

Dyonic (A)dS black holes in Einstein-Born-Infeld theory in diverse dimensions

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ABSTRACT: We study Einstein-Born-Infeld gravity and construct the dyonic (A)dS planar black holes in general even dimensions, that carry both the electric charge and magnetic fluxes along the planar space. In four dimensions, the solution can be constructed with also spherical and hyperbolic topologies. We study the black hole thermodynamics and obtain the first law. We also classify the singularity structure.

KEYWORDS: Black Holes, Classical Theories of Gravity, Spacetime Singularities

ARXIV EPRINT: [1606.02733](https://arxiv.org/abs/1606.02733)

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1 Introduction

In 1934, Born and Infeld [1] proposed an elegant nonlinear version of electrodynamics that successfully removes the divergence of self-energy of a point-like charge in Maxwell's theory of electrodynamics. The Lagrangian density of the Born-Infeld (BI) theory in D -dimensional Minkowski spacetime is given by

$$\mathcal{L} = -b^2 \sqrt{-\det \left(\eta_{\mu\nu} + \frac{F_{\mu\nu}}{b} \right)} + b^2, \tag{1.1}$$

where $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$ is the Minkowski metric, $F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]}$ is the Faraday tensor and $A = A_\mu dx^\mu$ is the Maxwell gauge potential. BI theory contains a dimensionful parameter b , and in the limit $b \rightarrow \infty$, BI theory reduces to the Maxwell theory,

$$\mathcal{L} = -\frac{1}{4}F^2 + \mathcal{O} \left(\frac{1}{b^2} \right). \tag{1.2}$$

In the limit $b \rightarrow 0$, the Lagrangian in four dimensions becomes $F \wedge F$ which is a total derivative. The limit is generally singular in higher dimensions.

BI theory has enjoyed further attentions since the invention of string theory. It turns out that the BI action can arise from string theory [2], describing the low energy dynamics of D-branes [3]. We refer to e.g. [4, 5] for some comprehensive reviews on the BI theory in string theory. The special Born-Infeld-like nonlinear form is also very useful to construct analogous new theories, such as Dirac-Born-Infeld (DBI) inflation theory [6, 7] and Eddington-inspired Born-Infeld (EiBI) cosmologies [8]. BI theory can also be adopted to explore issues of dark energy [9, 10].

In this paper, we focus on the study of black holes in Einstein-Born-Infeld (EBI) theory. The most general static type- D metric of the BI theory in four dimensions was constructed in [11]. (See also [12] and [13].) The spherically-symmetric solution was generalized to arbitrary D dimensions in [14] where the black hole thermodynamics was studied. The black hole solutions was also generalized to include different topologies [15]. The Born-Infeld black hole solutions were also studied in Einstein theory with a dilaton field [16] and in the modified gravity theories such as Gauss-Bonnet theory [17], Lovelock theory [18], Brans-Dicke theory [19], $f(T)$ theory [20], massive gravity [21] and so on. The extended thermodynamics [14–20, 22–24], geodesics [25, 26], and AdS/CFT correspondence properties [27–29] were studied too. Other Born-Infeld solutions are also studied, for example, thin-shell wormholes [30].

(A)dS black hole solutions in BI theory considered in literature typically involves only either the electric or magnetic charges. Although the dyonic black hole in EBI theory was constructed in [11], it is written in the general (static) type- D form. The global structure in the spherically symmetric form was analysed in [25] for the asymptotically-flat case. In this paper, we shall first study the dyonic (A)dS black holes in the EBI theory in four dimensions with general topologies, focus on analysing the black hole thermodynamics and singularity structure. We then construct dyonic AdS planar black holes in arbitrary even dimensions, where the solutions carry both the electric flux as well as the magnetic 2-form flux along the planar space.

Interestingly in almost all the previous works on constructing black holes, the equivalent action in four dimensions was used, rather than the original one. In $D = 4$, the Lagrangian can be equivalently expressed as [1]

$$\mathcal{L} = b^2 - b^2 \sqrt{1 + I_1 + I_2}, \tag{1.3}$$

where

$$I_1 = \frac{1}{2b^2} F_{\mu\nu} F^{\mu\nu} = \frac{B^2 - E^2}{b^2}, \quad I_2 = -\frac{1}{16b^4} \left(F_{\mu\nu} \tilde{F}^{\mu\nu} \right)^2 = -\frac{(E \cdot B)^2}{b^4}, \tag{1.4}$$

in which E and B are electric and magnetic fields, and

$$\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} = \frac{1}{2\sqrt{-\det(\eta_{ab})}} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}, \tag{1.5}$$

where $\epsilon^{\mu\nu\rho\sigma}$ is a tensor density with $\epsilon^{0123} = 1$.

The equivalence of (1.3) and (1.1) is only true in four dimensions; it is no longer valid in higher dimensions. However, if one considers only static solutions carrying electric charges,

one can nevertheless use the reduced Lagrangian (1.3). In fact in this case, one can even ignore the I_2 term. This was indeed done in many previous works, for example [12–15]. Since one of our purposes is to construct dyonic black holes in higher dimensions, the Lagrangian (1.3) is not suitable for this purpose and we shall use the original Lagrangian (1.1) instead for all our constructions.

The paper is organized as follows. In section 2, we review the EBI theory and then derive the equations of motion for all dimensions. In section 3, we obtain the exact dyonic (A)dS black hole solutions in four dimensions with a generic topological horizon. Then we study the global structure, black hole thermodynamics and the singularity structures. In section 4, we generalize the results to all even dimensions. We conclude the paper in section 5.

2 EBI and its equations of motion

In this section, we consider the EBI theory. The Lagrangian of BI theory can be naturally generalized to curved spacetimes and the Lagrangian is given by

$$\mathcal{L} = -b^2 \sqrt{-\det\left(g_{\mu\nu} + \frac{F_{\mu\nu}}{b}\right)} + b^2 \sqrt{-\det(g_{\mu\nu})}, \quad (2.1)$$

where $g_{\mu\nu}$ is the metric. The Lagrangian of the EBI theory with a bare cosmological constant Λ_0 can be written by

$$\mathcal{L} = \sqrt{-g}(R - 2\Lambda_0) - b^2 \sqrt{-\det\left(g_{\mu\nu} + \frac{F_{\mu\nu}}{b}\right)}, \quad (2.2)$$

where $\Lambda_0 = \Lambda - b^2/2$. Here, Λ is the effective cosmological constant. The variation of Lagrangian (2.2) gives rise to

$$\delta\mathcal{L} = \sqrt{-g}(-E^{\mu\nu}\delta g_{\mu\nu} + E_A^\nu \delta A_\nu + \nabla_\mu J^\mu), \quad (2.3)$$

where $g = \det(g_{\mu\nu})$, J^μ is the surface term and

$$E^{\mu\nu} = G^{\mu\nu} + g^{\mu\nu}\Lambda_0 + \frac{b^2}{2} \frac{\sqrt{-h}}{\sqrt{-g}} (h^{-1})^{(\mu\nu)}, \quad (2.4)$$

$$E_A^\nu = \nabla_\mu \left[\frac{\sqrt{-h}}{\sqrt{-g}} b (h^{-1})^{[\mu\nu]} \right], \quad (2.5)$$

in which $G^{\mu\nu} = R^{\mu\nu} - g^{\mu\nu}R/2$, $h_{\mu\nu} = g_{\mu\nu} + F_{\mu\nu}/b$, $h \equiv \det(h_{\mu\nu})$, and $(h^{-1})^{\mu\nu}$ denotes the inverse of $h_{\mu\nu}$, satisfying

$$(h^{-1})^{\mu\rho} h_{\rho\nu} = \delta_\nu^\mu, \quad h_{\nu\rho} (h^{-1})^{\rho\mu} = \delta_\nu^\mu. \quad (2.6)$$

We further defined

$$(h^{-1})^{(\mu\nu)} = \frac{1}{2} [(h^{-1})^{\mu\nu} + (h^{-1})^{\nu\mu}], \quad (h^{-1})^{[\mu\nu]} = \frac{1}{2} [(h^{-1})^{\mu\nu} - (h^{-1})^{\nu\mu}]. \quad (2.7)$$

The equations of motion are then given by $E^{\mu\nu} = 0$ and $E_A^\mu = 0$. These equations are derived from the original Lagrangian (2.2) of the EBI theory and hence are applicable in all dimensions and for all charge configurations.

3 Dyonic black hole in four dimensions

In the previous section, we obtained the equations of motion of the EBI theory. We now construct the static dyonic (A)dS black hole solution with a general topological horizon in four dimensions. We shall then study the global structure and the black hole thermodynamics.

3.1 Local solution

The static solution in the type- D form in the EBI theory was first constructed in [11]. The spherically-symmetric and asymptotically-flat solution was given in [25]. In this section, we study the properties of the dyonic (A)dS black holes. The most general static ansatz can be written as

$$ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 \left(\frac{du^2}{1 - ku^2} + (1 - ku^2)d\varphi^2 \right), \quad A = \phi(r)dt + pud\varphi, \quad (3.1)$$

where $k = 1, 0, -1$ denotes the metric for the unit 2-spheres, 2-torus or the unit hyperbolic 2-space, and p is magnetic charge parameter. It turns out that the equations of motion of the metric $g_{\mu\nu}$ imply that $h(r) = f(r)$ and the equations of motion for A_μ imply that $\phi(r)$ can be expressed as

$$\phi'(r) = \frac{q}{\sqrt{r^4 + \frac{Q^2}{b^2}}}, \quad \text{with} \quad Q = \sqrt{p^2 + q^2}, \quad (3.2)$$

where and thereafter, we use a prime to denote a derivative with respect to r , and q is a integral constant that is related to the electric charge. The function $f(r)$ satisfies

$$rf'(r) + f(r) = k - \Lambda_0 r^2 - \frac{b^2}{2} \sqrt{r^4 + \frac{Q^2}{b^2}}. \quad (3.3)$$

Thus $f(r)$ can be solved and expressed in terms of hypergeometric function ${}_2F_1$

$$f(r) = -\frac{1}{3}\Lambda_0 r^2 + k - \frac{\mu}{r} - \frac{b^2}{6} \sqrt{r^4 + \frac{Q^2}{b^2}} + \frac{Q^2}{3r^2} {}_2F_1 \left[\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{Q^2}{b^2 r^4} \right], \quad (3.4)$$

where μ is the integral constant corresponding to the mass of the solution. The electric potential $\phi(r)$ is expressed by

$$\phi(r) = \int_r^\infty \frac{qd\tilde{r}}{\sqrt{\tilde{r}^4 + \frac{Q^2}{b^2}}} = \frac{q}{r} {}_2F_1 \left[\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{Q^2}{b^2 r^4} \right]. \quad (3.5)$$

In the limit $b \rightarrow \infty$, the solution recovers the dyonic Reissner-Nordström-(A)dS black hole,

$$f(r) = -\frac{\Lambda}{3}r^2 + k - \frac{\mu}{r} + \frac{Q^2}{4r^2}, \quad \phi(r) = \frac{q}{r}. \quad (3.6)$$

On the other hand, in the limit $b \rightarrow 0$, the Born-Infeld field vanishes and the solution is reduced to the Schwarzschild-(A)dS black hole in pure cosmological Einstein theory,

$$f(r) = k - \frac{\mu}{r} - \frac{\Lambda}{3}r^2. \quad (3.7)$$

In the large- r expansion, we have

$$f(r) = -\frac{\Lambda}{3}r^2 + k - \frac{\mu}{r} + \frac{Q^2}{4r^2} + \mathcal{O}\left(\frac{1}{r^6}\right), \quad \phi(r) = \frac{q}{r} + \mathcal{O}\left(\frac{1}{r^5}\right). \quad (3.8)$$

Thus we see that the first few leading-order expansions match those of the Reissner-Nordström-(A)dS black hole.

3.2 Thermodynamics

Now we discuss black hole thermodynamics. The event horizon is defined through $f(r_+) = 0$, where r_+ denotes the largest root of f . It is convenient to express the constant μ in terms of r_+ , namely

$$\mu = -\frac{1}{3}\Lambda_0 r_+^3 + k r_+ - \frac{b^2 r_+}{6} \sqrt{r_+^4 + \frac{Q^2}{b^2}} + \frac{Q^2}{3r_+} {}_2F_1\left[\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{Q^2}{b^2 r_+^4}\right]. \quad (3.9)$$

Since the metric is asymptotically (A)dS, according to the definition of mass in asymptotically (A)dS space by Abbott-Deser-Tekin (ADT) formalism [31], we find

$$M = \frac{\omega_2}{8\pi} \mu, \quad (3.10)$$

where $\omega_2 = \int dud\varphi$. For $k = 1$, corresponding the unit S^2 , we have $\omega_2 = 4\pi$.

The temperature T and entropy S on the horizon are easily calculated as

$$T = \frac{f'(r_+)}{4\pi} = \frac{k - \Lambda_0 r_+^2}{4\pi r_+} - \frac{b^2}{8\pi r_+} \sqrt{r_+^4 + \frac{Q^2}{b^2}}, \quad (3.11)$$

$$S = \frac{\mathcal{A}}{4} = \frac{r_+^2}{4} \omega_2. \quad (3.12)$$

The electric and magnetic charges are given by

$$Q_e = \frac{\omega_2}{16\pi} \sqrt{-h} (h^{-1})^{[tr]}|_{r \rightarrow \infty} = \frac{q}{16\pi} \omega_2, \quad Q_m = \frac{\omega_2}{16\pi} F_{u\phi}|_{r \rightarrow \infty} = \frac{p}{16\pi} \omega_2. \quad (3.13)$$

Note that the above electric charge as a conserved quantity follows from the equation of motion (2.5). The electric and magnetic potentials are given by

$$\Phi_e = \frac{q}{r_+} {}_2F_1\left[\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{Q^2}{b^2 r_+^4}\right], \quad \Phi_m = \frac{p}{r_+} {}_2F_1\left[\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{Q^2}{b^2 r_+^4}\right]. \quad (3.14)$$

The differential first law of black hole thermodynamics can be written as

$$dM = TdS + \Phi_e dQ_e + \Phi_m dQ_m. \quad (3.15)$$

One can further treat the cosmological constant as a generalized ‘‘pressure’’ $\mathcal{P}_{\Lambda_0} = -\Lambda_0/(8\pi)$ [32, 33]. The conjugate quantity \mathcal{V} can be viewed as a thermodynamical volume. The first law reads

$$dM = TdS + \Phi_e dQ_e + \Phi_m dQ_m + \mathcal{V} d\mathcal{P}_{\Lambda_0}, \quad (3.16)$$

where

$$\mathcal{V} = \frac{\omega_2}{3} r_+^3. \tag{3.17}$$

Since b is a dimensionful quantity, it will inevitably appearing in the Smarr relation. It is useful also to introduce it as a thermodynamical quantity. Since b^2 has the same dimension of the cosmological constant, we may define $\mathcal{P}_b = -b^2/(16\pi)$, The corresponding thermodynamical potential is

$$\mathcal{V}_b = \frac{\omega_2}{3} r_+^3 \left(\sqrt{1 + \frac{Q^2}{r_+^4 b^2}} - \frac{Q^2}{2b^2 r_+^4} {}_2F_1 \left[\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; -\frac{Q^2}{b^2 r_+^4} \right] \right). \tag{3.18}$$

The extended differential first law of black hole thermodynamics is given by

$$dM = TdS + \Phi_e dQ_e + \Phi_m dQ_m + \mathcal{V} d\mathcal{P}_{\Lambda_0} + \mathcal{V}_b d\mathcal{P}_b. \tag{3.19}$$

The above first law can also be expressed as

$$dM = TdS + \Phi_e dQ_e + \Phi_m dQ_m + \mathcal{V} d\mathcal{P}_\Lambda + \frac{b}{8\pi} (\mathcal{V} - \mathcal{V}_b) db, \tag{3.20}$$

as was proposed in [22]. The integral first law of black hole thermodynamics, also called Smarr formula, is given by

$$M = 2(TS - \mathcal{V}\mathcal{P}_{\Lambda_0} - \mathcal{V}_b\mathcal{P}_b) + \Phi_e Q_e + \Phi_m Q_m. \tag{3.21}$$

(See, also [34–36].) When the topological parameter $k = 0$, corresponding to AdS planar black holes, there exists an additional generalized Smarr relation [37]

$$M = \frac{2}{3}(TS + \Phi_e Q_e + \Phi_m Q_m). \tag{3.22}$$

3.3 Wald formalism

Now we will calculate the conserved charge M by using Wald formalism [31]. The conserved charges of AdS black hole has been calculated by many different methods such as the covariant phase space approach [38, 39] developed by Wald, ADT formalism [31], and quasi-local ADT formalism [40–45]. The Wald formalism has been used to study the first law of thermodynamics for asymptotically-AdS formalism in lots of theories, including Einstein-scalar theory [46, 47], Einstein-Proca [48], Einstein-Yang-Mills [49], Einstein-Horndeski [50–52], in gravities extended with quadratic-curvature invariants [53], and also for Lifshitz black hole [54].

Since the conserved charge of dyonic black hole has the same as that with pure electric case, so for simplicity we calculate the conserved charge for the back hole with the pure electric charge. The effective Lagrangian is

$$\mathcal{L} = \sqrt{-g}L, \quad L = R - 2\Lambda_0 - b^2 \sqrt{1 + \frac{F^2}{2b^2}}, \tag{3.23}$$

A general variation of the Lagrangian (3.23) was given in (2.3). The equations of motion are given by

$$E_{\mu\nu} = G_{\mu\nu} + g_{\mu\nu}\Lambda_0 + \frac{1}{2}g_{\mu\nu}b^2\sqrt{1 + \frac{F^2}{2b^2}} - \frac{F_{\mu\rho}F_{\nu}^{\rho}}{2\sqrt{1 + \frac{F^2}{2b^2}}}, \quad (3.24)$$

$$E_A^{\nu} = \nabla_{\mu} \left(\frac{F^{\mu\nu}}{\sqrt{1 + \frac{F^2}{2b^2}}} \right). \quad (3.25)$$

The surface term $J^{\mu} = J_g^{\mu} + J_A^{\mu}$ is given by

$$\begin{aligned} J_g^{\mu} &= g^{\mu\rho}g^{\nu\sigma}(\nabla_{\sigma}\delta g_{\nu\rho} - \nabla_{\rho}\delta g_{\nu\sigma}), \\ J_A^{\mu} &= -\frac{F^{\mu\nu}\delta A_{\nu}}{\sqrt{1 + \frac{F^2}{2b^2}}}. \end{aligned} \quad (3.26)$$

From this one can define a 1-form $J_{(1)} = J_{\mu}dx^{\mu}$ and its Hodge dual $\Theta_{(D-1)} = (-1)^{(D-1)}\star J_{(1)}$. Considering the infinitesimal diffeomorphism $x^{\mu} \rightarrow x^{\mu} + \xi^{\mu}$, one can get

$$J_{(D-1)} \equiv \Theta_{(D-1)} - i_{\xi}\star\mathcal{L} = E_{\Phi}\delta\Phi - d\star J_{(2)}, \quad (3.27)$$

where i_{ξ} denotes a contraction of ξ^{μ} on the first index of the D -form $\star\mathcal{L}$. One can thus define an $(D-2)$ -form $\mathcal{Q}_{(D-2)} = \star J_2$ and $J_{(D-1)} = d\mathcal{Q}_{(D-2)}$. Here we use the subscript notation “ (p) ” to denote a p -form. To make contact with the first law of black hole thermodynamics, we take $\xi^{\mu} = (\partial_t)^{\mu}$. Wald shows that the variation of the Hamiltonian with respect to the integration constants of a specific solution is given by

$$\delta\mathcal{H} = \frac{1}{16\pi}\delta\int_c J_{(D-1)} - \frac{1}{16\pi}\int_c d(i_{\xi}\Theta_{(D-1)}) = \frac{1}{16\pi}\int_{\Sigma^{(D-2)}} (\delta\mathcal{Q}_{(D-2)} - i_{\xi}\Theta_{(D-1)}), \quad (3.28)$$

where c denotes a Cauchy surface and $\Sigma^{(D-2)}$ is its boundary, which has two components, one at infinity and one on the horizon. Thus according to the Wald formalism, the first law of black hole thermodynamics is a consequence of

$$\delta\mathcal{H}_{\infty} = \delta\mathcal{H}_{+}. \quad (3.29)$$

For four dimensional EBI theory, we have

$$J_{\alpha_1\alpha_2\alpha_3} = \text{E.O.M.} + \epsilon_{\alpha_1\alpha_2\alpha_3\mu}\nabla_{\nu} \left(2\nabla^{[\nu}\xi^{\mu]} - \frac{F^{\mu\nu}A_{\lambda}\xi^{\lambda}}{\sqrt{1 + \frac{F^2}{2b^2}}} \right). \quad (3.30)$$

To specialise to our static black hole ansatz (3.1) in $D=4$ dimensions (note that $h(r) = f(r)$), the result for Lagrangian is well established and is given by

$$\delta\mathcal{Q} - i_{\xi}\Theta = -\omega_2 r^2 \left(\frac{2\delta f}{r} + \left[1 - \frac{\phi'^2}{b^2} \right]^{-\frac{3}{2}} r^2 \phi \delta\phi' \right). \quad (3.31)$$

Choosing the gauge such that the electrostatic potential ϕ vanishes on the horizon, it is straightforward to verify that

$$\delta\mathcal{H}_{+} = T\delta S, \quad \delta\mathcal{H}_{\infty} = \delta M - \Phi_e\delta Q_e, \quad (3.32)$$

which yields the first law of black hole thermodynamics $dM = TdS + \Phi_e dQ_e$.

3.4 Singularity structures

Although the vector field is singularity free, the general solution has a curvature singularity at the origin $r = 0$. To study the nature of the singularity, we consider small- r expansion near the origin:

$$f = -\frac{2(M - M^*)}{r} + k - \frac{1}{2}bQ + \frac{1}{6}(b^2 - 2\Lambda_0)r^2 - \frac{b^3}{20Q}r^4 + \mathcal{O}(r^8), \quad (3.33)$$

where

$$M^* = \frac{\Gamma(\frac{1}{4})^2 \sqrt{b}}{24\sqrt{\pi}} Q^{\frac{3}{2}}. \quad (3.34)$$

The Riemann-tensor squared is given by

$$R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} = \frac{48(M - M^*)^2}{r^6} + \frac{8bQ(M - M^*)}{r^5} + \frac{b^2Q^2}{r^4} + \mathcal{O}\left(\frac{1}{r^2}\right). \quad (3.35)$$

Thus we see that when $M > M^*$, the spacetime has a space-like singularity analogous to the Schwarzschild black hole, whilst it has a time-like singularity. Note that the time-like singularity arising from $M < M^*$ is different from that in the Reissner-Nordström black hole which has a $1/r^8$ divergence. When $M = M^*$, the solution has a conical singularity where g_{tt} is non-vanishing. Thus, for spherically-symmetric solutions with $k = 1$, we have the following classifications:

- $Q > \frac{2}{b}$:
 - $M > M^*$: Schwarzschild-like black hole with space-like $1/r^6$ singularity.
 - $M = M^*$: black hole with space-like $1/r^4$ conical singularity.
 - $M_{\text{ext}} < M < M^*$: black hole with time-like $1/r^6$ singularity, with outer and inner horizons.
 - $M = M_{\text{ext}}$: extremal black hole with time-like $1/r^6$ singularity.
 - $M < M_{\text{ext}}$: naked time-like $1/r^6$ singularity.
- $Q < \frac{2}{b}$:
 - $M > M^*$: Schwarzschild-like black hole with space-like $1/r^6$ singularity.
 - $M = M^*$: naked time-like $1/r^4$ singularity.
 - $M < M^*$: naked time-like $1/r^6$ singularity.
- $Q = \frac{2}{b}$:
 - $M > M^*$: Schwarzschild-like black hole with space-like $1/r^6$ singularity.
 - $M = M^*$: a null singularity where the horizon and curvature singularity coincide.
 - $M < M^*$: naked time-like $1/r^6$ singularity.

It is worth noting that extremal black hole arises only for $Q > 2/b$.

As mentioned earlier, the matter field A is singularity free at $r = 0$, one would then expect that there exists a parameter like $M = M^*$ such that the spacetime solution is free from singularity. However, there is a singularity for the general solutions. To understand this phenomenon, we note from (2.4) that the matter energy-momentum tensor is

$$T_{\text{mat}}^{\mu\nu} = -\frac{b^2 \sqrt{-h}}{2 \sqrt{-g}} (h^{-1})^{(\mu\nu)}. \quad (3.36)$$

It follows that even if $h_{\mu\nu}$ is non-singular and non-vanishing at $r = 0$, the matter energy-momentum tensor diverges at $r = 0$ since $\sqrt{-g}$ vanishes there. The singularity however becomes much milder, and the solution with $Q < 2/b$ and $M = M^*$ may be viewed as a quasi-soliton.

4 Generalization to higher dimensions

In previous sections, we studied the dyonic black hole solutions in the four-dimensional EBI theory, and obtained the first law of thermodynamics for these black holes. Now in this section, we will generalize these results to arbitrary even dimensions $D = 2 + 2n$.

4.1 Local solutions

The general ansatz for AdS planar black holes in $D = 2 + 2n$ dimensions is given by

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 (dx_1^2 + dx_2^2 + \cdots + dx_{2n-1}^2 + dx_{2n}^2), \quad (4.1)$$

$$F = \phi'(r)dr \wedge dt + p(dx_1 \wedge dx_2 + \cdots + dx_{2n-1} \wedge dx_{2n}).$$

The equations of motion of A_ν imply that

$$\phi(r) = \int_r^\infty \frac{qdr}{\sqrt{(r^4 + \frac{p^2}{b^2})^n + \frac{q^2}{b^2}}}. \quad (4.2)$$

It reduces to the previous $D = 4$ case when $n = 1$. The Einstein equations imply that

$$(r^{2n-1}f)' = -\frac{\Lambda_0}{n}r^{2n} - \frac{b^2}{2n}\sqrt{\left(r^4 + \frac{p^2}{b^2}\right)^n + \frac{q^2}{b^2}}. \quad (4.3)$$

Note that $\Lambda_0 = \Lambda - \frac{1}{2}b^2$ and hence there is a smooth $b \rightarrow \infty$ limit. However, the limit $b \rightarrow 0$ is singular for $n \geq 3$.

Thus we see that the solution to the metric function can be expressed in terms of a quadrature. In order to read off the thermodynamical quantities, we would like to write the solution in terms of a well-defined quadrature as a definite integration. To do so, we may define a function $U_n(r)$, which is convergent at $r = 0$, such that $U_n(r) - \sqrt{(r^4 + \frac{p^2}{b^2})^n + \frac{q^2}{b^2}}$ has a falloff that is faster than $1/r$. This choice is not unique, and we may choose $U_n = (r^4 + p^2/b^2)^{\frac{n}{2}}$. Making use of the identity

$$\int_0^r d\tilde{r} \left(\tilde{r}^{2n} - \left(\tilde{r}^4 + \frac{p^2}{b^2} \right)^{\frac{n}{2}} \right) = \frac{r^{2n+1}}{2n+1} \left(1 - {}_2F_1 \left[-\frac{1}{4} - \frac{n}{2}, -\frac{n}{2}; \frac{3}{4} - \frac{n}{2}; -\frac{p^2}{b^2 r^4} \right] \right), \quad (4.4)$$

we find that the function f can now be expressed as

$$f(r) = -\frac{\mu}{r^{2n-1}} - \frac{\Lambda_0}{n(2n+1)}r^2 - \frac{b^2r^2}{2n(2n+1)}{}_2F_1\left[-\frac{1}{4} - \frac{n}{2}, -\frac{n}{2}; \frac{3}{4} - \frac{n}{2}; -\frac{p^2}{b^2r^4}\right] + \frac{b^2}{2nr^{2n-1}} \int_{\infty}^r d\tilde{r} \left\{ \left(\tilde{r}^4 + \frac{p^2}{b^2}\right)^{\frac{n}{2}} - \sqrt{\left(\tilde{r}^4 + \frac{p^2}{b^2}\right)^n + \frac{q^2}{b^2}} \right\}. \quad (4.5)$$

4.2 Thermodynamics

Now we study the thermodynamics of the dyonic AdS planar black holes in $D = 2 + 2n$ dimensions, constructed in the previous subsection. In the large- r expansion, the term associated with graviton condensation has the falloff of $1/r^{2n-1}$. It follows from (4.5) that its coefficient is $-\mu$, with no other terms giving any further contribution. Although there are slower falloffs due to the presence of the magnetic charges, one can nevertheless, following from the Wald formalism, define a ‘‘gravitational mass’’ associated with only the condensation of the graviton modes [37]. It is given by

$$M = \frac{n\omega_2^n}{8\pi} \mu, \quad (4.6)$$

where

$$\mu = -\frac{\Lambda_0 r_+^{2n+1}}{n(2n+1)} - \frac{b^2 r_+^{2n+1}}{2n(2n+1)}{}_2F_1\left[-\frac{1}{4} - \frac{n}{2}, -\frac{n}{2}; \frac{3}{4} - \frac{n}{2}; -\frac{p^2}{b^2 r_+^4}\right] + \frac{b^2}{2n} \int_{\infty}^{r_+} dr \left\{ \left(r^4 + \frac{p^2}{b^2}\right)^{\frac{n}{2}} - \sqrt{\left(r^4 + \frac{p^2}{b^2}\right)^n + \frac{q^2}{b^2}} \right\}. \quad (4.7)$$

Here, for simplicity, we assume that $\int dx_1 dx_2 = \int dx_3 dx_4 = \dots = \int dx_{2n-1} dx_{2n} \equiv \omega_2$. The relation between the mass and the horizon radius r_+ can be determined by $f(r_+) = 0$. We can now treat (q, p, r_+) as independent parameters of the solution. In terms of these parameters, the temperature and entropy are given by

$$T = \frac{f'(r_+)}{4\pi} = -\frac{\Lambda_0}{4n\pi} r_+ - \frac{b^2 r_+^{1-2n}}{8n\pi} \sqrt{\left(r_+^4 + \frac{p^2}{b^2}\right)^n + \frac{q^2}{b^2}}, \quad (4.8)$$

$$S = \frac{\mathcal{A}}{4} = \frac{\omega_2^n}{4} r_+^{2n}. \quad (4.9)$$

The electric and magnetic charges are given by

$$Q_e = \frac{\omega_2^n}{16\pi} \sqrt{-h} (h^{-1})^{[tr]}|_{r \rightarrow \infty} = \frac{q}{16\pi} \omega_2^n, \quad Q_m = \frac{n\omega_2}{16\pi} \int F = \frac{np}{16\pi} \omega_2, \quad (4.10)$$

It follows from (4.2) that the electric potential is given by

$$\Phi_e = \int_{r_+}^{\infty} \frac{q dr}{\sqrt{\left(r^4 + \frac{p^2}{b^2}\right)^n + \frac{q^2}{b^2}}}. \quad (4.11)$$

It is easy to verify that $\Phi_e = \partial M / \partial Q_e$. By assuming the differential first law of black hole thermodynamics (3.15) is still hold, we can obtain the magnetic potential

$$\begin{aligned} \Phi_m = \frac{\partial M}{\partial Q_m} = & 2\omega_2^{n-1} \left[\frac{b^2 r_+^{2n+1}}{4np} \left(\left[1 + \frac{p^2}{b^2 r_+^4} \right]^{\frac{n}{2}} - {}_2F_1 \left[-\frac{1}{4} - \frac{n}{2}, -\frac{n}{2}; \frac{3}{4} - \frac{n}{2}; -\frac{p^2}{b^2 r_+^4} \right] \right) \right. \\ & \left. + \frac{p}{2} \int_{\infty}^{r_+} \left(r^4 + \frac{p^2}{b^2} \right)^{\frac{n}{2}-1} \left(1 - \sqrt{\frac{\left(r^4 + \frac{p^2}{b^2} \right)^n}{\left(r^4 + \frac{p^2}{b^2} \right)^n + \frac{q^2}{b^2}}} \right) dr \right]. \end{aligned} \quad (4.12)$$

The generalized ‘‘pressure’’ $\mathcal{P}_{\Lambda_0} = -\Lambda_0 / (8\pi)$, its conjugate quantity \mathcal{V}

$$\mathcal{V} = \frac{\partial M}{\partial \mathcal{P}_{\Lambda_0}} = \frac{\omega_2^n}{2n+1} r_+^{2n+1}. \quad (4.13)$$

The conjugate term of $\mathcal{P}_b = -b^2 / (16\pi)$ is given by

$$\begin{aligned} \mathcal{V}_b = & -2n\omega_2^n \left[-\frac{r_+^{2n+1}}{8n} \left(1 + \frac{p^2}{b^2 r_+^4} \right)^{\frac{n}{2}} + \frac{2n-3}{8n(2n+1)} r_+^{2n+1} {}_2F_1 \left[-\frac{1}{4} - \frac{n}{2}, -\frac{n}{2}; \frac{3}{4} - \frac{n}{2}; -\frac{p^2}{b^2 r_+^4} \right] \right. \\ & + \frac{1}{2n} \int_{\infty}^{r_+} \left\{ \left(r^4 + \frac{p^2}{b^2} \right)^{\frac{n}{2}} - \sqrt{\left(r^4 + \frac{p^2}{b^2} \right)^n + \frac{q^2}{b^2}} \right\} dr + \frac{q^2}{4nb^2} \int_{\infty}^{r_+} \frac{dr}{\sqrt{\left(r^4 + \frac{p^2}{b^2} \right)^n + \frac{q^2}{b^2}}} \\ & \left. - \frac{p^2}{4b^2} \int_{\infty}^{r_+} \left(r^4 + \frac{p^2}{b^2} \right)^{\frac{n}{2}-1} \left(1 - \sqrt{\frac{\left(r^4 + \frac{p^2}{b^2} \right)^n}{\left(r^4 + \frac{p^2}{b^2} \right)^n + \frac{q^2}{b^2}}} \right) dr \right], \end{aligned} \quad (4.14)$$

The extended differential first law of black hole thermodynamics is given by

$$dM = TdS + \Phi_e dQ_e + \Phi_m dQ_m + \mathcal{V} d\mathcal{P}_{\Lambda_0} + \mathcal{V}_b d\mathcal{P}_b. \quad (4.15)$$

The above first law can also be expressed as

$$dM = TdS + \Phi_e dQ_e + \Phi_m dQ_m + \mathcal{V} d\mathcal{P}_{\Lambda} + \frac{b}{8\pi} (\mathcal{V} - \mathcal{V}_b) db. \quad (4.16)$$

The Smarr formula is given by

$$M = \frac{2n}{2n-1} TS - \frac{2}{2n-1} \mathcal{V} \mathcal{P} + \Phi_e Q_e + \frac{1}{2n-1} \Phi_m Q_m - \frac{2}{2n-1} \mathcal{V}_b \mathcal{P}_b. \quad (4.17)$$

The generalized Smarr formula is given by

$$M = \frac{2n}{2n+1} (TS + \Phi_e Q_e) + \frac{2}{2n+1} \Phi_m Q_m. \quad (4.18)$$

In order to show the above identities, we need to use

$$\begin{aligned}
 & \int_{\infty}^{r_+} \left\{ \left(r^4 + \frac{p^2}{b^2} \right)^{\frac{n}{2}} - \sqrt{\left(r^4 + \frac{p^2}{b^2} \right)^n + \frac{q^2}{b^2}} \right\} dr \\
 &= \frac{r_+}{2n+1} \left[\left(r_+^4 + \frac{p^2}{b^2} \right)^{\frac{n}{2}} - \sqrt{\left(r_+^4 + \frac{p^2}{b^2} \right)^n + \frac{q^2}{b^2}} \right] - \frac{2nq^2}{(2n+1)b^2} \int_{\infty}^{r_+} \frac{dr}{\sqrt{\left(r^4 + \frac{p^2}{b^2} \right)^n + \frac{q^2}{b^2}}} \\
 &+ \frac{2np^2}{(2n+1)b^2} \int_{\infty}^{r_+} \left(r^4 + \frac{p^2}{b^2} \right)^{\frac{n}{2}-1} \left(1 - \sqrt{\frac{\left(r^4 + \frac{p^2}{b^2} \right)^n}{\left(r^4 + \frac{p^2}{b^2} \right)^n + \frac{q^2}{b^2}}} \right) dr. \tag{4.19}
 \end{aligned}$$

Note that the definite integrations in all the above equations are well-defined with no divergence. It is remarkable that although the general solution is given up to a well-defined quadrature, the first law of thermodynamics, and Smarr relations can nevertheless be fully established.

4.3 Some explicit examples

We obtained the general dyonic AdS planar black holes, up to a quadrature. Here we present some explicit examples where the quadrature can be integrated in terms of some special functions.

4.3.1 Pure electric solutions

In this case, we set $p = 0$, and we find

$$f(r) = -\frac{\mu}{r^{2n-1}} - \frac{\Lambda_0 r^2}{n(2n+1)} - \frac{b^2 r^2}{2n(2n+1)} {}_2F_1 \left[-\frac{1}{2}, -\frac{1}{2} - \frac{1}{4n}; \frac{1}{2} - \frac{1}{4n}; -\frac{q^2}{b^2 r^{4n}} \right]. \tag{4.20}$$

Although our ansatz is for even $D = 2n + 2$ dimensions, the above solution is applicable for odd dimensions as well so we rewrite it in terms of D :

$$\begin{aligned}
 f(r) &= -\frac{\mu}{r^{D-3}} - \frac{2\Lambda_0 r^2}{(D-2)(D-1)} \\
 &+ \frac{b^2 r^2}{(D-2)(D-1)} {}_2F_1 \left[-\frac{1}{2}, -\frac{D-1}{2(D-2)}; \frac{D-3}{2(D-2)}; -\frac{q^2}{b^2 r^{2D-4}} \right]. \tag{4.21}
 \end{aligned}$$

Furthermore, we can add a topological parameter k to f so that $f \rightarrow f + k$. The solution becomes that for general topologies and was obtained in [15].

4.3.2 Pure magnetic solutions

In this case, we set $q = 0$, and we find

$$f(r) = -\frac{\mu}{r^{2n-1}} - \frac{\Lambda_0}{n(2n+1)} r^2 - \frac{b^2 r^2}{2n(2n+1)} {}_2F_1 \left[-\frac{1}{4} - \frac{n}{2}, -\frac{n}{2}; \frac{3}{4} - \frac{n}{2}; -\frac{p^2}{b^2 r^4} \right]. \tag{4.22}$$

Note that when $n = 1$, corresponding to four dimensions, the two solutions (4.22) and (4.21) takes the same form with $q \leftrightarrow p$, indicating electric and magnetic duality.

Note also that when $n = 2m$ is even, corresponding to $D = 4m + 2 = 6, 10, \dots$ dimensions, the hypergeometric function in (4.22) solution reduces to some polynomial functions. Here are some low-lying examples in 6, 10, 14 respective dimensions:

$$\begin{aligned}
 n = 2 : f(r) &= -\frac{\Lambda}{10}r^2 - \frac{\mu}{r^3} - \frac{p^2}{4r^2}; \\
 n = 4 : f(r) &= -\frac{\Lambda}{36}r^2 - \frac{\mu}{r^7} - \frac{p^2}{20r^2} - \frac{p^4}{8b^2r^6}; \\
 n = 6 : f(r) &= -\frac{\Lambda}{78}r^2 - \frac{\mu}{r^{11}} - \frac{p^2}{36r^2} - \frac{p^4}{20b^2r^6} - \frac{p^6}{12b^4r^{10}}.
 \end{aligned}
 \tag{4.23}$$

Note that when $n = 2$, corresponding to $D = 6$, the metric is independent of b , which implies that the energy-momentum tensor for the Born-Infeld model is the same as that of the Maxwell theory.

Here we present the thermodynamical properties of pure magnetic AdS planar black holes

$$\mu = -\frac{\Lambda_0 r_+^{2n+1}}{n(2n+1)} - \frac{b^2 r_+^{2n+1}}{2n(2n+1)} {}_2F_1 \left[-\frac{1}{4} - \frac{n}{2}, -\frac{n}{2}; \frac{3}{4} - \frac{n}{2}; -\frac{p^2}{b^2 r_+^4} \right], \tag{4.24}$$

$$T = -\frac{\Lambda_0}{4n\pi} r_+ - \frac{b^2 r_+^{1-2n}}{8n\pi} \left(r_+^4 + \frac{p^2}{b^2} \right)^{\frac{n}{2}}, \quad S = \frac{\omega_2^n}{4} r_+^{2n}, \quad Q_m = \frac{np}{16\pi} \omega_2, \tag{4.25}$$

$$\Phi_m = 2\omega_2^{n-1} \left[\frac{b^2 r_+^{2n+1}}{4np} \left(\left[1 + \frac{p^2}{b^2 r_+^4} \right]^{\frac{n}{2}} - {}_2F_1 \left[-\frac{1}{4} - \frac{n}{2}, -\frac{n}{2}; \frac{3}{4} - \frac{n}{2}; -\frac{p^2}{b^2 r_+^4} \right] \right) \right], \tag{4.26}$$

$$\begin{aligned}
 \mathcal{V}_b &= -2n\omega_2^n \left(-\frac{r_+^{2n+1}}{8n} \left(1 + \frac{p^2}{b^2 r_+^4} \right)^{\frac{n}{2}} \right. \\
 &\quad \left. + \frac{2n-3}{8n(2n+1)} r_+^{2n+1} {}_2F_1 \left[-\frac{1}{4} - \frac{n}{2}, -\frac{n}{2}; \frac{3}{4} - \frac{n}{2}; -\frac{p^2}{b^2 r_+^4} \right] \right).
 \end{aligned}
 \tag{4.27}$$

4.3.3 Dyonic solutions

The quadrature cannot be integrated for general n in terms of a special function, except for $n = 1$ and $n = 2$. The $n = 1$ example was discussed earlier. Now let us consider $n = 2$. The function $f(r)$ is given by

$$\begin{aligned}
 f(r) &= -\frac{\Lambda_0}{10}r^2 - \frac{\tilde{\mu}}{r^3} \\
 &\quad - \frac{1}{4r^2} \sqrt{p^4 + b^2 q^2} F_1 \left[\frac{1}{4}; -\frac{1}{2}, -\frac{1}{2}; \frac{5}{4}; -\frac{b^2 r^4}{\sqrt{-b^2 q^2 + p^2}}; \frac{b^2 r^4}{\sqrt{-b^2 q^2 - p^2}} \right],
 \end{aligned}
 \tag{4.28}$$

where F_1 is the Appell hypergeometric function. This form of the solution is not convenient for extracting the asymptotic infinite behavior. Another equivalent form of the solution is

given by

$$\begin{aligned}
 f(r) = & -\frac{\Lambda_0}{10}r^2 - \frac{\mu}{r^3} - \frac{4p^2 + b^2r^4}{20r^6} \sqrt{\frac{q^2}{b^2} + \left(r^4 + \frac{p^2}{b^2}\right)^2} \\
 & + \frac{3p^4 + b^2q^2}{15b^2r^6} F_1 \left[\frac{3}{4}; \frac{1}{2}, \frac{1}{2}; \frac{7}{4}; \frac{\sqrt{-b^2q^2 - p^2}}{b^2r^4}, -\frac{\sqrt{-b^2q^2 - p^2}}{b^2r^4} \right] \\
 & + \frac{3p^2(p^4 + b^2q^2)}{35b^4r^{10}} F_1 \left[\frac{7}{4}; \frac{1}{2}, \frac{1}{2}; \frac{11}{4}; \frac{\sqrt{-b^2q^2 - p^2}}{b^2r^4}, -\frac{\sqrt{-b^2q^2 - p^2}}{b^2r^4} \right].
 \end{aligned} \tag{4.29}$$

The large- r expansion of $f(r)$ is given by

$$\begin{aligned}
 f(r) = & -\frac{\Lambda}{10}r^2 - \frac{p^2}{4r^2} - \frac{\mu}{r^3} + \frac{q^2}{24r^6} - \frac{p^2q^2}{56b^2r^{10}} - \frac{q^2(b^2q^2 - 4p^4)}{352b^4r^{14}} \\
 & + \frac{p^2q^2(3b^2q^2 - 4p^4)}{480b^6r^{18}} + \dots
 \end{aligned} \tag{4.30}$$

It is then clear that the parameter μ is related to the gravitational mass. Since this solution is a special case of the general solutions, we shall not discuss its thermodynamics further.

4.3.4 A more general topology

We may consider more general ansatz with the following general topologies in $D = 2n + 2$ dimensions

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 \sum_{i=1}^n d\Omega_{i,k}^2, \tag{4.31}$$

$$F = \phi'(r)dr \wedge dt + p \sum_{i=1}^n dx_i \wedge dy_i, \tag{4.32}$$

where

$$d\Omega_{i,k}^2 = \frac{dx_i^2}{1 - kx_i^2} + (1 - kx_i^2)dy_i^2. \tag{4.33}$$

The $\phi(r)$ is given again the same as (4.2) and $f(r)$ is given by

$$\begin{aligned}
 f(r) = & \frac{k}{2n-1} - \frac{\mu}{r^{2n-1}} - \frac{\Lambda_0}{n(2n+1)}r^2 - \frac{b^2r^2}{2n(2n+1)^2} F_1 \left[-\frac{1}{4} - \frac{n}{2}, -\frac{n}{2}; \frac{3}{4} - \frac{n}{2}; -\frac{p^2}{b^2r^4} \right] \\
 & + \frac{b^2}{2nr^{2n-1}} \int_{\infty}^r d\tilde{r} \left\{ \left(\tilde{r}^4 + \frac{p^2}{b^2}\right)^{\frac{n}{2}} - \sqrt{\left(\tilde{r}^4 + \frac{p^2}{b^2}\right)^n + \frac{q^2}{b^2}} \right\}.
 \end{aligned} \tag{4.34}$$

It reduces to the previous result (4.5) for $k = 0$. The horizon topology now becomes $\mathcal{M}_2 \times \mathcal{M}_2 \times \dots \times \mathcal{M}_2$, where \mathcal{M}_2 can be sphere, torus or hyperbolic 2-space.

5 Conclusion

In this paper, we studied the EBI theory and derived the equations of motion that is valid in all dimensions and for all charge configurations. By contrast, the Lagrangian (1.3) considered in many previous works has limited application in higher dimensions. We then constructed the dyonic AdS black holes in four dimensions with a general topology. We analyzed the global structure and obtained the first law of thermodynamics. We classified the singularity structure of these solutions. We then constructed the dyonic AdS black holes in general even dimensions, where the solutions carry both the electric charge and also the magnetic fluxes along the planar space. The general solutions were given up to a quadrature; nevertheless, we show that the first law of black hole thermodynamics can be established. We also give many special examples where the quadrature can be integrated in terms of special functions. These solutions provide new gravity duals to study the AdS/CFT correspondence.

Acknowledgments

S.L.L. is grateful to Shuang-Qing Wu for useful discussions. He also thanks Xing-Hui Feng for kind help and discussions. S.L.L. and H.W. are supported in part by NSFC under Grants No. 11575022 and No. 11175016. H.L. is supported in part by NSFC grants No. 11175269, No. 11475024 and No. 11235003.

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