

Comments on supercurrent multiplets, supersymmetric field theories and supergravity

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ABSTRACT: We analyze various supersymmetry multiplets containing the supercurrent and the energy-momentum tensor. The most widely known such multiplet, the Ferrara-Zumino (FZ) multiplet, is not always well-defined. This can happen once Fayet-Iliopoulos (FI) terms are present or when the Kähler form of the target space is not exact. We present a new multiplet $\mathcal{S}_{\alpha\dot{\alpha}}$ which always exists. This understanding of the supersymmetry current allows us to obtain new results about the possible IR behavior of supersymmetric theories. Next, we discuss the coupling of rigid supersymmetric theories to supergravity. When the theory has an FZ-multiplet or it has a global R -symmetry the standard formalism can be used. But when this is not the case such simple gauging is impossible. Then, we must gauge the current $\mathcal{S}_{\alpha\dot{\alpha}}$. The resulting theory has, in addition to the graviton and the gravitino, another massless chiral superfield Φ which is essential for the consistency of the theory. Some of the moduli of various string models play the role of Φ . Our general considerations, which are based on the consistency of supergravity, show that such moduli cannot be easily lifted thus leading to constraints on gravity/string models.

KEYWORDS: Supersymmetric gauge theory, Supergravity Models, Supersymmetric Effective Theories

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1 Introduction

Supersymmetric theories¹ have a conserved supersymmetry current $S_{\mu\alpha}$

$$\partial^\mu S_{\mu\alpha} = 0 . \tag{1.1}$$

It is unique up to an improvement term of the form

$$S'_{\mu\alpha} = S_{\mu\alpha} + (\sigma_{\mu\nu})_{\alpha}^{\beta} \partial^\nu s_{\beta} . \tag{1.2}$$

Clearly, $S'_{\mu\alpha}$ is conserved and yields the same supercharge Q_{α} upon integrating over a space-like hypersurface. The supersymmetry current $S_{\mu\alpha}$ can be embedded in a supermultiplet. This multiplet should include the conserved energy-momentum tensor $T_{\mu\nu}$, which is also ambiguous due to a possible improvement of the form

$$T'_{\mu\nu} = T_{\mu\nu} + (\eta_{\mu\nu} \partial^2 - \partial_{\mu} \partial_{\nu}) t . \tag{1.3}$$

¹Throughout this note we will focus on four-dimensional theories. We will be using $\mathcal{N} = 1$ superspace, but its existence is not essential to our discussion. We simply use it to package supersymmetry multiplets in a convenient way.

The most widely known such multiplet is the Ferrara-Zumino (FZ) multiplet [1], $\mathcal{J}_{\alpha\dot{\alpha}}$. It is a real superfield² satisfying

$$\overline{D}^{\dot{\alpha}} \mathcal{J}_{\alpha\dot{\alpha}} = D_{\alpha} X, \quad \overline{D}_{\dot{\alpha}} X = 0. \quad (1.6)$$

(The component expressions of $\mathcal{J}_{\alpha\dot{\alpha}}$ and X appear below.) This multiplet includes six bosonic operators from the conserved $T_{\mu\nu}$, four bosonic operators in a (non-conserved) current $\mathcal{J}_{\mu}| = j_{\mu}$ and two bosonic operators in the complex scalar $X| = x$. Similarly, it has twelve fermionic operators in the conserved $S_{\mu\alpha}$ and its complex conjugate. As expected in a supersymmetric theory, the number of bosonic operators is the same as the number of fermionic operators.

The pair (\mathcal{J}_{μ}, X) can be transformed as

$$\begin{aligned} \mathcal{J}'_{\alpha\dot{\alpha}} &= \mathcal{J}_{\alpha\dot{\alpha}} - i\partial_{\alpha\dot{\alpha}} (\Xi - \overline{\Xi}) = \mathcal{J}_{\alpha\dot{\alpha}} + [D_{\alpha}, \overline{D}_{\dot{\alpha}}] (\Xi + \overline{\Xi}), \\ X' &= X + \frac{1}{2} \overline{D}^2 \overline{\Xi}, \\ \overline{D}_{\dot{\alpha}} \Xi &= 0. \end{aligned} \quad (1.7)$$

This preserves the defining equation (1.6) and acts on the components as improvement transformations like in (1.2), (1.3).

If $X = 0$ (or more precisely, if $X = -\frac{1}{2} \overline{D}^2 \overline{\Xi}$ for a well-defined chiral Ξ), the theory is superconformal and the bottom component of $\mathcal{J}_{\alpha\dot{\alpha}}$ is the superconformal R -symmetry. In fact, the bottom component of $\mathcal{J}_{\alpha\dot{\alpha}}$ is conserved if and only if the theory is superconformal.

Another multiplet, which is somewhat less known, exists whenever the theory has a continuous R -symmetry (see e.g. section 7 of [3]). We will refer to it as the \mathcal{R} -multiplet. Its bottom component is the conserved $U(1)_R$ current $j_{\mu}^{(R)}$. It is a real superfield, $\mathcal{R}_{\alpha\dot{\alpha}}$, satisfying

$$\begin{aligned} \overline{D}^{\dot{\alpha}} \mathcal{R}_{\alpha\dot{\alpha}} &= \chi_{\alpha}, \\ \overline{D}_{\dot{\alpha}} \chi_{\alpha} &= \overline{D}_{\dot{\alpha}} \overline{\chi}^{\dot{\alpha}} - D^{\alpha} \chi_{\alpha} = 0. \end{aligned} \quad (1.8)$$

(The component expressions of $\mathcal{R}_{\alpha\dot{\alpha}}$ and χ_{α} appear below.) Note that χ_{α} has the structure of a field strength chiral superfield. Equation (1.8) immediately implies $\partial^{\mu} \mathcal{R}_{\mu} = 0$. Therefore, $j_{\mu}^{(R)}$ is conserved. Like the FZ-multiplet, this multiplet also includes twelve bosonic operators and twelve fermionic operators.

It is often the case that a theory has several continuous R -symmetries. They differ by a continuous conserved non- R -symmetry. The latter is characterized by a real linear

²We follow the Wess and Bagger conventions [2]. A vector ℓ_{μ} , is often expressed in bi-spinor notation as

$$\ell_{\alpha\dot{\alpha}} = -2\sigma_{\alpha\dot{\alpha}}^{\mu} \ell_{\mu}, \quad \ell_{\mu} = \frac{1}{4} \overline{\sigma}_{\mu}^{\dot{\alpha}\alpha} \ell_{\alpha\dot{\alpha}}. \quad (1.4)$$

We sometimes use

$$\begin{aligned} \{D_{\alpha}, \overline{D}_{\dot{\alpha}}\} &= -2i\sigma_{\alpha\dot{\alpha}}^{\mu} \partial_{\mu} \equiv i\partial_{\alpha\dot{\alpha}}, \\ [D_{\alpha}, \overline{D}^2] &= 2i\overline{D}^{\dot{\alpha}} \partial_{\alpha\dot{\alpha}}, \quad [D^2, \overline{D}_{\dot{\alpha}}] = 2iD^{\alpha} \partial_{\alpha\dot{\alpha}}. \end{aligned} \quad (1.5)$$

superfield J ($D^2J = 0$). This ambiguity in the \mathcal{R} -multiplet is

$$\begin{aligned} \mathcal{R}'_{\alpha\dot{\alpha}} &= \mathcal{R}_{\alpha\dot{\alpha}} + [D_\alpha, \bar{D}_{\dot{\alpha}}]J, \\ \chi'_\alpha &= \chi_\alpha + \frac{3}{2}\bar{D}^2 D_\alpha J, \text{ nonnumber} \end{aligned} \tag{1.9}$$

$$D^2J = 0. \tag{1.10}$$

It affects the supercurrent and energy-momentum tensor through improvement terms (1.2), (1.3).

If a theory has an FZ-multiplet (1.6), it is easy to show that it has an exact $U(1)_R$ symmetry if and only if there exists a real and well-defined³ U such that $\bar{D}^2U = -2X$ (this normalization is for later convenience). Intuitively, U includes a non-conserved ordinary (non- R) current. The equation $\bar{D}^2U = -2X$ means that the violation of its conservation is similar to that of the R -current at the bottom of the FZ-multiplet. Therefore, the shift

$$R_{\alpha\dot{\alpha}} = \mathcal{J}_{\alpha\dot{\alpha}} + [D_\alpha, \bar{D}_{\dot{\alpha}}]U \tag{1.11}$$

leads to a conserved R -current. Indeed, it is easy to check that this current satisfies (1.8) with $\chi_\alpha = \frac{3}{2}\bar{D}^2 D_\alpha U$.

However, not every theory has such supersymmetry multiplets. First, it is clear that if the theory does not have a continuous R -symmetry, $\mathcal{R}_{\alpha\dot{\alpha}}$ does not exist. It is less obvious that the FZ-multiplet $\mathcal{J}_{\alpha\dot{\alpha}}$ is not always well-defined. It was pointed out in [4] that when the theory has Fayet-Iliopoulos terms the FZ-multiplet is not gauge invariant. We will show in section 2 that when the Kähler form of the target space is not exact the FZ-multiplet is not globally well-defined and hence does not correspond to a good operator in the theory.

This motivates us to look for another multiplet for the supersymmetry current and the energy-momentum tensor which exists in all theories. We propose to consider the multiplet⁴ $\mathcal{S}_{\alpha\dot{\alpha}}$ which “interpolates” between (1.6) and (1.8):

$$\begin{aligned} \bar{D}^{\dot{\alpha}} \mathcal{S}_{\alpha\dot{\alpha}} &= D_\alpha X + \chi_\alpha, \\ \bar{D}_{\dot{\alpha}} X &= 0, \\ \bar{D}_{\dot{\alpha}} \chi_\alpha &= \bar{D}_{\dot{\alpha}} \bar{\chi}^{\dot{\alpha}} - D^\alpha \chi_\alpha = 0. \end{aligned} \tag{1.12}$$

We will see that this multiplet exists for every supersymmetric theory. In some cases, if we can solve

$$\bar{D}^2U = -2X, \quad \text{or} \quad \bar{D}^2 D_\alpha U = -\frac{2}{3}\chi_\alpha \tag{1.13}$$

with a well-defined real U , it can be explicitly improved and reduced to either (1.8) or (1.6).

In section 2 we study the multiplet (1.12) in detail and clarify its relation to (1.6) and (1.8).

In section 3 we discuss the three multiplets (1.12), (1.6), (1.8) in simple cases and clarify when each of them exists.

³Note that formally it is always possible to solve this equation with a nonlocal real operator U .

⁴Such a multiplet was considered in [5], but was rejected as not having a conserved energy-momentum tensor. Our discussion below demonstrates that such a conserved tensor exists.

In section 4 we present field-theoretic applications of our multiplets. We review the discussion in [4] about FI-terms. We then present a similar argument for theories with nontrivial target spaces. In both cases we find that if the UV theory has neither FI-terms nor non-trivial target space topology, then it possesses an FZ-multiplet (1.6). Therefore, the low-energy theory must also have the same multiplet (since (1.6) is an operator equation). This immediately shows that FI-terms cannot be generated (even for emergent gauge groups in the IR), and also that the topology of the quantum moduli space of the theory is constrained. More explicitly, we show that starting with a renormalizable field theory without FI-terms and flowing to the IR the Kähler form of the quantum moduli space must be exact, and in particular, it cannot be compact (except of course isolated points).

In section 5 we couple supersymmetric field theories to supergravity. Here we study rigid theories whose parameters are independent of M_P and couple them to linearized supergravity at the leading order in $\frac{1}{M_P}$. This setup excludes theories with parameters of order the Planck scale such as FI-terms $\xi \sim M_P^2$ and nonlinear sigma models with $f_\pi \sim M_P$.

If the FZ-multiplet exists, it can be gauged; i.e. coupled to supergravity. This naturally gives rise to the “old minimal supergravity” [6–8] formalism. Theories without FZ-multiplets (e.g. theories with FI-terms or non-trivial target spaces) can still be coupled to supergravity using the old minimal formalism provided certain conditions are satisfied. For example, this is possible when the theory has a continuous R -symmetry. In this case the \mathcal{R} -multiplet exists and we can gauge it. The resulting theory is related to the “new minimal supergravity” [9, 10]. This way of constructing such theories leads to a new perspective on the construction of [11] and the results of [12–14] which were based on the “old minimal formalism” (see also the recent papers [15, 16]). We review the supergravity that we obtain by gauging the \mathcal{R} -multiplet in an appendix. It should be emphasized that as explained in [17], the resulting supergravity is the same as the one obtained using the old formalism.

However, gravity theories with continuous global symmetries are expected to be inconsistent. Therefore, we cannot base the consistency of the theory on the existence of an exact continuous R -symmetry. This leads us to the study of theories without an R -symmetry and without an FZ-multiplet. We emphasize that such theories cannot be coupled to minimal supergravity. The simplest possibility then is to couple the \mathcal{S} -multiplet to supergravity. This turns out to be related to “16/16 supergravity” [18–20]. We will limit ourselves to the linearized theory (leading order in $1/M_P$) and will derive the fact that in addition to the graviton and the gravitino the theory includes a propagating chiral “matter” superfield Φ (or equivalently a linear multiplet). We will study the constraints on the coupling of this superfield. In particular, the obstruction to the existence of the FZ-multiplet is that the equation $\overline{D}^2 D_\alpha U = -\frac{2}{3}\chi_\alpha$ (1.13) cannot be solved with a well-defined U . The new superfield Φ couples through the combination

$$\widehat{U} = U + \Phi + \Phi^\dagger \tag{1.14}$$

which is well-defined.

Special cases include the relation to the absence of supergravity theories with FI-terms [4] and a connection with the results of [21] about the quantization of Newton’s constant.

Section 6 summarizes our results. Here we discuss aspects of moduli stabilization and use our conclusions to constrain gravity/string models, including various string constructions like D-inflation, sequestered models, flux vacua, etc.

2 The \mathcal{S} -multiplet

The multiplet defined below is a new option for embedding the supercurrent and energy-momentum tensor in a superfield. The advantage of this multiplet is that it exists in many examples where the others do not. Its defining properties are

$$\begin{aligned}\bar{D}^{\dot{\alpha}}\mathcal{S}_{\alpha\dot{\alpha}} &= D_{\alpha}X + \chi_{\alpha}, \\ \bar{D}_{\dot{\alpha}}X &= 0, \\ \bar{D}_{\dot{\alpha}}\chi_{\alpha} &= \bar{D}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} - D^{\alpha}\chi_{\alpha} = 0.\end{aligned}\tag{2.1}$$

Clearly, this multiplet generalizes the FZ-multiplet (1.6) and the \mathcal{R} -multiplet (1.8). These special cases are obtained by setting $\chi_{\alpha} = 0$ or $X = 0$ in (2.1), respectively. In particular, the vector in the bottom component of this multiplet is typically not conserved.⁵

It is straightforward to work out the component expression for these superfields. The result after a little bit of algebra is

$$\begin{aligned}\mathcal{S}_{\mu} &= j_{\mu}^{(S)} + \theta\left(S_{\mu} - \frac{1}{\sqrt{2}}\sigma_{\mu}\bar{\psi}\right) + \bar{\theta}\left(\bar{S}_{\mu} + \frac{1}{\sqrt{2}}\bar{\sigma}_{\mu}\psi\right) + \frac{i}{2}\theta^2\partial_{\mu}x^{\dagger} - \frac{i}{2}\bar{\theta}^2\partial_{\mu}x \\ &+ (\theta\sigma^{\nu}\bar{\theta})\left(2T_{\mu\nu} - \eta_{\mu\nu}Z + \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\left(\partial^{\rho}j^{(S)\sigma} + F^{(S)\rho\sigma}\right)\right) \\ &+ \theta^2\left(\frac{i}{2}\partial_{\rho}S_{\mu}\sigma^{\rho} - \frac{i}{2\sqrt{2}}\partial_{\rho}\bar{\psi}\bar{\sigma}^{\rho}\sigma^{\mu}\right)\bar{\theta} + \bar{\theta}^2\theta\left(-\frac{i}{2}\sigma^{\rho}\partial_{\rho}\bar{S}_{\mu} + \frac{i}{2\sqrt{2}}\sigma^{\mu}\bar{\sigma}^{\rho}\partial_{\rho}\psi\right) \\ &+ \theta^2\bar{\theta}^2\left(\frac{1}{2}\partial_{\mu}\partial^{\nu}j_{\nu}^{(S)} - \frac{1}{4}\partial^2j_{\mu}^{(S)}\right),\end{aligned}\tag{2.2}$$

and

$$\begin{aligned}X &= x + \sqrt{2}\theta\psi + \theta^2\left(Z + i\partial^{\nu}j_{\nu}^{(S)}\right), \\ \chi_{\alpha} &= -i\lambda_{\alpha}^{(S)} + \left(\delta_{\alpha}^{\beta}D^{(S)} - 2i\sigma^{\rho}\bar{\sigma}^{\tau}F_{\rho\tau}^{(S)}\right)\theta_{\beta} + \theta^2\sigma_{\alpha\dot{\alpha}}^{\nu}\partial_{\nu}\bar{\lambda}^{(S)\dot{\alpha}},\end{aligned}\tag{2.3}$$

(x is the lowest component of the superfield X rather than a spacetime coordinate) satisfying the additional relations

$$\begin{aligned}D^{(S)} &= -4T_{\mu}^{\mu} + 6Z, \\ \lambda^{(S)} &= -2i\sigma^{\mu}\bar{S}_{\mu} + 3i\sqrt{2}\psi.\end{aligned}\tag{2.4}$$

In addition, the supercurrent $S_{\mu\alpha}$ is conserved and the energy-momentum tensor $T_{\mu\nu}$ is symmetric and conserved.

⁵This is reflected in two equations that follow from (2.1): $\bar{D}^2\mathcal{S}_{\alpha\dot{\alpha}} = 2i\partial_{\alpha\dot{\alpha}}X$ and $\partial^{\alpha\dot{\alpha}}\mathcal{S}_{\alpha\dot{\alpha}} = i(\bar{D}^2\bar{X} - D^2X)$. The latter equation shows that if $X = 0$ the bottom component is a conserved R -current.

We see that the multiplet includes the 12 + 12 operators in the FZ-multiplet, as well as one Weyl fermion ψ , a closed two-form $F_{\mu\nu}^{(S)}$ and a real scalar Z . Hence it has 16 + 16 physical operators. These additional 4 + 4 operators circumvent the no-go theorem of [5].

From the superfield (2.2) we can find the anticommutators

$$\begin{aligned} \{\bar{Q}_{\dot{\beta}}, S_{\mu\alpha}\} &= \sigma_{\alpha\dot{\beta}}^{\nu} \left(2T_{\mu\nu} - \frac{1}{2}\epsilon_{\nu\mu\rho\sigma} F^{(S)\rho\sigma} - i\eta_{\nu\mu} \partial^{\rho} j_{\rho}^{(S)} + i\partial_{\nu} j_{\mu}^{(S)} - \frac{1}{2}\epsilon_{\nu\mu\rho\sigma} \partial^{\rho} j^{(S)\sigma} \right), \\ \{Q_{\beta}, S_{\mu\alpha}\} &= 2i\epsilon_{\lambda\beta} (\sigma_{\mu\rho})_{\alpha}^{\lambda} \partial^{\rho} x^{\dagger}. \end{aligned} \tag{2.5}$$

Note that these anticommutators are consistent with the conservation equation $\partial^{\mu} S_{\mu\alpha} = 0$. The standard supersymmetry algebra follows provided the fields approach zero fast enough at spatial infinity and that $\int d^3x F_{ij}^{(S)}$ vanishes for all nonzero spatial i, j .

Given the operators $(\mathcal{S}_{\alpha\dot{\alpha}}, X, \chi_{\alpha})$ we can transform

$$\begin{aligned} \mathcal{S}_{\alpha\dot{\alpha}} &\rightarrow \mathcal{S}_{\alpha\dot{\alpha}} + [D_{\alpha}, \bar{D}_{\dot{\alpha}}]U, \\ X &\rightarrow X + \frac{1}{2}\bar{D}^2 U, \\ \chi_{\alpha} &\rightarrow \chi_{\alpha} + \frac{3}{2}\bar{D}^2 D_{\alpha} U, \end{aligned} \tag{2.6}$$

with any real superfield U and preserve the defining relations (2.1). This transformation shifts the energy-momentum tensor and the supersymmetry current by improvement terms

$$\begin{aligned} S_{\mu\alpha} &\rightarrow S_{\mu\alpha} - 2i(\sigma_{\mu\nu})_{\alpha}^{\beta} \partial^{\nu} U|_{\theta^{\beta}}, \\ T_{\mu\nu} &\rightarrow T_{\mu\nu} + \frac{1}{2}(\partial_{\mu}\partial_{\nu} - \eta_{\mu\nu}\partial^2)U, \end{aligned} \tag{2.7}$$

where $U = U| + \theta^{\beta}U|_{\theta^{\beta}} + \dots$.

We interpret the bottom component of $\mathcal{S}_{\alpha\dot{\alpha}}$ as an R -current which is not conserved. The $\theta\bar{\theta}$ component of U is an ordinary (non- R) current which is also not conserved. Hence, the transformation (2.6) shifts the non-conserved R -current and yields another non-conserved R -current.

We consider certain special cases:

1. If we can solve $X = -\frac{1}{2}\bar{D}^2 U$ with a well-defined (i.e. local and gauge invariant) real operator U , we can transform X away and find the \mathcal{R} -multiplet (1.8). Now, the bottom component of $\mathcal{S}_{\alpha\dot{\alpha}}$ is a conserved R -current. Conversely, if the theory has an exact $U(1)_R$ symmetry, the \mathcal{R} -multiplet (1.8) exists and therefore we can solve $X = -\frac{1}{2}\bar{D}^2 U$ in terms of a well-defined U . Therefore, we interpret an X which cannot be written as $\bar{D}^2 U$ with a real operator U as the obstruction to having an R -symmetry. Note that the remaining freedom in (2.6) which preserves $X = 0$ restricts U to satisfy $D^2 U = 0$, i.e. U is a conserved current multiplet. This has the effect of shifting the conserved R -current $j_{\mu}^{(R)}$ by a conserved non- R -current, as we explained around (1.10).
2. If we can solve $\chi_{\alpha} = -\frac{3}{2}\bar{D}^2 D_{\alpha} U$ with a well-defined real operator U , we can transform χ_{α} away and find the FZ-multiplet (1.6). The remaining freedom which preserves $\chi_{\alpha} = 0$ restricts U to the form $\Xi + \bar{\Xi}$ with a chiral Ξ . This is the ambiguity

in the FZ-multiplet we explained around (1.7). Hence we interpret a χ_α which cannot be written as $-\frac{3}{2}\overline{D}^2 D_\alpha U$ as the obstruction to the existence of the FZ-multiplet.

3. If we can write both $X = -\frac{1}{2}\overline{D}^2 U$ and $\chi_\alpha = -\frac{3}{2}\overline{D}^2 D_\alpha \tilde{U}$ but with $U \neq \tilde{U}$, both the FZ-multiplet and the \mathcal{R} -multiplet exist. In this case we can simply transform from one to the other

$$\mathcal{R}_{\alpha\dot{\alpha}} = \mathcal{J}_{\alpha\dot{\alpha}} + [D_\alpha, \overline{D}_{\dot{\alpha}}](U - \tilde{U}) . \tag{2.8}$$

This is equivalent to the discussion around (1.11).

4. If we can simultaneously solve $X = -\frac{1}{2}\overline{D}^2 U$ and $\chi_\alpha = -\frac{3}{2}\overline{D}^2 D_\alpha U$ (with the *same* U), we can set both X and χ_α to zero. Then the theory is superconformal.

3 Examples

3.1 Wess-Zumino models

As an example, let us first discuss the general sigma model, with Kähler potential $K(\Phi^i, \overline{\Phi}^{\bar{i}})$ and superpotential $W(\Phi^i)$. The expressions for $\mathcal{J}_{\alpha\dot{\alpha}}$ and X are

$$\begin{aligned} \mathcal{J}_{\alpha\dot{\alpha}} &= 2g_{i\bar{i}}(D_\alpha \Phi^i)(\overline{D}_{\dot{\alpha}} \overline{\Phi}^{\bar{i}}) - \frac{2}{3}[D_\alpha, \overline{D}_{\dot{\alpha}}]K , \\ X &= 4W - \frac{1}{3}\overline{D}^2 K . \end{aligned} \tag{3.1}$$

Kähler transformations shift $K \rightarrow K + \Lambda + \overline{\Lambda}$ with a chiral Λ . Of course, these transformations do not affect the physics; they change (3.1) by improvement transformations as in (1.7) with $\Xi = -\frac{2}{3}\Lambda$. The bottom component of $\mathcal{J}_{\alpha\dot{\alpha}}$ is an R -current. It includes a term which is bilinear in fermions and a purely bosonic term. The term bilinear in fermions is manifestly invariant under Kähler transformations. The bosonic part

$$j_\mu^{\text{bosonic}} = \frac{2i}{3} \left(\partial_\mu \phi^i \partial_i K - \partial_\mu \overline{\phi}^{\bar{i}} \partial_{\bar{i}} K \right) \tag{3.2}$$

is not invariant under Kähler transformations. This has the following geometric interpretation. The Kähler form $\omega \sim \partial_i \partial_{\bar{j}} K d\phi^i \wedge d\overline{\phi}^{\bar{j}}$ is globally well-defined. Locally it can be expressed in terms of the Kähler connection $\mathcal{A} \sim i\partial_i K d\phi^i - i\partial_{\bar{i}} K d\overline{\phi}^{\bar{i}}$ as $\omega = d\mathcal{A}$. Hence, we identify j_μ^{bosonic} as the pullback of \mathcal{A} to spacetime.

We learn that when ω is not exact, \mathcal{A} is not globally well-defined and hence the current j_μ is not a good operator. In this case, the whole FZ-multiplet is not well-defined. For example, if the target space has 2-cycles with non-vanishing integral of the Kähler form ω , the FZ-multiplet does not exist.

A point of clarification is in order here. If we can find a globally well-defined \mathcal{A} there is still freedom in performing Kähler transformations which affect the FZ-multiplet by improvement terms. The global obstruction we discuss here arises only when we must cover the target space with patches with nontrivial Kähler transformations between them.

We conclude that theories with a Kähler form that is not exact do not have an FZ-multiplet.

If the theory has a $U(1)_R$ symmetry (either spontaneously broken or not), we expect to find a globally well-defined $\mathcal{R}_{\alpha\dot{\alpha}}$ -multiplet. Let us see how this comes out. We can use a basis where our chiral superfields Φ_i have well-defined R -charges, R_i . The condition that there is an R -symmetry implies the following two constraints

$$\sum_i R_i \Phi^i \partial_i W = 2W, \quad \sum_i R_i \Phi^i \partial_i K = \sum_{\bar{i}} R_i \bar{\Phi}^{\bar{i}} \partial_{\bar{i}} K. \quad (3.3)$$

By writing $W = \frac{1}{2} \sum_i R_i \Phi^i \partial_i W$ and using the equations of motion

$$\bar{D}^2 \partial_i K = 4 \partial_i W, \quad (3.4)$$

we can express

$$X = 4W - \frac{1}{3} \bar{D}^2 K = \bar{D}^2 \left(\frac{1}{2} \sum_i R_i \Phi^i \partial_i K - \frac{1}{3} K \right). \quad (3.5)$$

Note that $\frac{1}{2} \sum_i R_i \Phi^i \partial_i K - \frac{1}{3} K$ is a real superfield because of the second constraint in (3.3).

Now we can perform the shift (2.8) and obtain the \mathcal{R} -multiplet. This leads to

$$\begin{aligned} \mathcal{R}_{\alpha\dot{\alpha}} &= 2g_{i\bar{j}} D_\alpha \Phi^i \bar{D}_{\dot{\alpha}} \bar{\Phi}^{\bar{j}} - [D_\alpha, \bar{D}_{\dot{\alpha}}] \sum_i R_i \Phi^i \partial_i K, \\ \chi_\alpha &= \bar{D}^2 D_\alpha \left(K - \frac{3}{2} \sum_i R_i \Phi^i \partial_i K \right). \end{aligned} \quad (3.6)$$

These operators are invariant under all Kähler transformations which preserve the R -symmetry. Therefore, even if the target space has a non-exact Kähler form, if the theory has an R -symmetry, the multiplet $\mathcal{R}_{\alpha\dot{\alpha}}$ is well-defined. Hence, the supersymmetry current and the energy-momentum tensor in this \mathcal{R} -multiplet are good operators.

Finally, let us discuss the most general case in which the target space has a nontrivial Kähler form and the theory does not have an R -symmetry. Our motivation is that we would like to eventually discuss supergravity, where exact continuous global symmetries are expected to be forbidden.

In this case neither the FZ-multiplet nor the \mathcal{R} -multiplet exist, but our $\mathcal{S}_{\alpha\dot{\alpha}}$ exists. Indeed, the operators

$$\begin{aligned} \mathcal{S}_{\alpha\dot{\alpha}} &= 2g_{i\bar{i}} (D_\alpha \Phi^i) (\bar{D}_{\dot{\alpha}} \bar{\Phi}^{\bar{i}}), \\ X &= 4W, \\ \chi_\alpha &= \bar{D}^2 D_\alpha K, \end{aligned} \quad (3.7)$$

are globally well-defined and satisfy (2.1). For example, $\mathcal{S}_{\alpha\dot{\alpha}}$ depends on the Kähler potential only through the Kähler metric, which is invariant under Kähler transformations.

The bottom component of $\mathcal{S}_{\alpha\dot{\alpha}}$ is an R -current under which all the chiral superfields have vanishing charge. It is not conserved unless $W = 0$. If $W = 0$, $\mathcal{S}_{\alpha\dot{\alpha}}$ coincides with the \mathcal{R} -multiplet (3.6). If $W \neq 0$, X measures the violation of the divergence of $j_\mu^{(S)}$.

3.2 Gauge fields with FI terms

We now consider a theory with a U(1) gauge field with an FI-term

$$\mathcal{L} = \dots + \int d^4\theta \xi V . \tag{3.8}$$

This case is easily handled by the substitution $K \rightarrow K + \xi V$ in the expressions (3.1), (3.6), (3.7)

$$\begin{aligned} \mathcal{J}_{\alpha\dot{\alpha}} &= \dots - \frac{2\xi}{3} [D_\alpha, \bar{D}_{\dot{\alpha}}] V , \\ X &= \dots - \frac{\xi}{3} \bar{D}^2 V , \\ \chi_\alpha &= \dots - 4\xi W_\alpha . \end{aligned} \tag{3.9}$$

From (3.6), (3.7) we see that $\mathcal{R}_{\alpha\dot{\alpha}}$ and $\mathcal{S}_{\alpha\dot{\alpha}}$ do not have explicit ξ dependence. They depend on ξ through the equations of motion.

Let us emphasize the analogy between an FI-term and nontrivial geometry. When ξ is nonzero the multiplet $\mathcal{J}_{\alpha\dot{\alpha}}$ is not gauge invariant [4]. If the theory has nonzero ξ but it has an R -symmetry, $\mathcal{R}_{\alpha\dot{\alpha}}$ is a good gauge invariant operator [15, 16, 22]. However, if $\xi \neq 0$ and the theory does not have an R -symmetry, we must use the multiplet $\mathcal{S}_{\alpha\dot{\alpha}}$. It includes gauge invariant and conserved $S_{\mu\alpha}$ and $T_{\mu\nu}$.

The similarities between the situation with a nontrivial target space and when there is a nonzero ξ are easily understood by considering a simple example. A U(1) gauge theory with n chiral superfields with charge one and negative ξ has as its classical moduli space of vacua \mathbf{CP}^{n-1} . (In four dimensions this theory is quantum mechanically anomalous, but this is irrelevant for this reasoning). The parameter ξ controls the size of the space. The peculiarities of the FI-term in the microscopic description which includes the gauge field translate to nontrivial transition functions in the macroscopic theory. Hence $\mathcal{J}_{\alpha\dot{\alpha}}$ is not gauge invariant in the short distance theory and it is not globally well-defined in the low-energy theory.

4 Applications to field theory

In the previous section we explained that theories with non-exact Kähler form or with an FI-term do not have a well-defined FZ-multiplet. This fact can be used to prove some non-renormalization theorems. Let us first review the argument in [4] for the FI-term.

A theory that has no FI-term gives rise to a well-defined FZ-multiplet satisfying the operator equation (1.6). Since this operator is well-defined, it behaves regularly along the renormalization group flow. This immediately implies that no FI-term can be generated for the original gauge group and even for gauge groups that emerge from the dynamics. This explains why models of SUSY breaking predominantly break SUSY through F -terms.

We can repeat the same idea for the moduli space. In the UV, we usually start from weakly interacting particles with canonical kinetic terms. Therefore, the Kähler metric is trivial and the FZ-multiplet exists. Since this multiplet must remain well-defined throughout the flow, it follows that the quantum moduli space is constrained. It has to

be such that the Kähler form $\omega \sim d\mathcal{A}$ is exact; i.e. \mathcal{A} is a globally well-defined. Hence the integral $\int \omega \wedge \omega \wedge \omega \cdots$ over any compact cycle must vanish. In particular, this means that the whole target space cannot be compact (it can, of course, be a set of points).⁶

Let us see how this works in the case of SQCD with $N_f = N_c$.⁷ The short distance theory is characterized by the classical moduli space

$$\mathcal{M}_c = \{M, B, \tilde{B} \mid \det M - B\tilde{B} = 0\} . \tag{4.1}$$

At long distance the theory flows to a theory of mesons and baryons with the quantum deformed moduli space [24]

$$\mathcal{M} = \{M, B, \tilde{B} \mid \det M - B\tilde{B} = \Lambda^{2N_c}\} . \tag{4.2}$$

We see that the topology of the moduli space changes. However, in accordance with the general result above, the Kähler form of \mathcal{M} is exact. In fact we can argue that even the Kähler potential on \mathcal{M} is single valued and is simply inherited from a well-defined Kähler potential in the embedding space parameterized by M, B, \tilde{B} . It is instructive to consider the theory with $N_f = N_c + 1$ with N_c massless quarks and a single light quark. This theory is described by a smooth Kähler potential for the mesons and the baryons [24]. Near the origin it is approximately canonical. Clearly, the massless modes in this theory are on the moduli space \mathcal{M} with a globally well-defined Kähler potential (which can actually be extended to the full embedding space). As we increase the mass of the light quark to infinity the Kähler potential changes but it remains well-defined. In the limit of infinitely large mass this is the Kähler potential of the $N_f = N_c$ theory on \mathcal{M} .

It is instructive to compare these nonrenormalization theorems to those about the FI-term. Three approaches to these nonrenormalization theorems are possible.

1. Both nonrenormalization theorems follow from the fact that the FZ-multiplet is not well-defined. This constrains the radiative corrections and the renormalization group flow in such theories. In both cases it prevents us from finding a macroscopic theory with a nonzero FI-term or non-exact Kähler form if they are absent in the short distance theory. This is the approach we have taken in this section.
2. The authors of [25, 26] followed [27] and promoted all coupling constants to background fields. The inability to do this for the FI-term leads to its non-renormalization.⁸ We can follow this approach also for the Kähler potential K . We introduce a coupling constant \hbar by replacing $K \rightarrow \frac{1}{\hbar}K$. If K is globally well-defined, we do not need to use Kähler transformations as we move from patch to patch. In this case we can trivially extend $\frac{1}{\hbar}$ to a real superfield (or to a chiral plus an antichiral superfield) and find complicated higher order radiative corrections. However, if we

⁶An argument for the non-compactness of moduli space has also been put forward by Witten, as referred to in [23]. It is based on supergravity considerations and the discussion in [21]. It is similar in spirit to our discussion here, which is purely field theoretical.

⁷We thank E. Witten for a useful discussion about this point.

⁸For an earlier related approach see [28].

need to cover the target space by patches which are related to each other by Kähler transformations, then $\frac{1}{\hbar}$ cannot be promoted to a background superfield; this would ruin the invariance of the Lagrangian under Kähler transformations⁹ Therefore, radiative corrections can arise only at one loop.¹⁰

3. Similar nonrenormalization theorems can be derived by weakly coupling the theory to supergravity and by using the non-existence of certain supergravity theories. We will discuss such supergravity theories in section 5 and in the appendix.

5 Coupling to supergravity

In this section we study the coupling to supergravity of the various supercurrent multiplets we presented above. We are only interested in linearized supergravity, namely the leading order in $\frac{1}{M_p}$. This approach to supergravity is taken, for example, in [32]. We begin with a review of the coupling of the FZ-multiplet to supergravity. We then explain the coupling of the \mathcal{S} -multiplet to supergravity. The case of the \mathcal{R} -multiplet is reviewed in the appendix.

5.1 Gauging the FZ-multiplet

We start by reviewing the coupling of the FZ-multiplet to linearized gravity. The FZ-multiplet (1.6) contains a conserved energy-momentum tensor and supercurrent and can therefore be coupled to supergravity. The supergravity multiplet is embedded in a real vector superfield $H_{\alpha\dot{\alpha}}$. The $\theta\bar{\theta}$ component of $H_{\alpha\dot{\alpha}}$ contains the metric field, $h_{\mu\nu}$, a two form field $B_{\mu\nu}$, and a real scalar. The coupling of gravity to matter is dictated at leading order by

$$\int d^4\theta \mathcal{J}_{\alpha\dot{\alpha}} H^{\alpha\dot{\alpha}} . \tag{5.1}$$

We should impose gauge invariance, namely, the invariance under coordinate transformations and local supersymmetry transformations. The gauge parameters are embedded in a complex superfield L_α , which so far obey no constraints. We assign a transformation law to the supergravity fields of the form

$$H'_{\alpha\dot{\alpha}} = H_{\alpha\dot{\alpha}} + D_\alpha \bar{L}_{\dot{\alpha}} - \bar{D}_{\dot{\alpha}} L_\alpha , \tag{5.2}$$

where $\bar{L}_{\dot{\alpha}}$ is the complex conjugate of L_α , and thus this maintains the reality condition.

Requiring that (5.1) be invariant under these coordinate transformations, we get a constraint on the superfield L_α . Indeed, invariance requires that $0 = \int d^4\theta \bar{D}^{\dot{\alpha}} \mathcal{J}_{\alpha\dot{\alpha}} L_\alpha = \int d^4\theta X D^\alpha L_\alpha$. Since X is an unconstrained chiral superfield we get the complex equation¹¹

$$\bar{D}^2 D^\alpha L_\alpha = 0 . \tag{5.3}$$

⁹The situation in $\mathcal{N} = 2$ supersymmetry in two dimensions is a bit different. Here both the coefficient of the FI-term and $\frac{1}{\hbar}$ in the case with nontrivial geometry can be promoted to the real part of a twisted chiral superfield. This allows us to write a supersymmetric effective action for these coupling constants. Such an analysis leads to a simple derivation [29] of the nonrenormalization theorems of [30, 31] about radiative corrections to the Kähler metric in sigma-models.

¹⁰In fact, in four dimensions these corrections are quadratically divergent and therefore ambiguous.

¹¹Equivalently, we can consider an unconstrained superfield L_α and add a compensator to cancel the variation.

The analog of the Wess-Zumino gauge is that the lowest components of H_μ vanish, i.e.

$$H_\mu| = H_\mu|_\theta = H_\mu|_{\bar{\theta}} = 0, \tag{5.4}$$

as well as the fact that $H_\mu|_{\theta\sigma\nu\bar{\theta}}$ is symmetric in μ and ν .

There is also some residual gauge freedom:

1. $H_\mu|_{\theta^2}$ can be shifted by any complex divergenceless vector. This leaves only one complex degree of freedom, $\partial^\mu H_\mu|_{\theta^2}$.
2. The metric field $h_{\mu\nu}$ transforms as

$$\delta h_{\mu\nu} = \partial_\nu \xi_\mu + \partial_\mu \xi_\nu, \tag{5.5}$$

where ξ_μ is a real vector.

3. The gravitino transforms as

$$\delta \Psi_{\mu\alpha} = \partial_\mu \omega_\alpha. \tag{5.6}$$

In this Wess-Zumino gauge the components containing the gravitino and metric take the form

$$H_\mu|_{\theta\sigma\nu\bar{\theta}} = h_{\mu\nu} - \eta_{\mu\nu} h, \tag{5.7}$$

and

$$H_\mu|_{\bar{\theta}^2\theta} = \Psi_{\mu\alpha} + \sigma_\mu \bar{\sigma}^\rho \Psi_\rho. \tag{5.8}$$

The top component of H_μ is a vector field which survives in the Wess-Zumino gauge. The bosonic off-shell degrees of freedom in H_μ consist of the complex scalar $\partial^\mu H_\mu|_{\theta^2}$, six real degrees of freedom in the graviton and the four real degrees of freedom in the top component of H_μ , for a total of 12 off-shell bosons. For the fermions, we have only the gravitino. It has $16-4 = 12$ off-shell degrees of freedom. This is the old minimal multiplet of supergravity [6–8]. This is in accordance with the 12 degrees of freedom in the FZ-multiplet.

A simple consistency check is to use (5.1) to check the leading couplings of the graviton and gravitino to matter. Recalling the formula for $\mathcal{J}_{\alpha\dot{\alpha}}$ (use (2.2) with $\chi_\alpha = 0$) we find

$$\mathcal{L}_{\text{graviton-matter}} \sim (h_{\mu\nu} - \eta_{\mu\nu} h) \left(T^{\mu\nu} - \frac{1}{3} \eta^{\mu\nu} T \right) = h_{\mu\nu} T^{\mu\nu}, \tag{5.9}$$

as expected. Similarly, for the coupling of the gravitino to matter we get

$$\mathcal{L}_{\text{gravitino-matter}} \sim \epsilon^{\alpha\beta} (\Psi_{\mu\alpha} + \sigma_\mu \bar{\sigma}^\rho \Psi_\rho) \left(S_\beta^\mu + \frac{1}{3} \sigma^\mu \bar{\sigma}^\rho S_\rho \right) = \Psi_{\mu\alpha} S^{\mu\alpha}. \tag{5.10}$$

We would also like to mention that in analogy with the situation in ordinary curved space, improvements of $\mathcal{J}_{\alpha\dot{\alpha}}$ as in (1.7) shift the coupling to gravity (5.1) by a term proportional to $\int d^4\theta (\Xi + \bar{\Xi}) [D_\alpha, \bar{D}_{\dot{\alpha}}] H^{\alpha\dot{\alpha}}$.

The last ingredient is the kinetic term for the graviton and gravitino. We begin by constructing a real superfield $E_{\alpha\dot{\alpha}}^{\text{FZ}}$ by covariantly differentiating $H_{\alpha\dot{\alpha}}$

$$E_{\alpha\dot{\beta}}^{\text{FZ}} = \bar{D}_{\dot{\tau}} D^2 \bar{D}^{\dot{\tau}} H_{\alpha\dot{\beta}} + \bar{D}_{\dot{\tau}} D^2 \bar{D}_{\dot{\beta}} H_{\alpha}^{\dot{\tau}} + D^\gamma \bar{D}^2 D_\alpha H_{\gamma\dot{\beta}} - 2\partial_{\alpha\dot{\beta}} \partial^{\gamma\dot{\tau}} H_{\gamma\dot{\tau}}. \tag{5.11}$$

This real expression¹² is equivalent to a different-looking expression in [33]. The gauge transformations (5.2) act as

$$E_{\alpha\dot{\beta}}^{\text{FZ}} = E_{\alpha\dot{\beta}}^{\text{FZ}} + [D_{\alpha}, \bar{D}_{\dot{\beta}}] \left(D^2 \bar{D}_{\dot{\alpha}} \bar{L}^{\dot{\alpha}} + \bar{D}^2 D^{\beta} L_{\beta} \right). \quad (5.12)$$

Note the similarity to the improvement transformations (1.7). We see that $E_{\alpha\dot{\beta}}^{\text{FZ}}$ is invariant if (5.3) is imposed.

$E_{\alpha\dot{\beta}}^{\text{FZ}}$ satisfies another important algebraic equation,

$$\bar{D}^{\dot{\beta}} E_{\alpha\dot{\beta}}^{\text{FZ}} = D_{\alpha} \left(\bar{D}^2 [D^{\gamma}, \bar{D}^{\dot{\gamma}}] H_{\gamma\dot{\gamma}} \right). \quad (5.13)$$

The superfield in parenthesis is chiral. Note the similarity of (5.13) to the defining property of the FZ-multiplet itself (1.6). The fact that E^{FZ} is invariant and satisfies an equation identical to the supercurrent superfield guarantees that the Lagrangian

$$\mathcal{L}_{\text{kinetic}} \sim M_P^2 \int d^4\theta H^{\mu} E_{\mu}^{\text{FZ}} \quad (5.14)$$

is invariant. This contains in components the linearized Einstein and Rarita-Schwinger terms. The six additional supergauge-invariant bosons, $\partial^{\mu} H_{\mu}|_{\theta^2}$, $H_{\mu}|_{\theta^4}$ are auxiliary fields which are easily integrated out yielding $\partial^{\mu} H_{\mu}|_{\theta^2} \sim ix$, $H_{\mu}|_{\theta^4} \sim j_{\mu}$ where x and j_{μ} are the matter operators in the supercurrent multiplet.

We conclude that theories which have a well-defined FZ-multiplet can be coupled to supergravity in this fashion. The coupling to supergravity adds to the original theory a propagating graviton and gravitino.

If there is no FZ-multiplet but there is an R -symmetry, one can still use the ill defined FZ-multiplet by slightly modifying the gauging procedure to construct a consistent supergravity theory. Alternatively, in this case we can construct the \mathcal{R} -multiplet and couple it to supergravity.¹³ For example, a free supersymmetric U(1) theory with an FI-term can be coupled to supergravity in this fashion, thus reproducing the component Lagrangian of [11]. This gives rise to a supergravity theory with a continuous global R -symmetry (unless there are no charged fields in the spectrum). This explains in a simple fashion the results about FI-terms [4, 11–14]. We expect that consistent theories of quantum gravity do not have such continuous symmetries. Hence, we will not pursue theories with an exact $U(1)_R$ symmetry here, but will describe them in the appendix.

5.2 Supergravity from the \mathcal{S} -multiplet

We emphasized above that various supersymmetric field theories do not have an FZ-multiplet and the energy-momentum tensor and the supersymmetry current must be embedded in a larger multiplet $S_{\alpha\dot{\alpha}}$. In such a case the only possible supergravity theory is the one in which this (or a larger) multiplet is gauged. In this section we analyze this theory and as in the previous subsection, we limit ourselves to the analysis of the linearized

¹²To see that the first term in (5.11) is real one can use $D^{\alpha} \bar{D}^2 D_{\alpha} = \bar{D}_{\dot{\alpha}} D^2 \bar{D}^{\dot{\alpha}}$.

¹³For some comments on this case see also [15, 16].

theory. We will see that this supergravity theory is not merely a different set of auxiliary fields, there are new on-shell modes.

We begin from the coupling to matter

$$\int d^4\theta \mathcal{S}_{\alpha\dot{\alpha}} H^{\alpha\dot{\alpha}} . \tag{5.15}$$

For this to be invariant under (5.2), we need to impose the constraints

$$\bar{D}^2 D^\alpha L_\alpha = 0, \quad \bar{D}_{\dot{\alpha}} D^2 \bar{L}^{\dot{\alpha}} = D_\alpha \bar{D}^2 L^\alpha . \tag{5.16}$$

The first of them already appeared in the gauging of the FZ-multiplet (5.3) and the second one is shown in the appendix to arise in the gauging of the \mathcal{R} -multiplet. Since L_α is more constrained here than in the previous subsection, we will find more gauge invariant degrees of freedom.

Using an arbitrary L_α subject to these constraints we can choose the Wess-Zumino gauge

$$H_\mu| = H_\mu|_\theta = H_\mu|_{\bar{\theta}} = 0 . \tag{5.17}$$

The residual gauge transformations allow us to transform $H_\mu|_{\theta^2}$ by any divergenceless vector so we remain with one complex gauge invariant operator $\partial^\mu H_\mu|_{\theta^2}$. The transformation law

$$\delta H_\mu|_{\theta\sigma\nu\bar{\theta}} + \delta H_\nu|_{\theta\sigma\mu\bar{\theta}} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu, \quad \partial^\nu \xi_\nu = 0 . \tag{5.18}$$

This means that the trace part of this symmetric tensor is invariant under the residual symmetries and therefore, the $\theta\bar{\theta}$ component contains the usual graviton but also an additional invariant scalar. The antisymmetric analog of (5.18) enjoys the usual gauge transformation for a two-form

$$\delta B_{\mu\nu} = \partial_\nu \omega_\mu - \partial_\mu \omega_\nu . \tag{5.19}$$

We also note that the top component of H_μ is invariant. Thus, we see that we have 16 off-shell bosonic degrees of freedom. The fermion is in the $\theta^2\bar{\theta}$ component (and its complex conjugate). It has residual gauge symmetry

$$\delta \Psi_{\mu\alpha} = i\partial_\mu \omega_\alpha, \quad \sigma_{\alpha\dot{\alpha}}^\mu \partial_\mu \bar{\omega}^{\dot{\alpha}} = 0 . \tag{5.20}$$

Since ω_α satisfies the Dirac equation it cannot be used to set any further components to zero. This is analogous to the discussion about the metric (5.18). Therefore, our theory includes a gravitino as well as an additional Weyl fermion. Thus, we have 16 off-shell fermionic degrees of freedom.

We conclude that the theory has 16 + 16 fields. This is in accord with the 16 + 16 operators in the multiplet $\mathcal{S}_{\alpha\dot{\alpha}}$. This (16, 16) supergravity multiplet has been recognized in the supergravity literature [18, 19].¹⁴ We will explain some of its important features below and then turn to derive some consequences.

¹⁴For an early discussion see also [34].

It is easy to construct a kinetic term; in fact $E_{\alpha\dot{\beta}}^{\text{FZ}}$ defined in (5.11) is still invariant because the set of transformations here is smaller than when the FZ-multiplet is gauged. However, This theory has another invariant. It is easy to see that

$$\delta \left([D^\beta, \bar{D}^{\dot{\beta}}] H_{\beta\dot{\beta}} \right) = \frac{3}{2} \left(D^\gamma \bar{D}^2 L_\gamma + \bar{D}_{\dot{\gamma}} D^2 \bar{L}^{\dot{\gamma}} \right) + \frac{1}{2} \left(\bar{D}^2 D^\gamma L_\gamma + D^2 \bar{D}_{\dot{\gamma}} \bar{L}^{\dot{\gamma}} \right) \quad (5.21)$$

vanishes upon imposing the constraints (5.16). Thus, $[D^\beta, \bar{D}^{\dot{\beta}}] H_{\beta\dot{\beta}}$ is invariant. We can use this observation to write an invariant kinetic term

$$\int d^4\theta H^{\alpha\dot{\alpha}} [D_\alpha, \bar{D}_{\dot{\alpha}}] [D^\beta, \bar{D}^{\dot{\beta}}] H_{\beta\dot{\beta}} = \int d^4\theta ([D, \bar{D}] H)^2 . \quad (5.22)$$

To summarize, we find that this theory admits two independent kinetic terms. Thus there is one free real parameter, r , and the most general kinetic term is¹⁵

$$\int d^4\theta \left(H^{\alpha\dot{\alpha}} E_{\alpha\dot{\alpha}}^{\text{FZ}} + \frac{1}{2r} H^{\alpha\dot{\alpha}} [D_\alpha, \bar{D}_{\dot{\alpha}}] [D^\beta, \bar{D}^{\dot{\beta}}] H_{\beta\dot{\beta}} \right) . \quad (5.23)$$

Our goal now is to identify the on-shell degrees of freedom in this theory and study their couplings to matter fields. One possibility is to substitute the most general $H_{\alpha\dot{\alpha}}$ in (5.15) and (5.23). Then we can identify the auxiliary fields and integrate them out. This is the approach we took in the previous subsection. Alternatively, we can enlarge the gauge symmetry, relaxing either one of the two constraints (5.16) or both, and add compensator fields. This makes the results more transparent and hence we will follow this approach here.

In order to contrast the situation with that in the previous subsection we choose to keep the constraint (5.3) (the first one in (5.16)) and relax the second one by adding a chiral compensator field λ_α which transforms as

$$\delta\lambda_\alpha = \frac{3}{2} \bar{D}^2 L_\alpha . \quad (5.24)$$

First, the non-invariance of the coupling to matter $\int d^4\theta H^{\alpha\dot{\alpha}} \mathcal{S}_{\alpha\dot{\alpha}}$ can be corrected by adding to the Lagrangian the term $-\frac{1}{6} \int d^2\theta \lambda^\alpha \chi_\alpha + c.c.$. Next, we move to the kinetic terms (5.23). The first term is invariant, but the second term is not. This is easily fixed by adding more terms to the Lagrangian. We end up with the invariant Lagrangian

$$\begin{aligned} \mathcal{L} = & \int d^4\theta \left(H^{\alpha\dot{\alpha}} E_{\alpha\dot{\alpha}}^{\text{FZ}} + \frac{1}{2r} H^{\alpha\dot{\alpha}} [D_\alpha, \bar{D}_{\dot{\alpha}}] ([D, \bar{D}] H) + H^{\alpha\dot{\alpha}} \mathcal{S}_{\alpha\dot{\alpha}} \right) \\ & - \left(\frac{1}{6} \int d^2\theta \lambda^\alpha \chi_\alpha + c.c. \right) \\ & - \frac{1}{r} \int d^4\theta \left((D^\gamma \lambda_\gamma + \bar{D}_{\dot{\gamma}} \bar{\lambda}^{\dot{\gamma}}) [D, \bar{D}] H - \frac{1}{2} (D^\gamma \lambda_\gamma + \bar{D}_{\dot{\gamma}} \bar{\lambda}^{\dot{\gamma}})^2 \right) . \quad (5.25) \end{aligned}$$

The first term in the second line corrects the non-invariance of the coupling to matter and the other two terms fix the transformation of the kinetic term (5.22).

¹⁵In order not to clutter the equations we set $M_p = 1$ and we suppress an overall constant in front of the Lagrangian.

In order to display the spectrum of (5.25) we introduce $G = D^\gamma \lambda_\gamma + \bar{D}_{\dot{\gamma}} \bar{\lambda}^{\dot{\gamma}}$ which is a real linear superfield (i.e. it satisfies $D^2 G = 0$). We also express $\chi_\alpha = -\frac{3}{2} \bar{D}^2 D_\alpha U$, with a real U . We should remember that this U might not be well-defined; e.g. it might not be globally well-defined or might not be gauge invariant. In fact, the need of gauging the \mathcal{S} -multiplet arises precisely when this U is not well-defined. The Lagrangian (5.25) becomes

$$\mathcal{L} = \int d^4\theta \left(H^{\alpha\dot{\alpha}} E_{\alpha\dot{\alpha}}^{\text{FZ}} + \frac{1}{2r} H^{\alpha\dot{\alpha}} [D_\alpha, \bar{D}_{\dot{\alpha}}] [D, \bar{D}] H + H^{\alpha\dot{\alpha}} \mathcal{S}_{\alpha\dot{\alpha}} \right) - \frac{1}{r} \int d^4\theta \left(G ([D, \bar{D}] H - rU) - \frac{1}{2} G^2 \right). \quad (5.26)$$

Now we can dualize G . This is done by viewing it as an arbitrary real superfield and imposing the constraint $D^2 G = 0$ by a Lagrange multiplier term $\int d^4\theta (\Phi + \Phi^\dagger) G$ where Φ is a chiral superfield which is invariant under the supergauge transformations subject to the constraint (5.3). This makes it easy to integrate out G using its equation of motion

$$G = r^2 (\Phi + \Phi^\dagger) + r^2 U + [D, \bar{D}] H \quad (5.27)$$

to find the Lagrangian

$$\mathcal{L} = \int d^4\theta \left(H^{\alpha\dot{\alpha}} E_{\alpha\dot{\alpha}}^{\text{FZ}} + (\Phi + \Phi^\dagger + U) [D, \bar{D}] H - \frac{r}{2} (\Phi + \Phi^\dagger + U)^2 + H^{\alpha\dot{\alpha}} \mathcal{S}_{\alpha\dot{\alpha}} \right). \quad (5.28)$$

In this presentation the theory looks like a standard supergravity theory based on the FZ-multiplet which is coupled to a matter system which includes the original matter as well as the chiral superfield Φ . This is consistent with the counting of degrees of freedom (4 + 4 degrees of freedom in addition to ordinary supergravity) and with the identification [20] of the 16/16 supergravity as an ordinary supergravity coupled to a chiral superfield. Note that even though the new superfield Φ originated from the gravity multiplet, its couplings are not completely determined. At the linear order we have freedom in the dimensionless parameter r and we expect additional freedom at higher orders.

The linear multiplet G in (5.26) or equivalently the chiral superfield Φ in (5.28) are easily recognized as the dilaton multiplet in string theory. There the graviton and the gravitino are accompanied by a dilaton, a two-form field and a fermion (dilatino). These are the degrees of freedom in G . After a duality transformation this multiplet turns into a chiral superfield Φ . Furthermore, as in string models, the second term in (5.28) mixes the dilaton and the trace of the linearized graviton h_μ^μ . Both this term and the term quadratic in Φ lead to the dilaton kinetic term.

As we mentioned above, the need for the multiplet $\mathcal{S}_{\alpha\dot{\alpha}}$ arises when the operator U is not a good operator in the theory. In this case the current $\mathcal{J}_{\alpha\dot{\alpha}}$ does not exist. The couplings in (5.28) explain how the chiral field Φ fixes this problem. Even though U is not a good operator, $\widehat{U} = \Phi + \Phi^\dagger + U$ is a good operator. If U is not gauge invariant, Φ transforms under gauge transformations such that \widehat{U} is gauge invariant. And if U is not globally well-defined because it undergoes Kähler transformations, Φ has similar Kähler transformations such that \widehat{U} is well-defined.

The result of this discussion can be presented in two different ways. First, as we did here, we started with a rigid theory without an FZ-multiplet and we had to gauge the \mathcal{S} -multiplet. This has led us to the Lagrangian (5.28). Alternatively, we could add the chiral superfield Φ to the original rigid theory such that the combined theory does have an FZ-multiplet. Then, this new rigid theory can be coupled to standard supergravity by gauging the FZ-multiplet.

Our discussion makes it clear that if we want to couple the theory to supergravity, the additional chiral superfield Φ is not an option — it must be added.

It is amusing to compare these conclusions with the discussion in section 4. There we used the fact that it is impossible to promote the FI-term or the coupling constant characterizing the geometry to background fields. The coupling of such theories to gravity forces us to turn these coupling constants to fields. However, these are not background classical fields but fluctuating dynamical fields.

6 Summary and applications

In most theories the supersymmetry current and the energy-momentum tensor can be embedded in the familiar FZ-multiplet (1.6). But in a number of situations this multiplet is not a good operator in the theory. It is either non-gauge invariant or not globally well-defined. In this case we must use the larger multiplet $\mathcal{S}_{\alpha\dot{\alpha}}$ which we analyzed in this paper.

These observations about the FZ-multiplet and the \mathcal{S} -multiplet allowed us to prove some non-renormalization theorems. For example, we have shown that starting with a renormalizable gauge theory, the moduli space of supersymmetric vacua cannot be compact. Similarly, the known non-renormalization theorems of theories with FI-terms trivially follow.

Of particular interest to us was the coupling of theories without an FZ-multiplet to supergravity. Here we have limited ourselves to supersymmetric field theories in which all dimensionful parameters are fixed and study the limit $M_p \rightarrow \infty$. We did not study theories in which the matter couplings depend on M_p . Since the FZ-multiplet does not exist, we have to gauge the \mathcal{S} -multiplet. The upshot of the analysis of this gauging is the following. We add to the rigid theory a chiral superfield Φ whose couplings are such that the combined system including Φ has an FZ-multiplet. This determines some but not all of the couplings of Φ to the matter fields. In the case of the FI-term Φ Higgses the symmetry and in the case of nontrivial target space geometry of the rigid theory it creates a larger total space in which the topology is simpler. Now that we have an FZ-multiplet we can simply gauge it using standard supergravity techniques. In particular, at the linearized level the couplings of Φ depend on only one free parameter — the normalization of its kinetic term.

Our results fit nicely with the many known examples of string vacua. We see that the ubiquity of moduli in string theory is a result of low energy consistency conditions in supergravity. As we emphasized above, the chiral superfield Φ is similar to the dilaton superfield in four dimensional supersymmetric string vacua. We often have field theory limits without an FZ-multiplet. For example, we can have a theory on a brane with an FI-term. The field theory limit does not have an FZ-multiplet and correspondingly,

$U \sim \xi V$ is not gauge invariant. This problem is fixed, as in (5.28), by coupling the matter theory to Φ which is not gauge invariant as in [35]. Similarly, we often consider field theory limits with a target space whose Kähler form is not exact. This happens, for instance, on D3-branes at a point in a Calabi-Yau manifold. If the latter is non-compact we find a supersymmetric field theory on the brane which typically does not have an FZ-multiplet because U is not globally well-defined. Coupling this system to supergravity corresponds to making M_p finite. In this case this is achieved by making the Calabi-Yau compact. Then in addition to the graviton, various moduli of the Calabi-Yau space become dynamical. They include fields like our Φ which couple as in (5.28), thus avoiding the problems with the FZ-multiplet and making the supergravity theory consistent.

This discussion has direct implications for moduli stabilization. It is often desirable to stabilize some moduli at energies above the supersymmetry breaking scale. In this case we have to make sure that the resulting supergravity theory is still consistent. In particular, it is impossible to stabilize Φ in a supersymmetric way and be left with a low energy theory without an FZ-multiplet.

For example, if the low energy theory includes a U(1) gauge field with an FI-term, this term must be Φ dependent. Furthermore, if the mass of Φ is above the scale of supersymmetry breaking, it must be the same as the mass of the gauge field it Higgses. Consequently, there is no regime in which it is meaningful to say that there is an FI-term. Similar comments hold for theories with a compact target space. It is impossible to stabilize the Kähler moduli while allowing moduli for the positions of branes to remain massless without supersymmetry breaking.¹⁶

The comments above have applications to many popular string constructions including D-inflation, flux compactifications, and sequestering. Some of these constructions might need to be revisited.

It would be nice to explore these ideas further, and to study in more detail specific examples in string theory. The question of moduli stabilization is crucial for understanding low energy aspects of string theory and may lead us to a better understanding of the space of vacua and SUSY breaking. It would also be nice to find additional results using our new tools. In particular, it is conceivable that sharp statements can be made about the masses of moduli by studying the full nonlinear supergravity theory.

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¹⁶This conclusion can also be obtained using the result of [21]. Such a putative stabilization leads at energies below the mass of Φ to an effective supergravity theory which violates the consistency conditions in [21]. This argument can be made in spite of the modifications to [21] we found in appendix A (and the general analysis in [36]).

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A Gauging the \mathcal{R} -multiplet

Theories that have an R -symmetry possess the \mathcal{R} -multiplet (1.8). Since the \mathcal{R} -multiplet contains a conserved supercurrent and an energy-momentum tensor, we can couple it to supergravity. We proceed along the lines of subsection 5.1. We postulate a coupling to a real superfield $H_{\alpha\dot{\alpha}}$

$$\int d^4\theta \mathcal{R}_{\alpha\dot{\alpha}} H^{\alpha\dot{\alpha}} . \tag{A.1}$$

The transformation law (5.2) leaves this action invariant only for a subset of all possible superfields L_α . Using the defining properties of the \mathcal{R} -multiplet (1.8) we immediately get the constraint

$$\bar{D}_{\dot{\alpha}} D^2 \bar{L}^{\dot{\alpha}} = D_\alpha \bar{D}^2 L^\alpha . \tag{A.2}$$

This means that the superfield $D_\alpha \bar{D}^2 L^\alpha$ is pure imaginary.

The analog of the Wess-Zumino gauge turns out to be

$$H_\mu| = H_\mu|_\theta = H_\mu|_{\bar{\theta}} = H_\mu|_{\theta^2} = H_\mu|_{\bar{\theta}^2} = 0 . \tag{A.3}$$

Note that in the $H_\mu|_{\theta^2}$ component there is a difference with subsection 5.1. The residual gauge transformations are

1. The vector in the top component $H_\mu|_{\theta^4} \equiv b_\mu$ transforms like a gauge field

$$\delta b_\mu \sim \partial_\mu \omega , \tag{A.4}$$

2. $B_{\mu\nu} = H_\mu|_{\theta\sigma\nu\bar{\theta}} - H_\nu|_{\theta\sigma\mu\bar{\theta}}$ transforms like a usual two-form field

$$\delta B_{\mu\nu} = \partial_\nu \omega_\mu - \partial_\mu \omega_\nu . \tag{A.5}$$

3. The graviton and gravitino are the same as in (5.5), (5.6).

Note that the transformation of the vector b_μ is consistent with the coupling (A.1) to the conserved current in \mathcal{R}_μ .

The two-form $B_{\mu\nu}$ has three off-shell degrees of freedom and the gauge field b_μ has three degrees of freedom as well. Together with the metric we find 12 bosonic degrees of freedom. The gravitino provides the 12 fermionic degrees of freedom. This is equivalent to the “new minimal multiplet” of supergravity [9, 10].

Our goal now is to construct the kinetic term for this theory. We again define a superfield $E_{\alpha\dot{\alpha}}^R$

$$E_{\alpha\dot{\alpha}}^R = i\partial_{\alpha\dot{\beta}} D^\gamma \bar{D}^{\dot{\beta}} H_{\gamma\dot{\alpha}} - i\partial_{\beta\dot{\alpha}} D^\beta \bar{D}^{\dot{\gamma}} H_{\alpha\dot{\gamma}} . \tag{A.6}$$

Replacing $D^\gamma \bar{D}^{\dot{\beta}}$ by $[D^\gamma, \bar{D}^{\dot{\beta}}]$ (and similarly for the second term), it is easy to show that the expression (A.6) is real.

Next, we have to study its transformation law under (5.2). This gives

$$\delta E_{\alpha\dot{\alpha}}^R = \frac{1}{2}[D_\alpha, \bar{D}_{\dot{\alpha}}] \left(D^\gamma \bar{D}^2 L_\gamma + \bar{D}_{\dot{\gamma}} D^2 \bar{L}^{\dot{\gamma}} \right). \quad (\text{A.7})$$

Thus, $E_{\alpha\dot{\alpha}}^R$ is invariant under the restricted gauge transformations (A.2). Note that the object in parentheses $\left(D^\gamma \bar{D}^2 L_\gamma + \bar{D}_{\dot{\gamma}} D^2 \bar{L}^{\dot{\gamma}} \right)$ is a linear multiplet as in (1.10).

Another important relation E^R satisfies is

$$D^\alpha E_{\alpha\dot{\alpha}}^R = \frac{1}{2} D^2 \bar{D}_{\dot{\alpha}} [D^\beta, \bar{D}^{\dot{\beta}}] H_{\beta\dot{\beta}}. \quad (\text{A.8})$$

These relations guarantee that

$$\mathcal{L}_{\text{kinetic}} \sim M_P^2 \int d^4\theta H^\mu E_\mu^R, \quad (\text{A.9})$$

is invariant.

We can now summarize the Lagrangian. We will not be careful about the coefficients since our goal is to explain the qualitative behavior. The bosonic couplings of supergravity to matter fields follow from (A.1)

$$\mathcal{L}_{\text{matter-garvity}} = h_{\mu\nu} T^{\mu\nu} + \epsilon^{\mu\nu\rho\sigma} \partial_\nu B_{\rho\sigma} \left(A_\mu^{(S)} + j_\mu^{(R)} \right) + b^\mu j_\mu^{(R)}. \quad (\text{A.10})$$

We have denoted $F_{\mu\nu}^{(S)} = \partial_\mu A_\nu^{(S)} - \partial_\nu A_\mu^{(S)}$, where $F_{\mu\nu}^{(S)}$ is the field strength appearing in the \mathcal{R} -multiplet.¹⁷ Note that this Lagrangian is invariant under all the residual gauge transformations. The (quadratic) kinetic term for the bosonic gravitational degrees of freedom are

$$\frac{1}{M_P^2} \mathcal{L}_{\text{kinetic}} = h\partial^2 h + (\epsilon^{\mu\nu\rho\sigma} \partial_\nu B_{\rho\sigma})^2 + b_\mu \epsilon^{\mu\nu\rho\sigma} \partial_\nu B_{\rho\sigma}. \quad (\text{A.11})$$

Here $h\partial^2 h$ is just a shorthand for the linearized Einstein theory. Note the absence of b_μ^2 due to gauge invariance.

We see that b_μ is an auxiliary field that can be easily integrated out to yield

$$\epsilon^{\mu\nu\rho\sigma} \partial_\nu B_{\rho\sigma} \sim j_\mu^{(R)}. \quad (\text{A.12})$$

Hence, $B_{\mu\nu}$ is an auxiliary field that is solved in terms of the R -current. We conclude that both the B -field and the vector field b_μ are auxiliary non-propagating degrees of freedom. Thus, the coupling to supergravity via the \mathcal{R} -multiplet does not introduce new propagating degrees of freedom beyond the graviton and gravitino.

Using (A.12) in the action we get

$$\mathcal{L}_{\text{total}} = h\partial^2 h + h_{\mu\nu} T^{\mu\nu} + A^{(S)\mu} j_\mu^{(R)} + \dots. \quad (\text{A.13})$$

As an example we can consider a pure U(1) gauge theory. It has an R -current given by $\mathcal{R}_{\alpha\dot{\alpha}} = -\frac{4}{g^2} W_\alpha \bar{W}_{\dot{\alpha}}$. We Denote by A_μ the elementary gauge field in the problem. For

¹⁷The expression in components for the \mathcal{R} -multiplet is obtained from (2.2) by setting $X = 0$.

this theory it easy to see that the operator $A_\mu^{(S)}$ in the \mathcal{R} -multiplet is just the FI-term times the fundamental gauge field, $A_\mu^{(S)} \sim \xi A_\mu$. Hence, the effect of (A.13) is to shift the gauge charges of all the fields in the problem by their R -charge (proportional to the FI-term and suppressed by the Planck scale). This reproduces the results of [12–14] which was derived in the old minimal formalism about the coupling of R -symmetric theories with an FI-term to supergravity. The necessity of an exact R -symmetry is the root of the incompatibility of these models with a complete quantum gravity theory.

Another interesting case is the \mathbf{CP}^1 sigma model. Since this theory has no superpotential, there is an R -symmetry such that all the fields carry R -charge zero. It is therefore guaranteed that this theory has a well-defined \mathcal{R} -multiplet (3.6). As we have explained in section 3, this multiplet is globally well-defined. It can therefore be (classically) coupled to supergravity for any value of the radius of the sphere.¹⁸

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¹⁸This conclusion differs from [21], which claims that the radius of the sphere has to be quantized in units of the Planck scale. The difference arises because of the existence of the R -symmetry. For a detailed discussion of this point see [36].

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