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Contribution of the Weinberg-type operator to atomic and nuclear electric dipole moments

Naohiro Osamura,[a] **Philipp Gubler**[b] **and Nodoka Yamanaka**[c,d]

[a] *Department of Physics, Nagoya University, Furo-cho Chikusa-ku, Nagoya, 464-8602 Japan*

[b] *Advanced Science Research Center, Japan Atomic Energy Agency, Tokai, Ibaraki 319-1195, Japan*

[c] *Kobayashi-Maskawa Institute for the Origin of Particles and the Universe (KMI), Nagoya University, Nagoya, 464-8602, Japan*

[d] *Nishina Center for Accelerator-Based Science, RIKEN, Wako 351-0198, Japan*

E-mail: osamura.naohiro.j2@s.mail.nagoya-u.ac.jp, gubler@post.j-parc.jp, nyamanaka@kmi.nagoya-u.ac.jp

Abstract: The contribution of the CP violating three-gluon Weinberg operator, $\frac{1}{3!}wf^{abc}\epsilon^{\nu\rho\alpha\beta}G^a_{\mu\nu}G^b_{\alpha\beta}G^{c\mu}_{\rho}$, to the atomic and nuclear EDMs is estimated using QCD sum rules. After calculating the transition matrix element between the pion and the vacuum through the Weinberg operator, we obtain the long-range CP-odd nuclear force by determining the isovector CP-odd pion-nucleon vertex, using chiral perturbation theory at NLO. The EDMs of ^{199}Hg, ^{129}Xe, and ^{225}Ra atoms, as well as those of ^{2}H and ^{3}He nuclei are finally given including comprehensive uncertainty analysis. While the leading contribution of the ^{199}Hg EDM is given by the intrinsic nucleon EDM, that of ^{129}Xe atom may be dominated by the one-pion exchange CP-odd nuclear force generated by the Weinberg operator. From current experimental data of the ^{199}Hg atomic EDM, we obtain an upper limit on the Weinberg operator magnitude of $|w| < 4 \times 10^{-10}\text{GeV}^{-2}$ if we assume that it is the only source of CP violation at the scale $\mu = 1\,\text{TeV}$.

Keywords: CP Violation, Electric Dipole Moments, Specific QCD Phenomenology, SMEFT

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Contents

1 Introduction

The matter dominant Universe is a cosmologically important phenomenon which cannot be explained by the standard model (SM). Indeed, a large CP violation is required to realize the matter dominance according to Sakharov [1]. However, the CP violation of the SM does not fulfill this criterion [2, 3] and hence the experimental search for new physics beyond the SM is actively pursued.

The electric dipole moment (EDM) [4–14] is a CP-violating observable sensitive to new physics which has been explored experimentally in various systems. The measurements of the EDMs in atomic systems are especially attracting attention, thanks to their high sensitivity [15–17], which can for certain regions of the parameter space, be higher than the experimental constraints obtained by the LHC experiments. We also note that their SM background is very small [18–22].

In practice one usually needs to integrate out new physics degrees of freedom to obtain an effective theory relevant at hadronic scales, which includes CP violating terms such as

$$\mathcal{L}_{\text{SMEFT,CP}} = \frac{g_s^2}{32\pi^2}\bar{\theta} G^a_{\mu\nu}\tilde{G}^{a\mu\nu} - \frac{i}{2}\sum_{i=u,d,s,e,\mu} d_i \bar{\psi}_i \sigma^{\mu} F_{\mu\nu}\gamma_5\psi_i - \frac{i}{2}\sum_{i=u,d,s}\tilde{d}_i\bar{\psi}_i g_s \sigma^{\mu\nu} G^a_{\mu\nu}\tau^a\gamma_5\psi_i$$
$$+ \frac{1}{3!} w f^{abc}\epsilon^{\nu\rho\alpha\beta} G^a_{\mu\nu} G^b_{\alpha\beta} G^{c\mu}_{\rho} + \sum_{i,j} C_{ij}\left(\bar{\psi}_i\psi_i\right)\left(\bar{\psi}_j i\gamma_5\psi_j\right) + \cdots . \tag{1.1}$$

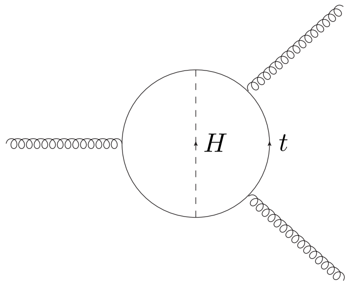


Figure 1. Two-loop level diagram contributing to the Weinberg operator generated in the two-Higgs doublet model.

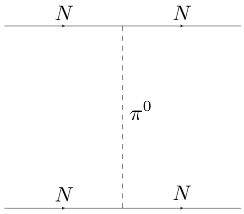
Here, we include only low dimension terms which give the leading contribution to low energy observables. As for the diamagnetic atoms, the effect of the electron EDM is suppressed due to the closed electron shell, and the CP violation of the quark-gluon sector is important [7, 13, 23]. Contributions of the θ term [24–27] and the quark EDM [28–34] (second term in the above equation) to the nucleon EDM have already been extensively analyzed in lattice QCD. The chromo-EDM (third term) is still difficult to handle on the lattice, but has been studied using QCD sum rules and chiral effective field theory (χEFT) [8, 35–43]. The purely gluonic CP-odd dimension-six Weinberg operator [44], is less studied, but nevertheless important because it appears in many well-known models such as the Higgs doublet model (as shown in figure 1) [44–61], supersymmetric models [62–75], and other models [76–84]. It is also generated by the heavy quark sector CP violation via renormalization group evolution [85–90]. The SM contribution to the Weinberg operator

$$\mathcal{L}_W \equiv \frac{1}{3!} w f^{abc} \epsilon^{\nu\rho\alpha\beta} G^a_{\mu\nu} G^b_{\alpha\beta} G^{c\mu}_{\rho}, \tag{1.2}$$

is known to be unobservably small [91–93]. While the effect of the Weinberg operator to the nucleon EDM has already been evaluated [66, 94–100], its contribution to the CP-odd nuclear force, which is expected to be one of the leading effects to the atomic and nuclear EDMs, is less known.

In χEFT, the leading contribution to the CP-odd nuclear force caused by the Weinberg operator is the contact nucleon-nucleon interaction [55, 101, 102], which is expected to have a small effect on the nuclear level CP violation due to the strong repulsion between nucleons. Similarly, the long range CP-odd nucleon-nucleon interaction generated by the pion exchange is also suppressed due to chiral symmetry. The long-range nucleon-nucleon interaction and the contact one are depicted in the left and right plots of figure 2, respectively. Indeed, the CP-odd pion-nucleon interaction, in contrast to the Weinberg operator, breaks chiral symmetry, so that the matching between them brings a suppression of at least one light quark mass factor. For these reasons, the contribution of the CP-odd nuclear forces has never been seriously considered in the context of atomic EDM, and it was long thought that the nucleon EDM is the leading process. However, a process generated by the pion-pole, as depicted in figure 3, is potentially significant because it may be enhanced by the large pion-nucleon sigma term. One also expects an enhancement of the CP-odd moments by the many-body effect for heavy atoms and nuclei.

JHEP06(2022)072



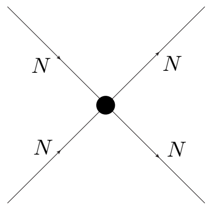


Figure 2. CP-odd nucleon-nucleon processes which induce nuclear level CP-odd moments. The left diagram is the long range nuclear force. It is generated by combining the CP-even and the CP-odd pion-nucleon interactions. The upper (lower) vertex represents the CP-even (CP-odd) interaction. The right figure depicts the CP-odd contact nucleon-nucleon interaction.

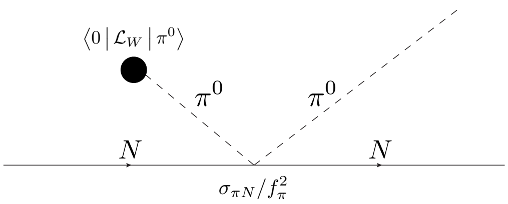


Figure 3. The leading order χEFT contribution to the isovector CP-odd πN interaction through the Weinberg operator.

Let us briefly mention here that the quark EDM and chromo-EDM are also generated at low energy scales through the renormalization group running of the Weinberg operator even if the latter is the sole source of CP-violation at high energy [62, 68, 103, 104]. However, the quark EDM contribution is small at the hadron level [28–33] and the calculation of the hadron matrix element of the chromo-EDM has a large theoretical uncertainty, some studies even predicting a null contribution [8, 36, 105].

To investigate potentially significant effect mentioned above, we in this paper estimate the contribution of the Weinberg operator to the isovector CP-odd pion-nucleon interaction. To calculate the pion-vacuum transition matrix element $\langle \pi^0 \,|\, \mathcal{L}_W \,|\, 0 \rangle$, we employ QCD sum rules, which allow us to perform a relatively simple and analytic analysis based on QCD [106–108]. In the future, it might be possible to compute this matrix element from lattice QCD. However, this task is currently still difficult because of the computational cost of treating gluonic operators on the lattice and their accurate renormalization, although some first trials of such computation already exist [109, 110].

This paper is organized as follows. In section 2, we set up the QCD sum rules formalism to compute the desired matrix element. The concrete calculation of the operator product expansion (OPE) and the numerical analysis follow in sections 3 and 4. In section 5, we derive the atomic and nuclear EDMs through hadron, nuclear and atomic level calculations, with a particular focus on the relative magnitudes between our newly calculated CP-odd nuclear force Weinberg operator contribution and that of the intrinsic nucleon EDM. The paper is concluded in section 6. Calculational details are provided in the appendices.

2 Formulation of the sum rules

The goal of this and following two sections is the estimation of the Weinberg operator matrix element between the contribution of a pion state and the non-perturbative QCD vacuum $|0\rangle$,

$$\left\langle \pi^0 \middle| \mathcal{L}_W \middle| 0 \right\rangle, \tag{2.1}$$

which, as mentioned in the introduction, contributes to atomic and nuclear EDMs through the isovector CP-odd pion-nucleon interaction. For this purpose, we will make use of the sum rule method. At first sight, the most direct approach to extract this matrix element would be to start from a correlator of $\mathcal{L}_W$ and the pionic operator $i\bar{q}\gamma_5 q$. However, such an estimation is challenging because it necessitates the computation of a three-loop diagram at leading order in the OPE, which cannot be easily carried out using configuration space techniques [111]. We will therefore follow a different strategy extracting the pion pole from the correlation function of two Weinberg operators,

$$\Pi(q^2) = i \int d^4x e^{-iq\cdot x} \langle 0 \,|\, T \left[\mathcal{L}_W(x)\mathcal{L}_W(0)\right] |\, 0\rangle\,, \tag{2.2}$$

rather than computing the matrix element directly. T here denotes the time ordering product.

We will estimate the correlator of eq. (2.2) at large $-q^2$ (the deep-Euclidean region) using the OPE (see next section for details) and then analyze the spectral function, defined as $\rho(s) = \frac{1}{\pi}\mathrm{Im}\Pi(s)$ numerically in Secion 4. The dispersion relation is useful to relate the correlator in the deep-Euclidean region to the spectral function at $q^2 > 0$, where it carries information about all physical states that couple to the operator $\mathcal{L}_W(x)$,

$$\begin{aligned} \Pi(q^2) &= \frac{1}{\pi}\int_0^\infty ds \frac{\mathrm{Im}\Pi(s)}{s-q^2} \\ &= \int_0^\infty ds \frac{\rho(s)}{s-q^2}. \end{aligned} \tag{2.3}$$

As it is not possible to extract detailed features of the spectral function from eq. (2.3) and the (limited) OPE information available at large $-q^2$, we here approximate the spectral function to consist of a simple pion pole and a continuum structure above some effective threshold s_{th} (for an alternative method making use of Bayesian statistics, see ref. [112]). We furthermore assume that this continuum can be described by the OPE expression. We hence have

$$\begin{aligned} \rho_{\mathrm{phen}}(s) &= \lambda_\pi^2 \delta(s - m_\pi^2) + \rho_{\mathrm{cont}}(s)\theta(s - s_{\mathrm{th}}) \\ &= \lambda_\pi^2 \delta(s - m_\pi^2) + \frac{1}{\pi}\mathrm{Im}\Pi_{\mathrm{OPE}}(s)\theta(s - s_{\mathrm{th}}), \end{aligned} \tag{2.4}$$

where $\lambda_\pi = \langle \pi^0 \,|\, \mathcal{L}_W \,|\, 0\rangle$ is the quantity we seek to compute in this work. Note that the matrix element of eq. (2.1) must be proportional to $m_u - m_d \equiv m_-$ due to the chiral symmetry, and therefore is strongly suppressed. On the other hand, a pseudoscalar, glueball state is expected to couple to the Weinberg operator and thus could contribute significantly

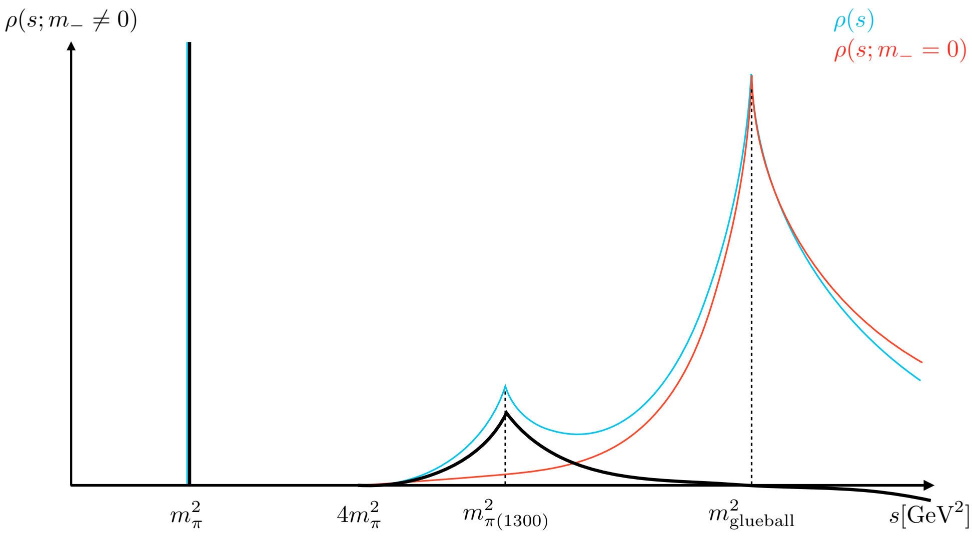


Figure 4. Schematic illustration of the isospin-breaking part of the spectral function $\rho(s; m_- \neq 0)$ (thick black line). The blue line depicts the complete spectral function $\rho(s)$ while the red one indicates its isospin-symmetric part $\rho(s; m_- = 0)$. $\pi(1300)$ is the first excited state of the pion. The m_-^2 differentiation causes only the isospin-breaking part to survive.

to the spectral function without any isospin suppression factor. Indeed, the correlator of eq. (2.2) is usually used to study the properties of such a glueball state (see, for example, refs. [113, 114]). Hence, to suppress the potentially large glueball contribution of the sum rules, we will in this work make use of the smallness of the quark mass dependence of this state, which is confirmed in recent lattice QCD calculations [115–118]. Specifically, we will study the quark mass dependence of the correlator by expanding it up to the second order in the up and down quark masses m_u and m_d and applying $m_-^2 \frac{\partial}{\partial m_-^2}$. This will strongly suppress the glueball contribution, while keeping the pion pole, for which λ_π^2 is proportional to m_-^2. More explicitly, in terms of the spectral function, $\rho(s)$ can be decomposed into two terms: $\rho(s) = \rho(s; m_- = 0) + \rho(s; m_- \neq 0)$. The isospin-symmetric part $\rho(s; m_- = 0)$ does not include the pion pole because of $\lambda_\pi = \langle \pi^0 \,|\, \mathcal{L}_W \,|\, 0 \rangle \propto m_-$. Thus, it at low energy only consists of a continuum spectrum beginning at the threshold $4m_\pi^2$ and at higher energy of the glueball peak mentioned above. $\rho(s; m_- = 0)$ furthermore will be positive definite since it is contains physical states in the limit of exact isospin symmetry. The isospin-breaking part $\rho(s; m_- \neq 0)$, illustrated as a thick black line in figure 4, can be obtained as $\rho(s) - \rho(s; m_- = 0)$ and does in contrast not need to be positive definite. As will be explicitly demonstrated later, the leading order (perturbative) OPE term leads to a negative high energy limit for $\rho(s; m_- \neq 0)$.

Finally, to both improve the convergence of the OPE and suppress the relative contributions of the higher energy states to the sum rules, we make use of the Borel transform, which in this work is defined as

$$\mathcal{B}\left[f(Q^2)\right] := \lim_{\substack{Q^2, n \to \infty \\ Q^2/n = M^2 = \text{const.}}} \frac{(Q^2)^n}{(n-1)!} \left(-\frac{d}{dQ^2}\right)^n f(Q^2), \tag{2.5}$$

where $Q^2 = -q^2$. After substituting eq. (2.4) into eq. (2.3), we obtain

$$\mathcal{B}\left[\Pi_{\text{phen}}\right](M^2) = \lambda_\pi^2 \cdot \frac{1}{M^2} e^{-m_\pi^2/M^2} + \frac{1}{\pi}\int_{s_{th}}^{\infty} ds \frac{e^{-s/M^2}}{M^2} \text{Im}\Pi_{\text{OPE}}(s; m_- \neq 0), \tag{2.6}$$

which gives the "phenomenological side" of the sum rules. The computation of the "theoretical side" will be discussed in the next section.

3 OPE for the Weinberg operator correlator

The OPE of the correlation function in eq. (2.2) can formally be expressed as

$$\Pi_{\text{OPE}}(q) = i\int d^4x e^{-iq\cdot x} \langle 0 \,|\, T\left[\mathcal{L}_W(x)\mathcal{L}_W(0)\right] |\, 0\rangle = \sum_d C_d(q;\mu) \langle 0 \,|\, O_d(\mu) \,|\, 0\rangle\,, \tag{3.1}$$

where O_d stand for all local operators allowed by gauge invariance and vacuum symmetries while $C_d(q;\mu)$ represent their respective Wilson coefficients. μ is the renormalization scale, which should be chosen large enough such that a perturbative calculation of the Wilson coefficient is possible [119]. Throughout this work, we will take $\mu = 1\text{GeV}$ if not explicitly stated otherwise. The Weinberg operator can for the purposes of the OPE calculation be rewritten as

$$\begin{aligned}\mathcal{L}_W &= \frac{1}{3!} w f^{abc} \epsilon^{\nu\rho\alpha\beta} G^a_{\mu\nu} G^b_{\alpha\beta} G^{c\mu}_{\rho} \\ &= \frac{1}{3} w f^{abc} \epsilon^{\nu\rho\alpha\beta} g^{\mu\sigma} \left[2\left(\partial_\mu A^a_\nu\right)\left(\partial_\rho A^c_\sigma\right) - \left(\partial_\mu A^a_\nu\right)\left(\partial_\sigma A^c_\rho\right) - \left(\partial_\nu A^a_\mu\right)\left(\partial_\rho A^c_\sigma\right)\right]\left(\partial_\alpha A^b_\beta\right) + \cdots\,.\end{aligned} \tag{3.2}$$

A^a_μ here stand for gluon fields and the ellipses indicate higher order contributions with respect to the strong coupling constant.

As mentioned in the previous section, we are in this work only interested in the m_--dependence of $\Pi(q^2)$ and hence decompose Π_{OPE} in the same way as the spectral function. Specifically, we have

$$\Pi_{\text{OPE}}(q^2) = \Pi_{\text{OPE}}(q^2; m_- = 0) + \Pi_{\text{OPE}}(q^2; m_- \neq 0), \tag{3.3}$$

where the second term can be expanded using the OPE as

$$\Pi_{\text{OPE}}(q^2; m_- \neq 0) = \Pi_{\text{loop}}(q^2) + \Pi_q(q^2) + \Pi_G(q^2) + \Pi_{\text{loop}+G}(q^2). \tag{3.4}$$

The four terms on the right-hand side correspond to the four diagrams shown in figure 5.

Essential parts of the computation of the four terms in eq. (3.4) are provided in appendix C. The results read

$$\Pi_{\text{loop}}(q^2) = -w^2 \frac{m_-^2 \alpha_s}{768\pi^4}(-q^2)^3 \ln\left(-q^2\right), \tag{3.5}$$

$$\Pi_q(q^2) = -w^2 \frac{m_-^2 B_0^2 h_3 \alpha_s}{4\pi^3}(-q^2)^2 \ln\left(-q^2\right), \tag{3.6}$$

$$\Pi_G(q^2) = 0, \tag{3.7}$$

$$\Pi_{\text{loop}+G}(q^2) = 0, \tag{3.8}$$

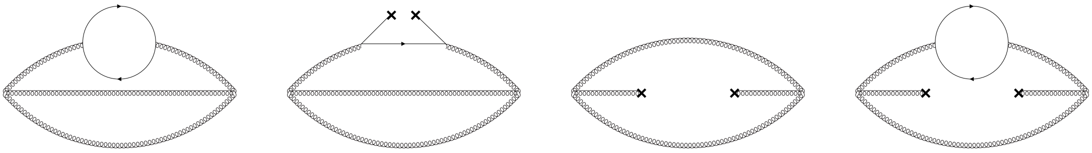


Figure 5. The diagrams corresponding to $\Pi_{\text{loop}}(q^2)$, $\Pi_q(q^2)$, $\Pi_G(q^2)$ and $\Pi_{\text{loop}+G}(q^2)$ of eq. (3.4), respectively. The crosses in the latter three diagrams represent the chiral condensate $\langle 0\,|\,\bar{q}q\,|\,0\rangle$, and the gluon condensate $\langle 0\,|\,\alpha_s G^2\,|\,0\rangle$. The two diagrams on the right, which both include a gluon condensate contribution, vanish in the OPE calculation of this work.

where B_0 is defined by the relation $\frac{1}{2}\langle 0|\bar{u}u + \bar{d}d|0\rangle = -B_0 f_\pi^2$. The parameter h_3 in eq. (3.6) is related to the difference between the up and down quark chiral condensates (see eqs. (C.17), (C.18)). Specifically, we have [120, 121]

$$h_3 = -\frac{\left\langle 0\,\middle|\,\bar{u}u - \bar{d}d\,\middle|\,0\right\rangle}{4B_0^2(m_u - m_d)}. \tag{3.9}$$

The m_- dependent part of the correlator is thus obtained as

$$\begin{aligned}\Pi_{\text{OPE}}(q^2; m_- \neq 0) &= \Pi_{\text{loop}}(q^2) + \Pi_q(q^2) + \Pi_G(q^2) + \Pi_{\text{loop}+G}(q^2) \\ &= w^2\left[-\frac{m_-^2\alpha_s}{128\pi^4}(-q^2)^3\ln\left(-q^2\right) - \frac{m_-^2 B_0 h_3 \alpha_s}{4\pi^3}(-q^2)^2\ln\left(-q^2\right)\right],\end{aligned} \tag{3.10}$$

which, after the Borel transform becomes

$$\mathcal{B}\left[\Pi_{\text{OPE}}(q^2; m_- \neq 0)\right] = w^2\left[-\frac{3m_-^2\alpha_s}{64\pi^4}M^6 + \frac{m_-^2 B_0^2 h_3 \alpha_s}{2\pi^3}M^4\right]. \tag{3.11}$$

4 Numerical sum rule analysis

Having estimated both phenomenological and theoretical sides of the correlator of eq. (2.2) in sections 2 and 3, we can now proceed to the numerical analysis of the sum rules, which after the Borel transform and the m_-^2 differentiation are obtained as

$$\frac{\lambda_\pi^2}{M^2}e^{-m_\pi^2/M^2} + \frac{1}{\pi}\int_{s_{th}}^{\infty} ds\frac{e^{-s/M^2}}{M^2}\text{Im}\Pi_{\text{OPE}}(s; m_- \neq 0) = \mathcal{B}\left[\Pi_{\text{OPE}}(q^2; m_- \neq 0)\right]. \tag{4.1}$$

From the above equation and eqs. (3.10) and (3.11), the pion pole residue λ_π^2 can be expressed as a function of the Borel mass M and the threshold parameter s_{th},

$$\begin{aligned}|\,\langle 0\,|\,\mathcal{L}_W\,|\,\pi\rangle\,|^2 &= \lambda_\pi^2 \\ &= w^2 e^{m_\pi^2/M^2}\left[\frac{m_-^2\alpha_s}{128\pi^4}M^2\left\{e^{-\frac{s_{\text{th}}}{M^2}}\left(6M^6 + 6M^4 s_{\text{th}} + 3M^2 s_{\text{th}}^2 + s_{\text{th}}^3\right) - 6M^6\right\}\right. \\ &\qquad \left. -\frac{m_-^2 B_0^2 h_3 \alpha_s}{4\pi^3}M^2\left\{e^{-\frac{s_{\text{th}}}{M^2}}\left(2M^2 s_{\text{th}} + 2M^4 + s_{\text{th}}^2\right) - 2M^4\right\}\right].\end{aligned} \tag{4.2}$$

JHEP06(2022)072

Input parameters	Values	Ref.
α_s	0.483 ± 0.016	[122, 123]
$-4(m_u - m_d)B_0 h_3/f_\pi^2$	$[0.014, 0.02]$	[124]
m_π	$135\,\text{MeV}$	
m_u	$2.9^{+0.66}_{-0.35}$	[125]
m_d	$6.0^{+0.65}_{-0.23}$	[125]
$B_0 f_\pi^2$	$(265\text{MeV})^3$	[125]

Table 1. Input parameters used in the numerical sum rule analysis. All are given at the renormalization scale of $\mu = 1\,\text{GeV}$.

In the numerical analysis of eq. (4.2), we employ the input parameters in table 1. For the value range of the parameter h_3, we have used the SU(2) estimate given in ref. [124]. The quark masses correspond to a renormalization scale of 1 GeV and were obtained by running their values from the 2 GeV value given in the PDG to 1 GeV using a two-loop renormalization group equation [125]. B_0, which is related to the quark condensate, similarly given at the renormalization scale of 1 GeV, was obtained from the Gell-Mann-Oakes-Renner relation and a renormalization group rescaling [125].

Let us here briefly discuss how to determine the parameter ranges of M and s_{th}. For the Borel mass M, one usually defines a so-called "Borel window", within the various approximations used to derive the sum rules are supposed to be valid. In our calculation, the term involving the quark condensate turns out to have the largest contribution. As this term has the largest mass dimension in the final OPE result of eq. (3.11) and the gluon condensate terms vanish, it would in this work not make much sense to determine the lower limit of the Borel window from the conventional condition of the OPE convergence. We thus here consider only the upper limit, which can be fixed from the pole dominance criterion often used in the QCD sum rule literature, and set the lower limit by hand. The pole dominance criterion can be given as

$$\frac{\frac{\lambda_\pi^2}{M^2} e^{-m_\pi^2/M^2}}{\frac{\lambda_\pi^2}{M^2} e^{-m_\pi^2/M^2} + \frac{1}{\pi}\int_{s_{th}}^\infty ds \frac{e^{-s/M^2}}{M^2} \text{Im}\Pi_{\text{OPE}}(s; m_- \neq 0)} > 0.5, \tag{4.3}$$

which demands that the pion pole gives the dominant contribution to the phenomenological side of the sum rules. The left-hand side of eq. (4.3) is shown in figure 6. We observe in this plot that the pole contribution decreases with increasing Borel mass M in the low M region, which is a typical behavior in conventional QCD sum rule calculations. For $M \gtrsim 0.8\,\text{GeV}$, the ratio of eq. (4.3) starts to increase because of a cancellation in its denominator and does not drop below 0.5 for too large values of the threshold parameter s_{th}. We hence fix the upper boundary of s_{th} such that this ratio can fall below 0.5, which gives $s_{\text{th}}^{\max} = 0.82\,\text{GeV}^2$. The lower limit of M is set up by hand as $M_{\min}^2 = 0.25\text{GeV}^2$. The Borel window fixed in this way is depicted as the blue shaded region in figure 7, where the colored curves show the right-hand side of eq. (4.2) as functions of the Borel mass for different values of s_{th}.

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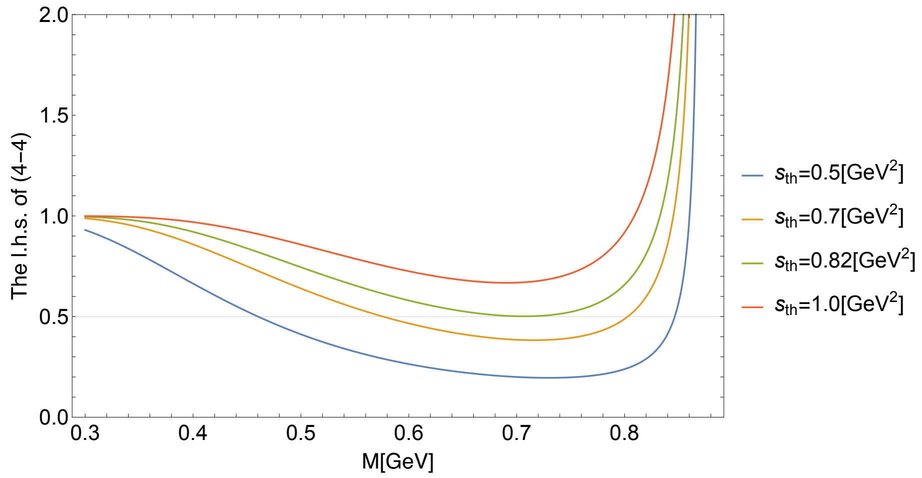

Figure 6. The left-hand side of eq. (4.3) as a function of the Borel mass M for different values of the threshold parameter $s_{\rm th}$.

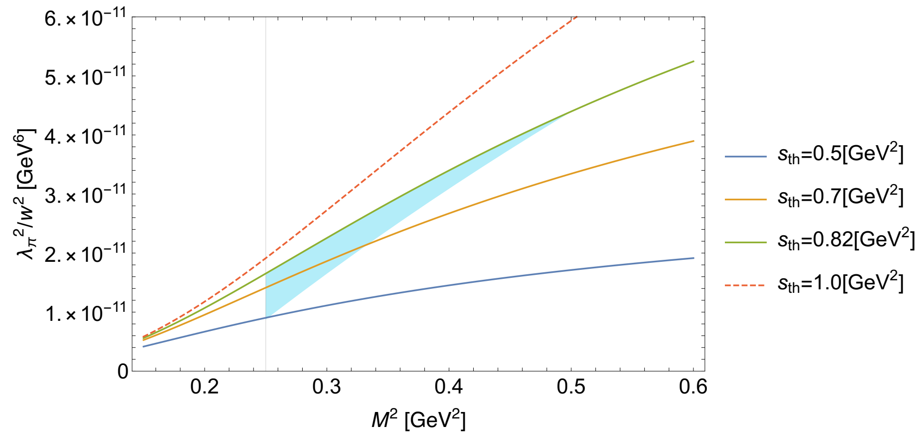

Figure 7. λ_π^2/w^2 as a function of the Borel mass M for various values of the threshold parameter $s_{\rm th}$. The blue shaded region shows the parameter space of M and $s_{\rm th}$ used to estimate the range of λ_π^2.

One can then see in this plot that the Borel window is no longer open for threshold parameter values below $s_{\rm th} = 0.5\,{\rm GeV}^2$, which thus fixes its lower boundary. All this then leads to the following range of values for the desired matrix element:

$$\left|\left\langle 0 \left| \mathcal{L}_W \right| \pi^0 \right\rangle\right| \in w \cdot [2.3, 8.3] \times 10^{-6} {\rm GeV}^5. \tag{4.4}$$

5 Derivation of the atomic and nuclear EDMs

The CP-odd effective Lagrangian needed in this work is

$$\begin{aligned}\mathcal{L}_{\rm CP} = & -\frac{i}{2} d_N \bar{N} \sigma^\mu F_{\mu\nu} \gamma_5 N + k \pi^0 + m_N \Delta_{3\pi} \pi_0 (\pi_0^2 + 2\pi^+ \pi^-) \\ & + \bar{g}^{(1)}_{\pi NN} \pi_0 \bar{N} N - \bar{C}_1 m_N \bar{N} N \, \bar{N} i \gamma_5 N,\end{aligned} \tag{5.1}$$

JHEP06(2022)072

where we implicitly assume that the sum over $N = n, p$ is taken. The first term of $\mathcal{L}_{\rm CP}$ is the intrinsic nucleon EDM. The Weinberg operator contribution to the neutron EDM is given by [48, 49]

$$\begin{aligned} d_n &= \frac{m_{\rm CP}}{m_N}\mu_n + d_n^{(\rm irr)} \\ &= w(20 \pm 12)e\,\text{MeV}. \end{aligned} \tag{5.2}$$

Here the first term corresponds to the chiral rotation of the neutron anomalous magnetic moment μ_n [66, 96], where $m_{\rm CP} = -\langle N|\mathcal{L}_w|N\rangle = -m_N w \frac{3g_s m_0^2}{32\pi^2}\ln\left(\frac{M^2}{\mu_{\rm IR}^2}\right) = w(-2.3 \pm 1.1) \times 10^{-2}\text{GeV}^3$ was calculated using QCD sum rules with 50% error according to ref. [60]. The second term stands for the irreducible term calculated in the quark model ($d_n^{(\rm irr)} \approx -5\,w\,e\,\text{MeV}$) [98]. We explicitly see from the above equation that the nucleon EDM generated by w has no chiral suppression.

The second term of $\mathcal{L}_{\rm CP}$ is the neutral pion tadpole, which we have calculated in the previous section. Its coupling constant k is the matrix element studied in this paper,

$$k = \langle 0\,|\,\mathcal{L}_W\,|\,\pi^0\rangle, \tag{5.3}$$

which generates a neutral pion from the vacuum, and yields the isovector CP-odd pion-nucleon interaction (the term with $\bar{g}^{(1)}_{\pi NN}$) in combination with a pion-nucleon scattering process (see figure 3). The coupling of the pion-nucleon scattering is given by the matrix element

$$\langle \pi N\,|\,\mathcal{L}_{\rm QCD}\,|\,\pi N\rangle \approx \frac{1}{f_\pi^2}\langle N\,|\,m_u\bar{u}u + m_d\bar{d}d\,|\,N\rangle \approx \frac{\sigma_{\pi N}}{f_\pi^2}, \tag{5.4}$$

where the first approximation is due to the partial conservation of the vector current, with $f_\pi = 93\,\text{MeV}$. The low energy constant $\sigma_{\pi N}$ appearing on the right-hand side of the above equation is the pion-nucleon sigma term, and represents the contribution of the current quark mass to the nucleon mass. Its value is still under debate, phenomenological studies yielding $\sigma_{\pi N} \approx 60\,\text{MeV}$ [126, 127] (see however ref. [128] for a smaller prediction), whereas lattice QCD results point to a smaller value $\sigma_{\pi N} \approx 30\,\text{MeV}$ [28, 30, 129]. A recent lattice QCD analysis is suggesting that this discrepancy may come from the contamination of excited states with additional pions of the correlators computed on the lattice [130]. Combining the neutral pion generated by eq. (5.3) and the pion-nucleon scattering (5.4), we obtain the isovector CP-odd pion-nucleon coupling as

$$\bar{g}^{(1)}_{\pi NN} = \langle 0\,|\,\mathcal{L}_W\,|\,\pi^0\rangle\frac{\sigma_{\pi N}}{f_\pi^2 m_\pi^2}. \tag{5.5}$$

To estimate the theoretical uncertainty of the above LO χEFT, we will in the following evaluate the NLO contribution to $\bar{g}^{(1)}_{\pi NN}$, which is in fact generated by the one-loop diagram given in figure 8, via the three-pion interaction (the term with $\Delta_{3\pi}$ in eq. (5.1)) [39, 102], which is part of the linear chiral symmetry and CP breaking term of the chiral Lagrangian

$$\mathcal{L}_{\rm 2CP} = \text{Tr}[UA - U^\dagger A] = \frac{4ia}{f_\pi}\pi_0 - \frac{4ia}{3!f_\pi^3}\pi_0(\pi_0^2 + 2\pi^+\pi^-) + \cdots, \tag{5.6}$$

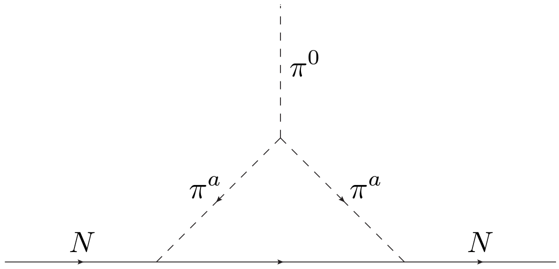

Figure 8. NLO correction to the isovector CP-odd pion-nucleon interaction. π^a may be a charged or neutral pion.

where $U \equiv e^{i\sqrt{2}M/f_\pi}$, with the meson matrix

$$M = \begin{pmatrix} \pi_0/\sqrt{2} & \pi^+ \\ \pi^- & -\pi_0/\sqrt{2} \end{pmatrix}. \tag{5.7}$$

Here we only considered the $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ subspace for simplicity. The matrix $A = a\tau_z$ expresses the explicit isospin breaking generated by the Weinberg operator. By matching eq. (5.6) and the pion one-point interaction (5.3), we obtain

$$a = \frac{f_\pi}{4i} \langle 0 | \mathcal{L}_W | \pi_0 \rangle. \tag{5.8}$$

The three-pion coupling generated by the Weinberg operator is then

$$m_N \Delta_{3\pi} = -\frac{\langle 0 | \mathcal{L}_w | \pi_0 \rangle}{6 f_\pi^2}. \tag{5.9}$$

Evaluating the zero-momentum transfer limit of the one-loop diagram of figure 8, we obtain the NLO contribution to the isovector CP-odd pion-nucleon coupling as [131, 132]

$$\bar{g}^{(1)}_{\pi NN,\mathrm{NLO}} = -m_N \Delta_{3\pi} \frac{15 g_A^2 m_\pi}{32\pi f_\pi^2} = +\frac{5 g_A^2 m_\pi}{64\pi f_\pi^4} \langle 0 | \mathcal{L}_w | \pi_0 \rangle, \tag{5.10}$$

where $g_A = 1.27$. The magnitude of the above NLO result amounts to about 20% of the LO expression of eq. (5.5) (if we assume $\sigma_{\pi N} = 60\,\mathrm{MeV}$). This relatively large one-loop correction is due to the factor 4π enhancement arising from the heavy baryon approximation. Regarding the sign of eq. (5.10), this one-loop level contribution is constructive, in accordance with refs. [131, 132] (for details, see appendix D).

The CP-odd pion-nucleon interaction (5.5) and (5.10) generates an isovector type CP-odd nuclear force through the one-pion exchange. Its Hamiltonian is given by

$$\mathcal{H} = \frac{\bar{G}^{(1)}_\pi}{8\pi m_N} (\vec{\sigma}_1 \tau_{1z} - \vec{\sigma}_2 \tau_{2z}) \cdot \vec{\nabla} \frac{e^{-m_\pi r}}{r} = \frac{-\bar{g}^{(1)}_{\pi NN} g_A}{8\pi f_\pi} (\vec{\sigma}_1 \tau_{1z} - \vec{\sigma}_2 \tau_{2z}) \cdot \vec{\nabla} \frac{e^{-m_\pi r}}{r}, \tag{5.11}$$

where $\vec{\sigma}_i$ and τ_{iz} $(i = 1, 2)$ are the spin and isospin operators acting on the i-th interacting nucleon, respectively. The coordinate $\vec{r}$ is directed to nucleon 1, and $\vec{\nabla}$ is the gradient defined accordingly. Here we defined $\bar{G}_\pi^{(1)} \equiv -\frac{g_A m_N}{f_\pi}\bar{g}_{\pi NN}^{(1)}$ as commonly done in nuclear level calculations.

The CP-odd nuclear force of eq. (5.11) polarizes the nucleus and leads to an observable effect. As for atomic systems, the EDMs of atomic nuclei are not directly observable due to the notorious Schiff's screening phenomenon [133]. The residual CP-odd moment, the nuclear Schiff moment $\vec{S}^{\mathrm{A}}$ (NSM), has to be evaluated using nuclear structure calculations. This has been done using several methods [7, 13, 134–140]. Here we quote the most recent results for ^{199}Hg [138, 139], ^{129}Xe [138, 140], and ^{225}Ra [135] nuclei:

$$S^{\mathrm{Hg}} = 2.65\, d_n\, \mathrm{fm}^2 - 0.075\, \bar{G}_\pi^{(1)} e\, \mathrm{fm}^3, \tag{5.12}$$

$$S^{\mathrm{Xe}} = 0.42\, d_n\, \mathrm{fm}^2 + 0.041\, \bar{G}_\pi^{(1)} e\, \mathrm{fm}^3, \tag{5.13}$$

$$S^{\mathrm{Ra}} = -6.0\, \bar{G}_\pi^{(1)} e\, \mathrm{fm}^3, \tag{5.14}$$

where we only display the contributions from the isovector CP-odd nuclear force and from the intrinsic nucleon EDM, for which we estimate the relative error to be about 30%. The contribution of the NSM to the atomic EDM has also been calculated within several frameworks. We here quote the results of the latest calculations [141–143],

$$\begin{aligned} d^{\mathrm{Hg}} &= -2.4 \times 10^{-17}\, \frac{S^{\mathrm{Hg}}}{e\, \mathrm{fm}^3} e\, \mathrm{cm} = -6.4 \times 10^{-4} d_n + 1.8 \times 10^{-5} \bar{G}_\pi^{(1)}\, e\, \mathrm{fm} \\ &= w\left(-1.3 \times 10^{-2} \pm [0.71, 4.4] \times 10^{-3}\right) e\ \mathrm{MeV}, \end{aligned} \tag{5.15}$$

$$\begin{aligned} d^{\mathrm{Xe}} &= 0.32 \times 10^{-17}\, \frac{S^{\mathrm{Xe}}}{e\, \mathrm{fm}^3} e\, \mathrm{cm} = 1.3 \times 10^{-5} d_n + 1.3 \times 10^{-6} \bar{G}_\pi^{(1)}\, e\, \mathrm{fm} \\ &= w\left(2.7 \times 10^{-4} \pm [0.52, 3.2] \times 10^{-4}\right) e\, \mathrm{MeV}, \end{aligned} \tag{5.16}$$

$$\begin{aligned} d^{\mathrm{Ra}} &= -6.3 \times 10^{-17}\, \frac{S^{\mathrm{Ra}}}{e\, \mathrm{fm}^3} e\, \mathrm{cm} = 3.8 \times 10^{-3} \bar{G}_\pi^{(1)}\, e\, \mathrm{fm} \\ &= w\left(\pm [1.5, 9.3] \times 10^{-1}\right) e\, \mathrm{MeV}. \end{aligned} \tag{5.17}$$

The atomic level calculations are in relatively good agreement among each other [7, 13, 144–146]. Their error is therefore negligible in the error budget.

It is also interesting to inspect the EDM of light nuclei for which experimental measurements are currently planned [147–149]. For the deuteron and the ^{3}He nucleus, we have [102]

$$\begin{aligned} d^{^2\mathrm{H}} &= 0.014\, \bar{G}_\pi^{(1)}\, e\, \mathrm{fm} \\ &= w\,(\pm[0.56, 3.4])\, e\, \mathrm{MeV}, \end{aligned} \tag{5.18}$$

$$\begin{aligned} d^{^3\mathrm{He}} &= 0.88\, d_n + 0.010\, \bar{G}_\pi^{(1)} + [1, 5] \times 10^{-4} \bar{C}_1\, \mathrm{GeV}^3\, e\, \mathrm{fm} \\ &= w\,(18 \pm [0.38, 2.3] + [1, 10])\, e\, \mathrm{MeV}. \end{aligned} \tag{5.19}$$

The nuclear level calculations for these light nuclei were performed by many groups [131, 150–155] and all are consistent, having errors of less than 10%. There are further results for other nuclei which may have larger sensitivities to CP violation, but are not considered here [102, 153, 154, 156–158]. We see that the deuteron EDM receives only contributions from the one-pion exchange CP-odd nuclear force, while the nucleon EDM and the contact CP-odd nuclear force (the last term of eq. (5.1)) also contribute to the ^{3}He EDM. The coupling constant of the contact CP-odd nuclear force is given by $\bar{C}_1 = \frac{m_{\rm CP} C_S}{m_N^2} = (3.1 \pm 1.6)\,\mathrm{GeV}^{-1}$ which was obtained by chirally rotating the CP-even contact nuclear force, in a similar way as eq. (5.2) [159], with $C_S = -120.8\,\mathrm{GeV}^{-2}$ [160]. The coefficient of $\bar{C}_1$ was calculated by matching a smeared delta function with the CP-odd ω-exchange nuclear force [150, 153, 154], and has a wide uncertainty band, due to the poorly known two-nucleon wavefunction near the origin. There is also a calculation within χEFT which yields a larger coefficient [131], but we do not further examine it here because of its large uncertainty. It has also recently been pointed out that the contact CP-odd nuclear force is required to unitarize the CP-odd nucleon-nucleon scattering with one-pion exchange [161], but we do not consider this effect in this work. We furthermore mention here that the coefficients of $\bar{C}_1$ are not available for atomic EDMs.

Finally, let us derive an explicit constraint on the magnitude of the Weinberg operator from the presently available experimental data, given by the EDM of ^{199}Hg. According to our analysis, the EDM of ^{199}Hg is $d^{\rm Hg} = w\,(\mu = 1\mathrm{GeV})\,(-1.3 \pm 0.96) \times 10^{-2} e\,\mathrm{MeV}$ where the central value is given only by the contribution of the intrinsic neutron EDM and the error is obtained by the taking the quadrature of the theoretical uncertainty of the Weinberg operator contribution of the neutron EDM (60%) [98], the error associated with the nuclear level calculation (30%) [139], and the maximal value of the our Weinberg operator result (the second term of (5.18)), which we consider here to be a systematic error. Combined with the experimental result, $\left|d^{\rm Hg\,(exp)}\right| < 7.4 \times 10^{-30} e\,\mathrm{cm}$ [15], this leads to

$$|w\,(\mu = 1\mathrm{TeV})| < 4 \times 10^{-10}\mathrm{GeV}^{-2}. \tag{5.20}$$

Here, we divided the experimental value by $(-1.3 + 0.96) \times 10^{-2} e\,\mathrm{MeV}$, so as to obtain an upper limit. Furthermore, $w\,(\mu = 1\mathrm{TeV}) = w\,(\mu = 1\mathrm{GeV})/0.3$ is renormalized at 1 TeV [68, 85, 103, 104, 162, 163]. It is interesting to compare this limit with that given by the direct measurement of the neutron EDM. The current experimental constraint $|d_n| < 1.8 \times 10^{-26} e$ cm [164] yields an upper limit of

$$|w\,(\mu = 1\mathrm{TeV})| < 4 \times 10^{-10}\mathrm{GeV}^{-2}, \tag{5.21}$$

where we took into account the uncertainty band of eq. (5.2). We see that the EDM of the ^{199}Hg atom and that of the neutron provide comparable constraints on the Weinberg operator, if we assume that it is the sole source of CP violation at the scale $\mu = 1\,\mathrm{TeV}$.

6 Conclusions

In this paper we calculated the pion-vacuum transition matrix element of the Weinberg operator using QCD sum rules and then the isovector CP-odd pion-nucleon interaction within χEFT at NLO. We finally derived relations between the Weinberg operator and the EDMs of several atoms and nuclei which are of experimental interest. For the case of atoms, the contribution of the nucleon-nucleon interaction was found to be of the same order as that induced by the intrinsic nucleon EDM.

JHEP06(2022)072

Possible improvements of this work include a more accurate investigation of the OPE convergence by taking into account condensates of higher dimension. Furthermore, using the correlator of two Weinberg operators, it was not possible to determine the sign of the matrix element studied in this work. Therefore, it will in the future become necessary to study the correlator between a Weinberg operator and a pion interpolating field.

Nevertheless, our results suggest that, depending on the sign, the CP violation may be enhanced in the ^{129}Xe EDM through the new Weinberg operator contribution. This experimental observable might therefore become crucial in the future determination of the magnitude of the Weinberg operator.

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A Borel transforms

We define the Borel transform of a function $f(Q^2)$ with $Q^2 = -q^2$ as

$$\mathcal{B}\left[f(Q^2)\right] := \lim_{\substack{Q^2,n\to\infty \\ Q^2/n=M^2=\text{const.}}} \frac{(Q^2)^n}{(n-1)!}\left(-\frac{d}{dQ^2}\right)^n f(Q^2), \tag{A.1}$$

where the unphysical parameter M is called the Borel mass. Below, we provide specific expressions for the Borel transforms used in this work.

$$\mathcal{B}\left[\left(\frac{1}{s+Q^2}\right)^k\right] = \frac{1}{(k-1)!}\left(\frac{1}{M^2}\right)^k e^{-s/M^2}, \tag{A.2}$$

$$\mathcal{B}\left[(Q^2)^k \log Q^2\right] = (-1)^{k+1}\Gamma(k+1)(M^2)^k. \tag{A.3}$$

Here, $k \in \mathbb{N}_+$ and $\Gamma(z)$ denotes the Euler gamma function.

B Fourier transforms

In this paper, we need to consider Fourier transforms from momentum space to configuration space and vice versa, which will in turn be discussed in this section, mostly following appendix E of ref. [96]. We define the Fourier transform of a function $f(p)$ to configuration space as

$$\mathcal{F}\left[f(p)\right] = \left(\frac{\mu_{\mathrm{IR}}^2}{4\pi e^{\gamma_E}}\right)^{-\epsilon_{\mathrm{IR}}} \int \frac{d^{4+2\epsilon_{\mathrm{IR}}}p}{(2\pi)^4} e^{-ip\cdot x} f(p), \tag{B.1}$$

where the divergent integral is treated using dimensional regularization with $d = 4 + 2\epsilon_{\mathrm{IR}}$. The reason for adopting this definition is to ensure the conservation of gauge invariance throughout the calculation. The OPE is based on the division of scales, in which higher regions of momentum space can be treated perturbatively to compute the Wilson coefficients [119]. Dimensional regularization provides the most economical renormalization scheme to do this in a gauge invariant manner. Any IR-pole can hence be removed as it should be considered to be part of the low momentum space regions and therefore can be absorbed into the renormalization of the condensates.

For functions of the form $1/(p^2)^k$ and $p^\mu p^\nu/(p^2)^k$, a simple calculation leads to

$$\mathcal{F}\left[\frac{1}{(p^2)^k}\right] = \frac{i}{4^k\pi^2}\left(-\frac{\mu_{\rm IR}^2 x^2}{4e^{\gamma_E}}\right)^{-\epsilon_{\rm IR}}\frac{\Gamma(2-k+\epsilon_{\rm IR})}{\Gamma(k)}(x^2)^{k-2}, \tag{B.2}$$

$$\mathcal{F}\left[\frac{p_\mu p_\nu}{(p^2)^k}\right] = (i\partial_\mu)(i\partial_\nu)\mathcal{F}\left[\frac{1}{(p^2)^k}\right], \tag{B.3}$$

where $k \in \mathbb{N}_+$. The Fourier transforms of type (B.2) and (B.3) relevant for our article are

$$\mathcal{F}\left[\frac{1}{p^2}\right] = \frac{i}{4\pi^2 x^2}, \tag{B.4}$$

$$\mathcal{F}\left[\frac{1}{(p^2)^2}\right] = \frac{i}{16\pi^2}\left[\frac{1}{\epsilon_{\rm IR}} - \ln\left(-\frac{\mu_{\rm IR}^2 x^2}{4}\right)\right], \tag{B.5}$$

$$\mathcal{F}\left[\frac{1}{(p^2)^3}\right] = -\frac{i}{2\cdot 4^3\pi^2}x^2\left[\frac{1}{\epsilon_{\rm IR}} + 1 - \ln\left(-\frac{\mu_{\rm IR}^2 x^2}{4}\right)\right], \tag{B.6}$$

$$\mathcal{F}\left[\frac{1}{(p^2)^4}\right] = \frac{i}{2^{10}\cdot 3\pi^2}(x^2)^2\left[\frac{1}{\epsilon_{\rm IR}} + \frac{3}{2} - \ln\left(-\frac{\mu_{\rm IR}^2 x^2}{4}\right)\right]. \tag{B.7}$$

In QCD sum rules, one often encounters ultraviolet divergent Fourier transforms of functions $f(x)$ from configuration space to momentum space. In this work, we define them as

$$\mathcal{F}\left[f(x)\right] = \left(\frac{\mu_{\rm UV}^2 e^{\gamma_E}}{4\pi}\right)^{-\epsilon_{\rm UV}}\int d^{4-2\epsilon_{\rm UV}} e^{ip\cdot x} f(x), \tag{B.8}$$

and use again dimensional regularization in treating the UV divergences. For the functions $1/(x^2)^k$ and $x^\mu x^\nu/(x^2)^k$ with $k \in \mathbb{N}_+$, we have

$$\mathcal{F}\left[\frac{1}{(x^2)^k}\right] = -\frac{i\pi^2}{4^{k-2}}\left(-\frac{\mu_{\rm UV}^2 e^{\gamma_E}}{p^2}\right)^{-\epsilon_{\rm UV}}\frac{\Gamma(2-k-\epsilon_{\rm UV})}{\Gamma(k)}(p^2)^{k-2}, \tag{B.9}$$

$$\mathcal{F}\left[\frac{x^\mu x^\nu}{(x^2)^k}\right] = \left(\frac{1}{i}\frac{\partial}{\partial p_\mu}\right)\left(\frac{1}{i}\frac{\partial}{\partial p_\nu}\right)\mathcal{F}\left[\frac{1}{(x^2)^k}\right]. \tag{B.10}$$

The resultant expressions will in the calculations of this paper be either Borel transformed or its imaginary part taken. As any polynomial of p^2 will vanish through the Borel transform, and the imaginary part of the correlation function originates only from logarithms, we only need to consider the log terms and can drop all polynomials in all practical calculations of this paper. The following simplified formulas will therefore be sufficient:

$$\mathcal{F}_{\rm pra}\left[\frac{1}{(x^2)^4}\right] = \frac{\pi^2 i}{2^6\cdot 3}(-p^2)^2\ln\left(-p^2\right), \tag{B.11}$$

$$\mathcal{F}_{\rm pra}\left[\frac{1}{(x^2)^5}\right] = \frac{\pi^2 i}{2^{10}\cdot 3^2}(-p^2)^3\ln\left(-p^2\right). \tag{B.12}$$

JHEP06(2022)072

C Details of the OPE calculation

The correlation function of eq. (3.1) can be expressed using the Weinberg operator of eq. (3.2) as

$$
\begin{aligned}
\Pi_{\rm OPE}(q) = & \, i\int d^4x e^{iq\cdot x}\cdot\frac{1}{9}w^2 f^{abc}f^{a'b'c'}\epsilon^{\nu\rho\alpha\beta}\epsilon^{\nu'\rho'\alpha'\beta'}g^{\mu\sigma}g^{\mu'\sigma'} \\
& \times\langle 0|T\left[2\left(\partial_\mu A^a_\nu\right)\left(\partial_\rho A^c_\sigma\right)-\left(\partial_\mu A^a_\nu\right)\left(\partial_\sigma A^c_\rho\right)-\left(\partial_\nu A^a_\mu\right)\left(\partial_\rho A^c_\sigma\right)\right]\left(\partial_\alpha A^b_\beta\right)(x) \\
& \cdot\left[2\left(\partial_{\mu'} A^{a'}_{\nu'}\right)\left(\partial_{\rho'} A^{c'}_{\sigma'}\right)-\left(\partial_{\mu'} A^{a'}_{\nu'}\right)\left(\partial_{\sigma'} A^{c'}_{\rho'}\right)-\left(\partial_{\nu'} A^{a'}_{\mu'}\right)\left(\partial_{\rho'} A^{c'}_{\sigma'}\right)\right]\left(\partial_{\alpha'} A^{b'}_{\beta'}\right)(0)|0\rangle \\
= & \, iw^2\int d^4x e^{iq\cdot x}\frac{1}{9}f^{abc}f^{a'b'c'} \\
& \times\Big[4\epsilon^{\nu\rho\alpha\beta}\epsilon^{\nu'\rho'\alpha'\beta'}g^{\mu\sigma}g^{\mu'\sigma'}-2\epsilon^{\nu\sigma\alpha\beta}\epsilon^{\nu'\rho'\alpha'\beta'}g^{\mu\rho}g^{\mu'\sigma'}-2\epsilon^{\nu\rho\alpha\beta}\epsilon^{\mu'\rho'\alpha'\beta'}g^{\mu\sigma}g^{\nu'\sigma'} \\
& -2\epsilon^{\nu\rho\alpha\beta}\epsilon^{\nu'\sigma'\alpha'\beta'}g^{\mu\sigma}g^{\mu'\rho'}-2\epsilon^{\mu\rho\alpha\beta}\epsilon^{\nu'\rho'\alpha'\beta'}g^{\nu\sigma}g^{\mu'\sigma'}+\epsilon^{\nu\sigma\alpha\beta}\epsilon^{\nu'\sigma'\alpha'\beta'}g^{\mu\rho}g^{\mu'\rho'} \\
& +\epsilon^{\nu\sigma\alpha\beta}\epsilon^{\mu'\rho'\alpha'\beta'}g^{\mu\rho}g^{\nu'\sigma'}+\epsilon^{\mu\rho\alpha\beta}\epsilon^{\nu'\sigma'\alpha'\beta'}g^{\nu\sigma}g^{\mu'\rho'}+\epsilon^{\mu\rho\alpha\beta}\epsilon^{\mu'\rho'\alpha'\beta'}g^{\nu\sigma}g^{\nu'\sigma'}\Big] \\
& \times\left\langle 0\left|T\left[(\partial_\mu A^a_\nu)(\partial_\alpha A^b_\beta)(\partial_\rho A^c_\sigma)(x)(\partial_{\mu'} A^{a'}_{\nu'})(\partial_{\alpha'} A^{b'}_{\beta'})(\partial_{\rho'} A^{c'}_{\sigma'})(0)\right]\right|0\right\rangle,
\end{aligned}
\tag{C.1}
$$

where, we have omitted all higher order contributions with respect to the strong coupling constant. The Wick contraction of the matrix element given in the last line of eq. (C.1) yields six distinct terms, which can be related to each other by simple permutations. It is hence sufficient to evaluate only one term, which we define as

$$
\begin{aligned}
& \Pi^{abc;a'b'c'}_{\mu\nu\alpha\beta\rho\sigma;\mu'\nu'\alpha'\beta'\rho'\sigma'}(x) \\
& \quad = \left(\partial^{(x)}_\mu\partial^{(y)}_{\mu'}S^{aa'}_{\nu\nu'}(x-y)\right)\left(\partial^{(x)}_\alpha\partial^{(y)}_{\alpha'}S^{bb'}_{\beta\beta'}(x-y)\right)\left(\partial^{(x)}_\rho\partial^{(y)}_{\rho'}S^{cc'}_{\sigma\sigma'}(x-y)\right)\Big|_{y=0},
\end{aligned}
\tag{C.2}
$$

where $S^{ab}_{\mu\nu}(x-y)$ stands for the gluon propagator. Other terms generated by the Wick contraction can be obtained by permutating the gauge and Lorentz indices in $\Pi^{abc;a'b'c'}_{\mu\nu\alpha\beta\rho\sigma;\mu'\nu'\alpha'\beta'\rho'\sigma'}(x)$. Gauge indices can be factored out as δ^{ab} from each gluon propagator. Eq. (C.1) can therefore be simplified as,

$$
\begin{aligned}
& \Pi_{\rm OPE}(q) \\
& = iw^2\int d^4x e^{iq\cdot x}\frac{1}{9}f^{abc}f^{a'b'c'} \\
& \quad\times\Big[4\epsilon^{\nu\rho\alpha\beta}\epsilon^{\nu'\rho'\alpha'\beta'}g^{\mu\sigma}g^{\mu'\sigma'}-2\epsilon^{\nu\sigma\alpha\beta}\epsilon^{\nu'\rho'\alpha'\beta'}g^{\mu\rho}g^{\mu'\sigma'}-2\epsilon^{\nu\rho\alpha\beta}\epsilon^{\mu'\rho'\alpha'\beta'}g^{\mu\sigma}g^{\nu'\sigma'} \\
& \quad -2\epsilon^{\nu\rho\alpha\beta}\epsilon^{\nu'\sigma'\alpha'\beta'}g^{\mu\sigma}g^{\mu'\rho'}-2\epsilon^{\mu\rho\alpha\beta}\epsilon^{\nu'\rho'\alpha'\beta'}g^{\nu\sigma}g^{\mu'\sigma'}+\epsilon^{\nu\sigma\alpha\beta}\epsilon^{\nu'\sigma'\alpha'\beta'}g^{\mu\rho}g^{\mu'\rho'} \\
& \quad +\epsilon^{\nu\sigma\alpha\beta}\epsilon^{\mu'\rho'\alpha'\beta'}g^{\mu\rho}g^{\nu'\sigma'}+\epsilon^{\mu\rho\alpha\beta}\epsilon^{\nu'\sigma'\alpha'\beta'}g^{\nu\sigma}g^{\mu'\rho'}+\epsilon^{\mu\rho\alpha\beta}\epsilon^{\mu'\rho'\alpha'\beta'}g^{\nu\sigma}g^{\nu'\sigma'}\Big] \\
& \quad\times\Big[\delta^{aa'}\delta^{bb'}\delta^{cc'}\Pi_{\mu\nu\alpha\beta\rho\sigma;\mu'\nu'\alpha'\beta'\rho'\sigma'}(x)+\delta^{aa'}\delta^{bc'}\delta^{cb'}\Pi_{\mu\nu\alpha\beta\rho\sigma;\mu'\nu'\rho'\sigma'\alpha'\beta'}(x) \\
& \quad +\delta^{ab'}\delta^{ba'}\delta^{cc'}\Pi_{\mu\nu\alpha\beta\rho\sigma;\alpha'\beta'\mu'\nu'\rho'\sigma'}(x)+\delta^{ab'}\delta^{bc'}\delta^{ca'}\Pi_{\mu\nu\alpha\beta\rho\sigma;\alpha'\beta'\rho'\sigma'\mu'\nu'}(x) \\
& \quad +\delta^{ac'}\delta^{bb'}\delta^{ca'}\Pi_{\mu\nu\alpha\beta\rho\sigma;\rho'\sigma'\alpha'\beta'\mu'\nu'}(x)+\delta^{ac'}\delta^{ba'}\delta^{cb'}\Pi_{\mu\nu\alpha\beta\rho\sigma;\rho'\sigma'\mu'\nu'\alpha'\beta'}(x)\Big] \\
& = iw^2\frac{2^3}{3}\int d^4x e^{iq\cdot x}C^{\mu\nu\alpha\beta\rho\sigma;\mu'\nu'\alpha'\beta'\rho'\sigma'}\left[\Pi_{\mu\nu\alpha\beta\rho\sigma;\mu'\nu'\alpha'\beta'\rho'\sigma'}(x)-\Pi_{\mu\nu\alpha\beta\rho\sigma;\mu'\nu'\rho'\sigma'\alpha'\beta'}(x)\right. \\
& \qquad -\Pi_{\mu\nu\alpha\beta\rho\sigma;\alpha'\beta'\mu'\nu'\rho'\sigma'}(x)+\Pi_{\mu\nu\alpha\beta\rho\sigma;\alpha'\beta'\rho'\sigma'\mu'\nu'}(x) \\
& \qquad \left.-\Pi_{\mu\nu\alpha\beta\rho\sigma;\rho'\sigma'\alpha'\beta'\mu'\nu'}(x)+\Pi_{\mu\nu\alpha\beta\rho\sigma;\rho'\sigma'\mu'\nu'\alpha'\beta'}(x)\right],
\end{aligned}
\tag{C.3}
$$

where $\Pi_{\mu\nu\alpha\beta\rho\sigma;\mu'\nu'\alpha'\beta'\rho'\sigma'}(x)$ is defined as

$$\Pi^{abc;a'b'c'}_{\mu\nu\alpha\beta\rho\sigma;\mu'\nu'\alpha'\beta'\rho'\sigma'}(x) = \delta^{aa'}\delta^{bb'}\delta^{cc'}\Pi_{\mu\nu\alpha\beta\rho\sigma;\mu'\nu'\alpha'\beta'\rho'\sigma'}(x), \tag{C.4}$$

and

$$\begin{aligned} &C^{\mu\nu\alpha\beta\rho\sigma;\mu'\nu'\alpha'\beta'\rho'\sigma'} \\ &:= \Big[4\epsilon^{\nu\rho\alpha\beta}\epsilon^{\nu'\rho'\alpha'\beta'}g^{\mu\sigma}g^{\mu'\sigma'} - 2\epsilon^{\nu\sigma\alpha\beta}\epsilon^{\nu'\rho'\alpha'\beta'}g^{\mu\rho}g^{\mu'\sigma'} - 2\epsilon^{\nu\rho\alpha\beta}\epsilon^{\mu'\rho'\alpha'\beta'}g^{\mu\sigma}g^{\nu'\sigma'} \\ &\quad -2\epsilon^{\nu\rho\alpha\beta}\epsilon^{\nu'\sigma'\alpha'\beta'}g^{\mu\sigma}g^{\mu'\rho'} - 2\epsilon^{\mu\rho\alpha\beta}\epsilon^{\nu'\rho'\alpha'\beta'}g^{\nu\sigma}g^{\mu'\sigma'} + \epsilon^{\nu\sigma\alpha\beta}\epsilon^{\nu'\sigma'\alpha'\beta'}g^{\mu\rho}g^{\mu'\rho'} \\ &\quad +\epsilon^{\nu\sigma\alpha\beta}\epsilon^{\mu'\rho'\alpha'\beta'}g^{\mu\rho}g^{\nu'\sigma'} + \epsilon^{\mu\rho\alpha\beta}\epsilon^{\nu'\sigma'\alpha'\beta'}g^{\nu\sigma}g^{\mu'\rho'} + \epsilon^{\mu\rho\alpha\beta}\epsilon^{\mu'\rho'\alpha'\beta'}g^{\nu\sigma}g^{\nu'\sigma'}\Big]. \end{aligned} \tag{C.5}$$

For convenience of the later calculations, we expand the gluon propagator in terms of condensates and other perturbative terms relevant for this work. As we are here especially interested in the quark-mass dependent part of the correlator, it is necessary to take into account the leading order quark loop insertion of the gluon propagator. The gluon propagator can thus be expanded as

$$S^{ab}_{\mu\nu}(x,y) = \left(S^{(0)}\right)^{ab}_{\mu\nu}(x-y) + \left(S^{(\mathrm{loop})}\right)^{ab}_{\mu\nu}(x-y) + \left(S^{(G)}\right)^{ab}_{\mu\nu}(x,y) + \left(S^{(q)}\right)^{ab}_{\mu\nu}(x-y), \tag{C.6}$$

where $S^{(0)}(x-y)$ is the free gluon propagator, $S^{(\mathrm{loop})}(x-y)$ its leading order perturbative quark loop correction, $S^{(G)}(x,y)$ stands for the gluon condensate insertion and $S^{(q)}(x-y)$ for the quark loop correction in which one of the quark propagator is replaced by the quark condensate. The analytic form of these terms can be given as

$$\left(S^{(0)}\right)^{ab}_{\mu\nu}(x-y) = \frac{\delta^{ab}g_{\mu\nu}}{4\pi^2(x-y)^2}, \tag{C.7}$$

$$\left(S^{(\mathrm{loop})}\right)^{ab}_{\mu\nu}(x-y) = \frac{m_q^2\alpha_s}{4^3\pi^2}\delta^{ab}\left[3g_{\mu\nu}\ln\left(-\frac{\mu_{\mathrm{IR}}^2(x-y)^2}{4}\right) - 2\frac{(x-y)_\mu(x-y)_\nu}{(x-y)^2}\right], \tag{C.8}$$

$$\begin{aligned} \left(S^{(q)}\right)^{ab}_{\mu\nu}(x-y) = \frac{m_q\alpha_s}{2^6\cdot 3^2\pi}\langle\bar{q}q\rangle\delta^{ab}\Bigg[&5g^{\mu\nu}(x-y)^2 + \Big(-5g^{\mu\nu}(x-y)^2 + 2(x-y)^\mu(x-y)^\nu\Big) \\ &\times\ln\left(-\frac{\mu_{\mathrm{IR}}^2(x-y)^2}{4}\right)\Bigg], \end{aligned} \tag{C.9}$$

$$\left(S^{(G)}\right)^{ab}_{\mu\nu}(x,y) = x^{\mu'}y^{\nu'}\frac{\langle G^2\rangle}{2^7\cdot 3}\delta^{ab}(g_{\mu'\nu'}g_{\mu\nu} - g_{\mu'\nu}g_{\mu\nu'}), \tag{C.10}$$

$$\begin{aligned} \left(S^{(q)}\right)^{ab}_{\mu\nu}(x-y) = \frac{m_q\alpha_s}{2^6\cdot 3^2\pi}\langle\bar{q}q\rangle\delta^{ab}\Bigg[&5g^{\mu\nu}(x-y)^2 + \Big(-5g^{\mu\nu}(x-y)^2 + 2(x-y)^\mu(x-y)^\nu\Big) \\ &\times\ln\left(-\frac{\mu_{\mathrm{IR}}^2(x-y)^2}{4}\right)\Bigg]. \end{aligned} \tag{C.11}$$

For obtaining $S^{(\mathrm{loop})}(x-y)$ we have expanded the full expression around the zero quark mass limit and only kept the second order m_q^2 term, since the leading order term does not contribute to the quark mass dependence and the first order term vanishes. Similarly, for $S^{(q)}(x-y)$, the leading order m_q^0 term vanishes and we keep only the first non-vanishing m_q term.

JHEP06(2022)072

Let us here, as an example, show the explicit further steps needed to calculate the OPE term corresponding to the left-most the diagram of figure 5. It can be written down as

$$
\begin{aligned}
&(\Pi'_{\text{loop}})^{abc;a'b'c'}_{\mu\nu\alpha\beta\rho\sigma;\mu'\nu'\alpha'\beta'\rho'\sigma'}(x)\\
&=\left(\partial^{(x)}_{\mu}\partial^{(y)}_{\mu'}(S^{(\text{loop})})^{aa'}_{\nu\nu'}(x-y)\right)\cdot\left(\partial^{(x)}_{\alpha}\partial^{(y)}_{\alpha'}(S^{(0)})^{bb'}_{\beta\beta'}(x-y)\right)\cdot\left(\partial^{(x)}_{\rho}\partial^{(y)}_{\rho'}(S^{(0)})^{cc'}_{\sigma\sigma'}(x-y)\right)\Big|_{y=0}\\
&\quad+\left(\partial^{(x)}_{\mu}\partial^{(y)}_{\mu'}(S^{(0)})^{aa'}_{\nu\nu'}(x-y)\right)\cdot\left(\partial^{(x)}_{\alpha}\partial^{(y)}_{\alpha'}(S^{(\text{loop})})^{bb'}_{\beta\beta'}(x-y)\right)\cdot\left(\partial^{(x)}_{\rho}\partial^{(y)}_{\rho'}(S^{(0)})^{cc'}_{\sigma\sigma'}(x-y)\right)\Big|_{y=0}\\
&\quad+\left(\partial^{(x)}_{\mu}\partial^{(y)}_{\mu'}(S^{(0)})^{aa'}_{\nu\nu'}(x-y)\right)\cdot\left(\partial^{(x)}_{\alpha}\partial^{(y)}_{\alpha'}(S^{(0)})^{bb'}_{\beta\beta'}(x-y)\right)\cdot\left(\partial^{(x)}_{\rho}\partial^{(y)}_{\rho'}(S^{(\text{loop})})^{cc'}_{\sigma\sigma'}(x-y)\right)\Big|_{y=0}\\
&=\delta^{aa'}\delta^{bb'}\delta^{cc'}(\Pi_{\text{loop}})_{\mu\nu\alpha\beta\rho\sigma;\mu'\nu'\alpha'\beta'\rho'\sigma'}(x),
\end{aligned}
\tag{C.12}
$$

where

$$
\begin{aligned}
&(\Pi'_{\text{loop}})_{\mu\nu\alpha\beta\rho\sigma;\mu'\nu'\alpha'\beta'\rho'\sigma'}(x)=\sum_{u,d}\frac{m_q^2\alpha_s}{2^7\pi^6}\\
&\times\Bigg\{g_{\beta\beta'}g_{\sigma\sigma'}\left(\frac{g_{\alpha\alpha'}}{x^4}-4\frac{x_\alpha x_{\alpha'}}{x^6}\right)\left(\frac{g_{\rho\rho'}}{x^4}-4\frac{x_\rho x_{\rho'}}{x^6}\right)\Bigg[(3g_{\mu\mu'}g_{\nu\nu'}-g_{\mu\nu}g_{\mu'\nu'}-g_{\mu\nu'}g_{\mu'\nu})\frac{1}{x^2}\\
&+2(g_{\mu\nu}x_{\mu'}x_{\nu'}+g_{\mu\nu'}x_{\mu'}x_{\nu}+g_{\mu'\nu}x_{\mu}x_{\nu'}+g_{\mu'\nu'}x_{\mu}x_{\nu}+g_{\mu\mu'}x_{\nu}x_{\nu'}-3g_{\nu\nu'}x_{\mu}x_{\mu'})\frac{1}{x^4}-8\frac{x_\mu x_{\mu'}x_\nu x_{\nu'}}{x^6}\Bigg]\\
&+g_{\nu\nu'}g_{\sigma\sigma'}\left(\frac{g_{\mu\mu'}}{x^4}-4\frac{x_\mu x_{\mu'}}{x^6}\right)\left(\frac{g_{\rho\rho'}}{x^4}-4\frac{x_\rho x_{\rho'}}{x^6}\right)\Bigg[(3g_{\alpha\alpha'}g_{\beta\beta'}-g_{\alpha\beta}g_{\alpha'\beta'}-g_{\alpha\beta'}g_{\alpha'\beta})\frac{1}{x^2}\\
&+2(g_{\alpha\beta}x_{\alpha'}x_{\beta'}+g_{\alpha\beta'}x_{\alpha'}x_{\beta}+g_{\alpha'\beta}x_{\alpha}x_{\beta'}+g_{\alpha'\beta'}x_{\alpha}x_{\beta}+g_{\alpha\alpha'}x_{\beta}x_{\beta'}-3g_{\beta\beta'}x_{\alpha}x_{\alpha'})\frac{1}{x^4}-8\frac{x_\alpha x_{\alpha'}x_\beta x_{\beta'}}{x^6}\Bigg]\\
&+g_{\beta\beta'}g_{\nu\nu'}\left(\frac{g_{\alpha\alpha'}}{x^4}-4\frac{x_\alpha x_{\alpha'}}{x^6}\right)\left(\frac{g_{\mu\mu'}}{x^4}-4\frac{x_\mu x_{\mu'}}{x^6}\right)\Bigg[(3g_{\rho\rho'}g_{\sigma\sigma'}-g_{\rho\sigma}g_{\rho'\sigma'}-g_{\rho\sigma'}g_{\rho'\sigma})\frac{1}{x^2}\\
&+2(g_{\rho\sigma}x_{\rho'}x_{\sigma'}+g_{\rho\sigma'}x_{\rho'}x_{\sigma}+g_{\rho'\sigma}x_{\rho}x_{\sigma'}+g_{\rho'\sigma'}x_{\rho}x_{\sigma}+g_{\rho\rho'}x_{\sigma}x_{\sigma'}-3g_{\sigma\sigma'}x_{\rho}x_{\rho'})\frac{1}{x^4}-8\frac{x_\rho x_{\rho'}x_\sigma x_{\sigma'}}{x^6}\Bigg]\Bigg\}.
\end{aligned}
\tag{C.13}
$$

Using eqs. (C.5) and (C.13), $\Pi'_{\text{loop}}(q^2)$, which denotes the contribution of these quark loop diagrams to eq. (C.3), can then be computed as

$$
\begin{aligned}
&\Pi'_{\text{loop}}(q^2)\\
&=iw^2\frac{2^3}{3}\int d^4xe^{iq\cdot x}C^{\mu\nu\alpha\beta\rho\sigma;\mu'\nu'\alpha'\beta'\rho'\sigma'}\\
&\quad\times\Big[(\Pi'_{\text{loop}})_{\mu\nu\alpha\beta\rho\sigma;\mu'\nu'\alpha'\beta'\rho'\sigma'}(x)-(\Pi'_{\text{loop}})_{\mu\nu\alpha\beta\rho\sigma;\mu'\nu'\rho'\sigma'\alpha'\beta'}(x)\\
&\quad\quad-(\Pi'_{\text{loop}})_{\mu\nu\alpha\beta\rho\sigma;\alpha'\beta'\mu'\nu'\rho'\sigma'}(x)+(\Pi'_{\text{loop}})_{\mu\nu\alpha\beta\rho\sigma;\alpha'\beta'\rho'\sigma'\mu'\nu'}(x)\\
&\quad\quad-(\Pi'_{\text{loop}})_{\mu\nu\alpha\beta\rho\sigma;\rho'\sigma'\alpha'\beta'\mu'\nu'}(x)+(\Pi'_{\text{loop}})_{\mu\nu\alpha\beta\rho\sigma;\rho'\sigma'\mu'\nu'\alpha'\beta'}(x)\Big]\\
&=iw^2\frac{m_q^2\alpha_s}{2^4\cdot 3\pi^6}\int d^4xe^{iq\cdot x}\frac{6912}{x^{10}}\\
&=-w^2\frac{m_q^2\alpha_s}{2^6\pi^4}(-q^2)^3\ln\left(-q^2\right).
\end{aligned}
\tag{C.14}
$$

JHEP06(2022)072

Taking into account the contributions of both u and d quarks, noting that

$$\begin{aligned} m_u^2 + m_d^2 &= \frac{1}{4}\left[(m_+ + m_-)^2 + (m_+ - m_-)^2\right] \\ &= \frac{1}{2}(m_+^2 + m_-^2), \end{aligned} \tag{C.15}$$

where $m_+ \equiv m_u + m_d$ and $m_- \equiv m_u - m_d$, and retaining only the m_--dependent part, we have

$$\Pi_{\text{loop}}(q^2, m_- \neq 0) = -w^2 \frac{m_-^2 \alpha_s}{2^7 \pi^4}(-q^2)^3 \ln\left(-q^2\right). \tag{C.16}$$

For the term involving the quark condensate, $\Pi^{(q)}(q^2)$, chirality demands that it must be proportional to $\sum_{u,d} m_q \langle \bar{q}q \rangle$ at leading non-vanishing order in the quark mass expansion. The m_--dependence can be extracted from this expression as

$$\begin{aligned} \sum_{q=u,d} m_q \langle \bar{q}q \rangle &= m_u \langle \bar{u}u \rangle + m_d \left\langle \bar{d}d \right\rangle \\ &= m_u(\langle \bar{q}q \rangle - 2m_- B_0^2 h_3) + m_d(\langle \bar{q}q \rangle + 2m_- B_0^2 h_3) \\ &= m_+ \langle \bar{q}q \rangle - 2m_-^2 B_0^2 h_3, \end{aligned} \tag{C.17}$$

where

$$\begin{aligned} \langle \bar{q}q \rangle &\equiv (\langle \bar{u}u \rangle + \langle \bar{d}d \rangle)/2, \\ \left\langle \bar{u}u - \bar{d}d \right\rangle &= -4(m_u - m_d) B_0^2 h_3. \end{aligned} \tag{C.18}$$

Numerically, we will use the values provided in ref. [124] (see table 1).

D One-loop correction to $\bar{g}_{\pi NN}^{(1)}$

In this appendix we recapitulate the calculation of the NLO correction to $\bar{g}_{\pi NN}^{(1)}$ induced by the three-pion interaction of eq. (5.1). We assume the following chiral $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ invariant effective Lagrangian

$$\begin{aligned} \mathcal{L}_0 = & \frac{f_\pi^2}{4} \mathrm{Tr}(\partial_\mu U \partial^\mu U^\dagger) + \mathrm{Tr}[\bar{B}(i\partial\!\!\!/ - m_B)B] \\ & + \frac{i}{2}\mathrm{Tr}[\bar{B}\gamma_\mu(\xi\partial^\mu\xi^\dagger + \xi^\dagger\partial^\mu\xi)B] + \frac{i}{2}\mathrm{Tr}[\bar{B}\gamma_\mu B(\partial^\mu\xi\xi^\dagger + \partial^\mu\xi^\dagger\xi)] \\ & + \frac{i}{2}(D+F)\mathrm{Tr}[\bar{B}\gamma_\mu\gamma_5(\xi\partial^\mu\xi^\dagger - \xi^\dagger\partial^\mu\xi)B] \\ & - \frac{i}{2}(D-F)\mathrm{Tr}[\bar{B}\gamma_\mu\gamma_5 B(\partial^\mu\xi\xi^\dagger - \partial^\mu\xi^\dagger\xi)], \end{aligned} \tag{D.1}$$

where $D + F = g_A = 1.27$, and

$$\xi = \exp\left(\frac{i\phi}{\sqrt{2}f_\pi}\right), \tag{D.2}$$

with $U = \xi^2$. The meson field ϕ is defined by

$$\phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -2\frac{\eta^0}{\sqrt{6}} \end{pmatrix}, \tag{D.3}$$

and the baryon field B by

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -2\frac{\Lambda^0}{\sqrt{6}} \end{pmatrix} . \tag{D.4}$$

We use the heavy baryon approximation where several simplifications occur. The 4-momentum of the nucleon can be split into two terms according to the scale separation between the heavy baryon mass and the nonrelativistic spatial 3-momentum, namely $p_\mu = m_N v_\mu + k_\mu$, where v is the velocity 4-vector of the nucleon. The baryon propagator is approximated as

$$\frac{i}{\not{p} - m_B + i\epsilon} \approx \frac{i\mathcal{P}_+}{v \cdot k + i\epsilon}, \tag{D.5}$$

where $\mathcal{P}_\pm \equiv \frac{1}{2}(1 \pm \not{v})$. The sign of the causal infinitesimal term $(i\epsilon)$ controls the sign of the final result.

The leading order meson-baryon interaction becomes

$$\begin{aligned} \mathcal{L}_{mB} &= \frac{i}{2}(D+F)\mathrm{Tr}[\bar{B}\gamma_\mu\gamma_5(\xi\partial^\mu\xi^\dagger - \xi^\dagger\partial^\mu\xi)B] \\ &= \frac{g_A}{\sqrt{2}f_\pi}\mathrm{Tr}[\bar{B}\gamma_\mu\gamma_5(\partial^\mu\phi)B] + \cdots \\ &= \frac{g_A}{\sqrt{2}f_\pi}\left[\bar{p}\gamma_\mu\gamma_5 n\partial^\mu\pi^+ + \bar{n}\gamma_\mu\gamma_5 p\partial^\mu\pi^- + \frac{1}{\sqrt{2}}\bar{p}\gamma_\mu\gamma_5 p\partial^\mu\pi^0 - \frac{1}{\sqrt{2}}\bar{p}\gamma_\mu\gamma_5 p\partial^\mu\pi^0\right] + \cdots \\ &\approx \frac{\sqrt{2}g_A}{f_\pi}\left[\bar{H}_p S_\mu H_n\partial^\mu\pi^+ + \bar{H}_n S_\mu H_p\partial^\mu\pi^- + \frac{1}{\sqrt{2}}\bar{H}_p S_\mu H_p\partial^\mu\pi^0 - \frac{1}{\sqrt{2}}\bar{H}_p S_\mu H_p\partial^\mu\pi^0\right] + \cdots, \end{aligned} \tag{D.6}$$

where the ellipses denote irrelevant higher order terms as well as interactions involving strangeness. The four-vector $S^\mu = \frac{1}{2}(0, \vec{\sigma})$ is the Pauli-Lubanski spin vector.

The NLO correction to $\bar{g}^{(1)}_{\pi NN}$ induced by the three-pion interaction of eq. (5.1) is then given by

$$i\mathcal{M}_{\pi^\pm} = -2m_N\Delta_{3\pi}\frac{2g_A^2}{f_\pi^2}\int\frac{d^4k}{(2\pi)^4}\frac{\bar{H}_N(-ik\cdot S)\mathcal{P}_+(ik\cdot S)H_N}{[k^2 - m_\pi^2]^2[v\cdot k + i\epsilon]}, \tag{D.7}$$

$$i\mathcal{M}_{\pi^0} = -6m_N\Delta_{3\pi}\frac{g_A^2}{f_\pi^2}\int\frac{d^4k}{(2\pi)^4}\frac{\bar{H}_N(-ik\cdot S)\mathcal{P}_+(ik\cdot S)H_N}{[k^2 - m_\pi^2]^2[v\cdot k + i\epsilon]}, \tag{D.8}$$

where $i\mathcal{M}_{\pi^\pm}$ and $i\mathcal{M}_{\pi^0}$ are the contributions from the charged and neutral pion loops, respectively (see figure 8).

JHEP06(2022)072

The loop integral can be transformed further, as

$$
\begin{aligned}
I &= \int \frac{d^4k}{(2\pi)^4} \frac{\bar{H}_N (S\cdot k)\mathcal{P}_+ (S\cdot k) H_N}{[k^2-m_\pi^2]^2[v\cdot k+i\epsilon]} \\
&= \int \frac{d^4k}{(2\pi)^4} \frac{\bar{H}_N (S\cdot k)^2 H_N}{[k^2-m_\pi^2]^2[v\cdot k+i\epsilon]} \\
&= \int \frac{d^4k}{(2\pi)^4} \frac{\bar{H}_N k_\alpha k_\beta \frac{1}{2}\Big([S^\alpha,S^\beta]+\{S^\alpha,S^\beta\}\Big) H_N}{[k^2-m_\pi^2]^2[v\cdot k+i\epsilon]} \\
&= \int \frac{d^4k}{(2\pi)^4} \frac{\bar{H}_N k_i k_j \frac{1}{4}\delta_{ij} H_N}{[k^2-m_\pi^2]^2[v\cdot k+i\epsilon]} \\
&= \frac{1}{4}\int \frac{d^4k}{(2\pi)^4} \frac{|\mathbf{k}|^2 \bar{H}_N H_N}{[k^2-m_\pi^2]^2[k_0+i\epsilon]} \\
&= \frac{1}{4}\int \frac{d^4k}{(2\pi)^4} \frac{|\mathbf{k}|^2 \bar{H}_N H_N[(\text{Principal value})-i\pi\delta(k_0)]}{[k_0^2-|\mathbf{k}|^2-m_\pi^2]^2} \\
&= \frac{-i}{8}\int \frac{d^3k}{(2\pi)^3} \frac{|\mathbf{k}|^2 \bar{H}_N H_N}{[|\mathbf{k}|^2+m_\pi^2]^2}.
\end{aligned} \tag{D.9}
$$

We then shift the dimension to use dimensional regularization:

$$
\begin{aligned}
I &= \frac{-i}{8}\int \frac{d^dk}{(2\pi)^d} \frac{|\mathbf{k}|^2 \bar{H}_N H_N}{[|\mathbf{k}|^2+m_\pi^2]^2} \\
&= \frac{-i}{8}\frac{1}{(4\pi)^{d/2}}\frac{d}{2}\frac{\Gamma\left(2-\frac{d}{2}-1\right)}{\Gamma(2)}\left(\frac{1}{m_\pi^2}\right)^{2-\frac{d}{2}-1}\bar{H}_N H_N.
\end{aligned} \tag{D.10}
$$

Taking the limit $d\to 3$, we derive

$$
\begin{aligned}
I &= \frac{-i}{8}\frac{3}{2(4\pi)^{3/2}}\Gamma\left(-\frac{1}{2}\right) m_\pi \bar{H}_N H_N \\
&= i\frac{3m_\pi}{64\pi}\bar{H}_N H_N,
\end{aligned} \tag{D.11}
$$

where we used $\Gamma\left(-\frac{1}{2}\right) = -\sqrt{4\pi}$.

The NLO correction to $\bar{g}^{(1)}_{\pi NN}$ induced by the three-pion interaction is thus obtained as

$$
\bar{g}^{(1)}_{\pi NN,\mathrm{NLO}} = \mathcal{M}_{\pi^\pm} + \mathcal{M}_{\pi^0} = -10 m_N \Delta_{3\pi}\frac{g_A^2}{f_\pi^2}\times\frac{I}{i} = -m_N\Delta_{3\pi}\frac{15 g_A^2 m_\pi}{32\pi f_\pi^2}, \tag{D.12}
$$

which reproduces the formula of eq. (5.10).

References

[1] A.D. Sakharov, *Violation of CP Invariance, C asymmetry, and baryon asymmetry of the universe*, *Pisma Zh. Eksp. Teor. Fiz.* **5** (1967) 32 [INSPIRE].

[2] G.R. Farrar and M.E. Shaposhnikov, *Baryon asymmetry of the universe in the standard electroweak theory*, *Phys. Rev. D* **50** (1994) 774 [hep-ph/9305275] [INSPIRE].

[3] P. Huet and E. Sather, *Electroweak baryogenesis and standard model CP-violation*, *Phys. Rev. D* **51** (1995) 379 [hep-ph/9404302] [INSPIRE].

[4] X.-G. He, B.H.J. McKellar and S. Pakvasa, *The Neutron Electric Dipole Moment*, *Int. J. Mod. Phys. A* **4** (1989) 5011 [*Erratum ibid.* **6** (1991) 1063] [INSPIRE].

[5] W. Bernreuther and M. Suzuki, *The electric dipole moment of the electron*, *Rev. Mod. Phys.* **63** (1991) 313 [*Erratum ibid.* **64** (1992) 633] [INSPIRE].

[6] I.B. Khriplovich and S.K. Lamoreaux, *CP violation without strangeness*, Springer, Berlin, Germany (1997) [DOI].

[7] J.S.M. Ginges and V.V. Flambaum, *Violations of fundamental symmetries in atoms and tests of unification theories of elementary particles*, *Phys. Rept.* **397** (2004) 63 [physics/0309054] [INSPIRE].

[8] M. Pospelov and A. Ritz, *Electric dipole moments as probes of new physics*, *Annals Phys.* **318** (2005) 119 [hep-ph/0504231] [INSPIRE].

[9] T. Fukuyama, *Searching for New Physics beyond the Standard Model in Electric Dipole Moment*, *Int. J. Mod. Phys. A* **27** (2012) 1230015 [arXiv:1201.4252] [INSPIRE].

[10] J.L. Hewett et al., *Fundamental Physics at the Intensity Frontier*, ANL-HEP-TR-12-25, SLAC-R-991 (2012) [arXiv:1205.2671] [INSPIRE].

[11] J. Engel, M.J. Ramsey-Musolf and U. van Kolck, *Electric Dipole Moments of Nucleons, Nuclei, and Atoms: The Standard Model and Beyond*, *Prog. Part. Nucl. Phys.* **71** (2013) 21 [arXiv:1303.2371] [INSPIRE].

[12] N. Yamanaka, *Analysis of the Electric Dipole Moment in the R-parity Violating Supersymmetric Standard Model*, Springer, Berlin, Germany (2014) [DOI].

[13] N. Yamanaka, B.K. Sahoo, N. Yoshinaga, T. Sato, K. Asahi and B.P. Das, *Probing exotic phenomena at the interface of nuclear and particle physics with the electric dipole moments of diamagnetic atoms: A unique window to hadronic and semi-leptonic CP-violation*, *Eur. Phys. J. A* **53** (2017) 54 [arXiv:1703.01570] [INSPIRE].

[14] T. Chupp, P. Fierlinger, M. Ramsey-Musolf and J. Singh, *Electric dipole moments of atoms, molecules, nuclei, and particles*, *Rev. Mod. Phys.* **91** (2019) 015001 [arXiv:1710.02504] [INSPIRE].

[15] B. Graner, Y. Chen, E.G. Lindahl and B.R. Heckel, *Reduced Limit on the Permanent Electric Dipole Moment of Hg199*, *Phys. Rev. Lett.* **116** (2016) 161601 [*Erratum ibid.* **119** (2017) 119901] [arXiv:1601.04339] [INSPIRE].

[16] M. Bishof et al., *Improved limit on the ^{225}Ra electric dipole moment*, *Phys. Rev. C* **94** (2016) 025501 [arXiv:1606.04931] [INSPIRE].

[17] F. Allmendinger et al., *Measurement of the Permanent Electric Dipole Moment of the ^{129}Xe Atom*, *Phys. Rev. A* **100** (2019) 022505 [arXiv:1904.12295] [INSPIRE].

JHEP06(2022)072

[18] M. Kobayashi and T. Maskawa, *CP Violation in the Renormalizable Theory of Weak Interaction*, *Prog. Theor. Phys.* **49** (1973) 652 [INSPIRE].

[19] C.-Y. Seng, *Reexamination of The Standard Model Nucleon Electric Dipole Moment*, *Phys. Rev. C* **91** (2015) 025502 [arXiv:1411.1476] [INSPIRE].

[20] N. Yamanaka and E. Hiyama, *Standard model contribution to the electric dipole moment of the deuteron,^{3}H, and^{3}He nuclei*, *JHEP* **02** (2016) 067 [arXiv:1512.03013] [INSPIRE].

[21] Y. Yamaguchi and N. Yamanaka, *Large long-distance contributions to the electric dipole moments of charged leptons in the standard model*, *Phys. Rev. Lett.* **125** (2020) 241802 [arXiv:2003.08195] [INSPIRE].

[22] Y. Ema, T. Gao and M. Pospelov, *Standard Model prediction for paramagnetic EDMs*, arXiv:2202.10524 [INSPIRE].

[23] T. Fleig and M. Jung, *Model-independent determinations of the electron EDM and the role of diamagnetic atoms*, *JHEP* **07** (2018) 012 [arXiv:1802.02171] [INSPIRE].

[24] M. Abramczyk, S. Aoki, T. Blum, T. Izubuchi, H. Ohki and S. Syritsyn, *Lattice calculation of electric dipole moments and form factors of the nucleon*, *Phys. Rev. D* **96** (2017) 014501 [arXiv:1701.07792] [INSPIRE].

[25] J. Dragos, T. Luu, A. Shindler, J. de Vries and A. Yousif, *Confirming the Existence of the strong CP Problem in Lattice QCD with the Gradient Flow*, *Phys. Rev. C* **103** (2021) 015202 [arXiv:1902.03254] [INSPIRE].

[26] C. Alexandrou, A. Athenodorou, K. Hadjiyiannakou and A. Todaro, *Neutron electric dipole moment using lattice QCD simulations at the physical point*, *Phys. Rev. D* **103** (2021) 054501 [arXiv:2011.01084] [INSPIRE].

[27] T. Bhattacharya, V. Cirigliano, R. Gupta, E. Mereghetti and B. Yoon, *Contribution of the QCD Θ-term to the nucleon electric dipole moment*, *Phys. Rev. D* **103** (2021) 114507 [arXiv:2101.07230] [INSPIRE].

[28] JLQCD collaboration, *Nucleon charges with dynamical overlap fermions*, *Phys. Rev. D* **98** (2018) 054516 [arXiv:1805.10507] [INSPIRE].

[29] R. Gupta, B. Yoon, T. Bhattacharya, V. Cirigliano, Y.-C. Jang and H.-W. Lin, *Flavor diagonal tensor charges of the nucleon from $(2+1+1)$-flavor lattice QCD*, *Phys. Rev. D* **98** (2018) 091501 [arXiv:1808.07597] [INSPIRE].

[30] C. Alexandrou et al., *Nucleon axial, tensor, and scalar charges and σ-terms in lattice QCD*, *Phys. Rev. D* **102** (2020) 054517 [arXiv:1909.00485] [INSPIRE].

[31] USQCD collaboration, *The Role of Lattice QCD in Searches for Violations of Fundamental Symmetries and Signals for New Physics*, *Eur. Phys. J. A* **55** (2019) 197 [arXiv:1904.09704] [INSPIRE].

[32] χQCD collaboration, *Nucleon isovector tensor charge from lattice QCD using chiral fermions*, *Phys. Rev. D* **101** (2020) 094501 [arXiv:2002.06699] [INSPIRE].

[33] Z. Davoudi et al., *Nuclear matrix elements from lattice QCD for electroweak and beyond-Standard-Model processes*, *Phys. Rept.* **900** (2021) 1 [arXiv:2008.11160] [INSPIRE].

[34] Nucleon Matrix Elements (NME) collaboration, *Precision nucleon charges and form factors using $(2+1)$-flavor lattice QCD*, *Phys. Rev. D* **105** (2022) 054505 [arXiv:2103.05599] [INSPIRE].

[35] M. Pospelov and A. Ritz, *Neutron EDM from electric and chromoelectric dipole moments of quarks*, Phys. Rev. D **63** (2001) 073015 [hep-ph/0010037] [INSPIRE].

[36] M. Pospelov, *Best values for the CP odd meson nucleon couplings from supersymmetry*, Phys. Lett. B **530** (2002) 123 [hep-ph/0109044] [INSPIRE].

[37] J. de Vries, R.G.E. Timmermans, E. Mereghetti and U. van Kolck, *The Nucleon Electric Dipole Form Factor From Dimension-Six Time-Reversal Violation*, Phys. Lett. B **695** (2011) 268 [arXiv:1006.2304] [INSPIRE].

[38] J. Hisano, J.Y. Lee, N. Nagata and Y. Shimizu, *Reevaluation of Neutron Electric Dipole Moment with QCD Sum Rules*, Phys. Rev. D **85** (2012) 114044 [arXiv:1204.2653] [INSPIRE].

[39] J. de Vries, E. Mereghetti, R.G.E. Timmermans and U. van Kolck, *The Effective Chiral Lagrangian From Dimension-Six Parity and Time-Reversal Violation*, Annals Phys. **338** (2013) 50 [arXiv:1212.0990] [INSPIRE].

[40] K. Fuyuto, J. Hisano and N. Nagata, *Neutron electric dipole moment induced by strangeness revisited*, Phys. Rev. D **87** (2013) 054018 [arXiv:1211.5228] [INSPIRE].

[41] E. Mereghetti and U. van Kolck, *Effective Field Theory and Time-Reversal Violation in Light Nuclei*, Ann. Rev. Nucl. Part. Sci. **65** (2015) 215 [arXiv:1505.06272] [INSPIRE].

[42] J. de Vries, E. Mereghetti and A. Walker-Loud, *Baryon mass splittings and strong CP-violation in* SU(3) *Chiral Perturbation Theory*, Phys. Rev. C **92** (2015) 045201 [arXiv:1506.06247] [INSPIRE].

[43] J. de Vries and U.-G. Meißner, *Violations of discrete space-time symmetries in chiral effective field theory*, Int. J. Mod. Phys. E **25** (2016) 1641008 [arXiv:1509.07331] [INSPIRE].

[44] S. Weinberg, *Larger Higgs Exchange Terms in the Neutron Electric Dipole Moment*, Phys. Rev. Lett. **63** (1989) 2333 [INSPIRE].

[45] D.A. Dicus, *Neutron Electric Dipole Moment From Charged Higgs Exchange*, Phys. Rev. D **41** (1990) 999 [INSPIRE].

[46] G. Boyd, A.K. Gupta, S.P. Trivedi and M.B. Wise, *Effective Hamiltonian for the Electric Dipole Moment of the Neutron*, Phys. Lett. B **241** (1990) 584 [INSPIRE].

[47] H.-Y. Cheng, *Is the Weinberg Model of CP Violation Really Excluded?*, Phys. Rev. D **42** (1990) 2329 [INSPIRE].

[48] I.I.Y. Bigi and N.G. Uraltsev, *Effective gluon operators and the dipole moment of the neutron*, Sov. Phys. JETP **73** (1991) 198 [INSPIRE].

[49] I.I.Y. Bigi and N.G. Uraltsev, *Induced Multi-Gluon Couplings and the Neutron Electric Dipole Moment*, Nucl. Phys. B **353** (1991) 321 [INSPIRE].

[50] T. Hayashi, Y. Koide, M. Matsuda and M. Tanimoto, *Neutron electric dipole moment in two Higgs doublet model*, Prog. Theor. Phys. **91** (1994) 915 [hep-ph/9401331] [INSPIRE].

[51] T. Hayashi, Y. Koide, M. Matsuda, M. Tanimoto and S. Wakaizumi, *Electric dipole moments of neutron and electron in two Higgs doublet model with maximal CP-violation*, Phys. Lett. B **348** (1995) 489 [hep-ph/9410413] [INSPIRE].

[52] Y.-L. Wu, *A Model of CP-violation*, hep-ph/9404241 [INSPIRE].

[53] M. Jung and A. Pich, *Electric Dipole Moments in Two-Higgs-Doublet Models*, JHEP **04** (2014) 076 [arXiv:1308.6283] [INSPIRE].

[54] J. Brod, U. Haisch and J. Zupan, *Constraints on CP-violating Higgs couplings to the third generation*, JHEP **11** (2013) 180 [arXiv:1310.1385] [INSPIRE].

[55] W. Dekens et al., *Unraveling models of CP-violation through electric dipole moments of light nuclei*, JHEP **07** (2014) 069 [arXiv:1404.6082] [INSPIRE].

[56] V. Cirigliano, W. Dekens, J. de Vries and E. Mereghetti, *Is there room for CP-violation in the top-Higgs sector?*, Phys. Rev. D **94** (2016) 016002 [arXiv:1603.03049] [INSPIRE].

[57] V. Cirigliano, W. Dekens, J. de Vries and E. Mereghetti, *Constraining the top-Higgs sector of the Standard Model Effective Field Theory*, Phys. Rev. D **94** (2016) 034031 [arXiv:1605.04311] [INSPIRE].

[58] G. Panico, M. Riembau and T. Vantalon, *Probing light top partners with CP-violation*, JHEP **06** (2018) 056 [arXiv:1712.06337] [INSPIRE].

[59] V. Cirigliano, A. Crivellin, W. Dekens, J. de Vries, M. Hoferichter and E. Mereghetti, *CP Violation in Higgs-Gauge Interactions: From Tabletop Experiments to the LHC*, Phys. Rev. Lett. **123** (2019) 051801 [arXiv:1903.03625] [INSPIRE].

[60] U. Haisch and A. Hala, *Bounds on CP-violating Higgs-gluon interactions: the case of vanishing light-quark Yukawa couplings*, JHEP **11** (2019) 117 [arXiv:1909.09373] [INSPIRE].

[61] K. Cheung, A. Jueid, Y.-N. Mao and S. Moretti, *Two-Higgs-doublet model with soft CP violation confronting electric dipole moments and colliders*, Phys. Rev. D **102** (2020) 075029 [arXiv:2003.04178] [INSPIRE].

[62] M. Dine and W. Fischler, *Constraints on New Physics From Weinberg's Analysis of the Neutron Electric Dipole Moment*, Phys. Lett. B **242** (1990) 239 [INSPIRE].

[63] J. Dai, H. Dykstra, R.G. Leigh, S. Paban and D. Dicus, *CP Violation From Three Gluon Operators in the Supersymmetric Standard Model*, Phys. Lett. B **237** (1990) 216 [*Erratum ibid.* **242** (1990) 547] [INSPIRE].

[64] R.L. Arnowitt, M.J. Duff and K.S. Stelle, *Supersymmetry and the Neutron Electric Dipole Moment*, Phys. Rev. D **43** (1991) 3085 [INSPIRE].

[65] S. Abel, S. Khalil and O. Lebedev, *EDM constraints in supersymmetric theories*, Nucl. Phys. B **606** (2001) 151 [hep-ph/0103320] [INSPIRE].

[66] D.A. Demir, M. Pospelov and A. Ritz, *Hadronic EDMs, the Weinberg operator, and light gluinos*, Phys. Rev. D **67** (2003) 015007 [hep-ph/0208257] [INSPIRE].

[67] D.A. Demir, O. Lebedev, K.A. Olive, M. Pospelov and A. Ritz, *Electric dipole moments in the MSSM at large tan beta*, Nucl. Phys. B **680** (2004) 339 [hep-ph/0311314] [INSPIRE].

[68] G. Degrassi, E. Franco, S. Marchetti and L. Silvestrini, *QCD corrections to the electric dipole moment of the neutron in the MSSM*, JHEP **11** (2005) 044 [hep-ph/0510137] [INSPIRE].

[69] S. Abel and O. Lebedev, *Neutron-electron EDM correlations in supersymmetry and prospects for EDM searches*, JHEP **01** (2006) 133 [hep-ph/0508135] [INSPIRE].

[70] J.R. Ellis, J.S. Lee and A. Pilaftsis, *Electric Dipole Moments in the MSSM Reloaded*, JHEP **10** (2008) 049 [arXiv:0808.1819] [INSPIRE].

[71] Y. Li, S. Profumo and M. Ramsey-Musolf, *A Comprehensive Analysis of Electric Dipole Moment Constraints on CP-violating Phases in the MSSM*, JHEP **08** (2010) 062 [arXiv:1006.1440] [INSPIRE].

JHEP06(2022)072

[72] S.-M. Zhao et al., *Neutron electric dipole moment in CP-violating BLMSSM*, *JHEP* **10** (2013) 020 [arXiv:1306.0664] [INSPIRE].

[73] F. Sala, *A bound on the charm chromo-EDM and its implications*, *JHEP* **03** (2014) 061 [arXiv:1312.2589] [INSPIRE].

[74] J. Hisano, D. Kobayashi, W. Kuramoto and T. Kuwahara, *Nucleon Electric Dipole Moments in High-Scale Supersymmetric Models*, *JHEP* **11** (2015) 085 [arXiv:1507.05836] [INSPIRE].

[75] Y. Nakai and M. Reece, *Electric Dipole Moments in Natural Supersymmetry*, *JHEP* **08** (2017) 031 [arXiv:1612.08090] [INSPIRE].

[76] D. Chang, C.S. Li and T.C. Yuan, *Larger neutron electric dipole moment in left-right symmetric models*, *Phys. Rev. D* **42** (1990) 867 [INSPIRE].

[77] I.Z. Rothstein, *Three gluon contribution to the neutron electric dipole moment in a model without the strong CP problem*, *Phys. Lett. B* **249** (1990) 467 [INSPIRE].

[78] F. Xu, H. An and X. Ji, *Neutron Electric Dipole Moment Constraint on Scale of Minimal Left-Right Symmetric Model*, *JHEP* **03** (2010) 088 [arXiv:0910.2265] [INSPIRE].

[79] K. Choi, S.H. Im, H. Kim and D.Y. Mo, 750 *GeV diphoton resonance and electric dipole moments*, *Phys. Lett. B* **760** (2016) 666 [arXiv:1605.00206] [INSPIRE].

[80] T. Abe, J. Hisano and R. Nagai, *Model independent evaluation of the Wilson coefficient of the Weinberg operator in QCD*, *JHEP* **03** (2018) 175 [*Erratum ibid.* **09** (2018) 020] [arXiv:1712.09503] [INSPIRE].

[81] W. Dekens, J. de Vries, M. Jung and K.K. Vos, *The phenomenology of electric dipole moments in models of scalar leptoquarks*, *JHEP* **01** (2019) 069 [arXiv:1809.09114] [INSPIRE].

[82] L. Di Luzio, R. Gröber and P. Paradisi, *Hunting for CP-violating axionlike particle interactions*, *Phys. Rev. D* **104** (2021) 095027 [arXiv:2010.13760] [INSPIRE].

[83] W. Dekens, L. Andreoli, J. de Vries, E. Mereghetti and F. Oosterhof, *A low-energy perspective on the minimal left-right symmetric model*, *JHEP* **11** (2021) 127 [arXiv:2107.10852] [INSPIRE].

[84] H. Gisbert, V. Miralles and J. Ruiz-Vidal, *Electric dipole moments from colour-octet scalars*, *JHEP* **04** (2022) 077 [arXiv:2111.09397] [INSPIRE].

[85] E. Braaten, C.-S. Li and T.-C. Yuan, *The Evolution of Weinberg's Gluonic CP Violation Operator*, *Phys. Rev. Lett.* **64** (1990) 1709 [INSPIRE].

[86] D. Chang, W.-Y. Keung, C.S. Li and T.C. Yuan, *QCD Corrections to CP Violation From Color Electric Dipole Moment of b Quark*, *Phys. Lett. B* **241** (1990) 589 [INSPIRE].

[87] J.F. Kamenik, M. Papucci and A. Weiler, *Constraining the dipole moments of the top quark*, *Phys. Rev. D* **85** (2012) 071501 [*Erratum ibid.* **88** (2013) 039903] [arXiv:1107.3143] [INSPIRE].

[88] H. Gisbert and J. Ruiz Vidal, *Improved bounds on heavy quark electric dipole moments*, *Phys. Rev. D* **101** (2020) 115010 [arXiv:1905.02513] [INSPIRE].

[89] B. Yan, S.-M. Zhao and T.-F. Feng, *Electric dipole moments of neutron and heavy quarks c, t in CP-violating* $\mathrm{U}(1)_X$ *SSM*, *Nucl. Phys. B* **975** (2022) 115671 [arXiv:2011.08533] [INSPIRE].

[90] U. Haisch and G. Koole, *Beautiful and charming chromodipole moments*, *JHEP* **09** (2021) 133 [arXiv:2106.01289] [INSPIRE].

[91] M.J. Booth, *A Note on Weinberg operators in the Standard Model*, *Phys. Rev. D* **48** (1993) 1248 [INSPIRE].

[92] M.E. Pospelov, *CP odd effective gluonic Lagrangian in the Kobayashi-Maskawa model*, *Phys. Lett. B* **328** (1994) 441 [hep-ph/9402317] [INSPIRE].

[93] Y. Yamaguchi and N. Yamanaka, *Quark level and hadronic contributions to the electric dipole moment of charged leptons in the standard model*, *Phys. Rev. D* **103** (2021) 013001 [arXiv:2006.00281] [INSPIRE].

[94] M. Chemtob, *Nucleon electric dipole moment and gluonic content of light hadrons*, *Phys. Rev. D* **45** (1992) 1649 [INSPIRE].

[95] C. Dib et al., *The Neutron electric dipole form-factor in the perturbative chiral quark model*, *J. Phys. G* **32** (2006) 547 [hep-ph/0601144] [INSPIRE].

[96] U. Haisch and A. Hala, *Sum rules for CP-violating operators of Weinberg type*, *JHEP* **11** (2019) 154 [arXiv:1909.08955] [INSPIRE].

[97] Y. Hatta, *CP-odd gluonic operators in QCD spin physics*, *Phys. Rev. D* **102** (2020) 094004 [arXiv:2009.03657] [INSPIRE].

[98] N. Yamanaka and E. Hiyama, *Weinberg operator contribution to the nucleon electric dipole moment in the quark model*, *Phys. Rev. D* **103** (2021) 035023 [arXiv:2011.02531] [INSPIRE].

[99] Y. Hatta, *Nucleon electric dipole moment from polarized deep inelastic scattering*, *Phys. Lett. B* **814** (2021) 136126 [arXiv:2012.01865] [INSPIRE].

[100] C. Weiss, *Nucleon matrix element of Weinberg's CP-odd gluon operator from the instanton vacuum*, *Phys. Lett. B* **819** (2021) 136447 [arXiv:2103.13471] [INSPIRE].

[101] J. de Vries et al., *Electric Dipole Moments of Light Nuclei From Chiral Effective Field Theory*, *Phys. Rev. C* **84** (2011) 065501 [arXiv:1109.3604] [INSPIRE].

[102] N. Yamanaka, *Review of the electric dipole moment of light nuclei*, *Int. J. Mod. Phys. E* **26** (2017) 1730002 [arXiv:1609.04759] [INSPIRE].

[103] J. Kley, T. Theil, E. Venturini and A. Weiler, *Electric dipole moments at one-loop in the dimension-6 SMEFT*, arXiv:2109.15085 [INSPIRE].

[104] W. Dekens and J. de Vries, *Renormalization Group Running of Dimension-Six Sources of Parity and Time-Reversal Violation*, *JHEP* **05** (2013) 149 [arXiv:1303.3156] [INSPIRE].

[105] V.M. Khatsimovsky, I.B. Khriplovich and A.S. Yelkhovsky, *Neutron Electric Dipole Moment, T Odd Nuclear Forces and Nature of CP Violation*, *Annals Phys.* **186** (1988) 1 [INSPIRE].

[106] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *QCD and Resonance Physics. Theoretical Foundations*, *Nucl. Phys. B* **147** (1979) 385 [INSPIRE].

[107] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *QCD and Resonance Physics: Applications*, *Nucl. Phys. B* **147** (1979) 448 [INSPIRE].

[108] P. Gubler and D. Satow, *Recent Progress in QCD Condensate Evaluations and Sum Rules*, *Prog. Part. Nucl. Phys.* **106** (2019) 1 [arXiv:1812.00385] [INSPIRE].

[109] V. Cirigliano, E. Mereghetti and P. Stoffer, *Non-perturbative renormalization scheme for the CP -odd three-gluon operator*, *JHEP* **09** (2020) 094 [arXiv:2004.03576] [INSPIRE].

[110] SymLat collaboration, *Short flow-time coefficients of CP-violating operators*, *Phys. Rev. D* **102** (2020) 034509 [arXiv:2005.04199] [INSPIRE].

JHEP06(2022)072

[111] S. Groote, J.G. Korner and A.A. Pivovarov, *On the evaluation of a certain class of Feynman diagrams in x-space: Sunrise-type topologies at any loop order*, Annals Phys. **322** (2007) 2374 [hep-ph/0506286] [INSPIRE].

[112] P. Gubler and M. Oka, *A Bayesian approach to QCD sum rules*, Prog. Theor. Phys. **124** (2010) 995 [arXiv:1005.2459] [INSPIRE].

[113] G. Hao, C.-F. Qiao and A.-L. Zhang, 0^{-+} *trigluon glueball and its implication for a recent BES observation*, Phys. Lett. B **642** (2006) 53 [hep-ph/0512214] [INSPIRE].

[114] H.-X. Chen, W. Chen and S.-L. Zhu, *Two- and three-gluon glueballs of* $C = +$, Phys. Rev. D **104** (2021) 094050 [arXiv:2107.05271] [INSPIRE].

[115] C.J. Morningstar and M.J. Peardon, *The Glueball spectrum from an anisotropic lattice study*, Phys. Rev. D **60** (1999) 034509 [hep-lat/9901004] [INSPIRE].

[116] Y. Chen et al., *Glueball spectrum and matrix elements on anisotropic lattices*, Phys. Rev. D **73** (2006) 014516 [hep-lat/0510074] [INSPIRE].

[117] W. Sun et al., *Glueball spectrum from* $N_f = 2$ *lattice QCD study on anisotropic lattices*, Chin. Phys. C **42** (2018) 093103 [arXiv:1702.08174] [INSPIRE].

[118] F. Chen, X. Jiang, Y. Chen, K.-F. Liu, W. Sun and Y.-B. Yang, *Glueballs at Physical Pion Mass*, arXiv:2111.11929 [INSPIRE].

[119] V.A. Novikov, M.A. Shifman, A.I. Vainshtein, M.B. Voloshin and V.I. Zakharov, *Use and Misuse of QCD Sum Rules, Factorization and Related Topics*, Nucl. Phys. B **237** (1984) 525 [INSPIRE].

[120] J. Gasser and H. Leutwyler, *Chiral Perturbation Theory to One Loop*, Annals Phys. **158** (1984) 142 [INSPIRE].

[121] J. Gasser and H. Leutwyler, *Chiral Perturbation Theory: Expansions in the Mass of the Strange Quark*, Nucl. Phys. B **250** (1985) 465 [INSPIRE].

[122] S. Bethke, *World Summary of* α_s *(2012)*, Nucl. Phys. B Proc. Suppl. **234** (2013) 229 [arXiv:1210.0325] [INSPIRE].

[123] Particle Data Group collaboration, *Review of Particle Physics*, PTEP **2020** (2020) 083C01 [INSPIRE].

[124] A. Gomez Nicola and R. Torres Andres, *Isospin-Breaking Quark Condensates in Chiral Perturbation Theory*, J. Phys. G **39** (2012) 015004 [arXiv:1009.2170] [INSPIRE].

[125] N. Yamanaka and E. Hiyama, *Electric dipole moment of the deuteron in the standard model with* $NN - \Lambda N - \Sigma N$ *coupling*, Nucl. Phys. A **963** (2017) 33 [arXiv:1605.00161] [INSPIRE].

[126] M. Hoferichter, J. Ruiz de Elvira, B. Kubis and U.-G. Meißner, *High-Precision Determination of the Pion-Nucleon* σ *Term from Roy-Steiner Equations*, Phys. Rev. Lett. **115** (2015) 092301 [arXiv:1506.04142] [INSPIRE].

[127] E. Friedman and A. Gal, *The pion-nucleon* σ *term from pionic atoms*, Phys. Lett. B **792** (2019) 340 [arXiv:1901.03130] [INSPIRE].

[128] B.-L. Huang and J. Ou-Yang, *Pion-nucleon scattering to* $\mathcal{O}(p^3)$ *in heavy baryon* SU(3)*-flavor chiral perturbation theory*, Phys. Rev. D **101** (2020) 056021 [arXiv:1911.00846] [INSPIRE].

[129] xQCD collaboration, πN *and strangeness sigma terms at the physical point with chiral fermions*, Phys. Rev. D **94** (2016) 054503 [arXiv:1511.09089] [INSPIRE].

JHEP06(2022)072

[130] R. Gupta, S. Park, M. Hoferichter, E. Mereghetti, B. Yoon and T. Bhattacharya, *Pion-Nucleon Sigma Term from Lattice QCD*, Phys. Rev. Lett. **127** (2021) 242002 [arXiv:2105.12095] [INSPIRE].

[131] J. Bsaisou et al., *Nuclear Electric Dipole Moments in Chiral Effective Field Theory*, JHEP **03** (2015) 104 [*Erratum ibid.* **05** (2015) 083] [arXiv:1411.5804] [INSPIRE].

[132] J. Bsaisou, U.-G. Meißner, A. Nogga and A. Wirzba, *P- and T-Violating Lagrangians in Chiral Effective Field Theory and Nuclear Electric Dipole Moments*, Annals Phys. **359** (2015) 317 [arXiv:1412.5471] [INSPIRE].

[133] L.I. Schiff, *Measurability of Nuclear Electric Dipole Moments*, Phys. Rev. **132** (1963) 2194 [INSPIRE].

[134] V.F. Dmitriev, R.A. Sen'kov and N. Auerbach, *Effects of core polarization on the nuclear Schiff moment*, Phys. Rev. C **71** (2005) 035501 [nucl-th/0408065] [INSPIRE].

[135] J. Dobaczewski and J. Engel, *Nuclear time-reversal violation and the Schiff moment of Ra-225*, Phys. Rev. Lett. **94** (2005) 232502 [nucl-th/0503057] [INSPIRE].

[136] S. Ban, J. Dobaczewski, J. Engel and A. Shukla, *Fully self-consistent calculations of nuclear Schiff moments*, Phys. Rev. C **82** (2010) 015501 [arXiv:1003.2598] [INSPIRE].

[137] N. Yoshinaga, K. Higashiyama, R. Arai and E. Teruya, *Nuclear Schiff moments for the lowest $1/2^+$ states in Xe isotopes*, Phys. Rev. C **87** (2013) 044332 [*Erratum ibid.* **89** (2014) 069902] [INSPIRE].

[138] K. Yanase and N. Shimizu, *Large-scale shell-model calculations of nuclear Schiff moments of ^{129}Xe and ^{199}Hg*, Phys. Rev. C **102** (2020) 065502 [arXiv:2006.15142] [INSPIRE].

[139] K. Yanase, *Screening of nucleon electric dipole moments in atomic systems*, Phys. Rev. C **103** (2021) 035501 [arXiv:2008.03678] [INSPIRE].

[140] K. Yanase, private communication.

[141] A. Sakurai, B.K. Sahoo, K. Asahi and B.P. Das, *Relativistic many-body theory of the electric dipole moment of ^{129}Xe and its implications for probing new physics beyond the standard model*, Phys. Rev. A **100** (2019) 020502 [arXiv:1908.04151] [INSPIRE].

[142] M. Hubert and T. Fleig, *Electric dipole moments due to nuclear Schiff moment interactions: A reassessment of the atoms ^{129}Xe, ^{199}Hg, and the molecule ^{205}TlF*, arXiv:2203.04618 [INSPIRE].

[143] V.S. Prasannaa, R. Mitra and B.K. Sahoo, *Reappraisal of P, T-odd parameters from the improved calculation of electric dipole moment of 225Ra atom*, J. Phys. B **53** (2020) 195004 [INSPIRE].

[144] V.A. Dzuba, V.V. Flambaum and S.G. Porsev, *Calculation of P,T-odd electric dipole moments for diamagnetic atoms ^{129}Xe, ^{171}Yb, ^{199}Hg, ^{211}Rn, and ^{225}Ra*, Phys. Rev. A **80** (2009) 032120 [arXiv:0906.5437] [INSPIRE].

[145] L. Radziute, G. Gaigalas, P. Jonsson and J. Biero, *Multiconfiguration Dirac-Hartree-Fock calculations of atomic electric dipole moments of ^{225}Ra, ^{199}Hg, and ^{171}Yb*, Phys. Rev. A **90** (2014) 012528 [arXiv:1312.6517] [INSPIRE].

[146] B.K. Sahoo and B.P. Das, *Relativistic Normal Coupled-Cluster Theory for Accurate Determination of Electric Dipole Moments of Atoms: First Application to the Hg199 Atom*, Phys. Rev. Lett. **120** (2018) 203001 [arXiv:1801.07045] [INSPIRE].

JHEP06(2022)072

[147] F.J.M. Farley et al., *A New method of measuring electric dipole moments in storage rings*, *Phys. Rev. Lett.* **93** (2004) 052001 [hep-ex/0307006] [INSPIRE].

[148] V. Anastassopoulos et al., *A Storage Ring Experiment to Detect a Proton Electric Dipole Moment*, *Rev. Sci. Instrum.* **87** (2016) 115116 [arXiv:1502.04317] [INSPIRE].

[149] JEDI collaboration, *How to Reach a Thousand-Second in-Plane Polarization Lifetime with* 0.97-*GeV/c Deuterons in a Storage Ring*, *Phys. Rev. Lett.* **117** (2016) 054801 [INSPIRE].

[150] Y.-H. Song, R. Lazauskas and V. Gudkov, *Nuclear electric dipole moment of three-body systems*, *Phys. Rev. C* **87** (2013) 015501 [arXiv:1211.3762] [INSPIRE].

[151] A. Gnech and M. Viviani, *Time Reversal Violation in Light Nuclei*, *Phys. Rev. C* **101** (2020) 024004 [arXiv:1906.09021] [INSPIRE].

[152] Z. Yang, E. Mereghetti, L. Platter, M.R. Schindler and J. Vanasse, *Electric dipole moments of three-nucleon systems in the pionless effective field theory*, *Phys. Rev. C* **104** (2021) 024002 [arXiv:2011.01885] [INSPIRE].

[153] P. Froese and P. Navratil, *Ab initio calculations of electric dipole moments of light nuclei*, *Phys. Rev. C* **104** (2021) 025502 [arXiv:2103.06365] [INSPIRE].

[154] N. Yamanaka and E. Hiyama, *Enhancement of the CP-odd effect in the nuclear electric dipole moment of* ^6Li, *Phys. Rev. C* **91** (2015) 054005 [arXiv:1503.04446] [INSPIRE].

[155] J. de Vries, E. Epelbaum, L. Girlanda, A. Gnech, E. Mereghetti and M. Viviani, *Parity- and Time-Reversal-Violating Nuclear Forces*, *Front. in Phys.* **8** (2020) 218 [arXiv:2001.09050] [INSPIRE].

[156] N. Yamanaka, T. Yamada, E. Hiyama and Y. Funaki, *Electric dipole moment of* ^{13}C, *Phys. Rev. C* **95** (2017) 065503 [arXiv:1603.03136] [INSPIRE].

[157] J. Lee, N. Yamanaka and E. Hiyama, *Effect of the Pauli exclusion principle in the electric dipole moment of* ^9Be *with* $|\Delta S| = 1$ *interactions*, *Phys. Rev. C* **99** (2019) 055503 [arXiv:1811.00329] [INSPIRE].

[158] N. Yamanaka, T. Yamada and Y. Funaki, *Nuclear electric dipole moment in the cluster model with a triton:* ^7Li *and* ^{11}B, *Phys. Rev. C* **100** (2019) 055501 [arXiv:1907.08091] [INSPIRE].

[159] N. Yamanaka, *Weinberg Operator Contribution to the Hadronic CP-violation*, in *Proceedings of the 24th International Spin Symposium (SPIN2021)*, Matsue, Japan, October 2021 [arXiv:2112.06478] [INSPIRE].

[160] E. Epelbaum, H.-W. Hammer and U.-G. Meissner, *Modern Theory of Nuclear Forces*, *Rev. Mod. Phys.* **81** (2009) 1773 [arXiv:0811.1338] [INSPIRE].

[161] J. de Vries, A. Gnech and S. Shain, *Renormalization of CP-violating nuclear forces*, *Phys. Rev. C* **103** (2021) L012501 [arXiv:2007.04927] [INSPIRE].

[162] E. Braaten, C.S. Li and T.C. Yuan, *The Gluon Color-Electric Dipole Moment and Its Anomalous Dimension*, *Phys. Rev. D* **42** (1990) 276 [INSPIRE].

[163] J. de Vries, G. Falcioni, F. Herzog and B. Ruijl, *Two- and three-loop anomalous dimensions of Weinberg's dimension-six CP-odd gluonic operator*, *Phys. Rev. D* **102** (2020) 016010 [arXiv:1907.04923] [INSPIRE].

[164] C. Abel et al., *Measurement of the Permanent Electric Dipole Moment of the Neutron*, *Phys. Rev. Lett.* **124** (2020) 081803 [arXiv:2001.11966] [INSPIRE].

JHEP06(2022)072