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Analytic approaches to anisotropic holographic superfluids

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Abstract: We construct an analytic solution of the Einstein-SU(2)-Yang-Mills system as the holographic dual of an anisotropic superfluid near its critical point, up to leading corrections in both the inverse Yang-Mills coupling and a symmetry breaking order parameter. We have also calculated the ratio of shear viscosity to entropy density in this background, and shown that the universality of this ratio is lost in the broken symmetry direction. The ratio displays a scaling behavior near the critical point with critical exponent $\beta = 1$, at the leading order in the double expansion.

Keywords: Gauge-gravity correspondence, AdS-CFT Correspondence, Holography and condensed matter physics (AdS/CMT)

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1 Introduction

Gauge/gravity duality has led to many useful insights into strongly coupled field theories. Recently, fluid/gravity duality has been widely studied, providing useful information about conformal fluid dynamics, the effectively long wavelength description of conformal field theory. Many crucial quantities characterizing conformal fluids can be obtained as real-time Green's functions in dual gravitational theories [1, 3].

The most celebrated example from holographic fluid dynamics is the ratio of shear viscosity η to entropy density s, $\frac{\eta}{s} = \frac{1}{4\pi}$ [4–7]. The ratio seems to be universal for theories with weakly coupled gravity duals.[1] However, in recent studies of anisotropic conformal fluids [11, 13], the universality of the ratio turns out to be violated [14, 17]. These studies have considered an exact solution of the bulk Einstein-SU(2)-Yang-Mills action[2] for an AdS black brane in 5 dimensions with nonzero chemical potential. Corresponding to the chemical potential, the temporal component of the gauge potential is proportional to σ_3 of the SU(2) gauge group. The boundary metric enjoys SO(3) global symmetry (symmetry under spatial rotations). At high temperature (equivalently small chemical potential μ), the system stays in the isometric phase. However at a certain chemical potential $\mu = \mu_c$, the SO(3) symmetry is *spontaneously* broken to SO(2) because one of the spatial components of the Yang-Mills field develops a non-trivial zero mode (as a solution of the linearized

[1]This universality can be violated when string effects or quantum effects are to be taken into account in dual gravity [8–10].

[2]See [2, 20–22] for pioneering works on connection between Einstein-SU(2)-Yang-Mills and p-wave holographic superfluids.

Yang-Mills field equations) and it is normalizable solution. It turns out that in the region of $\mu > \mu_c$, this mode condenses and there is a new anisotropic superfluid phase.

As long as we are looking at a solution with SO(3) symmetry, the universality of the ratio of entropy density and shear viscosity holds. *This is because the shear viscosity depends on gravitational perturbations in tensor modes of* SO(3) *in the dual gravity.* Each tensor mode satisfies a massless scalar field equation decoupled from the others, which ensures universality. The anisotropic symmetry-broken phase provides an anisotropic shear viscosity to entropy density ratio. The reason is that once SO(3) symmetry is broken to SO(2), the gravitational wave modes in the broken-symmetry directions are no longer SO(2) tensor modes. They are not decoupled from other fields and in fact interact with Yang-Mills fields. Therefore, in this case the gravitational modes do not display universal behavior.

Near the critical point($\mu = \mu_c$), the phase transition is expected to depend on $\alpha^2 \equiv \frac{\kappa_5^2}{g^2}$ [14], where κ_5 is 5-dimensional gravity constant and g is Yang-Mills coupling. If α is less than a certain critical value α_{crit}, the phase transition becomes second order. Near the second order phase transition, the ratio displays a scaling behavior with some critical exponent β,

$$1 - 4\pi\frac{\eta}{s} \sim \left(1 - \frac{T}{T_c}\right)^{\beta}, \tag{1.1}$$

where the value $\beta = 1.00 \pm 0.03$ has been calculated numerically, taking into account the back reaction to the background metric. For $\alpha > \alpha_{crit}$, the phase transition is first order. Unfortunately, the large-α region has not been explored very well numerically, due to technical difficulties. Another interesting remark from [14] is that the critical exponent, β does not seem to depend on α.

To examine such properties rigorously an analytic approach is needed. Our starting point for such an approach is a zero-mode solution at zero gravitational coupling, which is known exactly at the critical point [12]. In this note, we perturbatively analyze the properties of the anisotropic fluid near the critical point. We obtain the back reaction to the space time metric and also solve linearized equations of the gravitational perturbations and Yang-Mills fields from the back-reacted metric, using a double expansion of inverse Yang-Mills coupling α^2 and $\varepsilon\tilde{D}_1$. ε is a dimensionless small parameter and $\tilde{D}_1$ is an SO(3) symmetry-breaking scale appearing in the anisotropic part of the Yang-Mills field. Similar perturbative expansions for different holographic models have been discussed by various authors [15, 16, 18, 19]. However unlike those works, we consider both an anisotropy and the gravitational back reaction from the Yang-Mills fields.

In our double expansion, the nontrivial leading order turns out to be $O(\alpha^2\varepsilon^2)$. We also get the shear viscosity and entropy density ratio up to this order. Nonuniversality of the ratio of shear viscosity to entropy density shows up in the directions of broken symmetry while the ratio in the unbroken direction displays the expected universality. In our perturbative analysis we do not see the first order phase transition because α is small in the perturbative regime. Near the critical point, our solution also presents the scaling behavior as eq. (1.1) and it turns out that the critical exponent $\beta = 1$ up to the leading order correction, which is consistent with the numerical result in [14]. In general, our perturbative results agree with and complement the numerical results in [14].

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2 Holographic setup and large coupling expansion

We consider the Einstein-SU(2) Yang-Mills system in asymptotically AdS_5 spacetime. The action is

$$S = \int d^5x\sqrt{-G}\left(\frac{1}{\kappa_5^2}\left(R+\frac{12}{L^2}\right)-\frac{1}{4g^2}F^a_{MN}F^{aMN}\right), \tag{2.1}$$

where $M,\ N\ldots$ are 5-dimensional space-time indices, $a\ldots$ are SU(2) indices and g is the Yang-Mills coupling. We conventionally choose $L=1$. The Yang-Mills field strength F^a_{MN} is given by

$$F^a_{MN} = \partial_M A^a_N - \partial_N A^a_M - \epsilon^{abc}A^b_M A^c_N, \tag{2.2}$$

where ϵ^{abc} is anti-symmetric tensor with $\epsilon^{123}=1$. The equations of motion from the action are obtained as

$$W_{MN} \equiv R_{MN} + 4G_{MN} - \kappa_5^2\left(T_{MN} - \frac{1}{3}T^P_P G_{MN}\right) = 0, \tag{2.3}$$

$$Y^{aN} \equiv \nabla_M F^{aMN} - \epsilon^{abc}A^b_M F^{cMN} = 0, \tag{2.4}$$

where T_{MN} is energy-momentum tensor, of which form is

$$T_{MN} = \frac{1}{g^2}\left(F^a_{MP}F^{Pa}_N - \frac{1}{4}F_{PQa}F^{PQa}G_{MN}\right). \tag{2.5}$$

Our ansatz for the metric and Yang-Mills field are given by

$$\begin{aligned} A &= \phi(r)\tau^3 dt + \omega(r)\tau^1 dx, \\ ds^2 &= -N(r)\sigma^2(r)dt^2 + \frac{dr^2}{N(r)} + r^2 f^{-4}(r)dx^2 + r^2 f^2(r)\left(dy^2+dz^2\right), \end{aligned} \tag{2.6}$$

where $\tau^a = \frac{s^a}{2}$ and s^a are Pauli-matrices. The Yang-Mills field equations of motion in terms of the above ansatz are

$$\begin{aligned} \frac{r^2 Y^1_y}{f^4(r)N(r)} &= \omega''(r) + \left(\frac{1}{r}+\frac{\sigma'(r)}{\sigma(r)}+\frac{N'(r)}{N(r)}+4\frac{f'(r)}{f(r)}\right)\omega'(r) + \frac{\phi^2(r)\omega(r)}{N^2(r)\sigma^2(r)} = 0, \\ \sigma^2(r)Y^3_t &= \phi''(r) + \left(\frac{3}{r}-\frac{\sigma'(r)}{\sigma(r)}\right)\phi'(r) - \frac{f^4(r)\omega^2(r)}{r^2N(r)}\phi(r) = 0, \end{aligned} \tag{2.7}$$

and the Einstein equations are

$$\begin{aligned} \frac{2W_{tt}}{N^2(r)\sigma^2(r)} &= 2\frac{\sigma''(r)}{\sigma(r)} + \frac{6}{r}\frac{\sigma'(r)}{\sigma(r)} + \frac{N''(r)}{N(r)} + \frac{3}{r}\frac{N'(r)}{N(r)} - \frac{8}{N(r)} + 3\frac{\sigma'(r)}{\sigma(r)}\frac{N'(r)}{N(r)} \\ &\quad -\frac{2\kappa_5^2}{3g^2}\left(\frac{f^4(r)\omega'^2(r)}{r^2} + \frac{2\phi'^2(r)}{N(r)\sigma^2(r)} + \frac{2f^4(r)\phi^2(r)\omega^2(r)}{r^2\sigma^2(r)N^2(r)}\right) = 0 \end{aligned} \tag{2.8}$$

$$\begin{aligned} \bar{W} &\equiv \frac{2r}{\sigma^2(r)N(r)}W_{tt} + 2r^2N(r)W_{rr} \\ &= -12r\frac{f'^2(r)}{f^2(r)} + 6\frac{\sigma'(r)}{\sigma(r)} - \frac{2\kappa_5^2 f^4(r)}{g^2 r}\left(\omega'^2(r) + \frac{\phi^2(r)\omega^2(r)}{N^2(r)\sigma^2(r)}\right) = 0, \end{aligned} \tag{2.9}$$

$$\tilde{W} \equiv 2W_{yy} + f^6(r)W_{xx} \tag{2.10}$$

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$$
\begin{aligned}
&= 2 - \frac{4r^2}{N(r)} + r\frac{N'(r)}{N(r)} + r\frac{\sigma'(r)}{\sigma(r)} + \frac{r^2\kappa_5^2\phi'^2(r)}{3g^2\sigma^2(r)N(r)} = 0, \\
\frac{f^4(r)W_{xx}}{2r^2N(r)} &= \frac{f''(r)}{f(r)} + \left(\frac{\sigma'(r)}{\sigma(r)} + \frac{3}{r} + \frac{N'(r)}{N(r)}\right)\frac{f'(r)}{f(r)} - \frac{1}{r^2} + \frac{2}{N(r)} - \frac{N'(r)}{2rN(r)} - \frac{\sigma'(r)}{2r\sigma(r)} \qquad (2.11) \\
&\quad - \frac{\kappa_5^2}{g^2}\left(\frac{\omega'^2(r)f^4(r)}{3r^2} - \frac{\phi^2(r)\omega^2(r)f^4(r)}{3r^2\sigma^2(r)N^2(r)} + \frac{\phi'^2(r)}{6N(r)\sigma^2(r)}\right) - \frac{f'^2(r)}{f^2(r)} = 0 \\
W_{yy} &= W_{zz}. \qquad (2.12)
\end{aligned}
$$

A known exact solution of the equations of motion is the AdS charged-black-brane solution given by

$$
\begin{aligned}
\phi(r) &= \tilde{\mu}\left(1 - \frac{r_h^2}{r^2}\right), \quad \omega(r) = 0, \qquad (2.13) \\
\sigma(r) &= f(r) = 1 \text{ and } N(r) = N_0(r) \equiv r^2 - \frac{m}{r^2} + \frac{2\tilde{\mu}^2\alpha^2 r_h^4}{3r^4},
\end{aligned}
$$

where $\tilde{\mu}$ is chemical potential, r_h is the black brane horizon and $m \equiv r_h^4 + \frac{2\mu^2\alpha^2 r_h^2}{3}$. In the infinite Yang-Mills coupling limit as $g \to \infty$, the last term in $N(r)$ vanishes and the solution becomes uncharged.

2.1 Large coupling expansion and its leading order corrections

In this section, we develop corrections to the metric and Yang-Mills field perturbatively in a double expansion in $\varepsilon\tilde{D}_1$ and $\alpha^2 \equiv \frac{\kappa_5^2}{g^2}$. ε is a dimensionless small parameter and $\tilde{D}_1$ is the SO(3) rotational symmetry-breaking order parameter. We choose the horizon of the black brane to be conventionally located at $r = 1$ by scaling $r \to r_h r$ and $\{t, x, y, z\} \to \frac{1}{r_h}\{t, x, y, z\}$ and defining a new chemical potential $\mu \equiv \frac{\tilde{\mu}}{r_h}$. The equations of motion enjoy a certain scaling symmetry [11, 13]. By means of these rescalings, we can choose the asymptotic values of $\sigma(r = \infty) = 1$ and $f(r = \infty) = 1$ at the large r boundary where the spacetime becomes asymptotically AdS_5. The value of the chemical potential in the dual boundary field theory is taken to be $\mu = 4$ at the phase transition point. To obtain corrections, we expand any fields $a(r)$ appearing in the ansatz (2.6) as

$$
a(r) = a_0(r) + \varepsilon a_1(r) + \varepsilon^2 a_2(r)\dots \qquad (2.14)
$$

Each term in the expression can in turn be expanded as

$$
a_i(r) = a_{i,0}(r) + \alpha^2 a_{i,2} + \alpha^4 a_{i,4}(r)\dots \qquad (2.15)
$$

The zeroth-order solution in ε is given in eq. (2.13), where only N_0 contains a subleading correction of order α^2 in the sense of the above expansion. $N_{0,2} = \frac{32}{3}\left(\frac{1}{r^4} - \frac{1}{r^2}\right)$ and the higher-order terms in α^2 vanish, $N_{0,i} = 0$ for $i = 4, 6\dots$. The detailed computations of the nontrivial leading order corrections to metric and Yang-Mills field are given in appendix A. Here, we briefly list the leading-order back-reaction corrections to the metric, which are given by

$$
\sigma(r) = 1 - \varepsilon^2\alpha^2\frac{2\tilde{D}_1^2}{9(1+r^2)^3}, \quad f(r) = 1 - \varepsilon^2\alpha^2\frac{\tilde{D}_1^2(1-2r^2)}{18(1+r^2)^4} \qquad (2.16)
$$

$$\text{and}\quad N(r) = r^2 - \frac{1}{r^2} + \frac{32\alpha^2}{3}\left(\frac{1}{r^4} - \frac{1}{r^2}\right) - \varepsilon^2\alpha^2\frac{4\tilde{D}_1^2}{9r^2}\left(\frac{1+2r^2}{r^2(1+r^2)^3} - \frac{3r^2}{2(1+r^2)^2} + \frac{281}{560}\left(1 - \frac{1}{r^2}\right)\right).$$

Any subleading corrections to the Yang-Mills field in α^2 would not contribute to the leading back-reaction corrections to the metric (Our aim is to get metric corrections up to $O(\alpha^2\varepsilon^2)$). Therefore, we obtain the Yang-Mills field solutions up to $\phi_{i,0}$ and $\omega_{i,0}$ only. These are given by

$$\omega(r) = \varepsilon\frac{\tilde{D}_1 r^2}{(r^2+1)^2} + O(\varepsilon^2), \tag{2.17}$$

$$\phi(r) = 4\left(1 - \frac{1}{r^2}\right) + \frac{\varepsilon^2\tilde{D}_1^2}{4}\left(\frac{(1+2r^2)}{3r^2(1+r^2)^3} - \frac{1}{8} + \frac{281}{1680}\left(1 - \frac{1}{r^2}\right)\right) + O(\varepsilon^3). \tag{2.18}$$

The black brane temperature is modified by the leading corrections to

$$T = \frac{1}{\pi}\left(1 - \frac{16}{3}\alpha^2 + \frac{17}{1260}\tilde{D}_1^2\varepsilon^2\alpha^2\right), \tag{2.19}$$

where $T_c \equiv \frac{1}{\pi}\left(1 - \frac{16}{3}\alpha^2\right)$ is the critical temperature at the phase transition from the isotropic phase to the anisotropic phase. The black brane entropy is

$$S = \frac{2\pi}{\kappa_5^2}V_3, \tag{2.20}$$

where V_3 is spatial coordinate volume of the boundary space-time, $V_3 = \int dxdydz$, in this rescaled coordinate.

3 Anisotropy of shear viscosities

In this section, we calculate the ratio of shear viscosity to entropy density via the Kubo formula, by considering fluctuations h_{MN} and δA_M^a around the background metric and the background Yang-Mills field, respectively. We choose the gauge $h_{Mr} = \delta A_r^a = 0$. In the anisotropic phase, the bulk gravity system enjoys residual SO(2) and Z_2 symmetries. The modes may be decomposed according to their SO(2) representations as

- Tensor modes in SO(2): h_{yz}; $h_{yy} - h_{zz}$,
- Vector modes in SO(2): h_{yt},δA_y^3; h_{xy},δA_y^1,δA_y^2; h_{zt}, δA_z^3; h_{xz},δA_z^1,δA_z^2,
- Scalar Modes in SO(2): h_{tt},$h_{yy} + h_{zz}$,h_{xx},h_{xt},δA_t^a, δA_x^a,

where each decoupled mode is categorized by semicolons. h_{yz} is totally decoupled from any other modes and satisfies a massless scalar field equation showing universality. However h_{xy} interacts with δA_y^1 and δA_y^2, leading to nonuniversal behavior. In the following, we will obtain solutions for h_{yz} and h_{xy} and show this explicitly. Other modes can be calculated by the similar methods.

3.1 Universality of $\frac{\eta_{yz}}{s}$

In this subsection, we calculate the ratio of shear viscosity to entropy density for the shear mode h_{yz}, using the double expansion that we introduced in the previous section. We consider fluctuations of the Yang-Mills field and metric fields around the background metric (2.16) and obtain perturbative corrections up to $O(\varepsilon^2\alpha^2)$. We only consider time dependent fluctuations with frequency ν and use small frequency expansion up to first subleading order in ν. Even in the presence of a nonzero symmetry breaking parameter $\tilde{D}_1$, the rotational symmetry in $y-z$ plane is not broken and the ratio $\frac{\eta_{yz}}{s}$ is universal. In the following, as a warm up, we will explicitly calculate this ratio to be $\frac{1}{4\pi}$ up to leading-order corrections in $\alpha^2\varepsilon^2$. To show this, we begin with the linearized equation of motion of $h_{yz}(r) \equiv r^2 f^2(r)\Phi(r,t)$,

$$0 = \Phi''_\nu(r) + \left(\frac{1}{r} + \frac{4r}{N(r)} - \frac{\alpha^2 r\phi'^2(r)}{3\sigma^2(r)N(r)}\right)\Phi'_\nu(r) + \frac{\nu^2\Phi_\nu(r)}{N^2(r)\sigma^2(r)}, \tag{3.1}$$

where the prime denotes the radial derivative. For the field $\Phi(r)$, we have used the Fourier transform from real time to frequency, as

$$\Phi(r,t) = \int_{-\infty}^{\infty} e^{-i\nu t}\Phi_\nu(r)d\nu \tag{3.2}$$

The near horizon behavior of $\Phi_\nu(r)$ should be a purely ingoing solution

$$\Phi_\nu(r) \sim \left(1-\frac{1}{r}\right)^{-i\frac{\nu}{4}\left(1+\frac{16}{3}\alpha^2-\frac{17}{1260}\varepsilon^2\alpha^2\tilde{D}_1^2\right)+O(\varepsilon^k\alpha^l)}, \tag{3.3}$$

where k and l are integers with $k>2$ or $l>2$. With this boundary condition, the solution $\Phi_\nu(r)$ is obtained as

$$\Phi_\nu(r) = \left(\frac{N(r)}{r^2}\right)^{-i\frac{\nu}{4}\left(1+\frac{16}{3}\alpha^2-\frac{17}{1260}\varepsilon^2\alpha^2\tilde{D}_1^2\right)+O(\varepsilon^k\alpha^l)} F(\varepsilon,\alpha^2), \tag{3.4}$$

where

$$F(\varepsilon,\alpha^2) = \sum_{i,j=0}^{\infty}\Phi_{i,2j}(r)\varepsilon^i\alpha^{2j}. \tag{3.5}$$

Each $\Phi_{i,2j}$ and its near-AdS boundary expansion are given in appendix B. Here, we briefly discuss the near boundary expansion of the solution to get $\frac{\eta_{yz}}{s}$. Defining the boundary value $\tilde{\Phi} \equiv \Phi_\nu(\infty)$, $\Phi_\nu(r)$ can be expanded as

$$\Phi_\nu(r\to\infty) = \tilde{\Phi} + \frac{i\nu}{4r^4}\tilde{\Phi} + O(r^i\nu^j\varepsilon^k\alpha^l) \tag{3.6}$$

in the large r limit (see eq. (B.6) in appendix B), where $i<-4$, $j>1$,$k>2$ or $l>2$. Using the prescription to get retarded green's function in [14], we get

$$G^R_{yz,yz}(\nu,\vec{k}=0) = \frac{-i\nu}{2\kappa_5^2} + O(\nu^2) \tag{3.7}$$

The shear viscosity in $y-z$ direction is given by

$$\eta_{yz} \equiv \lim_{\nu\to 0} \frac{1}{2\nu i}[G^{R\star}_{yz,yz} - G^{R}_{yz,yz}] = \frac{1}{2\kappa_5^2}, \tag{3.8}$$

where star indicates complex conjugate. Using the entropy of the black brane (2.20), the ratio of shear viscosity to entropy density is obtained as

$$\frac{\eta_{yz}}{s} = \frac{1}{4\pi}. \tag{3.9}$$

This value turns out to be universal up to $O(\varepsilon^2\alpha^2)$.

3.2 Nonuniversality of $\frac{\eta_{xy}}{s}$

We start with a set of equations with $h_{xy} \equiv r^2 f^2(r)\Psi(r,t)$, δA^1_y and δA^2_y. The superscripts on δA fields note SU(2) indices and subscripts do space-time indices. We also solve these equations in the frequency space by Fourier transform as in the previous subsection. The equations in the frequency space are given by

$$\begin{aligned}
0 &= \Psi''(r) + \left(\frac{1}{r} + \frac{4r}{N(r)} + \frac{6f'(r)}{f(r)} - \frac{r\alpha^2\phi'^2(r)}{3N(r)\sigma^2(r)}\right)\Psi'(r) + \frac{\nu^2\Psi(r)}{N^2(r)\sigma^2(r)} \qquad (3.10)\\
&\quad + \frac{2\alpha^2}{r^2 f^2(r)}\left(\omega'(r)\delta A^{1\prime}_y(r) - \frac{\omega(r)\phi^2(r)\delta A^1_y(r)}{N^2(r)\sigma^2(r)} + \frac{i\nu\omega(r)\phi(r)\delta A^2_y(r)}{N^2(r)\sigma^2(r)}\right),\\
0 &= \delta A^{1\prime\prime}_y(r) + \left(\frac{1}{r} - \frac{2f'(r)}{f(r)} + \frac{N'(r)}{N(r)} + \frac{\sigma'(r)}{\sigma(r)}\right)\delta A^{1\prime}_y(r) + \left(\frac{\nu^2+\phi^2(r)}{N^2(r)\sigma^2(r)}\right)\delta A^1_y(r) \qquad (3.11)\\
&\quad - f^6(r)\omega'(r)\Psi'(r) - \frac{2i\nu\phi(r)\delta A^2_y(r)}{N^2(r)\sigma^2(r)},\\
0 &= \delta A^{2\prime\prime}_y(r) + \left(\frac{1}{r} - \frac{2f'(r)}{f(r)} + \frac{N'(r)}{N(r)} + \frac{\sigma'(r)}{\sigma(r)}\right)\delta A^{2\prime}_y(r) + \left(\frac{\nu^2+\phi^2(r)}{N^2(r)\sigma^2(r)}\right)\delta A^2_y \qquad (3.12)\\
&\quad - \frac{f^4(r)\omega^2(r)}{r^2 N(r)}\delta A^2_y + \frac{i\nu\phi(r)}{N^2(r)\sigma^2(r)}(-f^6(r)\omega(r)\Psi(r) + 2\delta A^1_y(r)).
\end{aligned}$$

We expand each field with the same fashion as eq. (3.4):

$$\begin{aligned}
\Psi(r) &= \left(\frac{N(r)}{r^2}\right)^{-i\frac{\nu}{4}\left(1+\frac{16}{3}\alpha^2-\frac{17}{1260}\varepsilon^2\alpha^2\tilde{D}_1^2\right)+O(\varepsilon^k\alpha^l)} G(\varepsilon,\alpha^2), \qquad (3.13)\\
\delta A^1_y(r) &= \left(\frac{N(r)}{r^2}\right)^{-i\frac{\nu}{4}\left(1+\frac{16}{3}\alpha^2-\frac{17}{1260}\varepsilon^2\alpha^2\tilde{D}_1^2\right)+O(\varepsilon^k\alpha^l)} H(\varepsilon,\alpha^2),\\
\delta A^2_y(r) &= \left(\frac{N(r)}{r^2}\right)^{-i\frac{\nu}{4}\left(1+\frac{16}{3}\alpha^2-\frac{17}{1260}\varepsilon^2\alpha^2\tilde{D}_1^2\right)+O(\varepsilon^k\alpha^l)} I(\varepsilon,\alpha^2),
\end{aligned}$$

where the functions $G(\varepsilon,\alpha^2)$, $H(\varepsilon,\alpha^2)$ and $I(\varepsilon,\alpha^2)$ are expanded as

$$G(\varepsilon,\alpha^2) = \sum_{i,j=0}^{\infty} \Psi_{i,2j}(r)\varepsilon^i\alpha^{2j}, \tag{3.14}$$

$$H(\varepsilon,\alpha^2) = \sum_{i,j=0}^{\infty} \delta A^1_{i,2j}(r)\varepsilon^i\alpha^{2j},$$
$$\text{and} \quad I(\varepsilon,\alpha^2) = \sum_{i,j=0}^{\infty} \delta A^2_{i,2j}(r)\varepsilon^i\alpha^{2j}.$$

The solutions of the equations are listed in appendix C. Corrections to the Yang-Mills fields which are subleadings in α^2 do not contribute to the leading order corrections of the shear viscosity, so we get $\delta A^1_{i,0}(r)$ and $\delta A^2_{i,0}(r)$ only (see eq. (3.10)). We also use small frequency expansion as in the last subsection, and obtain the solutions up to $O(\nu)$. We specify purely ingoing boundary conditions for the fields, the form of which are the same as eq. (3.3). Here, we discuss the near boundary expansion of $\Psi(r)$, obtaining the retarded correlator of h_{xy} and the shear viscosity to entropy density ratio, $\frac{\eta_{xy}}{s}$.

The near-AdS boundary expansions of $\Psi(r)$, δA^1_y and δA^2_y are given by

$$\delta A^1_y(r=\infty) = -\frac{i\varepsilon\nu}{192}\left(6\tilde{A}^{(0)}_{1,0} - 22\bar{A}^{(0)}_{1,0} - \tilde{D}_1\psi^{(0)}_{0,0}\right) + O(\varepsilon\nu^2), \tag{3.15}$$
$$\delta A^2_y(r=\infty) = \frac{i\varepsilon\nu}{192}\left(22\tilde{A}^{(0)}_{1,0} - 6\bar{A}^{(0)}_{1,0} - 11\tilde{D}_1\psi^{(0)}_{0,0}\right) + O(\varepsilon\nu^2),$$

and

$$\begin{aligned}\Psi(r) = {} & (\psi^{(0)}_{0,0} + \varepsilon\psi^{(0)}_{1,0} + \varepsilon^2\psi^{(0)}_{2,0}) + \alpha^2(\psi^{(0)}_{0,2} + \varepsilon\psi^{(0)}_{1,2} + \varepsilon^2\psi^{(0)}_{2,2}) \\ & + \nu(\psi^{(1)}_{0,0} + \varepsilon\psi^{(1)}_{1,0} + \varepsilon^2\psi^{(1)}_{2,0}) + \nu\alpha^2(\psi^{(1)}_{0,2} + \varepsilon\psi^{(1)}_{1,2} + \varepsilon^2\psi^{(1)}_{2,2}) \\ & + \frac{\nu}{r^4}\left(\frac{i}{4}(\psi^{(0)}_{0,0} + \varepsilon\psi^{(0)}_{1,0} + \varepsilon^2\psi^{(0)}_{2,0}) + \frac{i\alpha^2}{4}(\psi^{(0)}_{0,2} + \varepsilon\psi^{(0)}_{1,2} + \varepsilon^2\psi^{(0)}_{2,2})\right. \\ & \left. + \frac{i\alpha^2\varepsilon^2\tilde{D}_1}{192}(5\tilde{A}^{(0)}_{1,0} - 11\bar{A}^{(0)}_{1,0}) + O(\nu^2\varepsilon^3\alpha^3)\right).\end{aligned} \tag{3.16}$$

The SO(3) symmetry is broken *spontaneously*, so the Yang-Mills field should not provide any source terms to the dual field theory system. Therefore, $A^1_y(r)$ and $A^2_y(r)$ should become normalizable modes of the solutions, then we have

$$6\tilde{A}^{(0)}_{1,0} - 22\bar{A}^{(0)}_{1,0} - \tilde{D}_1\psi^{(0)}_{0,0} = 0, \quad \text{and} \quad 22\tilde{A}^{(0)}_{1,0} - 6\bar{A}^{(0)}_{1,0} - 11\tilde{D}_1\psi^{(0)}_{0,0} = 0. \tag{3.17}$$

The solutions of these equations are

$$\tilde{A}^{(0)}_{1,0} = \frac{59}{112}\tilde{D}_1\psi^{(0)}_{0,0}, \quad \text{and} \quad \bar{A}^{(0)}_{1,0} = \frac{11}{112}\tilde{D}_1\psi^{(0)}_{0,0}. \tag{3.18}$$

Using eq. (3.18) and defining $\Psi(\infty) \equiv \Psi$ as a boundary value of $\Psi(r)$, the near boundary expansion of $\Psi(r)$ is given by

$$\Psi(r\to\infty) = \Psi + \frac{i\nu}{4r^4}\Psi + \varepsilon^2\alpha^2\nu\frac{29i\tilde{D}_1^2\Psi}{3584r^4}. \tag{3.19}$$

The prescription of the retarded green's function in [14] provides

$$G^R_{xy,xy}(\nu,\vec{k}=0) = \frac{-i\nu}{2\kappa_5^2}\left(1 + \frac{29}{896}\varepsilon^2\alpha^2\tilde{D}_1^2\right) + O(\nu^2) \tag{3.20}$$

and the shear viscosity is calculated as

$$\eta_{xy} \equiv \lim_{\nu\to 0}\frac{1}{2\nu i}[G^{R\star}_{xy,xy} - G^{R}_{xy,xy}] = \frac{1}{2\kappa_5^2}\left(1+\frac{29}{896}\varepsilon^2\alpha^2\tilde{D}_1^2\right). \tag{3.21}$$

Using entropy of the black brane (2.20), the ratio of shear viscosity and entropy density obtained as

$$\frac{\eta_{xy}}{s} = \frac{1}{4\pi}\left(1+\frac{29}{896}\varepsilon^2\alpha^2\tilde{D}_1^2\right). \tag{3.22}$$

Therefore, the shear viscosity and entropy ratio in $x-y$ direction is not universal, and we have shown this up to non-trivial leading order correction in α and ε. Using the temperature of the black brane, eq. (3.22) can be written as

$$1-4\pi\frac{\eta_{xy}}{s} = \frac{1305\pi T_c}{544}\left(1-\frac{T}{T_c}\right)^{\beta}, \tag{3.23}$$

where $\beta = 1$. It is also shown that the critical exponent $\beta = 1$ up to corrections of order $\varepsilon^2\alpha^2$, near the phase transition point $T = T_c$.

For the final remark, we note that non-universalities of the shear viscosities are only valid for normal fluids. As followed by the discussion in [23], the dissipation parts of the boundary energy momentum tensor may be given by

$$T^{\alpha\beta}_{diss} = -\eta^{\alpha\beta\gamma\delta}\partial_\gamma u_\delta - \tilde{\eta}^{\alpha\beta\gamma\delta}\partial_\gamma v_\delta, \tag{3.24}$$

where η and $\tilde{\eta}$ are viscosity tensors for normal fluids and superfluids and u_α and v_α are their velocities respectively. The velocity of superfluids will be formally given by $v_\alpha = \partial_\alpha\varphi$, where φ is an order parameter in the superfluid phase. In our case, however, the anisotropic order parameter, $\omega(r)$ does not depend on the boundary coordinate at all and for our computations of the shear viscosities, there are no fluctuations around it, i.e. $\delta A^1_x = 0$. Therefore, any boundary derivatives on the superfluid order parameter will vanish and which does not contribute to the shear viscosities.

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A Leading order solutions

In this section, we solve eq. (2.3) and eq. (2.4) to get leading order back reaction to the metric by symmetry breaking order parameter in Yang-Mills field, $\tilde{D}_1$. Using metric and Yang-Mills fields ansatz (2.6) and their expansions (2.14) and (2.15), we obtain the equations order by order in ε and α. Since the aim is to get leading back reaction to the metric, we get Yang-Mills field solutions up to the zeroth order in α and the first order in ε for $\omega(r)$ but the second order in ε for $\phi(r)$, both of which provide first leading order corrections to the metric correctly. The first order Yang-Mills equations in ε are given by

$$0 = \phi_1''(r) + \frac{3}{r}\phi_1'(r) - \frac{8}{r^3}\sigma_1'(r), \tag{A.1}$$

$$0 = \omega_1''(r) + \left(\frac{1}{r} + \frac{2(r^4+1) + r^3 N_0'(r)}{r(r^4-1) + r^3 N_0(r)}\right)\omega_1'(r) + \frac{16\omega_1(r)}{(r^2+1+\frac{r^2 N_0(r)}{r^2-1})^2}, \tag{A.2}$$

and the first order Einstein equations in ε are

$$0 = \sigma_1'(r), \tag{A.3}$$

$$0 = 2N_1(r) + rN_1'(r) + \frac{64\alpha^2}{3r^4}\left(\frac{r^3}{4}\phi_1'(r) - 2\sigma_1(r)\right), \tag{A.4}$$

$$\begin{aligned} 0 &= 2rN_1(r) + r^2N_1'(r) + 2(1-5r^4)f_1'(r) + 2r(1-r^4)f_1''(r) \\ &\quad - \frac{16\alpha^2}{3r^3(1+r^2)}\left(-2r^2(2N_1(r) + rN_1'(r)) - 8r(1+r^2-2r^4)f_1'(r) - r^3(1+r^2)\phi_1'(r)\right). \end{aligned} \tag{A.5}$$

We expand every field appearing in above equations using eq. (2.15). With this expansion, the solutions of metric are trivial, which are given by

$$\begin{aligned} \sigma_{1,0}(r) &= \tilde{\sigma}_{1,0}, & \sigma_{1,2}(r) &= 0, \\ f_{1,0}(r) &= \tilde{f}_{1,0}, & f_{1,2}(r) &= 0, \\ N_{1,0}(r) &= \frac{\tilde{N}_{1,0}}{r^2}, & N_{1,2}(r) &= \frac{8}{3}\tilde{C}_1\left(\frac{1}{r^4} - \frac{\tilde{N}_{1,2}}{r^2}\right). \end{aligned} \tag{A.6}$$

where we set $\tilde{\sigma}_{1,0} = \tilde{f}_{1,0} = 0$ for the boundary values $\sigma(\infty) = f(\infty) = 1$. $\tilde{N}_{1,0} = 0$ and $\tilde{N}_{1,2} = 1$ for the space-time has its horizon at $r = 1$. The Yang-Mills fields equations (A.2) up to $O(\alpha^2)$ are given by

$$\begin{aligned} 0 &= \omega_{1,0}''(r) + \frac{1+3r^4}{r(r^4-1)}\omega_{1,0}'(r) + \frac{16\omega_{1,0}(r)}{(r^2+1)^2}, \\ 0 &= \omega_{1,2}''(r) + \frac{1+3r^4}{r(r^4-1)}\omega_{1,2}'(r) + \frac{16\omega_{1,2}(r)}{(r^2+1)^2} + \frac{128\tilde{D}_1(1+9r^2-2r^4)}{3r^2(1+r^2)^5}. \end{aligned} \tag{A.7}$$

The Yang-Mills field solutions are solved as

$$\omega_{1,0}(r) = \frac{\tilde{D}_1 r^2}{(r^2+1)^2}, \tag{A.8}$$

$$\phi_{1,0}(r) = \frac{\tilde{C}_1}{2}\left(1 - \frac{1}{r^2}\right), \tag{A.9}$$

$$\omega_{1,2}(r) = \frac{\tilde{D}_2 r^2}{(r^2+1)^2} + \frac{128\tilde{D}_1 r^2}{3(r^2+1)^2}\int_\infty^r dy \frac{(y^2+1)^3}{(y^2-1)y^3}\left(-\frac{136}{384}+\frac{1}{2}\ln\left(\frac{2y^2}{y^2+1}\right)\right.$$
$$\left. + \frac{7+53y^2+43y^4+27y^6+6y^8}{12(1+y^2)^5}\right). \quad \text{(A.10)}$$

The second order equation of Yang-Mills field in ε is

$$0 = \phi''_{2,0}(r) + \frac{3}{r}\phi'_{2,0}(r) - \frac{8}{r^3}\sigma'_{2,0}(r) - \frac{4r^2\tilde{D}_1^2}{(r^2+1)^5}, \quad \text{(A.11)}$$

and Einstein equations are obtained as

$$0 = 2N_2(r) + rN'_2(r) + 2\sigma'_2(r)\left(r^2 - \frac{1}{r^2} - \frac{32\alpha^2}{3r^2} + \frac{32\alpha^2}{3r^4}\right) \quad \text{(A.12)}$$
$$+\frac{64\alpha^2}{3r^4}\left(\frac{r^6}{64}\phi_1'^2(r) + \frac{r^3}{4}\phi'_2(r)\right),$$
$$0 = 6\sigma'_2(r) - \frac{8\alpha^2\tilde{D}_1^2}{3}\frac{r}{(r^2+1)^4}, \quad \text{(A.13)}$$
$$0 = r(-1+r^4)f''_2(r) + (-1+5r^4)f'_2(r) - \frac{4\alpha^2\tilde{D}_1^2}{3}\frac{r(r^2-1)(1-6r^2+r^4)}{(1+r^2)^5}, \quad \text{(A.14)}$$

where the second order equation of $\omega(r)$ is not given, because which provides subleading corrections to the metric backreaction.

The Yang-Mills field solution of $\phi(r)$ in $O(\varepsilon^2)$ are given by

$$\phi_{2,0}(r) = \tilde{\phi}_{2,0} + \tilde{C}_2\left(1 - \frac{1}{r^2}\right) + \frac{(1+2r^2)\tilde{D}_1^2}{12r^2(1+r^2)^3} \quad \text{(A.15)}$$

and the metric corrections are

$$\sigma_{2,0} = \tilde{\sigma}_{2,0}, \quad \text{(A.16)}$$
$$\sigma_{2,2} = \frac{2\tilde{D}_1^2}{9}\left(\tilde{\sigma}_{2,2} - \frac{1}{(1+r^2)^3}\right), \quad \text{(A.17)}$$
$$N_{2,0}(r) = \frac{\tilde{N}_{2,0}}{r^2}, \quad \text{(A.18)}$$
$$N_{2,2}(r) = -\frac{16}{3r^2}\left(\tilde{N}_{2,2} - \frac{\tilde{C}_1^2}{32r^2} - \frac{\tilde{C}_2}{r^2} + \frac{\tilde{D}_1^2}{12}\left(\frac{1+2r^2}{r^2(1+r^2)^3} - \frac{3r^2}{2(1+r^2)^2}\right)\right), \quad \text{(A.19)}$$
$$f_{2,0}(r) = \tilde{f}_{2,0}, \quad \text{(A.20)}$$
$$f_{2,2}(r) = \tilde{f}_{2,2} - \frac{(1-2r^2)\tilde{D}_1^2}{18(1+r^2)^4}, \quad \text{(A.21)}$$

where $\tilde{\phi}_{2,0} = -\frac{\tilde{D}_1^2}{32}$ for the regularity of the Yang-Mills field at the horizon and $\tilde{N}_{2,0}$, $\tilde{f}_{2,0}$, $\tilde{\sigma}_{2,2}$ and $\tilde{f}_{2,2}$ are $O(1)$ constants which are set to be vanished for $f(\infty) = \sigma(\infty) = 1$. $\tilde{N}_{2,2} = \frac{\tilde{C}_1^2}{32} + \tilde{C}_2$ for the space-time has its horizon at $r = 1$. $\tilde{C}_1$ and $\tilde{C}_2$ are coefficients of zero modes of Yang-Mills field equations. Without loss of generality, we can set $\tilde{C}_1 = 0$. However, $\tilde{C}_2 = \frac{281}{6720}\tilde{D}_1^2$ requesting $\omega_{3,0}(r)$ to be normalizable mode and regular at the black brane horizon. Then, we have metric backreaction only with $\tilde{D}_1$ as a SU(2) symmetry breaking scale.

B Leading order correction of h_{yz}

In this section, we briefly describe the solutions of eq. (3.1) using the form of expansion (3.4) and (3.5). Each term in expansion (3.5) is given by

$$\Phi_{0,0}(r) = \phi^{(0)}_{0,0} + \nu\phi^{(1)}_{0,0} + O(\nu^2), \tag{B.1}$$

$$\Phi_{0,2}(r) = \phi^{(0)}_{0,2} + \nu\left(\phi^{(1)}_{0,2} + 8i\phi^{(0)}_{0,0}\left(\ln\left(1+\frac{1}{r^2}\right) - \frac{1}{r^2}\right)\right) + O(\nu^2), \tag{B.2}$$

$$\Phi_{1,0}(r) = \phi^{(0)}_{1,0} + \nu\phi^{(1)}_{1,0} + O(\nu^2), \quad \Phi_{2,0}(r) = \phi^{(0)}_{2,0} + \nu\phi^{(1)}_{2,0} + O(\nu^2), \tag{B.3}$$

$$\Phi_{1,2}(r) = \phi^{(0)}_{1,2} + \nu\left(\phi^{(1)}_{1,2} + 8i\phi^{(0)}_{1,0}\left(\ln\left(1+\frac{1}{r^2}\right) - \frac{1}{r^2}\right)\right) + O(\nu^2), \tag{B.4}$$

and

$$\begin{aligned}\Phi_{2,2}(r) = {}& \phi^{(0)}_{2,2} + \nu\phi^{(1)}_{2,2} - \frac{i\nu}{840}\left(\phi^{(0)}_{0,0}\tilde{D}_1^2\left(-\frac{105}{2(1+r^2)} + \frac{175}{2(1+r^2)^2} - \frac{70}{3(1+r^2)^3}\right.\right. \\ & \left.\left. - \frac{35}{(1+r^2)^4} - \frac{279}{2r^2}\right) + \frac{6720\phi^{(0)}_{2,0}}{r^2} + 192\left(\tilde{D}_1^2\phi^{(0)}_{0,0} - 35\phi^{(0)}_{2,0}\right)\ln(1+\frac{1}{r^2})\right).\end{aligned} \tag{B.5}$$

We also obtain the near AdS boundary expansion of $\Phi(r)$ as

$$\begin{aligned}\Phi_\nu(r\to\infty) = {}& \Phi^{(0)}_{0,0} + \varepsilon\Phi^{(0)}_{1,0} + \varepsilon^2\Phi^{(0)}_{2,0} + \alpha^2\left(\Phi^{(0)}_{0,2} + \varepsilon\Phi^{(0)}_{1,2} + \varepsilon^2\Phi^{(0)}_{2,2}\right) \\ & +\nu\left(\Phi^{(1)}_{0,0} + \varepsilon\Phi^{(1)}_{1,0} + \varepsilon^2\Phi^{(1)}_{2,0} + \alpha^2\left(\Phi^{(1)}_{0,2} + \varepsilon\Phi^{(1)}_{1,2} + \varepsilon^2\Phi^{(1)}_{2,2}\right)\right) \\ & +\frac{i\nu}{4r^4}\left(\Phi^{(0)}_{0,0} + \varepsilon\Phi^{(0)}_{1,0} + \varepsilon^2\Phi^{(0)}_{2,0} + \alpha^2\left(\Phi^{(0)}_{0,2} + \varepsilon\Phi^{(0)}_{1,2} + \varepsilon^2\Phi^{(0)}_{2,2}\right)\right) \\ & +O(r^i\nu^j\varepsilon^k\alpha^l),\end{aligned} \tag{B.6}$$

where $i < -4$, $j > 1$, $k > 2$ or $l > 2$.

C Leading order correction of h_{xy}

In this section, we list the solutions of the set of equations (3.10), (3.11) and (3.12). We listed our solution using the expansion (3.13) and (3.14). As explained in section 3.2, we get $\delta A^1_{i,0}$ and $\delta A^2_{i,0}$ only for the Yang-Mills field solution. $\delta A^1_{0,0}$ and $\delta A^2_{0,0}$ are zero modes of the solutions. Without loss of any generality, we set $\delta A^1_{0,0} = \delta A^2_{0,0} = 0$. The first subleading corrections of Yang-Mills fields in ε are given by

$$\begin{aligned}\delta A^1_{1,0}(r) = {}& \frac{r^2}{(1+r^2)^2}\left(\tilde{A}^{(0)}_{1,0} + \nu\tilde{A}^{(1)}_{1,0} - \frac{i\nu}{192r^2}\left(6\tilde{A}^{(0)}_{1,0}(1+r^4+24r^2\ln(r)\right.\right. \\ & -8r^2\ln(1+r^2)) + 2\bar{A}^{(0)}_{1,0}(5-11r^4-40r^2\ln(r)-24r^2\ln(1+r^2)) \\ & \left.\left. - \tilde{D}_1\psi^{(0)}_{0,0}(17+r^4+56r^2\ln(r)-24r^2\ln(1+r^2))\right) + O(\nu^2)\right),\end{aligned} \tag{C.1}$$

and

$$\delta A^1_{1,0}(r) = \frac{r^2}{(1+r^2)^2}\left(\bar{A}^{(0)}_{1,0} + \nu\bar{A}^{(1)}_{1,0} + \frac{i\nu}{192r^2}\left(2\tilde{A}^{(0)}_{1,0}(-5+11r^4+40r^2\ln(r)\right.\right. \tag{C.2}$$

$$+24r^2\ln(1+r^2)) - 6\bar{A}^{(0)}_{1,0}(1+r^4+24r^2\ln(r)-8r^2\ln(1+r^2))$$
$$+\ \tilde{D}_1\psi^{(0)}_{0,0}(5-11r^4-40r^2\ln(r)-24r^2\ln(1+r^2))\Big) + O(\nu^2)\Big)\,.$$

$\Psi(r)$ solution is also obtained as

$$\Psi_{0,0}(r) = \psi^{(0)}_{0,0} + \nu\psi^{(1)}_{0,0} + O(\nu^2), \tag{C.3}$$
$$\Psi_{1,0}(r) = \psi^{(0)}_{1,0} + \nu\psi^{(1)}_{1,0} + O(\nu^2), \tag{C.4}$$
$$\Psi_{2,0}(r) = \psi^{(0)}_{2,0} + \nu\psi^{(1)}_{2,0} + O(\nu^2), \tag{C.5}$$
$$\Psi_{0,2}(r) = \psi^{(0)}_{0,2} + \nu\psi^{(1)}_{0,2} + 8i\nu\psi^{(0)}_{0,0}\left(\ln\left(1+\frac{1}{r^2}\right)-\frac{1}{r^2}\right) + O(\nu^2), \tag{C.6}$$
$$\Psi_{1,2}(r) = \psi^{(0)}_{1,2} + \nu\psi^{(1)}_{1,2} + 8i\nu\psi^{(0)}_{1,0}\left(\ln\left(1+\frac{1}{r^2}\right)-\frac{1}{r^2}\right) + O(\nu^2), \tag{C.7}$$

and

$$\begin{aligned}
\Psi_{2,2}(r) = {}& \psi^{(0)}_{2,2} + \frac{\tilde{A}^{(0)}_{1,0}\tilde{D}_1}{(1+r^2)^4} - \frac{2\tilde{A}^{(0)}_{1,0}\tilde{D}_1}{3(1+r^2)^3} \\
&+\nu\psi^{(1)}_{2,2} + \frac{\nu}{576}\Bigg(\frac{8i\tilde{D}_1(-3\tilde{A}^{(0)}_{1,0}+48i\tilde{A}^{(1)}_{1,0}+17\bar{A}^{(0)}_{1,0}+2\tilde{D}_1\psi^{(0)}_{0,0})}{(1+r^2)^3} \\
&+\frac{2i\tilde{D}_1(3\tilde{A}^{(0)}_{1,0}-5\bar{A}^{(0)}_{1,0}+4\tilde{D}_1\psi^{(0)}_{0,0})}{1+r^2} - \frac{2i\tilde{D}_1(-9\tilde{A}^{(0)}_{1,0}+19\bar{A}^{(0)}_{1,0}+37\tilde{D}_1\psi^{(0)}_{0,0})}{(1+r^2)^2} \\
&+\frac{12\tilde{D}_1(3i\tilde{A}^{(0)}_{1,0}+48\tilde{A}^{(1)}_{1,0}-3i\bar{A}^{(0)}_{1,0}-i\tilde{D}_1\psi^{(0)}_{0,0})}{(1+r^2)^4} + \frac{36i(93\tilde{D}_1^2\psi^{(0)}_{0,0}-4480\psi^{(0)}_{2,0})}{35r^2} \\
&-\frac{8i\tilde{D}_1(2r^2-1)\ln(r)}{(1+r^2)^4}(-18\tilde{A}^{(0)}_{1,0}+10\bar{A}^{(0)}_{1,0}+7\tilde{D}_1\psi^{(0)}_{0,0}) \\
&-4i(-3\tilde{A}^{(0)}_{1,0}\tilde{D}_1+5\bar{A}^{(0)}_{1,0}\tilde{D}_1-\frac{1814}{35}\tilde{D}_1^2\psi^{(0)}_{0,0}+2304\psi^{(0)}_{2,0})\ln(r) \\
&+\frac{24i\tilde{D}_1(2r^2-1)\ln(1+r^2)}{(1+r^2)^4}(-2\tilde{A}^{(0)}_{1,0}-2\bar{A}^{(0)}_{1,0}+\tilde{D}_1\psi^{(0)}_{0,0}) \\
&+i\ln(1+r^2)(-6\tilde{A}^{(0)}_{1,0}+10\bar{A}^{(0)}_{1,0}-\frac{3628}{35}\tilde{D}_1^2\psi^{(0)}_{0,0}+4608\psi^{(0)}_{2,0})\Bigg) \\
&+O(\nu^2).
\end{aligned} \tag{C.8}$$

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