Decoupling can revive minimal supersymmetric SU(5)

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Abstract: We revisit proton decay via the color-triplet Higgs multiplets in the minimal supersymmetric grand unified model with heavy sfermions. Although the model has been believed to be excluded due to the too short lifetime of proton, we have found that it is possible to evade the experimental constraints on the proton decay rate if the supersymmetric particles have masses much heavier than the electroweak scale. With such heavy sfermions, the 126 GeV Higgs boson is naturally explained, while they do not spoil the gauge coupling unification and the existence of dark matter candidates. Since the resultant proton lifetime lies in the regions which may be reached in the future experiments, proton decay searches may give us a chance to verify the scenario as well as the supersymmetric grand unified models.

Contents

1 Introduction

The discovery of the Higgs boson [1, 2] has opened the way for physics beyond the Standard Model (SM). It is certainly a striking hint for understanding the high-energy physics, as elementary scalar particles may play an important role in realizing our complicated world with apparent broken symmetries. The theories with supersymmetry (SUSY) naturally include such scalar particles; the Higgs boson might be a superpartner of chiral fermions, called higgsinos. Besides, there exists a set of scalar particles for each SM fermion, as well as adjoint fermions for the SM gauge bosons. Then, astonishingly enough, we find that with these extra particles the gauge coupling constants of the SM are to be unified at a certain high-energy scale with great accuracy [3–7]. This observation motivates us to study the supersymmetric grand unified theories (SUSY GUTs) [8, 9].

The SUSY GUTs predict an exciting phenomenon: proton decay. It is induced by the exchanges of the color-triplet Higgs multiplets and the X-bosons, and their effects are described in terms of the dimension-five and -six effective operators, respectively. In the minimal SUSY SU(5) GUT [8, 9], which is a simple supersymmetric extension of the original SU(5) GUT [10], the former process yields the dominant decay modes, such as $p \to K^+ \bar{\nu}$. The lifetime of the channel is estimated as $\tau(p \to K^+ \bar{\nu}) \lesssim 10^{30}$ yrs [11, 12], with the SUSY particles, in particular those of the third generation, assumed to have masses of around the electroweak scale. On the other hand, the Super-Kamiokande experiment gives stringent limits on the channels: $\tau(p \to K^+ \bar{\nu}) > 4.0 \times 10^{33}$ yrs [13]. This contradiction makes it widely believed that the minimal SUSY SU(5) GUT has been already excluded and, therefore, needs some extensions in order to suppress the dimension-five proton decay.

As is often the case with SUSY models, the SUSY GUTs are usually discussed within the context of the low-scale supersymmetry. Recently, experiments at the Large Hadron Collider (LHC) provide limits on the SUSY models. The ATLAS and CMS Collaborations have been searching for the SUSY particles and imposed severe constraints on their masses, especially those of squarks and gluino [14–16]. The mass bounds have began to exceed 1 TeV and, thus, the low-energy SUSY models are confronted with difficulties. Moreover, the observed mass of the Higgs boson around 126 GeV [1, 2] might also indicate the SUSY scale is considerably higher than the electroweak scale; in the minimal SUSY Standard Model (MSSM), the mass of the lightest Higgs boson is below the Z-boson mass at tree level, so sufficient mass difference between stops and top quark is required in order to raise the Higgs boson mass through the radiative corrections [17–21].

In fact, the SUSY models with heavy SUSY particles have a lot of attractive features [22–28]. First of all, since the flavor changing neutral current (FCNC) processes and/or the electric dipole moments induced by SUSY particles are suppressed by their masses, the SUSY flavor and CP problems [29] are relaxed when the masses are considerably heavy. As mentioned to above, the heavy sfermions yield sufficient radiative corrections to lift the Higgs mass up to 126 GeV [30–33]. They do not spoil the gauge coupling unification since the sfermions form complete SU(5) multiplets. Actually, it turns out that the unification is improved in the sense that the required threshold corrections at the GUT scale tend to be reduced [34]. As for the cosmology, the gravitino problem is avoided because of the high-scale SUSY breaking, and the thermal leptogenesis scenario well works with high reheating temperature [35]. Further, this high-scale SUSY scenario naturally accommodates dark matter (DM) candidates, which might be detected in future dark matter experiments directly [36–39] and indirectly [32, 40, 41]. Thus, with the recent LHC results considered, the high-scale SUSY scenario is even promising from a phenomenological point of view.

Interestingly, this scenario also provides an alternative solution to the problem regarding the dimension-five proton decay in the minimal SUSY GUT. The dimension-five operators generated via the color-triplet Higgs exchange contain squarks and/or sleptons in their external lines. These fields are to be integrated out below the SUSY scale through the wino or higgsino exchanging processes, and then the four-Fermi operators, suppressed by the sfermion masses, are induced. Hence, their effects are expected to be extremely reduced when the SUSY scale is much higher than the electroweak scale.

In this paper, we study such possibilities within the context of the high-scale SUSY scenario. We will find that the minimal SUSY SU(5) GUT actually evades the constraints from the proton decay experiments with the SUSY braking scales which naturally explain the 126 GeV Higgs boson and the existence of dark matter in the Universe. The resultant proton lifetime lies in the regions which may be reached in the future proton decay experiments. Therefore, although the high-scale SUSY scenario is hard to be probed in the collider experiments, the proton decay searches may give us a chance to verify the scenario as well as the existence of supersymmetry and the grand unification.

This paper is organized as follows: In section 2, a high-scale SUSY model which we discuss in this work and its phenomenology are briefly explained. In the next section, we

give a set of formulae for evaluating the proton decay rate via the dimension-five operators. Then, we show the resultant proton lifetime in the model, and compare it with current experimental limits in section 4. Section 5 is devoted to conclusions and discussion.

2 High-scale SUSY

To begin with, we describe a high-scale SUSY model which we deal with in the following discussion. Assume that there exists a SUSY breaking hidden sector where the SUSY breaking is triggered by a chiral superfield Z which is not a gauge singlet. Then, with a generic form of Kähler potential, all the scalar bosons except the lightest Higgs boson acquire masses of

$$M_S \sim \frac{F_Z}{M_*}\,, \tag{2.1}$$

with F_Z and M_* the F-component vacuum expectation value (VEV) of the field Z and the mediation scale of the SUSY breaking, respectively. The gravitino mass is $m_{3/2} = F_Z/\sqrt{3}M_{\mathrm{Pl}}$, and it is the same order as the scalar masses when M_* is around the Planck scale, M_{Pl}. The soft masses for the two doublet Higgses and the μ-term are fine-tuned in order to realize the electroweak symmetry breaking at the proper scale. The gaugino masses, on the other hand, are not generated by the dimension-five operators like

$$\int d^2\theta\ Z\mathrm{Tr}[W^\alpha W_\alpha]\,, \tag{2.2}$$

since the symmetry under which the superfield Z is charged prohibits such an operator. Here, W_α denotes the gauge field-strength chiral superfield. Instead, they are induced by the anomaly mediation mechanism [42, 43]:

$$M_a = \frac{b_a g_a^2}{16\pi^2} m_{3/2}\,, \tag{2.3}$$

where M_1, M_2, and M_3 are the masses of bino, wino, and gluino, respectively, and b_a are the one-loop beta-function coefficients of the gauge coupling constants g_a ($a = 1$, 2, and 3 for $\mathrm{U}(1)_Y$, $\mathrm{SU}(2)_L$, and $\mathrm{SU}(3)_C$, respectively). This expression tells us that the gaugino masses are suppressed by one-loop factors compared with the gravitino mass. Similarly, since we assume that the superfield which breaks supersymmetry is not a gauge-singlet, the A-terms are generated with the suppression by the loop factors. Finally, the higgsino mass, μ_H, is somewhat model-dependent; it might lie around the same order of gaugino masses when some additional symmetries exist, or be as large as gravitino masses if it is generated by the Kähler potential. Thus we regard it as a free parameter in the following discussion.

In this scenario, the lightest SUSY particle (LSP) is either wino, which turns out to be the lightest gaugino, or the lighter higgsino. Then, it is found that in any case the LSP may explain the dark matter in the Universe. Indeed, the thermal relic abundance of wino and higgsino DM with a mass of 2.7–3.0 TeV [44] and 1 TeV [45], respectively, accounts for the observed density of DM. With relatively small masses, the non-thermal production of them also might be consistent with the observation [46, 47]. The wino or higgsino DM in this model implies sfermion masses lie around 10^2–10^3 TeV.

From now on, we assume the sfermions are nearly degenerate in mass, and their masses are collectively denoted by M_S. The mass is supposed to be $M_S \simeq 10^2$–10^3 TeV, and either wino or higgsino is assumed to be the LSP. Models with such a mass spectrum and their phenomenology have been enthusiastically studied in the previous literature [48–55].

As mentioned to in the Introduction, the mass spectrum does not spoil the gauge coupling unification [23, 24], since the sfermions are embedded in the complete multiplets of SU(5). In fact, the unification may be improved [34]; a renormalization group analysis reveals that all of the GUT scale particles, especially the color-triplet Higgs multiplets, possibly lie around the GUT scale $\simeq 10^{16}$ GeV, contrary to the case of the low-energy SUSY. It indicates that the threshold corrections to the gauge coupling constants at the GUT scale are reduced in the high-scale SUSY scenario. Thus, the SUSY GUTs are still well-motivated.

3 Proton decay in the minimal SUSY SU(5) GUT

In this section, we review the proton decay in the minimal SUSY SU(5) GUT. In this model, the MSSM matter fields are embedded in a $\bar{\mathbf{5}} \oplus \mathbf{10}$ representation. The SU(2)$_L$ singlet down-type quarks $\bar{D}$ and doublet leptons L are incorporated into the $\bar{\mathbf{5}}$ fields, Φ, while the SU(2)$_L$ singlet up-type quarks, $\bar{U}$, doublet quarks, Q, and singlet leptons, $\bar{E}$, are formed into the $\mathbf{10}$ representations, Ψ. Here all the superfields are expressed in terms of the left-handed chiral superfields. The explicit form of the multiplets is

$$\Phi = \begin{pmatrix} \bar{D}_1 \\ \bar{D}_2 \\ \bar{D}_3 \\ E \\ -N \end{pmatrix}, \qquad \Psi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \bar{U}_3 & -\bar{U}_2 & U^1 & D^1 \\ -\bar{U}_3 & 0 & \bar{U}_1 & U^2 & D^2 \\ \bar{U}_2 & -\bar{U}_1 & 0 & U^3 & D^3 \\ -U^1 & -U^2 & -U^3 & 0 & \bar{E} \\ -D^1 & -D^2 & -D^3 & -\bar{E} & 0 \end{pmatrix}, \tag{3.1}$$

with

$$L = \begin{pmatrix} N \\ E \end{pmatrix}, \qquad Q^\alpha = \begin{pmatrix} U^\alpha \\ D^\alpha \end{pmatrix} . \tag{3.2}$$

Here, $\alpha = 1, 2, 3$ denotes the color index. The MSSM Higgs superfields, on the other hand, are embedded into a pair of $\mathbf{5}$ and $\bar{\mathbf{5}}$ superfields accompanied with the new Higgs superfields H_C^α and $\bar{H}_{C\alpha}$ called the color-triplet Higgs multiplets:

$$H = \begin{pmatrix} H_C^1 \\ H_C^2 \\ H_C^3 \\ H_u^+ \\ H_u^0 \end{pmatrix}, \qquad \bar{H} = \begin{pmatrix} \bar{H}_{C1} \\ \bar{H}_{C2} \\ \bar{H}_{C3} \\ H_d^- \\ -H_d^0 \end{pmatrix}, \tag{3.3}$$

where the last two components are corresponding to the MSSM Higgs superfields,

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}, \qquad H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix} . \tag{3.4}$$

Exchanges of the color-triplet Higgs multiplets induce the baryon-number violating interactions. They are coupled with the ordinary matter fields by the Yukawa coupling terms in the superpotential:[1]

$$W_{\text{Yukawa}} = \frac{1}{4} h^{ij} \epsilon_{abcde} \Psi_i^{ab} \Psi_j^{cd} H^e - \sqrt{2} f^{ij} \Psi_i^{ab} \Phi_{ja} \bar{H}_b \,, \tag{3.5}$$

where $i, j = 1, 2, 3$ and $a, b, c, \ldots = 1$–5 represent the generations and the SU(5) indices, respectively. The Yukawa couplings h^{ij} and f^{ij} in eq. (3.5) have redundant degrees of freedom, most of which are eliminated by the field re-definition of Ψ and Φ [56]. We parametrize the couplings according to ref. [57] as

$$h^{ij} = f_{u_i} e^{i\varphi_i} \delta_{ij} \,,$$
$$f^{ij} = V_{ij}^* f_{d_j} \,, \tag{3.6}$$

with V_{ij} the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The phase factors φ_i are subject to a constraint

$$\varphi_1 + \varphi_2 + \varphi_3 = 0 \,, \tag{3.7}$$

and thus two of them are independent parameters. The Yukawa coupling terms in eq. (3.5) are written with the component fields as follows:

$$W_{\text{Yukawa}} = f_{u_i} e^{i\varphi_i} \epsilon_{rs} \overline{U}_{i\alpha} Q_i^{\alpha r} H_u^s - V_{ij}^* f_{d_j} \epsilon_{rs} Q_i^{\alpha r} \overline{D}_{j\alpha} H_d^s - f_{d_j} \epsilon_{rs} V_{ij}^* \overline{E}_i L_j^r H_d^s$$
$$- \frac{1}{2} f_{u_i} e^{i\varphi_i} \epsilon_{\alpha\beta\gamma} \epsilon_{rs} Q_i^{\alpha r} Q_i^{\beta s} H_C^\gamma + V_{ij}^* f_{d_j} \epsilon_{rs} Q_i^{\alpha r} L_j^s \overline{H}_{C\alpha}$$
$$+ f_{u_i} e^{i\varphi_i} \overline{U}_{i\alpha} \overline{E}_i H_C^\alpha - V_{ij}^* f_{d_j} \epsilon^{\alpha\beta\gamma} \overline{U}_{i\alpha} \overline{D}_{j\beta} \overline{H}_{C\gamma} \,. \tag{3.8}$$

Here, r, s are the SU(2)$_L$ indices. In the following discussion, we also use the flavor basis for the matter fields. The relations between the flavor and gauge eigenstates are given as

$$Q_i = \begin{pmatrix} U_i' \\ V_{ij} D_j' \end{pmatrix}, \qquad L_i = \begin{pmatrix} N_i' \\ E_i' \end{pmatrix},$$
$$\bar{U}_i = e^{-i\varphi_i} \bar{U}_i', \qquad \bar{D}_i = \bar{D}_i', \qquad \bar{E}_i = V_{ij} \bar{E}_j', \tag{3.9}$$

where primes represent the flavor eigenstates.

The Yukawa interactions of color-triplet Higgs multiplets give rise to the dimension-five baryon-number violating operators. The processes in which the operators are induced are illustrated by the diagrams in figure 1. After the color-triplet Higgs multiplets are decoupled, we obtain the effective superpotential as

$$W_5 = + \frac{1}{2M_{H_C}} f_{u_i} f_{d_l} V_{kl}^* e^{i\varphi_i} \epsilon_{\alpha\beta\gamma} \epsilon_{rs} \epsilon_{tu} Q_i^{\alpha r} Q_i^{\beta s} Q_k^{\gamma t} L_l^u$$
$$+ \frac{1}{M_{H_C}} f_{u_i} e^{i\varphi_i} f_{d_l} V_{kl}^* \epsilon^{\alpha\beta\gamma} \overline{U}_{i\alpha} \overline{E}_i \overline{U}_{k\beta} \overline{D}_{l\gamma} \,, \tag{3.10}$$

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[1]In this paper we evaluate the proton decay rate with the SU(5) symmetric Yukawa couplings which are evaluated with up-type and down-type quark masses and the Cabibbo-Kobayashi-Maskawa matrix, though the ratios of charged lepton and down-type quark masses are not necessarily consistent with them. We have checked that, even if the lepton masses are used for f_{d_i}, our consequence presented below is not changed significantly.
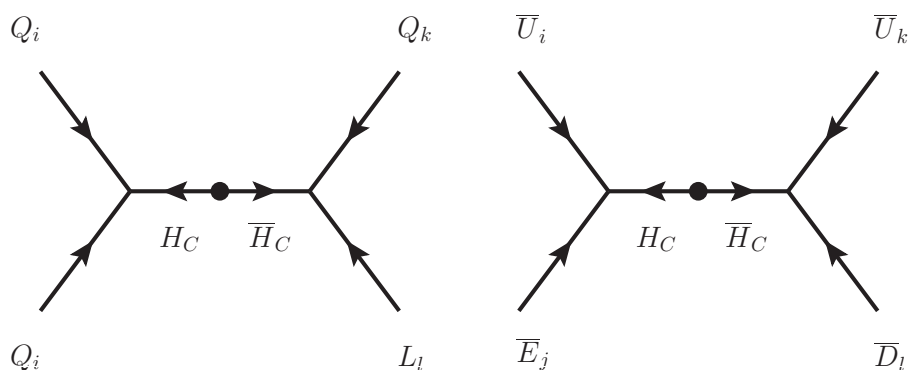
Figure 1. Supergraphs which illustrate color-triplet Higgs exchanging processes where dimension-five effective operators for proton decay are induced. Bullets indicate color-triplet Higgs mass term.
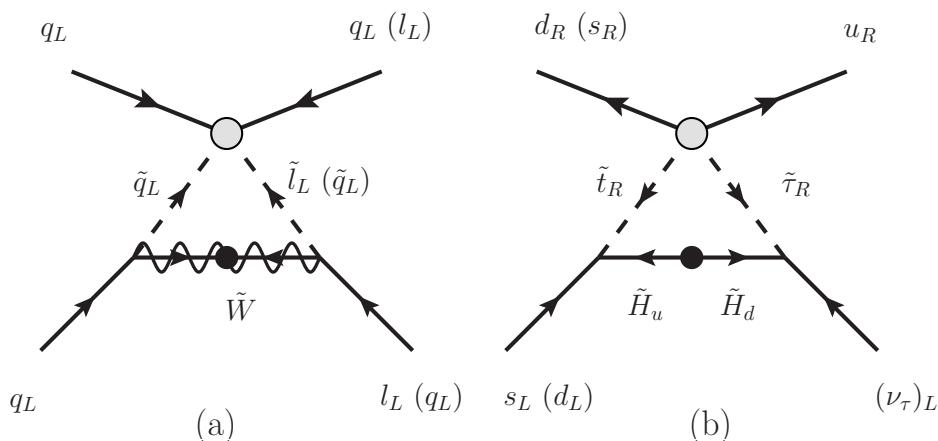

Figure 2. One-loop diagrams which yield the baryon-number violating four-Fermi operators. Diagrams (a) and (b) are generated by charged wino and higgsino exchanging processes, respectively. Gray dots indicate dimension-five effective interactions, while black dots represent wino or higgsino mass terms.

which yields the dimension-five effective operators,

$$\mathcal{L}_5 = \int d^2\theta W_5 \quad + \quad \text{h.c.}. \tag{3.11}$$

The effective operators contain sfermions in their external lines. Below the SUSY breaking scale, M_S, these sfermions turn into the SM fermions via the charged wino and higgsino exchanging processes shown in figure 2. In this figure, the gray and black dots indicate the dimension-five effective interactions and the mass terms for wino or higgsino, respectively. The first operator in eq. (3.10) contributes to the diagram (a), while the second one induces the diagram (b). Although the contribution of the diagram (b) is suppressed by the CKM matrix elements as it is generated in the flavor changing process, it is found to be sizable because of the large Yukawa couplings of the third generation fermions [11, 58]. The contributions of flavor-conserving neutral gauginos and higgsino

exchange are in general suppressed by the Yukawa couplings of the first generation, thus negligible. Among them, the gluino contribution might be sizable because of the large coupling. It turns out, however, that the gluino contribution vanishes in the limit where squarks are degenerate in mass, and we consider such a case in the following calculation. When sfermion mass matrices have large flavor mixing, the contributions may also be significant, though we do not take into account such a situation for simplicity.

After the electroweak symmetry breaking, the charged wino and higgsino are mixed with each other. In the following calculation, however, we neglect the effect since we mainly consider the case where $M_2, \mu_H \gg m_W$ with m_W the mass of W-boson. When the masses of wino and higgsino are nearly degenerate, the mixing effects might be significant. It is straight-forward to modify the formulae obtained below in such a case.

Evolving the four-Fermi operators from the SUSY breaking scale to the hadron scale ($\sim 1\,\mathrm{GeV}$) according to the renormalization group equations (RGEs), we finally obtain the effective operators for proton decay as

$$
\mathcal{L}_6 = \frac{\alpha_2^2}{M_{H_C} m_W^2 \sin 2\beta} \Bigg[2F(M_2, M_S^2) \sum_{i,j=2,3} \overline{m}_{u_i} \overline{m}_{d_j} V_{u_i d} V_{u_i s} V_{u d_j}^* e^{i\varphi_i}
$$

$$
\times A_R^{(i,j)} \epsilon_{\alpha\beta\gamma} \big\{ (u_L^\alpha d_L^\beta)(\nu_{Lj} s_L^\gamma) + (u_L^\alpha s_L^\beta)(\nu_{Lj} d_L^\gamma) \big\}
$$

$$
- \frac{\overline{m}_t^2 \overline{m}_\tau V_{tb}^* e^{i\varphi_1}}{m_W^2 \sin 2\beta} F(\mu_H, M_S^2) \overline{A}_R \epsilon_{\alpha\beta\gamma}
$$

$$
\times \big\{ \overline{m}_d V_{ud} V_{ts} (u_R^\alpha d_R^\beta)(\nu_\tau s_L^\gamma) + \overline{m}_s V_{us} V_{td} (u_R^\alpha s_R^\beta)(\nu_\tau d_L^\gamma) \big\} \Bigg]
$$

$$
+ \text{h.c.,} \tag{3.12}
$$

where we use the two-component spinor notation for the SM fermion fields; all of the quarks are written in the flavor basis though primes are omitted for simplicity; M_S is the mass of sfermions with all the sfermions assumed to be degenerate in mass; $\overline{m}_q$ are the masses of quarks defined in the $\overline{\mathrm{DR}}$ scheme at the scale of $\mu = 2\,\mathrm{GeV}$; u_2, d_2, u_3, and d_3 denote c, s, t, and b quarks, respectively; $\alpha_2 \equiv g_2^2/4\pi$ with g_2 the $\mathrm{SU}(2)_L$ gauge coupling constant at the electroweak scale; $\tan\beta \equiv \langle H_u^0 \rangle / \langle H_d^0 \rangle$; $A_R^{(i,j)}$ and $\overline{A}_R$ in eq. (3.12) represent the renormalization factors. These factors include the renormalization effects for both the couplings and the effective operators. The estimation of these factors is carried out in appendix A.

The loop function $F(M, M_S^2)$ is given as

$$
F(M, M_S^2) = M \left[\frac{1}{M_S^2 - M^2} - \frac{M^2}{(M_S^2 - M^2)^2} \ln\left(\frac{M_S^2}{M^2} \right) \right], \tag{3.13}
$$

where M is either the wino mass, M_2, or the higgsino mass, μ_H. In the limit of $M \ll M_S$, the function leads to

$$
F(M, M_S^2) \to \frac{M}{M_S^2}, \qquad (M_S \gg M), \tag{3.14}
$$

while in the limit of $M \to M_S$, it follows that

$$
F(M, M_S^2) \to \frac{1}{2M_S}, \qquad (M \to M_S). \tag{3.15}
$$

Note that the function is proportional to M, when $M \lesssim M_S$. For this reason, the contribution of the diagrams in figure 2 is enhanced when the masses of the exchanged particles are large. In particular, in the case of $\mu_H \gg M_2$, the higgsino exchange contribution (the diagram (b) in figure 2) dominates the wino exchange one. We also find from the behavior of the loop function that the transition amplitude is considerably suppressed when the sfermions have sufficiently large masses. Thus, we expect that the experimental constraints on the proton decay rate may be avoided in the high-scale SUSY scenario.

The effective operators in eq. (3.12) are written in terms of partons. In order to derive the decay amplitude for proton, we need to obtain the matrix elements of the quarks appearing in the operators between the proton state $|p\rangle$ and the kaon state $|K^+\rangle$. With the matrix elements, we finally obtain the partial decay widths, $\Gamma(p \to K^+ \bar{\nu}_\mu)$ and $\Gamma(p \to K^+ \bar{\nu}_\tau)$, and the sum of them well approximates the total decay rate of the $p \to K^+ \bar{\nu}$ channel. Explicit formulae for the decay widths as well as the determination of the matrix elements are presented in appendix. B.

Before concluding this section, we briefly comment on the proton decay via the SU(5) gauge boson exchange. The SU(5) gauge bosons, called X-bosons, give rise to the dimension-six operators which contribute to proton decay by the $p \to \pi^0 e^+$ channel. In ref. [34], it is pointed out that the GUT scale in the high-scale SUSY tends to be slightly lower than that in the low-energy SUSY. Thus, the proton decay rate in this channel is expected to be enhanced. It turns out, however, that the resultant lifetime is generally long enough [59] to evade the current experimental bound, $\tau(p \to \pi^0 e^+) > 1.29 \times 10^{34}$ yrs [60]. Thus, we ignore the contribution in the following calculation.

4 Results

Now we show numerical results of the proton decay lifetime in the high-scale SUSY scenario. We will see below that the resultant lifetime is well above the current experimental limits in a wide range of parameter region.

First, we consider the case where the higgsino mass is of the order of the sfermion masses, M_S. In this case, the higgsino exchange contribution (the diagram (b) in figure 2) dominates the wino exchange one, as mentioned to in the previous section. For this reason, the lifetime has little dependence on the additional phases, φ_i in eq. (3.6), as well as the wino mass. Thus, it is possible to make a robust prediction for the proton decay lifetime. As the right-handed stop and stau run in the loop in the higgsino exchanging diagram, M_S should be regarded as their masses, which we assume to be degenerate for brevity. For $M_S = \mu_H$, the proton lifetime τ_p is approximately given as[2]

$$\tau_p \simeq 4 \times 10^{35} \times \sin^4 2\beta \left(\frac{0.1}{\bar{A}_R} \right)^2 \left(\frac{M_S}{10^2 \text{ TeV}} \right)^2 \left(\frac{M_{H_C}}{10^{16} \text{ GeV}} \right)^2 \text{ yrs} , \qquad (4.1)$$

and found to be well above the current experimental limits, $\tau(p \to K^+ \bar{\nu}) > 4.0 \times 10^{33}$ yrs [13], with the SUSY scale being much higher than the electroweak scale.

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[2]The renormalization factor reduces the proton decay rate for larger M_S when $M_S = \mu_H = (10^2$–$10^5)$ TeV, as described in figure 8 in appendix. A.
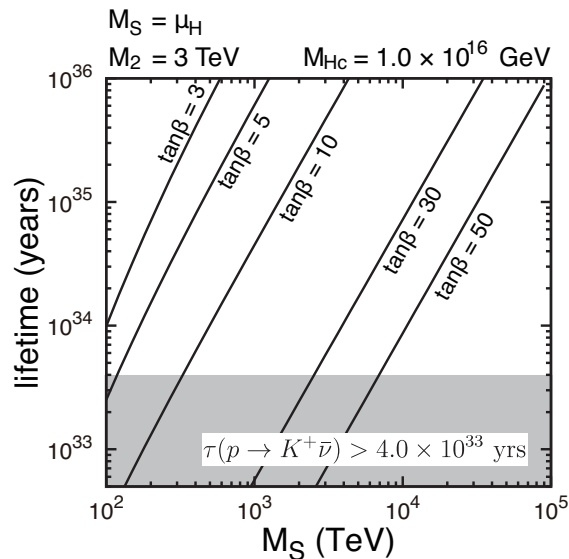
Figure 3. Lifetime of $p \rightarrow K^{+}\bar{\nu}$ mode as functions of $M_S = \mu_H$. Wino mass is set to be $3\,\text{TeV}$ and $M_{H_C} = 1.0 \times 10^{16}$ GeV. Solid lines correspond to $\tan\beta = 3, 5, 10, 30$, and 50 from left-top to right-bottom, respectively. Shaded region is excluded by the current experimental bound, $\tau(p \rightarrow K^{+}\bar{\nu}) > 4.0 \times 10^{33}$ yrs [13].

Let us investigate it in detail. To begin with, we consider the mass of the color-triplet Higgs multiplets, M_{H_C}. As mentioned to in section 2, through the RGE analysis discussed in refs. [57, 61] with requiring the gauge coupling unification, one finds that M_{H_C} may be around the GUT scale in the case of high-scale SUSY [34]. The prediction is, however, quite sensitive to the mass spectrum below the GUT scale, especially to the masses of higgsinos and gauginos. Thus, in the following discussion, we just fix M_{H_C} to be around the GUT scale. One easily obtains proton lifetimes corresponding to other values of M_{H_C} by using the power law given in eq. (4.1).

In figure 3, we present the lifetime of the $p \rightarrow K^{+}\bar{\nu}$ mode as functions of $M_S = \mu_H$. Here, the wino mass is set to be $3\,\text{TeV}$, while the result scarcely depends on the mass as long as $M_2 \ll M_S$. The color-triplet Higgs mass is fixed to $M_{H_C} = 1.0 \times 10^{16}$ GeV. The solid lines are for $\tan\beta = 3, 5, 10, 30$, and 50 from left-top to right-bottom, respectively. The shaded region is excluded by the current experimental bound, $\tau(p \rightarrow K^{+}\bar{\nu}) > 4.0 \times 10^{33}$ yrs [13]. The figure illustrates the behavior presented in eq. (4.1). Moreover, it is found that the proton decay lifetime in the high-scale SUSY scenario may evade the experimental constraints, especially for small $\tan\beta$ and high SUSY breaking scales. We also show a similar plot for a relatively small value of M_{H_C} in figure 4, where the mass is taken to be 1.0×10^{15} GeV. The wino mass is again set to be $M_2 = 3\,\text{TeV}$, and the solid lines correspond to $\tan\beta = 3, 5, 10$, and 30 from left-top to right-bottom, respectively. We see that the relation between the results presented in figures 3 and 4 is well explained by the simple power law in eq. (4.1), though the renormalization factors may also be changed with different values of M_{H_C}. For this reason, we just fix $M_{H_C} = 1.0 \times 10^{16}$ GeV in the following analysis. One easily read other results with different values of M_{H_C} by using the relation given in eq. (4.1).
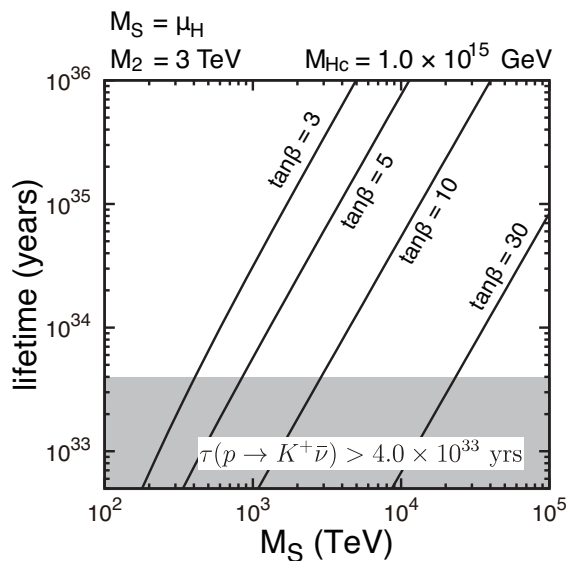
Figure 4. Lifetime of $p \to K^+ \bar{\nu}$ mode as functions of $M_S = \mu_H$. Wino mass is set to be $3\,\mathrm{TeV}$ and $M_{H_C} = 1.0 \times 10^{15}$ GeV. Solid lines correspond to $\tan\beta = 3, 5, 10$, and 30 from left-top to right-bottom, respectively. Shaded region is excluded by the current experimental bound, $\tau(p \to K^+ \bar{\nu}) > 4.0 \times 10^{33}$ yrs [13].
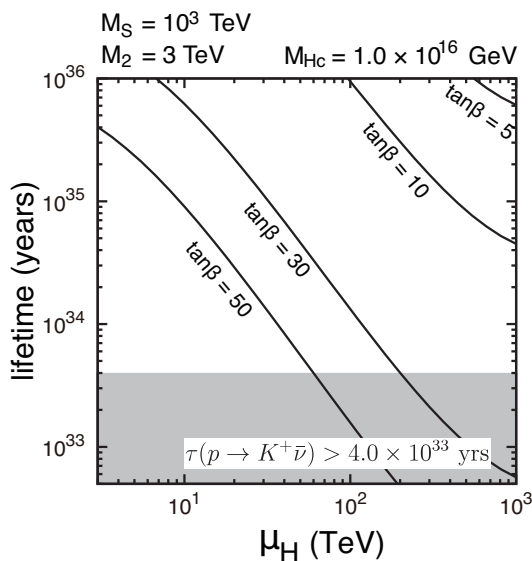

Figure 5. Lifetime of $p \to K^+ \bar{\nu}$ mode as functions of μ_H. Wino, sfermion and color-triplet Higgs masses are set to be $M_2 = 3\,\mathrm{TeV}$, $M_S = 10^3\,\mathrm{TeV}$, and $M_{H_C} = 1.0 \times 10^{16}$ GeV, respectively. Solid lines correspond to $\tan\beta = 5, 10, 30$, and 50 from right-top to left-bottom, respectively. Shaded region is excluded by the current experimental bound, $\tau(p \to K^+ \bar{\nu}) > 4.0 \times 10^{33}$ yrs [13].

Next, we consider the case where the higgsinos are lighter than the sfermions. In this case, the lifetime depends on the new phases appearing in eq. (3.6). Here, we take the phases so that they yield the maximal amplitude for the proton decay rate, i.e., we require that each term in eqs. (B.5) and (B.6) be constructive. This requirement together with
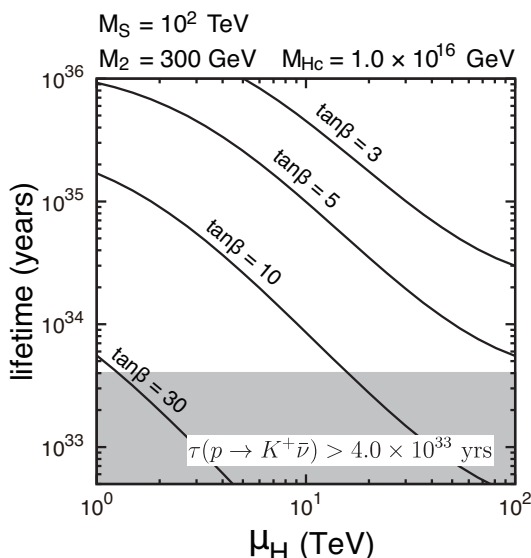
Figure 6. Lifetime of $p \to K^+ \bar{\nu}$ mode as functions of μ_H. Wino, sfermion, and color-triplet Higgs masses are set to be $M_2 = 300 \, \text{GeV}$, $M_S = 10^2 \, \text{TeV}$, and $M_{H_C} = 1.0 \times 10^{16}$ GeV, respectively. Solid lines correspond to $\tan\beta = 3, 5, 10$, and 30 from right-top to left-bottom, respectively. Shaded region is excluded by the current experimental bound, $\tau(p \to K^+ \bar{\nu}) > 4.0 \times 10^{33}$ yrs [13].

the constraint (3.7) uniquely determines all of the phases φ_i. Since the choice of phases gives the maximal proton decay rate, we are to obtain the most stringent limit on the parameters. In addition, we assume that both the higgsino and wino mass parameters are real and positive. However, as long as one chooses the phases constructively, the results would not change since it is possible to include the extra phases of the higgsino and wino masses into the redefinition of the phases φ_i.

In figure 5, we plot the proton lifetime as functions of the higgsino mass. Here, the wino, sfermion,[3] and color-triplet Higgs masses are set to be $M_2 = 3 \, \text{TeV}$, $M_S = 10^3 \, \text{TeV}$, and $M_{H_C} = 1.0 \times 10^{16}$ GeV, respectively. The solid lines correspond to $\tan\beta = 5, 10, 30$, and 50 from right-top to left-bottom, respectively. Again the shaded region is excluded by the current experimental bound, $\tau(p \to K^+ \bar{\nu}) > 4.0 \times 10^{33}$ yrs [13]. It is found that the lifetime considerably depends on the mass of higgsino as well as the value of $\tan\beta$. It illustrates that the higgsino contribution is dominant in a wide range of parameter region. Indeed, the contribution gets more significant as the higgsino mass is raised up. We also show a similar plot in the case of $M_2 = 300 \, \text{GeV}$ and $M_S = 100 \, \text{TeV}$ in figure 6. In this case, large $\tan\beta$ region is excluded even if the higgsino mass is around 1 TeV, while with a rather small value of $\tan\beta$ the proton lifetime easily exceeds the experimental limit. Anyway, we have found that in the high-scale SUSY scenario, the minimal SUSY SU(5) GUT is still alive without any conspiracy of suppressing the dimension-five operators.

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[3]To be concrete, we regard M_S as the stop mass, and all of the other sfermion masses are assumed to be degenerate with M_S. Generally speaking, stops are lighter than other sfermions, especially those of the first and second generations. So, even though one relaxes the degeneration assumption, one ends up obtaining a smaller proton decay rate.

5 Conclusions and discussion

In this work, we have evaluated the proton decay lifetime via the dimension-five operators in the high-scale SUSY scenario. It is found that the higgsino exchanging diagram gives rise to the dominant contribution in a wide range of parameter region. After all, we have revealed that the proton lifetime may evade the current experimental limit and, thus, the minimal SUSY SU(5) GUT is not excluded in the high-scale SUSY scenario.

In the $\mu_H \simeq M_S$ case, after M_{H_C} being fixed, the proton lifetime depends only on M_S and $\tan\beta$. In fact, these two parameters are also crucial for the prediction of the Higgs boson mass in the high-scale SUSY scenario. Therefore, since now we know that the mass of Higgs boson is 126 GeV, it is possible to relate the M_S and $\tan\beta$ in the present scenario [30–33]. Further, if the mass spectrum is somehow fixed, we are able to constraint M_{H_C} by requiring the gauge coupling unification [34, 57, 61]. Precise analyses in this direction enable us to predict proton decay rate in this scenario, and future experiments may examine the prediction. Such kind of model-dependent study is carried out on another occasion.

While the dimension-five proton decay is suppressed by the heavy sfermion masses, the dimension-six one through the X-boson exchange does not suffer from such a suppression. As referred to above, the GUT scale in the high-scale SUSY is slightly lower than the ordinary one. Since the dimension-six proton decay lifetime scales as $\propto M_X^4$ with M_X the mass of X-boson, it may be significantly enhanced even by a small change in the GUT scale. In such a case, the $p \to \pi^0 e^+$ mode may dominate the $p \to K^+ \bar\nu$ mode, and both of the modes might be searched in future experiments. For instance, the expected sensitivities of the Hyper-Kamiokande with ten years exposure [13] are 1.3×10^{35} and 2.5×10^{34} years at 90 % confidence level for the $p \to e^+ \pi^0$ and $p \to \bar\nu K^+$ modes,[4] respectively, which enable us to explore a wide range of parameter region in high-scale SUSY models.

After all, the minimal SUSY SU(5) GUT is still quite promising, and the proton decay experiments may reveal the existence of supersymmetry as well as the grand unification.

Note added: while this work was being finalized, we noticed the authors in refs. [55, 63] discussed the dimension-five proton decay in a similar context. In ref. [55], they have just shown dimensional analysis to constraint the dimension-five operators, while in ref. [63], the proton lifetime is examined in supergravity unified models.

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[4]Recent improvements in the analysis of the $K^+ \to \pi^+ \pi^0$ decay channel may provide a better sensitivity for the $p \to \bar\nu K^+$ mode: 3.2×10^{34} years at 90 % confidence level [62].

A Renormalization factors

Here we present the explicit expressions for the renormalization factors defined in eq. (3.12). First, we write the renormalization factors $A_R^{(i,j)}$ and $\overline{A}_R$ as the products of the long- and short-distance renormalization factors:

$$A_R^{(i,j)} \equiv A_L A_S^{(i,j)} \,,$$

$$\overline{A}_R \equiv \overline{A}_L \overline{A}_S \,, \tag{A.1}$$

where A_L and $\overline{A}_L$ represent the long-distance QCD renormalization factors between the electroweak scale ($\mu = m_Z$ with m_Z the Z-boson mass) and the scale of $\mu = 2\,\mathrm{GeV}$, while $A_S^{(i,j)}$ and $\overline{A}_S$ correspond to the short-distance renormalization effects between the electroweak and GUT scales. All of the effects are to be computed at one-loop level.

A.1 Long-range factors

First, we discuss the long-distance renormalization factors. The factors consist of two effects; one is the running of the quark masses from $\mu = 2\,\mathrm{GeV}$ to $\mu = m_Z$, and the other is the renormalization effect of the four-Fermi operators in eq. (3.12) from $\mu = m_Z$ to $\mu = 2\,\mathrm{GeV}$. We neglect the QED corrections since the electromagnetic coupling is much smaller than the strong coupling.

Before analyzing the renormalization effects, we first discuss the input parameters for quark masses. In ref. [64], the values of the light quark masses m_q ($q = u, d, s$) are given in the $\overline{\mathrm{MS}}$ scheme at $\mu \simeq 2\,\mathrm{GeV}$, while those for c- and b-quarks are presented in the $\overline{\mathrm{MS}}$ at $\mu = m_c$ and m_b, respectively. Since we define $\overline{m}_q$ in eq. (3.12) in the $\overline{\mathrm{DR}}$ scheme, we convert the input parameters into those in the $\overline{\mathrm{DR}}$ scheme. We use the one-loop relation [65]:

$$\overline{m}_q(\mu) = m_q(\mu)\left(1 - \frac{\alpha_s(\mu)}{3\pi}\right) \,, \tag{A.2}$$

where $\alpha_s \equiv g_s^2/4\pi$ with g_s the strong coupling constant. For c- and b-quarks, we evolve the masses to $\mu = 2\,\mathrm{GeV}$ by using the RGEs. For top quark, on the other hand, the pole mass is displayed in ref. [64]. The relation between the pole mass and the $\overline{\mathrm{DR}}$ mass is given by [65]

$$m_t = \overline{m}_t(\mu)\left[1 + \frac{\alpha_s(\mu)}{3\pi}\left(6\log\frac{\mu}{m_t} + 5\right)\right] \,. \tag{A.3}$$

Now we consider the QCD renormalization effects. The RGEs for the Wilson coefficients of the effective operators in eq. (3.12) at one-loop [66] are given as[5]

$$\mu\frac{\partial}{\partial\mu}C = -\frac{4g_s^2}{16\pi^2}C \,. \tag{A.4}$$

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[5]Two-loop effects are also calculated in ref. [67], though their contribution is found to be small.

By using the equation, as well as the RGEs for the quark masses, we readily obtain the long-range renormalization factors A_L and $\overline{A}_L$:

$$A_L = \frac{\overline{m}_{u_i}\overline{m}_{d_i}(m_Z) \cdot C(2 \text{ GeV})}{\overline{m}_{u_i}\overline{m}_{d_i}(2 \text{ GeV}) \cdot C(m_Z)} = \left(\frac{\alpha_s(2 \text{ GeV})}{\alpha_s(m_b)}\right)^{-\frac{18}{25}}\left(\frac{\alpha_s(m_b)}{\alpha_s(m_Z)}\right)^{-\frac{18}{23}},$$

$$\overline{A}_L = \frac{\overline{m}_t^2\overline{m}_{d_i}(m_Z) \cdot C(2 \text{ GeV})}{\overline{m}_t^2\overline{m}_{d_i}(2 \text{ GeV}) \cdot C(m_Z)} = \left(\frac{\alpha_s(2 \text{ GeV})}{\alpha_s(m_b)}\right)^{-\frac{6}{5}}\left(\frac{\alpha_s(m_b)}{\alpha_s(m_Z)}\right)^{-\frac{30}{23}}. \tag{A.5}$$

Numerically, we have

$$A_L = 0.53, \qquad \overline{A}_L = 0.34, \tag{A.6}$$

at one-loop level.

A.2 Short-range factors

Next, we evaluate the short-distance renormalization factors. They are composed of three factors. First, the effective operators given at the GUT scale receive the renormalization effects as they are taken down to the electroweak scale. Second, the Yukawa couplings in the color-triplet Higgs exchanging process are determined through the running of the couplings from the electroweak scale to the GUT scale. Third, the interaction vertices in the one-loop diagrams in figure 2 are obtained by evolving the $SU(2)_L$ gauge coupling and the Yukawa couplings according to the RGEs from the electroweak scale to the SUSY breaking scale, $\mu = M_S$.

Let us begin with the running of the Yukawa couplings. Initial values for the Yukawa coupling constants are given by

$$y_{u_i}(m_Z) = \frac{g_2}{\sqrt{2}m_W}\overline{m}_{u_i}(m_Z),$$

$$y_{d_i}(m_Z) = \frac{g_2}{\sqrt{2}m_W}\overline{m}_{d_i}(m_Z),$$

$$y_{e_i}(m_Z) = \frac{g_2}{\sqrt{2}m_W}\overline{m}_{e_i}(m_Z). \tag{A.7}$$

In the SM, the Yukawa couplings flow according to the following RGEs at one-loop level:

$$\mu\frac{\partial}{\partial\mu}y_{u_i} = \frac{1}{16\pi^2}y_{u_i}\left[\frac{3}{2}(y_{u_i}^2 - y_{d_i}^2) + Y_2 - \frac{17}{20}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2\right],$$

$$\mu\frac{\partial}{\partial\mu}y_{d_i} = \frac{1}{16\pi^2}y_{d_i}\left[\frac{3}{2}(y_{d_i}^2 - y_{u_i}^2) + Y_2 - \frac{1}{4}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2\right],$$

$$\mu\frac{\partial}{\partial\mu}y_{e_i} = \frac{1}{16\pi^2}y_{e_i}\left[\frac{3}{2}y_{e_i}^2 + Y_2 - \frac{9}{4}g_1^2 - \frac{9}{4}g_2^2\right], \tag{A.8}$$

where Y_2 is given as

$$Y_2 = \sum_i (3y_{u_i}^2 + 3y_{d_i}^2 + y_{e_i}^2), \tag{A.9}$$

and we neglect the off-diagonal components for simplicity. The equations are also applicable when the renormalization scale exceeds the gaugino masses. Above the higgsino mass,

eq. (A.8) is valid except that Y_2 is modified to

$$Y_2 = \sum_i (3y_{u_i}^2 + 3y_{d_i}^2 + y_{e_i}^2) + \frac{3}{10}g_1^2 + \frac{3}{2}g_2^2 . \tag{A.10}$$

Here we assume the ordinary supersymmetric relation for the gaugino-Higgs-higgsino couplings. The couplings may deviate the relation when the SUSY breaking scale is much higher than the gaugino and higgsino masses, but it is found that the deviation is usually not so significant [23, 24].

At the SUSY breaking scale, the Yukawa couplings y_f are matched with the supersymmetric ones, $\overline{y}_f$, as follows:

$$\overline{y}_{u_i}(M_S) = \frac{1}{\sin\beta}y_{u_i}(M_S) ,$$
$$\overline{y}_{d_i}(M_S) = \frac{1}{\cos\beta}y_{d_i}(M_S) ,$$
$$\overline{y}_{e_i}(M_S) = \frac{1}{\cos\beta}y_{e_i}(M_S) . \tag{A.11}$$

Above $\mu = M_S$, the RGEs for the Yukawa couplings are given as

$$\mu\frac{\partial}{\partial\mu}\overline{y}_{u_i} = \frac{1}{16\pi^2}\overline{y}_{u_i}\left[3\sum_j \overline{y}_{u_j}^2 + 3\overline{y}_{u_i}^2 + \overline{y}_{d_i}^2 - \frac{13}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2\right],$$
$$\mu\frac{\partial}{\partial\mu}\overline{y}_{d_i} = \frac{1}{16\pi^2}\overline{y}_{d_i}\left[\sum_j(3\overline{y}_{d_j}^2 + \overline{y}_{e_j}^2) + 3\overline{y}_{d_i}^2 + \overline{y}_{u_i}^2 - \frac{7}{15}g_1^2 - 3g_2^2 - \frac{16}{3}g_3^2\right],$$
$$\mu\frac{\partial}{\partial\mu}\overline{y}_{e_i} = \frac{1}{16\pi^2}\overline{y}_{e_i}\left[\sum_j(3\overline{y}_{d_j}^2 + \overline{y}_{e_j}^2) + 3\overline{y}_{e_i}^2 - \frac{9}{5}g_1^2 - 3g_2^2\right]. \tag{A.12}$$

Then, at the GUT scale, the Yukawa couplings f_{u_i} and f_{d_i} in eq. (3.6) are defined by

$$f_{u_i} \equiv \overline{y}_{u_i}(M_{H_C}) ,$$
$$f_{d_i} \equiv \overline{y}_{d_i}(M_{H_C}) . \tag{A.13}$$

For the gauge couplings, the one-loop gauge coupling beta function coefficients are given in the SM as

$$b_a = (41/10, -19/6, -7) , \tag{A.14}$$

for $U(1)_Y$, $SU(2)_L$, and $SU(3)_C$, respectively. Above the gaugino threshold, the coefficients are converted to[6]

$$b_a = (41/10, -11/6, -5) , \tag{A.15}$$

and after the higgsinos showing up, they lead to

$$b_a = (9/2, -7/6, -5) . \tag{A.16}$$

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[6]When evaluating the renormalization factors, we take the gluino mass equal to wino mass for simplicity.

Finally, in the MSSM, they are given as

$$b_a = (33/5, 1, -3) . \tag{A.17}$$

The short-distance renormalization factors for the four-Fermi operators in eq. (3.12) are presented in ref. [66]. For the effective operators generated by the wino exchanging diagram, the renormalization factor for the Wilson coefficient is

$$C(\mu) = \left(\frac{\alpha_3(\mu)}{\alpha_3(\mu_0)}\right)^{-\frac{2}{b_3}} \left(\frac{\alpha_2(\mu)}{\alpha_2(\mu_0)}\right)^{-\frac{15}{2b_2}} \left(\frac{\alpha_1(\mu)}{\alpha_1(\mu_0)}\right)^{-\frac{1}{10b_1}} C(\mu_0) , \tag{A.18}$$

while for those induced by the higgsino exchange, we have

$$\overline{C}(\mu) = \left(\frac{\alpha_3(\mu)}{\alpha_3(\mu_0)}\right)^{-\frac{2}{b_3}} \left(\frac{\alpha_2(\mu)}{\alpha_2(\mu_0)}\right)^{-\frac{9}{4b_2}} \left(\frac{\alpha_1(\mu)}{\alpha_1(\mu_0)}\right)^{-\frac{11}{20b_1}} \overline{C}(\mu_0) . \tag{A.19}$$

In the case of $\mu > M_S$, the theory is to be regarded as supersymmetric, and the renormalization factors are obtained as the product of the wave function renormalizations of the fields in the effective operators [68]. The wino contribution to the effective operators in eq. (3.12) is induced by the effective operators with a form like

$$C_{ij} \int d^2\theta \epsilon_{\alpha\beta\gamma} \epsilon_{rs} \epsilon_{tu} Q_i^{\alpha r} Q_i^{\beta s} Q_1^{rt} L_j^u , \tag{A.20}$$

with $i, j = 2, 3$, while the higgsino contribution is generated by

$$\overline{C} \int d^2\theta \epsilon^{\alpha\beta\gamma} \overline{U}_{3\alpha} \overline{E}_3 \overline{U}_{1\beta} \overline{D}_{l\gamma} , \tag{A.21}$$

with $l = 1, 2$. Then, the REGs for the Wilson coefficients of the operators are

$$\mu \frac{\partial}{\partial \mu} C_{ij} = \frac{1}{16\pi^2} \left[2(\overline{y}_{u_i}^2 + \overline{y}_{d_i}^2) + \overline{y}_{e_j}^2 - 8g_3^2 - 6g_2^2 - \frac{2}{5}g_1^2 \right] C_{ij} , \tag{A.22}$$

and

$$\mu \frac{\partial}{\partial \mu} \overline{C} = \frac{1}{16\pi^2} \left[2(\overline{y}_t^2 + \overline{y}_\tau^2) - 8g_3^2 - \frac{12}{5}g_1^2 \right] \overline{C} . \tag{A.23}$$

Note that in this case it is important to take the Yukawa interactions into account since the effective operators contain the third generation chiral superfields.

With the RGEs presented above, we finally compute the short-distance renormalization factors as follows:

$$A_S^{(i,j)} = \frac{m_W^2 \sin 2\beta f_{u_i} f_{d_j}}{4\pi \ \overline{m}_{u_i} \overline{m}_{d_j}(m_Z)} \frac{\alpha_2(M_S)}{\alpha_2^2(m_Z)} \cdot \frac{C(m_Z)}{C(M_S)} \frac{C_{ij}(M_S)}{C_{ij}(M_{H_C})} ,$$
$$\overline{A}_S = \frac{m_W^4 \sin^2 2\beta f_t \overline{y}_t(M_S) \overline{y}_\tau(M_S) f_{d_i}}{(4\pi)^2 \alpha_2^2(m_Z) \overline{m}_t^2 \overline{m}_\tau \overline{m}_{d_i}(m_Z)} \cdot \frac{\overline{C}(m_Z)}{\overline{C}(M_{H_C})} . \tag{A.24}$$

With the results obtained above, we compute the renormalization factors. We present $A_R^{(i,j)}$ and $\overline{A}_R$ in figures 7 and 8, respectively, as functions of M_S. In both figures, we take $M_S = \mu_H$, $M_2 = 3\,\text{TeV}$, and $M_{H_C} = 1.0 \times 10^{16}\,\text{GeV}$. In the left graph in figure 7,
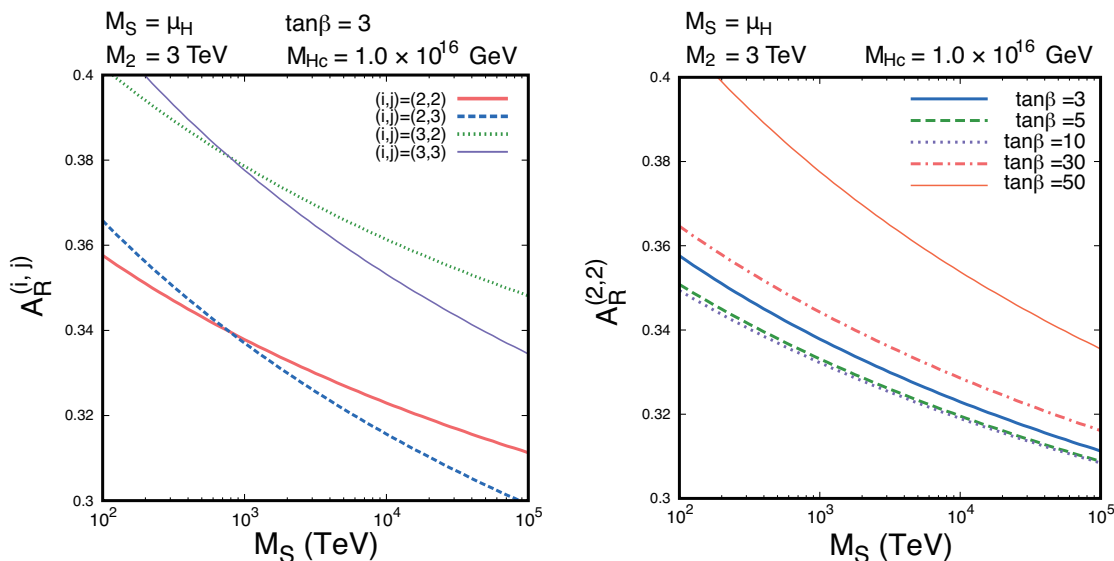
Figure 7. Left: $A_R^{(i,j)}$ with $(i,j) = (2,2)$, $(2,3)$, $(3,2)$, $(3,3)$ as functions of M_S. Here we set $\tan\beta = 3$, $M_S = \mu_H$, $M_2 = 3\,\mathrm{TeV}$, and $M_{H_C} = 1.0 \times 10^{16}\,\mathrm{GeV}$. Right: $A_R^{(2,2)}$ as functions of M_S. Same parameters as in the left graph are used except for $\tan\beta$. Each line corresponds to different values of $\tan\beta$ ($\tan\beta = 3, 5, 10, 30, 50$).
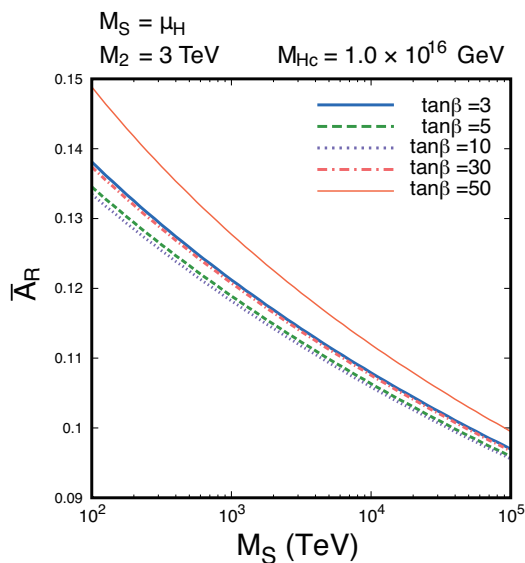

Figure 8. $\overline{A}_R$ as functions of M_S. Here we set $M_S = \mu_H$, $M_2 = 3\,\mathrm{TeV}$, and $M_{H_C} = 1.0 \times 10^{16}\,\mathrm{GeV}$. Each line shows different values of $\tan\beta$.

each $A_R^{(i,j)}$ is presented with $\tan\beta$ fixed to be $\tan\beta = 3$, while in the right graph the behavior of $A_R^{(2,2)}$ is shown for different values of $\tan\beta$ ($\tan\beta = 3, 5, 10, 30, 50$). Similarly, each line in figure 8 corresponds to $\overline{A}_R$ evaluated with various $\tan\beta$'s. It is found that the renormalization factors decrees as the SUSY scale increases, while their dependence on $\tan\beta$ is somewhat complicated. In addition, the left panel in figure 7 illustrates that the effects of the third generation Yukawa couplings are significant.

B Formulae for proton decay rate

In this section, we display the formulae for the partial decay widths of $p \to K^+ \bar{\nu}$ channels as well as the hadronic matrix elements which we need to evaluate the decay widths. Let us start with the matrix elements. We divide the derivation into two steps.[7] First, we express them in terms of the low-energy constants α_p and β_p by using the chiral perturbation techniques [71–73]:

$$\langle K^+ | \epsilon_{\alpha\beta\gamma}(u_L^\alpha d_L^\beta) s_L^\gamma | p \rangle = \frac{\beta_p}{\sqrt{2} f_\pi} \left(1 + \frac{D + 3F}{3} \frac{m_p}{M_B} \right) P_L u_p \,,$$

$$\langle K^+ | \epsilon_{\alpha\beta\gamma}(u_L^\alpha s_L^\beta) d_L^\gamma | p \rangle = \frac{\beta_p}{\sqrt{2} f_\pi} \left(\frac{2D}{3} \frac{m_p}{M_B} \right) P_L u_p \,,$$

$$\langle K^+ | \epsilon_{\alpha\beta\gamma}(u_R^\alpha d_R^\beta) s_L^\gamma | p \rangle = \frac{\alpha_p}{\sqrt{2} f_\pi} \left(1 + \frac{D + 3F}{3} \frac{m_p}{M_B} \right) P_L u_p \,,$$

$$\langle K^+ | \epsilon_{\alpha\beta\gamma}(u_R^\alpha s_R^\beta) d_L^\gamma | p \rangle = \frac{\alpha_p}{\sqrt{2} f_\pi} \left(\frac{2D}{3} \frac{m_p}{M_B} \right) P_L u_p \,, \tag{B.1}$$

where $f_\pi \simeq 92.2 \, \text{MeV}$ [64] is the pion decay constant and the baryon-meson couplings D and F are given as $D \simeq 0.80$ and $F \simeq 0.47$, respectively. m_p denotes the proton mass, while M_B represents the baryon mass parameter in the chiral Lagrangian, which we choose as $M_B \simeq (m_{\Sigma^0} + m_{\Lambda^0})/2$ with m_{Σ^0} and m_{Λ^0} the masses of Σ^0 and Λ^0, respectively. u_p is the four-component spinor wave function of proton, and P_L is the projection operator defined by $P_L \equiv (1 - \gamma_5)/2$. The low-energy constants α_p and β_p are defined as

$$\langle 0 | \epsilon_{\alpha\beta\gamma}(u_R^\alpha d_R^\beta) u_L^\gamma | p \rangle = \alpha_p P_L u_p \,,$$

$$\langle 0 | \epsilon_{\alpha\beta\gamma}(u_L^\alpha d_L^\beta) u_L^\gamma | p \rangle = \beta_p P_L u_p \,, \tag{B.2}$$

with $|0\rangle$ the vacuum state. Second, we determine the constants α_p and β_p. We extract them from the results of lattice simulations [74]:

$$\alpha_p = -0.0112 \pm 0.0012_{(\text{stat})} \pm 0.0022_{(\text{syst})} \, \text{GeV}^3 \,,$$

$$\beta_p = 0.0120 \pm 0.0013_{(\text{stat})} \pm 0.0023_{(\text{syst})} \, \text{GeV}^3 \,, \tag{B.3}$$

where they are evaluated at $\mu = 2 \, \text{GeV}$.

With the matrix elements and the effective operators in eq. (3.12), it is straightforward to derive the partial decay widths of the $p \to K^+ \bar{\nu}_\mu$ and $p \to K^+ \bar{\nu}_\tau$ channels. The result is

$$\Gamma(p \to K^+ \bar{\nu}_i) = \frac{m_p \alpha_2^4 |C_i|^2}{64\pi f_\pi^2 M_{H_C}^2 m_W^4 \sin^2 2\beta} \left(1 - \frac{m_K^2}{m_p^2} \right)^2 \,, \tag{B.4}$$

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[7]Calculation of the matrix elements is also conducted by using the direct method [69], in which the three-point correlation functions relevant for proton decay are directly computed on the lattice. Preliminary for the recent progress is reported in ref. [70].

with $(i = \mu, \tau)$ and

$$C_\mu = 2\beta_p F(M_2, M_S^2)\left\{1 + (D+F)\frac{m_p}{M_B}\right\}\overline{m}_s V_{us}^* \sum_{i=2,3} \overline{m}_{u_i} V_{u_i d} V_{u_i s} e^{i\varphi_i} A_R^{(i,2)}, \qquad \text{(B.5)}$$

$$C_\tau = 2\beta_p F(M_2, M_S^2)\left\{1 + (D+F)\frac{m_p}{M_B}\right\}\overline{m}_b V_{ub}^* \sum_{i=2,3} \overline{m}_{u_i} V_{u_i d} V_{u_i s} e^{i\varphi_i} A_R^{(i,3)}$$

$$- \alpha_p \frac{\overline{m}_t^2 \overline{m}_\tau V_{tb}^* e^{i\varphi_1}}{m_W^2 \sin 2\beta} F(\mu_H, M_S^2)\overline{A}_R \times$$

$$\times \left\{\overline{m}_d V_{ud} V_{ts}\left(1 + \frac{D+3F}{3}\frac{m_p}{M_B}\right) + \overline{m}_s V_{us} V_{td}\frac{2D}{3}\frac{m_p}{M_B}\right\}. \qquad \text{(B.6)}$$

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