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Marginal deformations of a class of AdS_3 $\mathcal{N} = (0, 4)$ holographic backgrounds

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ABSTRACT: We discuss marginal deformations of warped $\text{AdS}_3 \times \text{S}^2$ solutions preserving small $\mathcal{N} = (0, 4)$ supersymmetry in massive IIA and eleven-dimensional supergravity and obtain a whole family of new solutions. We characterise these new backgrounds by studying some observables like the quantised charges, associated Hannany-Witten brane set-ups and the holographic central charge, the latter is shown to be invariant under the deformation. The study of the preservation of supersymmetry shows that the new backgrounds support a dynamical $\text{SU}(2)$ structure on the internal five-dimensional space.

KEYWORDS: AdS-CFT Correspondence, D-branes, Supergravity Models

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Contents

1	Introduction and summary	1
2	The seed AdS_3 $\mathcal{N} = (0, 4)$ holographic backgrounds	3
3	The marginally deformed backgrounds	4
3.1	Quantised charges and brane set-ups	6
4	The deformation in eleven dimensions	8
5	Holographic central charge	10
6	Comments on supersymmetry and G-structure of the solutions	11
7	Conclusions	14
A	Supersymmetry conventions	15

1 Introduction and summary

Supersymmetric solutions with AdS_{p+1} factors in type II and eleven-dimensional supergravities play a prominent role in the context of the AdS/CFT correspondence since they provide a holographic description of p-dimensional superconformal field theories (SCFTs) at strong coupling [1]. The discovery of this holographic duality has ever since triggered a number of efforts to construct and classify AdS vacua for any dimension allowed and preserving various amounts of (super)symmetries that have been used to study and characterise SCFTs.

More recently, the case of AdS_3 backgrounds has gained a lot of attention. There are several motivations for this. For instance, the near horizon geometries of five-dimensional extremal black holes have AdS_3 factors. Using SCFT_2 data it is then possible to understand microscopic features of black holes, like their entropy by computing the central charge of the SCFT_2 [2], among other aspects (see for instance [3–7]). On the other hand, two-dimensional SCFTs are special on their own since they can, in certain cases, be fully solvable due to the structure of the superconformal algebra. It is therefore interesting to explore deeply each side of this dual pair in order to shed some light on new phenomena via holography. For a sample of works regarding AdS_3 supersymmetric backgrounds in ten and eleven-dimensional supergravity preserving different amounts of supersymmetry and their holographic applications see [8–41].

Moreover, on the geometrical side, attempts to constructing and classifying supersymmetric AdS_3 solutions have been mostly focused on the G-structure formalism [42] for

which one extracts geometric constraints for the fields of the solutions according to the number of (super)symmetries and geometrical structures, etc, we impose in the internal space. It has also been considered back-reacting D-brane arrangements which are known to produce AdS solutions in the near horizon limit [34, 36], among others. However, these efforts have been non-exhaustive due to the many choices we have on the number of supersymmetries, and superconformal algebras, supported by the solutions constraining the internal space submanifolds. Thus the approach has been focussed on searching and classifying all supergravity solutions preserving given amounts of supersymmetry, choices of internal structures, etc. This program has allowed to expand significantly our knowledge of new string backgrounds which may have very interesting applications in the context of holography. In this vein, another possibility to explore the landscape of AdS₃ vacua is to consider AdS-preserving deformations of well-known supergravity solutions. Depending on the details of the deformation these solutions may preserve supersymmetry whilst changing the structure of the internal space, and in some cases escape from presently known classifications of supergravity solutions.

In this work we will use TsT transformations [43] and the analog to eleven dimensions [44] in order to generate a larger class of warped AdS₃ supersymmetric solutions. The seed backgrounds we will consider are a subclass of the solutions constructed in [22] which are solutions of massive IIA supergravity of the warped form AdS₃ × S² × CY₂ foliated over an interval, the two-sphere realising geometrically the SU(2)_R R-charge of the solution. They are given in terms of three linear functions, preserve small $\mathcal{N} = (0, 4)$ supersymmetry and an SU(2) structure in the internal five-dimensional space, analogously, for these solutions, an SU(3) structure in the seven-dimensional space transverse to AdS₃. The above solutions appear in the near horizon limit of D₂-D₄-NS5-D₆-D₈ brane arrangements. D₂ and D₆ branes are colour branes and are suspended between the NS5 branes whilst D₄ and D₈ correspond to localised sources and provide flavour groups attached to the gauge nodes which leave the dual quiver CFT anomaly free [23, 24]. For vanishing Romans mass, the uplift to eleven dimensions of the above solution gives rise to a class of AdS₃ × S³/Z_k × CY₂ foliated over an interval, which preserve the same amount of supersymmetry and internal structure group [31]. The brane configuration for this solution involves M2 branes and KK monopoles suspended between M5' branes as well as extra flavour M5 branes.

Given the internal symmetries of the seed solutions above, we have two choices which produce inequivalent backgrounds after TsT transformations. Namely, if we consider or not the azimuthal direction inside the S² for the process. In the latter case we are left with solutions for which supersymmetry is fully preserved. Of course more generic supersymmetric solutions can be generated by a sequence of TsT's not involving the U(1) inside the S², but we will explore this more generic case in the future. For the ten-dimensional solutions, we study the brane configurations that we propose generate our solutions in the near-horizon limit. Holographically, these new backgrounds are dual to marginal deformations of the seed (undeformed) SCFT, both theories having the same central charge in the holographic limit. The latter can be understood since their degrees of freedom, in the aforementioned limit, are associated to the weighted volume of the internal spaces (to be defined below). The deformation changed the internal space enriching the geometric structure but left in-

variant its weighted volume. We prove this using the holographic calculation and left the specification of the SCFT for a forthcoming publication.

The content of this paper is organised as follows. In section 2 we start by briefly reviewing the seed solutions in [22]. We then proceed to apply the TsT transformation in section 3 in order to obtain the new family of backgrounds in massive IIA. We study the quantised charges and present brane configurations which we argue give rise to our solutions in the near horizon limit. In section 4 we study the eleven-dimensional analog of the TsT transformation for the solutions in section 2 with vanishing romans mass uplifted to eleven dimensions. One of the solutions obtained correspond to the uplift of the TsT-deformed IIA solution in the massless case. We then prove the invariance of the central charge under the deformation in section 5 using the holographic computation. Finally, In section 6 we study the preservation of supersymmetry for the solutions obtained in sections 3 and 4. This analysis suggest the new supersymmetric solutions support a dynamical SU(2) structure in the internal five-dimensional space. Some comments and final remarks are addressed in section 7. In appendix A we give our conventions for supersymmetry.

2 The seed AdS₃ $\mathcal{N} = (0, 4)$ holographic backgrounds

In this section we shall briefly review the AdS₃ solutions in massive IIA supergravity preserving small $\mathcal{N} = (0, 4)$ supersymmetry obtained in [22]. They will constitute our starting point from which we will obtain the marginally deformed solutions via a transformation involving dualities.

The solutions in [22] are of the warped form AdS₃ × S² × M₅, supporting an SU(2) structure on M₅, equivalently, for these solutions, an SU(3) structure in seven dimensions. Moreover, the five-dimensional space M₅ locally splits into a four-dimensional piece M₄ and an interval. There are two classes of solutions. In this work we will concentrate on a subclass of *class I* solutions for which M₄ is (conformally) CY₂. From now on we will consider CY₂ = T⁴. The NS sector of the solution in the string frame reads

$$\begin{aligned}
 ds^2 &= \frac{u}{\sqrt{h_4 h_8}} \left(ds_{\text{AdS}_3}^2 + \frac{h_8 h_4}{f_1} ds_{\text{S}^2}^2 \right) + \sqrt{\frac{h_4}{h_8}} ds_{\text{T}^4}^2 + \frac{\sqrt{h_4 h_8}}{u} d\rho^2, \\
 e^\Phi &= \frac{2h_4^{1/4}}{h_8^{3/4}} \sqrt{\frac{u}{f_1}}, \quad B_2 = f_2 d\text{vol}_{\text{S}^2},
 \end{aligned}
 \tag{2.1}$$

where the functions u, h_4, h_8 are functions of ρ only. This is supported with the following RR field strengths

$$\begin{aligned}
 F_0 &= h'_8, \quad F_2 = -\frac{1}{2} \left(h_8 - \frac{h'_8 u u'}{f_1} \right) d\text{vol}_{\text{S}^2}, \quad F_8 = \star_{10} F_2, \quad F_{10} = -\star_{10} F_0, \\
 F_4 &= -\left(\left(\frac{u u'}{2\hat{h}_4} \right)' + 2h_8 \right) d\rho \wedge d\text{vol}_{\text{AdS}_3} - h'_4 d\text{vol}_{\text{T}^4}, \quad F_6 = -\star_{10} F_4,
 \end{aligned}
 \tag{2.2}$$

where $' = \partial_\rho$ and

$$f_1 = 4h_4 h_8 + (u')^2, \quad f_2 = \frac{1}{2} \left(-\rho + \frac{u u'}{f_1} \right).
 \tag{2.3}$$

	$0 \leq \rho \leq 2\pi$	$2\pi j \leq \rho \leq 2\pi(j+1)$	$2\pi P \leq \rho \leq 2\pi(P+1)$
h_8	$\frac{\nu_0}{2\pi}\rho$	$\mu_j + \frac{\nu_j}{2\pi}(\rho - 2\pi j)$	$\mu_P - \frac{\nu_P}{2\pi}(\rho - 2P\pi)$
h_4	$\frac{\beta_0}{2\pi}\rho$	$\alpha_j + \frac{\beta_j}{2\pi}(\rho - 2\pi j)$	$\alpha_P - \frac{\beta_P}{2\pi}(\rho - 2P\pi)$
u	$\frac{b_0}{2\pi}\rho$		

Table 1. Piece-wise continuous functions satisfying the conditions in eq. (2.5). The value of $u(\rho)$ is the same in all intervals, as required by supersymmetry.

The above background is a supersymmetric solution of massive type IIA supergravity provided

$$h_4''(\rho) = 0, \quad h_8''(\rho) = 0, \quad u''(\rho) = 0, \quad (2.4)$$

the first two away from localised sources. The ρ coordinate parametrising the interval can be taken to be of finite range. This imposes additional constraints on the various functions of the solution. We require for $0 \leq \rho \leq 2\pi(P+1)$ that¹

$$h_8|_{\rho=0} = h_8|_{\rho=2\pi(P+1)} = h_4|_{\rho=0} = h_4|_{\rho=2\pi(P+1)} = 0. \quad (2.5)$$

The metric functions obeying the above conditions are then explicitly given in table 1.

The set of constants $(\alpha_j, \beta_j, \mu_j, \nu_j, b_0)$ for $j = 0, \dots, P$ parametrising the piece-wise continuous functions above are subject to certain constraints imposing continuity of the NS sector along the ρ intervals. The conditions are

$$\alpha_k = \sum_{j=0}^{k-1} \beta_j, \quad \mu_k = \sum_{j=0}^{k-1} \nu_j. \quad (2.6)$$

The supergravity solution is trustable whenever these constants as well as the number P have large values.

3 The marginally deformed backgrounds

In this section we will construct a family of solutions corresponding to deformations of the supergravity solutions in eqs. (2.1)–(2.2). Such deformations are built upon a sequence of T dualities and a change of coordinates [43]. The resulting backgrounds are considered to be holographic duals of the marginally deformed SCFTs dual to the original (undeformed) backgrounds.

In order to proceed, we first pick a two-torus in the geometry. For the solution in eq. (2.1) there are two options which will produce inequivalent solutions. They correspond to $U(1)_\varphi \times U(1)_{x_i}$ and $U(1)_{x_i} \times U(1)_{x_j}$ invariant sub-sectors, where φ is the azimuthal angle inside the S^2 and x_i the coordinates on T^4 . The deformation is achieved by performing a T-duality in one of the coordinates, a shift with parameter λ in the second and T duality back in the first. The solutions obtained will describe a family of solutions in terms of the functions u, h_4, h_8 and the parameter λ .

¹For other choices where some of these conditions are relaxed see [24, 35].

In the first case $T^2 : (\varphi, x_1)$, following the T duality rules in [45], the above procedure generates the following background

$$\begin{aligned}
 ds^2 &= \frac{u}{\sqrt{h_4 h_8}} \left(ds_{\text{AdS}_3}^2 + \frac{h_8 h_4}{f_1} d\theta^2 \right) + \sqrt{\frac{h_4}{h_8}} (dx_2^2 + dx_3^2 + dx_4^2) + \frac{\sqrt{h_4 h_8}}{u} d\rho^2 \\
 &\quad + \frac{1}{f_1 + \lambda^2 \sin^2 \theta h_4 u} \left(u \sqrt{h_4 h_8} \sin^2 \theta d\varphi^2 + \sqrt{\frac{h_4}{h_8}} f_1 (dx_1 - \lambda f_2 \sin \theta d\theta)^2 \right), \\
 e^\Phi &= \frac{2h_4^{1/4}}{h_8^{3/4}} \sqrt{\frac{u}{f_1 + \lambda^2 h_4 u \sin^2 \theta}}, \\
 B_2 &= \frac{\lambda h_4 u \sin^2 \theta}{f_1 + \lambda^2 h_4 u \sin^2 \theta} (dx_1 - \lambda f_2 \sin \theta d\theta) \wedge d\varphi + f_2 d\text{vol}_{S^2}, \\
 F_0 &= h'_8, \quad F_2 = \frac{\gamma h_4 u \sin^2 \theta h'_8}{f_1 + \lambda^2 h_4 u \sin^2 \theta} (dx_1 - \lambda f_2 \sin \theta d\theta) \wedge d\varphi - \frac{1}{2} \left(h_8 - \frac{h'_8 u u'}{f_1} \right) d\text{vol}_{S^2} \\
 F_4 &= - \left(\left(\frac{u u'}{2h_4} \right)' + 2h_8 \right) d\rho \wedge d\text{vol}_{\text{AdS}_3} - h'_4 d\text{vol}_{T^4} + \frac{1}{2} \gamma (h_4 - \rho h'_4) \sin \theta d\theta \wedge dx_2 \wedge dx_3 \wedge dx_4,
 \end{aligned} \tag{3.1}$$

where the higher fluxes are obtained via the lower ones as indicated in eq. (2.2). The background in eq. (3.1) is a solution of massive IIA supergravity if conditions in eq. (2.4) are imposed. We notice the original solution is recovered after turning off the deformation parameter, as expected.

For the second case $T^2 : (x_3, x_4)$, the procedure outlined above produces the following background

$$\begin{aligned}
 ds^2 &= \frac{u}{\sqrt{h_4 h_8}} \left(ds_{\text{AdS}_3}^2 + \frac{h_8 h_4}{f_1} ds_{S^2}^2 \right) + \frac{\sqrt{h_4 h_8}}{u} d\rho^2 + \sqrt{\frac{h_4}{h_8}} dx_{1,2}^2 + \sqrt{\frac{h_4}{h_8}} \frac{1}{1 + \lambda^2 \frac{h_4}{h_8}} dx_{3,4}^2, \\
 e^\Phi &= \frac{2h_4^{1/4}}{h_8^{3/4}} \sqrt{\frac{u}{(1 + \lambda^2 \frac{h_4}{h_8}) f_1}}, \quad B_2 = f_2 d\text{vol}_{S^2} - \lambda \frac{h_4}{h_8} \frac{dx_3 \wedge dx_4}{1 + \lambda^2 \frac{h_4}{h_8}}, \\
 F_0 &= h'_8, \quad F_2 = -\frac{1}{2} \left(h_8 - \frac{h'_8 u u'}{f_1} \right) d\text{vol}_{S^2} - \lambda h'_4 dx_1 \wedge dx_2 - \lambda \frac{h_4 h'_8}{h_8 (1 - \lambda^2 \frac{h_4}{h_8})} dx_3 \wedge dx_4, \\
 F_4 &= - \left(\left(\frac{u u'}{2h_4} \right)' + 2h_8 \right) d\rho \wedge d\text{vol}_{\text{AdS}_3} - \frac{h'_4}{1 + \lambda^2 \frac{h_4}{h_8}} d\text{vol}_{T^4} + \\
 &\quad \lambda \left(\frac{h_4}{2h_8 (1 + \lambda^2 \frac{h_4}{h_8})} \left(h_8 - \frac{u u' h'_8}{f_1} \right) dx_3 \wedge dx_4 + \frac{1}{2} \left(h_4 - \frac{u u' h'_4}{f_1} \right) dx_1 \wedge dx_2 \right) \wedge d\text{vol}_{S^2},
 \end{aligned} \tag{3.2}$$

which is a solution of massive IIA supergravity if conditions in eq. (2.4) are imposed. We notice since the S^2 is a spectator subspace for this deformation, we expect $\mathcal{N} = (0, 4)$ supersymmetry will be fully preserved as we will explicitly show in section 6.

3.1 Quantised charges and brane set-ups

In this section we will study the Page charges of the deformed backgrounds. Throughout, we shall use the following definitions² $Q_{D_p} = \frac{1}{(2\pi)^{7-p}} \int_{\Sigma_{8-p}} \hat{f}_{8-p}$, where $\hat{f} = e^{-B} \wedge f$, here f denotes the magnetic (internal) part of the RR polyform F .

We start with the solution in eq. (3.1) and consider the following non-trivial cycles of the geometry

$$\Sigma_2 = S^2, \quad \Sigma_4 = T^4, \quad \Sigma_3 = (\rho, S^2), \quad \Sigma_6 = (S^2, T^4), \quad \Sigma'_4 = (x_2, x_3, x_4, \theta), \quad \Sigma'_3 = (x_1, \rho, \varphi). \quad (3.3)$$

The Page charges read

$$\begin{aligned} Q_{\text{NS5}} &= \frac{1}{(2\pi)^2} \int_{\Sigma_3} H_3, & Q_{\text{NS5}'} &= \frac{1}{(2\pi)^2} \int_{\Sigma'_3} H_3, & Q_{D_8} &= 2\pi h'_8, & Q_{D_6} &= \frac{1}{2\pi} \int_{\Sigma_2} \hat{f}_2 \\ Q_{D_4} &= \frac{1}{(2\pi)^3} \int_{\Sigma_4} \hat{f}_4, & Q_{D'_4} &= \frac{1}{(2\pi)^3} \int_{\Sigma'_4} \hat{f}_4, & Q_{D_2} &= \frac{1}{(2\pi)^5} \int_{\Sigma_6} \hat{f}_6. \end{aligned} \quad (3.4)$$

If we allow large gauge transformations $B_2 \rightarrow B_2 + \pi k \, d\text{vol}_{S^2}$, the Page fluxes are those in eq. (3.1) except for

$$\begin{aligned} \hat{f}_2 &= -\frac{1}{2} (h_8 - h'_8 (\rho - 2\pi k)) d\text{vol}_{S^2}, \\ \hat{f}_6 &= \frac{1}{2} (h_4 - h'_4 (\rho - 2\pi k)) d\text{vol}_{S^2} \wedge d\text{vol}_{T^4}. \end{aligned} \quad (3.5)$$

The charges in eq. (3.4) computed in $2\pi k \leq \rho \leq 2\pi(k+1)$ are explicitly,

$$\begin{aligned} Q_{D_8} &= \nu_k, & Q_{D_6} &= \mu_k, & Q_{D_4} &= \beta_k, & Q_{D_2} &= \alpha_k, \\ Q_{D'_4} &= \lambda(k\beta_k - \alpha_k) & Q_{\text{NS5}} &= 1, & Q_{\text{NS5}'} &= 0. \end{aligned} \quad (3.6)$$

We notice that for finite $\rho \in [0, 2\pi(P+1)]$ we have $P+1$ parallel NS5 branes. From the above expressions we see in particular that no extra NS5' branes were generated by the deformation. In addition, the above charges are well-defined as long as the set of constants $\alpha_k, \beta_k, \mu_k, \nu_k$ as well as the combination $\lambda(k\beta_k - \alpha_k) \in \mathbb{Z}$.

As we pointed out before, the first two conditions in eq. (2.4) must be satisfied by the solutions everywhere except at points where we have localised sources. At those points, we have a change in gradient of the piece-wise linear functions proportional to $h''_{4,8}$ pointing the possible existence of a source for D_p branes via the modified Bianchi identities $d\hat{f} = j_s$. From table 1 we obtain

$$h''_4 = \sum_{k=1}^P \left(\frac{\beta_{k-1} - \beta_k}{2\pi} \right) \delta(\rho - 2\pi k), \quad h''_8 = \sum_{k=1}^P \left(\frac{\nu_{k-1} - \nu_k}{2\pi} \right) \delta(\rho - 2\pi k). \quad (3.7)$$

²We will be using units such that $g_s = \alpha' = 1$.

	t	x	r	θ	ϕ	x_1	x_2	x_3	x_4	ρ	N_{D_p}
D_8	•	•	•	•	•	•	•	•	•	•	$\nu_{k-1} - \nu_k$
D_6	•	•	•	•	•	•	•	•	•	•	μ_k
D'_4	•	•	•	•	•	•	•	•	•	•	$\lambda k N_{D_4}$
D_4	•	•	•	•	•	•	•	•	•	•	$\beta_{k-1} - \beta_k$
D_2	•	•	•	•	•	•	•	•	•	•	α_k
NS5	•	•	•	•	•	•	•	•	•	•	

Table 2. Brane configuration which in the near horizon limit gives the solution in eq. (3.1). We show the world-volume directions the branes are suspended as well as their number in the k -th interval.

Using this information as well as the Page fluxes of the solution we compute

$$d\hat{f}_0 = 2\pi h_8'' d\rho, \quad (3.8)$$

$$d\hat{f}_2 = \frac{1}{2}(\rho - 2\pi k)h_8'' d\rho \wedge d\text{vol}_{S^2} = 0, \quad (3.9)$$

$$d\hat{f}_4 = h_4'' d\rho \wedge \left(d\text{vol}_{T^4} + \lambda \frac{\rho}{2} \sin \theta d\theta \wedge dx_2 \wedge dx_3 \wedge dx_4 \right) \quad (3.10)$$

$$d\hat{f}_6 = \frac{h_4''}{2}(\rho - 2\pi k)d\rho \wedge d\text{vol}_{S^2} \wedge d\text{vol}_{T^4} = 0 \quad (3.11)$$

where we have used $x\delta(x) = 0$. From this we conclude that D_4 , D'_4 as well as D_8 , having non-zero sources, correspond to flavour branes whilst D_6 and D_2 are colour ones. Thus in addition to the D-branes of the seed solution, the deformation has induced (semi-localised) flavour $Q_{D'_4}$ branes. The brane configuration, before the near horizon limit is taken, we argue is associated to the solution above is shown in table 2.

For the second solution, we consider the following cycles

$$\Sigma'_2 = (x_1, x_2), \quad \Sigma_4 = T^4, \quad \Sigma'_4 = (S^2, x_1, x_2), \quad \Sigma_6 = (S^2, T^4), \quad \Sigma'_3 = (\rho, x_3, x_4), \quad (3.12)$$

and non-trivial Page forms

$$\begin{aligned} \hat{f}_2 &= -\frac{1}{2}(h_8 - h_8'(\rho - 2\pi k))d\text{vol}_{S^2} + \lambda h_4' dx_1 \wedge dx_2, \\ \hat{f}_4 &= h_4' d\text{vol}_{T^4} - \frac{\lambda}{2}(h_4 - h_4'(\rho - 2\pi k))d\text{vol}_{S^2} \wedge dx_1 \wedge dx_2, \\ \hat{f}_6 &= -\frac{1}{2}(h_4 - h_4'(\rho - 2\pi k))d\text{vol}_{S^2} \wedge d\text{vol}_{T^4}. \end{aligned} \quad (3.13)$$

An analysis as detailed above shows that in addition to the D-branes of the seed solution, the generated Page charges after the transformation (λ -dependent) are given by

$$Q_{D'_6} = \lambda\beta_k, \quad Q_{D'_4} = \lambda\alpha_k. \quad (3.14)$$

	t	x	r	θ	ϕ	x_1	x_2	x_3	x_4	ρ	N_{D_p}
D ₈	•	•	•	•	•	•	•	•	•	•	$\nu_{k-1} - \nu_k$
D ₆	•	•	•	•	•	•	•	•	•	•	μ_k
D' ₆	•	•	•	•	•	•	•	•	•	•	λN_{D_4}
D' ₄	•	•	•	•	•	•	•	•	•	•	λN_{D_2}
D ₄	•	•	•	•	•	•	•	•	•	•	$\beta_{k-1} - \beta_k$
D ₂	•	•	•	•	•	•	•	•	•	•	α_k
NS5	•	•	•	•	•	•	•	•	•	•	

Table 3. Brane configuration which in the near horizon limit gives the solution in eq. (3.2). We also show the world-volume directions the branes are suspended as well as their number in the k -th interval. We see the D'_6 and D'_4 branes are wrapped on $T^2 : (x_3, x_4)$.

This implies the quantisation conditions $\lambda\beta_k \in \mathbb{Z}, \lambda\alpha_k \in \mathbb{Z}$, which requires rational λ . In order to determine if the above charges correspond to colour or flavour branes, we compute

$$df_2 = \lambda h_4'' d\rho \wedge dx_1 \wedge dx_2 + \frac{1}{2}(\rho - 2k\pi)h_8'' d\rho \wedge d\text{vol}_{S^2} \tag{3.15}$$

$$df_4 = h_4'' d\rho \wedge d\text{vol}_{T^4} - \frac{\lambda}{2}(\rho - 2k\pi)h_4'' d\rho \wedge d\text{vol}_{S^2} \wedge dx_1 \wedge dx_2. \tag{3.16}$$

Using then (3.7) we find that the effect of the deformation was to add $Q_{D'_4}$ colour and $Q_{D'_6}$ flavour branes respectively. Therefore the original D_4 -NS5- D_2 and D_8 -NS5- D_6 brane arrangements are modified by the addition of D'_4 branes extended along (t, x, x_3, x_4, ρ) as well as semi-localised D'_6 branes in $(\text{AdS}_3, S^2, x_3, x_4)$ wrapped on $T^2 : (x_3, x_4)$. The brane set-up corresponding to this configuration is summarised in table 3.

4 The deformation in eleven dimensions

In this section we will study a generalisation to eleven dimensional supergravity of the TsT transformation studied in the previous section. The seed solutions will be the uplift of the background in eq. (2.1)–(2.2) for vanishing Romans mass. The backgrounds obtained will correspond to a family of supersymmetric solutions which are out of a subclass of the classification for AdS_3 eleven dimensional solutions studied in [31].

In order to proceed, we consider a vanishing Romans mass in the solution of eq. (2.1)–(2.2) which lead us to consider $h_8 = k$. The uplift of this solution to eleven dimensions was first constructed in [31]. For latter use we will present some details here. We determine the three and one-form potentials to be

$$A_3 = \left(\frac{uu'}{2h_4} + 2k\rho \right) d\text{vol}_{\text{AdS}_3} + h_4' x_1 d\text{vol}_{T^3}, \quad A_1 = \frac{k}{2} \cos\theta d\varphi. \tag{4.1}$$

We notice the 3-form potential above is not globally well-defined. This would be the case if h_4' were a continuous function. Using the usual KK ansatz eq. (A.2), the eleven-dimensional

solution reads

$$ds_{11}^2 = \Upsilon \left(\frac{u}{\sqrt{h_4 k}} ds_{\text{AdS}_3}^2 + \sqrt{\frac{h_4}{k}} ds_{\text{T}^4}^2 + \frac{\sqrt{h_4 k}}{u} d\rho^2 + \frac{k^2}{\Upsilon^3} ds_{\text{S}^3/\text{Z}_k}^2 \right), \quad (4.2)$$

$$G_4 = dC_3 = d \left(\left(\frac{uu'}{2h_4} + 2\rho k \right) d\text{vol}_{\text{AdS}_3} + 2k \left(-\rho + 2\pi k + \frac{uu'}{f_1} \right) d\text{vol}_{\text{S}^3/\text{Z}_k} \right) + h'_4 d\text{vol}_{\text{T}^4},$$

where

$$ds_{\text{S}^3/\text{Z}_k}^2 = \frac{1}{4} \left(ds_{\text{S}^2}^2 + \left(\frac{2}{k} dx_{11} + \cos \theta d\varphi \right)^2 \right), \quad \Upsilon = \frac{f_1^{1/3} \sqrt{k}}{(4u\sqrt{h_4})^{1/3}}. \quad (4.3)$$

This solution preserves small $\mathcal{N} = (0, 4)$ supersymmetry and supports an $\text{SU}(2)$ structure. We will now generalise this class of solutions by performing an $\text{SL}(3, \mathbb{R})$ transformation of coordinates.

For a solution which is $\text{SL}(3, \mathbb{R})$ invariant we use the ansatz

$$\begin{aligned} ds_{11}^2 &= \Delta^{-1/6} g_{\mu\nu} dx^\mu dx^\nu + \Delta^{1/3} M_{ab} \mathcal{D}\phi^a \mathcal{D}\phi^b, \\ C_3 &= C_{(0)} \mathcal{D}\phi^1 \wedge \mathcal{D}\phi^2 \wedge \mathcal{D}\phi^3 + \frac{1}{2} C_{(1)ab} \wedge \mathcal{D}\phi^a \wedge \mathcal{D}\phi^b + C_{(2)a} \wedge \mathcal{D}\phi^a + C_{(3)}, \\ \mathcal{D}\phi^a &= d\phi^a + A_\mu^a dx^\mu, \end{aligned} \quad (4.4)$$

where the a, b indices correspond to the three-torus directions, $g_{\mu\nu}$ is the transverse eight dimensional metric and $\det M = 1$. We have two possible choices for which we can apply the transformation. Namely $\text{T}^3: (x_2, x_3, x_4)$ and $\text{T}^3: (x_3, x_4, x_{11})$.

In the first case, the background in eq. (4.2) can be brought into the form of eq. (4.4) provided we identify

$$\begin{aligned} A_\mu^a &= 0, & M_{ab} &= \Delta^{-1/3} \left(\frac{h_4 f_1}{4u} \right)^{1/3} \delta_{ab}, & \Delta &= \frac{h_4 f_1}{4u} \\ C_{(0)} &= -h'_4 x_1, & C_{(3)} &= - \left(\frac{uu'}{2h_4} + 2k\rho \right) d\text{vol}_{\text{AdS}_3} + B_2 \wedge dx_{11}, \\ C_{(1)ab} &= C_{(2)a} = 0, \end{aligned} \quad (4.5)$$

$$\begin{aligned} \Delta^{-1/6} g_{\mu\nu} dx^\mu dx^\nu &= \sqrt{k} \left(\frac{f_1}{4\sqrt{h_4 u}} \right)^{1/3} \left(\frac{u}{\sqrt{h_4 k}} \left(ds_{\text{AdS}_3}^2 + \frac{h_4 k}{f_1} ds_{\text{S}^2}^2 \right) + \sqrt{\frac{h_4}{k}} dx_1^2 + \frac{\sqrt{h_4 k}}{u} d\rho^2 \right) \\ &\quad + \frac{k}{4} \left(\frac{16u^2 h_4}{f_1^2} \right)^{1/3} \left(\frac{2}{k} dx_{11} + \cos \theta d\varphi \right)^2, \end{aligned}$$

We then use the transformation rules spelled out in [44] to obtain the new background parametrised by λ . The transformation for the one-form A^a , using (4.5), gives $\tilde{A}^a = A^a + \frac{1}{2} \lambda \epsilon^{abc} C_{(1)bc} = 0$ and therefore $\tilde{\mathcal{D}}\phi^a = \mathcal{D}\phi^a = d\phi^a$. On the other hand, the non-trivial transformation associated to $\tau = -C_{(0)} + i\Delta^{1/2}$ reads $\tilde{\tau} = \tau / (1 + \lambda\tau)$, from which we obtain

$$\tilde{\Delta} = G^2 \Delta, \quad \tilde{C}_{(0)} = G \left(C_{(0)} - \lambda (C_{(0)}^2 + \Delta) \right), \quad G = \left(1 + 2\lambda C_{(0)} - \lambda^2 (C_{(0)}^2 + \Delta) \right)^{-1}. \quad (4.6)$$

The deformed background then reads

$$ds_{11}^2 = G^{-1/3} \left[\Upsilon \left(\frac{u}{\sqrt{h_4 k}} ds_{\text{AdS}_3}^2 + \sqrt{\frac{h_4}{k}} dx_1^2 + \frac{\sqrt{h_4 k}}{u} d\rho^2 + \frac{k^2}{\Upsilon^3} ds_{\text{S}^3/\text{Z}_k}^2 \right) + G \Upsilon \sqrt{\frac{h_4}{k}} ds_{\text{T}^3}^2 \right],$$

$$G_4 = dC_3, \tag{4.7}$$

where

$$C_3 = - \left(\frac{uu'}{2h_4} + 2k\rho + \lambda \frac{x_1}{2h_4} (h_4 f_1 + uu' h'_4) \right) d\text{vol}_{\text{AdS}_3} + \tilde{C}_0 d\text{vol}_{\text{T}^3}$$

$$+ 4k \left(f_2 - \lambda \frac{x_1}{2} \left(h_4 - \frac{uu' h'_4}{f_1} \right) \right) d\text{vol}_{\text{S}^3/\text{Z}_k}. \tag{4.8}$$

This background is a solution of 11d supergravity when conditions in eq. (2.4) are imposed, and reduces to the undeformed solution for $\lambda = 0$, as expected.

In the second case $\text{T}^3 : (x_3, x_4, x_{11})$, the solution obtained following the procedure spelled out above corresponds to the uplift to eleven dimensions of the solution in eq. (3.2). The eleven-dimensional background reads

$$ds_{11}^2 = \Upsilon (1 + \lambda^2 \frac{h_4}{h_8})^{1/3} \left(\frac{u}{\sqrt{h_4 k}} ds_{\text{AdS}_3}^2 + \frac{h_8 h_4}{f_1} ds_{\text{S}^2}^2 + \frac{\sqrt{h_4 h_8}}{u} d\rho^2 + \sqrt{\frac{h_4}{h_8}} (dx_1^2 + dx_2^2) \right. \tag{4.9}$$

$$\left. + \sqrt{\frac{h_4}{h_8}} \frac{1}{1 + \lambda^2 \frac{h_4}{h_8}} (dx_3^2 + dx_4^2) + \frac{k^2}{4\Upsilon^3} \left(ds_{\text{S}^2}^2 + \frac{Dy^2}{1 + \lambda^2 \frac{h_4}{h_8}} \right) \right),$$

$$G_4 = - \left(\partial_\rho \left(\frac{uu'}{2h_4} \right) + 2k \right) d\rho \wedge d\text{vol}_{\text{AdS}_3} - \frac{h'_4}{1 + \lambda^2 \frac{h_4}{k}} d\text{vol}_{\text{T}^4} - \frac{\lambda}{2} \frac{h'_4}{\left(1 + \lambda^2 \frac{h_4}{k} \right)^2} d\rho \wedge dx_3 \wedge dx_4 \wedge Dy$$

$$+ \frac{\lambda}{2} \left(\frac{h_4}{1 + \lambda^2 \frac{h_4}{k}} dx_3 \wedge dx_4 + \left(h_4 - \frac{uu' h'_4}{f_1} \right) dx_1 \wedge dx_2 \right) \wedge d\text{vol}_{\text{S}^2} + \frac{k}{2} \partial_\rho f_2 d\rho \wedge d\text{vol}_{\text{S}^2} \wedge Dy,$$

where

$$Dy = \frac{2}{k} dy + \cos \theta d\varphi - \frac{2}{k} \lambda x_1 h'_4 dx_2, \tag{4.10}$$

and Υ was defined in eq. (4.3). This background is a solution of 11d supergravity when conditions in eq. (2.4) are imposed. In section 6 we will show that the solutions presented in this section preserve $\mathcal{N} = (0, 4)$ supersymmetry supporting a dynamical $\text{SU}(2)$ structure.

Before to close this section, it is worth noticing that the solutions in eqs. (4.7) and (4.9) can be used as seed solutions in order to generate other families of supersymmetric solutions. For instance, after appropriate analytical continuations we can generate solutions with $\text{AdS}_3/\text{Z}_k \times \text{S}^3$ factors which further reduction to IIA along the Hopf-fibre direction of AdS_3 will generate new $\text{AdS}_2 \times \text{S}^3$ solutions in IIA supergravity, which can be further extended to massive IIA, generalising those studied in [31], etc.

5 Holographic central charge

The main goal of this section will be to compute the central charge characterising the new family of solutions. For the seed solutions this was done in [24, 31] and using the

analysis of the spin-2 spectrum in [29]. A generic result involving the deformations discussed above is that they leave the internal space volume transverse to AdS₃ -weighted by the dilaton- invariant. We then anticipate the central charges will be the same before and after the deformation.

In order to see this explicitly, we consider the metric of the solutions written in the following way

$$ds^2 = a(r, \vec{y}\hat{1}) \left(dx_{1,d}^2 + b(r)dr^2 \right) + g_{ij}(r, \vec{y})dy^i dy^j, \tag{5.1}$$

where $x_{1,d}$ parametrises $\mathcal{M}^{1,d}$ Minkowski space and g_{ij} the metric of the internal space. The holographic central charge is then given by the following expression [46]

$$c_{\text{hol}} = \frac{d^d}{G_N} \frac{b(r)^{d/2} H^{\frac{2d+1}{2}}}{H^d}, \tag{5.2}$$

where

$$H = \left(\int d\vec{y} \sqrt{e^{-4\Phi} \det(g_{ij}) a^d} \right). \tag{5.3}$$

For the ten dimensional solution, since the deformations acted on the internal space of the solutions, we clearly see the quantities $a(r, \vec{y}\hat{1}), b(r)$ in eq. (5.1) are spectator under the deformations. In addition, we find that

$$e^{\tilde{\Phi}} = \frac{e^{\Phi}}{\sqrt{1 + \lambda^2 \det(g_{T^2})}}, \quad \tilde{g}_{T^2} = \frac{g_{T^2}}{1 + \lambda^2 \det(g_{T^2})} \tag{5.4}$$

where tilde denotes fields after the deformation. It is then easy to see that $e^{-4\tilde{\Phi}} \det(\tilde{g}_{ij}) \tilde{a}^d = e^{-4\Phi} \det(g_{ij}) a^d$, giving $\tilde{c}_{\text{hol}} = c_{\text{hol}}$ as anticipated. This result goes through for the eleven-dimensional solutions after considering the relation between the ten and eleven-dimensional quantities in the KK ansatz (A.2) and $H = \left(\int d\vec{y} \sqrt{\det(\hat{g}_{ij}) \hat{a}^d} \right)$, where quantities with hat are eleven-dimensional ones.

After we have characterised the backgrounds by computing their central charges, the goal is to compare them with the central charges obtained from the putative dual field theories to these solutions, in the holographic limit. Some comments are in order. For instance, in the case of the field theory read off from the brane configuration in table 3, we can achieve an anomaly free quiver field theory following the rules in [23, 24]. Nevertheless, this gives a central charge that is apparently changing due to the extra gauge and flavour group insertions. We would expect cancelations among them that will give the same central charge as before the deformation, or that their contributions are sub-leading in the holographic limit. We will elaborate more on this in a forthcoming publication.

6 Comments on supersymmetry and G-structure of the solutions

In this section we will study the supersymmetries preserved by the supergravity solutions in eqs. (3.1), (3.2) and (4.7), (4.9), based on the explicit form of the Killing spinors of the original solution (2.1). The conventions we follow for supersymmetry are detailed in appendix A. The solution in eq. (2.1) preserves small $\mathcal{N} = (0, 4)$ supersymmetry by

construction. In the conventional approach, this implies the existence of two algebraic conditions on the ten-dimensional Majorana-Weyl (MW) spinor ensuring the vanishing of the supersymmetry variations.

In order to see this explicitly, we decompose the ten-dimensional gamma matrices as follows

$$\Gamma^\alpha = \sigma_1 \otimes \rho^\alpha \otimes \mathbb{I}, \quad \Gamma^\mu = \sigma_2 \otimes \mathbb{I} \otimes \gamma^\mu, \quad (6.1)$$

where ρ^α and γ^μ are three and seven-dimensional gamma matrices respectively and the σ_i are the usual Pauli matrices. In this notation the chirality matrix is $\Gamma^{11} = -\sigma_3 \otimes \mathbb{I} \otimes \mathbb{I}$. After plugging the solution in eq. (2.1) (in the natural frame) into eq. (A.11) we find the MW Killing spinor takes the form

$$\epsilon_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \zeta \otimes \chi_1, \quad \epsilon_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \zeta \otimes \chi_2, \quad (6.2)$$

where ζ is the AdS₃ Killing spinor and

$$\chi_{1,2} = e^{\frac{A}{2}} e^{i\frac{\theta}{2}\sigma_2\gamma^7} e^{\frac{\varphi}{2}\gamma^{67}} e^{-\frac{1}{2}\arctan 2\frac{\sqrt{h_4h_8}}{u'}\gamma^{67}\sigma_3} \chi_{(0)1,2}, \quad A = \frac{1}{2} \log \frac{u}{\sqrt{h_4h_8}}, \quad (6.3)$$

a seven-dimensional spinor satisfying the projection conditions

$$\gamma^{1234}\chi_{1,2} = -\chi_{1,2}, \quad \frac{1}{\sqrt{f_1}} \left(2\sqrt{h_4h_8}\gamma^5\sigma_1 + u'\gamma^{567}i\sigma_2 \right) \chi_{1,2} = \chi_{1,2} \quad (6.4)$$

where the indices on the gamma matrices 1, . . . 4, 5 and 6, 7 are flat indices corresponding to the M₅=(T⁴, ρ) and S² directions respectively. The purpose of this section, is to find the number of spinor components which are compatible with the TsT transformation.

Since the deformation involves a sequence of T dualities, a condition for preserving Killing spinors reduces to their invariance by the action of the Kosmann-Lie derivative along the Killing vector K associated to the isometric direction we picked to perform the duality $\mathcal{L}_K\epsilon = 0$, where ϵ is the Killing spinor of the un-dualised solution. By considering $K = \partial_y$, the above condition reduces to $\partial_y\epsilon = 0$ [47]. Moreover, invariance under the change of coordinates in the second direction also requires independence of it on the spinor. Therefore, supersymmetry is compatible with TsT transformations as long as the spinor is uncharged under the directions used for the transformation [48].

For the first solution in eq. (3.1), there is a residual U(1)_φ which we may think of as a candidate R-charge for $\mathcal{N} = (0, 2)$ preserved supersymmetry. However, the spinors (6.2) are charged under this coordinate and T duality along this direction will project out this dependence. The residual U(1)_φ is therefore a global symmetry and supersymmetry is completely broken. In other words, compatibility with the TsT transformation imposes the projection condition $\gamma^{67}\chi_{1,2} = 0$ breaking all supersymmetries. Despite the breaking of supersymmetries, this solution is interesting in its own since it still solves the BPS condition in eq. (2.4).

For the second marginally deformed solution in eq. (3.2), the spinor is independent of the T⁴ directions, so we ensure supersymmetry is fully preserved. To be more precise,

working with the supersymmetry transformations for the solution in eq. (3.2), we find

$$\begin{aligned} \delta\tilde{\lambda}_1 &= e^{-\arctan\lambda\sqrt{\frac{h_4}{h_8}}\gamma^{34}}\delta\lambda_1, & \delta\tilde{\lambda}_2 &= \delta\lambda_2, \\ \delta\tilde{\psi}_{\mu 1} &= e^{-\arctan\lambda\sqrt{\frac{h_4}{h_8}}\gamma^{34}}\delta\psi_{\mu 1}, & \delta\tilde{\psi}_{\mu 2} &= \delta\psi_{\mu 2}, \end{aligned} \tag{6.5}$$

where tilde denotes fields after the transformation, provided we identify

$$\tilde{\chi}_1 = e^{-\arctan\lambda\sqrt{\frac{h_4}{h_8}}\gamma^{34}}\chi_1, \quad \tilde{\chi}_2 = \chi_2, \tag{6.6}$$

ensuring supersymmetry is preserved as the original solution does. This is along the lines of the generic result in [48], which in addition showed that the entire information of the transformation is encoded in an antisymmetric bi-vector associated to classical r-matrices solving the Yang-Baxter equation.

Let us now turn to the G-structure characterising the above background. To begin with, the solutions in [24] were constructed by imposing that they support an SU(2) structure on the five-dimensional internal space M_5 transverse to $AdS_3 \times S^2$. For these solutions, this implies that the internal five-dimensional spinors are globally parallel. The deformed Killing spinors allow for a point-dependent phase between the undeformed spinors defining a dynamical SU(2) structure. To be more precise, given the rotation of the internal spinor under TsT eq. (6.6), the transformed MW spinor takes the form (6.2) with the internal spinor transformed accordingly $\chi_{1,2} \rightarrow \tilde{\chi}_{1,2}$. In addition, the seven-dimensional spinor can be further decomposed into $S^2 \times M_5$ factors according to eq. (6.3). Namely, $\chi_{1,2} = e^{\frac{A}{2}}\xi \otimes \eta_{1,2}$, where ξ is a Killing spinor on the S^2 charged under $SU(2)_R$. In a common basis, the five-dimensional spinors can in general be written as

$$\eta_1 = \eta, \quad \eta_2 = a\eta + b\eta^c + \frac{c}{2}\bar{\omega}\eta, \tag{6.7}$$

where a, b and c are complex and real coefficients respectively and ω a complex one-form. An SU(2) structure on M_5 fixes $c = 0$. In addition, without loss of generality, we can choose for these solutions $b = 0$ [22]. *Class I* solutions are further characterised by $a=1$ and therefore

$$\eta_1 = \eta, \quad \eta_2 = \eta. \tag{6.8}$$

Using (6.6) the TsT MW spinors are given by

$$\epsilon_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \zeta \otimes \tilde{\chi}_1, \quad \epsilon_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \zeta \otimes \tilde{\chi}_2, \tag{6.9}$$

where using the 2+5 split of the internal spinors $\chi_{1,2}$ and gamma matrices, we find

$$\tilde{\eta}_1 = \frac{1}{\sqrt{1 + \lambda^2 \frac{h_4}{h_8}}} \left(\eta - \lambda \sqrt{\frac{h_4}{h_8}} \gamma_{(5)}^{34} \eta \right), \quad \tilde{\eta}_2 = \eta. \tag{6.10}$$

where $\gamma_{(5)}$ are five dimensional gamma matrices. It is then easy to see the first projection in eq. (6.4) implies that $\gamma_{(5)}^{12}\eta = \gamma_{(5)}^{34}\eta = i\eta$. Using this information the spinor in eq. (6.10) reads

$$\tilde{\eta}_1 = e^{-i\arctan\lambda\sqrt{\frac{h_4}{h_8}}}\eta, \quad \tilde{\eta}_2 = \eta. \tag{6.11}$$

The two spinors then differ by a point-dependent phase defining an $SU(2)$ structure that we will refer to it as dynamical. Notice a solution with an identity structure on M_5 can be obtained if the TsT transformation involves the ρ direction. For this to be possible we need ρ to be an isometry such that all warping factors in the solution are constant.³ Moreover we could have also analysed the G-structure of the solution in terms of the seven-dimensional spinor. In this case the seed solution supports an $SU(3)$ structure. It would then be possible to understand the fate of the seven-dimensional G-structure following [49]. The analysis of this section suggest this may give a dynamical $SU(3)$ structure, and will provide a new example of AdS_3 solutions with dynamical $SU(3)$ structure. We plan to report on this in a forthcoming publication.

For the eleven dimensional solutions in eqs. (4.7) and (4.9) the preserved Majorana Killing spinors can be ascertained just as we did for the ten dimensional case. Namely, the preserved Killing spinors are those which are independent of the directions along which we performed the transformation. Using the relation between the eleven and ten-dimensional spinors (A.8) together with (A.4) we easily see that small $\mathcal{N} = (0, 4)$ supersymmetry is preserved. Once again whenever the deformation parameter is turned off we recover the undeformed Majorana Killing spinor defining an $SU(2)$ structure. In the case at hand we have a dynamical $SU(2)$ structure instead.

7 Conclusions

In this paper we have presented new solutions in massive IIA and eleven-dimensional supergravity obtained via TsT transformations and the analog in eleven dimensions. The solutions obtained preserve small $\mathcal{N} = (0, 4)$ supersymmetry and support a dynamical $SU(2)$ structure on the five-dimensional internal submanifold of the solution, as long as we do not use the azimuthal angle inside the S^2 in the procedure. The new backgrounds in ten and eleven dimensional supergravity constitute a whole family of solutions parametrised by the deformation parameter λ and linear functions satisfying the conditions in eq. (2.4). To the best of our knowledge, a complete classification of these solutions is still missing in the literature. One can in principle follow the same procedure as the one outlined in [22] for the $SU(2)$ structure case. That is to say construct bispinors out of seven-dimensional spinors supporting a (dynamical) $SU(2)$ structure in the internal five-dimensional space and obtain geometrical constraints in the form of the solution from the differential conditions implied by supersymmetry. Moreover, in terms of seven-dimensional G-structure, the seed solutions support an $SU(3)$ structure. The analysis we followed in section 6 suggests this becomes a dynamical $SU(3)$ structure after the transformation. Progress on classification of AdS_3 geometries supporting a dynamical $SU(3)$ structure was recently reported in [37].

For the ten-dimensional solutions, we studied the Page charges and associated brane configurations. We showed that depending on the two-torus chosen, the deformation adds either colour or flavour branes or both to the seed configuration. Holographically, The backgrounds obtained correspond to marginal deformations of the SCFT dual to the seed solutions. We verified this by computing the central charge of the deformed backgrounds,

³I thank the anonymous referee for pointing this out.

showing they are the same before and after the transformation. In the field theory side, we can engineer a dual quiver quantum field theory with the information obtained from the Hannany-Witten brane set-ups associated to the solutions. The specification of the dual quantum field theories and more field theory aspects of the solutions are left for a forthcoming publication.

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A Supersymmetry conventions

In this appendix we will set the conventions for supersymmetry. We find useful to review how to obtain the ten-dimensional supersymmetry variations from the eleven dimensional one by dimensional reduction.

Let us start with the supersymmetry variation for the gravitino in eleven-dimensional supergravity. It is

$$\delta\hat{\psi}_M = \left[\nabla_M + \frac{1}{288} \left(\hat{\Gamma}_M^{N_1\dots N_4} - 8\delta_M^{N_1} \hat{\Gamma}^{N_2 N_3 N_4} \right) \hat{F}_{N_1\dots N_4} \right] \hat{\epsilon}, \quad (\text{A.1})$$

where $\nabla_M = \partial_M + \frac{1}{4}\omega_M^{BC}\hat{\Gamma}_{BC}$. We will study the reduction of the above supersymmetry variation along $x^{11} = z$. From now on objects with hat will denote eleven dimensional quantities. To proceed, we use the usual ansatz for the string frame metric and three-form potential, which read

$$\begin{aligned} ds_{11}^2 &= \eta_{AB} E_{N_1}^A E_{N_2}^B dx^{N_1} dx^{N_2} = e^{-\frac{2}{3}\Phi} ds_{10}^2 + e^{\frac{4}{3}\Phi} (dz + A_1)^2, \\ C_3 &= A_3 + B_2 \wedge dz, \end{aligned} \quad (\text{A.2})$$

where Φ is the dilaton and $M, N = (\mu, z)$, $A = (a, z)$ are curved and tangent space indices respectively. The spin-connection components of the above geometry are thus given by

$$\begin{aligned} \tilde{\omega}_{ab} &= e^{\frac{\Phi}{3}} \left(\omega_{ab} - \frac{2}{3} \eta_{c[a} \partial_{b]} \phi e^c \right) - \frac{1}{2} e^{\frac{4}{3}\Phi} F_{ab} e^z, \\ \tilde{\omega}_{za} &= \frac{2}{3} e^{\frac{\Phi}{3}} \partial_a \Phi e^z + \frac{1}{2} e^{\frac{4}{3}\Phi} F_{ab} e^b, \end{aligned} \quad (\text{A.3})$$

where $F_2 = 2\partial A_1$. The dimensional reduction of the eleven-dimensional gravitino $\hat{\psi}_M$ generates a ten-dimensional gravitino and the dilatino as follows

$$\hat{\psi}_\mu = e^{\frac{\Phi}{6}} \left(\psi_\mu - \frac{1}{6} \hat{\Gamma}_\mu \lambda \right), \quad \hat{\psi}_z = \frac{1}{3} e^{\frac{\Phi}{6}} \hat{\Gamma}^z \lambda. \quad (\text{A.4})$$

In the same vein, the $\hat{F}_4 = 4\partial C_3$ field strength contains two pieces the components of which are $\hat{F}_{\alpha\beta\gamma\delta}$ and $\hat{F}_{\alpha\beta\gamma z}$. Using flat indices we identify

$$\begin{aligned} \hat{F}_{abcd} &= E_a^{N_1} E_b^{N_2} E_c^{N_3} E_d^{N_4} \hat{F}_{N_1 N_2 N_3 N_4} = 4e^{\frac{4}{3}\Phi} \left(\partial_{[a} A_{bcd]} - A_{[a} H_{bcd]} \right) = e^{\frac{4}{3}\Phi} F_{abcd}, \\ \hat{F}_{abcz} &= E_a^{N_1} E_b^{N_2} E_c^{N_3} E_z \hat{F}_{N_1 N_2 N_3 z} = e^{\frac{\Phi}{3}} H_{abc}, \end{aligned} \quad (\text{A.5})$$

where $H = 3\partial B_2$. Moreover, the dimensional reduction of eq. (A.1) generates the terms

$$\begin{aligned} \delta\hat{\psi}_z &= \partial_z\hat{\epsilon} + \frac{1}{4}\left(\omega_{z\ ab}\hat{\Gamma}^{ab} + 2\omega_{z\ za}\hat{\Gamma}^{za}\right)\hat{\epsilon} + \frac{1}{288}\left(\hat{\Gamma}_z^{ABCD} - 8\delta_z^A\hat{\Gamma}^{BCD}\right)\hat{F}_{ABCD} \\ &= \frac{1}{3}e^{\Phi/6}\hat{\Gamma}^z\left(\not{\partial}\phi - \frac{3}{4\cdot 2!}e^\Phi\not{F}_2\hat{\Gamma}^z - \frac{1}{12}\not{H}_3\hat{\Gamma}^z + \frac{1}{4\cdot 4!}e^\Phi\not{F}_4\right)\epsilon, \end{aligned} \quad (\text{A.6})$$

$$\begin{aligned} \delta\hat{\psi}_a &= E_a^\mu\partial_\mu\hat{\epsilon} + \frac{1}{4}\omega_{aBC}\hat{\Gamma}^{BC}\hat{\epsilon} + \frac{1}{288}\left(\hat{\Gamma}_a^{BCDE} - 12\delta_a^B\hat{\Gamma}^{CDE}\right)\hat{F}_{BCDE}\hat{\epsilon} \\ &= e^{\frac{\Phi}{6}}\left(\left(e_a^\mu\partial_\mu + \frac{1}{4}\omega_{abc}\hat{\Gamma}^{bc} - \frac{1}{6}\partial_a\Phi - \frac{1}{6}\hat{\Gamma}_a^c\partial_c\Phi + \frac{1}{4}e^\Phi F_{ac}\hat{\Gamma}^{cz}\right)\right. \\ &\quad \left. + \frac{1}{288}e^\Phi\hat{\Gamma}_a\not{F}_4 - \frac{1}{24}e^\Phi\hat{\Gamma}^{bcd}F_{abcd} + \frac{1}{72}\hat{\Gamma}_a\not{H}_3\hat{\Gamma}^z - \frac{1}{8}\not{H}_\mu\hat{\Gamma}^z\right)\epsilon, \end{aligned} \quad (\text{A.7})$$

where we have introduced the notation $\not{F} = F_{\mu_1\dots\mu_n}\Gamma^{\mu_1\dots\mu_n}$ and the relation between eleven and ten dimensional spinors

$$\hat{\epsilon} = e^{-\frac{\Phi}{6}}\epsilon. \quad (\text{A.8})$$

We then decompose the eleven-dimensional Gamma matrices in terms of the ten-dimensional ones in the following way

$$\hat{\Gamma}^a = \Gamma^a\sigma_1, \quad \hat{\Gamma}^z = \sigma_3, \quad a = 1, \dots, 10, \quad (\text{A.9})$$

which is related to the decomposition of the ten-dimensional Majorana spinor into its chiral components

$$\epsilon = \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}, \quad (\text{A.10})$$

satisfying $\Gamma^{11}\epsilon = -\sigma_3\epsilon$. Using (A.4)–(A.9) we then identify the supersymmetry variations for the dilatino and gravitino. For non-zero Romans mass, the expressions obtained can be slightly generalised to

$$\begin{aligned} \delta\lambda &= \not{\partial}\Phi\epsilon - \frac{1}{2\cdot 3!}\not{H}\sigma_3\epsilon + \frac{e^\Phi}{4}\left(5F_0\sigma_1 + \frac{3}{2!}\not{F}_2i\sigma_2 + \frac{1}{4!}\not{F}_4\sigma_1\right)\epsilon, \\ \delta\Psi_\mu &= \nabla_\mu\epsilon - \frac{1}{8}\not{H}_\mu\sigma_3\epsilon + \frac{e^\Phi}{8}\left(5F_0\sigma_1 + \frac{1}{2!}\not{F}_2i\sigma_2 + \frac{1}{4!}\not{F}_4\sigma_1\right)\Gamma_\mu\epsilon. \end{aligned} \quad (\text{A.11})$$

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