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Renormalization of vacuum expectation values in spontaneously broken gauge theories

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Abstract: We compute one-loop and two-loop β-functions for vacuum expectation values (VEVs) in gauge theories. In R_ξ gauge the VEVs renormalize differently from the respective scalar fields. We focus particularly on the origin and behaviour of this difference and show that it can be interpreted as the anomalous dimension of a certain scalar background field, leading to simple direct computation and qualitative understanding. The results are given for generic as well as supersymmetric gauge theories. These complement the set of well-known γ- and β-functions of Machacek/Vaughn. As an application, we compute the β-functions for VEVs and $\tan\beta$ in the MSSM, NMSSM, and E_6SSM.

Keywords: Spontaneous Symmetry Breaking, Renormalization Group, Supersymmetric gauge theory

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Contents

1 Introduction

Local gauge invariance has been established as the underlying principle of all fundamental interactions. Spontaneously broken gauge invariance, together with the postulate of a perturbative Higgs sector, is the basis of the theoretical description of electroweak interactions in the Standard Model (SM) or extensions like the Minimal Supersymmetric Standard Model (MSSM). These models have successfully passed electroweak precision tests [1] and are in line with the discovery of a Higgs-like particle at the LHC [2, 3].

Quantum field theoretical foundations of spontaneously broken gauge theories like renormalizability, unitarity, and gauge independence of the S-matrix have been established in [4–8]. Later, BRS invariance and algebraic renormalization have been introduced as elegant tools [9–11]. They were used in the all-order treatments of the renormalizability of the SM [12–15] and the MSSM [16].

In the present paper, we focus on the scalar (Higgs) vacuum expectation values (VEVs) and their renormalization in spontaneously broken gauge theories. In spite of their obvious central role, the VEVs are no gauge invariant, physical quantities and therefore less comprehensively studied. Like refs. [12, 13, 16], we use the approach put forward in ref. [17], where the VEVs are treated as background fields, similar to the background field method [14, 15, 18–20]. We describe the renormalization of general gauge theories in this approach, determine Feynman rules, and compute relevant renormalization constants. We show that this framework yields several results of practical and theoretical interest in an elegant way:

1. The renormalization transformation for a VEV v can generically be written in two equivalent ways,[1]

$$v \to v + \delta v = \sqrt{Z}\,(v + \delta\bar{v})\,, \tag{1.1}$$

   where $\sqrt{Z}$ is the field renormalization constant of the respective scalar field and $\delta\bar{v}$ is an extra term, which characterizes to what extent the VEV renormalizes differently from the field. We will show that this extra term has an elegant interpretation in terms of the background field and can be computed easily from the background field Feynman rules. This will also clarify why the $\delta\bar{v}$-term does not appear in theories with only rigid (global) invariance, and that even in local gauge theories it is only required for particular gauges.

2. In many extensions of the SM with several Higgs doublets such as the MSSM, the ratio of two VEVs $\tan\beta = v_u/v_d$ is considered. The explicit MSSM calculations of refs. [21, 22] have found a cancellation, in the notation of eq. (1.1)

$$\frac{\delta\bar{v}_u}{v_u} - \frac{\delta\bar{v}_d}{v_d} = \text{finite} \tag{1.2}$$

   at the one-loop level. Our approach will make clear that this cancellation is not general. We will exhibit the origin of the one-loop cancellation and extend the discussion to the two-loop level in the MSSM, NMSSM, and E$_6$SSM. The latter two cases provide examples where eq. (1.2) is not valid (see also refs. [23, 24] for corresponding results on the $\tan\beta$ renormalization constant).

3. Finally, we compute the renormalization-group β-functions for all VEVs in the general gauge theory and a general supersymmetric gauge theory at the one-loop and leading two-loop level. These results complement the well-known β and γ functions of Machacek/Vaughn [25–27] and Martin/Vaughn, Yamada, Jack and Jones [28–30] for parameters and fields.

The outline of the present paper is the following. First, we will introduce the generic model together with its properties and renormalization in section 2. Section 3.1 discusses the crucial points of our formalism and its equivalence to the standard approach. The remainder of section 3 gives an overview of the necessary one-loop and two-loop computations and states our main results, the general β-functions. Finally, section 4 applies the general results to the MSSM, NMSSM, and E$_6$SSM in order to provide the results for $\tan\beta$ and the VEV β-functions.

[1] In refs. [21, 22], where the second form is chosen, our $\delta\bar{v}$ is called $-\delta v$, while our δv is not used.

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2 General gauge theory and scalar background fields

2.1 Lagrangian

The present paper investigates the renormalization of general, spontaneously broken gauge theories. Following refs. [25, 31, 32], we write the gauge invariant Lagrangian in terms of real scalar fields φ_a and Weyl 2-spinors $\psi_{p\alpha}$ as

$$
\begin{aligned}
\mathcal{L}_{\text{inv}} = &-\frac{1}{4}F^A_{\mu\nu}F^{A\mu\nu} + \frac{1}{2}\left(D_\mu\varphi\right)_a\left(D^\mu\varphi\right)_a + i\psi^\alpha_p\sigma^\mu_{\alpha\dot\alpha}\left(D^\dagger_\mu\bar\psi^{\dot\alpha}\right)_p \\
&-\frac{1}{2!}m^2_{ab}\varphi_a\varphi_b - \frac{1}{3!}h_{abc}\varphi_a\varphi_b\varphi_c - \frac{1}{4!}\lambda_{abcd}\varphi_a\varphi_b\varphi_c\varphi_d \\
&-\frac{1}{2}\left[\left(m_f\right)_{pq}\psi^\alpha_p\psi_{q\alpha} + \text{h.c.}\right] - \frac{1}{2}\left[Y^a_{pq}\psi^\alpha_p\psi_{q\alpha}\varphi_a + \text{h.c.}\right].
\end{aligned}
\tag{2.1}
$$

Here the covariant derivatives and field strength tensor are defined as

$$D_\mu\varphi_a = \left(\delta_{ab}\partial_\mu + igT^A_{ab}V^A_\mu\right)\varphi_b, \tag{2.2a}$$
$$D_\mu\psi_{p\alpha} = \left(\delta_{pq}\partial_\mu + igt^A_{pq}V^A_\mu\right)\psi_{q\alpha}, \tag{2.2b}$$
$$F^A_{\mu\nu} = \partial_\mu V^A_\nu - \partial_\nu V^A_\mu - gf^{ABC}V^B_\mu V^C_\nu, \tag{2.2c}$$

with antisymmetric, purely imaginary generators T^A_{ab} for the scalars; hermitian generators t^A_{pq} for the spinors; and structure constants f^{ABC}. The standard procedure in spontaneously broken gauge theories is to shift the scalar fields by a constant (the "VEV")

$$\varphi_a \to \varphi_a + v_a, \tag{2.3}$$

where v_a can be adjusted to the minimum of the scalar potential. After applying the shift (2.3), the Lagrangian $\mathcal{L}_{\text{inv}}$ is still invariant under both local and global gauge transformations, if $(\varphi_a + v_a)$ are transformed as a whole.

For quantization a gauge fixing is required. In QED and QCD, typical gauge fixing terms break local gauge invariance but leave global gauge invariance intact. In contrast, R_ξ-gauges, for example, as often used in the spontaneously broken case, break even global invariance. This breaking is crucial for the renormalization properties of v_a. It turns out that these properties can be studied well by using background fields [17] instead of the shift in eq. (2.3).

In the following we briefly introduce the setup for R_ξ gauges including background fields. In section 3.1 we will highlight its crucial points, relate it to the standard procedure, and draw consequences. We note here only that the background formalism is a tool that provides additional information but does not alter any results compared to the standard approach.

We introduce real scalar background fields $(\hat\varphi_a + \hat v_a)$; $\hat\varphi_a$ is treated as a classical background field, $\hat v_a$ as a constant. The combination $(\hat\varphi_a + \hat v_a)$, by definition, has the same gauge transformation properties as φ_a. By means of the replacement

$$\varphi_a \to \varphi^{\text{eff}}_a = \varphi_a + \hat\varphi_a + \hat v_a \tag{2.4}$$

we introduce a non-trivial ground state as well as the background field.

Gauge fixing and Slavnov-Taylor identities require BRS transformations. The background fields transform as a BRS doublet with another background field $\hat{q}_a$,

$$s\hat{\varphi}_a = \hat{q}_a, \qquad\qquad s\hat{q}_a = 0. \tag{2.5a}$$

This implies that the physics content of the theory is unchanged by including the background fields [33, 34]. The BRS transformations of the scalar fields read

$$s\varphi_a = -igT^A_{ab}c^A\varphi^{\text{eff}}_b - \hat{q}_a, \tag{2.5b}$$

such that φ^{eff}_a transforms homogeneously

$$s\varphi^{\text{eff}}_a = -igT^A_{ab}c^A\varphi^{\text{eff}}_b. \tag{2.5c}$$

All other BRS transformations are standard [9–11] and read in our notation

$$sV^A_\mu = \partial_\mu c^A - gf^{ABC}V^B_\mu c^C, \tag{2.5d}$$
$$s\psi_{p\alpha} = -igc^A t^A_{pq}\psi_{q\alpha}, \tag{2.5e}$$
$$sc^A = \frac{1}{2}gf^{ABC}c^Bc^C, \tag{2.5f}$$
$$s\bar{c}^A = B^A, \qquad\qquad sB^A = 0. \tag{2.5g}$$

Herein, c^A, $\bar{c}^A$, and B^A denote the Faddeev-Popov ghost, Faddeev-Popov antighost, and the Nakanishi-Lautrup auxiliary field, respectively.

If not stated otherwise, we will use R_ξ gauge fixing with gauge fixing function

$$F^A = \partial^\mu V^A_\mu + ig\xi\xi'\left(\hat{\varphi} + \hat{v}\right)_a T^A_{ab}\varphi_b. \tag{2.6}$$

We will always set $\xi' = 1$, but keep it as a variable because it is renormalized. The full gauge fixing and ghost Lagrangian is then given by evaluating

$$\mathcal{L}_{\text{fix, gh}} = s\left[\bar{c}^A\left(F^A + \frac{\xi}{2}B^A\right)\right], \tag{2.7a}$$

which yields after elimination of B^A

$$\begin{aligned}\mathcal{L}_{\text{fix, gh}} = &-\frac{1}{2\xi}(\partial^\mu V^A_\mu)(\partial^\nu V^A_\nu) - \bar{c}^A\Box c^A - gf^{ABC}(\partial^\mu\bar{c}^A)V^B_\mu c^C \\ &- ig\xi'(\partial^\mu V^A_\mu)(\hat{\varphi} + \hat{v})_a T^A_{ab}\varphi_b - ig\xi\xi'\bar{c}^A\hat{q}_a T^A_{ab}(\varphi + \hat{\varphi} + \hat{v})_b \\ &- g^2\xi\xi'\bar{c}^A c^B(\hat{\varphi} + \hat{v})_a T^A_{ab}T^B_{bc}(\varphi + \hat{\varphi} + \hat{v})_c \\ &+ \frac{1}{2}g^2\xi\xi'^2(\hat{\varphi} + \hat{v})_a T^A_{ab}\varphi_b(\hat{\varphi} + \hat{v})_c T^A_{cd}\varphi_d.\end{aligned} \tag{2.7b}$$

This modified R_ξ gauge fixing preserves the rigid invariance due to the background fields. Finally, the study of renormalization is streamlined by introducing sources K for the nonlinear BRS transformations

$$\mathcal{L}_{\text{ext}} = K_{\varphi_a}\, s\varphi_a + K_{V^A_\mu}\, sV^A_\mu + K_{c^A}\, sc^A + \left[K_{\psi_p}\, s\psi_p + \text{h.c.}\right]. \tag{2.8}$$

In summary, the total Lagrange density is the sum of all discussed parts,

$$\mathcal{L}_{\text{tot}} = \mathcal{L}_{\text{inv}}|_{\varphi\to\varphi^{\text{eff}}} + \mathcal{L}_{\text{fix, gh}} + \mathcal{L}_{\text{ext}}. \tag{2.9}$$

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2.2 Renormalization

Renormalization proceeds essentially as in the case without background fields.[2] In the usual case the divergence structure is controlled by identities such as (1) the Slavnov-Taylor identity, expressing BRS invariance on the level of Green functions; (2) the so-called gauge condition, fixing the B^A-dependence, i.e. the R_ξ gauge fixing term. All divergences can be absorbed by a multiplicative renormalization transformation of fields and parameters. The required form is obtained from the general classical solution of the Slavnov-Taylor identity and the gauge condition [9–11].

Furthermore, the behaviour of $\mathcal{L}_{\text{fix, gh}}$ under rigid gauge transformations is crucial for the renormalization of the shift v. By construction, $\mathcal{L}_{\text{fix, gh}}$ necessarily breaks local gauge invariance. Nevertheless, some gauges like Landau gauge ($\xi = 0$) respect rigid gauge invariance. In such cases the corresponding rigid Ward identity leads to the combined renormalization of $\varphi + v$, i.e. the additional counterterm $\delta\bar{v}$ in eq. (1.1) is forbidden by symmetry. If, on the other hand, $\mathcal{L}_{\text{fix, gh}}$ breaks rigid gauge invariance then no symmetry forbids $\delta\bar{v}$, implying that $\delta\bar{v}$ will in general be necessary and divergent. Without background fields we have no control of $\delta\bar{v}$.

With background fields, again divergences are controlled by the Slavnov-Taylor identity and gauge-condition — but, in addition, we have one more identity at our disposal: the rigid Ward identity expressing rigid gauge invariance in the presence of $\hat{\varphi}$ and $\hat{q}$. Since it holds in the modified R_ξ gauge (2.7b) we gain further information.

Going through the standard steps and taking the additional Ward identity into account we obtain the most general structure of the divergences. The resulting renormalization transformations required to absorb all divergences can be summarized as follows:

1. Parameter renormalization: all parameters $p \in \{g, \xi, \xi', m^2_{ab}, h_{abc}, \lambda_{abcd}, (m_f)_{pq}, Y^a_{pq}\}$ renormalize as $p \to p + \delta p$. The shift $\hat{v}$ does not.

2. Field renormalization: all fields transform multiplicatively with appropriate $\sqrt{Z}$ factors. In particular, the renormalization transformations of interest are

$$\varphi_a \to \sqrt{Z}_{ab}\varphi_b, \tag{2.10a}$$

$$(\hat{\varphi} + \hat{v})_a \to \sqrt{Z}_{ab}\sqrt{\hat{Z}}_{bc}(\hat{\varphi} + \hat{v})_c, \tag{2.10b}$$

$$\hat{q}_a \to \sqrt{Z}_{ab}\sqrt{\hat{Z}}_{bc}\hat{q}_c. \tag{2.10c}$$

   Eq. (2.10b) is a consequence of the unbroken rigid gauge invariance, and eq. (2.10c) stems from the fact that $\hat{q}_a$ is the BRS transformation of $\hat{\varphi}_a$. The BRS sources K transform with inverse $\sqrt{Z}$ factors, in particular

$$K_{\varphi_a} \to \left(\sqrt{Z}^{-1}\right)_{ba} K_{\varphi_b}. \tag{2.11}$$

[2]In particular cases, such as the SM and the MSSM in refs. [12, 13, 16], the background fields were essential in order to control the $\mathrm{U}(1)_{\mathrm{em}}$ Ward identity. In the MSSM an even more complicated background field structure has been used to allow for on-shell renormalization conditions separating unphysical from physical degrees of freedom. Here, however, we are concerned with the generic situation and minimal subtraction, where the background fields are optional.

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The above mentioned relations prohibit an additional $\hat{v}$ counterterm and simplify the counterterm structure of the two-point function between K_φ and $\hat{q}$.

Before moving on to the next section, we impose a further constraint on the theories we consider. We require that the theories possess some additional symmetry, at least at the dimension 4 level, that enforces the field renormalization for the real scalar fields to be diagonal

$$\sqrt{Z}_{ab} \to \sqrt{Z}(a)\delta_{ab} \qquad \text{and} \qquad \sqrt{\hat{Z}}_{ab} \to \sqrt{\hat{Z}}(a)\delta_{ab}. \tag{2.12}$$

As an example, in the MSSM and NMSSM this symmetry is realized by the so-called Peccei-Quinn (PQ) symmetry [35] (only softly broken by the so-called μ-term), whereas in the E_6SSM the additional $\mathrm{U}(1)_N$ gauge group plays this role.[3]

3 General results

3.1 Equivalence and general consequences for δv

The purpose of this section is to emphasize and exploit the features of the approach explained above in section 2 and explain its equivalence to the standard approach.

In the standard approach, without background fields, the most generic renormalization transformation of the scalar fields with shifts reads as

$$\varphi_a + v_a \to \sqrt{Z}_{ab}\left(\varphi_b + v_b + \delta\bar{v}_b\right), \tag{3.1a}$$

whereas we have, from eqs. (2.4), (2.10a) and (2.10b),

$$\varphi_a^{\text{eff}} \to \sqrt{Z}_{ab}\left(\varphi_b + \sqrt{\hat{Z}}_{bc}(\hat{\varphi}+\hat{v})_c\right). \tag{3.1b}$$

For the calculation of Green functions the background field has to be set to zero, $\hat{\varphi} = 0$. Hence, the comparison of eq. (3.1a) and eq. (3.1b) yields the following identification between the two formalisms:

$$v_a + \delta\bar{v}_a = \sqrt{\hat{Z}}_{ab}\hat{v}_b, \tag{3.2}$$

and equivalently

$$v_a = \hat{v}_a, \tag{3.3a}$$

$$\delta\bar{v}_a = \left(\sqrt{\hat{Z}}-1\right)_{ab}\hat{v}_b = \frac{1}{2}\delta\hat{Z}_{ab}\hat{v}_b + \mathcal{O}(\hbar^2), \tag{3.3b}$$

$$\delta v_a \overset{(1.1)}{=} \left(\sqrt{Z}\sqrt{\hat{Z}}-1\right)_{ab}\hat{v}_b = \frac{1}{2}\left(\delta Z + \delta\hat{Z}\right)_{ab}\hat{v}_b + \mathcal{O}(\hbar^2). \tag{3.3c}$$

The advantage of this method is that one possesses more information about the renormalization properties of the shift $\hat{v}$. First, the field renormalization $\hat{Z}$ is a dimension zero

[3] In these models these symmetries and eq. (2.12) are the reason why no off-diagonal kinetic counterterms like $\delta Z_{H_u H_d}\epsilon_{ij}(D^\mu H_u)_i(D_\mu H_d)_j$ (where i, j are SU(2) indices and ϵ_{ij} is antisymmetric) are necessary, in spite of their gauge invariance. Note that eq. (2.12) does not forbid introducing additional, finite off-diagonal Z-factors e.g. for defining mass eigenstate fields as discussed in detail e.g. in ref. [16].

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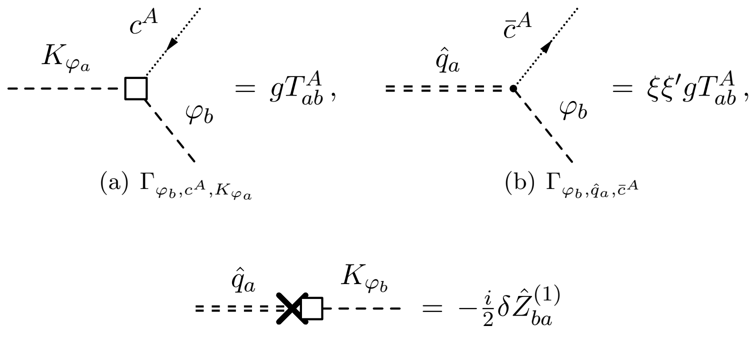


(a) $\Gamma_{\varphi_b, c^A, K_{\varphi_a}}$

(b) $\Gamma_{\varphi_b, \hat{q}_a, \bar{c}^A}$

(c) $\Gamma_{\hat{q}_a, K_{\varphi_b}}$

Figure 1. Feynman rules for $\sqrt{\hat{Z}}$ determination.

quantity which is at most logarithmic divergent. Second, the shift-counterterm $\delta\bar{v}_a$ is linear in the shift itself and otherwise determined by $\hat{Z}$. Third, $\hat{Z}$ is the field renormalization of the background field, meaning that it appears not only in eq. (3.1b) but also in other Green functions and, thus, can be directly evaluated by certain diagrams.

To elaborate on the last point, consider the computation of $\sqrt{\hat{Z}}$. It turns out that $\sqrt{\hat{Z}}$ is the only renormalization constant appearing in the two point function $\Gamma_{\hat{q}_a, K_{\varphi_b}}$,

$$\mathcal{L}_{\text{ext}} = -K_{\varphi_a}\hat{q}_a + \cdots \overset{\text{RT}}{\to} -K_{\varphi_a}\sqrt{\hat{Z}}_{ab}\hat{q}_b + \cdots\,, \tag{3.4}$$

leading to the Feynman rules in figure 1 (wherein the cross denotes the one-loop counterterm $\delta\hat{Z}$). Hence, $\sqrt{\hat{Z}}$ can be directly determined from the divergence to that two point function. Of course, $\Gamma_{\hat{q}_a, K_{\varphi_b}}$ is an unphysical Green function, which highlights its role as technical tool. However, the point is that very few Feynman diagrams with well localized origin contribute to it. The only coupling of the field $\hat{q}$ to propagating fields is the term $\sim \bar{c}\hat{q}\varphi$ in $\mathcal{L}_{\text{fix, gh}}$ (see figure 1(a)), which stems from BRS invariance. Its coefficient must be the same as the one of the $(\hat{\varphi}+\hat{v})\varphi$-term in the gauge fixing function F^A. Similarly, the term $\sim K_\varphi c\varphi$ is determined by inserting the BRS transformation $s\varphi$ in $\mathcal{L}_{\text{ext}}$, see eq. (2.8) and figure 1(b). Most important, all the mentioned Feynman rules are proportional to the gauge coupling g, hence, $\sqrt{\hat{Z}}$ is (at one-loop level) proportional to two powers of gauge couplings with coefficients fixed by BRS invariance. In contrast, all dimensionless couplings can contribute to the field renormalization constant $\sqrt{Z}$.

In summary, the method of background fields together with BRS and rigid invariance provides the following features: the VEV counterterm $\delta\bar{v}_a$ from eq. (1.1) is given by $\sqrt{\hat{Z}}$, see eq. (3.3b). Hence it is proportional to the VEV itself, and its divergence is given by a dimension zero quantity and at most logarithmic divergent. The relevant field renormalization $\hat{Z}$ can be determined by a single two point function to which few, specific Feynman rules contribute.

3.2 Field renormalization of scalar fields and background fields

This section will serve for the one- and leading two-loop renormalization computations. All calculations are performed in R_ξ gauge and the $\overline{\mathrm{MS}}/\overline{\mathrm{DR}}$ scheme.[4]

3.2.1 One-loop

The one-loop results for the divergent parts of Z and $\hat{Z}$ will be provided in this section together with a brief overview of the relevant diagrams.

The scalar field renormalization constant $\sqrt{Z}$ is given by the derivative of the scalar self-energy with respect to external momentum squared. At one-loop, there are ten (one-particle irreducible) diagrams and renormalization works as usual by requiring

$$= \text{finite}. \tag{3.5}$$

According to refs. [25, 32], the results are well-known; in our notation they are given by

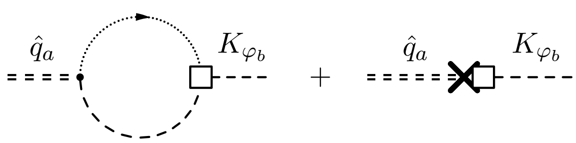
$$\delta_{ab}\delta Z^{(1)}(a) = \frac{1}{(4\pi)^2}\left[g^2\,(3-\xi)\,C^2_{ab}(\mathrm{S}) - Y^2_{ab}(\mathrm{S})\right]\cdot\frac{1}{\epsilon} \tag{3.6a}$$

$$= \gamma^{(1)}_{ab}(\mathrm{S})\cdot\frac{1}{\epsilon}, \tag{3.6b}$$

where $D = 4-2\epsilon$ in dimensional regularization, $\gamma^{(1)}_{ab}(\mathrm{S})$ is the one-loop anomalous dimension, and we introduced the following group invariants (for the scalar representation S)

$$C^2_{ab}(\mathrm{S}) = T^A_{ac}T^A_{cb}, \qquad Y^2_{ab}(\mathrm{S}) = \frac{1}{2}\left(Y^a_{pq}Y^{*b}_{pq} + Y^{*a}_{pq}Y^b_{pq}\right). \tag{3.7}$$

As described above, the new renormalization constant $\sqrt{\hat{Z}}$ can be determined easily because it appears as the counterterm Feynman rule for $\Gamma_{\hat{q}_a,K_{\varphi_b}}$. There exists only one diagram in one-loop order. Hence, requiring



$$= \text{finite} \tag{3.8}$$

leads to the result

$$\delta_{ab}\delta\hat{Z}^{(1)}(a) = \frac{1}{(4\pi)^2}2g^2\xi\xi' C^2_{ab}(\mathrm{S})\cdot\frac{1}{\epsilon}. \tag{3.9}$$

[4] $\overline{\mathrm{DR}}$ denotes modified minimal subtraction in regularization by dimensional reduction. There is no difference between the two schemes in any of the calculations carried out here.

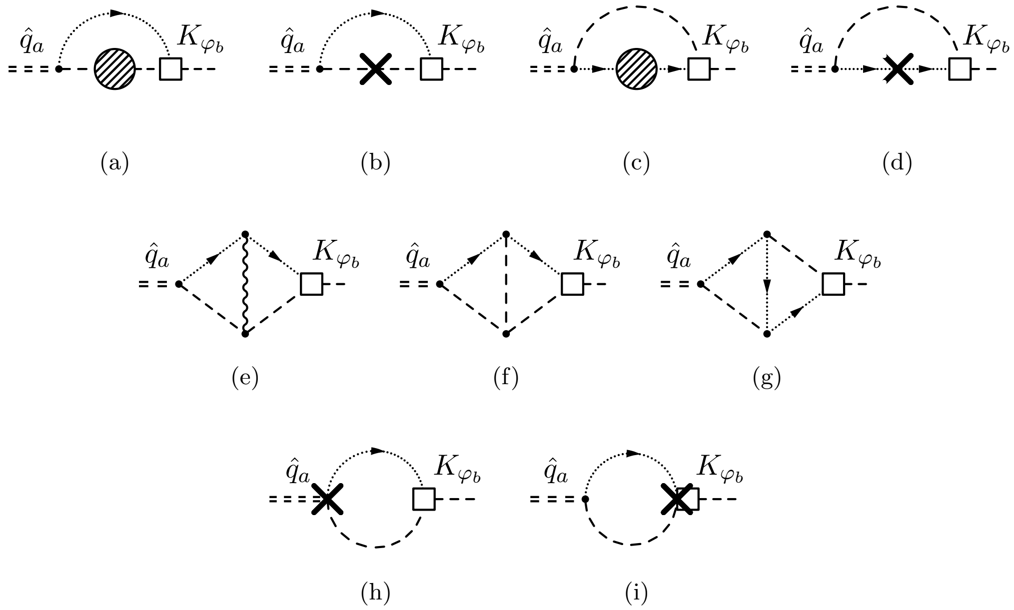


Figure 2. Feynman diagrams for full two-loop computation of $\delta\hat{Z}$. Note that figure 2(f) and 2(g) are power-counting finite. Furthermore, figure 2(h) is zero as the contained vertex counterterm vanishes due to non-renormalization of $\mathcal{L}_{\text{fix, gh}}$.

3.2.2 Two-loop

At the two-loop level, a similar computation of the two-loop self energy determines $\delta Z^{(2)}$. It is convenient to express the two-loop renormalization parameter in terms of the anomalous dimension $\gamma(\text{S})$ and β functions

$$\frac{1}{2}\delta Z^{(2)}_{ab} = \frac{1}{4}\gamma^{(2)}_{ab}(\text{S})\cdot\frac{1}{\epsilon} + \frac{1}{8}\left[\gamma^{(1)}_{ac}(\text{S})\gamma^{(1)}_{cb}(\text{S}) + \sum_x \beta^{(1)}(x)\left(\partial_x\gamma^{(1)}_{ab}(\text{S})\right)\right]\cdot\frac{1}{\epsilon^2}, \tag{3.10}$$

wherein $x \in \{g, \xi, \xi', Y^a_{pq}, Y^{*a}_{pq}\}$. The calculations have been performed in refs. [25, 32] and the anomalous dimension for the scalar fields (in DREG) is given as

$$\begin{aligned}\gamma^{(2)}_{ab}(\text{S}) = \frac{1}{(4\pi)^4}\Bigg\{ & g^4 C^2_{ab}(\text{S})\left[\left(\frac{35}{3} - 2\xi - \frac{1}{4}\xi^2\right)C_2(\text{G}) - \frac{10}{6}S_2(\text{F}) - \frac{11}{12}S_2(\text{S})\right] \\ & -\frac{3}{2}g^4 C^2_{ac}(\text{S})C^2_{cb}(\text{S}) + \frac{3}{2}H^2_{ab}(\text{S}) + \bar{H}^2_{ab}(\text{S}) - \frac{10}{2}g^2 Y^{2F}_{ab}(\text{S}) - \frac{1}{2}\Lambda^2_{ab}(\text{S})\Bigg\},\end{aligned} \tag{3.11}$$

where the group invariants $C_2(\text{G})$, $S_2(\text{F})$, $S_2(\text{S})$, $\Lambda^2_{ab}(\text{S})$, $H^2_{ab}(\text{S})$, $\bar{H}^2_{ab}(\text{S})$, and $Y^{2F}_{ab}(\text{S})$ are defined as in ref. [32].

Likewise, the two-loop value of $\delta\hat{Z}^{(2)}$ is determined by the two-loop part of $\Gamma_{\hat{q}_a,K_{\varphi_b}}$. The relevant diagrams for the full two-loop corrections to $\Gamma_{\hat{q}_a,K_{\varphi_b}}$ are shown in figure 2. Here, figure 2(a)–2(d) contain all insertion of one-loop self-energies (shaded circles) and corresponding one-loop counterterms (crosses). On the other hand, figure 2(e)–2(g) display

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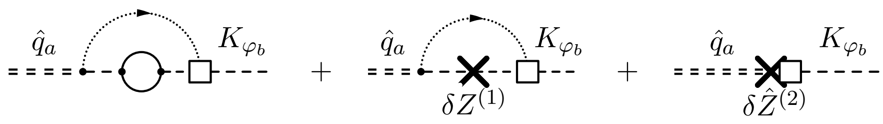
the exchange of intermediate fields and figure 2(h)–2(i) provide the one-loop vertex counterterms. The inspection of the diagrams leads to the conclusion that each contribution is either proportional to g^4 or $g^2 Y Y^\dagger$. Hence, terms with a different coupling structure cannot enter $\delta\hat{Z}^{(2)}$. In contrast, $\delta Z^{(2)}$ contains terms proportional to $\lambda^2 \leftrightarrow \Lambda^2_{ab}(\mathrm{S})$ or $(YY^\dagger)^2 \leftrightarrow H^2_{ab}(\mathrm{S}), \bar{H}^2_{ab}(\mathrm{S})$.

For the purpose of the present paper we restrict ourselves to the Yukawa-enhanced contributions of order $g^2 Y Y^\dagger$. They are obtained from requiring



$= \text{finite}, \qquad (3.12)$

where the crosses denote counterterm contributions from the indicated renormalization constants. The result reads as

$$\delta\hat{Z}^{(2)}(a)|^{g^2Y^2(\mathrm{S})}\,\delta_{ab} = \frac{1}{(4\pi)^4} g^2 \xi\xi' T^A_{ac} Y^2_{cd}(\mathrm{S}) T^A_{db} \left[\frac{1}{\epsilon^2} - \frac{1}{\epsilon}\right]. \tag{3.13}$$

3.3 Results for δv in generic models

The decomposition of δv_a in eq. (3.3c) leads to an equivalent decomposition of the β-function for v_a

$$\beta(v_a) = \mu\partial_\mu v_a = [\gamma_{ab}(\mathrm{S}) + \hat{\gamma}_{ab}(\mathrm{S})]\, v_b, \tag{3.14}$$

where μ is the $\overline{\mathrm{MS}}/\overline{\mathrm{DR}}$ renormalization scale, $\gamma(\mathrm{S})$ is the anomalous dimension of the scalar field,

$$\gamma_{ab}(\mathrm{S}) = \left(\mu\partial_\mu \sqrt{Z}^{-1}_{ac}\right)\sqrt{Z}_{cb}, \tag{3.15}$$

and $\hat{\gamma}(\mathrm{S})$ the analogous quantity for $\sqrt{\hat{Z}}$.

Hence, our main results for VEV renormalization constants and β and γ functions in the $\overline{\mathrm{MS}}$ and $\overline{\mathrm{DR}}$ schemes can be summarized as

$$\delta v^{(1)}_a = \frac{1}{(4\pi)^2}\left[g^2\left(\frac{3-\xi}{2} + \xi\xi'\right) C^2_{ab}(\mathrm{S}) - \frac{1}{2} Y^2_{ab}(\mathrm{S})\right] v_b \cdot \frac{1}{\epsilon}, \tag{3.16a}$$

$$\beta^{(1)}(v_a) = \frac{1}{(4\pi)^2}\left[g^2\left(3 - \xi + 2\xi\xi'\right) C^2_{ab}(\mathrm{S}) - Y^2_{ab}(\mathrm{S})\right] v_b, \tag{3.16b}$$

$$\gamma^{(1)}_{ab}(\mathrm{S}) = \frac{1}{(4\pi)^2}\left[g^2\left(3 - \xi\right) C^2_{ab}(\mathrm{S}) - Y^2_{ab}(\mathrm{S})\right], \tag{3.16c}$$

$$\hat{\gamma}^{(1)}_{ab}(\mathrm{S}) = \frac{1}{(4\pi)^2} 2 g^2 \xi\xi' C^2_{ab}(\mathrm{S}), \tag{3.16d}$$

at one-loop level, and

$$\beta^{(2)}(v_a) = \gamma^{(2)}_{ab}(\mathrm{S}) v_b - \frac{1}{(4\pi)^4} 2 g^2 \xi\xi' T^A_{ac} Y^2_{cd}(\mathrm{S}) T^A_{db} v_b + \mathcal{R}_{ab} v_b, \tag{3.17a}$$

$$\hat{\gamma}^{(2)}_{ab}(\mathrm{S}) = -\frac{1}{(4\pi)^4} 2 g^2 \xi\xi' T^A_{ac} Y^2_{cd}(\mathrm{S}) T^A_{db} + \mathcal{R}_{ab} \tag{3.17b}$$

at two-loop level. Here $\mathcal{R}_{ab}$ contains all $1/\epsilon$-pole contributions from $\sqrt{\hat{Z}}$ proportional to g^4. We remind the reader that $\xi' = 1$, but it is kept as a variable in order to visualize the origin of the different terms.

3.4 Results for δv in general SUSY models

Our results can now be specialized to general supersymmetric (SUSY) models with spontaneous gauge symmetry breaking. For supersymmetric models with or without soft SUSY breaking the results will be the same, since the general results (3.16)–(3.17) depend only on dimensionless couplings. We do not need to specify soft SUSY breaking terms explicitly, even though the later examples will be realistic models with softly broken SUSY.

The application to general supersymmetric models requires one to take gauginos λ^A into account. The generic supersymmetric Lagrangian in Wess-Zumino gauge with non-abelian gauge interactions contains the standard kinetic terms for complex scalar fields ϕ_a, their SUSY Weyl-spinor partners ψ_a, gauge fields V^A, and their SUSY Weyl-spinor partner λ^A. Besides a scalar potential with ϕ^n interactions for $n \in \{1,2,3,4\}$, the Lagrangian contains two sets of Yukawa-type interactions

$$\mathcal{L}_{\text{SUSY}} = -\frac{1}{2}\left[\psi_p^\alpha \psi_{q\alpha} \mathcal{W}_{pq} + \text{h.c.}\right] - \sqrt{2} g \left[\bar{\lambda}^A_{\dot{\alpha}} \bar{\psi}^{\dot{\alpha}}_p T^A_{pq} \phi_q + \text{h.c.}\right] + \cdots , \tag{3.18}$$

where $\mathcal{W}_{pq}$ denotes derivatives of the superpotential $\mathcal{W}$. Following ref. [28] the superpotential (formulated in chiral superfields Φ) is given by

$$\mathcal{W} = \frac{1}{3!} Y^{pqr} \Phi_p \Phi_q \Phi_r + \frac{1}{2!} \mu^{pq} \Phi_p \Phi_q + L^p \Phi_p. \tag{3.19}$$

Gaugino couplings are special in the sense that their form is given by the gauge coupling strength g times group generator. Hence, those Yukawa-type couplings contribute to the g^2 part of γ. The results can be expressed in terms of γ and β for the complex scalar fields ϕ_a. We obtain

$$\gamma^{(1)}_{aa}(\text{S})\Big|_{\text{SUSY}} = \frac{1}{(4\pi)^2}\left[g^2 (1-\xi)\, C^2_{aa}(\text{S}) - \frac{1}{2} Y^2_{aa}(\text{S})\right], \tag{3.20a}$$

$$\hat{\gamma}^{(1)}_{aa}(\text{S})\Big|_{\text{SUSY}} = \frac{1}{(4\pi)^2} 2 g^2 \xi \xi' C^2_{aa}(\text{S}), \tag{3.20b}$$

$$\beta^{(1)}(v_a)\Big|_{\text{SUSY}} = \frac{1}{(4\pi)^2}\left[g^2 \left(1 - \xi + 2\xi\xi'\right) C^2_{aa}(\text{S}) - \frac{1}{2} Y^2_{aa}(\text{S})\right] v_a, \tag{3.20c}$$

with the standard convention $Y^2_{ab}(\text{S}) = (Y^{pqa} Y^{*pqb} + Y^{*pqa} Y^{pqb})/2$. Note two changes here compared to the general results in eqs. (3.16): first, the g^2 pre-factor has changed from $(3-\xi)$ to $(1-\xi)$ due to the Gaugino couplings. Second, the overall normalization of the Yukawa couplings is different compared to the eq. (2.2a) and gives rise to the factor $1/2$ in front of $Y^2_{ab}(\text{S})$. Furthermore, we do not observe a change of the one-loop $\hat{\gamma}$ in a generic SUSY theory because Yukawa couplings do not contribute to it.

As a remark, we stress again that γ is the anomalous dimension of the component field ϕ in Wess-Zumino gauge, which is different from the corresponding quantity for a

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superfield Φ in a supersymmetric gauge (see ref. [36] for an explicit one-loop comparison). The latter can be found in ref. [28], but are not relevant in this analysis.

The same considerations are valid at two-loop level and lead to a corresponding change in $\gamma^{(2)}(\mathrm{S})$ as well as in $\hat{\gamma}^{(2)}(\mathrm{S})$. However, the $g^2 Y Y^\dagger$ part of $\hat{\gamma}^{(2)}(\mathrm{S})$, in which we are interested in, receives no SUSY contributions, because the gauginos couple via a gauge coupling and, thus, contribute merely to g^4 terms of $\hat{\gamma}^{(2)}(\mathrm{S})$. The generic result of eq. (3.17) is altered in a SUSY theory only by a factor of 1/2 due to the different normalization of the Yukawa couplings. We obtain

$$\hat{\gamma}^{(2)}_{aa}(\mathrm{S})\Big|_{\mathrm{SUSY}} = -\frac{1}{(4\pi)^4} g^2 \xi\xi' T^A_{ac} Y^2_{cd}(\mathrm{S}) T^A_{da} + \tilde{\mathcal{R}}_{aa}, \tag{3.21a}$$

$$\beta^{(2)}(v_a)\Big|_{\mathrm{SUSY}} = \gamma^{(2)}_{aa}(\mathrm{S}) v_a - \frac{1}{(4\pi)^4} g^2 \xi\xi' T^A_{ac} Y^2_{cd}(\mathrm{S}) T^A_{da} v_a + \tilde{\mathcal{R}}_{aa} v_a. \tag{3.21b}$$

Note that $\mathcal{R}$ is altered to $\tilde{\mathcal{R}}$ because the term $\sim g^2 Y Y^\dagger$ leads to a g^4 contribution due to the Gaugino coupling.

4 Application to concrete SUSY models

The aim of this section is to apply the results from section 3 to the MSSM and non-minimal supersymmetric models. We discuss the validity of eq. (1.2) and provide new, explicit results for the NMSSM and E$_6$SSM. For this application, we need to generalize our results to product gauge groups, see ref. [25], and we use model-specific expressions for the Yukawa couplings.

4.1 MSSM

The MSSM [37] contains two Higgs doublets H_u, H_d with opposite hypercharge $Y_{H_u}/2 = -Y_{H_d}/2 = 1/2$. Our calculations are based upon the following superpotential[5]

$$\mathcal{W}_{\mathrm{MSSM}} = \mu H_d \cdot H_u - y^e_{ij} H_d \cdot L_i \bar{E}_j - y^d_{ij} H_d \cdot Q_i \bar{D}_j - y^u_{ij} Q_i \cdot H_u \bar{U}_j. \tag{4.1}$$

Applying eq. (3.20) is in agreement with the known results for the divergent renormalization constants (and equivalent the β-functions)

$$\frac{\beta^{(1)}_{\mathrm{MSSM}}(v_u)}{v_u} = \frac{1}{(4\pi)^2}\left[\left(1 - \xi + 2\xi\xi'\right)\left(\frac{3}{20}g_1^2 + \frac{3}{4}g_2^2\right) - N_c \,\mathrm{Tr}\left(y^u y^{u\dagger}\right)\right]$$
$$= \gamma^{(1)}_{uu} + \frac{1}{(4\pi)^2}\xi\xi'\left(\frac{3}{10}g_1^2 + \frac{3}{2}g_2^2\right), \tag{4.2a}$$

$$\frac{\beta^{(1)}_{\mathrm{MSSM}}(v_d)}{v_d} = \frac{1}{(4\pi)^2}\left[\left(1 - \xi + 2\xi\xi'\right)\left(\frac{3}{20}g_1^2 + \frac{3}{4}g_2^2\right) - N_c \,\mathrm{Tr}\left(y^d y^{d\dagger}\right) - \mathrm{Tr}\left(y^e y^{e\dagger}\right)\right]$$
$$= \gamma^{(1)}_{dd} + \frac{1}{(4\pi)^2}\xi\xi'\left(\frac{3}{10}g_1^2 + \frac{3}{2}g_2^2\right). \tag{4.2b}$$

[5]The dot product $A \cdot B = \epsilon_{ij} A_i B_j$ denotes the SU(2) invariant product with antisymmetric ϵ_{ij}.

Here g_Y and g_2 are the $\mathrm{U}(1)_Y$ and $\mathrm{SU}(2)_L$ gauge couplings, and g_1 is GUT-normalized $g_1 = \sqrt{5/3}g_Y$. N_c is the number of colours.

The quantity $\tan\beta$ is defined as

$$\tan\beta = \frac{v_u}{v_d}, \tag{4.3}$$

and renormalization yields

$$\delta\tan\beta^{(1)} = \tan\beta\left(\frac{\delta v_u^{(1)}}{v_u} - \frac{\delta v_d^{(1)}}{v_d}\right), \tag{4.4a}$$

$$\beta^{(1)}(\tan\beta) = \tan\beta\left(\gamma_{uu}^{(1)} - \gamma_{dd}^{(1)} + \hat{\gamma}_{uu}^{(1)} - \hat{\gamma}_{dd}^{(1)}\right). \tag{4.4b}$$

There are two cancellations in this difference. First, the contribution from the $\hat{\gamma}$'s, equivalent to $\delta\bar{v}$ in eq. (1.1), cancels. The reason is that both doublets have the same $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ quantum numbers, up to a sign. Hence, eq. (1.2) is valid at the one-loop level and for the β-function the simplified result

$$\beta_{\mathrm{MSSM}}^{(1)}(\tan\beta) = \tan\beta\left(\gamma_{uu}^{(1)} - \gamma_{dd}^{(1)}\right) \tag{4.5}$$

holds. Second, even within this difference, the gauge coupling terms cancel, leaving

$$\frac{\beta_{\mathrm{MSSM}}^{(1)}(\tan\beta)}{\tan\beta} = -\frac{1}{(4\pi)^2}\left[N_c \operatorname{Tr}\left(y^u y^{u\dagger}\right) - N_c \operatorname{Tr}\left(y^d y^{d\dagger}\right) - \operatorname{Tr}\left(y^e y^{e\dagger}\right)\right]. \tag{4.6}$$

Obviously, both cancellations are group theoretical coincidences. As a remark, these results also provide further insight into the accidental gauge independence of $\tan\beta$ as discussed in ref. [38]. Going away from R_ξ-gauges, $\tan\beta$ becomes gauge dependent at the one-loop level [38, 39].

At two-loop level, the β-functions for the up and down type VEVs are given by

$$\frac{\beta_{\mathrm{MSSM}}^{(2)}(v_u)}{v_u} = \gamma_{uu}^{(2)} - \frac{1}{(4\pi)^4}\xi\xi'\left(\frac{3}{10}g_1^2 + \frac{3}{2}g_2^2\right)\left[N_c \operatorname{Tr}\left(y^u y^{u\dagger}\right)\right] + \mathcal{R}_u, \tag{4.7a}$$

$$\frac{\beta_{\mathrm{MSSM}}^{(2)}(v_d)}{v_d} = \gamma_{dd}^{(2)} - \frac{1}{(4\pi)^4}\xi\xi'\left(\frac{3}{10}g_1^2 + \frac{3}{2}g_2^2\right)\left[N_c \operatorname{Tr}\left(y^d y^{d\dagger}\right) + \operatorname{Tr}\left(y^e y^{e\dagger}\right)\right] + \mathcal{R}_d. \tag{4.7b}$$

Here $\mathcal{R}_{u,d}$ represent all contributions from $\delta\hat{Z}^{(2)} \sim g^4$ which we have not considered. However, it is clear that in the MSSM $\mathcal{R}_u = \mathcal{R}_d$ holds because the Higgs doublets have (up to a sign) the same quantum numbers with respect to all gauge groups. Our results are in agreement with ref. [23] if one simplifies the generation matrices $y_{ij}^{d/u/e}$ to complex numbers $y^{d/u/e}$.

The above result now implies that the g_2^2 and g_1^2 proportional parts of the divergent $\delta\tan\beta$ are not zero if the Yukawa couplings are different. Instead, we can write

$$\frac{\beta_{\mathrm{MSSM}}^{(2)}(\tan\beta)}{\tan\beta} = \gamma_{uu}^{(2)} - \gamma_{dd}^{(2)} + \frac{1}{(4\pi)^2}\xi\xi'\left(\frac{3}{10}g_1^2 + \frac{3}{2}g_2^2\right)\frac{\beta_{\mathrm{MSSM}}^{(1)}(\tan\beta)}{\tan\beta} \tag{4.8}$$

The second term is equivalent to a violation of eq. (1.2) at $\mathcal{O}(g^2 YY^\dagger)$. Note that $\mathcal{R}_u - \mathcal{R}_d$ vanishes in the MSSM as remarked earlier.

4.2 NMSSM

The NMSSM is a non-minimal SUSY model that attempts to solve the μ problem of the MSSM by introducing an additional gauge singlet S, which has a non-zero VEV v_s. The modified superpotential, see for example ref. [40], reads as

$$\mathcal{W}_{\text{NMSSM}} = \mathcal{W}_{\text{MSSM}}(\mu = 0) + \lambda S H_d \cdot H_u + \frac{1}{3}\kappa SSS + \frac{1}{2}\mu_s SS + \zeta S. \tag{4.9}$$

The one-loop β-functions are given by

$$\frac{\beta^{(1)}_{\text{NMSSM}}(v_s)}{v_s} = -\frac{1}{(4\pi)^2} 2\left(|\lambda|^2 + |\kappa|^2\right), \tag{4.10a}$$

$$\frac{\beta^{(1)}_{\text{NMSSM}}(v_{u,d})}{v_{u,d}} = \frac{\beta^{(1)}_{\text{MSSM}}(v_{u,d})}{v_{u,d}} - \frac{1}{(4\pi)^2}|\lambda|^2. \tag{4.10b}$$

The consequences of the singlet S for the Higgs VEVs are a change in the anomalous dimensions γ_{uu}, γ_{dd} due to the additional Yukawa coupling λ, whereas the $\hat{\gamma}_{uu}$, $\hat{\gamma}_{dd}$ are unchanged as S is a gauge singlet. In addition, the quantities $(\sqrt{\hat{Z}_s} - 1)$ and $\hat{\gamma}_{ss}$ vanish, because an additional counterterm for v_s is forbidden by the rigid invariance for S. Therefore, the one-loop β-function for $\tan\beta$ reads

$$\beta^{(1)}_{\text{NMSSM}}(\tan\beta) = \beta^{(1)}_{\text{MSSM}}(\tan\beta), \tag{4.11}$$

and eq. (1.2) holds.

Similarly, the changes in $\beta^{(2)}(v_{u,d,s})$ are

$$\frac{\beta^{(2)}_{\text{NMSSM}}(v_u)}{v_u} = \gamma^{(2)}_{uu} - \frac{1}{(4\pi)^4}\xi\xi'\left(\frac{3}{10}g_1^2 + \frac{3}{2}g_2^2\right)\left[N_c \operatorname{Tr}\left(y^u y^{u\dagger}\right) + |\lambda|^2\right] + \mathcal{R}_u, \tag{4.12a}$$

$$\frac{\beta^{(2)}_{\text{NMSSM}}(v_d)}{v_d} = \gamma^{(2)}_{dd} - \frac{1}{(4\pi)^4}\xi\xi'\left(\frac{3}{10}g_1^2 + \frac{3}{2}g_2^2\right)\left[N_c \operatorname{Tr}\left(y^d y^{d\dagger}\right) + \operatorname{Tr}\left(y^e y^{e\dagger}\right) + |\lambda|^2\right] + \mathcal{R}_d, \tag{4.12b}$$

$$\frac{\beta^{(2)}_{\text{NMSSM}}(v_s)}{v_s} = \gamma^{(2)}_{ss}. \tag{4.12c}$$

Again, $\mathcal{R}_u = \mathcal{R}_d$ because the gauge groups of NMSSM and MSSM are identical.

For the $\tan\beta$ two-loop β-function we obtain a result reminiscent of the MSSM eq. (4.8)

$$\frac{\beta^{(2)}_{\text{NMSSM}}(\tan\beta)}{\tan\beta} = \gamma^{(2)}_{uu} - \gamma^{(2)}_{dd} + \frac{1}{(4\pi)^2}\xi\xi'\left(\frac{3}{10}g_1^2 + \frac{3}{2}g_2^2\right)\frac{\beta^{(1)}_{\text{MSSM}}(\tan\beta)}{\tan\beta}. \tag{4.13}$$

Note that γ and $\mathcal{R}$ refer to the NMSSM quantities, i.e. they differ from the MSSM quantities. Nevertheless, the difference $\mathcal{R}_u - \mathcal{R}_d$ still vanishes in the NMSSM.

4.3 E$_6$SSM

The E$_6$SSM is based on the direct product gauge group $\text{SU}(3)_c \otimes \text{SU}(2)_L \otimes \text{U}(1)_Y \otimes \text{U}(1)_N$. Among the several Higgs doublet fields $\{H_{u,i}\}_{i=1,2,3}$ and $\{H_{d,i}\}_{i=1,2,3}$ only the third generation (i.e. $i = 3$) of up- and down-type Higgs acquire a non-zero VEV $v_{u,d}$ [41]. The $\text{U}(1)_N$

charges of the Higgs fields are given as $N_{H_{u,i}}/2 = -2$ and $N_{H_{d,i}}/2 = -3$. Similarly, there are three generations of SM-group singlets S_i, which carry $\mathrm{U}(1)_N$ charge $N_{S_i}/2 = 5$, and only S_3 has a non-zero VEV v_s. Following ref. [41] and using the same convention as in the MSSM, the approximated E_6SSM superpotential can be written as

$$\begin{aligned}\mathcal{W}_{\mathrm{E_6SSM}} \approx &- y^e_{ij} H_{d,3}\cdot L_i \bar{E}_j - y^d_{ij} H_{d,3}\cdot Q_i \bar{D}_j - y^u_{ij} Q_i \cdot H_{u,3}\bar{U}_j \\ &+ \lambda_i S_3 H_{d,i}\cdot H_{u,i} + \kappa_i S_3 X_i \bar{X}_i.\end{aligned} \tag{4.14}$$

The superfields X_i, $\bar{X}_i$ describe exotic colored matter and transform as triplet/anti-triplet under SU(3), as singlet under $\mathrm{SU}(2)_L$ and have $\mathrm{U}(1)_Y$ and $\mathrm{U}(1)_N$ quantum numbers $Y_{X_i}/2 = -1/3$, $Y_{\bar{X}_i}/2 = 1/3$, $N_{X_i}/2 = -2$, $N_{\bar{X}_i}/2 = -3$.

For $\beta(v_{u,d})$ and $\beta(v_s)$ we obtain a similar expression as in the MSSM/NMSSM with one profound difference: the $\mathrm{U}(1)_N$ charge difference leads to a non-vanishing contribution. The Casimir eigenvalue leads to $(N/2)^2$ contributions and, thus, the one-loop β-functions read

$$\frac{\beta^{(1)}_{\mathrm{E_6SSM}}(v_s)}{v_s} = \frac{1}{(4\pi)^2}\left[g_N^2\left(1-\xi+2\xi\xi'\right)\left(\frac{N_S}{2}\right)^2 - 2\,\mathrm{Tr}\left(\lambda\lambda^\dagger\right) - N_c\,\mathrm{Tr}\left(\kappa\kappa^\dagger\right)\right], \tag{4.15a}$$

$$\frac{\beta^{(1)}_{\mathrm{E_6SSM}}(v_{u,d})}{v_{u,d}} = \frac{\beta^{(1)}_{\mathrm{MSSM}}(v_{u,d})}{v_{u,d}} + \frac{1}{(4\pi)^2}\left[g_N^2\left(1-\xi+2\xi\xi'\right)\left(\frac{N_{H_u,H_d}}{2}\right)^2 - |\lambda_3|^2\right]. \tag{4.15b}$$

In contrast to the NMSSM, the SM-singlet S_3 has a non-vanishing $\hat{\gamma}_{ss}$-contribution due to the $\mathrm{U}(1)_N$ gauge coupling.

For $\tan\beta$ those one-loop results yield

$$\begin{aligned}\frac{\beta^{(1)}_{\mathrm{E_6SSM}}(\tan\beta)}{\tan\beta} = \frac{1}{(4\pi)^2}\Bigg\{&g_N^2\left(1-\xi+2\xi\xi'\right)\left[\left(\frac{N_{H_u}}{2}\right)^2 - \left(\frac{N_{H_d}}{2}\right)^2\right] \\ &- \left[N_c\,\mathrm{Tr}\left(y^u y^{u\dagger}\right) - N_c\,\mathrm{Tr}\left(y^d y^{d\dagger}\right) - \mathrm{Tr}\left(y^e y^{e\dagger}\right)\right]\Bigg\}.\end{aligned} \tag{4.16}$$

As a consequence, both MSSM one-loop cancellations do not occur in the E_6SSM: neither the $\hat{\gamma}$-terms nor the gauge-coupling terms within γ drop out because of the different Higgs $\mathrm{U}(1)_N$ charges. Eq. (4.16) is in agreement with the result of ref. [24] obtained from finiteness of the renormalized Yukawa couplings.

As a further application, we present the two-loop results for $\beta(\tan\beta)$ in the E_6SSM. First, the VEV β-functions read at two-loop level

$$\frac{\beta^{(2)}_{\mathrm{E_6SSM}}(v_s)}{v_s} = \gamma^{(2)}_{ss} - \frac{1}{(4\pi)^4}\xi\xi'\left(\frac{N_S}{2}\right)^2 g_N^2\left[2\,\mathrm{Tr}\left(\lambda\lambda^\dagger\right) + N_c\,\mathrm{Tr}\left(\kappa\kappa^\dagger\right)\right] + \mathcal{R}_s, \tag{4.17a}$$

$$\frac{\beta^{(2)}_{\mathrm{E_6SSM}}(v_u)}{v_u} = \gamma^{(2)}_{uu} - \frac{1}{(4\pi)^4}\xi\xi'\left(\frac{3}{10}g_1^2 + \frac{3}{2}g_2^2 + \left(\frac{N_{H_u}}{2}\right)^2 g_N^2\right)\left[N_c\,\mathrm{Tr}\left(y^u y^{u\dagger}\right) + |\lambda_3|^2\right] + \mathcal{R}_u, \tag{4.17b}$$

$$\frac{\beta^{(2)}_{\mathrm{E_6SSM}}(v_d)}{v_d} = \gamma^{(2)}_{dd} - \frac{1}{(4\pi)^4}\xi\xi'\left(\frac{3}{10}g_1^2 + \frac{3}{2}g_2^2 + \left(\frac{N_{H_d}}{2}\right)^2 g_N^2\right) \times \left[N_c \operatorname{Tr}\left(y^d y^{d\dagger}\right) + \operatorname{Tr}\left(y^e y^{e\dagger}\right) + |\lambda_3|^2\right] + \mathcal{R}_d. \tag{4.17c}$$

Unlike the MSSM and NMSSM case, $\mathcal{R}_u \neq \mathcal{R}_d$ in the $\mathrm{E_6SSM}$ as the Higgs doublets have different $\mathrm{U}(1)_N$ quantum numbers. Furthermore, the contributions from kinetic mixing of the $\mathrm{U}(1)_Y$ and $\mathrm{U}(1)_N$ groups [42] are not relevant for the $\mathcal{O}(g^2 YY^\dagger)$ contributions we have explicitely given.

Second, for $\tan\beta$ follows at two loop

$$\begin{aligned}\frac{\beta^{(2)}_{\mathrm{E_6SSM}}(\tan\beta)}{\tan\beta} = {} & \gamma^{(2)}_{uu} - \gamma^{(2)}_{dd} + \frac{1}{(4\pi)^2}\xi\xi'\left(\frac{3}{10}g_1^2 + \frac{3}{2}g_2^2\right)\frac{\beta^{(1)}_{\mathrm{MSSM}}(\tan\beta)}{\tan\beta} \\ & - \frac{1}{(4\pi)^4}\xi\xi' g_N^2 \Bigg\{\left(\frac{N_{H_u}}{2}\right)^2 N_c \operatorname{Tr}\left(y^u y^{u\dagger}\right) \\ & \qquad - \left(\frac{N_{H_d}}{2}\right)^2 \left[N_c \operatorname{Tr}\left(y^d y^{d\dagger}\right) + \operatorname{Tr}\left(y^e y^{e\dagger}\right)\right]\Bigg\} \\ & - \frac{1}{(4\pi)^4}\xi\xi' g_N^2 \left[\left(\frac{N_{H_u}}{2}\right)^2 - \left(\frac{N_{H_d}}{2}\right)^2\right]|\lambda_3|^2 + \mathcal{R}_u - \mathcal{R}_d.\end{aligned} \tag{4.18a}$$

Note the structure of eq. (4.18a): the second term corresponds to the MSSM $\mathcal{O}(g^2 YY^\dagger)$ term from eq. (4.8), whereas the second and third line represent further violations of eq. (1.2) due to the $\mathrm{U}(1)_N$ couplings.

5 Conclusions

We computed the $\overline{\mathrm{MS}}/\ \overline{\mathrm{DR}}$ β-function for VEVs in general gauge theories and general SUSY gauge theories (Wess-Zumino gauge) up to Yukawa-enhanced two-loop contributions. These results complement the β-functions of refs. [25–30]. In addition, we provided the β-functions for $\tan\beta$ in the MSSM, NMSSM, and $\mathrm{E_6SSM}$ up to this order in general R_ξ gauge. These β-functions are required in renormalization group studies of spontaneously broken gauge theories, and they can be implemented in computer codes, e.g. in spectrum-generator generators like *SARAH* [43, 44] or many existing MSSM or NMSSM spectrum generators.

Our results have been obtained by using the elegant approach of ref. [17], which is interesting in its own right. In the past, this approach has been applied in more abstract contexts, but we have shown that it also facilitates calculations and provides qualitative understanding. We therefore close by summarizing this approach and its consequences.

- The VEVs v are promoted to background fields. As a consequence, R_ξ gauge fixing can be formulated without breaking global gauge invariance.

- The renormalization of the VEVs is completely determined by the field renormalization of the fields and background fields. The VEV counterterm can be expressed in terms of the dimension-zero field renormalization constants $\sqrt{Z}$ and $\sqrt{\hat{Z}}$, and $(\sqrt{\hat{Z}}-1)\hat{v}$ replaces $\delta\bar{v}$ from eq. (1.1).

- The β-function of the VEV is similarly composed of the anomalous dimensions $\gamma(\mathrm{S})$ and $\hat{\gamma}(\mathrm{S})$ of the fields and background fields, see eq. (3.14).

- The approach leads to additional information since the new renormalization constant $\sqrt{\hat{Z}}$ appears in the Lagrangian in a well-defined manner. As a consequence, its computation requires to evaluate the Green function $\Gamma_{\hat{q}_a,K_{\varphi_b}}$, which is unphysical but very simple to compute.

- The vertices in figure 1(a), 1(b) contributing to $\Gamma_{\hat{q}_a,K_{\varphi_b}}$ loop corrections are dictated by BRS-invariance and, thus, are restricted to gauge couplings. In particular, non-trivial $\hat{q}$-vertices can only arise from eq. (2.7a) and are thus linked to the gauge fixing term. Gauges such as Landau gauge, where F^A is independent of $\hat{\varphi}$, do not lead to such $\hat{q}$-vertices and have $\sqrt{\hat{Z}}=1$ and $\delta\bar{v}=0$.

- The cancellation in $\tan\beta$, eq. (1.2), as observed in refs. [21, 22], is a group theoretic coincidence and can be understood from the general expression for $\hat{\gamma}$.

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