

A two-component dark matter model and its associated gravitational waves

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ABSTRACT: We consider an extension of the Standard Model that accounts for the muon $g - 2$ tension and neutrino masses and study in detail dark matter phenomenology. The model under consideration includes a WIMP and a FIMP scalar dark matter candidates and thus gives rise to two-component dark matter scenarios. We discuss different regimes and mechanisms of production, including the novel freeze-in semi-production, and show that the WIMP and FIMP together compose the observed relic density today. The presence of the extra scalar fields allows phase transitions of the first order. We examine the evolution of the vacuum state and discuss stochastic gravitational wave signals associated with the first-order phase transition. We show that the gravitational wave signals may be probed by future gravitational wave experiments which may serve as a complementary detection signal.

KEYWORDS: Cosmology of Theories BSM, Models for Dark Matter, Particle Nature of Dark Matter, Phase Transitions in the Early Universe

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1 Introduction

The Standard Model (SM) of particle physics proved to be very precise in describing the nature of the physical world. However, some of its problems were highlighted and studied in the past decades, including the neutrino masses, the existence of dark matter (DM), and the muon $g - 2$ tension. In the SM, neutrinos are massless. However, the evidence of neutrino oscillations indicates otherwise [1, 2]. The mass splitting from neutrino oscillation experiments is constrained to be $|\Delta m_{21}^2| = 7.42_{-0.20}^{+0.21} \times 10^{-5} \text{ eV}^2$ between the first and the second mass eigenstates, while it is $\Delta m_{32}^2 = 2.517_{-0.028}^{+0.026} \times 10^{-3} \text{ eV}^2$ for the second and the third [3]. Moreover, from cosmological data, we also have a bound on the sum of their masses $\sum_i m_{\nu_i} < 0.3 \text{ eV}$ [4].

The recent data coming from Fermilab [5] increased the tension between the SM theoretical prediction for the muon anomalous magnetic moment, the $g - 2$ factor, and the experimental data. At the moment there is a 4.2σ discrepancy,

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (2.51 \pm 0.59) \times 10^{-9}, \quad (1.1)$$

suggesting the presence of new physics at a scale of hundreds of GeV.

Finally, the SM fails to accommodate one or more particles that may play the role of the DM. Ever since the proposal by Zwicky for a dark, collision-less, and matter-like component of the energy budget of the universe [6, 7], evidences from different sources for a cold, particle-like DM have cumulated [8–10]. The most promising and studied solution to this problem is the Weakly Interacting Massive Particle (WIMP) [11–14]. The WIMP DM is, however, strongly constrained by experimental data [15–20]. Thus, more attention has been drawn to alternative DM production mechanisms. For example, the freeze-in mechanism has gained increasing interest [21–35]. The abundance of a WIMP DM is produced through the freeze-out mechanism, which is a thermal process, and it is generally inversely proportional to the thermal cross section. Instead, the DM produced via the freeze-in mechanism, called Feebly Interacting Massive Particle (FIMP), is out of equilibrium with respect to the thermal bath of the SM particles. A small coupling between the visible sector and the DM is predicted, making this candidate more difficult to detect and to constrain with direct detection experiments.¹ Nonetheless, both freeze-out and freeze-in production mechanisms are physically viable and not mutually exclusive. It is thus worth exploring the possibility of multi-component DM scenarios, where both the WIMP and FIMP DM contribute to the current relic density $\Omega_{\text{DM}}h^2 = 0.120 \pm 0.001$ observed by the Planck experiment [9]. Recent studies on multi-component DM scenarios include refs. [42–72].

In this paper, we consider an extension of the SM and explain the aforementioned three problems of the SM in a single unified framework. A novel set-up is proposed where we introduce three massive right-handed (RH) neutrinos N_R that, through the standard type-I seesaw mechanism [73, 74], provide a mass to the SM neutrinos. The SM is then also extended with two SM-singlet scalar fields ϕ_1 and ϕ_2 that play the role of the WIMP-like DM and FIMP-like DM, respectively. Finally, we introduce an extra $U(1)_{L_\mu-L_\tau}$ gauge symmetry with a related gauge boson $Z_{\mu\tau}$ which receives its mass from a second Higgs field ϕ_H . The presence of a new massive gauge boson, with the vacuum expectation value (VEV) of ϕ_H around hundreds of GeV can solve the $g-2$ tension [75–82].

Appropriately assigning $U(1)_{L_\mu-L_\tau}$ charges for the DM particles, we also address a novel production mechanism, namely the freeze-in mechanism by semi-production processes [83, 84] like $\phi_1\phi_2 \leftrightarrow \phi_2\phi_2$, that is the inverse of the semi-annihilation process [85]. This mechanism produces an exponentially increasing DM yield, and it typically requires a larger coupling than the standard freeze-in scenarios do, since the DM abundance is also suppressed by the small initial abundance which is generically required for the freeze-in production mechanism.

The evolution of the vacuum state of the scalar potential becomes non-trivial due to the three extra scalar fields. First-order phase transitions (FOPTs) may thus arise, producing stochastic gravitational wave (GW) signals [86] detectable by future GW experiments such as LISA [87] which is a space-based detector comprising of three spacecraft, utilising laser interferometry, DECIGO [88–92] which is a proposed GW antenna in space designed to observe GWs in the 0.1–10 Hz frequency range, consisting of four clusters of LISA-like three spacecraft, and BBO [93–95] which is a proposed follow-up of the LISA experiment,

¹In refs. [36–38], it was pointed out that such a small coupling can naturally arise in a clockwork framework [39–41].

aiming to form a triangular shape consisting of four LISA-like detectors, similar to the DECIGO. For recent studies on this subject, see, e.g., refs. [96–113]. This possibility gives a complementary detection signal to the standard (in-)direct detection and collider searches that potentially can probe our model and unveil the nature of the DM. We present a region of the model parameter space that produces detectable GW signals from a FOPT, relieves the muon $g - 2$ tension, gives masses to the SM neutrinos, and explains the correct DM abundance by a two-component WIMP-FIMP relic density.

The rest of the paper is organised as follows. We set up our model in section 2, introducing the particle content, the gauge groups, and the mass spectrum of the theory. We also present the standard type-I seesaw mechanism adopted to explain the neutrino masses, and we give a brief explanation of the muon $g - 2$ tension. In section 3, we discuss possible DM scenarios. We divide the parameter space into three regimes and study both one-component and two-component scenarios. In section 4, the FOPT and its associated GWs are studied. We showcase four benchmark points that explain the muon $g - 2$, neutrino masses, and correct DM relic density. The benchmark points predict GW signals within the detectability of future GW experiments, in particular Ultimate-DECIGO, which is an ultimate, idealised version of the DECIGO, whose sensitivity is only limited by quantum noises. We conclude in section 5.

2 Model

We consider the following Lagrangian:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\phi_H} + \mathcal{L}_N + \mathcal{L}_{\text{DM}} + \mathcal{L}_{\text{int}} - \frac{1}{4} F_{\mu\tau}^{\alpha\beta} F_{\mu\tau\alpha\beta}, \quad (2.1)$$

which obeys the symmetry of the complete gauge group $\text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y \times \text{U}(1)_{L_\mu - L_\tau}$, where \mathcal{L}_{SM} is the SM Lagrangian including the SM Higgs field ϕ_h , \mathcal{L}_{ϕ_H} is the Lagrangian for the $\text{U}(1)_{L_\mu - L_\tau}$ Higgs field ϕ_H ,

$$\mathcal{L}_{\phi_H} = (D_\mu \phi_H)^\dagger (D^\mu \phi_H) + \mu_H^2 |\phi_H|^2 - \lambda_H |\phi_H|^4, \quad (2.2)$$

and \mathcal{L}_N is the Lagrangian for the RH neutrinos containing their kinetic terms, mass terms, and Yukawa terms with the SM lepton doublets,

$$\begin{aligned} \mathcal{L}_N = & \sum_{i=e,\mu,\tau} \frac{i}{2} \bar{N}_i \gamma^\mu D_\mu^N N_i - \frac{1}{2} M_{ee} \bar{N}_e^c N_e - \frac{1}{2} M_{\mu\tau} (\bar{N}_\mu^c N_\tau + \bar{N}_\tau^c N_\mu) \\ & - h_{e\mu} (\bar{N}_e^c N_\mu + \bar{N}_\mu^c N_e) \phi_H^\dagger - h_{e\tau} (\bar{N}_e^c N_\tau + \bar{N}_\tau^c N_e) \phi_H - \sum_{i=e,\mu,\tau} y_i \bar{L}_i \tilde{\phi}_h N_i + \text{h.c.}, \end{aligned} \quad (2.3)$$

where $\tilde{\phi}_h = i\sigma_2 \phi_h^*$, and M_{ee} and $M_{\mu\tau}$ are constants whose mass-dimension is one, while $h_{e\mu}$, $h_{e\tau}$, and y_i are dimensionless coupling constants. In eq. (2.1), \mathcal{L}_{DM} is the DM Lagrangian that is given by

$$\mathcal{L}_{\text{DM}} = \sum_{i=1,2} (D^\mu \phi_i)^\dagger (D_\mu \phi_i) - \sum_{i=1,2} \mu_i^2 \phi_i^\dagger \phi_i - \sum_{i=1,2} \lambda_i (\phi_i^\dagger \phi_i)^2 - \lambda_{12} (\phi_1^\dagger \phi_1) (\phi_2^\dagger \phi_2) - \mu (\phi_1^\dagger \phi_2^3 + \text{h.c.}). \quad (2.4)$$

Furthermore, \mathcal{L}_{int} in eq. (2.1) contains all the interactions between the SM Higgs field ϕ_h , the second $U(1)_{L_\mu-L_\tau}$ Higgs field ϕ_H , and the DM fields $\phi_{1,2}$,

$$\mathcal{L}_{\text{int}} = -\lambda_{hH}(\phi_h^\dagger\phi_h)(\phi_H^\dagger\phi_H) - \sum_{i=1,2} \lambda_{ij}(\phi_i^\dagger\phi_i)(\phi_j^\dagger\phi_j). \quad (2.5)$$

The covariant derivatives in eqs. (2.1)–(2.4) can generically be written as $D_\nu X = (\partial_\nu + ig_{\mu\tau} Q_{\mu\tau}(X) Z_{\mu\tau\nu})X$, where X is a SM-singlet field whose $U(1)_{L_\mu-L_\tau}$ charge is $Q_{\mu\tau}(X)$ (see table 2), and $g_{\mu\tau}$ is the $U(1)_{L_\mu-L_\tau}$ gauge coupling. Finally, the kinetic term for the extra gauge boson $Z_{\mu\tau}$ is given by the last term in eq. (2.1) with its field strength tensor $F_{\mu\tau}^{\alpha\beta} = \partial^\alpha Z_{\mu\tau}^\beta - \partial^\beta Z_{\mu\tau}^\alpha$.

In general, the Lagrangian (2.1) may include the gauge kinetic mixing term [114],

$$\mathcal{L} \supset \frac{\zeta}{2} F_{\mu\tau}^{\alpha\beta} F_{\alpha\beta}, \quad (2.6)$$

between the $U(1)_{L_\mu-L_\tau}$ gauge boson and the SM $U(1)_Y$ gauge boson whose field-strength tensor is denoted by $F_{\alpha\beta}$. In the presence of the gauge kinetic mixing term, one may work with the physical gauge boson states instead of the original gauge boson states by diagonalising the mass matrix of the gauge bosons [115]. Furthermore, as we shall see shortly, the DM phenomenology as well as the FOPT-associated GWs are qualitatively indifferent to the gauge kinetic mixing term. Therefore, since the kinetic mixing term does not play an important role in our discussion, we assume, for simplicity, that $\zeta \ll 1$ in this work.²

The presence of the interaction term between ϕ_h and ϕ_H in eq. (2.5) introduces a mass mixing. In unitary gauge, the Higgs fields ϕ_h and ϕ_H after the spontaneous breaking of $SU(2)_L \times U(1)_Y \times U(1)_{L_\mu-L_\tau}$ gauge symmetry may be expressed as

$$\phi_h = \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix} \quad \text{and} \quad \phi_H = \begin{pmatrix} \frac{v_{\mu\tau}+H_{\mu\tau}}{\sqrt{2}} \end{pmatrix}, \quad (2.7)$$

where v and $v_{\mu\tau}$ are the VEVs of the Higgs fields ϕ_h and ϕ_H , respectively. The scalar mass matrix is then given by

$$\mathcal{M}_{\text{scalar}}^2 = \begin{pmatrix} 2\lambda_h v^2 & \lambda_{hH} v_{\mu\tau} v \\ \lambda_{hH} v_{\mu\tau} v & 2\lambda_H v_{\mu\tau}^2 \end{pmatrix}. \quad (2.8)$$

In the presence of the Higgs-portal coupling λ_{hH} , the physical states are obtained after diagonalising the matrix $\mathcal{M}_{\text{scalar}}^2$. The mass eigenstates h_1 and h_2 can be written as

$$h_1 = H \cos \theta + H_{\mu\tau} \sin \theta, \quad h_2 = -H \sin \theta + H_{\mu\tau} \cos \theta. \quad (2.9)$$

The mixing angle θ and the mass eigenvalues $M_{h_1}^2$ and $M_{h_2}^2$ are given by

$$\tan 2\theta = \frac{\lambda_{hH} v_{\mu\tau} v}{\lambda_h v^2 - \lambda_H v_{\mu\tau}^2}, \quad (2.10)$$

$$M_{h_1}^2 = \lambda_h v^2 + \lambda_H v_{\mu\tau}^2 - \sqrt{(\lambda_h v^2 - \lambda_H v_{\mu\tau}^2)^2 + (\lambda_{hH} v v_{\mu\tau})^2}, \quad (2.11)$$

$$M_{h_2}^2 = \lambda_h v^2 + \lambda_H v_{\mu\tau}^2 + \sqrt{(\lambda_h v^2 - \lambda_H v_{\mu\tau}^2)^2 + (\lambda_{hH} v v_{\mu\tau})^2}. \quad (2.12)$$

We identify the lighter scalar field h_1 with the observed SM Higgs field.

²In refs. [116, 117], it was shown that small values of the kinetic mixing parameter ζ are favoured from the muon $g-2$ aspect when taking into account the experimental constraint of Borexino [118, 119]. See also, e.g., ref. [120] for a comprehensive study on experimental constraints on the kinetic mixing parameter ζ .

Gauge Group	Baryon Fields			Lepton Fields			Scalar Fields			
	$Q_L^i = (u_L^i, d_L^i)^T$	u_R^i	d_R^i	$L_L^i = (\nu_L^i, e_L^i)^T$	e_R^i	N_R^i	ϕ_h	ϕ_H	ϕ_1	ϕ_2
SU(2) _L	2	1	1	2	1	1	2	1	1	1
U(1) _Y	1/6	2/3	-1/3	-1/2	-1	0	1/2	0	0	0

Table 1. Particle contents and their corresponding charges under the SM gauge group.

Gauge Group	Baryon Fields	Lepton Fields			Scalar Fields			
	(Q_L^i, u_R^i, d_R^i)	(L_L^e, e_R, N_R^e)	$(L_L^\mu, \mu_R, N_R^\mu)$	$(L_L^\tau, \tau_R, N_R^\tau)$	ϕ_h	ϕ_H	ϕ_1	ϕ_2
U(1) _{L_μ-L_τ}	0	0	1	-1	0	1	3n _{μτ}	n _{μτ}

Table 2. Particle contents and their corresponding charges under U(1)_{L_μ-L_τ}.

For the masses of the WIMP and FIMP, we obtain, with $\mu_{1,2}^2 > 0$, as

$$M_1^2 = \mu_1^2 + \lambda_{1h} \frac{v^2}{2} + \lambda_{1H} \frac{v_{\mu\tau}^2}{2}, \quad M_2^2 = \mu_2^2 + \lambda_{2h} \frac{v^2}{2} + \lambda_{2H} \frac{v_{\mu\tau}^2}{2}. \quad (2.13)$$

We summarise the particle contents of our model and their corresponding charges in table 1 and table 2. In the remaining part of this section, we present the standard type-I seesaw mechanism that we adopt to explain the neutrino masses, and we briefly explain how the muon $g - 2$ tension can be relieved in our model. For a detailed explanation, readers may refer to e.g. refs. [121, 122].

2.1 Neutrino masses

Once the SM and U(1)_{L_μ-L_τ} Higgs fields develop VEVs, the RH neutrino mass matrix can be expressed as

$$M_R = \begin{pmatrix} M_{ee} & \frac{h_{e\mu} v_{\mu\tau}}{\sqrt{2}} & \frac{h_{e\tau} v_{\mu\tau}}{\sqrt{2}} \\ \frac{h_{e\mu} v_{\mu\tau}}{\sqrt{2}} & 0 & M_{\mu\tau} e^{i\eta} \\ \frac{h_{e\tau} v_{\mu\tau}}{\sqrt{2}} & M_{\mu\tau} e^{i\eta} & 0 \end{pmatrix}, \quad (2.14)$$

where η is the only fermionic phase factor that cannot be absorbed by field redefinitions, and we see from the Yukawa terms in eq. (2.3) that the Dirac mass matrix can be written as

$$M_D = \begin{pmatrix} \frac{y_e v}{\sqrt{2}} & 0 & 0 \\ 0 & \frac{y_\mu v}{\sqrt{2}} & 0 \\ 0 & 0 & \frac{y_\tau v}{\sqrt{2}} \end{pmatrix}. \quad (2.15)$$

Therefore, the complete neutrino mass matrix is a 6×6 matrix in the basis (ν_l, N_l) ,

$$M_\nu = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}. \quad (2.16)$$

After diagonalisation, we can obtain the mass matrix for the mass eigenstates. Then, we can write the light neutrino mass and heavy mass matrix as follows:

$$m_\nu^{\text{light}} = -M_D^T M_R^{-1} M_D, \quad M_R^{\text{heavy}} = M_R. \quad (2.17)$$

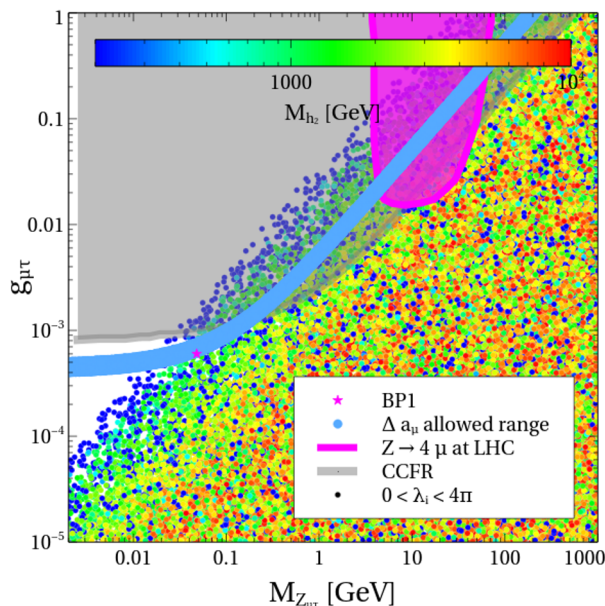


Figure 1. Constraints on $g_{\mu\tau}$ and $M_{Z_{\mu\tau}}$. The cyan region relieves the muon $g - 2$ tension. The magenta region is excluded by the $Z \rightarrow 4\mu$ searches from LHC [123–125], while the grey region is excluded by the neutrino trident experiments CHARM-II and CCFR [126–128]. See also ref. [129] for constraints coming from the lepton universality test and LEP searches. The colour of the scan points represents M_{h_2} . The star (*) corresponds to our benchmark point 1 (see table 3 in section 4).

With the RH neutrino mass matrix elements in GeV range and the Dirac mass matrix in keV range, one may easily obtain the neutrino mass in the correct experimental range [3]; see also refs. [121, 122] for details.

The RH neutrino mass matrix squared, $(M_R^{\text{heavy}})^2$, can be diagonalised analytically when $M_{ee} = M_{\mu\tau}$ and $h_{e\tau} = h_{e\mu}$, and we obtain the eigenvalues as

$$(M_{ee} - h_{e\tau}v_{\mu\tau})^2, \quad M_{ee}^2, \quad (M_{ee} + h_{e\tau}v_{\mu\tau})^2. \quad (2.18)$$

In the following, we assume that this is the case.

2.2 Muon $g - 2$

The presence of additional gauge boson $Z_{\mu\tau}$ can alleviate the $(g - 2)_\mu$ anomaly through the one-loop contribution, resulting in [130, 131]

$$\Delta a_\mu = \frac{g_{\mu\tau}^2}{8\pi^2} \int_0^1 \frac{2x(1-x)^2}{(1-x)^2 + rx} dx, \quad (2.19)$$

where $r = M_{Z_{\mu\tau}}^2/m_\mu^2$. Figure 1 shows the region that addresses the discrepancy between the experimental and theoretical values of muon $g - 2$, together with constraints from the neutrino trident experiments such as CHARM-II [126] and CCFR [127, 128] and the LHC $Z \rightarrow 4\mu$ searches [123–125]. We observe that $M_{Z_{\mu\tau}} \lesssim 0.1$ GeV region with $4 \times 10^{-4} \lesssim g_{\mu\tau} \lesssim 1 \times 10^{-3}$ successfully explains the muon $g - 2$ tension. We also present our scan points whose colour

represents the value of M_{h_2} . One may clearly see from figure 1 that $M_{h_2} \gtrsim 1.1$ TeV is disfavoured from the muon $g - 2$ point of view as long as the quartic couplings are in the perturbative regime. Therefore, throughout the paper, we consider $M_{h_2} \lesssim 1.1$ TeV. The star (*) in figure 1 depicts our benchmark point 1 (see table 3). Strong GW signals can be emitted from a region that explains the muon $g - 2$ tension. In section 4, we discuss possible GW signals in detail.

3 Two-component dark matter

In this section, we examine the possibility of having DM component(s) in the present model. As the model contains two scalar DM candidates, we may have a single-component or two-component DM scenario depending on the mass range of the WIMP and FIMP DM particles. We first discuss the production of DM when the μ term in eq. (2.4) is dominant and the other quartic terms associated with the DM are also significant. We also look at the scenario when the μ parameter is less significant and quartic terms are the ones which take part in the DM productions. In regime I and regime II, we study the effect of the μ term on the production of DM. In these cases, depending on the mass range of the WIMP DM, we obtain both the single-component and two-component DM scenarios. In regime III, we study DM productions when the μ term is small and the quartic terms are relevant. In this regime, we have a two-component DM scenario where one component is WIMP-type DM and another component is FIMP-type DM. In studying DM phenomenology, we have implemented our model in FEYNRULES [132] and generated the CALCHEP files [133]. We have then used MICROMEGAS [134] to solve the coupled Boltzmann equations relevant for our study. Some useful analytical expressions are derived and summarised in appendix A. We discuss the different regimes in detail below.

3.1 Regime I ($M_{h_2} < 2M_2$ and $M_1 > 3M_2$)

As the WIMP DM mass is larger than three times the mass of the FIMP DM, a three-body decay channel from the WIMP DM to the FIMP DM is open in this regime. Since the WIMP DM decays into the FIMP DM, this regime gives us a single-component DM scenario, unless the lifetime is larger than the age of the Universe. It would require extremely small couplings to make the lifetime larger than the age of the Universe, and we do not consider such a scenario. Additionally, we assume that the SM and BSM Higgs masses are such that the decay production of the FIMP DM is kinematically forbidden. Nevertheless, a freeze-in contribution through annihilation processes, $AB \rightarrow \phi_2^\dagger \phi_2$, where A and B are the SM particles, will be there.

The Boltzmann equations associated with the WIMP and FIMP DM productions are given by

$$\begin{aligned} \frac{dY_1}{dx} &= -\frac{2\pi^2}{45} \frac{M_{\text{Pl}} M_1 \sqrt{g_*(x)}}{1.66x^2} \langle \sigma v \rangle_{\text{th}} (Y_1^2 - Y_1^{\text{eq}2}) - \frac{3M_{\text{Pl}} x \sqrt{g_*(x)}}{1.66M_1^2 g_s(x)} \langle \Gamma \rangle (Y_1 - Y_2^3), \\ \frac{dY_2}{dx} &= \frac{4\pi^2}{45} \frac{M_{\text{Pl}} M_1 \sqrt{g_*}}{1.66x^2} \sum_{i,j \in \text{SM}, \phi_1} \langle \sigma v \rangle_{ij} (Y_i^{\text{eq}} Y_j^{\text{eq}} - Y_2^2) + \frac{3M_{\text{Pl}} x \sqrt{g_*(x)}}{1.66M_1^2 g_s(x)} \langle \Gamma \rangle (Y_1 - Y_2^3), \end{aligned} \tag{3.1}$$

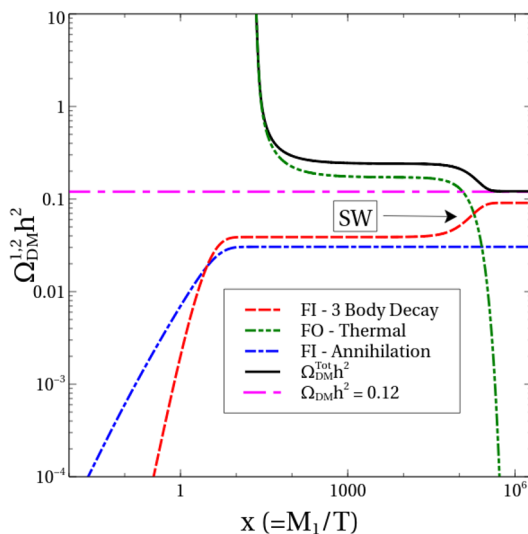


Figure 2. Evolution of DM relic density produced by different mechanisms: freeze-in production from three-body decay (red dashed line), thermal freeze-out of WIMP (green double-dot-dashed line), freeze-in production from annihilation (blue dot-dashed line), and the total contribution (black solid line). The model parameters are chosen as follows: $M_1 = 1650$ GeV, $M_2 = 500$ GeV, $M_{h_2} = 500$ GeV, $M_{Z_{\mu\tau}} = 0.1$ GeV, $g_{\mu\tau} = 9 \times 10^{-4}$, $\lambda_{2i} = 6 \times 10^{-12}$ ($i = h, H$), $\lambda_{12} = 6 \times 10^{-12}$, $\lambda_{1i} = 0.5$ ($i = h, H$), and $\mu = 7.5 \times 10^{-11}$. The magenta dot-dashed line corresponds to the correct value of DM relic density.

where $M_{Pl} = 1.22 \times 10^{19}$ GeV is the Planck mass, and $g_*(x)$ and $g_s(x)$ are the effective and entropic degrees of freedom of the Universe. Here, $Y_{1,2} \equiv n_{\phi_{1,2}}/S$ are the yields, with $n_{\phi_{1,2}}$ being the number densities and S the entropy density. The first equation corresponds to the evolution of the WIMP DM and the second equation represents the production of the FIMP DM. In the right hand side of the first equation, the first term is the annihilation of the WIMP DM to the SM particles. Here, $\langle\sigma v\rangle_{th}$ is the thermal average of cross section times velocity of DM annihilating to the SM particles. The second term implies the three-body decay of the WIMP DM to the FIMP DM, where $\langle\Gamma\rangle$ is the thermal average of the decay rate Γ , defined as $\langle\Gamma\rangle = \Gamma K_1(x)/K_2(x)$ with $K_{1,2}$ being the modified Bessel functions of the second kind. The analytical expression for the three-body decay is provided in appendix A; see eq. (A.8). Similarly, the first term in the right hand side of the second equation represents the annihilation contribution to the FIMP DM and the second term is the decay contribution of the WIMP DM to the FIMP DM. Here, $\langle\sigma v\rangle_{ij}$ is the thermal average of annihilations of i, j particles to FIMP DM. Due to the allowed decay term of the WIMP to the FIMP, we see that the WIMP DM eventually decays to the FIMP DM before big bang nucleosynthesis (BBN). We still do not have any contribution to visible energy even when the WIMP DM decays after BBN, and thus, our model remains safe from the constraints which come from light elements abundances [135].

In figure 2, we show the evolution of the DM relic density in terms of $x = M_1/T$. We examine the contributions of different production mechanisms. The blue dot-dashed line in figure 2 corresponds to the production of the FIMP DM through annihilation processes

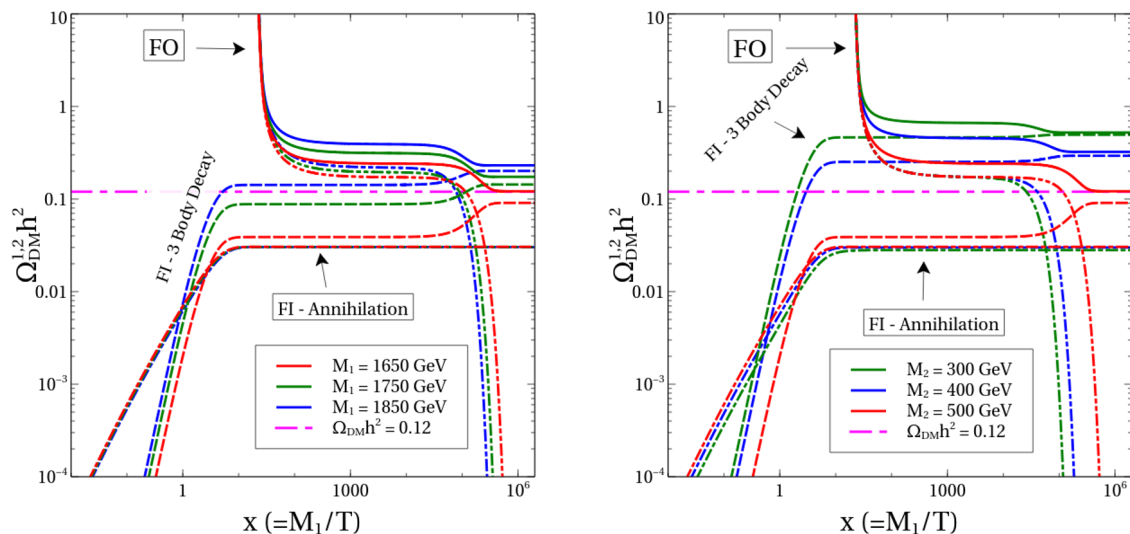


Figure 3. Evolutions of DM relic density for three different values of the WIMP DM mass M_1 (*left*) and for three different values of the FIMP DM mass M_2 (*right*). For the rest of the model parameters, see figure 2.

which saturate at $x \simeq 1$. The dominating processes in the annihilation contribution are the four-point contact terms which are $AB \rightarrow \phi_2^\dagger \phi_2$ ($A, B = h_{1,2}, \phi_1$) and are not propagator-suppressed. The green double-dot-dashed line represents the evolution of the WIMP DM which freezes out at $x \simeq 20$ and starts to decay into the FIMP DM at $x \simeq 10^5$. Since the WIMP DM decays into the FIMP DM, this regime corresponds to a single-component DM scenario. The red dashed line is the freeze-in production from the three-body decay of the WIMP DM i.e. $\phi_1 \rightarrow \phi_2 \phi_2 \phi_2$ that happens at $x \simeq 1$ which means that the WIMP DM is in thermal equilibrium with the cosmic soup. Moreover, there is also another contribution that is superWIMP (SW) contribution appears at $x \simeq 10^5$ [136]. It comes from the three-body decay of the WIMP DM. Finally, the black solid line corresponds to the total DM relic density which comes after summing all the contributions. The magenta dot-dashed line corresponds to the correct value of the DM relic density. We see that, for the choice of model parameters outlined in the caption of figure 2, our model correctly produces the exact amount of DM relic density.

Changes in the DM relic density with respect to the masses of the WIMP DM and FIMP DM are shown respectively in the left panel and the right panel of figure 3. The freeze-in production of the FIMP DM due to the annihilation is insensitive to the WIMP DM mass. This is consistent with the observation that there is no direct effect of the WIMP DM mass on the annihilation production of the FIMP DM apart from the annihilation process $\phi_1^\dagger \phi_1 \rightarrow \phi_2^\dagger \phi_2$ which has negligible dependence on the WIMP DM mass. The freeze-in production from the decay of the WIMP DM when it is in thermal equilibrium, i.e., $x \lesssim 20$, is in general inversely proportional to the WIMP DM mass in this regime. However, we observe the opposite behaviour, i.e., we get more production as the WIMP DM mass increases. This is due to the fact that, for a low value of the WIMP DM mass,

$M_1 = 1650$ GeV, we have a phase-space suppression in the decay. Thus, we get less amount of the FIMP DM, and when we increase the WIMP DM mass, the effect of phase space gets reduced, and we obtain more DM from decay. The WIMP DM freezes out at $x \simeq 20$ and starts to decay into the FIMP DM at $x \simeq 10^5$. We see that the WIMP DM starts to decay into the FIMP DM earlier as the WIMP DM mass increases as the double-dot-dashed lines in figure 3 indicate. This is because the decay width is linearly proportional to the WIMP DM mass. On top of that, there is also the phase-space suppression which further reduces the decay width and delays the WIMP decay. When the decay of the WIMP DM happens, we see a rise in the production of the FIMP DM at $x \simeq 10^5$ which is similar to the superWIMP production mechanism.

In the right panel of figure 3, we show the dependence on the FIMP DM mass. For the FIMP production due to the annihilation which is represented by the dot-dashed lines, we see a slight variation in the relic density. This is because $h_2 h_2 \rightarrow \phi_2^\dagger \phi_2$ is the dominant process, and we have taken the BSM Higgs mass to be $M_{h_2} = 500$ GeV which is comparable to the FIMP DM mass. Therefore, suppression due to the phase-space factor and increment due to mass compensate each other. The FIMP DM production due to the three-body decay is shown by the dashed lines. We observe one order of magnitude difference in the DM production when we vary the FIMP DM mass from 500 GeV to 400 GeV. This happens purely because the effect of phase space is small, and the same effect continues when we decrease the FIMP DM mass further. Finally, let us discuss the production of the WIMP DM which decays into the FIMP DM at $x \simeq 10^5$. Again, we see that, as the FIMP DM mass decreases from 500 GeV to 400 GeV, WIMP decay width increases due to lower phase-space suppression which indicates that the WIMP decays earlier. This is visible by the double-dot-dashed lines. When the WIMP DM decay happens, we have further production of the FIMP DM similar to the superWIMP production. In both the left and right panels of figure 3, solid lines represent the total DM relic density after summing all the production contributions.

The left and right panels of figure 4, shows the dependence of DM relic density for three different values of the μ and λ_{12} parameters, respectively. In the left panel, from the freeze-in contribution through annihilation, we see that there is no change in the relic density coming from the annihilation contribution which is represented by the dot-dashed line. This is because the annihilation process associated with the μ term is proportional to $Y_1^{\text{eq}} Y_2$ ($Y_2 = 0$ at initial value of x), while other annihilation terms are proportional to $Y_A^{\text{eq}} Y_B^{\text{eq}}$ (A, B are the annihilating particles). Let us turn to the production of the FIMP DM from the three-body decay of the WIMP DM. The production of the FIMP DM before $x \simeq 10$ occurs in the domain when the WIMP DM is still in thermal equilibrium, and from its decay, the FIMP DM is produced. We clearly see that the production has a quadratic dependence on the μ parameter which is perfectly consistent with the analytical expression given in the appendix A; see eq. (A.8). We note that the freeze-out temperature of the WIMP DM does not depend on the μ parameter while the WIMP DM decay does. Decay of the WIMP happens earlier (later) for a higher (lower) value of μ . This is consistent with the analytical expression; see appendix A for details. Depending on the decay occurrence, the superWIMP contribution to the FIMP DM happens earlier or later and has an equal

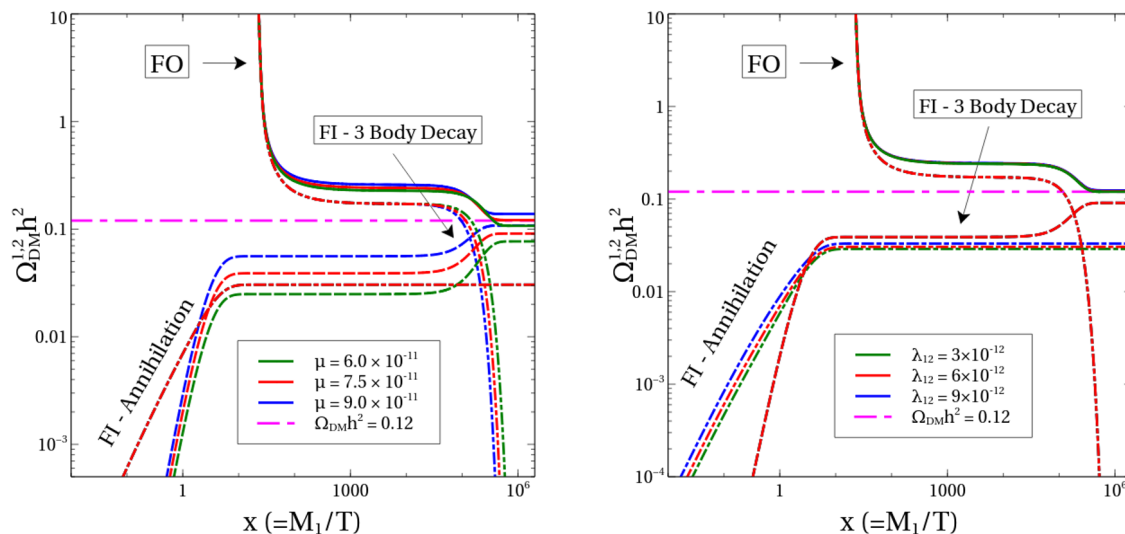


Figure 4. Dependence of the DM relic density on the μ (left) and λ_{12} (right) parameters. For the rest of the model parameters, see figure 2.

contribution for all three values, as the freeze-out contributions do not depend on the μ parameter, and this contribution is equal to $\Omega_{\text{DM}}^2 h^2 = \Omega_{\text{DM}}^1 h^2 (M_2/M_1)$.

From the right panel of figure 4, one may clearly see that the three-body decay and the superWIMP production do not depend on λ_{12} . However, we see changes in the production coming from the annihilation process, $\phi_1^\dagger \phi_1 \rightarrow \phi_2^\dagger \phi_2$. In both the left and right panels of figure 4, the solid lines correspond to the total contribution in DM relic density.

Finally, the dependence of the DM relic density on the quartic couplings of the DM with the Higgses, λ_{ij} ($i = 1, 2$ and $j = h, H$), is shown in figure 5. From the left panel, we see that the production of the FIMP DM from the freeze-in by the three-body decay and annihilation does not change; see the red dashed and dot-dashed lines. We can also see that there is a significant change in the final value when we consider the freeze-out production of the WIMP DM. This can be explained in a very simple way. The WIMP relic density is determined from the inverse of thermal average of cross section times velocity, namely $\Omega_{\text{DM}}^1 h^2 \propto 1/\lambda_{1i}^2$ ($i = h, H$). Therefore, larger values of λ_{1j} ($j = h, H$) imply that we face the situation when most of the particles annihilate away, and thus, we have less abundance for WIMP. The relative strength of the WIMP DM relic density due to different values of quartic couplings is given by $\Omega_{\text{DM}}^1 h^2|_A / \Omega_{\text{DM}}^1 h^2|_B = \lambda_{1i}^2|_B / \lambda_{1i}^2|_A$ ($i = h, H$). This is consistent with our numerical results as one may see from the blue, red, and green lines for the freeze-out production of the WIMP DM. Since there is no variation of the FIMP DM production from the three-body decay and annihilation, most of the changes in the FIMP DM production comes after $x \simeq 10^5$ when the WIMP DM decays into the FIMP DM. Solid lines correspond to the total contribution after taking into account all the production mechanisms.

On the other hand, from the right panel of figure 5, we see that the FIMP DM production from the three-body decay of the WIMP when the WIMP is in thermal equilibrium does not change as the quartic couplings vary. We can also see that the freeze-out production

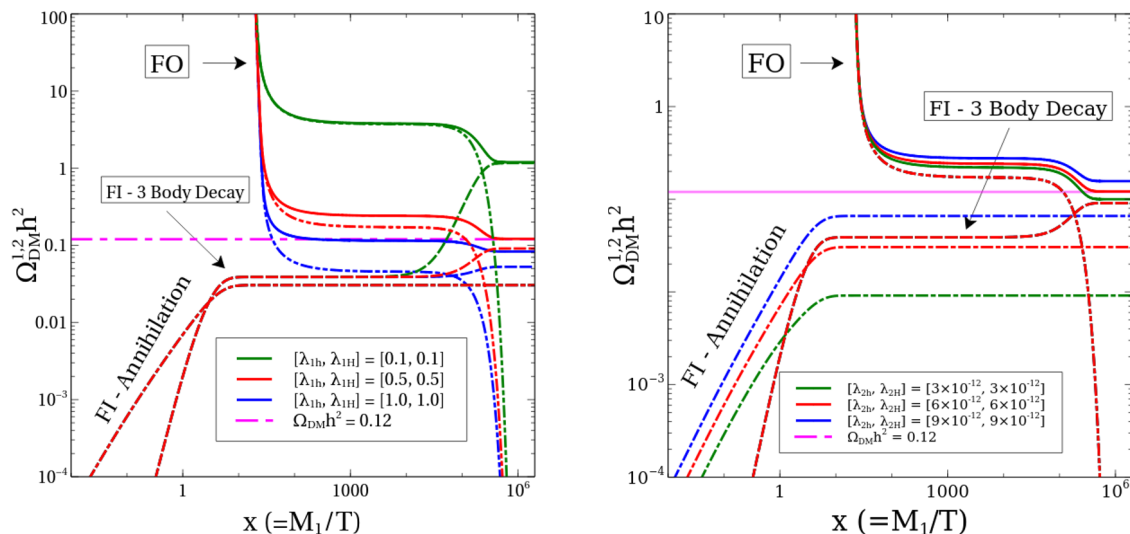


Figure 5. Dependence of the DM relic density on the quartic coupling of the WIMP with the Higgses λ_{1i} ($i = h, H$) (left) and the quartic coupling of the FIMP with Higgses λ_{2i} ($i = h, H$) (right). For the rest of the model parameters, see figure 2.

of the WIMP DM shown by the double-dot-dashed line is not affected by the change of the quartic couplings. However, the FIMP production from annihilation changes. This is indeed consistent with the observation that the freeze-in contribution by annihilation processes is proportional to the quartic couplings. In this case, the production is directly proportional to the quartic coupling which is visible by the dot-dashed lines; see eqs. (A.15) and (A.16) in appendix A.

3.2 Regime II ($M_{h_2} < 2M_2$ and $M_1 < 3M_2$)

When the WIMP DM mass is less than the three times the FIMP DM mass, the three-body decay channel of the WIMP DM to the FIMP DM is kinematically forbidden. Therefore, in this case, we have a two-component DM scenario, one WIMP-type DM and one FIMP-type DM.

The WIMP DM freezes out at $x \simeq 20$, and we obtain relic density in the experimentally allowed range put by Planck [9] near the Higgs resonance region. For the FIMP DM, we examine the effect of the μ term on the FIMP DM production and choose the FIMP DM mass in such a way that the decay channel $h_2 \rightarrow \phi_2^\dagger \phi_2$ is kinematically forbidden. Nevertheless, we have an annihilation contribution in the production of the FIMP DM through the freeze-in mechanism. Due to the presence of the μ term, there exists $\phi_1 \phi_2 \rightarrow \phi_2 \phi_2$ annihilation process, and it will exhibit an exponential growth. In the production of the FIMP, at $x \simeq 0.01$, we have a tiny amount of FIMP DM produced from the annihilation processes of the SM particles, and at $x \simeq 1$, the exponential enhancement will take place which will be discussed in detail below.

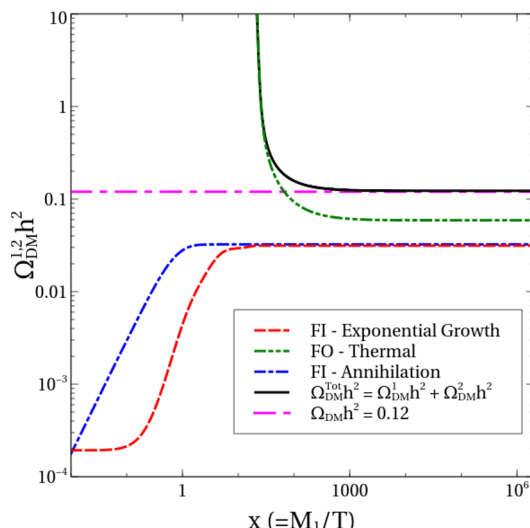


Figure 6. Evolution of the DM relic density in regime II. In this regime, we have a two-component DM scenario with one WIMP DM and one FIMP DM. The evolution of the WIMP DM is shown in the green double-dot-dashed line, while the freeze-in productions of FIMP DM through annihilation and exponential growth are shown in the blue dot-dashed line and red dashed line, respectively. The black solid line corresponds to the sum of the WIMP and FIMP DM relic densities, and the magenta dot-dashed line indicates the observed value of DM relic density. The model parameters are chosen as $M_1 = 500$ GeV, $M_2 = 700$ GeV, $M_{h_2} = 1000$ GeV, $\lambda_{2i} = 6 \times 10^{-12}$ ($i = h, H$), $\lambda_{12} = 6 \times 10^{-12}$, $\lambda_{1h} = 0.05$, $\lambda_{1H} = 0.125$, and $\mu = 10^{-6}$.

The governing Boltzmann equations in this regime are given by

$$\begin{aligned}
 \frac{dY_1}{dx} &= -\frac{2\pi^2}{45} \frac{M_{\text{Pl}} M_1}{1.66x^2} \sqrt{g_*} \langle \sigma v \rangle_{\text{th}} \left(Y_1^2 - Y_1^{\text{eq}2} \right), \\
 \frac{dY_2}{dx} &= -\frac{2\pi^2}{45} \frac{M_{\text{Pl}} M_1}{1.66x^2} \sqrt{g_*} \langle \sigma v \rangle_{\text{exp}} \left(Y_2^2 - Y_1^{\text{eq}} Y_2 \right) \\
 &\quad + \frac{4\pi^2}{45} \frac{M_{\text{Pl}} M_1}{1.66x^2} \sqrt{g_*} \sum_{i,j \in \text{SM}, \phi_1} \langle \sigma v \rangle_{ij} \left(Y_i^{\text{eq}} Y_j^{\text{eq}} - Y_2^2 \right). \tag{3.2}
 \end{aligned}$$

The first (second) equation represents the evolution of the WIMP (FIMP) DM. Here, $\langle \sigma v \rangle_{\text{exp}}$ is the thermal average associated with the exponential growth computed using the prescription described in appendix A.1.1. We now discuss the effect of model parameters on the production of WIMP and FIMP DM by different mechanisms.

In figure 6, the evolution of the WIMP and FIMP DM relic densities is shown. The model parameters are chosen in such a way that the WIMP and FIMP DM relic densities contribute equally and generate a total DM relic density in the correct ballpark value as referred by the Planck collaboration [9]. The red dashed line in figure 6 corresponds to the exponential growth of FIMP DM due to the presence of the process $\phi_1 \phi_2 \rightarrow \phi_2 \phi_2$. This kind of process can be solved analytically. The co-moving number density can be expressed as

$$Y_2 = Y_2^{\text{ini}} e^{\int_{x_{\text{ini}}}^{\infty} \frac{2\pi^2}{45} \frac{M_{\text{Pl}} M_1}{1.66x^2} \sqrt{g_*} \langle \sigma v \rangle Y_1^{\text{eq}} dx}, \tag{3.3}$$

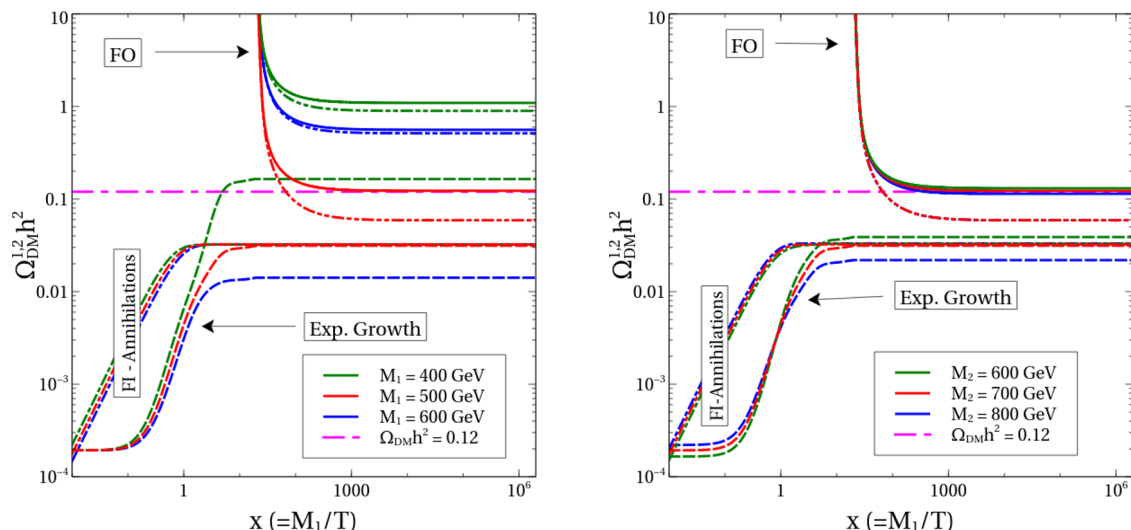


Figure 7. Dependence of the DM relic density on the WIMP DM mass M_1 (left) and FIMP DM mass M_2 (right). For the rest of the model parameters, see figure 6.

where $Y_2^{\text{ini}} = Y_2(x = x_{\text{ini}})$. Thus, we see an exponential enhancement of the FIMP DM. The blue dot-dashed line represents the FIMP DM production through the annihilation processes $AB \rightarrow \phi_2^\dagger \phi_2$ where A and B are the particles in thermal equilibrium. The green double-dot-dashed line indicates the WIMP DM production through the freeze-out mechanism which happens at $x \simeq 20$. The total sum of the WIMP and FIMP DM contributions is depicted by the black solid line which matches with the correct value of DM relic density $\Omega_{\text{DM}} h^2 = 0.12$.

Figure 7 shows the dependence of DM relic density on the WIMP and FIMP DM masses. In the left panel, we see that the WIMP DM mass has no observable effect on the freeze-in production of FIMP DM through annihilation. However, the WIMP DM mass affects the exponential growth of FIMP DM as the thermal average of cross section times velocity is inversely proportional to mass of the initial state particle, which is the WIMP DM in the present case. In the case of the WIMP DM production, when we increase or decrease the WIMP DM mass around $M_1 = 500$ GeV, we get more abundance for the WIMP DM. This is understood from the resonance behaviour of the Higgs-mediated diagram. In figure 7, we have considered $M_{h_2} = 1000$ GeV which is the resonance region for $M_1 = 500$ GeV DM. Thus, we get a large annihilation cross section which results in the reduction in WIMP abundance. If the WIMP DM mass deviates from 500 GeV, we get a smaller value of annihilation cross section and higher WIMP DM abundance. The solid lines correspond to the total contribution in DM relic density both from the WIMP and FIMP contributions.

In the right panel of figure 7, we may observe the effect of FIMP DM mass on the production of WIMP and FIMP DM by different mechanisms. The FIMP DM mass has little impact on the FIMP DM production through annihilation, while it has an observable effect on the exponentially enhanced production of FIMP DM. For $x \lesssim 1$, we see that the change in the FIMP DM relic density is proportional to the FIMP DM mass. On the other hand, for $x \gtrsim 1$, we get a similar kind of enhancement as discussed in the previous

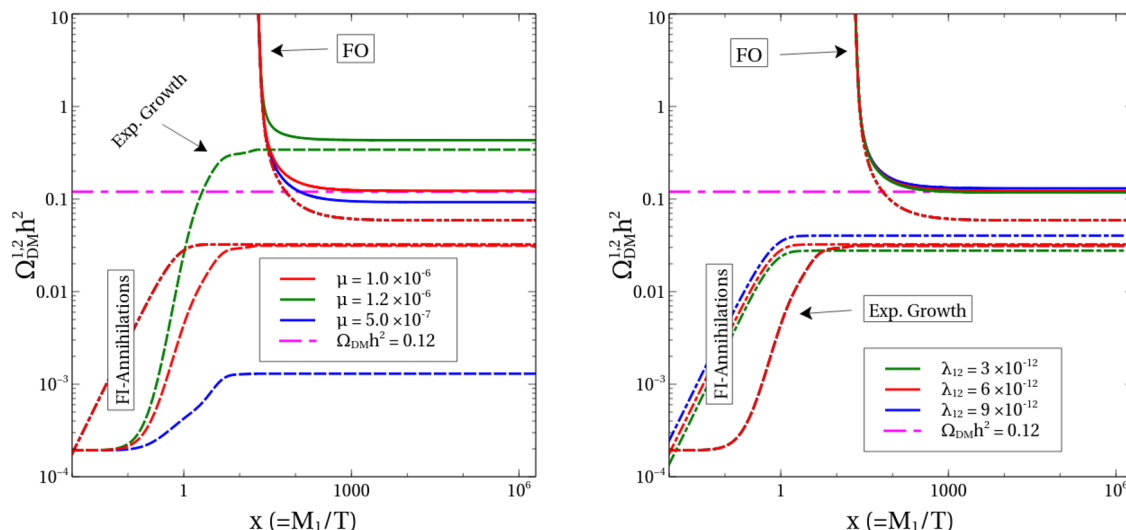


Figure 8. Dependence of the DM relic density on the μ parameter (*left*) and the quartic coupling λ_{12} between the WIMP DM and FIMP DM (*right*). For the rest of the model parameters, see figure 6.

paragraph, due to the dependence of the thermal average of the cross section on mass. As the chosen masses are in a large range, we see no big difference in the produced relic densities like before. There is also no effect of the FIMP DM mass on the production of WIMP DM through the freeze-out process. The solid lines are the total sum of WIMP and FIMP DM relic densities, and they all match the correct value of the DM relic density given by Planck.

The dependences of the production of WIMP and FIMP DM on the μ parameter and the quartic coupling between the WIMP DM and FIMP DM λ_{12} are respectively shown in the left and right panels of figure 8. The μ parameter only affects the process $\phi_1\phi_2 \rightarrow \phi_2\phi_2$, and thus, other DM productions do not change. Looking at the freeze-in production of FIMP DM from annihilation and WIMP DM production through the freeze-out process, we easily see that these production mechanisms do not vary when μ changes. However, we can see a strong dependence of the exponential enhancement on the μ parameter. If we take $\mu \lesssim 10^{-7}$, then the exponential enhancement is absent, while for $\mu \gtrsim 5 \times 10^{-6}$, there exists a tremendous exponential enhancement in the production which overproduces the DM. Thus, higher values of μ are disfavoured. The solid lines are again the total sum of WIMP and FIMP contributions, and the variation in their values are solely due to the effect of the exponential enhancement.

On the other hand, since the quartic coupling λ_{12} is in the feeble regime, it does not contribute to the WIMP DM production which is clearly visible by the double-dot-dashed line which is same for all the three values of λ_{12} . The dashed line, which accounts for the FIMP DM production through the exponential enhancement, is also unchanged for different values of λ_{12} . However, the freeze-in contribution through annihilation depends on the λ_{12} parameter as the dot-dashed lines indicate. The amount of DM production through annihilation depends quadratically on the λ_{12} parameter.

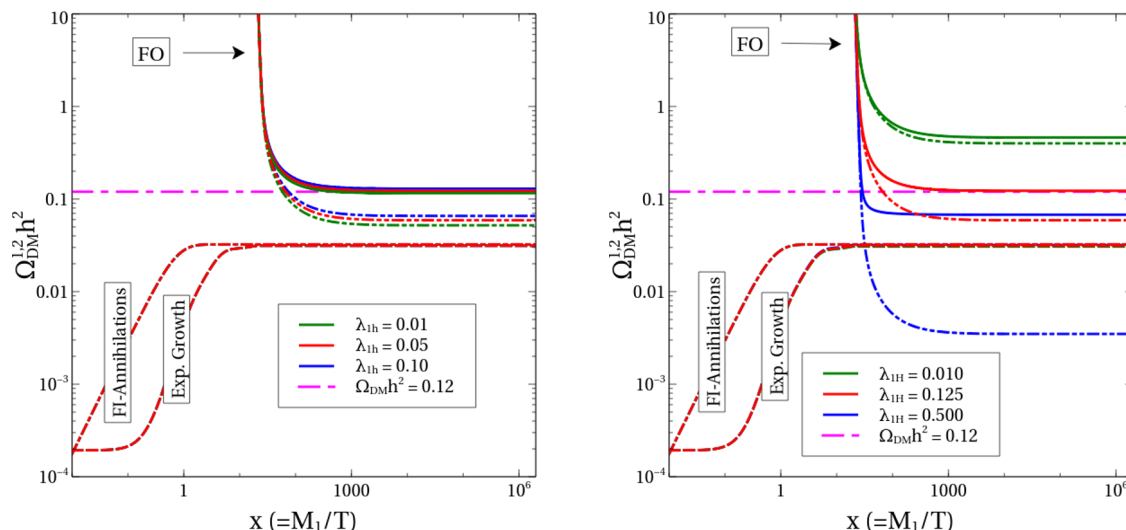


Figure 9. Dependence of the DM relic density on the quartic coupling between the WIMP DM and the SM Higgs (*left*) and the quartic coupling between the WIMP DM and the BSM Higgs (*right*). For the rest of the model parameters, see figure 6.

The left panel and the right panel of figure 9 show the dependence of the DM relic density on the quartic coupling between the WIMP DM and the SM Higgs λ_{1h} and the quartic coupling between the WIMP DM and the BSM Higgs λ_{1H} , respectively. The coupling λ_{1h} connects the WIMP DM to the visible sector through the SM Higgs. Since this quartic coupling does not affect the exponential growth of the FIMP DM and has a negligible effect on the FIMP DM through annihilation, there is no change in the FIMP DM production for different values of λ_{1h} . We can see, however, changes in the WIMP DM production, although the difference is small. The small dependence on λ_{1h} is due to the fact that the WIMP DM mass is chosen in such a way that it lies in the BSM Higgs resonance regime. Moreover, we have kept λ_{1h} below 0.1. Otherwise, the WIMP DM will be ruled out by the direct detection experiments.

The right panel of figure 9 indicates that the quartic coupling λ_{1H} has no effect on the FIMP DM production as well. However, we see that a change in λ_{1H} results in an order of magnitude variation in the WIMP DM relic density. This is because our parameters are chosen such that the WIMP mass is in the resonance region for the second, BSM Higgs, $M_1 \simeq M_{h_2}/2$. Therefore, a change in λ_{1H} which measures the coupling strength for $\phi_1^\dagger \phi_1 h_2^2$ has a direct impact on the thermal DM relic density.

The left panel of figure 10 shows the dependence of the DM relic density on the quartic coupling between the FIMP DM and the SM and BSM Higgses λ_{2i} ($i = h, H$). Since λ_{2h} and λ_{2H} are associated with the FIMP DM, it does not affect the WIMP DM production as one may easily see from the figure. The quartic coupling λ_{2i} ($i = h, H$) also does not alter the FIMP DM production by the exponential enhancement. On the other hand, we see that the FIMP production by annihilation gets affected due to the variation of λ_{2h} and λ_{2H} . This is because the associated annihilation processes $AB \rightarrow \phi_2^\dagger \phi_2$, where A and B

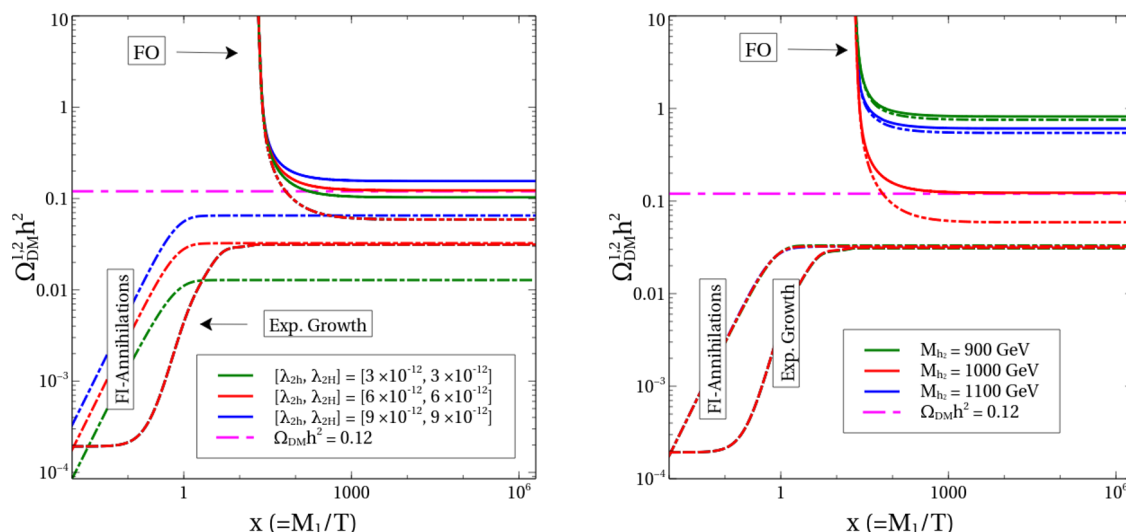


Figure 10. Dependence of the DM relic density on the quartic coupling between the FIMP DM and the SM and BSM Higgses (*left*) and the BSM Higgs mass (*right*). For the rest of the model parameters, see figure 6.

belong to the SM and BSM particles, directly depend on the strength of the λ_{2h} and λ_{2H} couplings. The changes in the solid line, which is the total sum of both the FIMP and WIMP contributions, are due to the variation in FIMP DM relic density coming from the annihilation part.

The right panel of figure 10 shows the dependence of the DM relic density on the BSM Higgs mass M_{h_2} . Since in this regime, the decay process $h_2 \rightarrow \phi_2^\dagger \phi_2$ is not allowed, we do not see any observable effect on the FIMP DM production. However, we see an effect on the production of WIMP DM. The reason is exactly the same as the one we discussed earlier for the left panel of figure 7. Here as well, since $M_1 = 500$ GeV, if M_{h_2} deviates from $M_{h_2} = 1000$ GeV, we are basically going away from the resonance region. This means that DM freezes out earlier due to the reduction in the thermal cross section, and we get higher WIMP DM relic density. The changes in the solid line are purely due to variation in the WIMP contribution to the DM relic density.

3.3 Regime III ($M_{h_2} > 2M_2$ with μ negligible)

In this regime, one should take into account the FIMP DM production from the decay of the Higgses as well. Throughout the discussion, we assume that μ is negligible and focus on two-component DM scenarios. Since μ is negligible, we may neglect the exponential enhancement in the FIMP DM production. We note that this scenario is different from the individual study of WIMP [121, 137, 138] and FIMP [122] as the FIMP DM can also be produced from the annihilation of the WIMP DM through the process $\phi_1^\dagger \phi_1 \rightarrow \phi_2^\dagger \phi_2$. This annihilation contribution can be increased or decreased with the strength of the λ_{12} parameter as discussed in the right panel of figure 8 and can produce the FIMP DM with the correct DM relic density. Therefore, our study on the two-component DM scenario in

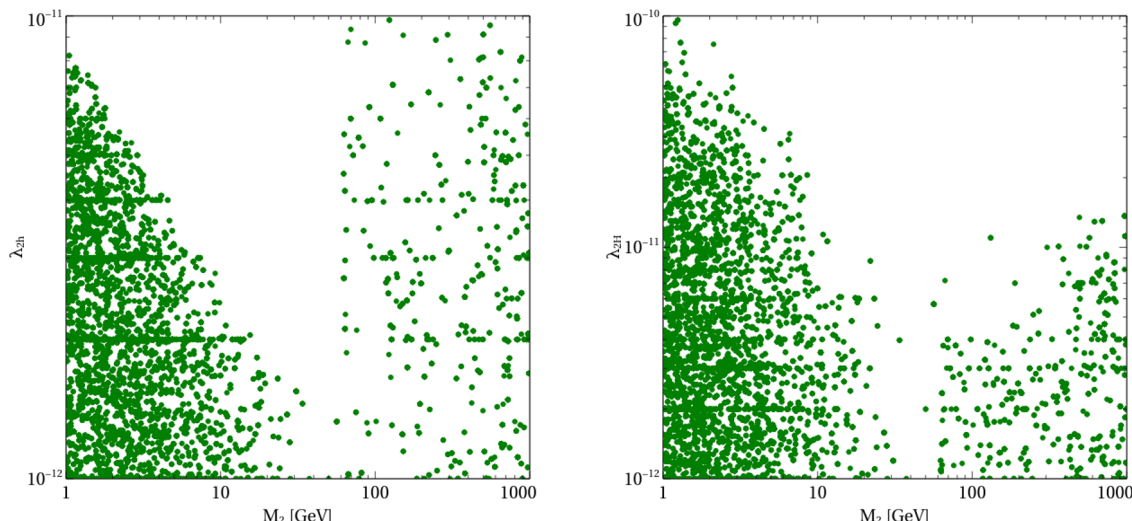


Figure 11. Allowed parameter space in the M_2 - λ_{2h} plane (left) and in the M_2 - λ_{2H} plane (right) with the total DM relic density in the range $0.01 \leq \Omega_{\text{DM}} h^2 \leq 0.12$. The parameter ranges used in the scan are given in eq. (3.5).

the regime III is new and interesting. We provide analytical expressions for the decay and $2 \rightarrow 2$ contact annihilation processes in appendix A; see eqs. (A.13)–(A.16).

The Boltzmann equations associated with the WIMP and FIMP DM are given by

$$\begin{aligned} \frac{dY_1}{dx} &= -\frac{2\pi^2}{45} \frac{M_{\text{Pl}} M_1}{1.66x^2} \sqrt{g_*} \langle \sigma v \rangle_{\text{th}} \left(Y_1^2 - Y_1^{\text{eq}2} \right), \\ \frac{dY_2}{dx} &= \frac{2M_{\text{Pl}}}{1.66M_1^2} \frac{x\sqrt{g_*(x)}}{g_s(x)} \sum_{i=1,2} \langle \Gamma_{h_i \rightarrow \phi_1^\dagger \phi_1} \rangle \left(Y_{h_i}^{\text{eq}} - Y_2^2 \right) \\ &\quad + \frac{4\pi^2}{45} \frac{M_{\text{Pl}} M_1 \sqrt{g_*(x)}}{1.66x^2} \sum_{i,j \in \text{SM}, \phi_1} \langle \sigma v \rangle_{ij} \left(Y_i^{\text{eq}} Y_j^{\text{eq}} - Y_2^2 \right). \end{aligned} \tag{3.4}$$

In the following, we solve the above Boltzmann equations and discuss the correlation between the model parameters by performing scans with the following range:

$$\begin{aligned} 10^{-3} \leq \theta \leq 10^{-1}, \quad & 10^{-3} \leq \lambda_{1h}, \lambda_{1H} \leq 10^{-1}, \quad 10^{-12} \leq \lambda_{2h}, \lambda_{2H}, \lambda_{12} \leq 10^{-10}, \\ 10^{-10} \leq n_{\mu\tau} \leq 10^{-8}, \quad & 10^{-4} \leq g_{\mu\tau} \leq 10^{-2}, \\ 10^{-3} \leq M_{Z_{\mu\tau}} [\text{GeV}] \leq 1, \quad & 200 \leq M_{h_2} [\text{GeV}] \leq 1100, \quad 1 \leq M_{1,2} [\text{GeV}] \leq 1000. \end{aligned} \tag{3.5}$$

When performing the scans, we demand the total DM relic density to be in the range $0.01 \leq \Omega_{\text{DM}} h^2 \leq 0.12$. We stress that, when the sum of the WIMP and FIMP DM relic densities is smaller than $\Omega_{\text{DM}} h^2 = 0.12$, the rest of the amount can easily be obtained by suitably adjusting the μ parameter which we neglect at the moment.

The allowed parameter regions in the M_2 - λ_{2h} and M_2 - λ_{2H} planes after imposing $0.01 \leq \Omega_{\text{DM}} h^2 \leq 0.12$ are shown in figure 11. From the left panel, we see a sharp correlation between the FIMP DM mass M_2 and the coupling λ_{2h} for $M_2 \lesssim 62$ GeV. The sharp

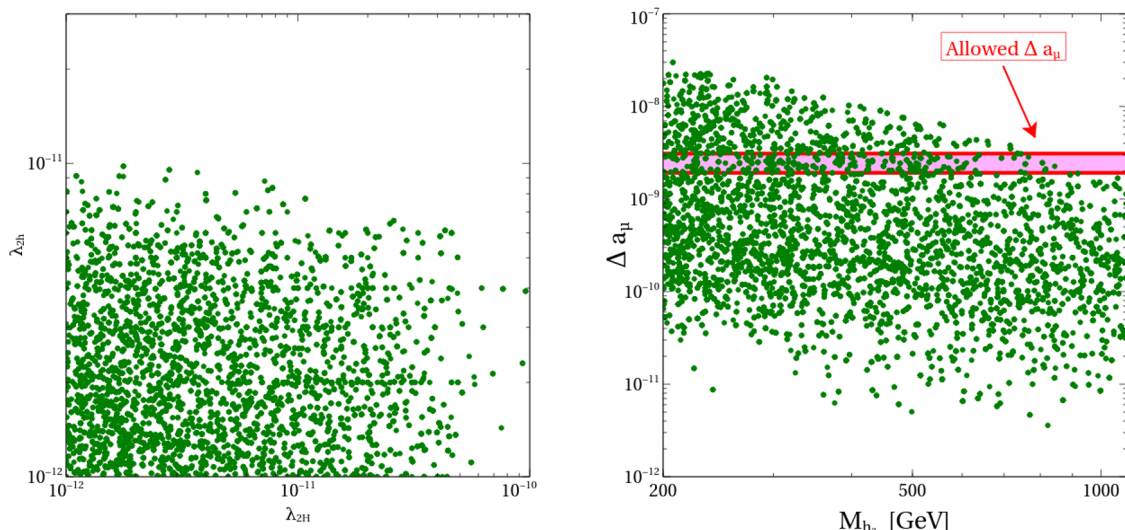


Figure 12. Allowed parameter space in the λ_{2h} - λ_{2H} plane (left) and in the M_{h_2} - Δa_μ plane (right) with the total DM relic density in the range $0.01 \leq \Omega_{\text{DM}} h^2 \leq 0.12$. The muon $g - 2$ tension may be relieved in the magenta-coloured region. The parameter ranges used in the scan are given in eq. (3.5).

correlation may be understood as follows. For $M_2 \lesssim 62$ GeV, the DM can be produced from the SM Higgs decay. The decay mode $h_1 \rightarrow \phi_2^\dagger \phi_2$ is proportional to λ_{1h} and the phase-space factor $\sqrt{1 - 4M_2^2/M_{h_1}^2}$. Moreover, the DM relic density is proportional to the DM mass as well. Thus, as the DM mass increases, the λ_{2h} coupling needs to be decreased in order to obtain the correct DM relic density. For DM mass in the range 50–60 GeV, we have the phase-space suppression. Hence, in order to get DM in this range, we need a larger value of λ_{2h} . For $M_2 \gtrsim 62.5$ GeV, we do not have the decay channel of the SM Higgs into the FIMP DM, and annihilation processes take over. We thus do not have a sharp correlation in the large M_2 region.

From the right panel of figure 11, we also observe a similar kind of behaviour between M_2 and λ_{2H} . However, for $M_2 \gtrsim 62.5$ GeV, the λ_{2H} parameter cannot be arbitrarily large as the h_2 decay mode is present. Moreover, we see that λ_{2H} may become as large as 10^{-10} whereas λ_{2h} can go only up to 10^{-11} . The reason for this is that we varied M_{h_2} up to 1.1 TeV and that the FIMP DM relic density through decay is proportional to λ_{2H}^2/M_{h_2} . Therefore, λ_{2H} may become larger as M_{h_2} takes a larger value, which is impossible for the SM Higgs case.

The left panel of figure 12 shows the allowed parameter region in the λ_{2h} - λ_{2H} plane. Since we have considered both the WIMP and FIMP DM in the DM relic density bound, it is hard to bound the quartic couplings from below as there will always be a contribution from the WIMP DM. However, we may obtain an upper bound on λ_{2h} and λ_{2H} above which the DM is overproduced. We find the upper limits as $\lambda_{2h} \lesssim 10^{-11}$ and $\lambda_{2H} \lesssim 10^{-10}$ for the choice of model parameters used in the scan (3.5).

The right panel of figure 12 shows the allowed parameter region in the M_{h_2} - Δa_μ plane. The magenta-coloured region corresponds to the correct experimental range of muon

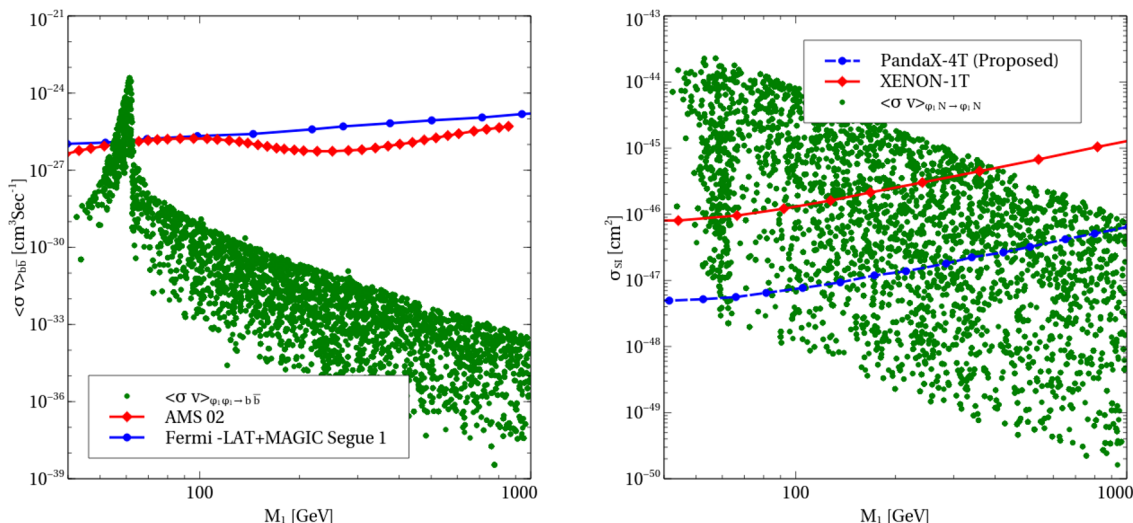


Figure 13. *Left:* results of the scans in the $M_1-\langle\sigma v\rangle_{b\bar{b}}$ plane. The points above the red or blue lines are ruled out by indirect detection experiments [17, 139]. *Right:* results of the scans in the $M_1-\sigma_{SI}$ plane. The points above the red line is ruled out from the direct detection experiment XENON-1T [15]. The blue line indicates the sensitivity of the future direct detection experiment PandaX [141]. The parameter ranges used in the scan are given in eq. (3.5).

$g - 2$. The parameter space above the allowed magenta-coloured band is ruled out, and the points below the band demand additional positive contributions in $(g - 2)_\mu$ to match the experimental range. One may see a correlation between Δa_μ and M_{h_2} . For a larger value of M_{h_2} , we get a lower value of Δa_μ . This is due to the fact that a higher value of M_{h_2} indicates a higher VEV of the BSM Higgs, $v_{\mu\tau}$. Since $v_{\mu\tau} = M_{Z_{\mu\tau}}/g_{\mu\tau}$, a higher VEV implies a lower value of $g_{\mu\tau}$ which reduces the Δa_μ contribution.

In figure 13, we present the indirect and direct detection bounds on the mass of the WIMP DM. In the left panel, the AMS 02 indirect detection bound coming from the WIMP DM annihilation to $b\bar{b}$ [17, 139] is indicated with a red line. We see a sharp rise around $M_1 \simeq 62$ GeV which corresponds to the SM Higgs resonance region. A part of the region is ruled out by the indirect detection bound and the rest of the region is to be probed shortly by different ongoing indirect detection experiments [17]. In the right panel, the spin-independent direct detection XENON-1T bound [15] on the WIMP DM is shown. One may see from figure 13 that a part of the $M_1 \leq 500$ GeV region is already ruled out from the direct detection experiments. The rest of the region will be explored in the future by different proposed experiments like Darwin [140] and PandaX [141].

4 Gravitational waves from phase transition

The presence of the extra scalar fields in addition to the SM Higgs field not only makes the phenomenology of DM much richer, but it also makes the evolution dynamics of the vacuum state non-trivial and may lead to a FOPT in the early universe as opposed to the SM case whose phase transition is of the cross-over type [142]. See, e.g., ref. [143] for a

recent review on the FOPT. As a consequence, stochastic GW signals may be emitted. The produced stochastic GW signals have a potential to be detected by future GW experiments such as LISA [87], DECIGO [88], and BBO [93], and this possibility gives a complementary detection signal to the standard (in-)direct detection and collider searches. In this section, we examine stochastic GW signals from a FOPT in our model and compare them with the sensitivity curves of future GW experiments. Furthermore, we present benchmark points that explain the muon $g - 2$ tension, neutrino masses, and correct DM relic density, while producing strong GW signals that are within the detectability of Ultimate-DECIGO.

We closely follow ref. [99] to estimate the stochastic GW signal from a FOPT. The three main sources of the GWs produced by a FOPT include the collision of bubble walls, the sound wave in the plasma, and the magneto-hydrodynamic turbulence in the plasma, and thus,

$$\Omega_{\text{GW}}h^2 \simeq \Omega_{\text{col}}h^2 + \Omega_{\text{sw}}h^2 + \Omega_{\text{turb}}h^2, \quad (4.1)$$

where

$$\Omega_{\text{col}}h^2 = 1.67 \times 10^{-5} \left(\frac{H_*}{\beta}\right)^2 \left(\frac{\kappa_\phi \alpha}{1+\alpha}\right)^2 \left(\frac{100}{g_*}\right)^{\frac{1}{3}} \left(\frac{0.11v_w^3}{0.42+v_w^2}\right) \left(\frac{3.8(f/f_{\text{col}})^{2.8}}{1+2.8(f/f_{\text{col}})^{3.8}}\right), \quad (4.2)$$

$$\Omega_{\text{sw}}h^2 = 2.65 \times 10^{-6} \left(\frac{H_*}{\beta}\right) \left(\frac{\kappa_v \alpha}{1+\alpha}\right)^2 \left(\frac{100}{g_*}\right)^{\frac{1}{3}} v_w (f/f_{\text{sw}})^3 \left(\frac{7}{4+3(f/f_{\text{sw}})^2}\right)^{\frac{7}{2}}, \quad (4.3)$$

and

$$\Omega_{\text{turb}}h^2 = 3.35 \times 10^{-4} \left(\frac{H_*}{\beta}\right) \left(\frac{\kappa_{\text{turb}} \alpha}{1+\alpha}\right)^{\frac{3}{2}} \left(\frac{100}{g_*}\right)^{\frac{1}{3}} \left(\frac{v_w (f/f_{\text{turb}})^3}{[1+(f/f_{\text{turb}})]^{\frac{11}{3}} (1+8\pi f/h_*)}\right), \quad (4.4)$$

with

$$h_* = 1.65 \times 10^{-5} \text{ Hz} \left(\frac{T_*}{100\text{GeV}}\right) \left(\frac{g_*}{100}\right)^{\frac{1}{6}}. \quad (4.5)$$

The expressions for f_{col} , f_{sw} , and f_{turb} are given as follows:

$$f_{\text{col}} = 1.65 \times 10^{-5} \text{ Hz} \left(\frac{0.62}{1.8 - 0.1v_w + v_w^2}\right) \left(\frac{\beta}{H_*}\right) \left(\frac{T_*}{100\text{GeV}}\right) \left(\frac{g_*}{100}\right)^{\frac{1}{6}}, \quad (4.6)$$

$$f_{\text{sw}} = 1.9 \times 10^{-5} \text{ Hz} \left(\frac{1}{v_w}\right) \left(\frac{\beta}{H_*}\right) \left(\frac{T_*}{100\text{GeV}}\right) \left(\frac{g_*}{100}\right)^{\frac{1}{6}}, \quad (4.7)$$

and

$$f_{\text{turb}} = 2.7 \times 10^{-5} \text{ Hz} \left(\frac{1}{v_w}\right) \left(\frac{\beta}{H_*}\right) \left(\frac{T_*}{100\text{GeV}}\right) \left(\frac{g_*}{100}\right)^{\frac{1}{6}}. \quad (4.8)$$

Here, g_* is the number of effective degrees of freedom at $T = T_*$. For the bubble wall velocity v_w , we use [144]

$$v_w = \frac{\sqrt{1/3} + \sqrt{\alpha^2 + 2\alpha/3}}{1 + \alpha}, \quad (4.9)$$

and we adopt [86]

$$\kappa = \frac{0.715\alpha + (4/27)\sqrt{3\alpha/2}}{1 + 0.715\alpha}, \quad \kappa_v = \frac{\alpha}{0.73 + 0.083\sqrt{\alpha} + \alpha}, \quad \kappa_{\text{turb}} = 0.1\kappa_v. \quad (4.10)$$

In estimating the sound-wave contribution to the GW signal, we have ignored the possible suppression factor associated with the lifetime of the sound-wave source.³ The suppression factor may be estimated as [145–148]

$$\mathcal{S}_{\text{sw}} \simeq 1.81 \times \min \left\{ 1, \frac{2(8\pi)^{1/3}}{\sqrt{3}} v_w \left(\frac{H_*}{\beta} \right) \sqrt{\frac{1 + \alpha}{\kappa_v \alpha}} \right\}, \quad (4.11)$$

which corresponds to $\mathcal{O}(10^{-2}\text{--}10^{-1})$ for the benchmark points (BPs) presented in table 3. It is also important to note that such a suppression may be followed by a possible enhancement in the turbulence contribution to the GW signal [145]. The precise determination requires dedicated and sophisticated numerical simulations which go beyond the scope of the present work.

From eqs. (4.2)–(4.8), one may see that the key parameters that control the GW signal are α , β/H_* , and T_n , where

$$\alpha = \frac{\rho_{\text{vac}}}{\rho_{\text{rad}}^*}, \quad \frac{\beta}{H_*} = T_* \left. \frac{dS_E}{dT} \right|_{T_*}, \quad (4.12)$$

with S_E being the Euclidean action of a bubble and ρ_{vac} the energy density released during the FOPT. We note that $\rho_{\text{rad}}^* = g_* \pi^2 T_*^4/30$. Throughout the section, we take T_* to be the nucleation temperature T_n , i.e., $T_* = T_n$.

To understand the dynamics of the FOPT, we use the one-loop effective potential,⁴

$$V_{\text{eff}}^{1\text{-loop}} = V^{\text{tree}} + V_{\text{eff},T=0}^{1\text{-loop}} + V_{\text{eff},T \neq 0}^{1\text{-loop}}. \quad (4.13)$$

Here, V^{tree} is the tree-level potential and $V_{\text{eff},T=0}^{1\text{-loop}}$ is the zero-temperature one-loop Coleman-Weinberg contribution [154] which, in the $\overline{\text{MS}}$ scheme, is given by

$$V_{\text{eff},T=0}^{1\text{-loop}} = \pm \sum_i n_i \frac{M_i^4}{64\pi^2} \left[\ln \frac{M_i^2}{\Lambda^2} - c_i \right], \quad (4.14)$$

where Λ is the renormalisation scale which we take to be $\Lambda^2 = (v^2 + v_{\mu\tau}^2)/2$, n_i is the number of degrees of freedom of the particle with field-dependent mass M_i , the constants c_i are 1/2 (3/2) for transverse gauge bosons (all other particles), and + (−) is for bosons (fermions). The last correction, $V_{\text{eff},T \neq 0}^{1\text{-loop}}$, is the finite-temperature one-loop correction given by [155]

$$V_{\text{eff},T \neq 0}^{1\text{-loop}} = \sum_i \frac{T^4}{2\pi^2} n_i I_{\pm} \left(\frac{m_i^2}{T^2} \right), \quad (4.15)$$

³We thank the anonymous referee for pointing out this.

⁴For a gauge dependence issue, readers may refer to refs. [149–153].

with

$$I_{\pm}(x) = \pm \int_0^{\infty} dy y^2 \ln \left(1 \mp e^{-\sqrt{y^2+x}} \right), \quad (4.16)$$

where $+$ ($-$) is for fermions (bosons). To take into account the re-summed ring diagrams, we replace the field-dependent masses as

$$M_i^2 \rightarrow \widetilde{M}_i^2 = M_i^2 + \Pi_i(T), \quad (4.17)$$

where $\Pi_i(T)$ are the thermal masses [156]. For the scalars in our model these corrections are

$$\Pi_h = \frac{T^2}{48} \left(3g_1^2 + 9g_2^2 + 12y_t^2 + 12\lambda_h + 2\lambda_{hH} + 4\lambda_{1h} + 4\lambda_{2h} \right), \quad (4.18)$$

$$\Pi_H = \frac{T^2}{24} \left(6g_{\mu\tau}^2 + 6\lambda_H + \lambda_{hH} + 2\lambda_{1H} + 2\lambda_{2H} + 4h_{e\tau}^2 \right), \quad (4.19)$$

$$\Pi_{\varphi_1} = \Pi_{\eta_1} = \frac{T^2}{24} (8\lambda_1 + 2\lambda_{12} + \lambda_{1h} + \lambda_{1H}), \quad (4.20)$$

$$\Pi_{\varphi_2} = \Pi_{\eta_2} = \frac{T^2}{24} (8\lambda_2 + 2\lambda_{12} + \lambda_{2h} + \lambda_{2H}), \quad (4.21)$$

where $\varphi_{1,2}$ ($\eta_{1,2}$) are the real (imaginary) components of the DM candidate $\phi_{1,2}$, and for the gauge boson, for which only the longitudinal mode receive corrections,

$$\Pi_{W_L^{1,2,3}} = \frac{11}{6} g_2^2 T^2, \quad \Pi_{B_L} = \frac{11}{6} g_1^2 T^2, \quad \Pi_{Z_{\mu\tau L}} = \frac{1}{3} g_{\mu\tau}^2 T^2. \quad (4.22)$$

Fermions do not receive any corrections.

In order to estimate the α and β/H_* parameters in our model, we performed a numerical analysis by using a modified version of COSMOTRANSITIONS [157] together with the mass spectra given above. We restrict our focus on the case where only the SM and the BSM Higgses develop VEVs, taking zero VEVs for the DM candidates ϕ_1 and ϕ_2 throughout the temperature evolution of the system. We work with the following input parameters:

$$\begin{aligned} &v_{\mu\tau}, \quad M_{h_2}, \quad \theta, \quad M_{ee}, \quad M_1, \quad M_2, \quad \mu, \quad g_{\mu\tau}, \quad h_{e\tau}, \\ &\lambda_1, \quad \lambda_2, \quad \lambda_{1h}, \quad \lambda_{1H}, \quad \lambda_{12}, \quad \lambda_{2h}, \quad \lambda_{2H}, \end{aligned} \quad (4.23)$$

with the assumptions $M_{ee} = M_{\mu\tau}$ and $h_{e\tau} = h_{e\mu}$ which allow us to analytically diagonalise the RH neutrino mass matrix as we discussed in section 2. In the following, we take $h_{e\tau} = \sqrt{2}M_{ee}/v_{\mu\tau}$. We impose the vacuum stability conditions,

$$\lambda_h > 0, \quad \lambda_H > 0, \quad 4\lambda_h\lambda_H - \lambda_{hH}^2 \geq 0, \quad (4.24)$$

as well as the perturbativity and unitarity bounds,

$$|\lambda_h| < 4\pi, \quad |\lambda_H| < 4\pi, \quad |\lambda_{hH}| < 8\pi, \quad 3\lambda_h + 2\lambda_H + \sqrt{(3\lambda_h - 2\lambda_H)^2 + 2\lambda_{hH}^2} < 8\pi. \quad (4.25)$$

We focus on the following range of the parameters:

$$\begin{aligned} &20 \leq v_{\mu\tau} [\text{GeV}] \leq 250, \quad 130 \leq M_{h_2} [\text{GeV}] \leq 1000, \quad 0 \leq \theta \leq 0.4, \\ &1 \leq M_{ee} [\text{GeV}] \leq 100, \quad 0 \leq \lambda_{1h} \leq 0.5, \quad 0 \leq \lambda_{1H} \leq 0.5, \quad 10^{-12} \leq \lambda_{2h} \leq 10^{-10}, \\ &100 \leq M_1 [\text{GeV}] \leq 300, \quad 50 \leq M_2 [\text{GeV}] \leq 250, \end{aligned} \quad (4.26)$$

BP	$v_{\mu\tau}$	M_{h_2}	M_{ee}	M_1	M_2	θ	λ_{2h}	λ_{1h}	λ_{1H}	α	β/H	T_n	$\frac{\Omega_{\text{DM}}^1}{\Omega_{\text{DM}}}$	$\frac{\Omega_{\text{DM}}^2}{\Omega_{\text{DM}}}$
1	80.14	408.06	98.45	250.31	169.75	0.388	6.0×10^{-12}	0.1	0.1	0.0036	4994.4	235.0	0.64	0.36
2	81.69	415.34	99.83	244.57	99.39	0.387	4.5×10^{-12}	0.1	0.1	0.0037	15293.2	238.8	0.58	0.42
3	81.66	398.97	98.19	210.0	209.9	0.002	4.5×10^{-12}	0.1	0.1	0.0068	4884.0	178.7	0.79	0.21
4	83.28	366.09	67.07	249.51	119.54	0.034	1.7×10^{-11}	0.289	0.228	0.0056	47146.8	189.2	0.15	0.85

Table 3. Four BPs. Values of the mass-dimensional parameters are given in units of GeV. We present the model parameters, the GW-related parameters, α , β/H , and T_n , and the DM relic densities $\Omega_{\text{DM}}^{1,2}$. For all of these four BPs, the muon $g - 2$ tension and the neutrino masses can be accounted for. The GW signals corresponding to the four BPs are highlighted in figure 14. The other parameters are chosen as $\lambda_{2H} = 6 \times 10^{-12}$, $\mu = 10^{-7}$, $\lambda_{12} = 6 \times 10^{-12}$, $g_{\mu\tau} = 6 \times 10^{-4}$, and $n_{\mu\tau} = 10^{-8}$. The neutrino masses can be obtained for the choice of parameters as studied in detail in ref. [121], and $(g - 2)_\mu$ will also be obtained in the correct ballpark value as shown in figure 1. In determining the WIMP and FIMP DM individual contribution, we have considered the total DM relic density of $\Omega_{\text{DM}} h^2 = 0.12$.

while fixing the other parameters as follows:

$$\lambda_1 = \lambda_2 = 0.1, \quad \lambda_{2H} = \lambda_{12} = 6 \times 10^{-12}, \quad g_{\mu\tau} = 6 \times 10^{-4}, \quad \mu = 10^{-7}. \quad (4.27)$$

We note that the upper bound of the mixing angle, $\theta = 0.4$, is chosen by considering the LHC constraints on the hVV couplings coming from the measurements of the Higgs decay into gauge bosons [158]. We observe FOPTs and its associated GW signals for a wide range of the parameter values, including the mixing angle. To show that GWs can be accompanied with both small and large values of the mixing angle θ , we present two BPs with a large value of θ and two BPs with a small value of θ .

In figure 14, we show the associated GW signals together with the sensitivity curves of future GW experiments. We select four BPs and present the results in table 3. From figure 14, we see that all of our four BPs, and many other signals, are well within the reach of detectability of Ultimate-DECIGO, while their signal strengths are below the sensitivity curves of BBO and DECIGO. Taking into account the suppression factor (4.11), we see that some of the GW signals for the chosen BPs fall below the sensitivity curve of the Ultimate-DECIGO, while some stay marginally within the sensitivity curve. However, we stress that the conclusion that the GW signals associated with the FOPT within our model, which simultaneously accounts for the muon $g - 2$ tension, neutrino masses, and two-component DM scenarios, are within the reach of the Ultimate-DECIGO sensitivity curve remains intact.

One may see from table 3 that, in the parameter space that can solve the muon $g - 2$ tension, generate the neutrinos masses, and produce the correct DM relic density, GWs are also expected whose signals are strong enough to be seen by Ultimate-DECIGO. The presented four BPs clearly showcase that the muon $g - 2$, neutrino masses, and two-component DM scenarios are accounted for in our model which, at the same time, predicts stochastic GWs associated with the FOPT in a single unified framework.

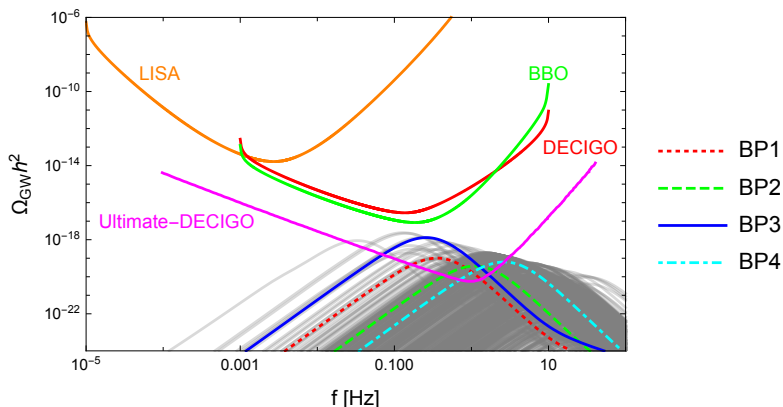


Figure 14. GW spectrum from the FOPT together with the sensitivity curves of future GW experiments. The predicted GW signals span in the frequency range $0.01 \text{ Hz} \lesssim f \lesssim 100 \text{ Hz}$ with the magnitude as large as $\Omega_{\text{GW}} h^2 \simeq 10^{-18}$. While the strengths of the signals are below the sensitivity curves of BBO and DECIGO, they are well within the reach of detectability of Ultimate-DECIGO. The data for the sensitivity curves of LISA, BBO, and DECIGO are obtained from ref. [159], and for the data for the sensitivity curve of Ultimate-DECIGO, we used ref. [160]. Earlier work on the sensitivity curves includes e.g. refs. [161–168]. The red dotted, green dashed, blue solid, and cyan dot-dashed lines correspond to our four benchmark points 1, 2, 3, and 4, respectively, that are summarised in table 3.

5 Conclusion

In this paper, we studied an extension of the Standard Model that accounts for the dark matter, the muon $g - 2$ tension, and the neutrino masses, in a single unified framework. We introduced three massive right-handed neutrinos which, through the type-I seesaw mechanism, provide a mass to the Standard Model neutrinos. We then extended the Standard Model by introducing two scalar fields that play the role of the dark matter. Finally, an extra $U(1)_{L_\mu - L_\tau}$ gauge symmetry is imposed, where the associated gauge boson $Z_{\mu\tau}$ alleviates the muon $g - 2$ tension.

As the model we considered contains two Standard Model-singlet scalar dark matter candidates, ϕ_1 and ϕ_2 , we examined the possibility of a single-component as well as two-component dark matter scenarios. Focusing on three different regimes, we showed how a single-component or two-component dark matter scenario can emerge by numerically solving the coupled Boltzmann equations. We found that, when the $\mu\phi_1^\dagger\phi_2^3$ term is not negligible, both the single- and two-component scenarios may be obtained, depending on the mass range. When the μ parameter is small or absent, we showed that a two-component dark matter scenario naturally arises without dependence on the mass range of the WIMP and FIMP DM. In the case of a two-component scenario, one component becomes the WIMP-type dark matter and the other component is the FIMP-type dark matter. We performed a numerical scan and presented viable parameter spaces which are compatible with the current experimental bounds such as the direct and indirect detections, relieving the muon $g - 2$ tension at the same time.

The presence of the extra scalar fields not only makes the dark matter phenomenology richer. It also affects the evolution dynamics of the vacuum state. As opposed to the Standard Model case whose phase transition is of the cross-over type, a first-order phase transition may be realised in our model. Consequently, stochastic GW signals may be emitted. We investigated the parameter space where the first-order phase transition occurs and scrutinised the associated stochastic gravitational wave signals. Performing a numerical scan, we showed that the predicted gravitational waves are strong enough to be probed by future gravitational wave experiments such as Ultimate-DECIGO.

We explicitly demonstrated that our model is capable of accommodating the three problems of the Standard Model, namely the dark matter, neutrino masses, and the muon $g - 2$ tension, by presenting four benchmark points. The chosen four benchmark points give rise to the first-order phase transition, and consequently, we observe the associated gravitational wave signals. All of the four benchmark points are within the reach of detectability of Ultimate-DECIGO. Furthermore, the chosen benchmark points realise two-component dark matter scenarios. We expect that the gravitational wave feature of our model may serve as a complementary detection signal to the standard (in-)direct detection and collider searches.

Acknowledgments

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A Analytical expressions for the freeze-in dark matter

A.1 FIMP-WIMP interactions

We summarise different channels for the FIMP production, considering the interaction between the FIMP DM and the WIMP DM.

A.1.1 Exponential yield

In the regime where $M_1 < 3M_2$, the decay of the WIMP DM is kinetically forbidden. For the scattering process $\phi_2\phi_2 \leftrightarrow \phi_1\phi_2$, the Boltzmann equation is given by

$$\dot{n}_{\phi_2} + 3Hn_{\phi_2} = \langle\sigma v\rangle n_{\phi_2} n_{\phi_1}^{\text{eq}}, \quad (\text{A.1})$$

to a good approximation.⁵ Since ϕ_2 is in a FIMP regime, i.e., out of equilibrium, its number density is considerably low, and we can thus neglect the quadratic term in n_{ϕ_2} . Notice that if the decay were allowed, the decay channel would become the dominant process as it may

⁵The viability of the use of number densities in the Boltzmann equations is questioned and checked in, for example, ref. [169] by considering the backreaction effects and solving the Boltzmann equations at the level of the phase-space distribution.

produce the total relic density of DM with a coupling orders of magnitude smaller than the scattering process as it is shown in section 3. In terms of the yield $Y_2 = n_{\phi_2}/S$, where S is the entropy density, the Boltzmann equation can be re-written as

$$\frac{dY_2}{dT} = -\frac{1}{HT} \langle \sigma v \rangle n_{\phi_1}^{\text{eq}} Y_2. \quad (\text{A.2})$$

The solution has an exponential behaviour,

$$Y_2 = Y_0 \exp \left[\int dT \frac{1}{HT} \langle \sigma v \rangle n_{\phi_1}^{\text{eq}} \right]. \quad (\text{A.3})$$

The thermal average of cross section times velocity, $\langle \sigma v \rangle$, can be obtained by

$$\langle \sigma v \rangle_{AB \rightarrow CD} = \frac{1}{8M_A^2 M_B^2 K_2(M_A/T) K_2(M_B/T)} \int_{(M_A+M_B)^2}^{\infty} ds \frac{\sigma_{AB \rightarrow CD}}{\sqrt{s}} p_{AB} K_1 \left(\frac{\sqrt{s}}{T} \right), \quad (\text{A.4})$$

where $M_{A,B}$ are the masses of A and B , s is the centre-of-mass energy, $p_{AB} = [s - (M_A - M_B)^2][s - (M_A + M_B)^2]$, and $K_{1,2}$ are the modified Bessel functions of the second kind. In our model, the cross section is given by

$$\sigma = \frac{|\mathcal{M}|^2}{32\pi s} \sqrt{\frac{s(s - 4M_2^2)}{(s - (M_1 + M_2)^2)(s - (M_1 - M_2)^2)}}, \quad (\text{A.5})$$

where the matrix element is given by $|\mathcal{M}|^2 = 36\mu^2$.

A.1.2 Three-body decay

In the opposite regime, $M_1 > 3M_2$, the decay channel of the WIMP DM to the FIMP DM is open. The differential decay rate is given by

$$d\Gamma = \frac{M_1 \pi}{32(2\pi)^5} |\mathcal{M}|^2 dx_3 dx_1 d\cos\theta d\phi, \quad (\text{A.6})$$

where

$$0 \leq \phi \leq 2\pi, \quad -1 \leq \cos\theta \leq 1, \quad 2\sqrt{c} \leq x_3 \leq 1+c-a-b-2\sqrt{a}\sqrt{b}, \quad x_1^- \leq x_1 \leq x_1^+,$$

$$x_1^\pm = 1+a-b+c-x_3 - \frac{1}{2}(2c-x_3) \left(1 + \frac{a-b}{1+c-x_3} \right) \pm \frac{1}{2} y_3 \sqrt{1 - 2\frac{a+b}{1+c-x_3} + \frac{(a-b)^2}{(1+c-x_3)^2}}, \quad (\text{A.7})$$

with $a = b = c = M_2^2/M_1^2$ and $y_3 = \sqrt{x_3 - 4c}$. Therefore, we obtain the decay rate as follows:

$$\Gamma = \frac{M_1}{256\pi^3} |\mathcal{M}|^2 \int_{2\sqrt{a}}^{1-3a} dx_3 \sqrt{\frac{(x_3^2 - 4a)(1 - 3a - x_3)}{1 + a - x_3}}, \quad (\text{A.8})$$

with $|\mathcal{M}|^2 = 36\mu^2$.

A.2 FIMP-SM/BSM Higgs interactions

We consider now different FIMP production channels through the interactions with the SM and BSM Higgses. Rotating into the mass eigenstates and considering the electroweak broken phase, we have the following interaction:

$$\begin{aligned} \mathcal{L} \supset & 2v|\phi_2|^2 h_2 (\lambda_{2H} \cos \theta - \lambda_{2h} \sin \theta) + 2v_{\mu\tau} |\phi_2|^2 h_1 (\lambda_{2H} \sin \theta + \lambda_{2h} \cos \theta) \\ & + |\phi_2|^2 h_1 h_2 (\lambda_{2H} \cos \theta \sin \theta - \lambda_{2h} \cos \theta \sin \theta) + \frac{1}{2} |\phi_2|^2 h_2^2 (\lambda_{2H} \cos^2 \theta - \lambda_{2h} \sin^2 \theta) \\ & + \frac{1}{2} |\phi_2|^2 h_1^2 (\lambda_{2H} \cos^2 \theta - \lambda_{2h} \sin^2 \theta) \end{aligned} \quad (\text{A.9})$$

$$= 2\lambda_a v |\phi_2|^2 h_2 + 2\lambda_b v_{\mu\tau} |\phi_2|^2 h_1 + \lambda_c |\phi_2|^2 h_1 h_2 + \frac{1}{2} \lambda_d |\phi_2|^2 h_2^2 + \frac{1}{2} \lambda_e |\phi_2|^2 h_1^2, \quad (\text{A.10})$$

where we have introduced new coupling constants as

$$\begin{aligned} \lambda_a & \equiv \lambda_{2H} \cos \theta - \lambda_{2h} \sin \theta, \\ \lambda_b & \equiv \lambda_{2H} \sin \theta + \lambda_{2h} \cos \theta, \\ \lambda_c & \equiv \lambda_{2H} \cos \theta \sin \theta - \lambda_{2h} \cos \theta \sin \theta, \\ \lambda_d & \equiv \lambda_{2H} \cos^2 \theta - \lambda_{2h} \sin^2 \theta, \\ \lambda_e & \equiv \lambda_{2H} \cos^2 \theta - \lambda_{2h} \sin^2 \theta. \end{aligned} \quad (\text{A.11})$$

A.2.1 Decay contribution

In the parameter space where $M_{h_1, h_2} > 2M_2$, the decays of the SM and the BSM Higgs fields are allowed, with the decay rates

$$\Gamma_{\text{SM}} = \frac{v_{\mu\tau}^2}{4\pi M_{h_1}} \lambda_b \sqrt{M_{h_1}^2 - 4M_2^2}, \quad \Gamma_{\text{BSM}} = \frac{v^2}{4\pi M_{h_2}} \lambda_a \sqrt{M_{h_2}^2 - 4M_2^2}. \quad (\text{A.12})$$

Solving the Boltzmann equation, we obtain

$$\begin{aligned} Y_2 & = \int dT \frac{M_{h_1}^2}{2\pi^2 HS} K_1 \left(\frac{M_{h_1}}{T} \right) \Gamma_{\text{SM}} + \int dT \frac{M_{h_2}^2}{2\pi^2 HS} K_1 \left(\frac{M_{h_2}}{T} \right) \Gamma_{\text{BSM}} \\ & = \frac{135 M_{\text{Pl}}}{1.66 \cdot 8\pi^3 g_*^S \sqrt{g_*}} \left(\frac{\Gamma_{\text{SM}}}{M_{h_1}^2} + \frac{\Gamma_{\text{BSM}}}{M_{h_2}^2} \right). \end{aligned} \quad (\text{A.13})$$

We note that, when the decay channel is kinematically open, it dominates the production.

A.2.2 Scattering contribution

When the decay is inactive, the dominant contribution to the FIMP production comes from the scattering process. The yield in this case is given by

$$\frac{dY_2}{dT} = -\frac{1}{32\pi^4 HS} \int_{4M_{h_{12}}^2}^{\infty} ds \sigma(s - 4M_{h_{12}}^2) K_1 \left(\frac{\sqrt{s}}{T} \right), \quad (\text{A.14})$$

where $M_{h_{12}}$ is M_{h_1} (M_{h_2}) if the relevant interaction is governed by the coupling λ_e (λ_d). In the $M_{h_1} \ll M_2$ limit, we get

$$Y_2 = \frac{135 \lambda_e^2 M_{\text{Pl}}}{1.66 \cdot 4096 \pi^4 g_*^S \sqrt{g_*} M_2^2}, \quad (\text{A.15})$$

while in the $M_{h_2} \ll M_2$ limit, we find

$$Y_2 = \frac{135\lambda_d^2 M_{\text{Pl}}}{1.66 \cdot 4096\pi^4 g_*^S \sqrt{g_*} M_2^2}. \quad (\text{A.16})$$

As h_1 represents the SM Higgs in our consideration, the first case, namely $M_{h_1} \ll M_2$, always holds to be the case.

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References

- [1] SUPER-KAMIOKANDE collaboration, *Evidence for oscillation of atmospheric neutrinos*, *Phys. Rev. Lett.* **81** (1998) 1562 [[hep-ex/9807003](#)] [[INSPIRE](#)].
- [2] M.C. Gonzalez-Garcia and Y. Nir, *Neutrino Masses and Mixing: Evidence and Implications*, *Rev. Mod. Phys.* **75** (2003) 345 [[hep-ph/0202058](#)] [[INSPIRE](#)].
- [3] I. Esteban, M.C. Gonzalez-Garcia, M. Maltoni, T. Schwetz and A. Zhou, *The fate of hints: updated global analysis of three-flavor neutrino oscillations*, *JHEP* **09** (2020) 178 [[arXiv:2007.14792](#)] [[INSPIRE](#)].
- [4] A. Goobar, S. Hannestad, E. Mortsell and H. Tu, *A new bound on the neutrino mass from the sdss baryon acoustic peak*, *JCAP* **06** (2006) 019 [[astro-ph/0602155](#)] [[INSPIRE](#)].
- [5] MUON $g - 2$ collaboration, *Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm*, *Phys. Rev. Lett.* **126** (2021) 141801 [[arXiv:2104.03281](#)] [[INSPIRE](#)].
- [6] F. Zwicky, *Die Rotverschiebung von extragalaktischen Nebeln*, *Helv. Phys. Acta* **6** (1933) 110 [[INSPIRE](#)].
- [7] G. Bertone and D. Hooper, *History of dark matter*, *Rev. Mod. Phys.* **90** (2018) 045002 [[arXiv:1605.04909](#)] [[INSPIRE](#)].
- [8] J.P. Ostriker and P.J.E. Peebles, *A Numerical Study of the Stability of Flattened Galaxies: or, can Cold Galaxies Survive?*, *Astrophys. J.* **186** (1973) 467 [[INSPIRE](#)].
- [9] PLANCK collaboration, *Planck 2018 results. VI. Cosmological parameters*, *Astron. Astrophys.* **641** (2020) A6 [*Erratum ibid.* **652** (2021) C4] [[arXiv:1807.06209](#)] [[INSPIRE](#)].
- [10] E. Corbelli and P. Salucci, *The Extended Rotation Curve and the Dark Matter Halo of M33*, *Mon. Not. Roy. Astron. Soc.* **311** (2000) 441 [[astro-ph/9909252](#)] [[INSPIRE](#)].
- [11] J.E. Gunn, B.W. Lee, I. Lerche, D.N. Schramm and G. Steigman, *Some Astrophysical Consequences of the Existence of a Heavy Stable Neutral Lepton*, *Astrophys. J.* **223** (1978) 1015 [[INSPIRE](#)].
- [12] P. Hut, *Limits on Masses and Number of Neutral Weakly Interacting Particles*, *Phys. Lett. B* **69** (1977) 85 [[INSPIRE](#)].
- [13] B.W. Lee and S. Weinberg, *Cosmological Lower Bound on Heavy Neutrino Masses*, *Phys. Rev. Lett.* **39** (1977) 165 [[INSPIRE](#)].
- [14] G. Bertone, D. Hooper and J. Silk, *Particle dark matter: Evidence, candidates and constraints*, *Phys. Rept.* **405** (2005) 279 [[hep-ph/0404175](#)] [[INSPIRE](#)].

- [15] XENON collaboration, *Dark Matter Search Results from a One Ton-Year Exposure of XENON1T*, *Phys. Rev. Lett.* **121** (2018) 111302 [[arXiv:1805.12562](#)] [[INSPIRE](#)].
- [16] CMS collaboration, *Phenomenological MSSM interpretation of CMS searches in pp collisions at $\sqrt{s} = 7$ and 8 TeV*, *JHEP* **10** (2016) 129 [[arXiv:1606.03577](#)] [[INSPIRE](#)].
- [17] MAGIC and FERMI-LAT collaborations, *Limits to Dark Matter Annihilation Cross-Section from a Combined Analysis of MAGIC and Fermi-LAT Observations of Dwarf Satellite Galaxies*, *JCAP* **02** (2016) 039 [[arXiv:1601.06590](#)] [[INSPIRE](#)].
- [18] G. Arcadi et al., *The waning of the WIMP? A review of models, searches, and constraints*, *Eur. Phys. J. C* **78** (2018) 203 [[arXiv:1703.07364](#)] [[INSPIRE](#)].
- [19] PANDAX-II collaboration, *Dark Matter Results from First 98.7 Days of Data from the PandaX-II Experiment*, *Phys. Rev. Lett.* **117** (2016) 121303 [[arXiv:1607.07400](#)] [[INSPIRE](#)].
- [20] LUX collaboration, *Results from a search for dark matter in the complete LUX exposure*, *Phys. Rev. Lett.* **118** (2017) 021303 [[arXiv:1608.07648](#)] [[INSPIRE](#)].
- [21] J. McDonald, *Thermally generated gauge singlet scalars as selfinteracting dark matter*, *Phys. Rev. Lett.* **88** (2002) 091304 [[hep-ph/0106249](#)] [[INSPIRE](#)].
- [22] K.-Y. Choi and L. Roszkowski, *E-WIMPs*, *AIP Conf. Proc.* **805** (2005) 30 [[hep-ph/0511003](#)] [[INSPIRE](#)].
- [23] A. Kusenko, *Sterile neutrinos, dark matter, and the pulsar velocities in models with a Higgs singlet*, *Phys. Rev. Lett.* **97** (2006) 241301 [[hep-ph/0609081](#)] [[INSPIRE](#)].
- [24] L.J. Hall, K. Jedamzik, J. March-Russell and S.M. West, *Freeze-In Production of FIMP Dark Matter*, *JHEP* **03** (2010) 080 [[arXiv:0911.1120](#)] [[INSPIRE](#)].
- [25] C. Cheung, G. Elor and L. Hall, *Gravitino Freeze-In*, *Phys. Rev. D* **84** (2011) 115021 [[arXiv:1103.4394](#)] [[INSPIRE](#)].
- [26] F. Elahi, C. Kolda and J. Unwin, *UltraViolet Freeze-in*, *JHEP* **03** (2015) 048 [[arXiv:1410.6157](#)] [[INSPIRE](#)].
- [27] G. Arcadi, L. Covi and M. Nardecchia, *Gravitino Dark Matter and low-scale Baryogenesis*, *Phys. Rev. D* **92** (2015) 115006 [[arXiv:1507.05584](#)] [[INSPIRE](#)].
- [28] N. Bernal, M. Heikinheimo, T. Tenkanen, K. Tuominen and V. Vaskonen, *The Dawn of FIMP Dark Matter: A Review of Models and Constraints*, *Int. J. Mod. Phys. A* **32** (2017) 1730023 [[arXiv:1706.07442](#)] [[INSPIRE](#)].
- [29] K. Benakli, Y. Chen, E. Dudas and Y. Mambrini, *Minimal model of gravitino dark matter*, *Phys. Rev. D* **95** (2017) 095002 [[arXiv:1701.06574](#)] [[INSPIRE](#)].
- [30] N. Bernal, M. Dutra, Y. Mambrini, K. Olive, M. Peloso and M. Pierre, *Spin-2 Portal Dark Matter*, *Phys. Rev. D* **97** (2018) 115020 [[arXiv:1803.01866](#)] [[INSPIRE](#)].
- [31] N. Bernal, F. Elahi, C. Maldonado and J. Unwin, *Ultraviolet Freeze-in and Non-Standard Cosmologies*, *JCAP* **11** (2019) 026 [[arXiv:1909.07992](#)] [[INSPIRE](#)].
- [32] L. Covi, A. Ghosh, T. Mondal and B. Mukhopadhyaya, *Models of decaying FIMP Dark Matter: potential links with the Neutrino Sector*, [arXiv:2008.12550](#) [[INSPIRE](#)].
- [33] S. Khan, *Explaining Xenon-1T signal with FIMP dark matter and neutrino mass in a $U(1)_X$ extension*, *Eur. Phys. J. C* **81** (2021) 598 [[arXiv:2007.13008](#)] [[INSPIRE](#)].
- [34] M.A.G. Garcia, Y. Mambrini, K.A. Olive and S. Verner, *Case for decaying spin-3/2 dark matter*, *Phys. Rev. D* **102** (2020) 083533 [[arXiv:2006.03325](#)] [[INSPIRE](#)].

- [35] N. Bernal, J. Rubio and H. Veermäe, *UV Freeze-in in Starobinsky Inflation*, *JCAP* **10** (2020) 021 [[arXiv:2006.02442](#)] [[INSPIRE](#)].
- [36] J. Kim and J. McDonald, *Clockwork Higgs portal model for freeze-in dark matter*, *Phys. Rev. D* **98** (2018) 023533 [[arXiv:1709.04105](#)] [[INSPIRE](#)].
- [37] J. Kim and J. McDonald, *Freeze-In Dark Matter from a sub-Higgs Mass Clockwork Sector via the Higgs Portal*, *Phys. Rev. D* **98** (2018) 123503 [[arXiv:1804.02661](#)] [[INSPIRE](#)].
- [38] A. Goudelis, K.A. Mohan and D. Sengupta, *Clockworking FIMPs*, *JHEP* **10** (2018) 014 [[arXiv:1807.06642](#)] [[INSPIRE](#)].
- [39] K. Choi and S.H. Im, *Realizing the relaxation from multiple axions and its UV completion with high scale supersymmetry*, *JHEP* **01** (2016) 149 [[arXiv:1511.00132](#)] [[INSPIRE](#)].
- [40] D.E. Kaplan and R. Rattazzi, *Large field excursions and approximate discrete symmetries from a clockwork axion*, *Phys. Rev. D* **93** (2016) 085007 [[arXiv:1511.01827](#)] [[INSPIRE](#)].
- [41] G.F. Giudice and M. McCullough, *A Clockwork Theory*, *JHEP* **02** (2017) 036 [[arXiv:1610.07962](#)] [[INSPIRE](#)].
- [42] K.M. Zurek, *Multi-Component Dark Matter*, *Phys. Rev. D* **79** (2009) 115002 [[arXiv:0811.4429](#)] [[INSPIRE](#)].
- [43] S. Profumo, K. Sigurdson and L. Ubaldi, *Can we discover multi-component WIMP dark matter?*, *JCAP* **12** (2009) 016 [[arXiv:0907.4374](#)] [[INSPIRE](#)].
- [44] D. Feldman, Z. Liu, P. Nath and G. Peim, *Multicomponent Dark Matter in Supersymmetric Hidden Sector Extensions*, *Phys. Rev. D* **81** (2010) 095017 [[arXiv:1004.0649](#)] [[INSPIRE](#)].
- [45] L. Bian, R. Ding and B. Zhu, *Two Component Higgs-Portal Dark Matter*, *Phys. Lett. B* **728** (2014) 105 [[arXiv:1308.3851](#)] [[INSPIRE](#)].
- [46] A. Biswas, D. Majumdar, A. Sil and P. Bhattacharjee, *Two Component Dark Matter: A Possible Explanation of 130 GeV γ -Ray Line from the Galactic Centre*, *JCAP* **12** (2013) 049 [[arXiv:1301.3668](#)] [[INSPIRE](#)].
- [47] S. Bhattacharya, A. Drozd, B. Grzadkowski and J. Wudka, *Two-Component Dark Matter*, *JHEP* **10** (2013) 158 [[arXiv:1309.2986](#)] [[INSPIRE](#)].
- [48] L. Bian, T. Li, J. Shu and X.-C. Wang, *Two component dark matter with multi-Higgs portals*, *JHEP* **03** (2015) 126 [[arXiv:1412.5443](#)] [[INSPIRE](#)].
- [49] G. Bélanger, F. Boudjema, A. Pukhov and A. Semenov, *MicrOMEGAs4.1: two dark matter candidates*, *Comput. Phys. Commun.* **192** (2015) 322 [[arXiv:1407.6129](#)] [[INSPIRE](#)].
- [50] S. Esch, M. Klasen and C.E. Yaguna, *A minimal model for two-component dark matter*, *JHEP* **09** (2014) 108 [[arXiv:1406.0617](#)] [[INSPIRE](#)].
- [51] S. Bhattacharya, P. Poulose and P. Ghosh, *Multipartite Interacting Scalar Dark Matter in the light of updated LUX data*, *JCAP* **04** (2017) 043 [[arXiv:1607.08461](#)] [[INSPIRE](#)].
- [52] G. Arcadi, C. Gross, O. Lebedev, Y. Mambrini, S. Pokorski and T. Toma, *Multicomponent Dark Matter from Gauge Symmetry*, *JHEP* **12** (2016) 081 [[arXiv:1611.00365](#)] [[INSPIRE](#)].
- [53] A. Dutta Banik, M. Pandey, D. Majumdar and A. Biswas, *Two component WIMP-FIMP dark matter model with singlet fermion, scalar and pseudo scalar*, *Eur. Phys. J. C* **77** (2017) 657 [[arXiv:1612.08621](#)] [[INSPIRE](#)].

- [54] S. Bhattacharya, P. Ghosh, T.N. Maity and T.S. Ray, *Mitigating Direct Detection Bounds in Non-minimal Higgs Portal Scalar Dark Matter Models*, *JHEP* **10** (2017) 088 [[arXiv:1706.04699](#)] [[INSPIRE](#)].
- [55] A. Ahmed, M. Duch, B. Grzadkowski and M. Igllicki, *Multi-Component Dark Matter: the vector and fermion case*, *Eur. Phys. J. C* **78** (2018) 905 [[arXiv:1710.01853](#)] [[INSPIRE](#)].
- [56] N. Bernal, D. Restrepo, C. Yaguna and O. Zapata, *Two-component dark matter and a massless neutrino in a new $B - L$ model*, *Phys. Rev. D* **99** (2019) 015038 [[arXiv:1808.03352](#)] [[INSPIRE](#)].
- [57] S. Bhattacharya, P. Ghosh and N. Sahu, *Multipartite Dark Matter with Scalars, Fermions and signatures at LHC*, *JHEP* **02** (2019) 059 [[arXiv:1809.07474](#)] [[INSPIRE](#)].
- [58] F. Elahi and S. Khatibi, *Multi-Component Dark Matter in a Non-Abelian Dark Sector*, *Phys. Rev. D* **100** (2019) 015019 [[arXiv:1902.04384](#)] [[INSPIRE](#)].
- [59] D. Borah, R. Roshan and A. Sil, *Minimal two-component scalar doublet dark matter with radiative neutrino mass*, *Phys. Rev. D* **100** (2019) 055027 [[arXiv:1904.04837](#)] [[INSPIRE](#)].
- [60] S. Bhattacharya, P. Ghosh, A.K. Saha and A. Sil, *Two component dark matter with inert Higgs doublet: neutrino mass, high scale validity and collider searches*, *JHEP* **03** (2020) 090 [[arXiv:1905.12583](#)] [[INSPIRE](#)].
- [61] C.E. Yaguna and O. Zapata, *Multi-component scalar dark matter from a Z_N symmetry: a systematic analysis*, *JHEP* **03** (2020) 109 [[arXiv:1911.05515](#)] [[INSPIRE](#)].
- [62] W. Abdallah, S. Choubey and S. Khan, *FIMP dark matter candidate(s) in a $B - L$ model with inverse seesaw mechanism*, *JHEP* **06** (2019) 095 [[arXiv:1904.10015](#)] [[INSPIRE](#)].
- [63] G. Bélanger, A. Pukhov, C.E. Yaguna and O. Zapata, *The Z_5 model of two-component dark matter*, *JHEP* **09** (2020) 030 [[arXiv:2006.14922](#)] [[INSPIRE](#)].
- [64] S.-M. Choi, J. Kim, P. Ko and J. Li, *A multi-component SIMP model with $U(1)_X \rightarrow Z_2 \times Z_3$* , *JHEP* **09** (2021) 028 [[arXiv:2103.05956](#)] [[INSPIRE](#)].
- [65] B. Díaz Sáez, K. Möhling and D. Stöckinger, *Two real scalar WIMP model in the assisted freeze-out scenario*, *JCAP* **10** (2021) 027 [[arXiv:2103.17064](#)] [[INSPIRE](#)].
- [66] B. Díaz Sáez, P. Escalona, S. Norero and A.R. Zerwekh, *Fermion singlet dark matter in a pseudoscalar dark matter portal*, *JHEP* **10** (2021) 233 [[arXiv:2105.04255](#)] [[INSPIRE](#)].
- [67] B.D. Sáez and K. Ghorbani, *Singlet scalars as dark matter and the muon $(g - 2)$ anomaly*, *Phys. Lett. B* **823** (2021) 136750 [[arXiv:2107.08945](#)] [[INSPIRE](#)].
- [68] G. Bélanger, A. Mjallal and A. Pukhov, *Two dark matter candidates: The case of inert doublet and singlet scalars*, *Phys. Rev. D* **105** (2022) 035018 [[arXiv:2108.08061](#)] [[INSPIRE](#)].
- [69] S.-Y. Ho, P. Ko and C.-T. Lu, *Reshuffled SIMP Dark Matter*, [arXiv:2107.04375](#) [[INSPIRE](#)].
- [70] S.-Y. Ho, P. Ko and C.-T. Lu, *Scalar and fermion two-component SIMP dark matter with an accidental Z_4 symmetry*, *JHEP* **03** (2022) 005 [[arXiv:2201.06856](#)] [[INSPIRE](#)].
- [71] S. Bhattacharya, P. Ghosh, J. Lahiri and B. Mukhopadhyaya, *Distinguishing two dark matter component particles at e^+e^- colliders*, [arXiv:2202.12097](#) [[INSPIRE](#)].
- [72] A. Das, S. Gola, S. Mandal and N. Sinha, *Two-component scalar and fermionic dark matter candidates in a generic $U(1)_X$ model*, *Phys. Lett. B* **829** (2022) 137117 [[arXiv:2202.01443](#)] [[INSPIRE](#)].

- [73] P. Minkowski, $\mu \rightarrow e\gamma$ at a Rate of One Out of 10^9 Muon Decays?, *Phys. Lett. B* **67** (1977) 421 [INSPIRE].
- [74] M. Gell-Mann, P. Ramond and R. Slansky, *Complex Spinors and Unified Theories*, *Conf. Proc. C* **790927** (1979) 315 [arXiv:1306.4669] [INSPIRE].
- [75] W. Abdallah, A. Awad, S. Khalil and H. Okada, *Muon Anomalous Magnetic Moment and $\mu \rightarrow e\gamma$ in $B - L$ Model with Inverse Seesaw*, *Eur. Phys. J. C* **72** (2012) 2108 [arXiv:1105.1047] [INSPIRE].
- [76] S. Khalil and C.S. Un, *Muon Anomalous Magnetic Moment in SUSY $B-L$ Model with Inverse Seesaw*, *Phys. Lett. B* **763** (2016) 164 [arXiv:1509.05391] [INSPIRE].
- [77] M. Lindner, M. Platscher and F.S. Queiroz, *A Call for New Physics: The Muon Anomalous Magnetic Moment and Lepton Flavor Violation*, *Phys. Rept.* **731** (2018) 1 [arXiv:1610.06587] [INSPIRE].
- [78] E.J. Chun and J. Kim, *Leptonic Precision Test of Leptophilic Two-Higgs-Doublet Model*, *JHEP* **07** (2016) 110 [arXiv:1605.06298] [INSPIRE].
- [79] L. Calibbi, R. Ziegler and J. Zupan, *Minimal models for dark matter and the muon $g - 2$ anomaly*, *JHEP* **07** (2018) 046 [arXiv:1804.00009] [INSPIRE].
- [80] P. Arnan, A. Crivellin, M. Fedele and F. Mescia, *Generic Loop Effects of New Scalars and Fermions in $b \rightarrow s\ell^+\ell^-$, $(g - 2)_\mu$ and a Vector-like 4th Generation*, *JHEP* **06** (2019) 118 [arXiv:1904.05890] [INSPIRE].
- [81] L. Calibbi, M.L. López-Ibáñez, A. Melis and O. Vives, *Muon and electron $g - 2$ and lepton masses in flavor models*, *JHEP* **06** (2020) 087 [arXiv:2003.06633] [INSPIRE].
- [82] P. Athron, C. Balázs, D.H.J. Jacob, W. Kotlarski, D. Stöckinger and H. Stöckinger-Kim, *New physics explanations of a_μ in light of the FNAL muon $g - 2$ measurement*, *JHEP* **09** (2021) 080 [arXiv:2104.03691] [INSPIRE].
- [83] T. Bringmann, P.F. Depta, M. Hufnagel, J.T. Ruderman and K. Schmidt-Hoberg, *Dark Matter from Exponential Growth*, *Phys. Rev. Lett.* **127** (2021) 191802 [arXiv:2103.16572] [INSPIRE].
- [84] A. Hryczuk and M. Laletin, *Dark matter freeze-in from semi-production*, *JHEP* **06** (2021) 026 [arXiv:2104.05684] [INSPIRE].
- [85] F. D’Eramo and J. Thaler, *Semi-annihilation of Dark Matter*, *JHEP* **06** (2010) 109 [arXiv:1003.5912] [INSPIRE].
- [86] M. Kamionkowski, A. Kosowsky and M.S. Turner, *Gravitational radiation from first order phase transitions*, *Phys. Rev. D* **49** (1994) 2837 [astro-ph/9310044] [INSPIRE].
- [87] J. Baker et al., *The Laser Interferometer Space Antenna: Unveiling the Millihertz Gravitational Wave Sky*, arXiv:1907.06482 [INSPIRE].
- [88] N. Seto, S. Kawamura and T. Nakamura, *Possibility of direct measurement of the acceleration of the universe using 0.1-Hz band laser interferometer gravitational wave antenna in space*, *Phys. Rev. Lett.* **87** (2001) 221103 [astro-ph/0108011] [INSPIRE].
- [89] S. Kawamura et al., *The Japanese space gravitational wave antenna DECIGO*, *Class. Quant. Grav.* **23** (2006) S125 [INSPIRE].
- [90] S. Sato et al., *The status of DECIGO*, *J. Phys. Conf. Ser.* **840** (2017) 012010 [INSPIRE].

- [91] S. Isoyama, H. Nakano and T. Nakamura, *Multiband Gravitational-Wave Astronomy: Observing binary inspirals with a decihertz detector, B-DECIGO*, *PTEP* **2018** (2018) 073E01 [[arXiv:1802.06977](#)] [[INSPIRE](#)].
- [92] S. Kawamura et al., *Current status of space gravitational wave antenna DECIGO and B-DECIGO*, *PTEP* **2021** (2021) 05A105 [[arXiv:2006.13545](#)] [[INSPIRE](#)].
- [93] V. Corbin and N.J. Cornish, *Detecting the cosmic gravitational wave background with the big bang observer*, *Class. Quant. Grav.* **23** (2006) 2435 [[gr-qc/0512039](#)] [[INSPIRE](#)].
- [94] J. Crowder and N.J. Cornish, *Beyond LISA: Exploring future gravitational wave missions*, *Phys. Rev. D* **72** (2005) 083005 [[gr-qc/0506015](#)] [[INSPIRE](#)].
- [95] G.M. Harry, P. Fritschel, D.A. Shaddock, W. Folkner and E.S. Phinney, *Laser interferometry for the big bang observer*, *Class. Quant. Grav.* **23** (2006) 4887 [Erratum *ibid.* **23** (2006) 7361] [[INSPIRE](#)].
- [96] C. Grojean and G. Servant, *Gravitational Waves from Phase Transitions at the Electroweak Scale and Beyond*, *Phys. Rev. D* **75** (2007) 043507 [[hep-ph/0607107](#)] [[INSPIRE](#)].
- [97] S.J. Huber and T. Konstandin, *Gravitational Wave Production by Collisions: More Bubbles*, *JCAP* **09** (2008) 022 [[arXiv:0806.1828](#)] [[INSPIRE](#)].
- [98] J.R. Espinosa, T. Konstandin, J.M. No and M. Quirós, *Some Cosmological Implications of Hidden Sectors*, *Phys. Rev. D* **78** (2008) 123528 [[arXiv:0809.3215](#)] [[INSPIRE](#)].
- [99] C. Caprini et al., *Science with the space-based interferometer eLISA. II: Gravitational waves from cosmological phase transitions*, *JCAP* **04** (2016) 001 [[arXiv:1512.06239](#)] [[INSPIRE](#)].
- [100] M. Artymowski, M. Lewicki and J.D. Wells, *Gravitational wave and collider implications of electroweak baryogenesis aided by non-standard cosmology*, *JHEP* **03** (2017) 066 [[arXiv:1609.07143](#)] [[INSPIRE](#)].
- [101] I. Baldes, *Gravitational waves from the asymmetric-dark-matter generating phase transition*, *JCAP* **05** (2017) 028 [[arXiv:1702.02117](#)] [[INSPIRE](#)].
- [102] A. Beniwal, M. Lewicki, M. White and A.G. Williams, *Gravitational waves and electroweak baryogenesis in a global study of the extended scalar singlet model*, *JHEP* **02** (2019) 183 [[arXiv:1810.02380](#)] [[INSPIRE](#)].
- [103] K. Hashino, M. Kakizaki, S. Kanemura, P. Ko and T. Matsui, *Gravitational waves from first order electroweak phase transition in models with the $U(1)_X$ gauge symmetry*, *JHEP* **06** (2018) 088 [[arXiv:1802.02947](#)] [[INSPIRE](#)].
- [104] C. Caprini and D.G. Figueroa, *Cosmological Backgrounds of Gravitational Waves*, *Class. Quant. Grav.* **35** (2018) 163001 [[arXiv:1801.04268](#)] [[INSPIRE](#)].
- [105] L. Bian and Y.-L. Tang, *Thermally modified sterile neutrino portal dark matter and gravitational waves from phase transition: The Freeze-in case*, *JHEP* **12** (2018) 006 [[arXiv:1810.03172](#)] [[INSPIRE](#)].
- [106] L. Bian and X. Liu, *Two-step strongly first-order electroweak phase transition modified FIMP dark matter, gravitational wave signals, and the neutrino mass*, *Phys. Rev. D* **99** (2019) 055003 [[arXiv:1811.03279](#)] [[INSPIRE](#)].
- [107] L. Bian, W. Cheng, H.-K. Guo and Y. Zhang, *Cosmological implications of a $B - L$ charged hidden scalar: leptogenesis and gravitational waves*, *Chin. Phys. C* **45** (2021) 113104 [[arXiv:1907.13589](#)] [[INSPIRE](#)].

- [108] L. Bian, Y. Wu and K.-P. Xie, *Electroweak phase transition with composite Higgs models: calculability, gravitational waves and collider searches*, *JHEP* **12** (2019) 028 [[arXiv:1909.02014](#)] [[INSPIRE](#)].
- [109] C. Caprini et al., *Detecting gravitational waves from cosmological phase transitions with LISA: an update*, *JCAP* **03** (2020) 024 [[arXiv:1910.13125](#)] [[INSPIRE](#)].
- [110] Y. Di, J. Wang, R. Zhou, L. Bian, R.-G. Cai and J. Liu, *Magnetic Field and Gravitational Waves from the First-Order Phase Transition*, *Phys. Rev. Lett.* **126** (2021) 251102 [[arXiv:2012.15625](#)] [[INSPIRE](#)].
- [111] R. Zhou, L. Bian and J. Shu, *Probing new physics for $(g - 2)_\mu$ and gravitational waves*, [arXiv:2104.03519](#) [[INSPIRE](#)].
- [112] A. Mohamadnejad, *Electroweak phase transition and gravitational waves in a two-component dark matter model*, *JHEP* **03** (2022) 188 [[arXiv:2111.04342](#)] [[INSPIRE](#)].
- [113] L. Bian, Y.-L. Tang and R. Zhou, *FIMP dark matter mediated by massive gauge boson around the phase transition period and the gravitational waves production*, [arXiv:2111.10608](#) [[INSPIRE](#)].
- [114] B. Holdom, *Two U(1)'s and Epsilon Charge Shifts*, *Phys. Lett. B* **166** (1986) 196 [[INSPIRE](#)].
- [115] K.S. Babu, C.F. Kolda and J. March-Russell, *Implications of generalized Z - Z' mixing*, *Phys. Rev. D* **57** (1998) 6788 [[hep-ph/9710441](#)] [[INSPIRE](#)].
- [116] W. Altmannshofer, S. Gori, J. Martín-Albo, A. Sousa and M. Wallbank, *Neutrino Tridents at DUNE*, *Phys. Rev. D* **100** (2019) 115029 [[arXiv:1902.06765](#)] [[INSPIRE](#)].
- [117] A. Biswas and S. Khan, *$(g - 2)_{e, \mu}$ and strongly interacting dark matter with collider implications*, [arXiv:2112.08393](#) [[INSPIRE](#)].
- [118] R. Harnik, J. Kopp and P.A.N. Machado, *Exploring ν Signals in Dark Matter Detectors*, *JCAP* **07** (2012) 026 [[arXiv:1202.6073](#)] [[INSPIRE](#)].
- [119] BOREXINO collaboration, *First Simultaneous Precision Spectroscopy of pp , ${}^7\text{Be}$, and pep Solar Neutrinos with Borexino Phase-II*, *Phys. Rev. D* **100** (2019) 082004 [[arXiv:1707.09279](#)] [[INSPIRE](#)].
- [120] M. Bauer, P. Foldenauer and J. Jaeckel, *Hunting All the Hidden Photons*, *JHEP* **07** (2018) 094 [[arXiv:1803.05466](#)] [[INSPIRE](#)].
- [121] A. Biswas, S. Choubey and S. Khan, *Neutrino Mass, Dark Matter and Anomalous Magnetic Moment of Muon in a $U(1)_{L_\mu - L_\tau}$ Model*, *JHEP* **09** (2016) 147 [[arXiv:1608.04194](#)] [[INSPIRE](#)].
- [122] A. Biswas, S. Choubey and S. Khan, *FIMP and Muon $(g - 2)$ in a $U(1)_{L_\mu - L_\tau}$ Model*, *JHEP* **02** (2017) 123 [[arXiv:1612.03067](#)] [[INSPIRE](#)].
- [123] CMS collaboration, *Observation of Z Decays to Four Leptons with the CMS Detector at the LHC*, *JHEP* **12** (2012) 034 [[arXiv:1210.3844](#)] [[INSPIRE](#)].
- [124] ATLAS collaboration, *Measurements of Four-Lepton Production at the Z Resonance in pp Collisions at $\sqrt{s} = 7$ and 8 TeV with ATLAS*, *Phys. Rev. Lett.* **112** (2014) 231806 [[arXiv:1403.5657](#)] [[INSPIRE](#)].
- [125] CMS collaboration, *Search for an $L_\mu - L_\tau$ gauge boson using $Z \rightarrow 4\mu$ events in proton-proton collisions at $\sqrt{s} = 13$ TeV*, *Phys. Lett. B* **792** (2019) 345 [[arXiv:1808.03684](#)] [[INSPIRE](#)].

- [126] CHARM-II collaboration, *First observation of neutrino trident production*, *Phys. Lett. B* **245** (1990) 271 [[INSPIRE](#)].
- [127] CCFR collaboration, *Neutrino tridents and WZ interference*, *Phys. Rev. Lett.* **66** (1991) 3117 [[INSPIRE](#)].
- [128] W. Altmannshofer, S. Gori, M. Pospelov and I. Yavin, *Neutrino Trident Production: A Powerful Probe of New Physics with Neutrino Beams*, *Phys. Rev. Lett.* **113** (2014) 091801 [[arXiv:1406.2332](#)] [[INSPIRE](#)].
- [129] E.J. Chun, A. Das, J. Kim and J. Kim, *Searching for flavored gauge bosons*, *JHEP* **02** (2019) 093 [*Erratum ibid.* **07** (2019) 024] [[arXiv:1811.04320](#)] [[INSPIRE](#)].
- [130] S.N. Gninenko and N.V. Krasnikov, *The Muon anomalous magnetic moment and a new light gauge boson*, *Phys. Lett. B* **513** (2001) 119 [[hep-ph/0102222](#)] [[INSPIRE](#)].
- [131] S. Baek, N.G. Deshpande, X.G. He and P. Ko, *Muon anomalous $g-2$ and gauged $L_\mu - L_\tau$ models*, *Phys. Rev. D* **64** (2001) 055006 [[hep-ph/0104141](#)] [[INSPIRE](#)].
- [132] A. Alloul, N.D. Christensen, C. Degrande, C. Duhr and B. Fuks, *FeynRules 2.0 — A complete toolbox for tree-level phenomenology*, *Comput. Phys. Commun.* **185** (2014) 2250 [[arXiv:1310.1921](#)] [[INSPIRE](#)].
- [133] A. Belyaev, N.D. Christensen and A. Pukhov, *CalcHEP 3.4 for collider physics within and beyond the Standard Model*, *Comput. Phys. Commun.* **184** (2013) 1729 [[arXiv:1207.6082](#)] [[INSPIRE](#)].
- [134] G. Bélanger, F. Boudjema, A. Goudelis, A. Pukhov and B. Zaldivar, *MicrOMEGAs5.0: Freeze-in*, *Comput. Phys. Commun.* **231** (2018) 173 [[arXiv:1801.03509](#)] [[INSPIRE](#)].
- [135] M. Kawasaki, K. Kohri, T. Moroi and Y. Takaesu, *Revisiting Big-Bang Nucleosynthesis Constraints on Long-Lived Decaying Particles*, *Phys. Rev. D* **97** (2018) 023502 [[arXiv:1709.01211](#)] [[INSPIRE](#)].
- [136] J.L. Feng, A. Rajaraman and F. Takayama, *Superweakly interacting massive particles*, *Phys. Rev. Lett.* **91** (2003) 011302 [[hep-ph/0302215](#)] [[INSPIRE](#)].
- [137] W. Rodejohann and C.E. Yaguna, *Scalar dark matter in the $B-L$ model*, *JCAP* **12** (2015) 032 [[arXiv:1509.04036](#)] [[INSPIRE](#)].
- [138] A. Biswas, S. Choubey and S. Khan, *Galactic gamma ray excess and dark matter phenomenology in a $U(1)_{B-L}$ model*, *JHEP* **08** (2016) 114 [[arXiv:1604.06566](#)] [[INSPIRE](#)].
- [139] A. Reinert and M.W. Winkler, *A Precision Search for WIMPs with Charged Cosmic Rays*, *JCAP* **01** (2018) 055 [[arXiv:1712.00002](#)] [[INSPIRE](#)].
- [140] DARWIN collaboration, *DARWIN: towards the ultimate dark matter detector*, *JCAP* **11** (2016) 017 [[arXiv:1606.07001](#)] [[INSPIRE](#)].
- [141] PANDAX collaboration, *Dark matter direct search sensitivity of the PandaX-4T experiment*, *Sci. China Phys. Mech. Astron.* **62** (2019) 31011 [[arXiv:1806.02229](#)] [[INSPIRE](#)].
- [142] K. Kajantie, M. Laine, K. Rummukainen and M.E. Shaposhnikov, *Is there a hot electroweak phase transition at $m_H \gtrsim m_W$?*, *Phys. Rev. Lett.* **77** (1996) 2887 [[hep-ph/9605288](#)] [[INSPIRE](#)].
- [143] M.B. Hindmarsh, M. Lüben, J. Lumma and M. Pauly, *Phase transitions in the early universe*, *SciPost Phys. Lect. Notes* **24** (2021) 1 [[arXiv:2008.09136](#)] [[INSPIRE](#)].

- [144] P.J. Steinhardt, *Relativistic Detonation Waves and Bubble Growth in False Vacuum Decay*, *Phys. Rev. D* **25** (1982) 2074 [INSPIRE].
- [145] J. Ellis, M. Lewicki and J.M. No, *On the Maximal Strength of a First-Order Electroweak Phase Transition and its Gravitational Wave Signal*, *JCAP* **04** (2019) 003 [arXiv:1809.08242] [INSPIRE].
- [146] J. Ellis, M. Lewicki, J.M. No and V. Vaskonen, *Gravitational wave energy budget in strongly supercooled phase transitions*, *JCAP* **06** (2019) 024 [arXiv:1903.09642] [INSPIRE].
- [147] J. Ellis, M. Lewicki and J.M. No, *Gravitational waves from first-order cosmological phase transitions: lifetime of the sound wave source*, *JCAP* **07** (2020) 050 [arXiv:2003.07360] [INSPIRE].
- [148] H.-K. Guo, K. Sinha, D. Vagie and G. White, *Phase Transitions in an Expanding Universe: Stochastic Gravitational Waves in Standard and Non-Standard Histories*, *JCAP* **01** (2021) 001 [arXiv:2007.08537] [INSPIRE].
- [149] N.K. Nielsen, *On the Gauge Dependence of Spontaneous Symmetry Breaking in Gauge Theories*, *Nucl. Phys. B* **101** (1975) 173 [INSPIRE].
- [150] R. Fukuda and T. Kugo, *Gauge Invariance in the Effective Action and Potential*, *Phys. Rev. D* **13** (1976) 3469 [INSPIRE].
- [151] H.H. Patel and M.J. Ramsey-Musolf, *Baryon Washout, Electroweak Phase Transition, and Perturbation Theory*, *JHEP* **07** (2011) 029 [arXiv:1101.4665] [INSPIRE].
- [152] C.-W. Chiang and E. Senaha, *On gauge dependence of gravitational waves from a first-order phase transition in classical scale-invariant $U(1)'$ models*, *Phys. Lett. B* **774** (2017) 489 [arXiv:1707.06765] [INSPIRE].
- [153] D. Croon, O. Gould, P. Schicho, T.V.I. Tenkanen and G. White, *Theoretical uncertainties for cosmological first-order phase transitions*, *JHEP* **04** (2021) 055 [arXiv:2009.10080] [INSPIRE].
- [154] S.R. Coleman and E.J. Weinberg, *Radiative Corrections as the Origin of Spontaneous Symmetry Breaking*, *Phys. Rev. D* **7** (1973) 1888 [INSPIRE].
- [155] L. Dolan and R. Jackiw, *Symmetry Behavior at Finite Temperature*, *Phys. Rev. D* **9** (1974) 3320 [INSPIRE].
- [156] M.E. Carrington, *The Effective potential at finite temperature in the Standard Model*, *Phys. Rev. D* **45** (1992) 2933 [INSPIRE].
- [157] C.L. Wainwright, *CosmoTransitions: Computing Cosmological Phase Transition Temperatures and Bubble Profiles with Multiple Fields*, *Comput. Phys. Commun.* **183** (2012) 2006 [arXiv:1109.4189] [INSPIRE].
- [158] ATLAS and CMS collaborations, *Measurements of the Higgs boson production and decay rates and constraints on its couplings from a combined ATLAS and CMS analysis of the LHC pp collision data at $\sqrt{s} = 7$ and 8 TeV*, *JHEP* **08** (2016) 045 [arXiv:1606.02266] [INSPIRE].
- [159] K. Schmitz, *New Sensitivity Curves for Gravitational-Wave Signals from Cosmological Phase Transitions*, *JHEP* **01** (2021) 097 [arXiv:2002.04615] [INSPIRE].
- [160] A. Ringwald, K. Saikawa and C. Tamarit, *Primordial gravitational waves in a minimal model of particle physics and cosmology*, *JCAP* **02** (2021) 046 [arXiv:2009.02050] [INSPIRE].
- [161] S.L. Larson, W.A. Hiscock and R.W. Hellings, *Sensitivity curves for spaceborne gravitational wave interferometers*, *Phys. Rev. D* **62** (2000) 062001 [gr-qc/9909080] [INSPIRE].

- [162] H. Kudoh, A. Taruya, T. Hiramatsu and Y. Himemoto, *Detecting a gravitational-wave background with next-generation space interferometers*, *Phys. Rev. D* **73** (2006) 064006 [[gr-qc/0511145](#)] [[INSPIRE](#)].
- [163] K. Yagi and N. Seto, *Detector configuration of DECIGO/BBO and identification of cosmological neutron-star binaries*, *Phys. Rev. D* **83** (2011) 044011 [Erratum *ibid.* **95** (2017) 109901] [[arXiv:1101.3940](#)] [[INSPIRE](#)].
- [164] E. Thrane and J.D. Romano, *Sensitivity curves for searches for gravitational-wave backgrounds*, *Phys. Rev. D* **88** (2013) 124032 [[arXiv:1310.5300](#)] [[INSPIRE](#)].
- [165] C.J. Moore, R.H. Cole and C.P.L. Berry, *Gravitational-wave sensitivity curves*, *Class. Quant. Grav.* **32** (2015) 015014 [[arXiv:1408.0740](#)] [[INSPIRE](#)].
- [166] S. Kuroyanagi, K. Nakayama and J. Yokoyama, *Prospects of determination of reheating temperature after inflation by DECIGO*, *PTEP* **2015** (2015) 013E02 [[arXiv:1410.6618](#)] [[INSPIRE](#)].
- [167] K. Saikawa and S. Shirai, *Primordial gravitational waves, precisely: The role of thermodynamics in the Standard Model*, *JCAP* **05** (2018) 035 [[arXiv:1803.01038](#)] [[INSPIRE](#)].
- [168] T. Robson, N.J. Cornish and C. Liu, *The construction and use of LISA sensitivity curves*, *Class. Quant. Grav.* **36** (2019) 105011 [[arXiv:1803.01944](#)] [[INSPIRE](#)].
- [169] Y. Du, F. Huang, H.-L. Li, Y.-Z. Li and J.-H. Yu, *Revisiting dark matter freeze-in and freeze-out through phase-space distribution*, *JCAP* **04** (2022) 012 [[arXiv:2111.01267](#)] [[INSPIRE](#)].