

Glauber gluons and multiple parton interactions

Jonathan R. Gaunt

*Theory Group, Deutsches Elektronen-Synchrotron (DESY),
D-22607 Hamburg, Germany*

E-mail: jonathan.gaunt@desy.de

ABSTRACT: We show that for hadronic transverse energy E_T in hadron-hadron collisions, the classic Collins-Soper-Sterman (CSS) argument for the cancellation of Glauber gluons breaks down at the level of two Glauber gluons exchanged between the spectators. Through an argument that relates the diagrams with these Glauber gluons to events containing additional soft scatterings, we suggest that this failure of the CSS cancellation actually corresponds to a failure of the ‘standard’ factorisation formula with hard, soft and collinear functions to describe E_T at leading power. This is because the observable receives a leading power contribution from multiple parton interaction (or spectator-spectator Glauber) processes. We also suggest that the same argument can be used to show that a whole class of observables, which we refer to as MPI sensitive observables, do not obey the standard factorisation at leading power. MPI sensitive observables are observables whose distributions in hadron-hadron collisions are disrupted strongly by the presence of multiple parton interactions (MPI) in the event. Examples of further MPI sensitive observables include the beam thrust $B_{a,b}^+$ and transverse thrust.

KEYWORDS: QCD Phenomenology, Hadronic Colliders

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Contents

1	Introduction	1
2	Violation of CSS factorisation for MPI sensitive observables	3
2.1	Hadronic transverse energy	3
2.2	Other observables	15
2.3	MPI sensitive variable of same order as the hard scale	16
3	Conclusions	17
A	Generalised optical theorem	18

1 Introduction

Since the LHC is a proton-proton collider, collisions at the LHC are necessarily a QCD-rich environment. A key component in making predictions at the LHC are factorisation formulae, which separate out the short-distance interaction we are interested in from the long-distance QCD physics (pieces of which we may not be able to calculate perturbatively, but are universal — e.g. PDFs, fragmentation functions, etc.). However, factorisation has only been rigorously proven in hadron-hadron collisions for the inclusive cross section $p + p \rightarrow V + X$, where V is some hard final state and X can be anything, as well as for the p_T distribution of V , when V is colourless [1–4]. These factorisation formulae are true at least to the leading power in Λ_{QCD}/Q (where Q is the hard scale associated with the production of V).

In the standard type of factorisation formula one has hard, collinear and (central) soft functions describing high virtuality particles, particles approximately collinear to some beam or jet direction, and particles with all components of momentum small and of the same order respectively. Examples of collinear functions include PDFs, fragmentation functions, transverse momentum dependent PDFs (TMD PDFs) [4–12], and beam functions [13, 14]. Another momentum region that can potentially contribute at the leading power is the Glauber region. Let us decompose a general momentum A according to $A = A^+p + A^-n + \mathbf{A}_T$, where $p \cdot n = 1$, $n^2 = p^2 = 0$ and \mathbf{A}_T is a vector perpendicular to n and p . Then a Glauber momentum r satisfies $|r^+r^-| \ll \mathbf{r}_T^2 \ll Q^2$. In order to obtain a ‘standard’ factorisation formula one has to show that the effects of the Glauber region cancel. In the proofs of factorisation for the inclusive total cross section and p_T of V [1–4], the effect of the Glauber region is shown to cancel using an argument that relies on us being able to sum inclusively over the undetected particles X (equivalently it relies on us being able to sum over the cuts of graphs containing Glauber gluons — see below). We shall refer to this argument as the Collin-Soper-Sterman (CSS) argument [3] (see also [15]).

In the following we shall also say that an observable factorises if we can write down a standard ‘hard \otimes collinear \otimes central soft’ factorisation formula for it, and for brevity write ‘factorisation formula’ rather than ‘standard factorisation formula’.¹

A number of leading power factorisation formulae in hadron-hadron collisions have been written down for other observables, and some partially proven (especially in the context of soft-collinear effective field theory (SCET) [16–21]). Typically these proofs are limited in the sense that they only apply in the absence of Glauber modes — but see [22–30] for progress in incorporating Glauber modes into SCET.

In this paper we investigate in detail the effect of Glauber gluons on, and factorisation properties of, two observables — hadronic transverse energy E_T , and beam thrust $B_{a,b}^+$, when the hard process is the production of a colourless particle V ($V = H, W, Z \dots$) with associated scale Q . To obtain the E_T in the event $pp \rightarrow V + X$ we take every particle i in X , compute $\sqrt{m_i^2 + \mathbf{p}_{Ti}^2}$, and then sum over i . To define beam thrust we divide the event into two hemispheres, one of which contains p and the other of which contains n (with p oriented along one proton direction and n oriented along the other). In the p hemisphere we take each non- V particle i with momentum p_i , compute $\sqrt{2}p \cdot p_i$, and then sum over i to obtain B_a^+ . To obtain B_b^+ we look in the n hemisphere and sum over $\sqrt{2}n \cdot p_i$. A standard factorisation formula for the E_T distribution has been written down in [31–33] (although in [31, 33] it is referred to as a resummation formula), and a standard factorisation formula for $B_{a,b}^+$ has been written down in [13]. In the latter case, additional arguments were given to rule out possible Glauber effects, relying in part on the original CSS arguments [3, 15]. Our analysis below indicates that the given arguments are not sufficient to rule out Glauber contributions from spectator-spectator interactions, which as we will see are related to multiparton interactions (MPI). When one makes plots of these observables using Monte Carlo generators, one observes a significant impact on their shapes arising from MPI in the underlying event (UE) [31, 33, 34], and the resulting shapes look very much broader than the predictions from the factorisation formula. One might take this as a suggestion that the standard factorisation formulae may be inadequate for these observables, and this is one reason why we choose to re-visit their factorisation properties.

To begin with, in section 2.1, we will just consider the E_T case in detail. We will see if the CSS argument (or some slight modification thereof), which functioned successfully for the inclusive cross section and p_T cases, can also be applied to the observable E_T , when this variable is parametrically smaller than the hard scale Q . What we shall see is that a slight modification of the argument works at the level of one Glauber gluon exchange, but fails when we have two Glauber gluon exchanges. Our discussion very closely follows that of section 14.3 of [4], and involves using the Libby-Sterman analysis [35, 36]. This approach involves identifying pinch surfaces of Feynman graphs and then using power counting to identify the pinches that contribute to leading power. As part of our discussion we will review the cancellation of Glauber gluon effects according to the CSS argument for the observables p_T of V and the total cross section.

¹We leave open the possibility for the observables we study in this paper, which do not satisfy the standard factorisation, that a more general type of leading power factorisation incorporating Glauber modes may be possible.

We will also highlight the connection between this type of two Glauber gluon exchange graph and the soft MPI modelled in Monte Carlo generators. The connection between Glauber gluons and soft MPI has been made before [4, 37] but here we devote it particular attention. Using this connection we will suggest that the failure of the CSS argument strongly indicates a breakdown in the standard factorisation.

Due to the connection between Glauber interactions and MPI, there are problems with applying the standard factorisation formula for a wider class of observables. We refer to these variables as MPI sensitive observables and choose the symbol O_S to refer to a generic MPI sensitive observable. In section 2.2 we discuss some of these observables, including beam thrust $B_{a,b}^+$ and transverse thrust. We also discuss how jet-based observables are less MPI sensitive than global observables such as E_T and $B_{a,b}^+$.

Our main discussion is for the case where O_S is parametrically smaller than the hard scale Q (where here we assume always that O_S has mass dimension 1 — if this is not the case then the statements below have to be altered in a straightforward manner). We shall however devote some discussion to the case $O_S \sim Q$ in section 2.3. Here one might expect the factorisation breaking effects not to be so significant, as the soft Glauber scattering/additional low-scale interaction affects the observable comparatively less. However, since multiple additional scatterings are possible (and tend to occur) and have a cumulative effect on O_S , there is the possibility of the Glauber gluons/MPI having a significant effect on these observables even when they are of order Q . As an example we shall discuss the case of E_T , where detailed Monte Carlo studies have been performed [31, 33] and show a strong disruption of the E_T spectrum up to values of order Q and beyond, when Q was of order 100 GeV. This discussion will be of a somewhat less formal nature than that in sections 2.1 and 2.2.

2 Violation of CSS factorisation for MPI sensitive observables

2.1 Hadronic transverse energy

In this section we will demonstrate in detail that the CSS method of cancelling the Glauber region by summing over cuts for the Glauber subgraph (or some straightforward extension of the CSS method) fails for the ‘MPI sensitive’ observable E_T . At the same time we will review the success of the method for p_T and the total cross section.

For our calculation we will use the model discussed in section 14.3 of [4]. In particular, we will take the proton to be composed of only two constituents (say a quark and an anti-quark). In our discussion we will assume very little about the coupling of the constituents to the proton. In many past calculations the coupling of the partons to the proton is taken to be ‘soft’ in the sense that it suppresses large transverse momenta of the outgoing constituents more strongly than the QCD vertex (such that there is no UV divergence at this coupling for large k_T) — see for example section 2.7 of [38], or any of [39–41]. If one takes this model for the coupling, then in the parlance of [41] the quark and antiquark are a ‘nonperturbatively generated’ pair, and this scenario is most appropriate when E_T is of order Λ_{QCD} . Alternatively we could replace the proton by a gluon from the proton,

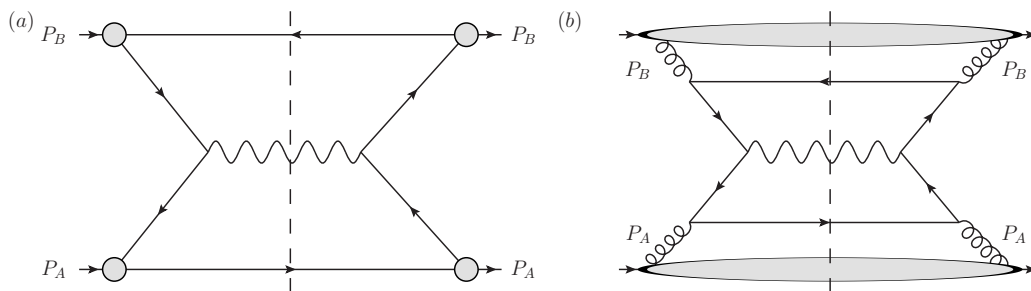


Figure 1. Figure (a) shows the ‘parton-model’ graph we consider for the production of V from the two ‘protons’ P_A and P_B . The only physical cut of this diagram is denoted on the graph using a dashed line. Each grey blob in this figure can either be a nonperturbative coupling of the quark-antiquark pair to the proton (appropriate when E_T is of order of Λ_{QCD}) or it can represent the quark-antiquark-gluon coupling in figure (b) (appropriate when E_T is perturbative). In figure (b) the external lines to the left and right of the figure are protons, and the long grey blobs contain collinear processes occurring at scales lower than E_T .

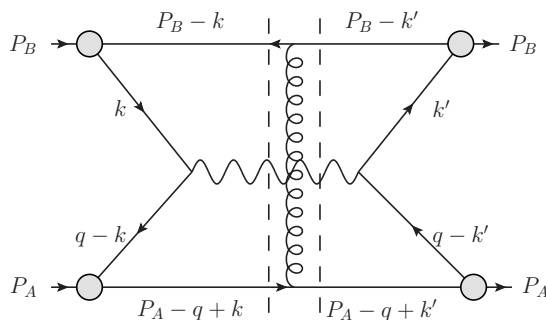


Figure 2. Graph with one Glauber exchange between the spectators. We use the same momentum assignments as figure 14.4 in [4]. The two cuts that allow the gluon to remain in the Glauber region are denoted on the graph as dashed lines.

and take the quark-antiquark-‘proton’ coupling to be the QCD coupling. Then the quark-antiquark pair is ‘perturbatively generated’, and this scenario is more appropriate when E_T is perturbative. We will denote the coupling of the constituents to the ‘proton’ by a grey blob.

We will take the hard process to be $q\bar{q} \rightarrow V$ where the hard scale associated with this is Q , and to avoid the complexities associated with final-state colour, we take V to be colourless. Then the lowest order ‘parton model’ process for $P_A + P_B \rightarrow V + X$ is shown in figure 1(a) (where we also draw the more explicit picture for perturbative E_T in figure 1(b)). For simplicity we take all partons (gluons + quarks) in the calculation to be massless. We take the spatial momentum of P_A to point along that of p , and the spatial momentum of P_B to point along that of n . Protons A and B are taken to have a small mass such that P_A^- and P_B^+ are nonzero but small compared to P_A^+ and P_B^- (this is not essential to our argument).

We start with the most simple type of diagram that contains a Glauber gluon, which

is the diagram in figure 2. In this diagram we have a Glauber gluon attached to two ‘final-state’ spectators (where we use quotation marks because, as has been pointed out by Collins [4] and as we shall review later, this is not an interaction in the final state). Note that in this paper we will not consider Glauber gluon attachments between the two initial state lines (i.e. the lines leading into the hard interaction) and between the initial state lines and the spectator lines. It is well known that these type of interactions do not have a pinch in the Glauber region [4, 42, 43], so these types of interactions are handled satisfactorily using the usual collinear and soft functions and there is no need to study them further.

It is well known and easy to show that this diagram has a leading pinch singularity with the gluon exchanged between the spectators being trapped in the Glauber region (leading means that this diagram contributes at the same power in Λ_{QCD}/Q as the ‘parton model’ diagram of figure 1) [4]. Let us take the parameterisation of the loop momenta to be as in figure 3, with k as the Glauber momentum. We take k_A and k_B to be collinear to P_A and P_B — i.e. their momenta are of order $(1, \lambda^2, \lambda)Q$ and $(\lambda^2, 1, \lambda)Q$ respectively (where λ is a ‘normal coordinate’ that describes the degree of collinearity of the lines). Let us take the transverse component of k to be of order $\lambda_S Q$. Which constraints do we have on k^+ and k^- ? The structure of the integral over these components is as follows:

$$\int \frac{dk^+ dk^-}{(2\pi)^2} \frac{\text{numerator}}{2k^+ k^- - \mathbf{k}_T^2 + i0} \tag{2.1}$$

$$\times \frac{1}{[-2k^+(P_B^- - k_B^-) + \dots + i0][2k^+ k_B^- + \dots + i0]}$$

$$\times \frac{1}{[-2k^- k_A^+ + \dots + i0][2k^-(P_A^+ - k_A^+) + \dots + i0]}$$

where the terms indicated by “...” are all independent of k^+ and k^- and of order $Q(\lambda^2, \lambda\lambda_S, \lambda_S^2)$. We see that if λ_S is of order λ then k^+ and k^- are trapped at small values of order $\lambda_S^2 Q$ — this is the Glauber pinch. The region around this pinch is leading (for $\lambda_S \sim \lambda$) — compared to the leading parton model graph of figure 1 we have an enhancement of $1/\lambda^6$ from the extra 3 propagator denominators (there are no extra factors of λ from the numerator), plus a suppression from the (Glauber) phase space of k which is of order λ^6 (λ^2 from each of dk^+ and dk^- , and λ^2 from $d^2\mathbf{k}_T$). This gives λ^0 overall — i.e. leading. For more detail see [4].

There are two possible cuts of this diagram which leave the gluon in the Glauber region, one to the left of the Glauber gluon, and one to the right. These are depicted using dashed lines in figure 2. Of course we could also have a cut that runs through the ‘Glauber’ gluon itself, but then the gluon would be forced into the central soft region by the on-shellness condition, and would therefore no longer be in the region we want to consider here. It was shown in [4] in the leading power approximation that for each cut of figure 2, we can replace the momentum in the two spectator lines that are not already on shell by an on-shell momentum. This replacement is given pictorially for one of the cuts in figure 4.

Let us briefly review this argument. In the region we are considering (close to the pinch, in which the lines connected to P_A are approximately collinear to P_A , whilst the

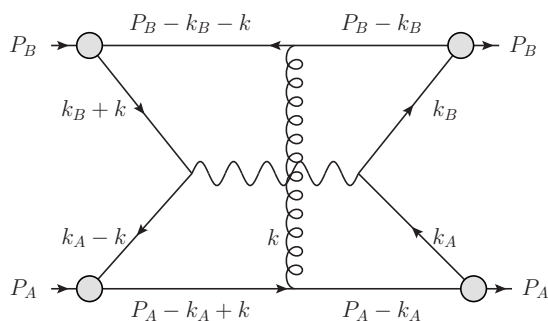


Figure 3. Parameterisation of loop momentum in one-Glauber diagram that is particularly convenient to show the Glauber pinch.

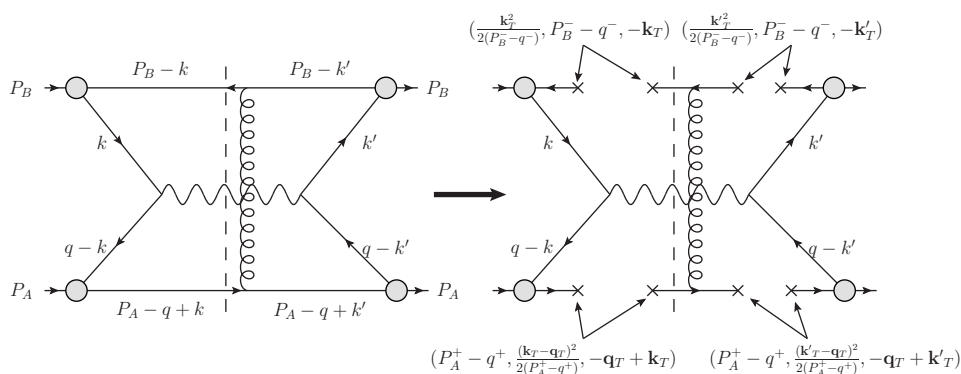


Figure 4. Leading power decomposition of one of the cut graphs in figure 2 — the decomposition for the other cut is the same, except with the cut on the right hand side of the Glauber gluon. The lines with crosses on are now on-shell lines - we write the $(+, -, T)$ components of these momenta near the lines.

lines connected to P_B are approximately collinear to P_B) the components k^+ and k'^+ are small, and we can neglect them in the bottom half of the graph compared to the large momentum components q^+ and $P_A^+ - q^+$. Similarly, in the top half of the graph we can neglect $q^- - k^-$ and $q^- - k'^-$ compared to the large components q^- and $P_B^- - q^-$. Finally, we can neglect the Glauber-trapped components k^\pm and k'^\pm compared to the larger components \mathbf{k}_T and \mathbf{k}'_T .

Now let us consider the k^+ integration. After the approximations, the only dependence on k^+ in this integration is in the lines k and $P_B - k$, and we can perform the integral by

closing the k^+ contour on the pole of $(P_B - k)^2$:

$$\begin{aligned}
 & \int \frac{dk^+}{2\pi} \frac{i}{(2q^- k^+ - \mathbf{k}_T^2 + i0)} \frac{i}{(2(-k^+ + P_B^+)(P_B^- - q^-) - \mathbf{k}_T^2 + i0)} \\
 &= \frac{i}{2(P_B^- - q^-)} \frac{i}{2q^- k_{\text{on-shell}}^+ - \mathbf{k}_T^2 + i0} \\
 &= \int \frac{dk^+}{2\pi} \frac{i}{(2q^- k^+ - \mathbf{k}_T^2 + i0)} 2\pi\delta(2(-k^+ + P_B^+)(P_B^- - q^-) - \mathbf{k}_T^2 + i0)
 \end{aligned} \tag{2.2}$$

where

$$k_{\text{on-shell}}^+ = P_B^+ - \frac{\mathbf{k}_T^2}{2(P_B^- - q^-)} \tag{2.3}$$

In the above we have ignored the numerator factor which is not relevant for the present discussion. The effect of the integral over k^+ is to set $P_B - k$ on shell. The same argument can be repeated for the integrations over k'^+ , k^- and k'^- to set the lines $P_B - k'$, $P_B - q + k$ and $P_B - q + k'$ on shell.

We can therefore separate the sum of two cut graphs into two factors, one of which is just the ‘parton model’ graph with no Glauber gluon (but with slightly different momenta to the left and right of the cut, for general $\mathbf{k}_T, \mathbf{k}'_T$), and the second of which is the sum over the two external cuts for the $q\bar{q} \rightarrow q\bar{q}$ t -channel gluon exchange diagram. These two factors are integrated over $\mathbf{k}_T, \mathbf{k}'_T$, together with any measurement delta functions.

Let’s first say that we make no measurement on the final state (i.e. we are interested only in the fully inclusive cross section for the production of V). Then there are no measurement delta functions to worry about. Now, the gluon exchange graph with the external cut to the right of the graph is just the graph itself $L(\mathbf{k}_T \rightarrow \mathbf{k}'_T)$. On the other hand, the graph with the external cut to the left is the complex conjugate graph for the time reversed process, $L^*(\mathbf{k}'_T \rightarrow \mathbf{k}_T)$. Graphs L and the corresponding amplitudes \mathcal{M} are related by $\mathcal{M} = -iL$, so at the amplitude level we have $i\mathcal{M}(\mathbf{k}_T \rightarrow \mathbf{k}'_T) - i\mathcal{M}^*(\mathbf{k}'_T \rightarrow \mathbf{k}_T)$. This is equal to minus the sum over internal cuts of the graph according to the Cutkosky rules [44, 45], which are the generalised form of the optical theorem described in appendix A. However there are no internal cuts of the graph, so the sum of external cuts gives zero and the effect of the Glauber gluons cancels at this order for the total cross section.

Now let’s consider what happens when we add a measurement. What we would ideally like to happen in order to cancel the Glauber diagrams is, that for fixed momenta $\mathbf{k}_T, \mathbf{k}'_T$, the two cuts have the same value of the measurement. Then the two cuts of the gluon exchange diagram with the same momenta contribute to the same measurement, and we can add them together to give zero as for the total cross section. This certainly works if the measurement is the p_T of V (or equivalently the p_T of all other final state particles) — the core reason behind this is simply momentum conservation, which means that for given $\mathbf{k}_T, \mathbf{k}'_T$ the total p_T of all final state particles not including V is equal both ‘before’ and ‘after’ the gluon exchange.

On the other hand, this does not work for our ‘MPI sensitive’ variable, the total E_T . There the E_T is $|\mathbf{k}_T| + |\mathbf{q}_T - \mathbf{k}_T|$ for the cut to the left of the Glauber gluon, and

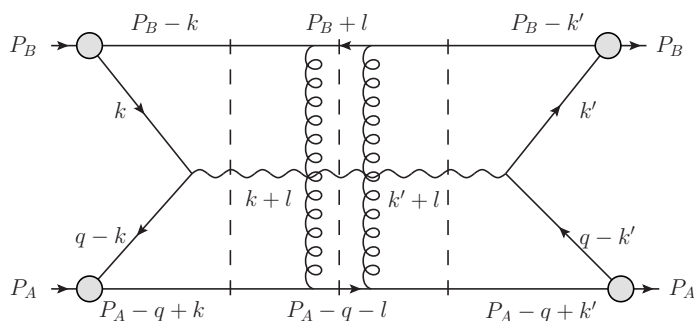


Figure 5. Graph with two Glauber exchanges between the spectators. The three cuts that allow the gluon to remain in the Glauber region are denoted on the graph as dashed lines.

$|\mathbf{k}'_T| + |\mathbf{q}_T - \mathbf{k}'_T|$ for the cut to the right. However we can still attempt to arrange a cancellation of the Glauber region at this simplest order as follows. We write the sum over two cuts as follows:

$$\int d^{d-2}\mathbf{k}_T d^{d-2}\mathbf{k}'_T f_P(\mathbf{k}_T) f_P^*(\mathbf{k}'_T) L(\mathbf{k}_T \rightarrow \mathbf{k}'_T) \delta(E_T = |\mathbf{k}'_T| + |\mathbf{k}'_T - \mathbf{q}_T|) + \quad (2.4)$$

$$\int d^{d-2}\mathbf{k}_T d^{d-2}\mathbf{k}'_T f_P(\mathbf{k}_T) f_P^*(\mathbf{k}'_T) L^*(\mathbf{k}'_T \rightarrow \mathbf{k}_T) \delta(E_T = |\mathbf{k}_T| + |\mathbf{k}_T - \mathbf{q}_T|)$$

where f_P is half the parton model graph (the piece to the left of the cut in figure 1), and L is the one-Glauber exchange graph. We have suppressed a dependence on P_A , P_B , and q in these factors to avoid too many function arguments.

In the second term we re-label $\mathbf{k}_T \leftrightarrow \mathbf{k}'_T$, and then combine the terms, obtaining:

$$\int d^{d-2}\mathbf{k}_T d^{d-2}\mathbf{k}'_T \delta(E_T = |\mathbf{k}'_T| + |\mathbf{k}'_T - \mathbf{q}_T|) \times \quad (2.5)$$

$$[f_P(\mathbf{k}_T) f_P^*(\mathbf{k}'_T) L(\mathbf{k}_T \rightarrow \mathbf{k}'_T) + f_P(\mathbf{k}'_T) f_P^*(\mathbf{k}_T) L^*(\mathbf{k}_T \rightarrow \mathbf{k}'_T)]$$

There are no physical cuts of f_P — therefore $f_P^* = -f_P$ using the Cutkosky rules. Also to leading power the gluon exchange graph L only depends on the transverse momenta through the gluon propagator denominator, which is $\sim (\mathbf{k}_T - \mathbf{k}'_T)^2$ at leading power. L is therefore symmetric in \mathbf{k}_T and \mathbf{k}'_T . Using this fact we can write (2.5) as:

$$\int d^{d-2}\mathbf{k}_T d^{d-2}\mathbf{k}'_T \delta(E_T = |\mathbf{k}'_T| + |\mathbf{k}'_T - \mathbf{q}_T|) \times \quad (2.6)$$

$$f_P(\mathbf{k}_T) f_P^*(\mathbf{k}'_T) [L(\mathbf{k}_T \rightarrow \mathbf{k}'_T) + L^*(\mathbf{k}'_T \rightarrow \mathbf{k}_T)]$$

Now $L + L^*$ can cancel as before, giving zero for the contribution from the Glauber region.

So, we have seen that the Glauber region cancels for one Glauber gluon exchange both for the MPI insensitive variable p_T , and for the MPI sensitive variable E_T , though considerably more gymnastics was required to show this for the latter case. Let us now add in one more Glauber gluon, giving rise to the graph shown in figure 5. It is once again

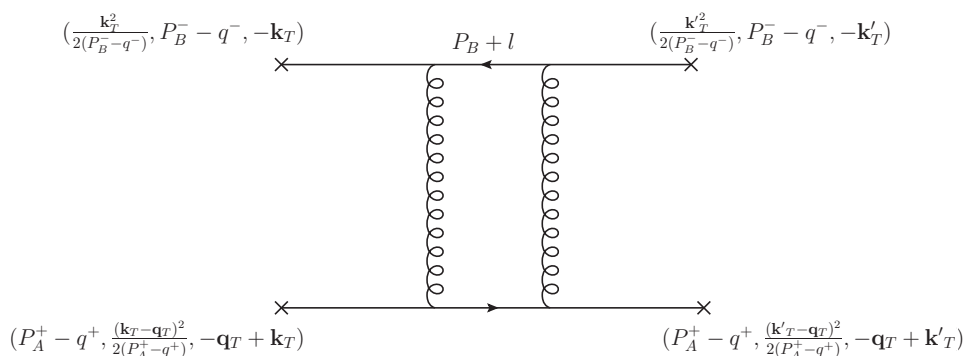


Figure 6. ‘Glauber gluon’ subgraph of the two Glauber exchange graph in figure 5.

simple to show that both gluons in this graph can be trapped in the Glauber region (by taking a loop momentum looping round the left hand side for the left-hand gluon, and a loop momentum looping around to the right for the right-hand gluon), and that this region contributes at leading power of Λ_{QCD}/Q . Does the Glauber region cancel for this graph as well?

There are now 3 cuts of the graph (‘Glauber cuts’) that allow both gluons to remain in the Glauber region, which are drawn in figure 5. We can follow the same reasoning as we did for the one-Glauber-exchange graph and factor a parton-model graph from each cut graph, leaving us with the sum over (internal + external) physical cuts of the 2-gluon exchange graph depicted in figure 6. By external cuts we mean cuts running through external legs of the Glauber subgraph (we shall later refer to these also as ‘absorptive’ cuts), and by internal cuts we mean cuts running through internal legs of the Glauber subgraph (these will later also be referred to as ‘real’ cuts). If we just measure the total cross section (no measurement function) then the sum over cuts cancels for given $\mathbf{k}_T, \mathbf{k}'_T$. As mentioned before, the sum over the two external cuts gives:

$$L(\mathbf{k}_T \rightarrow \mathbf{k}'_T; l) + L^*(\mathbf{k}'_T \rightarrow \mathbf{k}_T; l) = i\mathcal{M}(\mathbf{k}_T \rightarrow \mathbf{k}'_T; l) - i\mathcal{M}^*(\mathbf{k}'_T \rightarrow \mathbf{k}_T; l) \quad (2.7)$$

where $L(\mathbf{k}_T \rightarrow \mathbf{k}'_T; l)$ [$\mathcal{M}(\mathbf{k}_T \rightarrow \mathbf{k}'_T; l)$] is the one-loop graph [amplitude piece] corresponding to figure 6. The sole internal cut gives

$$\begin{aligned} & \int d\Phi_2 L(\mathbf{k}_T \rightarrow \mathbf{l}_T) L^*(\mathbf{k}'_T \rightarrow \mathbf{l}_T) \\ &= \int d\Phi_2 \mathcal{M}(\mathbf{k}_T \rightarrow \mathbf{l}_T) \mathcal{M}^*(\mathbf{k}'_T \rightarrow \mathbf{l}_T) \end{aligned} \quad (2.8)$$

where $d\Phi_2$ is the two-particle on-shell phase space, as in equation (A.7), and the L and \mathcal{M} here are the tree-level quantities. However (2.7) is equal to minus (2.8) according to the diagram-by-diagram version of (A.6) (i.e. the Cutkosky rules), so the sum of three cuts gives zero. Also, if we have the MPI insensitive observable p_T then all cuts contribute to the same value of the observable by momentum conservation (for arbitrary loop transverse momentum \mathbf{l}_T), and cancel against each other just as occurs for the total cross section.

Now, what happens if we have the MPI sensitive observable E_T ? Here the cancellation fails because the ‘real’ cut of the diagram between the Glauber gluons gives a value of E_T that depends on the transverse component of the ‘internal’ loop momentum \mathbf{l}_T (where by internal we mean internal to the $q\bar{q} \rightarrow q\bar{q}$ scattering process), whilst the other two ‘absorptive’ cuts either side of the Glauber gluons give an E_T that does not depend on this internal loop momentum.

One might wonder if there is some clever simultaneous change of variables involving both the ‘external’ variables \mathbf{k}_T and \mathbf{k}'_T and the ‘internal’ loop variable \mathbf{l}_T that can be used to bring about the cancellation, as we found in the one-Glauber case. In this case we argue that no such variable transform is possible. Making a change in the internal variable \mathbf{l}_T is inadvisable, as the Cutkosky cancellation to which we would like to appeal is a cancellation that occurs point by point in spatial momentum (as is shown in appendix B of [46]). Therefore we should match the parameterisation of spatial momentum between the external and internal cuts of the graph. This essentially leaves us with the relabelling $\mathbf{k}_T \leftrightarrow \mathbf{k}'_T$, which in this case does not help (due to the internal cut).

Let us see this more explicitly. We consider the three cuts of figure 6 with this figure embedded in the larger diagram figure 5:

$$\int d^{d-2}\mathbf{k}_T d^{d-2}\mathbf{k}'_T f_P(\mathbf{k}_T) f_P^*(\mathbf{k}'_T) \left\{ \int \frac{d^d l}{(2\pi)^d} [L(\mathbf{k}_T \rightarrow \mathbf{k}'_T; l) \delta(E_T = |\mathbf{k}'_T| + |\mathbf{k}'_T - \mathbf{q}_T|) + L^*(\mathbf{k}'_T \rightarrow \mathbf{k}_T; l) \delta(E_T = |\mathbf{k}_T| + |\mathbf{k}_T - \mathbf{q}_T|)] \right. \tag{2.9}$$

$$\left. + \int d\Phi_2 L(\mathbf{k}_T \rightarrow \mathbf{l}_T) L^*(\mathbf{k}'_T \rightarrow \mathbf{l}_T) \delta(E_T = |\mathbf{l}_T| + |\mathbf{l}_T + \mathbf{q}_T|) \right\}$$

In the Glauber region at leading power the numerator factor in the L factors just gives a constant and is irrelevant to the present discussion. The graph $L(\mathbf{k}_T, \mathbf{k}'_T; l)$ is invariant under $\mathbf{k}_T \leftrightarrow \mathbf{k}'_T$ at leading power. This is because when we flip $\mathbf{k}_T \leftrightarrow \mathbf{k}'_T$ we get back the original graph but with the directions of all momentum arrows reversed. Flipping the directions of the momenta affects only the numerator, but in the region we are interested in, the numerator is just a constant and is unchanged by the flip. Using this, plus the previously established fact that $f_P^* = -f_P$, in the second term:

$$\int d^{d-2}\mathbf{k}_T d^{d-2}\mathbf{k}'_T f_P(\mathbf{k}_T) f_P^*(\mathbf{k}'_T) \left\{ \int \frac{d^d l}{(2\pi)^d} [L(\mathbf{k}_T \rightarrow \mathbf{k}'_T; l) \delta(E_T = |\mathbf{k}'_T| + |\mathbf{k}'_T - \mathbf{q}_T|) + L^*(\mathbf{k}'_T \rightarrow \mathbf{k}_T; l) \delta(E_T = |\mathbf{k}'_T| + |\mathbf{k}'_T - \mathbf{q}_T|)] \right. \tag{2.10}$$

$$\left. + \int d\Phi_2 L(\mathbf{k}_T \rightarrow \mathbf{l}_T) L^*(\mathbf{k}'_T \rightarrow \mathbf{l}_T) \delta(E_T = |\mathbf{l}_T| + |\mathbf{l}_T + \mathbf{q}_T|) \right\}$$

Now L and L^* have the same value of the measurement and we can combine them together using the Cutkosky rule:

$$\int \frac{d^d l}{(2\pi)^d} [L(\mathbf{k}_T \rightarrow \mathbf{k}'_T; l) + L^*(\mathbf{k}'_T \rightarrow \mathbf{k}_T; l)] = - \int d\Phi_2 L(\mathbf{k}_T \rightarrow \mathbf{l}_T) L^*(\mathbf{k}'_T \rightarrow \mathbf{l}_T) \tag{2.11}$$

Then we obtain:

$$\int d^{d-2}\mathbf{k}_T d^{d-2}\mathbf{k}'_T f_P(\mathbf{k}_T)f_P^*(\mathbf{k}'_T) \int d\Phi_2 L(\mathbf{k}_T \rightarrow \mathbf{l}_T)L^*(\mathbf{k}'_T \rightarrow \mathbf{l}_T) \times \quad (2.12)$$

$$[-\delta(E_T = |\mathbf{k}'_T| + |\mathbf{k}'_T - \mathbf{q}_T|) + \delta(E_T = |\mathbf{l}_T| + |\mathbf{l}_T + \mathbf{q}_T|)]$$

The explicit expression for $\int d\Phi_2 L(\mathbf{k}_T \rightarrow \mathbf{l}_T)L^*(\mathbf{k}'_T \rightarrow \mathbf{l}_T)$ at leading power is as follows:

$$\int d\Phi_2 L(\mathbf{k}_T \rightarrow \mathbf{l}_T)L^*(\mathbf{k}'_T \rightarrow \mathbf{l}_T) \quad (2.13)$$

$$= \int \frac{d^d l}{(2\pi)^d} \frac{\text{const.}}{(l+k)^2(l+k')^2} (2\pi)\delta((P_B+l)^2)(2\pi)\delta((P_A-q-l)^2)$$

$$\simeq \int \frac{d^{d-2}\mathbf{l}_T}{(2\pi)^{(d-2)}} \frac{\text{const.}}{2(P_A^+ - q^+)2(P_B^- - q^-)(\mathbf{l}_T + \mathbf{k}_T)^2(\mathbf{l}_T + \mathbf{k}'_T)^2}$$

Due to the mismatch in the delta function arguments, (2.12) does not cancel to zero for general $f_P(\mathbf{k}_T)$.

The lack of a cancellation for fixed \mathbf{k}_T and \mathbf{k}'_T , which worked in the total cross section and p_T cases, can be illustrated by looking at a simpler observable which will shall refer to as T_n . To obtain T_n one just measures the sum of squared transverse momenta of all non- V particles in the n -hemisphere. For simplicity let us restrict ourselves for the moment to the case $\mathbf{k}_T = \mathbf{k}'_T$. Then the function inside the square brackets in (2.12) is:

$$[-\delta(T_n = \mathbf{k}_T^2) + \delta(T_n = \mathbf{l}_T^2)] \quad (2.14)$$

If $\mathbf{k}_T^2 \neq T_n$ then the first delta function in (2.14) does not contribute. The second gives a finite result when inserted into the integral over l , equation (2.13):

$$\begin{cases} \text{const.} \times \frac{T_n + \mathbf{k}_T^2}{(\mathbf{k}_T^2 - T_n)^3} & \text{for } \mathbf{k}_T^2 > T_n \\ \text{const.} \times \frac{T_n + \mathbf{k}_T^2}{(T_n - \mathbf{k}_T^2)^3} & \text{for } \mathbf{k}_T^2 < T_n \end{cases} \quad (2.15)$$

with the constant equal in both cases. Note that this function appears to have an (infra-red) divergence as $\mathbf{k}_T^2 \rightarrow T_n$ — however this divergence is cancelled by the integral associated with the absorptive cuts (first delta function in (2.14)).

It is clear from (2.15) that the integral over l for the observable T_n does not give zero for fixed $\mathbf{k}_T = \mathbf{k}'_T$. Even when integrating along $\mathbf{k}_T = \mathbf{k}'_T$ we will not get zero unless the weight function $f_P(\mathbf{k}_T)f_P^*(\mathbf{k}'_T)$ happens to have exactly the right shape such that the contribution from the absorptive cuts at $\mathbf{k}_T^2 = T_n$ can cancel the contribution from the real cuts distributed over all \mathbf{k}_T^2 (e.g. if $f_P(\mathbf{k}_T)f_P^*(\mathbf{k}'_T) = 1$, which of course is not at all realistic). Such a cancellation seems therefore implausible.

In the more general case $\mathbf{k}_T \neq \mathbf{k}'_T$ the maths is more complicated, but the qualitative picture remains similar. The contribution from the central real cut is the only nonzero one for $T_n \neq \mathbf{k}_T^2, \mathbf{k}'_T^2$. As either \mathbf{k}_T^2 or \mathbf{k}'_T^2 tend to T_n , this contribution becomes divergent, but the divergence is cancelled by the contribution from the absorptive cuts, if we symmetrise

the delta function in this contribution: $\delta(T_n = \mathbf{k}_T^2) \rightarrow \frac{1}{2}[\delta(T_n = \mathbf{k}_T^2) + \delta(T_n = \mathbf{k}'_T^2)]$ (we are allowed to do this due to the symmetry of the rest of the integrand). Let us write out the integral over l explicitly, dropping the constant factors and implementing the symmetrisation in the delta function:

$$\int \frac{d^{d-2}\mathbf{l}_T}{(2\pi)^{(d-2)}} \frac{1}{(\mathbf{l}_T + \mathbf{k}_T)^2(\mathbf{l}_T + \mathbf{k}'_T)^2} \times [-\frac{1}{2}\delta(T_n = \mathbf{k}_T^2) - \frac{1}{2}\delta(T_n = \mathbf{k}'_T^2) + \delta(T_n = \mathbf{l}_T^2)] \tag{2.16}$$

Now we consider integrating T_n over a small region around $T_n = \mathbf{k}_T^2$ (for example). Only one of the two delta functions associated with the absorptive cuts gives a nonzero contribution, which according to a straightforward calculation has the following divergent piece:

$$\frac{\pi}{(2\pi)^2\epsilon} \frac{1}{(\mathbf{k}_T - \mathbf{k}'_T)^2} + \mathcal{O}(\epsilon^0) \tag{2.17}$$

where we take $d = 4 - 2\epsilon$ to regulate the integral.

In the integral associated with the real cut, the divergence comes from the region around $\mathbf{l}_T = -\mathbf{k}_T$. We can therefore replace the denominator factor $(\mathbf{l}_T + \mathbf{k}'_T)^2$ by $(\mathbf{k}_T - \mathbf{k}'_T)^2$ for the divergent part. Then we get from the contribution from the real cut:

$$\frac{1}{(\mathbf{k}_T - \mathbf{k}'_T)^2} \int \frac{d^{d-2}\mathbf{l}_T}{(2\pi)^{(d-2)}} \frac{1}{(\mathbf{l}_T + \mathbf{k}_T)^2} + \mathcal{O}(\epsilon^0) = -\frac{\pi}{(2\pi)^2\epsilon} \frac{1}{(\mathbf{k}_T - \mathbf{k}'_T)^2} + \mathcal{O}(\epsilon^0) \tag{2.18}$$

where the integral on the first line is only over a small region around $\mathbf{l}_T = -\mathbf{k}_T$. We see that the divergent parts (2.17) and (2.18) cancel.

We end up with some more complicated version of (2.15) with all divergences regularised. When we integrate over \mathbf{k}_T and \mathbf{k}'_T a very particular form for the parton model weight function $f_P(\mathbf{k}_T)f_P^*(\mathbf{k}'_T)$ would be required to achieve a cancellation — such a cancellation is again not very plausible.

One might wonder if it is still possible to cancel figure 5 against other graphs in which the ‘separate’ $q\bar{q} \rightarrow q\bar{q}$ two-gluon exchange process is one of the processes depicted in figure 7. We can see that this will not happen, because these graphs only have external or absorptive cuts, and no real cuts. We can follow through the same logic as we did in equations (2.4) to (2.6), and cancel the contribution of these graphs to the E_T spectrum individually. In these cases we obtain $L(\mathbf{k}_T \rightarrow \mathbf{k}'_T; l) = L(\mathbf{k}'_T \rightarrow \mathbf{k}_T; l)$ using the same argument as was applied to figure 6.

Another possibility one might think of is that figure 5 could be cancelled against other graphs containing central soft or collinear connections from the Glauber subgraph to the main scattering process. We have not analysed this possibility in detail but find it implausible for such a cancellation to occur. Very shortly we will see that the Glauber gluon process in figure 5 is intimately related to soft MPI in the hadron-hadron scattering process. The cancellation of figure 5 by graphs with soft or collinear modes would in some sense be equivalent to the suppression of soft MPI by soft or collinear interactions, which we do not find very plausible. Note that in the inclusive cross section or p_T of V cases, the

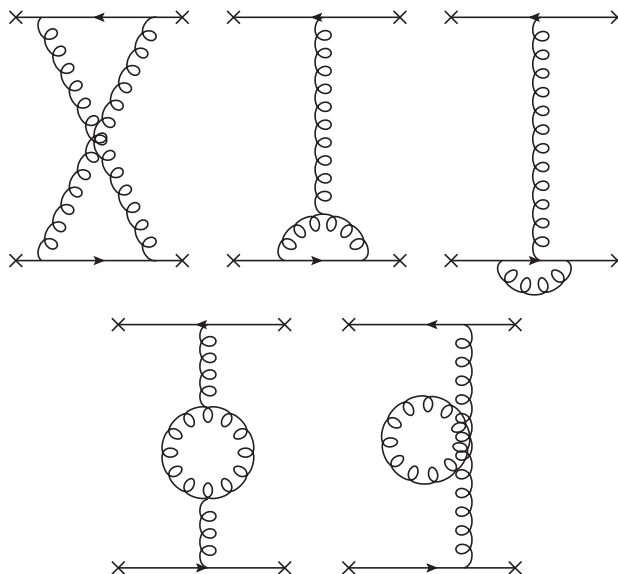


Figure 7. Alternative two-gluon-exchange diagrams. We have not drawn diagrams that are related to the ones above by reflection with respect to a horizontal axis.

cancellation of figure 5 does not imply a suppression of soft MPI, but an insensitivity of the observables to whether additional soft interactions occurred or not. We know from the above discussion that E_T is certainly sensitive to whether an additional scattering occurred or not.

So how does all of this discussion of factorisation breaking relate to MPI, and why do we refer to E_T as MPI sensitive? This should be clear already from our model calculation involving two Glauber gluons. Here, the process effectively splits up into two scattering processes — the high-scale hard process, and a lower scale scattering process (for the cuts internal to the Glauber system) or absorptive interaction (for the external cuts to the Glauber system). The possibility of the real cut in the Glauber subgraph, or MPI, and the fact that the MPI changes the observable, is what results in the factorisation breaking. Note that in the Monte Carlo one does not calculate the virtual graphs explicitly, but that they are included implicitly via unitarity constraints.

Note that with an interpretation of figure 5 as a high scale primary interaction producing V plus a much lower scale secondary QCD interaction, one might expect the lower scale interaction to be initiated at a smaller ‘shower time’ in the initial state shower than the hard interaction (where ‘shower time’ represents the virtuality of the partons in the shower and increases towards the hard interaction). This is the situation in (for example) the `Pythia` initial state shower + MPI model (see figure 1 of [47]). In [4] an analysis of the spacetime locations of the interactions is used to show that the additional interaction is actually initiated first in space-time as well as shower time, and so cannot be thought of as an interaction purely in the final state.

Many models of MPI in Monte Carlo programs — notably the ones in `Pythia` [47–49], `Herwig(++)` [50–52], and the `Pythia`-inspired `AMISIC++` model in `Sherpa` [53] — take

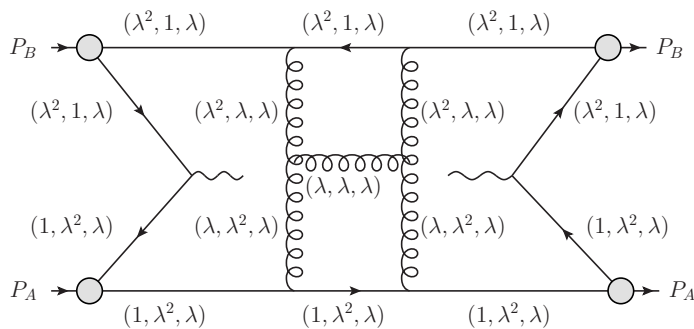


Figure 8. Interaction between spectators composed from ‘collinear Glauber’ uprights and a central soft rung. The scaling of the momenta at the collinear Glauber + central soft pinch is denoted on the figure. Each momentum r_i on the uprights is now referred to as ‘collinear Glauber’ because one of $|r_i^+|$ or $|r_i^-|$ is now non-negligible compared to the transverse momentum, but the momentum satisfies the basic condition for a Glauber momentum $|r_i^+ r_i^-| \ll \mathbf{r}_{Ti}^2$.

additional interactions to be just extra $2 \rightarrow 2$ scattering processes. It may be important to consider more general $2 \rightarrow n$ processes. Let us go back for the moment to our model, and consider the next step up in complexity from our ‘two Glauber gluon’ example. In this example we make the simplest gluon ladder possible by inserting a horizontal gluon ‘rung’ between the vertical gluons in figure 5, to obtain figure 8. This graph can have a leading pinch in the region in which the vertical gluons are in the ‘collinear Glauber’ region, and the rung is central soft — the scalings of all the particles at the pinch are demonstrated on figure 8. In the E_T case, we again do not achieve a cancellation between the central ‘real’ cut and the ‘absorptive’ cuts to either side of the Glauber subgraph, as we also found for the simpler two-Glauber-gluon graph.

Figure 8 is suppressed by an additional power of α_S , but it is known from BFKL physics that when the rapidities are strongly ordered (as indeed they are in figure 8) then one picks up an additional enhancement from rapidity logarithms that acts to counterbalance the α_S suppression (see for example [54]). Indeed, we can actually insert arbitrary numbers of gluon rungs into the Glauber process, forming something akin to a Pomeron, and still be at leading logarithmic order in the BFKL sense (the connection between Glauber gluons and the Pomeron/BFKL physics is also highlighted in [4, 27–29, 37]). These objects, as well as other effects from Pomeron physics such as Pomeron splitting and merging, are not included in e.g. default `Pythia` and `Herwig` (except in the parts of these programs designed to describe diffractive observables), but could well be important. Due to this fact we cannot be completely confident that the contribution to E_T from the underlying event generated by `Herwig++` in the studies [31, 33] will be completely accurate. We note here that the Monte Carlo codes `SHRiMPS` [55] and `DIPSY` [56] do incorporate some of the necessary BFKL/Pomeron-type effects.

2.2 Other observables

With the connection between standard factorisation violation, two Glauber exchange and soft MPI, we expect any observable sensitive to MPI O_S not to obey factorisation when the value of that observable is parametrically small compared to the hard scale (where here again we have taken O_S to have mass dimension 1). Consider for example beam thrust. In this case the function inside the square brackets in (2.12) becomes:

$$\left[-\delta\left(b_a^+ = \frac{(\mathbf{k}'_T - \mathbf{q}_T)^2}{2(P_A^+ - q^+)}\right) \delta\left(b_b^+ = \frac{\mathbf{k}'_T{}^2}{2(P_B^- - q^-)}\right) + \delta\left(b_a^+ = \frac{(\mathbf{l}_T + \mathbf{q}_T)^2}{2(P_A^+ - q^+)}\right) \delta\left(b_b^+ = \frac{\mathbf{l}_T^2}{2(P_B^- - q^-)}\right) \right] \quad (2.19)$$

with $B_a^+ = \sqrt{2}b_a^+$, $B_b^+ = \sqrt{2}b_b^+$. We can see that also in this case the integrals associated with the real and absorptive cuts will not cancel.²

It is particularly interesting in the case of beam thrust to consider figure 8. For beam thrust the central real cut of figure 8 gives a power suppressed contribution compared to the absorptive cuts (which are leading power), so in this case the lack of a cancellation between real and absorptive cuts becomes particularly clear. The reason why the real cut gives a power suppression for beam thrust is that its contribution to beam thrust is spread over a much larger phase space than that from the parton model or absorptive graphs, being of order λQ rather than $\lambda^2 Q$.

One example of a further observable which can be considered to be MPI sensitive is transverse thrust. This is defined by computing $\sum_i |\mathbf{q}_{iT} \cdot \mathbf{n}_{iT}| / \sum_i |\mathbf{q}_{iT}|$ and then maximising over the direction of the transverse vector \mathbf{n}_{iT} . One can verify by following the steps above that the CSS argument does not go through for this variable either. In fact we have experimental data for this observable in hadron-hadron collisions [57–60]. In the CDF study [57] it was shown that an NLO+NLL result [61] from the standard factorisation formula for this observable disagreed strongly with the shape of the transverse thrust from data, and that the Monte Carlo curves including MPI agreed much better with the data. This might be regarded as experimental evidence that the standard factorisation does not work for this observable.

Another class of observables that is interesting to consider is the set of jet based observables — e.g. the p_T or mass of an individual jet. If we denote the momentum of the

²Imagine if we add some initial state radiation (ISR) to the ‘primary’ $q\bar{q} \rightarrow V$ process in figure 5. The effect of this initial state radiation is what is resummed in the standard factorisation formula. Then if x_p corresponds to the x values in the primary interaction, then one expects the contribution to the beam thrust from this ISR to be roughly of order transverse momentum squared divided by $x_p P$ (with $P \sim P_A^+ \sim P_B^-$). By contrast the contribution from the Glauber miscancellation in (2.19) is of order transverse momenta squared divided by $x_s P$, with x_s the x values taken by the spectators (and we have $x_s + x_p = 1$). If $q^+, q^- \ll P_A^+, P_B^-$ (as is often the case for processes measured at the LHC) such that $x_p \ll 1$, and $x_s \gg x_p$ in our model, then one might be tempted to argue that the Glauber miscancellation for beam thrust is energy suppressed. However, this is neglecting the fact that in reality in the proton we can have many spectators, and also have ISR from the spectators, such that the x values of the spectators can be (and often are) as small or smaller than the x values for the primary interaction. So one cannot argue that the Glauber miscancellation is energy suppressed for beam thrust in this way.

jet after the clustering algorithm has been applied as p_J , then the p_T of the jet is $|\mathbf{p}_{TJ}|$ and the mass squared of the jet is p_J^2 . Another example is \mathcal{T}_j , which is given by identifying jets $j(R)$ of size R , and then taking the beam thrust of the jet which gives the largest contribution to this quantity [32]. For these variables, the equivalent to (2.12) has a sum of extra delta functions in each term expressing the fact that at least one of the emitted particles in the MPI or absorptive process should be inside the jet. For some general jet observable the contributions associated with the real and absorptive cuts in figure 5 will again not cancel, so once again we get a $\mathcal{O}(\lambda^0)$ contribution from the Glauber region/MPI and we cannot claim that the standard factorisation formulae for these observables holds at leading power in λ either. However, for jet observables, we have another parameter, the jet radius R . In order for the Glauber miscancellation to happen, at least one of the absorptive or real sets of delta functions in the equivalent to (2.12) must be satisfied, a possibility that gets rarer and rarer the smaller the jet radius R is. So for jet-based observables the Glauber miscancellation is suppressed by the jet size R (as opposed to λ), and these observables are less MPI sensitive such that we may usefully apply the standard factorisation formulae to predict them (provided R is not too large). The physical picture corresponding to this is clear — for jet-based observables, we only collect up MPI over the area of the jet(s), and so are much less sensitive to them than we would be for the global observables discussed above (with the sensitivity decreasing as the jet radius R decreases). The notion of MPI sensitivity in jet-based observables has been discussed before, in [62].

2.3 MPI sensitive variable of same order as the hard scale

In our discussion so far we did not specify the size of the MPI sensitive observable O_S . When the MPI sensitive variable is parametrically small with respect to the hard scale Q (e.g. of order λQ in the case of E_T or of order $\lambda^2 Q$ in the case of $B_{a,b}^+$) then the effect of the Glauber gluon miscancellation is large. On the other hand, naive application of the CSS (or more recent Aybat-Sterman [15]) argument might lead one to believe that when one considers O_S to be of values of order Q , then the standard factorisation formula should apply (this is in particular for the cumulant distribution in O_S , where by the term ‘cumulant’ here we mean the integral of the O_S distribution from zero up to some value of O_S). Then one seems to be ‘inclusive’ up to the hard scale Q , and a standard factorisation formula should then be satisfactory. When $O_S \sim Q$, then the Glauber miscancellation in figure 5 only smears the observable by some power suppressed amount, which especially in the cumulant is not a significant effect.

However, the problem in practice is that an MPI sensitive observable O_S can receive a cumulative contribution from many additional soft scatters, and there is the possibility of Glauber miscancellations on multiple spectator legs adding together to disrupt (broaden) the O_S distribution significantly for not too large O_S (even when it is of order Q). In this sense the O_S distribution/cumulant can be sensitive to whether much lower scale scatterings occur or not (i.e. not be sufficiently inclusive) even for $O_S \sim Q$.

This possibility of multiple additional interactions is built into the Monte Carlo generators. The effect of MPI on the MPI sensitive observable E_T has been studied using the Monte Carlo generator `Herwig++` in [31, 33]. From section 5.3 of [31] and section 4.3

of [33] we can see that the E_T differential distribution (and also infer that the cumulant) changes drastically when we add MPI according to the `Herwig++` model, even when E_T is of order Q (and beyond). This is when Q is of order of the typical electroweak scales that are probed at the LHC, and no minimum p_T cut on detected hadrons is imposed. By imposing some minimum p_T cut on detected hadrons of the order of $1-2$ GeV (and thereby effectively becoming insensitive to the lowest scale MPI), the shape of the E_T distributions becomes much narrower such that the cumulant in E_T agrees better with the result from the factorisation formula when integrating up to Q . However, one could argue that this is not terribly useful since the value they are then agreeing on is the total cross section for $p+p \rightarrow V+X$, which we know we can predict via a factorisation formula. Also the strong sensitivity of the E_T distribution/cumulant on a very low scale p_T cut off on hadrons shows that the E_T distribution/cumulant is strongly sensitive on very much lower scale MPI occurring, even when $E_T \sim Q$. Or, to put it another way, the E_T distribution/cumulant is not a sufficiently inclusive observable even when $E_T \sim Q$.

One might expect that there is some limit to how much E_T can be generated by the MPI, since one cannot have an infinite number of extra interactions (that generate enough p_T to be measurable) and each extra interaction tends to only contribute a small amount to the observable. Then if one measured E_T at values much larger than this limit one would expect to get a good description from the standard factorisation formula (up to some smearing of the order of the maximum value of the observable you can obtain from MPI). However, in the study [31] it was observed that the MPI in `Herwig++` are capable of generating an E_T of $200-300$ GeV (this is consistent with the statement above that E_T spectra for $E_T \sim Q$ were being strongly disrupted in `Herwig++` even with $Q = \mathcal{O}(100\text{GeV})$). Taking this as a guideline we can see that at least for the MPI sensitive observable E_T , finding a regime where the standard factorisation formula can be applied at the LHC is very difficult.

Due to the preference of additional interactions for small values of the observable, we expect typical events that contribute to the observable to contain a large number of MPI. This means it is not possible to simply apply the framework being developed for double parton scattering (DPS) [41, 63–67] to this problem. To put it another way, MPI sensitive observables are in general not a good tool to measure DPS because of strong pollution of the observable by MPI with much higher numbers of scatters.

3 Conclusions

We have shown explicitly that the classic Collins-Soper-Sterman (CSS) method of cancelling the contribution from non-factorising Glauber gluons (or a straightforward extension thereof) fails at the level of two Glauber gluons for hadronic transverse energy E_T , when E_T is much less than the hard scale of the primary interaction Q . By connecting diagrams with such Glauber gluon exchanges to events with soft MPI, we argued that this failure of the CSS technique corresponds to an actual breakdown in the factorisation of the cross section for these observables into hard, collinear, and central soft functions at leading power. Using the same connection, we also argued that such factorisation fails for a larger

class of variables, when these observables are $\ll Q$. We refer to these observables as MPI sensitive observables — they are the ones whose distributions in hadron-hadron collisions can be disrupted strongly by the cumulative contribution of a large number of soft quasi-uncorrelated multiple parton interactions. Other examples of MPI sensitive variables are beam thrust $B_{a,b}^+$ and transverse thrust.

We suggested that since MPI sensitive observables can receive a cumulative contribution from MPI, there might be a problem in applying the standard ‘hard \otimes collinear \otimes central soft’ factorisation to these observables even when they are of order of the hard scale. The Monte Carlo studies of (for example) the E_T spectrum when $Q = \mathcal{O}(100\text{GeV})$ [31, 33] give results consistent with this expectation.

Describing MPI sensitive variables correctly is a difficult problem. On the one hand, for a given value of the observable, there should be a significant contribution from events with a large number of very soft MPI. Monte Carlo MPI models include these effects at some level (although of course the description of the very soft scatters can only be a model). On the other hand, MPI sensitive variables can be affected not just by $2 \rightarrow 2$ MPI but also $2 \rightarrow n$ scatters and a whole range of related effects familiar to the BFKL (or high energy) community. These effects are not described so well by default *Pythia* and *Herwig* (which just have just have $2 \rightarrow 2$ MPI) but are included to some degree in other Monte Carlo codes such as *SHRiMPS* and *DIPSY*. Including the full range of these effects is of course highly nontrivial.

An alternative approach is of course to avoid MPI sensitive observables. Global observables such as the ones we have discussed tend by their nature to be MPI sensitive (because they collect up all the particles produced in the event, including the huge number of soft MPI distributed more or less evenly over the event). Jet-based observables are much less ‘MPI sensitive’ in this regard, because they only collect radiation from MPI over the area of the jets, which is much smaller than the whole area of the event in general. This is of course the strategy that has been favoured at the LHC and other hadron colliders. One should be cautious not to use jet areas that are too large — then the observable starts to become more MPI sensitive [62].

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A Generalised optical theorem

Here we briefly review the most simple derivation of the generalised optical theorem (see for example [69]). The Cutkosky rules [44–46] are a more generalised form of this relation.

From the unitarity of the scattering matrix S we can infer that S must satisfy:

$$S^\dagger S = \mathbb{I} \tag{A.1}$$

We separate S as usual into the identity matrix \mathbb{I} (which corresponds to the particles passing through each other with no scattering) and a transition matrix T (which always appears accompanied by the factor i):

$$S = \mathbb{I} + iT \tag{A.2}$$

Substituting (A.2) into (A.1) we get:

$$iT - iT^\dagger + T^\dagger T = 0 \tag{A.3}$$

Now we sandwich (A.3) between two different states $\langle f|$ and $|i\rangle$. We use the fact that:

$$\langle f|T|i\rangle = i(2\pi)^d \delta^{(d)}(p_i = p_f) \mathcal{M}(i \rightarrow f) \tag{A.4}$$

and:

$$\langle f|T^\dagger|i\rangle = \langle i|T|f\rangle^* \tag{A.5}$$

to obtain:

$$i[\mathcal{M}(i \rightarrow f) - \mathcal{M}^*(f \rightarrow i)] = - \sum_X \int d\Phi_X \mathcal{M}(i \rightarrow X) \mathcal{M}^*(f \rightarrow X) \tag{A.6}$$

where X is any possible intermediate state, and $d\Phi_X$ is the on-shell phase space element for X . Denoting the momenta in X as p_j , this is given by:

$$d\Phi_X = (2\pi)^d \delta^{(d)}\left(p_{i(f)} - \sum_j p_j\right) \prod_j \frac{d^d p_j}{(2\pi)^d} (2\pi) \delta(p_j^2 - m_j^2) \tag{A.7}$$

The relation (A.6) is the generalised optical theorem. The Cutkosky rules are more general than this, and state that (A.6) applies also on a diagram-by-diagram basis (with the sum over X being replaced by a sum over physical cuts of the graph), and point-by-point in the spatial momenta of all (external+loop) particles [44–46].

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