Do non-relativistic neutrinos oscillate?

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Abstract: We study the question of whether oscillations between non-relativistic neutrinos or between relativistic and non-relativistic neutrinos are possible. The issues of neutrino production and propagation coherence and their impact on the above question are discussed in detail. It is demonstrated that no neutrino oscillations can occur when neutrinos that are non-relativistic in the laboratory frame are involved, except in a strongly mass-degenerate case. We also discuss how this analysis depends on the choice of the Lorentz frame. Our results are for the most part in agreement with Hinchliffe's rule.

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Contents

1 Introduction

In virtually all theoretical studies of neutrino oscillations it is assumed that neutrinos are extremely relativistic, and for good reason: we normally do not deal with non-relativistic neutrinos in our everyday life. This does not mean, however, that non-relativistic neutrinos do not exist in nature. From neutrino oscillation experiments it follows that either two mass-eigenstate neutrino species have mass $\gtrsim 0.05\,\text{eV}$, or one of them has mass $\gtrsim 0.05\,\text{eV}$ and one other $\gtrsim 8 \times 10^{-3}\,\text{eV}$. The present-day temperature of the cosmic neutrino background $T_\nu \simeq 1.95\,\text{K} \simeq 1.68 \times 10^{-4}\,\text{eV}$ therefore means that at least two relic neutrino species are currently non-relativistic. Moreover, there are some indications of possible existence of (predominantly) sterile neutrinos with an eV-scale mass, and chiefly sterile neutrinos with keV — MeV — GeV scale masses are also being discussed [1–3]. If exist, such neutrinos may well be non-relativistic in some situations of practical interest. The question of whether non-relativistic neutrinos can oscillate is therefore not of completely academic nature. In addition, as we shall see, it is related to some fundamental aspects of the theory of neutrino oscillations and therefore is of considerable conceptual interest.

When asking whether non-relativistic neutrinos can oscillate, one should obviously specify the reference frame in which neutrinos are considered. Neutrinos that are non-relativistic in one Lorentz frame may be highly relativistic in another, and vice versa.

If not otherwise specified, we shall be considering neutrinos in the laboratory frame, by which we mean the frame in which the neutrino source is at rest or is slowly moving.[1] For neutrinos produced in decays, the source is just the parent particle; for neutrinos produced in collisions, the definition of the velocity of the source is more complicated and involves a consideration of the velocities and wave packet lengths of all particles participating in neutrino production [4].

The question of whether non-relativistic neutrinos can take part in neutrino oscillations has been discussed in refs. [4–10], with varying degree of detail and differing conclusions. In refs. [4–6] it was argued that non-relativistic neutrinos cannot participate in neutrino oscillations, whereas in refs. [7–10] the opposite conclusion has been reached (or the opposite was implicitly assumed). In this paper we demonstrate that, unless non-relativistic neutrinos are highly degenerate in mass, their large energy and momentum differences prevent coherence of different neutrino mass eigenstates and therefore preclude flavour oscillations. Some of the arguments presented here have already been discussed in the literature, though mostly at a qualitative level. We put them on a more quantitative basis in this paper. In addition, we discuss in detail how these arguments depend on the choice of the Lorentz frame. To the best of the present author's knowledge, this issue has not been previously addressed in the literature.

2 Non-relativistic neutrinos and coherence conditions

It is well known that neutrino oscillations can only be observable if neutrino production and detection processes cannot discriminate between different neutrino mass eigenstates. This is because only flavour eigenstates undergo oscillations — mass eigenstates do not oscillate.[2] Flavour eigenstates are coherent linear superpositions of mass eigenstates, and therefore the question of observability of neutrino oscillations is closely related to the question of coherence of neutrino production, propagation and detection. If at any of these stages coherence of different mass eigenstates is violated, oscillations will not be observable. We therefore examine here if the coherence conditions are satisfied for non-relativistic neutrinos. The answer to this question depends on whether the neutrino mass spectrum is hierarchical or quasi-degenerate. We first consider the latter case.

2.1 Quasi-degenerate in mass non-relativistic neutrinos

Conceptual issues of neutrino oscillation theory can only be consistently studied within the quantum-mechanical (QM) wave packet framework [4, 6, 11–25][3] or in a formalism based on the quantum field theoretic (QFT) approach [4, 26–37]. As has been mentioned

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[1]This definition may sometimes differ from what one would normally consider to be the laboratory frame in a neutrino experiment. For instance, in accelerator neutrino experiments the conventional definition would likely be that this is the frame in which the neutrino detector is at rest, whereas neutrinos are produced in decays of relativistic pions.

[2]In this paper we shall only deal with neutrino oscillations in vacuum. Mass eigenstates can oscillate in matter, but this does not change our conclusions about the observability of oscillations of non-relativistic neutrinos.

[3]For reviews and further references see refs. [4, 25].

above, it is usually assumed in these studies that neutrinos are ultra-relativistic. However, careful examination of the derivations of the neutrino oscillation probability in both QM and QFT-based approaches shows that these derivations apply without any modifications also to the case of non-relativistic but highly degenerate in mass neutrinos, i.e. when

$$\frac{|\Delta m^2|}{2E} \ll E\,, \tag{2.1}$$

where $\Delta m^2 \equiv m_i^2 - m_k^2$ and E are the neutrino mass squared difference and the average energy of the neutrino mass eigenstates composing a produced neutrino flavour state, respectively. As an example, in [22] it has been demonstrated that the conditions for neutrino oscillations to occur and to be described by the standard oscillation probability are that (i) the energy difference of the neutrino mass eigenstates composing the produced flavour eigenstate,

$$\Delta E \equiv \Delta E_{ik} = \sqrt{p_i^2 + m_i^2} - \sqrt{p_k^2 + m_k^2}\,, \tag{2.2}$$

is related to their momentum difference $\Delta p \equiv p_i - p_k$, mass squared difference Δm^2 and average group velocity v_g by

$$\Delta E \approx \frac{\partial E}{\partial p}\Delta p + \frac{\partial E}{\partial m^2}\Delta m^2 = v_g \Delta p + \frac{\Delta m^2}{2E}\,, \tag{2.3}$$

and (ii) the coherence conditions for neutrino production, propagation and detection (to be discussed in detail in sections 2.2 and 2.3) are satisfied. The relation in eq. (2.3) is satisfied with very good accuracy for ultra-relativistic neutrinos; however, it is easy to see that for its validity it is sufficient that the neutrino mass eigenstates be quasi-degenerate in mass, i.e. condition (2.1) be satisfied. In this case different mass eigenstates are produced under essentially the same kinematic conditions, i.e. their energy difference ΔE and momentum difference Δp are small compared to their average energy E and average momentum p, respectively: $|\Delta E| \ll E$, $|\Delta p| \ll p$. This means that one can expand ΔE in Δp and Δm^2, which yields eq. (2.3).

The conditions of coherent neutrino production, propagation and detection put upper limits on $\Delta m^2/E^2$ and $\Delta m^2/(2E)$ and actually do not require neutrinos to be ultra-relativistic; they can also be satisfied if neutrinos are sufficiently degenerate in mass.

Thus, the standard formalism of neutrino oscillations developed for ultra-relativistic neutrinos applies without any modifications also to non-relativistic but highly degenerate in mass neutrinos. Such neutrinos undergo the usual flavour oscillations provided that the standard coherence conditions are satisfied.

In the rest of this paper we shall be assuming that neutrinos are not quasi-degenerate in mass.

2.2 Non-relativistic neutrinos and production coherence

Let us now consider neutrino production coherence in the case when non-relativistic neutrinos are produced. We start with a general discussion of neutrino production coherence.

2.2.1 General case

Different neutrino mass eigenstates can be emitted coherently and compose a flavour state only if the intrinsic QM uncertainties of their energies and momenta, σ_E and σ_p, are sufficiently large to accommodate their differing energies in momenta. Assuming absence of certain cancellations (as will be explained towards the end of this subsection), this condition can be written as

$$\frac{|\Delta E|}{\sigma_E} \ll 1, \qquad \frac{|\Delta p|}{\sigma_p} \ll 1. \qquad (2.4)$$

If, on the contrary, the uncertainties σ_E and σ_p are very small, then by measuring 4-momenta of the other particles involved in neutrino production and using energy-momentum conservation, one could in principle determine the energy and momentum of the produced neutrino state with high accuracy. This would allow one to accurately infer the neutrino mass, which would mean that a mass (rather than flavour) eigenstate has been emitted, and neutrino oscillations would not take place.

Indeed, assume that by measuring the energies and momenta of all particles taking part in neutrino production we determined neutrino energy and momentum with some accuracy. According to QM uncertainty relations, the uncertainties of the determined neutrino energy and momentum cannot be smaller than the intrinsic energy and momentum uncertainties σ_E and σ_p related to localization of the production process in finite space-time region. Assuming σ_E and σ_p to be independent, from the on-shell dispersion relation $E^2 = p^2 + m^2$ one can then find the minimum error in the determination of the squared neutrino mass [12]:

$$\sigma_{m^2} = \left[(2E\sigma_E)^2 + (2p\sigma_p)^2 \right]^{1/2}. \qquad (2.5)$$

If σ_{m^2} satisfies $\sigma_{m^2} \gg |\Delta m^2| = |m_i^2 - m_k^2|$, one cannot kinematically distinguish between the mass eigenstates ν_i and ν_k in the production process, i.e. they can be emitted coherently. Conversely, for $\sigma_{m^2} \lesssim |\Delta m^2|$ one can find out which neutrino eigenstate has actually been produced; this means that ν_i and ν_k cannot be produced coherently. Since neutrino oscillations are a result of interference of amplitudes corresponding to different mass eigenstates, absence of their coherence means that no oscillations will take place. The flavour transition probabilities would then correspond to averaged neutrino oscillations.

This situation is quite similar to that with the electron interference in double slit experiments. If there is no way to find out which slit the detected electron has passed through, the detection probability will exhibit an interference pattern; however if such a determination is possible, the interference pattern will be washed out.

In the general case when more than two neutrino species are involved, the mass eigenstates ν_i and ν_k can be produced coherently if their mass difference satisfies $|\Delta m_{ik}^2| \ll \sigma_{m^2}$, even if the other pairs of mass eigenstates do not satisfy such a condition and therefore cannot be produced coherently. In this case partial decoherence takes place.

As mentioned above, intrinsic energy and momentum uncertainties σ_E and σ_p characterizing a produced neutrino state are related to space-time localization of the production process: the better the localization, the larger the σ_E and σ_p, and the easier it is to satisfy

the production coherence condition. It is instructive to formulate this condition in configuration space [12, 20, 22]; this will also allow us to find out when eq. (2.4) represents the coherent production condition. To this end, consider the oscillation phase acquired over the distance x and the time interval t from the space-time point at which neutrino was produced:[4]

$$\phi_{\text{osc}} = \Delta E \cdot t - \Delta \vec{p} \cdot \vec{x}. \tag{2.6}$$

The 4-coordinate of the neutrino production point has an intrinsic uncertainty related to the finite space-time extension of the production process; this leads to fluctuations of the oscillation phase

$$\delta \phi_{\text{osc}} = \Delta E \cdot \delta t - \Delta \vec{p} \cdot \delta \vec{x}, \tag{2.7}$$

where δt and $|\delta \vec{x}|$ are limited by the duration of the neutrino production process σ_t and its spatial extension σ_X: $\delta t \lesssim \sigma_t$, $|\delta \vec{x}| \lesssim \sigma_X$. For oscillations to be observable, the fluctuations of the oscillation phase must satisfy $|\delta \phi_{\text{osc}}| \ll 1$ — otherwise oscillations will be washed out upon averaging of the phase over the 4-coordinate of neutrino production. That is, observability of neutrino oscillations requires that the condition

$$|\Delta E \cdot \delta t - \Delta \vec{p} \cdot \delta \vec{x}| \ll 1 \tag{2.8}$$

be satisfied. Barring accidental cancellations between the two terms in (2.8) and taking into account that $\sigma_t \sim \sigma_E^{-1}$, $\sigma_X \sim \sigma_p^{-1}$, we arrive at eq. (2.4). Therefore,

> *different neutrino mass eigenstates are produced coherently and hence neutrino oscillations may be observable only if the oscillation phase acquired over the space-time extension of the production region is much smaller than unity.*

This condition essentially coincides with the obvious requirement that the size of the neutrino production region be much smaller than the oscillation length (which corresponds to $\phi_{\text{osc}} = 2\pi$).

It should be noted that coherent neutrino production is necessary for observability of neutrino oscillations, but it is not by itself sufficient: for the oscillations to take place, also the propagation and detection coherence conditions must be satisfied. Detection coherence can be considered quite similarly to the production one; propagation coherence will be discussed in section 2.3.

2.2.2 Non-relativistic neutrinos

Let us now discuss the production coherence condition in the case when one or more neutrino mass eigenstates are non-relativistic in the frame where the neutrino source is at rest or is slowly moving. In this case different neutrino mass eigenstates are produced under very different kinematic conditions, and therefore have vastly differing energies and momenta. We shall demonstrate that large energy and momentum differences will then prevent coherent neutrino production.

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[4]The phase ϕ_{osc} (as well as the energy difference ΔE and the momentum difference $\Delta \vec{p}$) is actually defined for each pair of neutrino mass eigenstates ν_i and ν_k and should carry the indices ik. We suppress them to simplify the notation.

For illustration, we start with a concrete example. Consider neutrino production in 2-body decays at rest $X \to l\nu_i$, where l denotes a charged lepton, ν_i is the ith neutrino mass eigenstate, and X is either a charged pseudoscalar meson (π, K, ...), or W-boson, or a charged scalar particle. The energies and momenta of the produced neutrino mass eigenstates are

$$E_i = \frac{m_X^2 - m_l^2 + m_i^2}{2m_X}, \qquad p_i = \frac{\left\{[m_X^2 - (m_l^2 + m_i^2)]^2 - 4m_l^2 m_i^2\right\}^{1/2}}{2m_X}. \qquad (2.9)$$

For the energy difference $\Delta E \equiv E_i - E_k$ this gives

$$\Delta E = \frac{\Delta m^2}{2m_X}. \qquad (2.10)$$

Non-relativistic neutrinos are produced in X-boson decay if their mass nearly coincides with the energy release $m_X - m_l$. An example is the (now defunct) KARMEN time distribution anomaly [38], which has been interpreted as a production in $\pi \to \mu\nu$ decay of a non-relativistic neutrino with mass $m \simeq 33.9\,\text{MeV}$ and velocity $v \simeq 0.02$. Assuming that the heaviest of the neutrino mass eigenstates produced in X-boson decay is non-relativistic and barring near degeneracy of the charged lepton and X-boson masses, from eq. (2.10) we then find

$$|\Delta E| \sim m_X. \qquad (2.11)$$

Consider now the energy uncertainty σ_E of the produced neutrino state. For neutrinos born in decay of a free particle at rest, σ_E is given by the energy uncertainty of the parent particle, i.e. by its decay width Γ_X. The first of the two coherence conditions in eq. (2.4) thus reduces to

$$|\Delta E| \ll \Gamma_X. \qquad (2.12)$$

The decay rates $\Gamma(X \to l\nu_i)$ for the processes under discussion are given in appendix A. Their common feature is that they can be written as $\Gamma_X = \kappa_X m_X$, where m_X is the mass of the X-boson and $\kappa_X \ll 1$. The smallness of κ_X is due to the fact that it contains a product of small numerical and dynamical factors; in the cases when non-relativistic neutrinos are produced, the coefficients κ_X are additionally suppressed by a small kinematic factor which comes from the suppression of the phase space volume available to the final-state particles. We thus have

$$\Gamma_X \ll m_X. \qquad (2.13)$$

From eq. (2.11) it then follows that the coherent production condition (2.12) is strongly violated, i.e. different neutrino mass eigenstates cannot be produced coherently. Note that (2.13) is a general property of all unstable particles — their decay width is small compared to their mass. The only exception are decays of very broad resonances, for which $\Gamma_X \sim m_X$; however, even in this case the coherent production condition (2.12) is violated if a non-relativistic neutrino is involved.

This result is actually quite general and holds also when neutrinos are produced in more complicated decays or in reactions. To see this, recall that the produced neutrino state is described by a wave packet, whose energy dispersion σ_E is determined by the

temporal localization of the production process. The mean energy of the neutrino state $\bar{E}$ can be much larger than σ_E or of the order of σ_E, but can never be much smaller than the energy dispersion σ_E. For processes with production of a non-relativistic neutrino, the differences ΔE between its energy and the energies of the other neutrino mass eigenstates are of the order of the corresponding mean energies. Therefore,

$$|\Delta E| \sim \bar{E} \gtrsim \sigma_E , \qquad (2.14)$$

which means that the first of the two coherent production conditions in eq. (2.4) is not met.

What about the second condition in eq. (2.4)? When a non-relativistic neutrino is produced, the differences Δp between its momentum and momenta of the other mass eigenstates satisfy $|\Delta p| \gtrsim \bar{p}$, where $\bar{p}$ is the mean momentum, similarly to what we found for neutrino energy differences and energy uncertainty. However, since momentum is a vector whose projections on coordinate axes can be of either sign, one cannot in general claim that the modulus of its mean value satisfies $\bar{p} \gtrsim \sigma_p$. In particular, for a wave packet describing neutrino at rest, $\bar{p} = 0$ while σ_p is finite. The momentum dispersion of the produced neutrino state is determined by the momentum uncertainty inherent in the production process, which in turn depends on the spatial localization of this process. The latter depends on how the source particles were created and on other features of neutrino production [4], and there are no simple and general arguments that would allow one to tell if the condition $\Delta p \ll \sigma_p$ is satisfied for non-relativistic neutrinos, in contrast to the situation with the requirement $\Delta E \ll \sigma_E$. However, for slow neutrino sources the temporal duration and spatial localization of the neutrino production process are not directly related. This means that in general no cancellations between the two terms in (2.8) occur, and for neutrino production to be coherent both conditions in eq. (2.4) must be separately satisfied. Hence, violation of the first of these two conditions is sufficient to prevent coherent neutrino emission and thus neutrino oscillations.

One might naturally wonder what happens if we consider the usual neutrino oscillations (such as e.g. oscillations of reactor, accelerator or atmospheric neutrinos)[5] in a reference frame where one of the neutrino mass eigenstates is slowly moving or at rest. Indeed, in that case the relations in eq. (2.14) should also be valid, and yet neutrinos must be oscillating: the answer to the question of whether neutrinos oscillate cannot depend on the choice of the reference frame in which neutrinos are considered. We shall discuss this issue in section 3.1.

2.3 Wave packet separation and propagation coherence

In addition to neutrino production and detection coherence, there is another important coherence condition that has to be satisfied for neutrino oscillations to be observable: propagation coherence. Coherence may be lost on the way between the neutrino source and detector because the wave packets of different neutrino mass eigenstates propagate with different group velocities. After long enough time (coherence time) they will separate

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[5]In what follows by the 'usual neutrino oscillations' we shall always mean oscillations of neutrinos which are ultra-relativistic in the rest frame of their source.

by a distance exceeding the spatial length σ_x of the wave packets, which then cease to overlap. The coherence time can therefore be found from the relation

$$|\Delta v_g| \cdot t_{\mathrm{coh}} \simeq \sigma_x \,, \tag{2.15}$$

where Δv_g is the difference of the group velocities of different neutrino mass eigenstates. The corresponding coherence distance is given by

$$L_{\mathrm{coh}} \simeq v_g \cdot t_{\mathrm{coh}} \simeq \frac{v_g}{|\Delta v_g|}\sigma_x \,, \tag{2.16}$$

where v_g is the average group velocity of different neutrino mass eigenstates. Since in the case of ultra-relativistic or quasi-degenerate in mass neutrinos different mass eigenstates are produced under essentially the same kinematic conditions, the lengths of their wave packets σ_{xi} are practically the same. For processes with emission of non-relativistic neutrinos, the lengths of the wave packets of different mass eigenstates may differ; the quantity σ_x in eqs. (2.15) and (2.16) should then be understood as the largest among σ_{xi}. In all known cases the lengths of the neutrino wave packets are tiny (microscopic);[6] still, in the case of the usual neutrino oscillations (with relativistic or highly degenerate in mass neutrinos) the coherence distance L_{coh} is macroscopic and very long because $v_g/|\Delta v_g| \simeq 2E^2/\Delta m^2$ is extremely large. The situation is quite different when one or more of the produced neutrinos are non-relativistic in the laboratory frame, and neutrinos are not quasi-degenerate in mass. The velocity differences between different non-relativistic neutrino mass eigenstates (or between relativistic and non-relativistic states) in that case are $|\Delta v_g| \sim 1$; the coherence distance is therefore microscopic, $L_{\mathrm{coh}} \sim \sigma_x$. This means that, even if non-relativistic neutrinos were produced coherently, they would have lost their coherence due to wave packet separation practically immediately, before getting a chance of being detected.

3 Lorentz boosts

We have found that in the case when one or more of the produced neutrino mass eigenstates are non-relativistic in the reference frame where their source is at rest or is slowly moving, the production coherence condition is violated and therefore neutrino oscillations cannot take place. It is interesting to see how this analysis changes and what prevents neutrinos from oscillating if we go to a frame where all neutrinos are ultra-relativistic.

A different but related question is this: how would the usual neutrino oscillations (such as oscillations of reactor, accelerator or atmospheric neutrinos) look like in a reference frame where one of the neutrino mass eigenstates is at rest? Neutrinos must obviously oscillate in that frame as well, but it is very instructive to see where our previous arguments against oscillations of non-relativistic neutrinos fail in this case. We study this issue first.

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[6]A possible exception is the hypothetical recoilless neutrino emission from crystals in Mössbauer-type experiments, in which σ_x could actually be as long as a few meters [37]. It is not, however, clear if it will ever be possible to realize such experiments.

3.1 Usual neutrino oscillations in the rest frame of ν_2

Consider for simplicity 2-flavour neutrino oscillations in 1-dimensional approach, i.e. assuming that $\vec{p} \parallel \vec{x}$. For definiteness, we shall assume that neutrinos are produced in pion decays at rest. Extensions to the cases of more then two flavours and of moving neutrino source are straightforward; extension to the full 3-dimensional picture of neutrino oscillations is somewhat more involved but does not pose any problems [4].

Consider first the neutrino oscillation phase in the frame where the neutrino source and detector are at rest. By making use of eq. (2.3) valid for ultra-relativistic neutrinos one can rewrite eq. (2.6) as

$$\phi_{\rm osc} \simeq -\frac{1}{v_g}(x - v_g t)\Delta E + \frac{\Delta m^2}{2p}x \,. \tag{3.1}$$

The distance x and the time t between neutrino production and detection may both be very large, but the difference $x - v_g t$ is always small. It vanishes for pointlike neutrinos; in the case when neutrinos are described by finite-size wave packets, it is less than or of the order of the spatial length of the neutrino wave packet σ_x: $|x - v_g t| \lesssim \sigma_x$. The quantity σ_x is, in turn, determined by the space-time extension of the neutrino production region and is typically dominated by its temporal localization or, equivalently, by the energy uncertainty σ_E inherent in the neutrino production process [4, 22, 34]. In particular, for neutrinos produced by non-relativistic sources $\sigma_x \simeq v_g/\sigma_E$. The first term on the right hand side of eq. (3.1) is therefore $\lesssim \sigma_x |\Delta E|/v_g \simeq |\Delta E|/\sigma_E$. If the first of the coherent production conditions in eq. (2.4) is satisfied, this term can be neglected, and we obtain the standard expression for the oscillation phase $\phi_{\rm osc} \simeq [\Delta m^2/(2p)]x$.

Let us demonstrate that under very general assumptions the second condition in eq. (2.4) actually follows from the first one. From eq. (2.3) we find that the condition $|\Delta E|/\sigma_E \ll 1$ is equivalent to

$$\left| v_g \frac{\Delta p}{\sigma_E} + \frac{\Delta m^2}{2E\sigma_E} \right| \ll 1 \,. \tag{3.2}$$

Barring accidental cancellations, this gives

$$v_g \frac{|\Delta p|}{\sigma_E} \ll 1 \,, \qquad \frac{|\Delta m^2|}{2E\sigma_E} \simeq \frac{|\Delta m^2|}{2p}\sigma_x \ll 1 \,. \tag{3.3}$$

In refs. [4, 22, 34] it has been shown that $\sigma_E \leq \sigma_p$; taking also into account that $v_g \simeq 1$, we find that the first strong inequality in (3.3) yields the second condition in (2.4), as advertised. Note that the second condition in (3.3), $[|\Delta m^2|/(2p)]\sigma_x \ll 1$, has a simple meaning: the size of the neutrino wave packet σ_x should be much smaller than the neutrino oscillation length $l_{\rm osc} = 4\pi p/\Delta m^2$. In the example we consider (free pion decay at rest), we have $\sigma_E \simeq \Gamma_\pi \simeq 2.5 \times 10^{-8}\,{\rm eV}$, $E \simeq p \simeq 29.8\,{\rm MeV}$, and for $\Delta m^2 = \Delta m_{\rm atm}^2 \simeq 2.5 \times 10^{-3}\,{\rm eV}^2$ we find that the coherent production conditions (2.4) are satisfied with a very large margin.

Let us now go to a frame where the heavier of the two neutrino mass eigenstates (which we choose to be ν_2) is at rest. This is certainly not the best frame to consider neutrino

oscillations, as the whole setup will look rather weird in it! Indeed, assume that in the initial frame where the neutrino source and detector are at rest neutrinos are moving in the positive direction of the x-axis. Then in the new frame ν_2 will be at rest, ν_1 will still be moving in the positive direction of x (though with a smaller velocity), the parent pion will be moving in the negative x-direction, and the detector will also be moving in the negative direction of x towards the neutrinos. On top of that, the wave packet describing the state of ν_2 will be fast spreading. Indeed, being in the rest frame of ν_2 means that the mean momentum of its wave packet vanishes. Still, the neutrino wave packets are characterized by a finite momentum spread, which means that in its rest frame ν_2 will have both positive and negative momentum components along the x-axis, i.e. its wave packet will quickly spread. Even though this will not affect observability of neutrino oscillations because the neutrino detector will "collide" with neutrinos before a significant spreading occurs (see appendix B),[7] this adds weirdness to the whole picture. Still, considering neutrino oscillations in the rest frame of one of the mass eigenstates is very instructive for understanding when and why non-relativistic neutrinos can actually oscillate.

Let us go to a reference frame in which the whole neutrino source — detector setup is boosted with velocity u along the x-axis. The standard Lorentz transformations read

$$x' = \gamma_u(x + ut), \qquad t' = \gamma_u(t + ux), \tag{3.4}$$

$$E_i' = \gamma_u(E_i + up_i), \qquad p_i' = \gamma_u(p_i + uE_i), \tag{3.5}$$

where $\gamma_u = (1 - u^2)^{-1/2}$ is the Lorentz factor of the boost and the prime refers to the quantities in the new frame. To go to the rest frame of ν_2 we choose $u = -v_{g2} = -(p_2/E_2)$, which gives $\gamma_u = E_2/m_2$. In the new frame we then have:[8]

$$E_2' = m_2, \qquad p_2' = 0, \qquad E_1' \simeq \frac{m_2^2 + m_1^2}{2m_2}, \qquad p_1' \simeq \frac{m_2^2 - m_1^2}{2m_2}. \tag{3.6}$$

For $\Delta E' \equiv E_2' - E_1'$ and $\Delta p' \equiv p_2' - p_1'$ this gives

$$\Delta E' \simeq -\Delta p' \simeq \frac{m_2^2 - m_1^2}{2m_2}. \tag{3.7}$$

Next, we consider the transformation laws for neutrino energy and momentum uncertainties. For neutrinos produced in pion decay at rest, the energy uncertainty is given by the pion decay width: $\sigma_E = \Gamma_\pi$. In a moving frame in which the parent pion has velocity v_π', the energy uncertainty is given by the pion decay width in that frame, $\Gamma_\pi' = \Gamma_\pi/\gamma_{v_\pi'}$, where $\gamma_{v_\pi'} = (1 - v_\pi'^2)^{-1/2}$ is the Lorentz factor of the boost from the pion's rest frame. That is, upon going from the pion rest frame to a moving frame the neutrino energy uncertainty transforms as

$$\sigma_E' = \frac{\sigma_E}{\gamma_{v_\pi'}} = \frac{\Gamma_\pi}{\gamma_u}, \tag{3.8}$$

where we have taken into account that v_π' coincides with the boost velocity u.

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[7]The spreading of the wave packets of neutrinos that are ultra-relativistic in the rest frame of their source has negligible effect on their oscillations. Obviously, the same should also be true in any other frame, including the rest frame of one of the neutrino mass eigenstates. We discuss these points in appendix B.

[8]Here and below we take into account that neutrinos are ultra-relativistic in the original frame, with $E_1 \simeq E_2$ and $p_1 \simeq p_2$.

Let us consider now the neutrino momentum uncertainty σ_p. By the coordinate — momentum uncertainty relation, it is the reciprocal of the neutrino coordinate uncertainty. The latter essentially coincides with the length σ_x of the wave packet of the produced neutrino. It has been demonstrated in [20, 22] that the quantity $\sigma_{xj} E_j$ is invariant under Lorentz boosts, i.e.

$$\sigma'_{xj} = \sigma_{xj} \frac{E_j}{E'_j} = \frac{\sigma_{xj}}{\gamma_u(1 + u v_{gj})} \,, \tag{3.9}$$

where we have used eq. (3.5). For the momentum uncertainty $\sigma_{pj} \simeq 1/\sigma_{xj}$ we therefore have

$$\sigma'_{pj} = \sigma_{pj} \gamma_u (1 + u v_{gj}) \,, \tag{3.10}$$

i.e. the neutrino momentum uncertainty transforms in the same way as the neutrino energy.

To go from the rest frame of the parent pion to the ν_2 rest frame we choose $u = -v_{g2}$, and eqs. (3.9) and (3.10) give

$$\sigma'_{x2} = \sigma_x \gamma_u \,, \qquad \sigma'_{p2} = \frac{\sigma_p}{\gamma_u} \,. \tag{3.11}$$

Here we have taken into account that, although in the pion rest frame (where neutrinos are ultra-relativistic) all the neutrino mass eigenstates composing the produced flavour eigenstate have essentially the same momentum uncertainty σ_p and their wave packets have the same length σ_x, this is no longer true in reference frames where some of the neutrino mass eigenstates are non-relativistic. In particular, in the rest frame of ν_2 its wave packet is the longest one and therefore it is characterized by the smallest momentum uncertainty, $\sigma'_{p\min} = \sigma'_{p2}$. Note that it is actually the smallest momentum uncertainty that is of interest to us from the viewpoint of possible violation of the production coherence condition.

Combining eqs. (3.7), (3.8) and (3.11), we find that in the rest frame of ν_2

$$\frac{|\Delta E'|}{\sigma'_E} \simeq \frac{\Delta m^2}{2m_2} \frac{\gamma_u}{\Gamma_\pi} \simeq \frac{\Delta m^2}{2E\Gamma_\pi} \gamma_u^2 \,, \qquad \frac{|\Delta p'|}{\sigma'_{p\min}} \simeq \frac{\Delta m^2}{2m_2} v_{g2} \frac{\gamma_u}{\Gamma_\pi} \simeq \frac{\Delta m^2}{2E\Gamma_\pi} \gamma_u^2 \,, \tag{3.12}$$

where $E \simeq \frac{m_\pi^2 - m_l^2}{2m_\pi}$ is the mean neutrino energy in the pion rest frame and we have taken into account that $\gamma_u = E_2/m_2 \simeq E/m_2$. From eq. (3.12) it follows that both $|\Delta E'|/\sigma'_E$ and $|\Delta p'|/\sigma'_{p\min}$ scale as γ_u^2. Therefore, even though conditions (2.4) are satisfied in the original frame where the parent pion is at rest, they may be badly violated in the rest frame of ν_2 provided that the boost factor γ_u is large enough, i.e. that the group velocity of the second neutrino mass eigenstate in the pion rest frame v_{g2} is sufficiently close to 1.

So, something went wrong here. To understand the root of the problem, let us note that the primary condition of coherent neutrino production is the requirement (2.8) that the variation of the oscillation phase with varying 4-coordinate of the neutrino emission point be small. Condition (2.4) is secondary and obtains from eq. (2.8) only under the assumption that the two terms in (2.8) are uncorrelated and do not cancel (or approximately cancel) each other. It is easy to see that it is actually this seemingly innocent assumption that led to the above problem. To show this, let us note that the Lorentz transformation (3.4) with $u = -v_{g2} \simeq -1$ gives

$$\delta t' \simeq \gamma_u(\delta t - \delta x) \,, \qquad \delta x' \simeq \gamma_u(\delta x - \delta t) \,, \tag{3.13}$$

i.e. $\delta t' \simeq -\delta x'$. Thus, even if in the original frame δt and δx are completely independent, the corresponding quantities in the rest frame of ν_2 are highly correlated. In addition, eq. (3.7) tells us that $\Delta E' \simeq -\Delta p'$.[9] Therefore, in the rest frame of ν_2 the two terms in (2.7) approximately cancel each other:

$$\delta\phi'_{\rm osc} = \Delta E' \cdot \delta t' - \Delta p' \cdot \delta x' \simeq \Delta E' \cdot (\delta t' + \delta x') \simeq 0 \,. \qquad (3.14)$$

This shows that (i) eq. (2.8) does not lead to the conditions in eq. (2.4) in this case and (ii) no enhancement of $\delta\phi'_{\rm osc}$ actually occurs. More accurate calculation taking into account the small deviation of $u = -v_{g2}$ from -1 yields $\delta\phi'_{\rm osc} = \delta\phi_{\rm osc} \ll 1$, so that the coherent neutrino production condition is satisfied in both frames.

This is exactly as it must be: both the oscillation phase and its variation, being products of two 4-vectors, are Lorentz invariant. So must be the coherence conditions: the answer to the question of whether different mass eigenstates are emitted coherently cannot depend on the choice of the Lorentz frame in which we look at neutrinos. The conditions in eq. (2.4), which are often used in the literature as the coherent production conditions, are not Lorentz invariant; they follow from the Lorentz invariant condition (2.8) only in reference frames where the neutrino source is non-relativistic. Obviously, they cannot be automatically extrapolated from one Lorentz frame to another.

So, we can now answer the question posed at the end of section 2.2.2. In the reference frame in which neutrino source is at rest or is slowly moving the two terms in the expression $\delta\phi_{\rm osc} = \Delta E \cdot \delta t - \Delta p \cdot \delta x$ do not in general cancel, and the coherent production condition (2.8) reduces to (2.4). Since the usual neutrinos produced in pion decay at rest are highly relativistic with very small energy and momentum differences of their mass eigenstates, the coherence conditions (2.4) are very well satisfied for them. In the frame where one of the produced neutrino mass eigenstates is at rest, the energy and momentum differences of neutrino mass eigenstates become large, and conditions (2.4) are no longer satisfied, as discussed in section 2.2.2. However, in this case eq. (2.4) does not represents the coherent production condition and is actually irrelevant. This happens because the boost with a very large Lorentz factor which is necessary to go to the new frame leads to near cancellation of the two terms in eq. (2.8) in that frame. As a result, conditions (2.4) no longer follow from the coherent production condition (2.8).

3.2 Boosting non-relativistic neutrinos

After we have studied in great detail coherence of the usual neutrino oscillations in the rest frame of one of the neutrino mass eigenstates, it is easy to understand what happens when neutrinos produced as non-relativistic in the rest frame of their source are boosted to become relativistic. In the original (laboratory) frame, the variations of temporal and spatial coordinates of the neutrino production point within the production region are not correlated, and neither are the energy and momentum differences of the neutrino mass

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[9]While eq. (3.7) is specific to neutrinos produced in pion decays (or more generally in 2-body decays), the fact that in the new frame $\Delta E' \simeq -\Delta p'$ is actually quite general. It directly follows from the Lorentz transformation (3.5) with $u \simeq -1$.

eigenstates. Under these circumstances the coherent production condition (2.8) leads to eq. (2.4). Violation of the first of the constraints in (2.4), $|\Delta E| \ll \sigma_E$, which, as discussed in section 2.2.2, takes place in this case, therefore means that the coherent production condition (2.8) is not met.

Assume now we go to a fast moving frame in which all neutrino mass eigenstates are highly relativistic and have nearly the same energies and momenta. Because of Lorentz invariance of $\delta\phi_{\text{osc}}$, the production coherence condition will be violated in the new frame as well. Thus, boosting neutrinos that were non-relativistic in the laboratory frame to make them relativistic will not let them oscillate, as expected.

4   Summary and discussion

We have studied in detail the question of whether neutrinos that are non-relativistic in a reference frame in which their source is at rest or is slowly moving can oscillate. The answer to this question depends on the neutrino mass spectrum. If neutrinos are highly degenerate in mass, the standard formalism of neutrino oscillations applies to them without any modifications, and they do oscillate provided that the standard coherence conditions are satisfied. This also answers the question why non-relativistic neutral K, B and D mesons oscillate: this is because their corresponding mass eigenstates are highly degenerate in mass.

If, however, non-relativistic neutrinos are not quasi-degenerate in mass, their large energy and momentum differences prevent different mass eigenstates from being produced coherently. As a result, no oscillations with participation of non-relativistic neutrinos are possible. The flavour transition probabilities would correspond to averaged-out oscillations in that case, and in particular survival probabilities would exhibit a constant suppression.[10]

We have also shown that even if non-relativistic neutrinos were produced coherently, they would have lost coherence due to their wave packet separation practically immediately, at microscopic distances from their birthplace. Although propagation decoherence may in principle be undone by a very coherent neutrino detection [14], in the case of non-relativistic neutrinos this would require a completely unrealistic degree of coherence of the detection process. In addition, even though in general detection may restore neutrino coherence if it was lost on the way between the source and the detector, the coherence can never be restored if it was violated at neutrino production.

We have also considered in detail how the choice of the Lorentz frame influences our arguments and explicitly demonstrated that the coherence conditions are Lorentz invariant, as they should be. In particular, since neutrinos which are non-relativistic in the rest frame of their source are produced incoherently and do not oscillate in that frame, they will also be incoherent and will not oscillate upon a boost to a reference frame where they are all ultra-relativistic. On the other hand, the usual neutrinos that are ultra-relativistic and oscillate

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[10]When the production (or detection) coherence conditions are violated, the probability of the overall neutrino production — propagation — detection process does not factorize into the production rate, oscillation probability and detection cross section, so that the very notion of the oscillation probability loses its sense. In that case one could still, in principle, study the oscillatory behaviour of the overall probability (with or without lepton flavour change) as a function of the distance between the neutrino production and detection points. Decoherence, however, means that no such oscillatory behaviour will take place.

in the frame where their source is at rest or is slowly moving will maintain their coherence and will be oscillating also in the rest frame of any of the neutrino mass eigenstates.

Our discussion demonstrated that the conditions $|\Delta E|/\sigma_E \ll 1$, $|\Delta p|/\sigma_p \ll 1$ that are often employed as criteria of neutrino production coherence are not Lorentz invariant and should be used with caution. They can only serve as the coherent production conditions in the case of non-relativistic neutrino sources, and in general should be replaced by the Lorentz-invariant constraint on the variation of the oscillation phase over the neutrino production region (2.8).

The main reason why neutrinos that are non-relativistic in the frame where their source is at rest or is slowly moving do not oscillate is their very large energy and momentum differences, which significantly exceed the corresponding energy and momentum uncertainties inherent in the neutrino production process. This is very similar to the reason why charged leptons do not oscillate [39].

The results of our study are for the most part in agreement with Hinchliffe's rule [40, 41].

A   Decay rates

We present here the rates of 2-body decays $X \to l\nu_i$, where l denotes a charged lepton, ν_i stands for ith neutrino mass eigenstate, and X is either a charged pseudoscalar meson $(\pi, K, \dots)$, or W-boson, or a charged scalar particle. All the rates are given in the rest frame of the parent particle and are calculated to leading order in electroweak interaction, initially without neglecting any masses of the involved particles.

We start with the rate of the charged pion decays $\pi \to l\nu_i$. Direct calculation yields

$$\Gamma(\pi \to l\nu_i) = \frac{g^4}{256\pi} \frac{m_\pi^4}{m_W^4} \frac{f_\pi^2}{m_\pi^2} m_\pi |U_{li}|^2 \frac{m_l^2 + m_i^2}{m_\pi^2} \left(1 - \frac{m_l^2 + m_i^2}{m_\pi^2}\right) \left\{ \left(1 - \frac{m_l^2 + m_i^2}{m_\pi^2}\right)^2 - \frac{4m_l^2 m_i^2}{m_\pi^4} \right\}^{1/2}. \tag{A.1}$$

Here U_{li} is the element of the leptonic mixing matrix, $g \simeq 0.65$ is the SU(2)$_L$ gauge coupling constant, m_W is the W-boson mass, $f_\pi \simeq 130\,\text{MeV}$ is the pion decay constant, the rest of notation being self-explanatory. Note that the pion decay rate is usually expressed through the Fermi constant $G_F = \sqrt{2}g^2/(8m_W^2)$; we prefer to express it here through the dimensionless gauge coupling constant g. For other charged pseudoscalar bosons ($X = K$, $B, \dots$) the decay rates $\Gamma(X \to l\nu_i)$ can be obtained from (A.1) by the obvious substitution $m_\pi \to m_X$, $f_\pi \to f_X$.

Usually, the decay rates of charged pseudoscalar mesons are calculated under the assumption that all neutrino masses are very small and can be neglected from the outset. The $X \to l\nu_l$ decay rates are then obtained by summing over all the neutrino mass eigenstates. The resulting expressions are independent of U_{li} due to unitarity of the leptonic mixing matrix. Such an approximation is not applicable if neutrinos with mass $m_i \sim m_X$ exist.

For small lepton masses the factor $\frac{m_l^2 + m_i^2}{m_\pi^2}$ in (A.1) describes chiral suppression of the X-meson decay; however, for decays with production of non-relativistic neutrinos this factor is not small, i.e. there is no chiral suppression. The factor in the curly brackets in

eq. (A.1) (and similar factors in eqs. (A.2) and (A.4) below) is of kinematic origin; it is just the magnitude of the momentum of the produced neutrino (and of equal in magnitude but opposite in sign momentum of the charged lepton) in units of the mass of the parent particle. It vanishes when $m_l + m_i$ approaches the parent particle's mass.

Consider next leptonic decays of W boson. The leading order $W \to l\nu_i$ decay rate reads

$$\Gamma(W \to l\nu_i) = \frac{g^2}{48\pi}m_W|U_{li}|^2\left(1 - \frac{m_l^2 + m_i^2}{2m_W^2} - \frac{(m_l^2 - m_i^2)^2}{2m_W^4}\right)\left\{\left(1 - \frac{m_l^2 + m_i^2}{m_W^2}\right)^2 - \frac{4m_l^2 m_i^2}{m_W^4}\right\}^{1/2}. \tag{A.2}$$

Finally, we consider the rate of decay of a charged scalar ϕ caused by the Yukawa-type interaction

$$\mathcal{L}_{\text{int}} = y\bar{l}\nu_i\phi + \text{h.c.}, \tag{A.3}$$

where y is the Yukawa coupling constant. Note that such charged scalars exist in many extensions of the Standard Model, e.g. in 2 Higgs doublet models. Direct calculation to the leading order in the Yukawa coupling y yields

$$\Gamma(\phi \to l\nu_i) = \frac{|y|^2}{8\pi}m_\phi\left(1 - \frac{(m_l + m_i)^2}{m_\phi^2}\right)\left\{\left(1 - \frac{m_l^2 + m_i^2}{m_\phi^2}\right)^2 - \frac{4m_l^2 m_i^2}{m_\phi^4}\right\}^{1/2}. \tag{A.4}$$

The production coherence condition (2.12) is more easily satisfied for larger values of Γ_X; the latter are generally increased with decreasing mass of the produced charged lepton m_l (because this increases the phase space volume available to the final-state particles). It therefore may be useful to consider the decay widths of X-bosons also in an (unrealistic) limit $m_l \to 0$. The rates in eqs. (A.1), (A.2) and (A.4) then simplify to

$$\Gamma(\pi \to l\nu_i) = \frac{g^4}{256\pi}\frac{m_\pi^4}{m_W^4}\frac{f_\pi^2}{m_\pi^2}m_\pi|U_{li}|^2\frac{m_i^2}{m_\pi^2}\left(1 - \frac{m_i^2}{m_\pi^2}\right)^2, \tag{A.5}$$

$$\Gamma(W \to l\nu_i) = \frac{g^2}{48\pi}m_W|U_{li}|^2\left(1 - \frac{m_i^2}{m_W^2}\right)\left(1 - \frac{m_i^2}{2m_W^2} - \frac{m_i^4}{2m_W^4}\right), \tag{A.6}$$

$$\Gamma_\phi = \frac{|y|^2}{8\pi}m_\phi\left(1 - \frac{m_i^2}{m_\phi^2}\right)^2. \tag{A.7}$$

B Neutrino wave packet spreading

Consider the spreading of the neutrino wave packets in the case of the usual neutrino oscillations. We shall discuss how the effects of this spreading change when going from the rest frame of the neutrino source (where all neutrinos are ultra-relativistic) to the rest frame of one of the neutrino mass eigenstates.

The wave packet spreading is caused by the velocity dispersion, i.e. by the dependence of the group velocity $\vec{v}_{gi}$ of the neutrino mass eigenstate ν_i on its momentum. Indeed, from $E_i = (\vec{p}^2 + m_i^2)^{1/2}$ it follows that for $m_i \neq 0$ the group velocity $\vec{v}_{gi} = \partial E_i/\partial\vec{p} = \vec{p}/E_i$ is a function of $\vec{p}$. Therefore, the momentum spread within the neutrino wave packet means

that its different momentum components propagate with different velocities, leading with time to its spreading. The spreading velocity is thus[11]

$$\mathrm{v}^j \simeq \sum_k \frac{\partial v_g^j}{\partial p^k} \sigma_p^k = \sum_k \frac{1}{E} \big(\delta^{jk} - v_g^j v_g^k \big) \sigma_p^k \,, \tag{B.1}$$

where σ_p^k is the neutrino momentum dispersion in the kth direction. For spreading of the wave packet of the ith neutrino mass eigenstate in the direction of the neutrino propagation (longitudinal spreading) we obtain

$$\mathrm{v}_i^\parallel = \frac{m_i^2}{E_i^3} \sigma_p \,, \tag{B.2}$$

where σ_p is the momentum dispersion in the longitudinal direction and we have taken into account that for neutrinos that are ultra-relativistic in the rest frame of their source the momentum dispersion of the different neutrino mass eigenstates is practically the same.

As follows from eq. (B.2), for ultra-relativistic neutrinos the longitudinal spreading velocity is very small. As a result, the spreading of their wave packets is of no relevance to neutrino oscillations. To show this, let us define the characteristic spreading time $t_{\mathrm{spr}\,i}$ as the time over which the wave packet of ν_i spreads to about twice its initial length, $\sigma_x \sim 1/\sigma_p$: $\mathrm{v}_i^\parallel t_{\mathrm{spr}\,i} \simeq 1/\sigma_p$. This gives

$$t_{\mathrm{spr}\,i} \simeq \frac{E_i^3}{m_i^2 \sigma_p^2} \,. \tag{B.3}$$

Let us compare this time with the oscillation time t_{osc} (which for ultra-relativistic neutrinos coincides with the oscillation length $l_{\mathrm{osc}} = 4\pi E/\Delta m^2$):

$$\frac{t_{\mathrm{spr}\,i}}{t_{\mathrm{osc}}} \simeq \frac{E^2}{4\pi \sigma_p^2} \frac{\Delta m^2}{m_i^2} \,. \tag{B.4}$$

Barring quasi-degeneracy of the neutrino masses, from the oscillation data it follows that $\Delta m^2/m_i^2 \gtrsim \Delta m_{21}^2/\Delta m_{31}^2 \sim 1/30$. Next, we note that in realistic situations neutrino energy is always very large compared to the energy uncertainty: $E \gg \sigma_E$. As an example, for $\pi \to \mu\nu$ decay at rest $E \simeq (m_\pi^2 - m_\mu^2)/(2m_\pi) \simeq 29.8\,\mathrm{MeV}$ and $\sigma_E \simeq \Gamma_\pi = 2.5 \times 10^{-8}\,\mathrm{eV}$, so that $E/\sigma_E \simeq 1.2 \times 10^{15}$. Since for ultra-relativistic neutrinos $\sigma_p \simeq \sigma_E$, the ratio in eq. (B.4) is extremely large.[12] Thus, it takes a much longer time for the neutrino wave packet to spread by about a factor of two than for neutrino oscillation probability to reach its first maximum. This means that the effects of wave packet spreading on neutrino oscillations can be safely neglected in all realistic situations.

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[11]We use superscripts to label the Cartesian components of the vectors, whereas lower indices are used to mark the mass eigenstates. In eq. (B.1) the latter are omitted in order not to overload the notation.

[12]A possible exception are supernova neutrinos, for which E/σ_E can be as small as ~ 10. However, in this case the spreading of the neutrino wave packets is not relevant to neutrino oscillations either [24].

Note that the requirement $t_{\mathrm{spr}\,i} \gg l_{\mathrm{osc}}$ as a condition for neglecting the wave packet spreading effects is actually a very conservative one. Indeed, for the usual neutrino oscillations, the coherent production condition is satisfied with a large margin, which, in particular, means that the initial length of the neutrino wave packets satisfies $\sigma_x \ll l_{\mathrm{osc}}$ (see section 3.1). In these circumstances the value of σ_x has no effect on neutrino oscillations,[13] and neither will have its doubling.

One naturally expects that, if the wave packet spreading effect on neutrino oscillations is negligible in the rest frame of the neutrino source, the same will hold in all other Lorentz frames. We shall now demonstrate this explicitly. Let us go to the frame where the neutrino source moves with a velocity u in the direction of neutrino emission. In the new frame eq. (B.2) yields

$$\mathrm{v}_i^{\|\prime} = \frac{m_i^2}{E_i'^3}\sigma_p' \simeq \mathrm{v}_i^{\|} \frac{1}{\gamma_u^2(1 + uv_{gi})^2}\,,\tag{B.5}$$

where we have used eqs. (3.5) and (3.10). Note that in the rest frame of the neutrino mass eigenstate ν_i (i.e. for $u = -v_{gi}$) the longitudinal spreading velocity of its wave packet is a factor γ_u^2 larger than it is in the rest frame of the parent pion.

Next, let us find the characteristic spreading time $t_{\mathrm{spr}\,i}'$ in the moving frame. Eqs. (3.9) and (B.5) yield

$$t_{\mathrm{spr}\,i}' \simeq \frac{\sigma_x'}{\mathrm{v}_i^{\|\prime}} = t_{\mathrm{spr}\,i}\gamma_u(1 + uv_{gi})\,.\tag{B.6}$$

Let us now compare the spreading times $t_{\mathrm{spr}\,i}$ and $t_{\mathrm{spr}\,i}'$ with, respectively, the time intervals between the ν_i production and detection in the original frame and in the new frame, Δt_i and $\Delta t_i'$.[14] Taking into account that $\Delta x_i \simeq v_{gi}\Delta t_i$, from the Lorentz transformation law (3.4) one finds $\Delta t_i' = \Delta t_i\gamma_u(1 + uv_{gi})$. Together with eq. (B.6) this gives

$$\frac{t_{\mathrm{spr}\,i}'}{\Delta t_i'} = \frac{t_{\mathrm{spr}\,i}}{\Delta t_i}\,.\tag{B.7}$$

Thus, if the wave packet spreading time is much larger than the neutrino flight time in the rest frame of the neutrino source, the same will be true in any other Lorentz frame, including the rest frame of one of the neutrino mass eigenstates. Therefore, the relative effects of the wave packet spreading on neutrino oscillations is frame independent, as expected.

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[13]Except for neutrino propagation decoherence, for which the finite length of the neutrino wave packet σ_x is crucial, see section 2.3. However, as was shown in [24], the spreading of the neutrino wave packets does not affect the coherence length, which is therefore defined by the initial value of σ_x.

[14]Note that if we choose the new frame to be the rest frame of the neutrino mass eigenstate ν_i, the quantity $\Delta t_i'$ will actually be the interval between the neutrino production time and the time when the detector "collides" with the resting ν_i.

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