Published for SISSA by Springer

Received: May 11, 2011
Revised: June 15, 2011
Accepted: June 30, 2011
Published: July 6, 2011

ABJM baryon stability and Myers effect

Yolanda Lozano,[a] Marco Picos,[a] Konstadinos Sfetsos[b] and Konstadinos Siampos[c]

[a] *Department of Physics, University of Oviedo, Avda. Calvo Sotelo 18, 33007 Oviedo, Spain*

[b] *Department of Engineering Sciences, University of Patras, 26110 Patras, Greece*

[c] *Centre de Physique Théorique, Ecole Polytechnique, CNRS, 91128 Palaiseau, France*

E-mail: ylozano@uniovi.es, picosmarcos@uniovi.es, sfetsos@upatras.gr, ksiampos@cpht.polytechnique.fr

Abstract: We consider magnetically charged baryon vertex like configurations in $AdS_4 \times CP^3$ with a reduced number of quarks l. We show that these configurations are solutions to the classical equations of motion and are stable beyond a critical value of l. Given that the magnetic flux dissolves D0-brane charge it is possible to give a microscopical description in terms of D0-branes expanding into fuzzy CP^n spaces by Myers dielectric effect. Using this description we are able to explore the region of finite 't Hooft coupling.

Keywords: Gauge-gravity correspondence, D-branes, Bosonic Strings, Space-Time Symmetries

ArXiv ePrint: 1105.0939

doi:10.1007/JHEP07(2011)032

JHEP07(2011)032

Contents

1 Introduction

The AdS_4/CFT_3 duality relates the Type IIA superstring on $AdS_4 \times CP^3$ to an $\mathcal{N} = 6$ quiver Chern-Simons-matter theory with gauge group $\mathrm{U}(N)_k \times \mathrm{U}(N)_{-k}$ known as the ABJM model [1]. Like its AdS_5/CFT_4 counterpart it is a strong/weak coupling duality, with 't Hooft coupling $\lambda = N/k$. Being the superpotential coupling proportional to k^{-2}, an appropriate large k limit $N \ll k^5$ allows for a weak coupling regime. The Type IIA theory is then weakly curved when $k \ll N$.

The CP^3 space has $H^q(CP^3) = \mathbb{R}$ for even q. Therefore it is possible to have D2, D4 and D6 particle-like branes wrapping a topologically non-trivial cycle. In $AdS_4 \times CP^3$ these branes were already discussed in [1], and their interpretation in the context of the CFT dual given. The D6-brane wrapped on the entire CP^3 is the analogous of the baryon vertex in $AdS_5 \times S^5$ discussed by Witten in [2]. Due to the F_6 flux of the background it has a tadpole that has to be cancelled with N fundamental strings ending on it, which correspond to N external quarks on the boundary of AdS_4. Similarly, the D2-brane wrapped on a $CP^1 \subset CP^3$ captures the F_2 flux of the $AdS_4 \times CP^3$ background, and develops a tadpole that has to be cancelled with k fundamental strings ending on it. The field theory interpretation of this brane is as a 't Hooft monopole, realized as a Sym_k product of Wilson lines. The D4-brane wrapped on a $CP^2 \subset CP^3$ does not capture any of the background fluxes, and it is gauge invariant. It is dual to the di-baryon operator [3, 4], which has the same baryon charge and dimension to agree with the gravity result.

These gravitational configurations admit a natural generalization by allowing non-trivial worldvolume gauge fluxes [5]. These generalizations have been proposed as candidates for holographic anyons [6] in ABJM [7], and are therefore of potential interest for AdS/CMT applications. Allowing for a non-trivial worldvolume magnetic flux has the effect of adding lower dimensional brane charges to the configurations, in particular D0-brane charge. This modifies how the brane captures the background fluxes in a way that depends on the induced charges, such that, in some cases, additional fundamental strings are required to cancel the worldvolume tadpoles. The D2 and D6-branes are only stable if the induced charges lie below some upper bound. In turn, the D4-brane with flux behaves quite differently from the zero charge case, since it now requires fundamental strings ending on it. Given that in the presence of a non-trivial magnetic flux all these branes require fundamental strings ending on them we will loosely refer to them as baryon vertex like configurations.

In this paper we further generalize these constructions by reducing the number of strings that stretch between the brane and the boundary of AdS_4, i.e. the number of quarks. It was shown in [8, 9] that in $AdS_5 \times S^5$ perfect baryon vertex classical solutions to the equations of motion exist for a number of quarks l satisfying $5N/8 \leqslant l \leqslant N$. Although one would expect that bound states of quarks should be singlets of the gauge group the analysis of the stability against fluctuations confirms that the configurations are stable for a number of quarks $0.813N \leqslant l \leqslant N$ [10]. It is likely that this will not be the case in other theories with reduced supersymmetry.

It is one aim of this paper to perform a similar analysis for magnetically charged baryon vertex like configurations with reduced number of quarks in $AdS_4 \times CP^3$. Our analysis will reveal that also in this case baryon vertex like classical solutions exist that are moreover stable against fluctuations.

In order to be able to use the probe brane approximation in the study of the dynamics we will consider a uniform distribution of strings on a $CP^{\frac{p}{2}}$ geometrical shell, with $p = 2, 4, 6$. This will be our particular profile for the distribution of quarks inside the baryon vertex configuration. Although this choice completely breaks supersymmetry we will be able to ignore the strings backreaction [9, 11, 12].

The fact that the magnetized branes have dissolved D0-branes in their worldvolumes hints at the existence of a microscopical description in terms of non-Abelian n D0-branes polarizing due to Myers dielectric effect [13]. This description allows to explore the configurations in the region where $N \ll n^{\frac{4}{p}}\, k$, and is therefore complementary to the supergravity description in terms of probe branes. We will see that classical stable solutions still exist in this regime. Moreover, we will show that the flat half-integer B_2 field that is required by the Freed-Witten anomaly in the di-baryon [14] has to be introduced already at the classical level so that a CP^2 non-spin manifold can be recovered in the large n limit.

The organization of the paper is as follows: We start in section 2 by summarizing some of the properties of the magnetized baryon vertex like configurations constructed in [5]. In section 3 we reduce the number of quarks and find the values for which classical configurations still exist. In section 4 we perform the stability analysis under small fluctuations. Section 5 is devoted to the microscopical description. This description will confirm the existence of non-singlet classical stable solutions when $N \ll k^5$. An interesting output of this analysis will be the derivation of new higher curvature dielectric couplings not predicted before in the literature. Finally, in section 6 we summarize our results and discuss further directions. We have written appendix A, containing a number of useful results on the $AdS_4 \times CP^3$ background and also appendix B with the computation of the Kähler form for the fuzzy $CP^{\frac{p}{2}}$, used in the main text.

2 Magnetically charged baryon vertex configurations in $AdS_4 \times CP^3$ spaces

It was shown in [5] that it is possible to construct more general monopole, di-baryon and baryon vertex configurations in $AdS_4 \times CP^3$ if the particle like branes carry lower dimensional brane charges induced by a non-trivial magnetic flux $F = \mathcal{N} J$, where J is the Kähler form of the CP^3. For the D2 and D6 branes the effect of the magnetic flux is to allow the construction of similar monopole and baryon vertex configurations with D0-brane charge and a different number of fundamental strings attached. Indeed the study of the dynamics reveals that the configurations are stable if the magnetic flux does not exceed some maximum value, for which the configurations reduce to radial fundamental strings (free quarks) plus the wrapped D-brane.

The di-baryon is more substantially modified by the presence of the magnetic flux, capturing the F_2 flux and developing a tadpole. In this case the study of the dynamics shows that the D4-brane with the fundamental strings attached is stable if the magnetic flux takes values in a given interval, at the limits of which the configuration ceases to be stable and reduces to free quarks plus the D4-brane. This is consistent with the fact that the D4-brane with F-strings does not exist for zero magnetic flux. Moreover, since the D4-brane wraps a non-spin manifold it must carry a half-integer worldvolume magnetic flux due to the Freed-Witten anomaly [15]. In order to still keep its dual interpretation as a di-baryon it was proposed in [14] that a flat half-integer B_2-field should be switched on in the dual background in order to cancel the contribution of the Freed-Witten worldvolume magnetic flux.

A question that remained open after the study in [5] was the interpretation of the magnetized Dp-branes in the field theory. A difficulty comes from the expected lack of SUSY for the D2 and D6-branes. In turn the D4-brane with flux forms a threshold BPS intersection with the D0-branes. Therefore one could expect that a supersymmetric spiky solution exists and one could give an interpretation to the bounds in the gauge theory dual. As shown in [5] the maximum (and minimum, if applicable) values of the magnetic flux are functions of $\sqrt{\lambda}$, with λ the 't Hooft coupling, for all branes. This suggests an origin on the conformal symmetry of the gauge theory. Ultimately one would expect a connection between the existence of these bounds and the stringy exclusion principle of [16].

JHEP07(2011)032

We summarize next the energies and charges carried by the various branes. In order to set up the notation a short review of the $AdS_4 \times CP^3$ background is given in appendix A. We will use Poincaré coordinates to parameterize AdS_4 throughout the paper.

2.1 Charges and energies

The computation of the energy of a Dp-brane in $AdS_4 \times CP^3$ wrapped on a $CP^{\frac{p}{2}}$ cycle of the CP^3 with $p = 2, 4$ and 6, in the presence of a magnetic flux $F = \mathcal{N} J$, with $\mathcal{N} \in 2\mathbb{Z}$, was done in [5]. We review this result and show that the equations of motion are satisfied for $F = \mathcal{N} J$.

The DBI action is

$$S_p = -T_p \int d^{p+1}\xi \, e^{-\phi} \sqrt{|\det(P[g + 2\pi\mathcal{F}])|} \,, \qquad T_p = \frac{1}{(2\pi)^p} \,, \tag{2.1}$$

where $\mathcal{F} = F + \frac{1}{2\pi} B_2$ and we set $\ell_s = 1$. The equations of motion arising from varying the gauge potential are given by

$$\partial_\alpha \left(\sqrt{|\det P([g + 2\pi\mathcal{F}])|} \, (P[g + 2\pi\mathcal{F}])^{-1[\alpha\beta]} \right) = 0 \,, \tag{2.2}$$

where $[\alpha\beta]$ denotes the antisymmetric part. Identifying the world-volume coordinates with the angles of the various CP-cycles as indicated in appendix A and considering static solutions independent of the ξ^i's we find an induced metric

$$ds^2_{\text{ind}} = -\frac{16\rho^2}{L^2} d\tau^2 + L^2 ds^2_{CP^{\frac{p}{2}}} \,. \tag{2.3}$$

Using that in our case $\mathcal{F}$ is proportional to the Kähler form, since $F = \mathcal{N} J$ and $B_2 = -2\pi J$, we can easily prove that the equations of motion are satisfied. If M is an antisymmetric $p \times p$ matrix satisfying $M^2 = -c\,\mathbb{I}$, where for consistency $c = -\dfrac{1}{p}\text{Tr}(M^2)$, one can show that $(\mathbb{I} + M)^{-1} = \frac{\mathbb{I} - M}{1 + c}$, and, moreover, due to the fact that M is antisymmetric: $\det(\mathbb{I} + M) = (1 + c)^{\frac{p}{2}}$. Using these identities we find that $\partial_\alpha(\sqrt{g} J^{\alpha\beta}) = 0$, where g is the metric on $CP^{\frac{p}{2}}$, or equivalently $\nabla_\alpha J^{\alpha\beta} = 0$. The latter is the condition for having a Kähler manifold and therefore it is automatically satisfied. Also we find that the DBI action is given by (we use $c = (2\pi\mathcal{N})^2$)

$$S^{Dp}_{DBI} = -\frac{T_p}{g_s} \int d^{p+1}\xi \sqrt{-\det(g + 2\pi\mathcal{F})} = -Q_p \int d\tau \frac{2\rho}{L} \,, \tag{2.4}$$

where

$$Q_p = \frac{T_p}{g_s} \, \text{Vol}(CP^{\frac{p}{2}}) \, \left(L^4 + (2\pi)^2 (\mathcal{N} - 1)^2 \right)^{\frac{p}{4}} \,, \qquad \text{for} \quad p = 2, 6 \tag{2.5}$$

and

$$Q_4 = \frac{T_4}{g_s} \, \text{Vol}(CP^2) \, \left(L^4 + (2\pi\mathcal{N})^2 \right) \,, \tag{2.6}$$

since in this case B_2 cancels the contribution of the Freed-Witten vector field, such that $\mathcal{F} = F_{FW} + \mathcal{N} J + \frac{1}{2\pi} B_2 = \mathcal{N} J$. Also, in this case

$$S^{D4}_{DBI} = -\frac{T_4}{g_s} \int d^5\xi \sqrt{-\det(g + 2\pi\mathcal{F})} = -\frac{T_4}{g_s} \int d^5\xi \, \sqrt{|g_{tt}|} \sqrt{g_{\mathbb{P}^2}} \left(L^4 + 2(2\pi)^2 \mathcal{F}_{\alpha\beta} \mathcal{F}^{\alpha\beta} \right) \tag{2.7}$$

The volume of the $CP^{\frac{p}{2}}$ is given by

$$\mathrm{Vol}(CP^{\frac{p}{2}}) = \frac{\pi^{\frac{p}{2}}}{(\frac{p}{2})!}\,. \tag{2.8}$$

From (2.5) and (2.6) it is clear that $\mathcal{N}^2$ is comparable to $L^4 \gg 1$.

Analyzing the Chern-Simons actions one can also show that the magnetic flux has the effect of dissolving lower dimensional brane charge in the Dp-branes. For instance the D4-brane has D2 and D0-brane charges dissolved, as can be seen from the couplings:

$$S_{CS}^{D4} = 2\pi\, T_4 \int_{\mathbb{R}\times\mathbb{P}^2} C_3 \wedge F = \frac{\mathcal{N}}{2}\, T_2 \int C_3 \tag{2.9}$$

and

$$S_{CS}^{D4} = \frac{1}{2}(2\pi)^2\, T_4 \int_{\mathbb{R}\times\mathbb{P}^2} C_1 \wedge F \wedge F = \frac{\mathcal{N}^2}{8}\, T_0 \int_{\mathbb{R}} C_1\,, \tag{2.10}$$

respectively. In general the number of Ds-branes dissolved in the worldvolume of a Dp is given by [5]

$$n = \frac{\mathcal{N}^{\frac{p-s}{2}}}{2^{\frac{p-s}{2}}(\frac{p-s}{2})!}\,. \tag{2.11}$$

Both the D4 and D6-branes have CP^1 D2-branes dissolved. Therefore in the presence of a magnetic flux they capture the F_2 flux and develop a tadpole with charge

$$q = k\,\frac{\mathcal{N}^{\frac{p}{2}-1}}{2^{\frac{p}{2}-1}(\frac{p}{2}-1)!} \tag{2.12}$$

More explicitly, for the D4-brane we have that

$$\begin{aligned} S_{CS}^{D4} &= \frac{1}{2}(2\pi)^2\, T_4 \int_{\mathbb{R}\times\mathbb{P}^2} P[F_2] \wedge F \wedge A = 2\,(2\pi)^2 T_4\, k\mathcal{N} \int_{\mathbb{R}\times\mathbb{P}^2} J \wedge J \wedge A \\ &= k\,\frac{\mathcal{N}}{2}\, T_{F1} \int dtA_t \end{aligned} \tag{2.13}$$

The analogous coupling for the D6-brane is

$$S_{CS}^{D6} = \frac{1}{6}(2\pi)^3\, T_6 \int_{\mathbb{R}\times\mathbb{P}^3} P[F_2] \wedge F \wedge F \wedge A = k\,\frac{\mathcal{N}^2}{8}\, T_{F1} \int dtA_t\,. \tag{2.14}$$

Note however that for the D6-brane the couplings $\int_{D6} F_2 \wedge B_2 \wedge B_2 \wedge A$ and $\int_{D6} F_2 \wedge F \wedge B_2 \wedge A$ in its CS action contribute as well to its k charge. In the absence of magnetic flux it was shown in [14] that the contribution from $\int_{D6} F_2 \wedge B_2 \wedge B_2 \wedge A$ is cancelled from the higher curvature coupling [17–19]

$$S_{h.c.}^{D6} = \frac{3}{2}(2\pi)^5\, T_6 \int C_1 \wedge F \wedge \sqrt{\frac{\hat{\mathcal{A}}(T)}{\hat{\mathcal{A}}(N)}}\,, \tag{2.15}$$

where $\hat{\mathcal{A}}$ is the A-roof (Dirac) genus

$$\hat{\mathcal{A}} = 1 - \frac{\hat{p}_1}{24} + \frac{7\,\hat{p}_1^2 - 4\,\hat{p}_2}{5760} + \cdots \tag{2.16}$$

and the Pontryagin classes are written in terms of the curvature of the corresponding bundle as

$$\hat{p}_1 = -\frac{1}{8\pi^2}\,\mathrm{Tr}\,R^2\,, \qquad \hat{p}_2 = \frac{1}{256\,\pi^4}\Big((\mathrm{Tr}\,R^2)^2 - 2\,\mathrm{Tr}\,R^4\Big)\,. \tag{2.17}$$

This charge cancellation is consistent with the dual interpretation of the D6-brane as a baryon vertex. For a non-vanishing magnetic flux the term $\int_{D6} F_2 \wedge F \wedge B_2 \wedge A$ contributes however with $-k\mathcal{N}/4$ units of F-string charge, as shown in [5]. Therefore, adding the N units induced by the F_6 flux,

$$S^{D6}_{CS} = 2\pi\,T_6 \int_{R\times\mathbb{P}^3} P[F_6]\wedge A = N\,T_{F1}\int dt A_t\,, \tag{2.18}$$

not captured by the other branes, we find that the total F-string charge carried by the D6-brane is given by

$$q_{D6} = N + k\,\frac{\mathcal{N}(\mathcal{N}-2)}{8} \tag{2.19}$$

Note that this is always an integer due to the quantization condition

$$\frac{1}{2\pi}\int F = \frac{\mathcal{N}}{2} \in \mathbb{Z} \tag{2.20}$$

3 Varying the number of fundamental strings

It was shown in [8, 9] that the baryon vertex in $AdS_5 \times S^5$ can be generalized such that the number of quarks l lies in the interval $5N/8 \leqslant l \leqslant N$. These configurations are not only perfect classical solutions to the equations of motion but for $0.813\,N \leqslant l \leqslant N$ are stable against fluctuations [10]. In this section we generalize the construction in [8, 9] to the baryon vertex like configurations discussed in the previous section. We will see that in all cases there exist configurations with a reduced number of quarks that are solutions to the classical equations of motion.

We consider a classical configuration consisting on a Dp-brane wrapped on $CP^{\frac{p}{2}}$, located at $\rho = \rho_0$, l strings stretching from ρ_0 to the boundary of AdS_4 and $(q-l)$ straight strings that go from ρ_0 to 0. The configuration is depicted in figure 1. Further, we switch on the magnetic flux $F = \mathcal{N}J$, with J the Kähler form of the CP^3. Taking the gauge $\tau = t$, $\sigma = \rho$ for the worldsheet coordinates of the string, the Nambu-Goto action of the l fundamental strings is given by [20]

$$S_{lF1} = -l\,T_{F1}\int dt d\rho\,\sqrt{1+\frac{16\rho^4}{L^4}r'^2} \tag{3.1}$$

where r is the radius of the configuration at the boundary of AdS_4. The equations of motion then reduce to

$$\frac{16\rho^4 r'}{L^4\sqrt{1+\frac{16\rho^4}{L^4}r'^2}} = c = \frac{4\rho_1^2}{L^2} \tag{3.2}$$

where the constant has been fixed demanding that $r' = \infty$ at the turning point of each string, ρ_1. The turning point is such that $0 \leqslant \rho_1 \leqslant \rho_0$. From (3.2)

$$r' = \frac{L^2\rho_1^2}{4\rho^2\sqrt{\rho^4-\rho_1^4}} \equiv r'_{\rm cl} \tag{3.3}$$

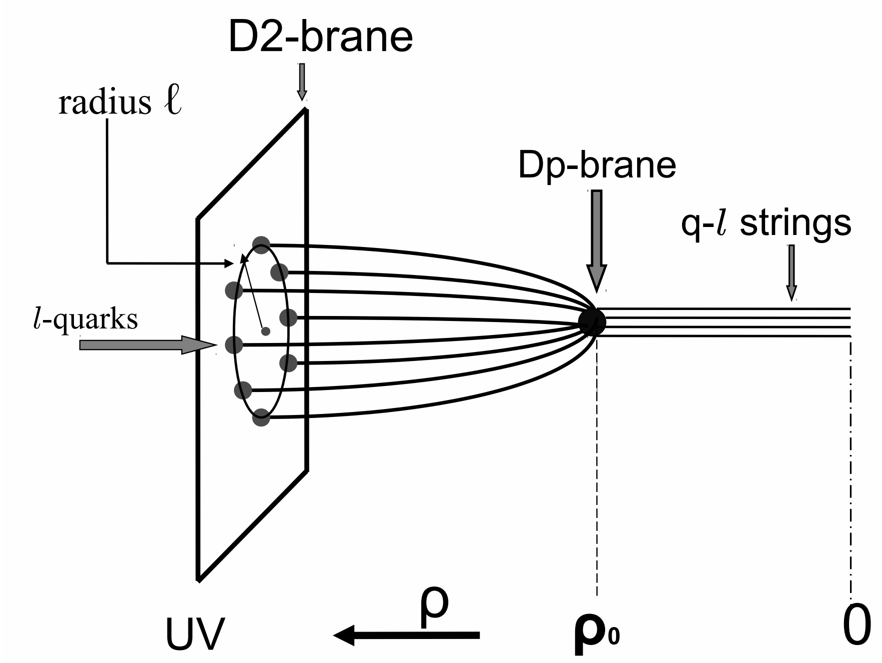

Figure 1. A baryon configuration with l-external quarks placed on a circle of radius ℓ at the boundary of AdS space, each connected to a Dp-brane wrapped on a $CP^{\frac{p}{2}}$ located at $\rho = \rho_0$, and $q - l$ straight strings ending at 0.

Defining $a \equiv \frac{l}{q}$, the boundary equation reads

$$\frac{1}{a}\sqrt{1-\beta^2} + \frac{1-a}{a} = \sqrt{1-\frac{\rho_1^4}{\rho_0^4}} \tag{3.4}$$

where we defined [5]

$$\sqrt{1-\beta^2} \equiv \frac{2Q_p}{L\,q\,T_{F_1}}\,. \tag{3.5}$$

We then must have

$$\frac{2Q_p}{L\,q\,T_{F_1}} \leqslant 1 \tag{3.6}$$

in order to find a stable configuration. Since Q_p (and also q, for the D4 and D6-branes), are functions of $\mathcal{N}$ this condition imposes a bound on the magnetic flux that can be dissolved on the worldvolume. For the D2 and D6-branes $\mathcal{N}$ must lie below some upper bound, for which $\beta = 0$. For the D4 the magnetic flux must also lie above a lower bound, for which $\beta = 0$ as well. This is consistent with the fact that the D4-brane with fundamental strings attached only exists for non-zero magnetic flux.

For the values of the magnetic flux allowed by equation (3.6) we must still fulfill the boundary equation (3.4), and this implies that

$$q\sqrt{1-\beta^2} + q - l \leqslant l \qquad \Leftrightarrow \qquad l \geqslant \frac{q}{2}(1+\sqrt{1-\beta^2}) = l_{\min} \tag{3.7}$$

JHEP07(2011)032

This condition determines the minimum value of strings that can form the baryon vertex like configuration. Note that $l_{\rm min}$ is a function of the magnetic flux, and is such that it decreases with β. For the D2 and D6-branes β is maximum for zero magnetic flux, for which $l_{\rm min}$ reaches its minimum value: $l_{\rm min} = \frac{q}{2}(1+\frac{1}{2\pi})$, $l_{\rm min} = \frac{q}{2}(1+\frac{1}{6\pi})$, respectively. Recall that for this value of the magnetic flux the configuration is maximally stable [5]. For the D4-brane β is maximum when $\frac{\mathcal{N}}{L^2} = \frac{1}{2\pi}$, which also corresponds to the most stable configuration. For this value of the magnetic flux l/q is minimum[1], and one finds the maximum range of values allowed for l: $\frac{q}{2}(1+\frac{1}{2\pi}) \leqslant l \leqslant q$. Again, this range is maximum for the most stable configuration. On the contrary, when $\beta = 0$ we can only have $l = q$, and therefore it is not possible to reduce the number of quarks. For this value of the magnetic flux the strings are no longer bounded and the configurations reduce to q free quarks. Indeed, $\beta = 0$, $l = q$ implies $\rho_1 = \rho_0 \to \rho' = \infty$, i.e. the fundamental strings become radial. Note as well that when $l = l_{\rm min}$ the strings become radial for any value of the magnetic flux. The conclusion is that the $(l, \mathcal{N})$ parameter space for which the classical configurations exist is bounded by those values corresponding to the free quarks case.

Equations (3.3) and (3.4) allow to calculate the radius of the configuration,

$$\ell = \frac{L^2\rho_1^2}{12\rho_0^3}\int_1^\infty \frac{dz}{z^2\sqrt{z^4-\frac{\rho_1^4}{\rho_0^4}}} = \frac{L^2\rho_1^2}{12\rho_0^3}\, {}_2F_1\left(\frac{1}{2},\frac{3}{4},\frac{7}{4};\frac{\rho_1^4}{\rho_0^4}\right)\,, \qquad \frac{\rho_1^4}{\rho_0^4} = 4\frac{l_{\rm min}}{l}\left(1-\frac{l_{\rm min}}{l}\right)\,, \tag{3.8}$$

where we have changed the integration variable as follows $z = \frac{\rho}{\rho_0}$ and ${}_2F_1(a,b,c;x)$ is a hypergeometric function. This expression has the same form than the size of the baryon vertex in $AdS_5 \times S^5$ [8, 21] and the $q\bar{q}$ system [20, 22]. Note that the dependence on the location of the Dp-brane, ρ_0, and on L^2 is also the same. This is a non-trivial prediction of the AdS/CFT correspondence for the strongly coupled CS-matter theory. Note as well that (3.8) reduces to the expression found in [5] when $l = q$.

The total on-shell energy is in turn given by

$$\begin{aligned} E &= E_{Dp} + E_{lF1} + E_{(q-l)F1} \\ &= l\, T_{F_1}\rho_0\left(\frac{q}{l}\sqrt{1-\beta^2} + \int_1^\infty dz \frac{z^2}{\sqrt{z^4-\frac{\rho_1^4}{\rho_0^4}}} + \frac{q-l}{l}\int_0^1 dz\right)\,. \end{aligned} \tag{3.9}$$

The binding energy can then be obtained by subtracting the (divergent) energy of the constituents. Note that, as we have discussed before, the free quarks configuration is degenerate, since it can be reached in three cases: when the Dp-brane is located at $\rho_0 = 0$ (at this location the energy of the Dp vanishes), as in [8], when $\beta = 0$ ($\Leftrightarrow l = q$) and ρ_0 is arbitrary, and when $l = l_{\rm min}$, for any β and any ρ_0. In all these cases the constituents contribute with an energy $l\, T_{F1}\int_0^\infty d\rho$ and the binding energy is given by:

$$E_{bin} = l\; T_{F1}\,\rho_0\left\{-\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4};4\frac{l_{\rm min}}{l}\left(1-\frac{l_{\rm min}}{l}\right)\right) + 2\frac{l_{\rm min}}{l} - 1\right\}\,. \tag{3.10}$$

[1] Recall that in this case $q = k\mathcal{N}/2$.

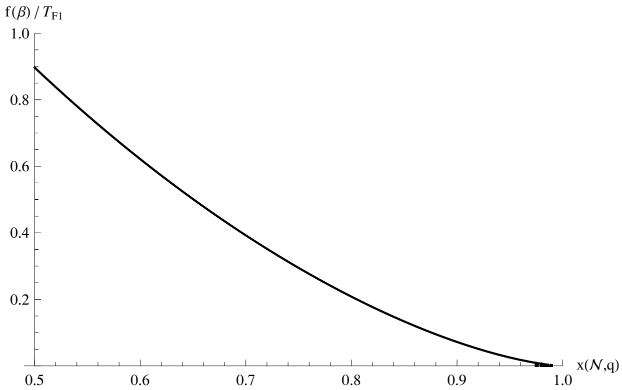

Figure 2. Positivity of $f(x)$ as a function of x

This expression has again the same form than the corresponding expressions in [5, 8, 20–22].[2] Setting $x = l_{\rm min}/l$ the configurations are maximally stable when x is minimum, i.e. when $l = q$ and β reaches its maximum value. This happens for zero magnetic flux for the D2 and D6-branes, and for $\frac{\mathcal{N}}{L^2} = \frac{1}{2\pi}$ for the D4.

From (3.8) and (3.10) we have that for all l and $\mathcal{N}$ the binding energy of the baryon reads

$$E_{bin} = -f(x)\frac{(g_s N)^{2/5}}{\ell} \leqslant 0 \tag{3.11}$$

since $f(x) \geqslant 0$. The behavior of $f(x)$ is depicted in figure 2. Moreover the binding energy satisfies the concavity condition $\dfrac{dE}{dL} \geqslant 0$, $\dfrac{d^2E}{dL^2} \leqslant 0$. Therefore the force is manifestly attractive and increasing in magnitude. Note however that this was not necessarily expected for baryons, since in this case there is no analogue of the concavity condition for heavy quark-antiquark pairs [23, 24]. The $1/\ell$ behavior is that dictated by conformal invariance, whereas the non-analytical dependence on the 't Hooft coupling λ is the one predicted in [25–31], which hints at a universal behavior based on the conformal symmetry of the gauge theory.

4 Stability analysis

We shall next consider the stability analysis of the classical solution. We know from [10] that the instabilities can emerge only from longitudinal fluctuations of the l strings, since only these possess a non-divergent zero mode, which is a sign of instability. To study the fluctuations about the classical solution we perturb the embedding according to

$$r = r_{\rm cl} + \delta r(\rho) \tag{4.1}$$

and expand the Nambu-Goto action to quadratic order in the fluctuations. δr is then solved from the equation

$$\frac{d}{d\rho}\left(\frac{(\rho^4 - \rho_1^4)^{3/2}}{\rho^2}\frac{d}{d\rho}\right)\delta r = 0\,, \tag{4.2}$$

[2] In this case we have added the on-shell energy of the Dp-brane.

from where we find

$$\delta r = A\int_\rho^\infty d\rho \frac{\rho^2}{(\rho^4-\rho_1^4)^{3/2}} = \frac{A}{3\rho^3}\,{}_2F_1\left(\frac{3}{2},\frac{3}{4};\frac{7}{4};\frac{\rho_1^4}{\rho^4}\right). \tag{4.3}$$

Supplementing with the boundary condition (eq. (3.12) in [10])

$$\rho_0\gamma^2\delta r' + 2(1+\gamma^2)\delta r = 0 \quad \text{at} \quad \rho=\rho_0 \quad \text{where} \quad \gamma\equiv\sqrt{1-\frac{\rho_1^4}{\rho_0^4}}\,, \tag{4.4}$$

we find that

$${}_2F_1\left(\frac{3}{2},\frac{3}{4};\frac{7}{4};1-\gamma^2\right) = \frac{3}{2\gamma(1+\gamma^2)}\,. \tag{4.5}$$

The numerical result for γ is then $\gamma_c = 0.538$. The critical value for a can be read from (3.4), and we find it is a function of the magnetic flux

$$a_c = \frac{1+\sqrt{1-\beta^2}}{1+\gamma_c} \tag{4.6}$$

Therefore, for the various configurations with magnetic flux there is a bound for the number of F-strings coming from stability

$$l \geqslant \frac{q}{1+\gamma_c}(1+\sqrt{1-\beta^2}) \tag{4.7}$$

which is more restrictive than the bound imposed by the existence of a classical solution

$$l \geqslant \frac{q}{2}(1+\sqrt{1-\beta^2})\,. \tag{4.8}$$

Note that in fact the stability condition (4.7) imposes a bound on the magnetic flux $1+\sqrt{1-\beta^2}\leqslant 1+\gamma_c$ which is also more restrictive than the one coming from (3.6), since now $\beta\geqslant\sqrt{1-\gamma_c^2}$ and therefore $\beta=0$, which was setting the condition for the maximum (and minimum, if applicable) magnetic flux, is not reached. Therefore stability further restricts the allowed values for the magnetic flux coming from the analysis of the equations of motion.

Finally, we turn to the fluctuations of the Dp-brane. We perturb the embedding according to

$$x^\mu = \delta x^\mu(t,\theta_\alpha)\,, \qquad \rho=\rho_0\,, \qquad x^\mu = x, y\,, \tag{4.9}$$

leaving the position of the Dp-brane at $\rho=\rho_0$ intact due to the gauge choice $\rho=\sigma$ for the strings. To be more precise, the ρ-fluctuations can be proven to be decoupled from the others both in the equations of motion and in the boundary equations; being periodic in the angles of $CP^{\frac{p}{2}}$. Moreover, leaving the position of the brane at $\rho=\rho_0$ can also be proven to be allowed for spaces for which $g_{tt}\sim\rho^2$ (as in AdS_4) at zero mode of the angular fluctuations, whereas for higher modes the $\delta\rho$ fluctuations are stable. Moreover, for the

CP^1 and CP^2 cases we have kept fixed the D2 and D4 embeddings on the CP^3. We then find that to second order in the fluctuations the expansion of the Dp-brane action reads

$$
\begin{aligned}
S_{Dp} = &-\frac{T_p}{g_s} L^p (1+c)^{\frac{p}{4}} \int dt\, d\Omega_p \sqrt{-g_{tt}} \sqrt{\gamma} \times \\
&\times \left\{ 1 + \frac{g_{\mu\nu}}{2(1+c)} \gamma^{\alpha\beta} \partial_\alpha \delta x^\mu \partial_\beta \delta x^\nu + \frac{g_{\mu\nu}}{2g_{tt}} \delta\dot{x}^\mu \delta\dot{x}^\nu \right\}, \qquad c = (2\pi\mathcal{N})^2 ,
\end{aligned}
\tag{4.10}
$$

where $c = (2\pi(\mathcal{N}-1))^2$ for the D2 and D6-branes, $c = (2\pi\mathcal{N})^2$ for the D4, $\gamma_{\alpha\beta}$ is the metric of $CP^{\frac{p}{2}}$ and the action is calculated at $\rho = \rho_0$. The subscripts α, μ refer to the angles of $CP^{\frac{p}{2}}$ and to the x, y coordinates, respectively. Expanding the fluctuations in terms of the spherical harmonics of the $CP^{\frac{p}{2}}$ coset manifold[3] as

$$
\delta x^\mu(t, \theta_\alpha) = \delta x^\mu(t) \Psi_\ell(\theta_\alpha) , \tag{4.11}
$$

we find from the Euler-Lagrange equations for the action that

$$
\frac{d^2 \delta x^\mu}{dt^2} + \Omega_\ell^2 \delta x^\mu = 0, \qquad \Omega_\ell^2 = -\frac{g_{tt}}{1+c} \omega_\ell^2 \geqslant 0 \, . \tag{4.12}
$$

Note that there are no boundary conditions for these fluctuations, the reason being that the $\mathbb{R} \times CP^{\frac{p}{2}}$ space has no boundary. The conclusion is that the Dp-brane is also stable against fluctuations.

5 The microscopical description

In the previous sections we have described magnetically charged baryon vertex like configurations with varying number of quarks using the probe brane approximation. This description is valid in the supergravity limit $L \gg 1$ (in string units), equivalently when $k \ll N$, and in the weakly coupled region in which $g_s \ll 1$, equivalently when $N \ll k^5$. In this section we show that it is possible to give a description for finite 't Hooft coupling in terms of fuzzy $CP^{\frac{p}{2}}$ manifolds built up out of dielectrically expanded D0-branes.

The fact that the magnetic flux induces D0-brane charge on the Dp-branes wrapped on $CP^{\frac{p}{2}}$ suggests a close analogy with the dielectric effect of [13, 32]. We then expect that a complementary description in terms of coincident D0-branes expanded into fuzzy $CP^{\frac{p}{2}}$ manifolds should be possible. This would be the 'microscopical' realization of the 'macroscopical' Dp-branes wrapping classical $CP^{\frac{p}{2}}$ spaces with magnetic flux. It is well known that the macroscopical and microscopical descriptions have complementary ranges of validity [13]. The first is valid in the supergravity limit $L \gg 1$, whereas the second is a good description when the mutual separation of the expanding D0-branes is much smaller than the string length. For n expanding such branes this is fixed by the condition $L \ll n^{\frac{1}{p}}$. The two descriptions are then complementary for finite n and should agree in the large n limit, where they have a common range of validity. In $AdS_4 \times CP^3$ the regime of validity

[3] Satisfying the eigenvalue equation $\nabla_\gamma^2 \Psi_\ell = -\omega_\ell^2 \Psi_\ell$ where ω_ℓ^2 is positive since the Laplace operator is defined on a compact manifold.

of the microscopical description is fixed by the condition that $N \ll n^{\frac{4}{p}}\, k$. Therefore this description allows to explore the region of finite 't Hooft coupling.

Dielectric branes expanding into fuzzy coset manifolds have been discussed in the literature in different contexts [21, 33–36]. G/H coset manifolds can be described as fuzzy surfaces if H is the isotropy group of the lowest weight state of a given irreducible representation of G [33, 37]. Since different irreducible representations have associated different isotropy subgroups they can give rise to different cosets G/H. For instance, CP^2 has $G = \mathrm{SU}(3)$, $H = \mathrm{U}(2)$, and this is precisely the isotropy group of the SU(3) irreducible representations $(m, 0)$, $(0, m)$, where we parameterize the irreducible representations of SU(3) by two integers (n, m) corresponding to the number of fundamental and anti-fundamental indices. Any other choice of (n, m) has isotropy group $\mathrm{U}(1) \times \mathrm{U}(1)$ and therefore yields a different coset, $\mathrm{SU}(3)/(\mathrm{U}(1) \times \mathrm{U}(1))$. One can also take a more geometrical view more suitable for our purposes. Using the fact that $CP^{\frac{p}{2}}$ spaces can be defined as the submanifolds of $\mathbb{R}^{\frac{p^2}{4}+p}$ determined by a given set of $p^2/4$ constraints, a fuzzy version arises by promoting the Cartesian coordinates that embed the $CP^{\frac{p}{2}}$ in $\mathbb{R}^{\frac{p^2}{4}+p}$ to $\mathrm{SU}(\frac{p}{2}+1)$ matrices in the irreducible totally symmetric representations $(m, 0)$ or $(0, m)$. Indeed only for these representations can the set of $p^2/4$ constraints be realized at the level of matrices. The Cartesian coordinates are then taken to play the role of the non-Abelian transverse scalars that couple in Myers action for coincident D-branes. Using this action one can then provide a microscopical description of a Dq-brane wrapped on the classical $CP^{\frac{p}{2}}$ space in terms of D$(q-p)$-branes expanding into a fuzzy $CP^{\frac{p}{2}}$. Exact agreement between the two descriptions is found in the large m limit.

5.1 The DBI action in the microscopical description

The DBI action describing the dynamics of n coincident D0-branes is given by [13]

$$S^{DBI}_{nD0} = -\int d\tau\, \mathrm{STr}\Big\{ e^{-\phi} \sqrt{|\det\Big(P[E_{\mu\nu} + E_{\mu i}(Q^{-1}-\delta)^i{}_j E^{jk} E_{k\nu}]\Big)\det Q|}\Big\} \tag{5.1}$$

where $E = g + B_2$,

$$Q^i{}_j = \delta^i{}_j + \frac{i}{2\pi}[X^i, X^k]E_{kj}\,, \tag{5.2}$$

and we have set the tension of the D0-branes to 1. We take $g_{\mu\nu}$ to be the metric in $AdS_4 \times CP^3$ and $B_2 = -2\pi J$, as in appendix A. The number of D0-branes, n, is related to the magnetic flux of the macroscopical description by (2.11), with $s = 0$

$$n = \frac{\mathcal{N}^{\frac{p}{2}}}{2^{\frac{p}{2}}(\frac{p}{2})!}\,. \tag{5.3}$$

We now let these D0-branes expand into a fuzzy $CP^{\frac{p}{2}}$ space to build up a Dp-brane. We find that

$$S^{DBI}_{nD0} = -\frac{1}{g_s}\int d\tau \frac{2\rho}{L}\, \mathrm{STr}\sqrt{\det(Q)}\,. \tag{5.4}$$

As we have mentioned, a fuzzy version of $CP^{\frac{p}{2}}$ is well-known. Here we will mainly follow [38]. $CP^{\frac{p}{2}}$ is the coset manifold $\mathrm{SU}(\frac{p}{2}+1)/\mathrm{U}(\frac{p}{2})$, and can be defined by the submanifold of $\mathbb{R}^{\frac{p^2}{4}+p}$ determined by the set of $p^2/4$ constraints

$$\sum_{i=1}^{\frac{p^2}{4}+p} x^i x^i = 1\,, \qquad \sum_{j,k=1}^{\frac{p^2}{4}+p} d^{ijk} x^j x^k = \frac{\frac{p}{2}-1}{\sqrt{\frac{p}{4}(\frac{p}{2}+1)}} x^i \tag{5.5}$$

where d^{ijk} are the components of the totally symmetric $\mathrm{SU}(\frac{p}{2}+1)$-invariant tensor. The Fubini-Study metric of the $CP^{\frac{p}{2}}$ is given by

$$ds^2_{CP^{\frac{p}{2}}} = \frac{p}{4(\frac{p}{2}+1)} \sum_{i=1}^{\frac{p^2}{4}+p} (dx^i)^2\,. \tag{5.6}$$

A fuzzy version of $CP^{\frac{p}{2}}$ can then be obtained by imposing the conditions (5.5) at the level of matrices. This is achieved with a set of coordinates X^i ($i = 1, \ldots, \frac{p^2}{4}+p$) in the irreducible totally symmetric representation of order m, $(m,0)$, satisfying

$$[X^i, X^j] = i\Lambda_{(m)} f_{ijk} X^k\,, \qquad \Lambda_{(m)} = \frac{1}{\sqrt{\frac{pm^2}{4(\frac{p}{2}+1)} + \frac{p}{4}m}} \tag{5.7}$$

with f_{ijk} the structure constants in the algebra of the generalized Gell-Mann matrices of $\mathrm{SU}(\frac{p}{2}+1)$. The dimension of the $(m,0)$ representation is given by

$$\dim(m,0) = \frac{(m+\frac{p}{2})!}{m!(\frac{p}{2})!}\,. \tag{5.8}$$

The Kähler form of the fuzzy $CP^{\frac{p}{2}}$ is given by (see appendix B):

$$J_{ij} = \frac{1}{\frac{p}{2}+1}\sqrt{\frac{p}{4(\frac{p}{2}+1)}} f_{ijk} X^k\,. \tag{5.9}$$

Substituting this non-commutative ansatz in (5.4) we can compute $\det(Q)$. This is however a difficult computation to perform in general, since $Q^i{}_j = \delta^i{}_j + M^i{}_j$ with M given by

$$M^i{}_j = -\frac{1}{\frac{p}{2}+1}\Lambda_{(m)} f_{ikl} X^l \left(\frac{pL^2}{8\pi}\delta^k{}_j - \sqrt{\frac{p}{4(\frac{p}{2}+1)}} f_{kjm} X^m\right), \tag{5.10}$$

and one has to compute traces of powers of M using the constraints above as well as (B). Given this we are going to start by making the comparison with the macroscopical calculation. For this purpose it is enough to work to leading order in m, to which the second term in (5.10), coming from B_2, does not contribute. This should match the macroscopical result for $B_2 = 0$. Indeed, recall from section 2.1 that B_2 contributes to (2.4) to order $O(1/\mathcal{N})$. Already in this case we find that

$$\mathrm{Tr}(M) = 0\,, \qquad \mathrm{Tr}(M^2) = -\frac{p}{2^4\pi^2} r\,\mathbb{I}\,, \qquad \mathrm{Tr}(M^3) = -i\,\frac{p(\frac{p}{2}+1)}{2^7\pi^3 L^2} r^2\,\mathbb{I}\,, \tag{5.11}$$
$$\mathrm{Tr}(M^4) = \frac{p}{2^8\pi^4} r^2\,\mathbb{I} + \frac{p}{2^{10}\pi^4 L^4}\left(\left(\frac{p}{2}+1\right)^2 - 4\right) r^3\,\mathbb{I}\,,$$

with

$$r = \frac{L^4}{m(m+\frac{p}{2}+1)} \, . \tag{5.12}$$

However, in the limit

$$L \gg 1 \, , \qquad m \gg 1 \, , \quad \text{with} \quad r \simeq \frac{L^4}{m^2} = \text{finite} \, , \tag{5.13}$$

some terms in the traces of higher powers of M drop out, and we find

$$\text{Tr}(M^{2n}) = p\,(-1)^n \left(\frac{r}{16\pi^2}\right)^n \mathbb{I} \, , \qquad \text{Tr}(M^{2n+1}) = 0 \, . \tag{5.14}$$

Substituting in (5.4) we then obtain that

$$\det(Q) = \left(1 + \frac{r}{16\pi^2}\right)^{\frac{p}{2}} \mathbb{I} \, . \tag{5.15}$$

The DBI action of n D0-branes expanding into a fuzzy $CP^{\frac{p}{2}}$ is then given to leading order in m by

$$S^{DBI}_{nD0} = -\frac{n}{g_s}\left(1 + \frac{L^4}{16\pi^2 m^2}\right)^{\frac{p}{4}} \int d\tau \frac{2\rho}{L} \tag{5.16}$$

where $n = \dim(m,0)$ arises as $\dim(m,0) = \text{STr}\,\mathbb{I}$. Note that in the regime of validity of the microscopical description $L \ll n^{\frac{1}{p}} \to L^4 \ll m^2$, and we could expand in powers of $\frac{L^4}{m^2}$. We will see however that the agreement with the macroscopical description still holds for the entire expression in (5.16). We encountered already this situation in the microscopical descriptions of giant gravitons in [34, 36, 39, 40]. Taking into account (5.8) and (5.3) we have that to leading order in m the label of the irreducible representation and the unit of magnetic flux are related through

$$m \sim \frac{\mathcal{N}}{2} \tag{5.17}$$

and (5.16) becomes

$$S^{DBI}_{nD0} = -\frac{T_p}{g_s} \text{Vol}(CP^{\frac{p}{2}}) \Big(L^4 + (2\pi\mathcal{N})^2\Big)^{\frac{p}{4}} \int d\tau \frac{2\rho}{L} \, , \tag{5.18}$$

which exactly matches the result (2.4) of the macroscopical calculation for $B_2 = 0$. Note that $\mathcal{N} \sim 2m$ is in agreement with the quantization condition $\mathcal{N} \in 2\mathbb{Z}$.

Let us now include the effect of the B_2 field. We know from the macroscopical calculation that B_2 produces a shift $\mathcal{N} \to \mathcal{N} - 1$ in the D2 and D6-branes, and cancels the contribution of the Freed-Witten worldvolume flux in the D4-brane. Its effect is therefore $O(1/m)$, and this is why we could ignore it in the leading order calculation above. Analytical and numerical results for $B_2 \neq 0$ and the agreement with the macroscopical calculation suggest that the complete expression for the determinant to order $O(1/m)$ can be obtained from the expansion of

$$\det(Q) = \left(\left(1 - \frac{1}{2\sqrt{m(m+\frac{p}{2}+1)}}\right)^2 + \frac{r}{16\pi^2}\right)^{\frac{p}{2}} . \tag{5.19}$$

This is the exact result for $p = 2$ in the limit (5.13) and correctly matches the macroscopical result to this order for all p. Indeed, using (5.19) we find that

$$S^{DBI}_{nD0} = -\frac{n}{g_s}\left(\left(1-\frac{1}{2\sqrt{m(m+\frac{p}{2}+1)}}\right)^2+\frac{L^4}{16\pi^2 m(m+\frac{p}{2}+1)}\right)^{\frac{p}{4}}\int d\tau\frac{2\rho}{L}\,, \tag{5.20}$$

which to order $O(1/m)$ yields

$$S^{DBI}_{nD0} = -\frac{T_p}{g_s}\,\mathrm{Vol}(CP^{\frac{p}{2}})\left(L^4+(2\pi)^2\left(2m+\frac{p}{2}+1-1\right)^2\right)^{\frac{p}{4}}\int d\tau\frac{2\rho}{L}\,. \tag{5.21}$$

Here we have not cancelled the two ones inside the parenthesis to emphasize their different origin, coming from the $1/m$ expansion of the second term in (5.20) (the +1) and the B_2 contribution (the -1). Comparing to the macroscopical calculation for $B_2 \neq 0$ this result suggests a redefinition of $\mathcal{N} = \mathcal{N}(m)$ to order $O(1/m)$:

$$\mathcal{N} = 2m+\frac{p}{2}+1 \qquad \text{for} \quad p = 2,6 \tag{5.22}$$

$$\mathcal{N} = 2m+\frac{p}{2} \qquad \text{for} \quad p = 4 \tag{5.23}$$

With these redefinitions we can, on the one hand, obtain a magnetic flux properly quantized, i.e. such that $\mathcal{N} \in 2\mathbb{Z}$, and, on the other hand, reproduce the expected shift of $\mathcal{N}$, $\mathcal{N} \to \mathcal{N} - 1$, for $p = 2, 6$. The $p = 4$ case is more interesting. Recall that in the macroscopical analysis B_2 was introduced in order to cancel the flux of the (Freed-Witten) vector field required by the Freed-Witten anomaly, such that $\mathcal{F} = F_{FW} + \frac{1}{2\pi}B_2 = 0$.[4] Microscopically we should see, in the absence of B_2, an obstacle to the expansion of the D0-branes into a CP^2, which should be absent for the CP^1 and CP^3. However, since the Freed-Witten field strength cannot couple in the worldvolume of D0-branes it is not clear a priori how exactly a non-vanishing B_2 could allow the construction of the CP^2. We have found through a simple classical computation that B_2 is required in order to get an even $\mathcal{N}$, that is later interpreted as (twice) the units of magnetic flux in the macroscopical description. This clarifies the precise way in which the flat half-integer B_2 allows for the correct construction of the di-baryon with magnetic charge at the microscopical level. We will see in the next section that the analysis of the charges carried by the different branes confirms the redefinitions (5.22), (5.23).

In conclusion, we have seen that it is indeed possible to give a microscopical description of the magnetic baryon vertex like configurations of [5] in terms of D0-branes expanding into fuzzy $CP^{\frac{p}{2}}$. This expansion is caused by the couplings in the Born-Infeld part of the action, and therefore it is entirely due to a gravitational dielectric effect, analogous to the one described in [21, 41]. The regime of validity is fixed by the condition

$$N \ll k\left[\frac{(m+\frac{p}{2})!}{m!(\frac{p}{2})!}\right]^{\frac{4}{p}}\,. \tag{5.24}$$

[4] In fact, the original argument supporting this B_2-field in [14] had to do with the analysis of the supergravity charges, while the analysis of the D4-brane worldvolume dynamics arose as a consistency check. We refer to the original paper for more details.

Therefore for finite m this description allows to explore the region of finite 't Hooft coupling. Note however that for $B_2 \neq 0$ we have not been able to give exact analytical expressions beyond the constant term in a $1/m$ expansion.

5.2 The F-strings in the microscopical description

An essential part of the baryon vertex-like configurations described in this paper are the fundamental strings that stretch from the Dp-brane to the boundary of AdS_4. In this section we show how these strings arise in the microscopic setup.

The CS action for n coincident D0-branes is given by

$$S_{CS} = \int_{\mathbb{R}} \mathrm{STr}\Big\{ P\Big(e^{\frac{i}{2\pi}(i_X i_X)} \sum_q C_q \, e^{B_2}\Big) e^{2\pi F}\Big\} \,. \tag{5.25}$$

In this expression the dependence of the background potentials on the non-Abelian scalars occurs through the Taylor expansion [42]

$$C_q(t,X) = C_q(t) + X^k \partial_k C_q(t) + \frac{1}{2} X^l X^k \partial_l \partial_k C_q(t) + \dots \tag{5.26}$$

and it is implicit that the pull-backs into the worldline are taken with gauge covariant derivatives $D_t X^\mu = \partial_t X^\mu + i[A_t, X^\mu]$.

In the $AdS_4 \times CP^3$ background we have

$$F_2 = \frac{2L}{g_s} J \,, \qquad F_6 = \frac{L^5}{g_s} J \wedge J \wedge J \,, \qquad B_2 = -2\pi J \tag{5.27}$$

with J the Kähler form of the CP^3. Therefore taking into account (5.26) the relevant CS couplings in this background are

$$\begin{aligned} S_{CS} = i \int d\tau \mathrm{STr}\Big\{ \Big[(i_X i_X) F_2 - \frac{1}{(2\pi)^2}(i_X i_X)^3 F_6 + \frac{i}{2\pi}(i_X i_X)^2 F_2 \wedge B_2 - \\ - \frac{1}{2}\frac{1}{(2\pi)^2}(i_X i_X)^3 F_2 \wedge B_2 \wedge B_2 \Big] A_\tau \Big\} \,. \end{aligned} \tag{5.28}$$

These terms arise, respectively, from

$$S_{CS} = \int \mathrm{STr}\Big\{ P\Big(C_1 - \frac{1}{2}\frac{1}{(2\pi)^2}(i_X i_X)^2 C_5 + \frac{i}{2\pi}(i_X i_X) C_1 \wedge B_2 - \frac{1}{4}\frac{1}{(2\pi)^2}(i_X i_X)^2 C_1 \wedge B_2 \wedge B_2 \Big)\Big\} \tag{5.29}$$

in (5.25).

The first coupling in (5.28) is non-vanishing when the D0-branes expand into a fuzzy CP^1, which can be that in which a D2-brane is wrapped or any of the CP^1 cycles of a CP^2 D4-brane or a CP^3 D6-brane. Since the Kähler form for a fuzzy $CP^{\frac{p}{2}}$ is given by (see the appendix B)

$$J_{ij} = \frac{1}{\frac{p}{2}+1} \sqrt{\frac{p}{4(\frac{p}{2}+1)}} f_{ijk} X^k \tag{5.30}$$

we find that

$$S_{CS_1} = i \int STr\{(i_X i_X) F_2 \wedge A\} = k \Big(m \Big(m + \frac{p}{2} + 1 \Big)\Big)^{-1/2} \frac{(m+\frac{p}{2})!}{m!(\frac{p}{2})!} \int d\tau A_\tau \tag{5.31}$$

which gives in the large m limit

$$S_{CS_1} = k\,\frac{m^{\frac{p}{2}-1}}{(\frac{p}{2})!}\int d\tau A_\tau \tag{5.32}$$

Taking into account that the dimension of the irreducible representation is related to the units of magnetic flux of the macroscopical description by $m=\frac{\mathcal{N}}{2}$, as we showed in the previous section, we find that the number of fundamental string charge in each CP^1 is given by:

$$q = \frac{2}{p}\,k\,\frac{\mathcal{N}^{\frac{p}{2}-1}}{2^{\frac{p}{2}-1}(\frac{p}{2}-1)!} \tag{5.33}$$

which is in agreement with the macroscopical result (2.12).

Let us now look at the second term in (5.28). This term is non-vanishing when the D0-branes expand into a fuzzy CP^3, so it should give the fundamental string charge carried by the CP^3 D6-brane in the large m limit. The explicit computation gives

$$S_{CS_2} = -\frac{i}{(2\pi)^2}\int STr\{(i_X i_X)^3 F_6\wedge A\} = N\Big(m(m+4)\Big)^{-3/2}\frac{(m+3)!}{m!}\int d\tau A_\tau \tag{5.34}$$

and, in the large m limit

$$S_{CS_2} = N\int d\tau A_\tau\,, \tag{5.35}$$

in agreement with the macroscopical result.

The third and fourth terms in (5.28) contribute when we take into account the B_2 field that is necessary to compensate the Freed-Witten worldvolume field of the D4-brane. Therefore they contribute to the k charge to order $O(1/m)$ relative to (5.32). We find, explicitly:

$$S_{CS_3} = -\frac{1}{2\pi}\int \mathrm{STr}\Big\{(i_X i_X)^2 F_2\wedge B_2\wedge A\Big\} = -k\Big(m\Big(m+\frac{p}{2}+1\Big)\Big)^{-1}\frac{(m+\frac{p}{2})!}{m!\,(\frac{p}{2})!}\int d\tau A_\tau \tag{5.36}$$

and

$$\begin{aligned} S_{CS_4} &= -\frac{i}{2}\frac{1}{(2\pi)^2}\int \mathrm{STr}\Big\{(i_X i_X)^3 F_2\wedge B_2\wedge B_2\wedge A\Big\} \\ &= \frac{3!}{8}k\Big(m\Big(m+\frac{p}{2}+1\Big)\Big)^{-3/2}\frac{(m+\frac{p}{2})!}{m!\,(\frac{p}{2})!}\int d\tau A_\tau \end{aligned} \tag{5.37}$$

These yield in the large m limit

$$S_{CS_3} = -k\,\frac{m^{\frac{p}{2}-2}}{(\frac{p}{2})!}\int d\tau A_\tau \tag{5.38}$$

and

$$S_{CS_4} = \frac{3!}{8}\,k\,\frac{m^{\frac{p}{2}-3}}{(\frac{p}{2})!}\int d\tau A_\tau \tag{5.39}$$

respectively. In order to find the total k charge to this (lower) order in m (relative to (5.32)) we have to add the contributions to this order coming from (5.31), that we have ignored in (5.32). Doing this we find that the total F-string charge for $p=2$ is still k, but for $p=4$ and $p=6$ it is given by $k(m+1)$, $N+\frac{k}{2}((m+2)^2-m-2+\frac{1}{4})$, respectively. Taking into account the redefinitions (5.22) and (5.23) we find precisely the $k\mathcal{N}/2$ units of F-string charge of the CP^2 D4-brane and the $N+k\frac{\mathcal{N}(\mathcal{N}-2)}{8}$ units of F-string charge of the CP^3 D6-brane, given respectively by equations (2.12) (for $p=4$) and (2.19). Note that we find in addition a $k/8$ contribution for the D6, coming from S_{CS_4}. Macroscopically we already encountered this charge when computing the contribution of the coupling $\int_{D6} F_2\wedge B_2\wedge B_2\wedge A$ to the D6-brane tadpole. Given that this charge was cancelled from the anomalous higher curvature coupling

$$S_{h.c.}=\frac{3}{2}(2\pi)^5\,T_6\int d^7\xi\, P\left(C_1\wedge\sqrt{\frac{\hat{A}(T)}{\hat{A}(N)}}\right)\wedge F\,, \tag{5.40}$$

a similar cancellation should occur microscopically. We will discuss in the next section how this can be achieved. Coming back to the D4-brane it is interesting that we need again at the classical level a flat half-integer B_2 in order to recover the right fundamental string charge of the macroscopic D4-brane.

5.3 Dielectric higher-curvature terms

In this section we show that generalizing the microscopical Chern-Simons action in [13] to include higher curvature terms [17–19] we can predict the existence of a dielectric higher curvature coupling in the action for multiple D0-branes that exactly cancels the $k/8$ contribution to the D6-brane tadpole that we obtained above.

Generalizing the Chern-Simons action for multiple Dp-branes in [13] to include higher curvature terms we find

$$S_{h.c.}=T_p\int d^{p+1}\xi\ \mathrm{Str}\left[P\left(e^{\frac{i}{2\pi}(i_Xi_X)}\sum_q C_q\, e^{B_2}\,\Omega\right)e^{2\pi F}\right]_{p+1}\,,\quad \Omega=\sqrt{\frac{\hat{A}(T)}{\hat{A}(N)}}\,. \tag{5.41}$$

Keeping the first term in the $\hat{A}$-roof (Dirac) genus expansion, a general term of the previous expression for $D0$-branes has the following form

$$[(i_Xi_X)^nC_q\ (B_2)^k\Omega_4]\wedge F^\ell\,,\quad (n,\ell,k)\in\mathbb{N}\,, \tag{5.42}$$

$$\underbrace{(q+2(k-n)+4)}_{\geqslant 0}+2\ell=1\,,$$

where Ω_4 is given in term of the Pontryagin classes of the normal and the tangent bundle of the three CP^2 circles of the CP^3 manifold [43, 44]; $\Omega_4=3(1-3)\frac{(2\pi)^4}{48\pi^2}J\wedge J$. To find the term of the expansion that contributes for the CP^3 we proceed as follows: We first note that $\ell=0$ and that in the macroscopic limit only terms with $n+1=3\rightarrow n=2$ contribute, thus we have to solve $q+2k=1$, which has solution $(k,q)=(0,1)$. Thus the term reads

$$S_{h.c.}=-\frac{1}{2(2\pi)^2}\int_{\mathbb{R}}P[(i_Xi_X)^2C_1\wedge\Omega_4]=-\frac{i}{(2\pi)^2}\int_{\mathbb{R}}[(i_Xi_X)^3(F_2\wedge\Omega_4)]A \tag{5.43}$$

and substituting F_2 and Ω_4:

$$S_{h.c.} = -\frac{\kappa}{8}(m(m+4))^{-3/2}\frac{(m+3)!}{m!}\int_{\mathbb{R}} d\tau A_\tau \simeq -\frac{\kappa}{8}\int_{\mathbb{R}} d\tau A_\tau\,, \tag{5.44}$$

where we took into account that there are three CP^2 circles in CP^3. Thus this higher curvature coupling cancels the S_{CS_4} contribution as in the macroscopical case.

Anomalous dielectric couplings as those predicted by (5.41) have, to the best of our knowledge, not been discussed before in the literature. Furthermore, acting with T-duality on the A-roof in (5.41) one can obtain dielectric terms that couple the RR-potentials to derivatives of B_2 and the metric that generalize the anomalous terms derived in [45, 46] for a single Dp-brane. It would be interesting to confirm the existence of all these new couplings through string amplitude calculations.

5.4 Stability analysis

The study of the stability goes along the same lines than in the macroscopical set-up. Note that also in the microscopical description the DBI action can be written as (2.4), where Q_p depends now on the label of the irreducible representation, m, in the precise way given by (5.20). The number of F-strings that must end on the Dp-brane is in turn given by the sum of the contributions from equations (5.31), (5.34), (5.36) and (5.37), where some of these terms have to be multiplied by the number of CP^1 or CP^2 cycles in the CP^3 as appropriate. Other than these differences we can vary the number of quarks, study the dynamics and the stability exactly along the same lines as in sections 3 and 4. Only now equation (3.6) will impose a bound on m, that is, on the number of D0-branes that can expand into a fuzzy $CP^{\frac{p}{2}}$ by Myers dielectric effect. In the large m limit this is the bound that we encountered for $\mathcal{N}$ in the macroscopical description. As in there the existence of this bound should be related in some way to the stringy exclusion principle of [16], although we have not been able to find a direct interpretation.

The conclusion is that also in the microscopical set-up there exist perfect baryon vertex classical solutions to the equations of motion that are stable against fluctuations.

6 Conclusions

We have analyzed various configurations of magnetically charged particle-like branes in ABJM with reduced number of quarks. We have shown that 't Hooft monopole, di-baryon and baryon vertex configurations with magnetic charge and reduced number of quarks can be constructed which are not only perfect classical solutions to the equations of motion but also stable against small fluctuations.

The magnetic flux has to satisfy some upper bound (also some lower bound for the di-baryon, consistently with the fact that the D4 with fundamental strings only exists for non-zero magnetic flux), and once this bound is fixed it is possible to reduce the number of quarks to a minimum value determined by $\mathcal{N}$ (or β):

$$l \geqslant \frac{q}{2}(1+\sqrt{1-\beta^2})$$

JHEP07(2011)032

From here we can see that the number of quarks is maximally reduced when the energy of the configuration is minimum, that is, for those values of the flux for which $\beta = 0$.

The analysis of the stability against small fluctuations reveals that the configurations are stable if

$$l \geqslant \frac{q}{1+\gamma_c}(1+\sqrt{1-\beta^2})$$

where γ_c is fixed numerically to $\gamma_c = 0.538$. Stability therefore increases the classical lower bound for each value of the magnetic flux. This is the same effect encountered in [10] for asymptotically $AdS_5 \times S^5$ spaces. It is worth mentioning that in fact following [10] it is trivial to extend our analysis to asymptotically $AdS_4 \times CP^3$ backgrounds and non-zero temperature.

The previous analysis is based on a probe brane approximation, and is therefore valid in the supergravity limit $k \ll N$. Using the fact that we can consistently add dissolved D0-branes to the configurations we have given an alternative description in terms of D0-branes expanded into fuzzy $CP^{\frac{p}{2}}$ spaces that allows to explore the finite 't Hooft coupling region. In this description the expansion is caused by a purely gravitational dielectric effect, while the Chern-Simons terms only indicate the need to introduce the number of fundamental strings required to build up the (generalized) vertex. The microscopical analysis confirms the existence of non-singlet classical stable solutions for finite 't Hooft coupling.

An output of this analysis is the prediction of dielectric higher curvature couplings that to the best of our knowledge have not been considered before in the literature. The particular explicit coupling in the action for multiple D0-branes that has come out in our analysis is necessary in order to obtain the right fundamental string charge of the baryon vertex. For the rest of branes they are predicted by T-duality. These couplings imply in turn new couplings of the RR-potentials to derivatives of B_2 and the metric, along the lines in [45, 46], with further implications for other branes via S and U dualities. It would be interesting to explore more closely these implications.

Finally, it would be interesting to extend the existence of non-singlet baryon vertex like configurations like the ones considered in this paper to theories with reduced supersymmetry, like the Klebanov-Strassler backgrounds [47], where the internal geometry is the $T^{1,1}$ conifold.

Acknowledgments

We would like to thank N. Gutiérrez, D. Rodríguez-Gómez, D.C. Thompson and P. Tzivel-oglou for useful discussions. The work of Y.L. and M.P. has been partially supported by the research grants MICINN-09-FPA2009-07122, MEC-DGI-CSD2007-00042 and COF10-03. K. Siampos has been supported by the ITN programme PITN-GA-2009-237920, the ERC Advanced Grant 226371, the IFCPAR CEFIPRA programme 4104-2 and the ANR programme blanc NT09-573739. K.Siampos would also like to thank the University of Patras for hospitality and the Theory Unit of CERN for hospitality and financial support where part of this work was done.

A Review of the $AdS_4 \times CP^3$ background

In this appendix we give a short review of the $AdS_4 \times CP^3$ background. In our conventions the $AdS_4 \times CP^3$ metric reads

$$ds^2 = L^2\left(\frac{1}{4}ds^2_{AdS_4} + ds^2_{\mathbb{CP}^3}\right), \tag{A.1}$$

with L the radius of curvature in string units

$$L = \left(\frac{32\pi^2 N}{k}\right)^{1/4} \tag{A.2}$$

and where we have normalized the two factors such that $R_{\mu\nu} = -3g_{\mu\nu}$ and $8g_{\alpha\beta}$ for AdS_4 and CP^3, respectively. The explicit parameterization of AdS_4 we use in the main text is

$$ds^2_{AdS_4} = \frac{16\,\rho^2}{L^2}d\vec{x}^2 + L^2\frac{d\rho^2}{\rho^2}, \quad d\vec{x}^2 = -d\tau^2 + dx_1^2 + dx_2^2\,. \tag{A.3}$$

For the metric on CP^3 we use the parameterization in [48, 49]

$$\begin{aligned} ds^2_{\mathbb{CP}^3} = d\mu^2 + \sin^2\mu & \left[d\alpha^2 + \frac{1}{4}\sin^2\alpha\left(\cos^2\alpha\,(d\psi - \cos\theta\,d\phi)^2 + d\theta^2 + \sin^2\theta\,d\phi^2\right)\right. \\ & \left. + \frac{1}{4}\cos^2\mu\left(d\chi + \sin^2\alpha\,(d\psi - \cos\theta\,d\phi)\right)^2\right], \end{aligned} \tag{A.4}$$

where

$$0 \leqslant \mu,\, \alpha \leqslant \frac{\pi}{2}, \quad 0 \leqslant \theta \leqslant \pi\,, \quad 0 \leqslant \phi \leqslant 2\pi\,, \quad 0 \leqslant \psi,\, \chi \leqslant 4\pi\,. \tag{A.5}$$

Inside CP^3 there is a CP^1 for $\mu = \alpha = \pi/2$ and fixed χ and ψ and also a CP^2 for fixed θ and ϕ.

In these coordinates the connection in $ds^2_{S^7} = (d\tau + \mathcal{A})^2 + ds^2_{\mathbb{CP}^3}$ reads

$$\mathcal{A} = \frac{1}{2}\sin^2\mu\left(d\chi + \sin^2\alpha\,(d\psi - \cos\theta\,d\phi)\right). \tag{A.6}$$

The Kähler form

$$J = \frac{1}{2}d\mathcal{A}\,, \tag{A.7}$$

is then normalized such that

$$\int_{CP^1} J = \pi\,, \qquad \int_{CP^2} J\wedge J = \pi^2\,, \qquad \int_{CP^3} J\wedge J\wedge J = \pi^3\,. \tag{A.8}$$

Therefore,

$$\frac{1}{6}J\wedge J\wedge J = d\mathrm{Vol}(\mathbb{P}^3) \qquad \text{and} \qquad \mathrm{Vol}(\mathbb{CP}^3) = \frac{\pi^3}{6}\,. \tag{A.9}$$

The $AdS_4 \times CP^3$ background fluxes can then be written as

$$F_2 = \frac{2L}{g_s}J\,, \qquad F_4 = \frac{3L^3}{8g_s}\,d\mathrm{Vol}(AdS_4)\,, \qquad F_6 = -(\star F_4) = \frac{6\,L^5}{g_s}\,d\mathrm{Vol}(\mathbb{P}^3)\,, \tag{A.10}$$

where $g_s = \dfrac{L}{k}$. The flux integrals satisfy

$$\int_{CP^3} F_6 = 32\,\pi^5\,N\,, \qquad \int_{CP^1} F_2 = 2\pi\,k\,. \tag{A.11}$$

The flat B_2-field that is needed to compensate for the Freed-Witten worldvolume flux in the D4-brane is given by [14]

$$B_2 = -2\pi J\ . \tag{A.12}$$

B Computation of the Kähler form for fuzzy $CP^{\frac{p}{2}}$

In this appendix we compute the Kähler form for the fuzzy $CP^{\frac{p}{2}}$ spaces considered in the paper. The Kähler form is given in terms of the exterior derivative of the one form U(1) gauge field [50, 51]

$$\begin{aligned} &J = J_{(i)}X^i\,, && J_{(i)} = \frac{1}{2}\,dA_i\,, && A_i = \sqrt{\frac{p}{\frac{p}{2}+1}}\,L_i, \\ &J \equiv \frac{1}{2}J_{ij}L_i\wedge L_j\,, && L_i = -i\,\mathrm{Tr}(t_i g^{-1}dg)\,, && g\in SU\left(\frac{p}{2}+1\right)\,, \end{aligned} \tag{B.1}$$

where t_i are the generators of SU($\frac{p}{2}$+1) in the adjoint representation, $(t_i)_{jk} = -if_{ijk}$. Using that $\mathrm{Tr}(t_it_j) = (\frac{p}{2}+1)\delta_{ij}$, $\mathrm{Tr}(t_it_jt_k) = i\,\frac{\frac{p}{2}+1}{2}f_{ijk}$, which result from the identities [52]

$$\begin{aligned} f_{ikm}f_{jkm} = N\delta_{ij}, \quad f_{iaj}f_{jbk}f_{kci} = -\frac{N}{2}f_{abc}, \quad d_{iaj}d_{jbk}f_{kci} = \frac{N^2-4}{2N}f_{abc}, \\ f_{iaj}f_{jbk}f_{kcm}f_{mdi} = \delta_{ab}\delta_{cd} + \delta_{ad}\delta_{bc} + \frac{N}{4}(d_{abe}d_{cde} + d_{ade}d_{bce} - d_{ace}d_{bde})\,, \end{aligned} \tag{B.2}$$

we compute the Kähler form as follows

$$\begin{aligned} J &= \frac{i}{2}\sqrt{\frac{p}{\frac{p}{2}+1}}\,\mathrm{Tr}(t_k g^{-1}dg\wedge g^{-1}dg)X^k\,, \qquad iL_it_i = \left(\frac{p}{2}+1\right)\,g^{-1}dg \\ \Rightarrow \quad J &= \frac{i^3}{2}\sqrt{\frac{p}{\frac{p}{2}+1}}\,\frac{\mathrm{Tr}(t_it_jt_k)}{(\frac{p}{2}+1)^2}\,X^k\,L_i\wedge L_j\,, \\ \Rightarrow \quad J_{ij} &= \frac{1}{\frac{p}{2}+1}\sqrt{\frac{p}{4(\frac{p}{2}+1)}}f_{ijk}X^k \end{aligned} \tag{B.3}$$

Then, for the n $D0$-branes expanding into a fuzzy $CP^{\frac{p}{2}}$ we find that

$$\begin{aligned} (i_Xi_X)J = X^jX^iJ_{ij} &= -\frac{i}{2}\sqrt{\frac{p}{4(\frac{p}{2}+1)}}\,\Lambda_{(m)}\mathbb{I}\,, \\ (i_Xi_X)^{\frac{p}{2}}\underbrace{J\wedge J\wedge\cdots\wedge J}_{\frac{p}{2}\text{ terms}} &= \left(\frac{p}{2}\right)!\left(-\frac{i}{2}\sqrt{\frac{p}{4(\frac{p}{2}+1)}}\,\Lambda_{(m)}\right)^{\frac{p}{2}}\mathbb{I}\,, \end{aligned} \tag{B.4}$$

so that the interior products are constant.

References

- [1] O. Aharony, O. Bergman, D.L. Jafferis and J. Maldacena, *$N = 6$ superconformal Chern-Simons-matter theories, M2-branes and their gravity duals*, *JHEP* **10** (2008) 091 [arXiv:0806.1218] [SPIRES].
- [2] E. Witten, *Baryons and branes in Anti de Sitter space*, *JHEP* **07** (1998) 006 [hep-th/9805112] [SPIRES].
- [3] S.S. Gubser and I.R. Klebanov, *Baryons and domain walls in an $N = 1$ superconformal gauge theory*, *Phys. Rev.* **D 58** (1998) 125025 [hep-th/9808075] [SPIRES].
- [4] D. Berenstein, C.P. Herzog and I.R. Klebanov, *Baryon spectra and AdS/CFT correspondence*, *JHEP* **06** (2002) 047 [hep-th/0202150] [SPIRES].
- [5] N. Gutierrez, Y. Lozano and D. Rodriguez-Gomez, *Charged particle-like branes in ABJM*, *JHEP* **09** (2010) 101 [arXiv:1004.2826] [SPIRES].
- [6] S.A. Hartnoll, *Anyonic strings and membranes in AdS space and dual Aharonov-Bohm effects*, *Phys. Rev. Lett.* **98** (2007) 111601 [hep-th/0612159] [SPIRES].
- [7] S. Kawamoto and F.-L. Lin, *Holographic anyons in the ABJM theory*, *JHEP* **02** (2010) 059 [arXiv:0910.5536] [SPIRES].
- [8] A. Brandhuber, N. Itzhaki, J. Sonnenschein and S. Yankielowicz, *Baryons from supergravity*, *JHEP* **07** (1998) 020 [hep-th/9806158] [SPIRES].
- [9] Y. Imamura, *Baryon mass and phase transitions in large-N gauge theory*, *Prog. Theor. Phys.* **100** (1998) 1263 [hep-th/9806162] [SPIRES].
- [10] K. Sfetsos and K. Siampos, *Stability issues with baryons in AdS/CFT*, *JHEP* **08** (2008) 071 [arXiv:0807.0236] [SPIRES].
- [11] Y. Imamura, *Supersymmetries and BPS configurations on Anti-de Sitter space*, *Nucl. Phys.* **B 537** (1999) 184 [hep-th/9807179] [SPIRES].
- [12] C.G. Callan Jr., A. Guijosa and K.G. Savvidy, *Baryons and string creation from the fivebrane worldvolume action*, *Nucl. Phys.* **B 547** (1999) 127 [hep-th/9810092] [SPIRES].
- [13] R.C. Myers, *Dielectric-branes*, *JHEP* **12** (1999) 022 [hep-th/9910053] [SPIRES].
- [14] O. Aharony, A. Hashimoto, S. Hirano and P. Ouyang, *D-brane charges in gravitational duals of $2 + 1$ dimensional gauge theories and duality cascades*, *JHEP* **01** (2010) 072 [arXiv:0906.2390] [SPIRES].
- [15] D.S. Freed and E. Witten, *Anomalies in string theory with D-branes*, hep-th/9907189 [SPIRES].
- [16] J.M. Maldacena and A. Strominger, *AdS_3 black holes and a stringy exclusion principle*, *JHEP* **12** (1998) 005 [hep-th/9804085] [SPIRES].
- [17] M.B. Green, J.A. Harvey and G.W. Moore, *I-brane inflow and anomalous couplings on D-branes*, *Class. Quant. Grav.* **14** (1997) 47 [hep-th/9605033] [SPIRES].
- [18] Y.-K.E. Cheung and Z. Yin, *Anomalies, branes and currents*, *Nucl. Phys.* **B 517** (1998) 69 [hep-th/9710206] [SPIRES].
- [19] C.P. Bachas, P. Bain and M.B. Green, *Curvature terms in D-brane actions and their M-theory origin*, *JHEP* **05** (1999) 011 [hep-th/9903210] [SPIRES].

JHEP07(2011)032

[20] J.M. Maldacena, *Wilson loops in large-N field theories*, *Phys. Rev. Lett.* **80** (1998) 4859 [hep-th/9803002] [SPIRES].

[21] B. Janssen, Y. Lozano and D. Rodriguez-Gomez, *The baryon vertex with magnetic flux*, *JHEP* **11** (2006) 082 [hep-th/0606264] [SPIRES].

[22] S.-J. Rey and J.-T. Yee, *Macroscopic strings as heavy quarks in large-N gauge theory and Anti-de Sitter supergravity*, *Eur. Phys. J.* **C 22** (2001) 379 [hep-th/9803001] [SPIRES].

[23] B. Baumgartner, H. Grosse and A. Martin, *Order of levels in potential models*, *Nucl. Phys.* **B 254** (1985) 528 [SPIRES].

[24] C. Bachas, *Convexity of the quarkonium potential*, *Phys. Rev.* **D 33** (1986) 2723 [SPIRES].

[25] N. Drukker, J. Plefka and D. Young, *Wilson loops in 3-dimensional $N = 6$ supersymmetric Chern-Simons theory and their string theory duals*, *JHEP* **11** (2008) 019 [arXiv:0809.2787] [SPIRES].

[26] B. Chen and J.-B. Wu, *Supersymmetric Wilson loops in $N = 6$ super Chern-Simons-matter theory*, *Nucl. Phys.* **B 825** (2010) 38 [arXiv:0809.2863] [SPIRES].

[27] S.-J. Rey, T. Suyama and S. Yamaguchi, *Wilson loops in superconformal Chern-Simons theory and fundamental strings in Anti-de Sitter supergravity dual*, *JHEP* **03** (2009) 127 [arXiv:0809.3786] [SPIRES].

[28] T. Suyama, *On large-N solution of ABJM theory*, *Nucl. Phys.* **B 834** (2010) 50 [arXiv:0912.1084] [SPIRES].

[29] M. Mariño and P. Putrov, *Exact results in ABJM theory from topological strings*, *JHEP* **06** (2010) 011 [arXiv:0912.3074] [SPIRES].

[30] A. Kapustin, B. Willett and I. Yaakov, *Exact results for Wilson loops in superconformal Chern-Simons theories with matter*, *JHEP* **03** (2010) 089 [arXiv:0909.4559] [SPIRES].

[31] N. Drukker and D. Trancanelli, *A supermatrix model for $N = 6$ super Chern-Simons-matter theory*, *JHEP* **02** (2010) 058 [arXiv:0912.3006] [SPIRES].

[32] R. Emparan, *Born-Infeld strings tunneling to D-branes*, *Phys. Lett.* **B 423** (1998) 71 [hep-th/9711106] [SPIRES].

[33] S.P. Trivedi and S. Vaidya, *Fuzzy cosets and their gravity duals*, *JHEP* **09** (2000) 041 [hep-th/0007011] [SPIRES].

[34] B. Janssen, Y. Lozano and D. Rodriguez-Gomez, *Giant gravitons and fuzzy $CP(2)$*, *Nucl. Phys.* **B 712** (2005) 371 [hep-th/0411181] [SPIRES].

[35] Y. Lozano and D. Rodriguez-Gomez, *Fuzzy 5-spheres and pp-wave matrix actions*, *JHEP* **08** (2005) 044 [hep-th/0505073] [SPIRES].

[36] B. Janssen, Y. Lozano and D. Rodriguez-Gomez, *A KK-monopole giant graviton in $AdS_5 \times Y_5$*, *JHEP* **06** (2007) 028 [arXiv:0704.1438] [SPIRES].

[37] . Madore, *An introduction to noncommutative differential geometry and its applications*, Cambridge University Press, Cambridge U.K. (1995).

[38] U. Carow-Watamura, H. Steinacker and S. Watamura, *Monopole bundles over fuzzy complex projective spaces*, *J. Geom. Phys.* **54** (2005) 373 [hep-th/0404130] [SPIRES].

[39] B. Janssen and Y. Lozano, *A microscopical description of giant gravitons*, *Nucl. Phys.* **B 658** (2003) 281 [hep-th/0207199] [SPIRES].

JHEP07(2011)032

[40] B. Janssen, Y. Lozano and D. Rodriguez-Gomez, *A microscopical description of giant gravitons. II: the $AdS_5 \times S^5$ background*, *Nucl. Phys.* **B 669** (2003) 363 [hep-th/0303183] [SPIRES].

[41] D. Rodriguez-Gomez, *Branes wrapping black holes as a purely gravitational dielectric effect*, *JHEP* **01** (2006) 079 [hep-th/0509228] [SPIRES].

[42] M.R. Garousi and R.C. Myers, *World-volume interactions on D-branes*, *Nucl. Phys.* **B 542** (1999) 73 [hep-th/9809100] [SPIRES].

[43] T. Eguchi, P.B. Gilkey and A.J. Hanson, *Gravitation, gauge theories and differential geometry*, *Phys. Rept.* **66** (1980) 213 [SPIRES].

[44] O. Bergman and S. Hirano, *Anomalous radius shift in AdS_4/CFT_3*, *JHEP* **07** (2009) 016 [arXiv:0902.1743] [SPIRES].

[45] K. Becker, G. Guo and D. Robbins, *Higher derivative brane couplings from T-duality*, *JHEP* **09** (2010) 029 [arXiv:1007.0441] [SPIRES].

[46] M.R. Garousi, *T-duality of anomalous Chern-Simons couplings*, arXiv:1007.2118 [SPIRES].

[47] I.R. Klebanov and M.J. Strassler, *Supergravity and a confining gauge theory: duality cascades and χ_{SB}-resolution of naked singularities*, *JHEP* **08** (2000) 052 [hep-th/0007191] [SPIRES].

[48] B.E.W. Nilsson and C.N. Pope, *Hopf fibration of eleven-dimensional supergravity*, *Class. Quant. Grav.* **1** (1984) 499 [SPIRES].

[49] C.N. Pope and N.P. Warner, *An* SU(4) *invariant compactification of $D = 11$ supergravity on a stretched seven sphere*, *Phys. Lett.* **B 150** (1985) 352 [SPIRES].

[50] D. Karabali and V.P. Nair, *The effective action for edge states in higher dimensional quantum Hall systems*, *Nucl. Phys.* **B 679** (2004) 427 [hep-th/0307281] [SPIRES].

[51] Y. Abe, *Construction of fuzzy spaces and their applications to matrix models*, arXiv:1002.4937 [SPIRES].

[52] J.A. de Azcarraga, A.J. Macfarlane, A.J. Mountain and J.C. Perez Bueno, *Invariant tensors for simple groups*, *Nucl. Phys.* **B 510** (1998) 657 [physics/9706006].

JHEP07(2011)032