is preserved. However, we also get

$$\frac{M_s^2}{M_{\text{KK,h}}^2} = \frac{M_s^2}{M_{\text{KK,h}}^2|_0} \exp\left(-\frac{\Theta}{\Theta_c}\right) \quad (4.48)$$

which means that for field excursions $\Theta/\Theta_c > 1$ all *heavy* KK-states are heavier than the string scale, i.e. $M_{\text{KK,h}} \gtrsim_p M_s$. This invalidates the usage of the low-energy effective supergravity action.

This is all consistent with the Swampland Conjecture. The question now is whether we also get constraints for the critical value $\Theta_c \sim \lambda^{-1}$. Can it really be tuned by fluxes to be larger than M_{pl} or do we find support for the *Refined Swampland Conjecture* that says Θ_c is close to M_{pl} ?

For this purpose, let us consider the quotient of the *light* KK-mass and the heavy moduli mass

$$\frac{M_{\text{KK,l}}^2}{M_{\text{mod}}^2} \sim \frac{1}{h q}. \quad (4.49)$$

This ratio is independent of f_0 and therefore of Θ in the large field regime. Now, we can distinguish two cases:

1. In the case that we could tune λ small by choosing the open string flux μ small, there is no problem with the mass hierarchies. As discussed in section 4.1.2, this would be in principle possible if one just considers the naive type IIB form of the open string superpotential.
2. However, in the backreacted F-theory picture μ is quantized. It is obvious that for large H -flux h (i.e. $\lambda \ll 1$) the ratio (4.49) is parametrically smaller than one and the moduli masses are heavier than the KK-mass. This spoils the usage of an effective four-dimensional effective action for studying the stabilization of the former massless moduli.⁸

⁸Recall that for the strongly isotropic torus, one has $\mu = h$ and therefore $\Theta_c = O(1)$ from the very beginning.

For case 2. one has $\lambda = O(1)$ and consequently $\Theta_c = O(1)$. Thus, we found evidence that the distance in proper field space Θ , where the logarithmic behavior sets in, is around the Planck-scale and cannot be much increased without invalidating the effective theory. In addition, this means that the inflaton cannot be kept parametrically lighter than the other moduli. Therefore, integrating out the latter first is not a self-consistent approach. We emphasize that this is precisely what the *Refined Swampland Conjecture* states.

With $\Theta_c = O(1)$ for trans-Planckian field excursions one gets the plateau-like potential (4.38). Analogous to the former closed string example, for the ratio of the KK-scale to the Hubble scale one finds

$$\frac{M_{\text{KK},1}}{H_{\text{inf}}} \sim \frac{1}{(qh)^{\frac{1}{2}}} \exp\left(-\frac{3\Theta_*}{2\Theta_c}\right). \tag{4.50}$$

We again find the parametric relation $H_{\text{inf}} \gtrsim_p M_{\text{KK},1}$. Having KK-modes lighter than the Hubble scale, spoils the possibility of realizing large field plateau-like inflation in a controlled way.

4.3 Open string model: O2

Let us now consider a model on the strongly isotropic torus. Unfortunately, it is not exactly solvable, but the intuition we gained from the previous examples, allows us to extract the value of λ at least in a perturbative approach. Here we follow the procedure described in section 4.1.4 and laid out in [31, 32], i.e. in a first step we freeze all moduli except the axionic inflaton candidate. Then we scale these fluxes up and introduce an additional order one flux to freeze the inflaton. As long as the initial values of the moduli are shifted only slightly, we can integrate them out and determine an effective potential for the inflaton. This allows us to read off the ratio of the heavy moduli masses and the inflaton masses. From the former analysis, we expect that this ratio is directly related to $\Theta_c = \lambda^{-1}$, the scale which determine the backreaction.

4.3.1 Moduli stabilization and masses

The model is defined by the same Kähler potential (4.22) and the superpotential

$$W = \Lambda \left(i\tilde{f}_1 U + i\tilde{f}_0 U^3 + ihS + iqT \right) - \mu_1 (3US - \Phi^2) - q_1 3UT, \tag{4.51}$$

where Λ is a large scaling factor of the four fluxes that, in the first step, will fix all four saxions and two axionic directions. It turns out that the effective approach is only justified if one choose $hq_1 - q\mu_1 = 0$, i.e. that only the axionic combination $hc + qp$ appears in the superpotential. Thus, the orthogonal combination will remain massless. Otherwise, we would not recover the old minimum when setting $\mu_1 = 0$ and the strong backreaction would imply $\Theta_c \sim \mathcal{O}(1)$ from the very beginning.

In the first step, we set $\mu_1 = q_1 = 0$ and find that there exist a tachyon-free non-supersymmetric minimum at

$$\begin{aligned}
 s_0 &= \frac{2^{\frac{5}{4}} \cdot 5^{\frac{1}{2}}}{3^{\frac{9}{4}}} \frac{f_1^{\frac{3}{2}}}{h \tilde{f}_0^{\frac{1}{2}}}, & \tau_0 &= \frac{5^{\frac{1}{2}}}{2^{\frac{3}{4}} \cdot 3^{\frac{5}{4}}} \frac{f_1^{\frac{3}{2}}}{q \tilde{f}_0^{\frac{1}{2}}} \\
 u_0 &= \frac{5^{\frac{1}{2}}}{2^{\frac{1}{4}} \cdot 3^{\frac{3}{4}}} \left(\frac{f_1}{\tilde{f}_0} \right)^{\frac{1}{2}}, & \varphi_0 &= 0 \\
 v_0 &= hc_0 + q\rho_0 = 0,
 \end{aligned}
 \tag{4.52}$$

leaving one axionic direction unconstrained. The masses of the massive moduli are all of the same scale

$$M_{\text{heavy}}^2 \sim \frac{\Lambda^2 h q^3 \tilde{f}_0^{\frac{5}{2}}}{f_1^{\frac{9}{2}}}. \tag{4.53}$$

In the second step we now scale Λ up and turn on the small fluxes μ_1 and q_1 . Since the axion $\theta = \text{Im}(\Phi)$ only appears in these extra term in W , we expect that it receives a small mass. In order to estimate it, we integrate out the former stabilized heavy moduli and compute an effective scalar potential for θ . In this regime, the canonically normalized mass of the axion Θ is

$$M_{\Theta}^2 \sim \frac{\mu_1^2 q^3 \tilde{f}_0^{\frac{3}{2}}}{h f_1^{\frac{7}{2}}}, \tag{4.54}$$

so that, for the scale where the backreaction is expected to become substantial, we obtain

$$\Theta_c \sim \frac{M_{\text{heavy}}}{M_{\Theta}} \sim \frac{\Lambda h \tilde{f}_0^{\frac{1}{2}}}{\mu_1 f_1^{\frac{1}{2}}} \gg 1. \tag{4.55}$$

This is large for a sufficiently large flux-scaling factor Λ . Note that at this stage, Θ_c is flux dependent and by appropriate choices can be tuned large.

As in the previous example O1, let us compute the various mass scales. We obtain for the string scale, the heavy and light KK-scales in the minimum

$$M_s^2 \sim \frac{h^{\frac{1}{2}} q^{\frac{3}{2}} \tilde{f}_0}{f_1^3}, \quad M_{\text{KK,h}}^2 \sim \frac{q^2 \tilde{f}_0^{\frac{1}{2}}}{f_1^{\frac{5}{2}}}, \quad M_{\text{KK,l}}^2 \sim \frac{q^2 \tilde{f}_0^{\frac{3}{2}}}{f_1^{\frac{7}{2}}}. \tag{4.56}$$

For the ratio of the string and the heavy KK-scale one finds

$$\frac{M_s^2}{M_{\text{KK,h}}^2} \sim \left(\frac{h \tilde{f}_0}{q f_1} \right)^{\frac{1}{2}} \gg 1, \tag{4.57}$$

that we require to be parametrically larger than one. However, the ratio of the light KK-scale and the heavy moduli mass is given by

$$\frac{M_{\text{KK,l}}^2}{M_{\text{heavy}}^2} \sim \frac{1}{\Lambda^2 q^2} \left(\frac{q f_1}{h \tilde{f}_0} \right)^{\frac{1}{p}} \lesssim 1 \tag{4.58}$$

which becomes parametrically small for large Λ . Therefore, even to get all the high scales in the correct order, we can at best work at the boundary of parametric control, where all fluxes are of order $O(1)$. However, in this case also the critical field distance becomes of order one $\Theta_c = O(1)$ for quantized flux μ_1 .

The only possible loop-hole could be that μ_1 is not quantized and can be significantly smaller than one. This will be analyzed next.

4.3.2 A comment on tuning in the landscape

From the discussed examples it is clear that a possible loop-hole is the assumption about the quantization of the fluxes. Of course, all the fluxes in the initial superpotential are quantized but, following the idea of the landscape, one could imagine that it is a linear combination of terms that leads to an effective flux μ_{eff} that eventually appears in Θ_c . This effective flux could depend, not only on flux integers but, also on vacuum expectation values of other fields. Here we present a model which exemplifies the above idea and discuss the difficulties to get a substantial tuning.

In the framework of the isotropic torus, we can extend the model O2 by additional flux induced terms⁹

$$W = \Lambda \left(i\mathfrak{f}_1 U + i\tilde{\mathfrak{f}}_0 U^3 + ih S + iq T \right) - \mu_1 (3US - \Phi^2) - q_1 3UT + \mu_3 U^2(US - \Phi^2) + q_3 U^3 T. \tag{4.59}$$

Again, to control the minimum of the potential we choose the fluxes such that only the combination $hc + q\theta$ appears in W , i.e. $hq_1 - q\mu_1 = hq_3 - q\mu_3 = 0$. This guarantees that all Bianchi identities are satisfied, as well. Integrating out the heavy moduli, the mass of the inflaton takes the same form as in (4.54)

$$M_{\Theta}^2 \sim \frac{\mu_{\text{eff}}^2 q^3 \tilde{\mathfrak{f}}_0^{\frac{3}{2}}}{h \mathfrak{f}_1^{\frac{7}{2}}}, \tag{4.60}$$

but with an effective flux parameter

$$\mu_{\text{eff}}^2 = \mu_1^2 - \frac{5}{12\sqrt{6}} \left(\frac{\mathfrak{f}_1}{\tilde{\mathfrak{f}}_0} \right) \mu_1 \mu_3 + \frac{25}{54} \left(\frac{\mathfrak{f}_1}{\tilde{\mathfrak{f}}_0} \right)^2 \mu_3^2. \tag{4.61}$$

As mentioned above, this effective parameter is also moduli dependent and therefore is certainly not an integer. The question is whether in the perturbative regime $\mathfrak{f}_1 > \tilde{\mathfrak{f}}_0$ (so $s_0, \tau_0 > 1$), the effective flux can be non-zero and significantly smaller than one. First, for $\mu_1 \neq 0$, the effective flux μ_{eff} can be expressed as

$$\mu_{\text{eff}}^2 = \frac{63}{64} \mu_1^2 + \frac{25}{54} \left(\frac{\mathfrak{f}_1}{\tilde{\mathfrak{f}}_0} \right)^2 \left(\mu_3 - \frac{3\sqrt{3}}{20\sqrt{2}} \left(\frac{\tilde{\mathfrak{f}}_0}{\mathfrak{f}_1} \right) \mu_1 \right)^2 \geq \frac{63}{64} \mu_1^2 \tag{4.62}$$

showing that μ_{eff} is larger than $63/64 \approx 1$. For $\mu_1 = 0$, it is also clear that $\mu_{\text{eff}} > 25/54$ giving us the total lower bound for the effective flux. Thus, we conclude that in this model

⁹Applying a T-duality in the three x -directions, the fluxes μ_3 and q_3 become non-geometric R-fluxes in type IIA.

one cannot substantially tune the effective flux in the landscape. As a consequence, the critical field distance is still of order one.

Up to now, we have analyzed all possible models arising from the brane superpotential (4.17) corresponding to a single D7-brane living on an isotropic torus T^6 , although the results also apply to the case of a Calabi-Yau with a single complex structure modulus. The natural forthcoming step would be to generalize the previous idea of tuning in the landscape to more elaborated models including more than one complex structure modulus, with the hope of getting a more intricate effective flux parameter μ_{eff} that can be tuned small.

However, the inclusion of more fields makes it necessary to extend the backreaction analysis to also these new fields and the corresponding KK scales. Of course, this issue cannot so easily be addressed in full generality, but we would like to emphasize a universal obstacle which seems difficult to overcome even if appealing to landscape arguments. This universal obstacle is the backreaction coming from the dilaton field. The best thing one can intend, is to stabilize the dilaton by inducing mixing terms between the latter and other complex structure moduli that do not couple to the open string modulus. In this way, one can hope to decouple the scale of S and Φ and delay the backreaction. As pointed out in [28], this tuning is in principle possible in the context of F-theory, where the D7 position moduli and the dilaton become part of the complex structure moduli of the Calabi-Yau four-fold. Let us remark, though, that this is precisely the mechanism underlying the model O1, in which in principle one can get a tunable flux-dependent λ . However, as we have seen, even in this case the model fails from realizing large field inflation. The required mass hierarchy cannot be achieved without getting into trouble with the KK scale. Therefore, we suspect similar results might hold for more generic models with more than one complex structure modulus. A more thorough analysis of Calabi-Yau geometries is surely interesting and deserves more investigation, so we leave it for future work.

4.4 Models with instanton corrections

Let us consider now the case of open string models within the framework of KKLT [60] and Large Volume Scenario (LVS) [61]. The inflaton is still a $D7$ -brane position modulus. The Kähler moduli are not stabilized by non-geometric fluxes, though, but by non-perturbative effects. These non-perturbative corrections can arise, for instance, from Euclidean D3-branes or gaugino condensation of a stack of distant D7-branes. As in the previous examples, the complex structure and axio-dilaton moduli will be stabilized by R-R and NS-NS fluxes.

The backreaction of a field excursion of the inflaton onto the complex structure and axio-dilaton moduli proceeds analogously to the previous section and leads to a logarithmic scaling of the proper field distance at large field. The critical value at which this happens is given by the mass ratio M_u/M_θ . In contrast to the previous models, now this value can in principle be tuned large, because the KK-scale entering (4.49) depends on the Kähler modulus whose stabilization is now disentangled from the stabilization of the complex structure and axio-dilaton moduli. In fact, in the analysis of the KKLT and LVS scenarios

we will assume a hierarchy of scales

$$M_u > M_\tau > M_\theta, \quad (4.63)$$

and analyze the effective models after integrating out the complex structure and the axio-dilaton moduli. The question is whether this effective field theory also shows the typical control issues that we found for the previously studied models. As opposed to the previous flux examples, here the backreaction can only be determined up to next-to-leading order. The relevant parameter controlling when the backreaction of the inflaton field onto the Kähler modulus becomes substantial is $\theta_c \sim (M_\tau/M_\theta)^p$. Notice that the saxions that determine the kinetic term for the inflaton have already been integrated out. Therefore, one does not see the logarithmic behavior from the swampland conjecture for very large field excursions. However, as before, we find a potential problem that can invalidate the possibility of large field inflation.

As already observed in [23, 71], in the presence of a dynamical uplifting term, the backreaction on the Kähler moduli can destabilize the vacuum. If the relative displacement of the Kähler moduli during inflation is of order one, the minimum and the maximum of the KKL T potential merge into a saddle point so that the minimum disappears and the theory decompactifies. This is the same effect that we also found in section 3.2.2 for an uplift for the closed string model C1. Thus, the trajectory does not extend into the regime $\theta > \theta_c$. The question is, then, whether one can parametrically obtain $\theta_c > 1$, i.e. the mass hierarchy between the inflaton and the Kähler modulus. This is an obvious challenge for KKL T and LVS as the open string modulus is stabilized at tree-level, whereas Kähler moduli are fixed by non-perturbative corrections.

We also believe that a full treatment of the backreaction, i.e. including the complex structure and axio-dilaton moduli, would also reveal behavior from the swampland conjecture.

4.4.1 KKL T scenario

Let us start analyzing the case of KKL T extended by an open string modulus Φ . The effective theory, once the dilaton and complex structure moduli are integrated out, is given by the Kähler potential

$$K = -3 \log(T + \bar{T}) + \frac{(\Phi + \bar{\Phi})^2}{2}, \quad (4.64)$$

and the superpotential

$$W = W_0 + \mu \Phi^2 + A e^{-aT}. \quad (4.65)$$

For simplicity we have set $4su = 1$ (in eq. (4.4)), as one can show that otherwise the constraints discussed below become even stronger. Moreover, we have approximated the Kähler potential by assuming a small real part of the open string modulus $\text{Re}(\Phi) = \phi$, which will in fact be stabilized at zero. W_0 and the Pfaffian A are determined in terms of fluxes and the stabilized values of the complex structure moduli. In the following we make the assumptions of KKL T, namely $A = O(1)$ and $W_0 \ll 1$. Moreover, we have in mind that μ is quantized so that we will work in the regime $W_0 \ll \mu$.

The interplay between large field inflation and KKLT moduli stabilization was already analyzed in [23] and further examined in [42]. Here we just borrow some of the relations derived there. The supersymmetric AdS minimum of the scalar potential is at $\Phi = 0$ and for a τ_0 satisfying the transcendental relation

$$W_0 = -Ae^{-a\tau_0} \left(1 + \frac{2a\tau_0}{3} \right). \tag{4.66}$$

The masses of the Kähler modulus and the inflaton $\theta = \text{Im}(\Phi)$ are given by

$$M_\tau^2 = \frac{(aW_0)^2}{2\tau_0}, \quad M_\theta^2 = \frac{1}{2\tau_0^3} \left(\mu^2 + \frac{3}{2}\mu W_0 \right) \tag{4.67}$$

where the latter is the sum of a supersymmetric mass and a soft mass. If the inflaton is displaced away from its minimum, the minimization condition for the Kähler modulus changes in such a way that the minimum for τ becomes θ -dependent with

$$\tau = \tau_0 \left[1 + \frac{1}{2} \left(\frac{\theta}{\theta_c} \right)^2 + \dots \right], \quad \theta_c^2 = \frac{a\tau_0 W_0}{\mu}. \tag{4.68}$$

The backreaction becomes substantial beyond the critical field distance θ_c . In the regime of interest $W_0 \ll \mu$, the supersymmetric mass term for M_θ is dominant so that one gets the relation

$$\theta_c = \sqrt{\frac{M_\tau}{M_\theta}}, \tag{4.69}$$

i.e., as for the previous examples, large field inflation is possible once we parametrically control the mass ratio $\frac{M_\tau}{M_\theta} > 1$. Let us now analyze the two possible obstructions mentioned above:

- **Controlling θ_c**

From (4.68) it is already clear that one cannot get $\theta_c > 1$ for μ quantized and $W_0 \ll 1$ (as required in KKLT). Employing the condition (4.66), we obtain an upper bound for the critical field distance¹⁰

$$\theta_c^2 = \frac{|A|}{\mu} (a\tau_0) e^{-a\tau_0} \left(1 + \frac{2a\tau_0}{3} \right) = \frac{|A|}{\mu} F(a\tau_0) \lesssim \frac{|A|}{\mu}. \tag{4.70}$$

Thus, for $A = O(1)$ one can get $\theta_c > 1$ only for a parametrically small value of μ . This was already noticed in [42]. Therefore, the situation is very similar to the cases studied before, where the Kähler moduli were stabilized via fluxes. This supports the conjecture that one cannot achieve single large field inflation in a parametrically controlled effective theory.

- **Destabilization due to dynamical uplift**

As shown in [23, 42, 71], in the presence of an uplift term (which goes to zero in

¹⁰Here we used the fact that the function $F(x) = x e^{-x} \left(1 + \frac{2x}{3} \right)$ is bounded from above by $F_{\max} = 3 \exp\left(-\frac{3}{2}\right) \sim 0.67$.

the decompactification limit) the relative displacement of the Kähler modulus $\delta\tau/\tau_0$ cannot be made larger than one since otherwise the AdS minimum and the maximum of the potential merge into a saddle point, destabilizing the Kähler modulus. Thus, around the critical value θ_c the inflationary trajectory stops before reaching the top of the backreacted potential.

Let us remark that, unlike in the previous models, there is no problem related to Kaluza-Klein states becoming light. Indeed, the Kaluza-Klein scale stays heavier than the rest of the scales as long as $W_0 \ll 1/(a\sqrt{\tau_0})$, which is satisfied for large volume.

4.4.2 Large volume scenario

One could think that the above problems can be avoided by considering a scheme in which W_0 is not necessarily small. This is indeed one of the ideas proposed in [42] to avoid the above control problems. As an example, we now consider the LVS scenario [61] extended by a D7-brane position modulus $\Phi = \phi + i\theta$. The important feature of LVS is that there exists a non-supersymmetric AdS minimum in which the leading order α' -correction to the Kähler potential is balanced against a non-perturbative correction to the superpotential. This leads to an exponentially large overall volume \mathcal{V} that parametrically controls the vacuum against higher order corrections.

After integrating out the complex structure and axio-dilaton moduli, we get an effective model for a typical swiss-cheese manifold with large and small Kähler moduli T_b and T_s , respectively,

$$\begin{aligned}
 W &= W_0 + Ae^{-aT_s} + \mu\Phi^2, \\
 K &= -2\ln\left[(T_b + \bar{T}_b)^{\frac{3}{2}} - (T_s + \bar{T}_s)^{\frac{3}{2}} + \xi\right] + \frac{(\Phi + \bar{\Phi})^2}{2}.
 \end{aligned}
 \tag{4.71}$$

Here, ξ denotes the usual α' -correction term and W_0 and A are treated as effective parameters of order one. In particular, denoting the overall volume by $\mathcal{V} \approx \tau_b^{3/2}$ and the small four-cycle volume as $\tau = \text{Re}(T_s)$, in the minimum one gets for their values

$$\mathcal{V}_0 = \frac{3W_0\sqrt{\tau_0}}{\sqrt{2}aA} e^{a\tau_0} \left(1 - \frac{3}{4a\tau_0}\right).
 \tag{4.72}$$

The relevant mass scales for this model are given by

$$M_{\mathcal{V}} \sim \frac{W_0}{\mathcal{V}_0^{\frac{3}{2}}}, \quad M_{\tau} \sim \frac{W_0}{\mathcal{V}_0}, \quad M_{\text{KK}} \sim \frac{1}{\mathcal{V}_0^{\frac{3}{2}}},
 \tag{4.73}$$

where, compared to \mathcal{V}_0 , we have treated the value of τ_0 as a number of order one. The requirement of having the small four-cycle Kähler modulus lighter than the Kaluza-Klein scale already imposes an upper bound for W_0 ,

$$W_0 < \mathcal{V}_0^{1/3}.
 \tag{4.74}$$

The mass of the open string inflaton was derived in [23] and at leading order in $1/\mathcal{V}$ it takes the simple form

$$M_{\theta}^2 \sim \frac{4\mu^2}{\mathcal{V}_0^2}.
 \tag{4.75}$$

The backreaction of an inflaton excursion onto the Kähler moduli has also been examined in [23](eq. (5.21)). At leading order in $1/\mathcal{V}$, it can be expressed as

$$\begin{aligned} \mathcal{V} &= \mathcal{V}_0 \left[1 + O(1) \frac{\mu^2 \mathcal{V}_0}{W_0^2} \theta^2 + \dots \right] \\ \tau &= \tau_0 \left[1 + O(1) \frac{\mu^2 \mathcal{V}_0}{W_0^2} \theta^2 + \dots \right], \end{aligned} \tag{4.76}$$

where the order one prefactors include powers of τ_0 and a . Thus, the critical field distance can be read of as

$$\theta_c \sim \frac{W_0}{\mu \mathcal{V}_0^{\frac{1}{2}}} \sim \frac{M_{\mathcal{V}}}{M_{\theta}}, \tag{4.77}$$

and, as usual, is related to the quotient of the masses. Finally, we are ready to consider the issues we have already encountered for KKLT:

- **Controlling θ_c**

Employing the condition (4.74), we immediately arrive at the constraint

$$\theta_c < \frac{1}{\mu \mathcal{V}_0^{\frac{1}{6}}}, \tag{4.78}$$

which for quantized μ and large volume is parametrically smaller than one. Only for very small values of μ with $\mu < \mathcal{V}^{-\frac{1}{6}}$ it could exceed the Planck-scale. Clearly, this problem just reflects the naive expectation that it is hard to control an inverted mass hierarchies, i.e. that a non-perturbative mass term should be larger than a tree-level mass.

- **Destabilization due to dynamical uplift**

As for the KKLT example, it was found in [23] that in the presence of a dynamical uplift, the overall volume gets destabilized and the theory decompactifies if the energy during inflation is bigger than the potential barrier. This occurs when the displacement of the overall volume field becomes comparable to the value at the minimum, i.e. at θ_c . Therefore, the trajectory does not extend in the regime $\theta > \theta_c$.

Hence, LVS does not provide a better framework than KKLT in this regard. We can conclude that for a quantized open string flux $\mu \geq 1$, the effective KKLT and LVS scenarios for Kähler moduli stabilization feature the similar control issues that we already saw for the previous example of tree-level Kähler moduli stabilization.

The loophole again comes from considering an effective μ -parameter depending on other scalars such that it could be tuned small in the landscape. Whether this tuning is indeed possible is still an open question and deserves more investigation. Notice that the difficulties outlined in section 4.3.2 also apply to these models. Let us also mention that here we are assuming that W_0, A can be disentangled from the mass scale of the complex structure moduli. But it could very well be that in a full fledged global compactification the two parameters controlling the backreaction of complex structure and Kahler moduli are related, which could reveal the behavior from the swampland conjecture at a lower

scale than naively expected. Unfortunately, the global 10d action of these scenarios is not known, so we cannot address this issue in more detail for the moment (see though [72] for an effective analysis of the effect of field-dependent Pfaffians A).

5 Conclusions

In this paper we have critically analyzed the possibility of realizing large field inflation in the framework of F-term axion monodromy inflation for concrete models of string moduli stabilization. This included revisiting some of the earlier attempts [31–34], where it was already observed that once one dials the flux parameters such that a model of single field inflation arises, one encounters major obstacles to parametrically control the various mass hierarchies in the chain

$$M_{\text{pl}} > M_s > M_{\text{KK}} > M_{\text{mod}} > H_{\text{inf}} > M_{\Theta}.$$

It was suggested in [37, 38], that these obstacles could be related to the axionic extension of the Swampland Conjecture, that was proposed to hold in a theory of quantum gravity. For large field inflation, the essential parameter in this conjecture is the critical scale $\Theta_c \sim \lambda^{-1}$, beyond which a field excursion imply an infinite tower of states to become exponentially light. The purpose of this paper was to continue the investigation initiated in [37, 38, 41, 42] by enlarging the class of models put under the microscope of the Swampland Conjecture.

Discussing both closed and open string models with flux induced superpotentials in the perturbative large volume regime, we found further evidence for this conjecture to hold in string theory, once moduli stabilization is taken into account. We explicitly saw the appearance of KK-towers of exponentially light states that could be traced back to the backreaction of a large field excursion on the other moduli, leading to the relation for the proper field distance $\Theta \sim \log(\theta)$.

Upon the addition of a constant uplift term, the backreaction of the inflaton onto the other moduli deforms the polynomial potential to a Starobinsky-like plateau above $\Theta > \Theta_c$. However, the appearance of KK-towers invalidates the effective theory in this regime, spoiling inflation. Figure 3 illustrates these issues for a typical backreacted axion potential. Furthermore, in the presence of a dynamical uplift which goes to zero at infinite volume, the minimum disappears and the trajectory destabilizes at a scale close to Θ_c , as already observed in the framework of large field inflation for KKLT and LVS. The only hope to achieve large field inflation is, thus, obtain a parametrically large value for the critical scale Θ_c .

Whenever the inflationary trajectory can be understood as an original flat direction stabilized by a subleading flux μ , the critical value Θ_c will depend on the mass hierarchy between the inflaton mass and the heavy moduli masses. If the theory is well behaved in the $\mu \rightarrow 0$ limit and the saxions are not destabilized, one can aim to delay the backreaction effects by increasing the aforementioned mass hierarchy. We have carefully analyzed effective theories arising from toroidal compactifications of type IIB in which all moduli (including the Kähler moduli) are stabilized at tree level by fluxes. Employing these features for promising models of large field inflation with an open string modulus, we find that

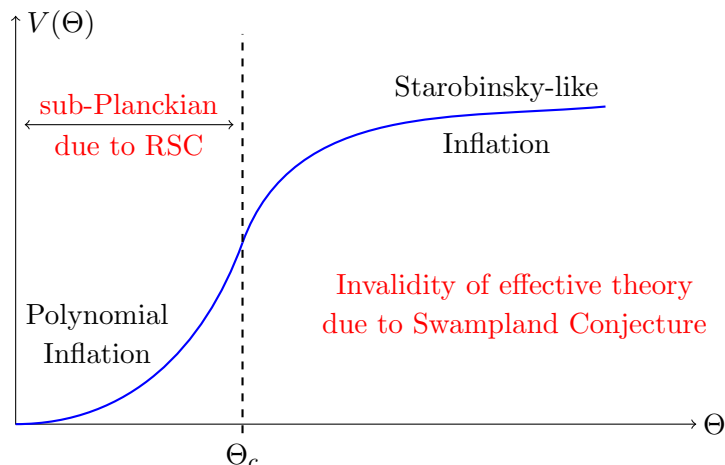


Figure 3. The plot depicts schematically a typical potential $V(\Theta)$ for an inflaton Θ achieved via axion monodromy. Above some critical value Θ_c the backreaction of the inflaton onto the other moduli deforms the polynomial potential to a Starobinsky-like plateau. However, in this regime the effective theory breaks down according to the swampland conjecture. The refined version of the conjecture (RSC) sets $\Theta_c \sim 1$, reducing the controllable inflaton field range to sub-Planckian distances.

parametric control over the effective supergravity theory eventually required that the critical scale is just at the Planck-scale, i.e. $\Theta_c \approx 1$. Consistent with the *Refined Swampland Conjecture*, we could only achieve a light axionic inflaton at the expense of spoiling the validity of the four-dimensional effective action due to a decrease of the Kaluza-Klein scale.

We also discussed two scenarios (KKLT and LVS) where the Kähler moduli are not stabilized at tree level by fluxes but by non-perturbative effects. Similar control issues arose in the effective theories after integrating out the complex structure and the axio-dilaton moduli at a higher scale.

Thus we conclude: all the previous failing attempts and the concrete string models discussed in this paper support the *Refined Swampland Conjecture* [37, 38]. The take home message is that even if the critical field value Θ_c at which the effective theory breaks down is in principle a tunable flux-dependent parameter, we find that it cannot be tuned larger than the Planck mass without losing parametric control of the effective theory in all the examples considered so far.

Since our analysis was focusing on obtaining parametric control, we cannot exclude that there might occur accidental coincidences where the numerical prefactors all work in favor of seemingly generating the right hierarchy of scales. Though, in all the examples we investigated this does not happen.

We think that it is satisfying to see that a general principle, the *Refined Swampland Conjecture*, explains the failure of all previous attempts to embed the idea of F-term axion monodromy inflation in the framework of string moduli stabilization. If true, it has huge implications for phenomenology, implying the following result:

In string theory (quantum gravity) it is impossible to achieve a parametrically controllable model of large (single) field inflation. The tensor-to-scalar ratio is thus bounded from above by $r \lesssim 10^{-3}$.

It is a task for the future to gather more evidence for the conjecture or find a model that challenges their implications. With this in mind, let us mention a few possible loop-holes that can trigger further investigation, even though we are not very confident that they will make large field inflation possible.

It could be that not all fluxes are quantized, as it naively seemed to arise for the type IIB open string superpotential. Alternatively this could happen after integrating out other more heavy moduli so that an effective parameter appears in front of a light modulus in the superpotential. This is what one usually means by fine-tuning in the landscape. We have analyzed a possible model of this kind within the toroidal framework, without succeed in getting a trans-Planckian field range. However, whether this can happen in a controlled way in a more generic F-theory compactification, remains to be seen. One related issue is that, introducing more moduli, also means introducing more KK-scales whose sizes cannot simply be ignored in a honest approach. Thus, by referring too early to the help of a fine-tuning property in the landscape, the danger is that one sweeps the dangerous control issues under the carpet.

Moreover, we were also restricting the analysis to the small string coupling, large radius and large complex structure regime. It could be that perturbing around other points in the moduli space works better, even though we expect that one faces serious control issues [73], as well.

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