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Four-loop QCD propagators and vertices with one vanishing external momentum

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Abstract: We have computed the self-energies and a set of three-particle vertex functions for massless QCD at the four-loop level in the $\overline{\text{MS}}$ renormalization scheme. The vertex functions are evaluated at points where one of the momenta vanishes. Analytical results are obtained for a generic gauge group and with the full gauge dependence, which was made possible by extensive use of the Forcer program for massless four-loop propagator integrals. The bare results in dimensional regularization are provided in terms of master integrals and rational coefficients; the latter are exact in any space-time dimension. Our results can be used for further precision investigations of the perturbative behaviour of the theory in schemes other than $\overline{\text{MS}}$. As an example, we derive the five-loop beta function in a relatively common alternative, the minimal momentum subtraction (MiniMOM) scheme.

Keywords: Perturbative QCD, Renormalization Group

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Contents

1 Introduction

Motivated by the goal to obtain predictions that are as precise as possible for quantities entering benchmark high-energy processes, there have been significant recent developments of computational techniques and methodologies in higher-order perturbative calculations for Quantum Chromodynamics (QCD). As a result, computations of the basic renormalization group functions of QCD [1–14] have recently reached the five-loop level [15–21]. In the modified minimal subtraction ($\overline{\text{MS}}$) scheme [22, 23], the standard renormalization scheme for perturbative QCD which is closely related to dimensional regularization [24, 25], these functions are obtained by computing single poles in corresponding Green's functions. Since only the poles are required, the above four- and five-loop results were first obtained using the method of infrared rearrangement [26–34] which simplifies computations without changing the ultraviolet singular structure, but modifies the finite parts.

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The main aim of this work is to provide the self-energies and a set of vertices with one vanishing external momentum for massless QCD at the four-loop level. The unrenormalized results are exact in terms of $\varepsilon = (D-4)/2$, where D is the space-time dimension, and four-loop master integrals [35, 36]. The renormalized $D=4$ results are given in the $\overline{\text{MS}}$ scheme. The computation has been performed for a general gauge group and in an arbitrary covariant linear gauge, by using the FORCER program [37–39] for massless four-loop propagator-type integrals. For the vertices, setting one of the momenta to zero effectively reduces vertex integrals to propagator-type integrals. In QCD this does not create IR divergences, which means the poles do not change. At the three-loop level, similar computations were performed in ref. [40], but with an expansion in ε. In addition, studies of QCD vertices in perturbation theory for various configurations include refs. [41–53].

We compute all QCD vertices in a general linear covariant gauge, with the exception of the four-gluon vertex for which there are at least three difficulties: first, two momenta have to be nullified before the diagrams become propagator-like. Second, the number of diagrams is large at four loops. Third, the colour structure for a generic group is no longer an overall factor, but will be term dependent. Additionally, the renormalization constant is completely determined through the Slavnov-Taylor identities [54, 55], which means the quartic gluon vertex does not provide extra information in this context.

A direct application of our results is to compute conversion factors from the $\overline{\text{MS}}$ scheme to momentum subtraction schemes, see, e.g., refs. [41, 44], for renormalization group functions. Unlike the $\overline{\text{MS}}$ scheme, momentum subtraction schemes are defined in a regularization-independent way. In these schemes, the field renormalizations are performed such that finite radiative corrections on propagators are absorbed as well as divergences and hence they coincide with their tree-level values at the renormalization point. Then one of (or an arbitrary linear combination of) the vertex functions is normalized to its tree-level value and the other vertices are fixed via the Slavnov-Taylor identities. Common choices for the subtraction point of the vertex are a symmetric point (referred as MOM schemes) and an asymmetric point where one of the momenta is nullified, sometimes referred as $\widetilde{\text{MOM}}$ schemes. The latter choice corresponds to our result for the vertex functions. Indeed, ref. [40] derived four-loop beta functions in four particular $\widetilde{\text{MOM}}$ schemes from that in the $\overline{\text{MS}}$ scheme by computing conversion factors via finite parts of two- and three-point functions in the $\overline{\text{MS}}$ scheme.

As an example application, we provide the five-loop beta function in the minimal momentum subtraction (MiniMOM) scheme introduced in ref. [56], thus extending previous results [56, 57] by one order in the coupling constant. This scheme, see the preceeding references for a detailed discussion, is more convenient than $\overline{\text{MS}}$ for extending analyses of the strong coupling constant and its scale dependence into the non-perturbative regime, e.g., via lattice QCD; for a recent analysis see ref. [58]. In the perturbative regime the MiniMOM scheme provides an alternative to $\overline{\text{MS}}$ for studying the behaviour and truncation uncertainty of the perturbation series for benchmark quantities such as the R-ratio in e^+e^- annihilation and the Higgs-boson decay to gluons, see refs. [59–61].

The remainder of this article is organized as follows: in section 2 we specify our notations for the self-energies and vertex functions and recall their renormalization. In section 3 we address technical details and checks of the computation. Due to their length, the results for the renormalized self-energies and vertex functions for a generic gauge group and a general linear covariant gauge are deferred to appendix A. In section 4 the five-loop beta function is presented in the MiniMOM scheme for QCD in the Landau gauge; the general result and the corresponding $\overline{\text{MS}}$-to-MiniMOM conversion factor for the coupling constant are provided in appendix B. In section 5 we summarize and give a brief outlook.

Our result can be obtained in a digital form as supplementary material to this article. They are also available from the authors upon request. The files contain the bare results for the self-energies and vertices in terms of master integrals with coefficients that are exact for any dimension D, as well as the results in the $\overline{\text{MS}}$ scheme for $D = 4$. The notations in these files can be found in appendix C.

2 Preliminaries

We first summarize the notations for self-energies and vertex functions with one vanishing momentum presented in this article. In most cases we follow the conventions in ref. [40].[1]

2.1 Self energies

The gluon, ghost and quark self-energies (figure 1) are of the form

$$\Pi^{ab}_{\mu\nu}(q) = -\delta^{ab}(q^2 g_{\mu\nu} - q_\mu q_\nu)\Pi(q^2), \tag{2.1}$$

$$\tilde{\Pi}^{ab}(q) = \delta^{ab} q^2 \tilde{\Pi}(q^2), \tag{2.2}$$

$$\Sigma^{ij}(q) = \delta^{ij} \not{q} \Sigma_V(q^2). \tag{2.3}$$

The colour indices are understood such that a and b are for the adjoint representation of the gauge group, i and j for the representation to which the quark belongs. The form factors $\Pi(q^2)$, $\tilde{\Pi}(q^2)$ and $\Sigma_V(q^2)$ can easily be extracted from contributions of the corresponding one-particle irreducible diagrams by applying projection operators [40] (the same holds for the vertex functions discussed below). They are related to the full gluon, ghost and quark propagators as follows:

$$D^{ab}_{\mu\nu}(q) = \frac{\delta^{ab}}{-q^2}\left[\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\frac{1}{1+\Pi(q^2)} - \xi\,\frac{q_\mu q_\nu}{q^2}\right], \tag{2.4}$$

$$\Delta^{ab}(q) = \frac{\delta^{ab}}{-q^2}\,\frac{1}{1+\tilde{\Pi}(q^2)}\,, \tag{2.5}$$

$$S^{ij}(q) = \frac{\delta^{ij}}{-q^2}\,\frac{\not{q}}{1+\Sigma_V(q^2)}\,. \tag{2.6}$$

Here the Landau gauge corresponds to $\xi = 0$, and the Feynman gauge to $\xi = 1$. We note that this convention differs from that in the widely used FORM version [62] of the MINCER program [63] for three-loop self-energies, where the symbol `xi` represents $1 - \xi$.

[1] We note that these conventions may be different from the ones commonly used in the literature. In fact, the Feynman rules in FORCER are different as well, and hence we occasionally had to convert intermediate results from one convention to the other and back.

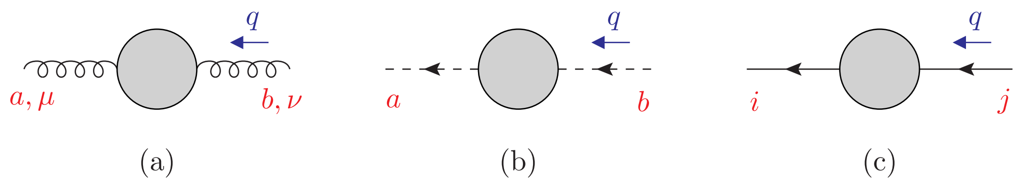

Figure 1. The gluon, ghost and quark self-energies $\Pi^{ab}_{\mu\nu}(q)$ (a), $\tilde{\Pi}^{ab}(q)$ (b) and $\Sigma^{ij}(q)$ (c).

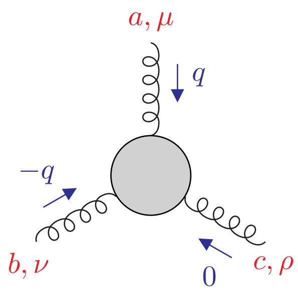

Figure 2. The triple-gluon vertex with one vanishing momentum, $\Gamma^{abc}_{\mu\nu\rho}(q, -q, 0)$.

2.2 Triple-gluon vertex

Without loss of generality, one can set the momentum of the third gluon to zero, as depicted in figure 2. Then the triple-gluon vertex can be written in the following form:

$$\Gamma^{abc}_{\mu\nu\rho}(q, -q, 0) = -igf^{abc}\left[(2g_{\mu\nu}q_\rho - g_{\mu\rho}q_\nu - g_{\rho\nu}q_\mu)\, T_1(q^2) - \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) q_\rho T_2(q^2)\right], \tag{2.7}$$

where g is the coupling constant and f^{abc} are the structure constants of the gauge group. The first term in the square bracket corresponds to the tree-level vertex while the second term arises from radiative corrections, i.e., at the tree-level the form factors $T_{1,2}(q^2)$ read

$$T_1(q^2)\big|_{\text{tree}} = 1, \qquad T_2(q^2)\big|_{\text{tree}} = 0. \tag{2.8}$$

Because of Furry's theorem [64] and the fact that we have no colour-neutral particles, symmetric invariants with an odd number of indices cannot occur for internal fermion lines. Neither can such invariants occur for the adjoint representation. Hence, if we project out a d^{abc} structure, we would get a scalar invariant with an odd number of f tensors, and such a combination must be zero. This has been checked explicitly to the equivalent of six-loop vertices in ref [65]. Due to the bosonic property of gluons, the totally antisymmetric colour factor f^{abc} leads to antisymmetric Lorentz structure as in eq. (2.7). One could consider another Lorentz structure,

$$-igf^{abc}q_\mu q_\nu q_\rho T_3(q^2). \tag{2.9}$$

However, a Slavnov-Taylor identity requires $T_3(q^2)$ to vanish.

2.3 Ghost-gluon vertex

Since the tree-level vertex is proportional to the outgoing ghost momentum, nullifying this momentum gives identically zero in perturbation theory. Therefore, we only have

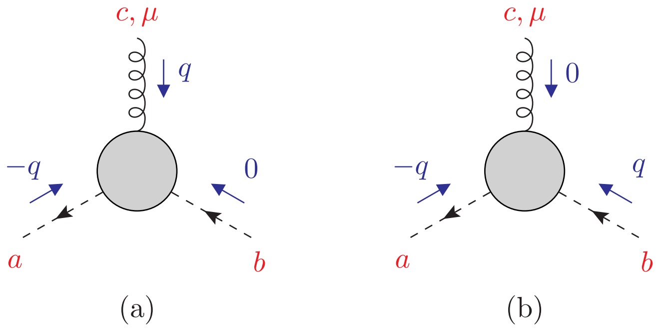

Figure 3. The ghost-gluon vertex: (a) $\tilde{\Gamma}^{abc}_{\mu}(-q, 0; q)$ with the vanishing incoming ghost momentum and (b) $\tilde{\Gamma}^{abc}_{\mu}(-q, q; 0)$ with the vanishing gluon momentum.

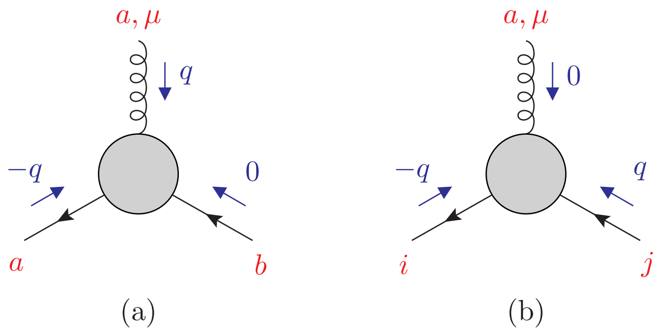

Figure 4. The quark-gluon vertex: (a) $\Lambda^{a}_{\mu,ij}(-q, 0; q)$ with the vanishing incoming quark momentum and (b) $\Lambda^{a}_{\mu,ij}(-q, q; 0)$ with the vanishing gluon momentum.

two possibilities to set one of the external momenta to zero. One is the incoming ghost momentum and the other is the gluon momentum (figure 3):

$$\tilde{\Gamma}^{abc}_{\mu}(-q, 0; q) = -igf^{abc}q_{\mu}\tilde{\Gamma}_{h}(q^2), \tag{2.10}$$

$$\tilde{\Gamma}^{abc}_{\mu}(-q, q; 0) = -igf^{abc}q_{\mu}\tilde{\Gamma}_{g}(q^2). \tag{2.11}$$

The subscript 'h' of $\tilde{\Gamma}_h(q^2)$ indicates the function with vanishing incoming ghost momentum, whereas 'g' of $\tilde{\Gamma}_g(q^2)$ denotes the vanishing gluon momentum. These functions are equal to one at the tree-level,

$$\tilde{\Gamma}_h(q^2)\big|_{\text{tree}} = \tilde{\Gamma}_g(q^2)\big|_{\text{tree}} = 1. \tag{2.12}$$

2.4 Quark-gluon vertex

We consider the case of a vanishing incoming quark momentum and the case of a vanishing gluon momentum (figure 4). Nullifying the outgoing quark momentum gives the same result as nullifying the incoming quark momentum. Then the vertex can be written as

$$\Lambda^{a}_{\mu,ij}(-q, 0; q) = gT^{a}_{ij}\left[\gamma_{\mu}\Lambda_{q}(q^2) + \gamma^{\nu}\left(g_{\mu\nu} - \frac{q_{\mu}q_{\nu}}{q^2}\right)\Lambda^{T}_{q}(q^2)\right], \tag{2.13}$$

$$\Lambda^{a}_{\mu,ij}(-q, q; 0) = gT^{a}_{ij}\left[\gamma_{\mu}\Lambda_{g}(q^2) + \gamma^{\nu}\left(g_{\mu\nu} - \frac{q_{\mu}q_{\nu}}{q^2}\right)\Lambda^{T}_{g}(q^2)\right]. \tag{2.14}$$

T_{ij} is the generator of the representation for the quark. The subscript 'q' indicates the functions with vanishing incoming quark momentum and 'g' indicates those with vanishing gluon momentum. At the tree-level we have

$$\Lambda_q(q^2)\big|_{\text{tree}} = \Lambda_g(q^2)\big|_{\text{tree}} = 1, \tag{2.15}$$

$$\Lambda_q^T(q^2)\big|_{\text{tree}} = \Lambda_g^T(q^2)\big|_{\text{tree}} = 0. \tag{2.16}$$

2.5 Renormalization

In a generic renormalization scheme R, the respective renormalizations of the gluon, ghost and quark fields can be written as

$$(A^B)_\mu^a = \sqrt{Z_3^R}\,(A^R)_\mu^a, \tag{2.17}$$

$$(\eta^B)^a = \sqrt{\tilde{Z}_3^R}\,(\eta^R)^a, \tag{2.18}$$

$$\psi_{if}^B = \sqrt{Z_2^R}\,\psi_{if}^R. \tag{2.19}$$

The superscript 'B' indicates a bare quantity and 'R' a renormalized one. For the coupling constant, we define $a = \alpha_\text{s}/(4\pi) = g^2/(16\pi^2)$. Then a and the gauge parameter ξ are renormalized in dimensional regularization ($D = 4 - 2\varepsilon$) as follows:

$$a^B = \mu^{2\varepsilon} Z_a^R a^R, \tag{2.20}$$

$$\xi^B = Z_3^R \xi^R. \tag{2.21}$$

Here μ is the 't Hooft mass scale. We have used the fact that the gauge parameter is also renormalized by the gluon field renormalization constant, $Z_\xi^R = Z_3^R$. The renormalization of the self-energies and vertex functions is performed as

$$1 + \Pi^R = Z_3^R(1 + \Pi^B), \tag{2.22}$$

$$1 + \tilde{\Pi}^R = \tilde{Z}_3^R(1 + \tilde{\Pi}^B), \tag{2.23}$$

$$1 + \Sigma_V^R = Z_2^R(1 + \Sigma_V^B), \tag{2.24}$$

and

$$T_i^R = Z_1^R T_i^B, \qquad i = 1, 2, \tag{2.25}$$

$$\tilde{\Gamma}_i^R = \tilde{Z}_1^R \tilde{\Gamma}_i^B, \qquad i = h, g, \tag{2.26}$$

$$\Lambda_i^R = \bar{Z}_1^R \Lambda_i^B, \qquad \Lambda_i^{T,R} = \bar{Z}_1^R \Lambda_i^{T,B}, \qquad i = q, g, \tag{2.27}$$

where the vertex renormalization constants are related to the field and coupling renormalization constants via the Slavnov-Taylor identities by

$$\sqrt{Z_a^R Z_3^R} = \frac{Z_1^R}{Z_3^R} = \frac{\tilde{Z}_1^R}{\tilde{Z}_3^R} = \frac{\bar{Z}_1^R}{Z_2^R}\,. \tag{2.28}$$

In MS-like schemes, the renormalization constants contain only pole terms with respect to ε and thus take the form

$$Z_i^{\text{MS}} = 1 + \sum_{l=1}^{\infty} a^l \sum_{n=1}^{l} \frac{Z_i^{\text{MS},(l,n)}}{\varepsilon^n}\,. \tag{2.29}$$

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The coefficients $Z_i^{\mathrm{MS},(l,n)}$ are determined order by order in such a way that any renormalized Green's function becomes finite. The choice of the subtraction point defines the specific (MS-like) renormalization scheme; the beta function is independent of this choice.

3 Computations and checks

The results in this article are obtained by direct computation using the Forcer package [37–39], written in Form [66–68]. Forcer is a four-loop equivalent of Mincer [62, 63]: for each topology a parametric reduction to simpler topologies is provided. Most topologies can be reduced using direct integration of one-loop insertions, by the triangle rule [69, 70] or by the diamond rule [71]. If these substructures are absent, a manually designed reduction scheme is required. Through parametric integration-by-parts (IBP) identities [69, 70], similar to the triangle rule, such reductions are found in a computer-assisted manner.

Great care has to be taken to prevent term blow-up from rewriting momenta to a new basis for the transitions between topologies. In Mincer this was done manually, but in Forcer a basis is found that minimizes the terms that are created automatically. The performance of the Forcer program has been demonstrated by computing the four-loop beta function in ten minutes in the Feynman gauge and in eight hours with all powers of the gauge parameter on a modern 24-core machine [37, 38].

The diagrams have been generated using Qgraf [72]. Subsequently, we filter propagator insertions. These insertions are known lower-loop propagators, and can be factorized out of the diagram, see ref. [73]. Next, the topology is mapped to a built-in Forcer topology, after nullifying a leg for the vertices, and the colour factor is determined using the Form program of ref. [65]. To extract the form factors defined above, a generalization of the projection operators in ref. [40] to a generic gauge group is used. Then the Feynman rules are applied. The remaining Lorentz-scalar integrals (which include loop-momenta in numerators) are computed by the Forcer program.

The computation time varied between an hour and a week, on a single computer. The easy cases, such as the ghost propagator and quark propagator took an hour. The gluon propagators and ghost-ghost-gluon vertex and quark-quark-gluon vertex took about eight hours per configuration. The triple gluon vertex was the hardest case and took a week per configuration on a single machine with 24 cores. Had we chosen to compute with an expansion in ε, the computations would have been much faster.

We have checked our setup and results in various ways:

- The longitudinal component of the gluon self-energy $\delta^{ab} q_\mu q_\nu \Pi_L(q^2)$ was shown to be zero by an explicit calculation at the four-loop level.

- The form factor $T_3(q^2)$ of the triple-gluon vertex in eq. (2.9) was computed and indeed vanished at the four-loop level.

- All the self-energies and vertex functions computed in this work were compared up to three loops with those in ref. [40]. Note that the finite parts of the vertex-function results in ref. [40] are only correct for SU(N) gauge groups, since the presence of quartic Casimir operators was not taken into account in the reconstruction of the general case. This fact was also noted in ref. [20].

- The four-loop renormalization constants and anomalous dimensions for the case of SU(N) and a general linear covariant gauge were provided in ancillary files [74] of ref. [13]. Directly after FORCER was completed, we established agreement with those results. For a generic group our results are in agreement with ancillary files of ref. [20].

- We remark that the ghost-gluon vertex is unrenormalized $\tilde{Z}_1^{\overline{\rm MS}} = \tilde{Z}_3^{\overline{\rm MS}}\sqrt{Z_a^{\overline{\rm MS}} Z_3^{\overline{\rm MS}}} = 1$ in the Landau gauge. Moreover, our results confirm that the vertex has no radiative corrections when the incoming ghost momentum is nullified (i.e., $\tilde{\Gamma}_h^{\overline{\rm MS}} = 1$) in the Landau gauge up to four loops.

The renormalized results for all self-energies and vertex functions are rather lengthy and are thus given in appendix A. The unrenormalized results in terms of master integrals, and the values of the master integrals in $4-2\varepsilon$ dimensions available online as supplementary material to this article. A description of the files is given in appendix C.

4 Five-loop Landau-gauge QCD beta function in the MiniMOM scheme

In the MiniMOM scheme [56], the self-energies are completely absorbed into the field renormalization constants at the subtraction point $q^2 = -\mu^2$:

$$1 + \Pi^{\rm MM}(-\mu^2) = Z_3^{\rm MM}\Big[1 + \Pi^B(-\mu^2)\Big] = 1, \tag{4.1}$$

$$1 + \tilde{\Pi}^{\rm MM}(-\mu^2) = \tilde{Z}_3^{\rm MM}\Big[1 + \tilde{\Pi}^B(-\mu^2)\Big] = 1, \tag{4.2}$$

$$1 + \Sigma_V^{\rm MM}(-\mu^2) = Z_2^{\rm MM}\Big[1 + \Sigma_V^B(-\mu^2)\Big] = 1. \tag{4.3}$$

Here the superscript 'MM' indicates a quantity in the MiniMOM scheme. In addition, motivated by the non-renormalization of the ghost-gluon vertex in the Landau gauge [54], the vertex renormalization constant for this vertex is chosen the same as that in $\overline{\rm MS}$,

$$\tilde{Z}_1^{\rm MM} = \tilde{Z}_1^{\overline{\rm MS}}, \tag{4.4}$$

which is equal to one in the Landau gauge.

The above renormalization conditions lead to the following relations for the coupling constant and gauge parameter in the two schemes:

$$a^{\rm MM}(\mu^2) = a^{\overline{\rm MS}}(\mu^2)\, \frac{1}{\big[1 + \Pi^{\overline{\rm MS}}(-\mu^2)\big]\big[1 + \tilde{\Pi}^{\overline{\rm MS}}(-\mu^2)\big]^2}\,, \tag{4.5}$$

$$\xi^{\overline{\rm MS}}(\mu^2) = \xi^{\rm MM}(\mu^2)\, \frac{1}{1 + \Pi^{\overline{\rm MS}}(-\mu^2)}\,. \tag{4.6}$$

Eq. (4.5) allows one to convert a value of $\alpha_s^{\overline{\rm MS}}$ to $\alpha_s^{\rm MM}$. For example, $\alpha_s^{\overline{\rm MS}}(M_Z^2) = 0.118$ leads to $\alpha_s^{\rm MM}(M_Z^2) = 1.096\,\alpha_s^{\overline{\rm MS}}(M_Z^2)$ for QCD in the Landau gauge with $n_f = 5$ quark flavours. The general expansion of eq. (4.5) is given in appendix B.

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The scale dependence of the coupling constant in eq. (4.5) in this scheme is given by

$$\beta^{\rm MM} = \mu^2 \frac{da^{\rm MM}}{d\mu^2} = \frac{\partial a^{\rm MM}}{\partial a^{\overline{\rm MS}}} \beta^{\overline{\rm MS}} + \frac{\partial a^{\rm MM}}{\partial \xi^{\overline{\rm MS}}} \gamma_3^{\overline{\rm MS}} \xi^{\overline{\rm MS}}, \tag{4.7}$$

where we have used the beta function and gluon field anomalous dimension in $\overline{\rm MS}$,

$$\beta^{\overline{\rm MS}} = \mu^2 \frac{da^{\overline{\rm MS}}}{d\mu^2}, \tag{4.8}$$

$$\gamma_3^{\overline{\rm MS}} \xi^{\overline{\rm MS}} = \mu^2 \frac{d\xi^{\overline{\rm MS}}}{d\mu^2}. \tag{4.9}$$

Note that the right-hand side of eq. (4.5), and hence that of eq. (4.7), is naturally given in terms of $a^{\overline{\rm MS}}$ and $\xi^{\overline{\rm MS}}$. One has to convert them into $a^{\rm MM}$ and $\xi^{\rm MM}$ by inverting the series of eq. (4.5) and by using eq. (4.6).[2]

Having results for the four-loop self-energies in the $\overline{\rm MS}$ scheme at hand, one can obtain the five-loop beta function in the MiniMOM scheme from the five-loop beta function [16, 19] and the four-loop gluon field anomalous dimension in the $\overline{\rm MS}$ scheme. The result for SU(3) in the Landau gauge ($\xi^{\rm MM} = 0$) reads

$$\beta^{\rm MM} = -\sum_{l=0}^{4} \left(a^{\rm MM}\right)^{l+2} \beta_l^{\rm MM} + \mathcal{O}\left(\left(a^{\rm MM}\right)^7\right), \tag{4.10}$$

with

$$\begin{aligned}
\beta_0^{\rm MM} &= 11 - \frac{2}{3} n_f, \\
\beta_1^{\rm MM} &= 102 - \frac{38}{3} n_f, \\
\beta_2^{\rm MM} &= \left(\frac{28965}{8} - \frac{3861}{8}\zeta_3\right) + n_f\left(-\frac{7715}{12} + \frac{175}{12}\zeta_3\right) + n_f^2\left(\frac{989}{54} + \frac{8}{9}\zeta_3\right), \\
\beta_3^{\rm MM} &= \left(\frac{1380469}{8} - \frac{625317}{16}\zeta_3 - \frac{772695}{32}\zeta_5\right) + n_f\left(-\frac{970819}{24} + \frac{516881}{72}\zeta_3 + \frac{1027375}{144}\zeta_5\right) \\
&\quad + n_f^2\left(\frac{736541}{324} - \frac{6547}{27}\zeta_3 - \frac{9280}{27}\zeta_5\right) + n_f^3\left(-\frac{800}{27} + \frac{16}{9}\zeta_3\right), \\
\beta_4^{\rm MM} &= \left(\frac{3248220045}{256} - \frac{1064190195}{512}\zeta_3 - \frac{4922799165}{512}\zeta_5 - \frac{7696161}{64}\zeta_3^2 + \frac{21619456551}{4096}\zeta_7\right) \\
&\quad + n_f\left(-\frac{115659378547}{31104} + \frac{10327103555}{20736}\zeta_3 + \frac{18219328375}{6912}\zeta_5 + \frac{82869}{32}\zeta_3^2\right. \\
&\quad \left. - \frac{24870449471}{18432}\zeta_7\right) + n_f^2\left(\frac{833934985}{2592} - \frac{13019053}{1296}\zeta_3 - \frac{65264845}{324}\zeta_5 + \frac{59531}{36}\zeta_3^2\right. \\
&\quad \left. + \frac{26952037}{432}\zeta_7\right) + n_f^3\left(-\frac{3249767}{324} - \frac{129869}{162}\zeta_3 + \frac{299875}{54}\zeta_5 - \frac{2240}{27}\zeta_3^2\right) + n_f^4\left(\frac{2617}{27}\right. \\
&\quad \left. + \frac{304}{27}\zeta_3 - \frac{1760}{27}\zeta_5\right).
\end{aligned} \tag{4.11}$$

[2] In ref. [56], the results are presented in $\xi^{\overline{\rm MS}}$ instead of $\xi^{\rm MM}$. On the other hand, in ref. [57] the conversion from $\xi^{\overline{\rm MS}}$ to $\xi^{\rm MM}$ was performed. The results become the same in the Landau gauge $\xi^{\overline{\rm MS}} = \xi^{\rm MM} = 0$. The same is true for the "$\widetilde{\rm MOMh}$" scheme of ref. [40].

The result for a generic group and in an arbitrary covariant linear gauge can be found in appendix B; it agrees with the result given in ref. [57] up to four loops. As is well known, the first coefficient $\beta_0^{\rm MM}$ is scheme independent. The next coefficient $\beta_1^{\rm MM}$ has a gauge dependence and the universal value is obtained only in the Landau gauge. The last coefficient $\beta_4^{\rm MM}$ is the new result. In the $\overline{\rm MS}$ scheme, some of higher values of the zeta function (e.g., ζ_3^2, ζ_6 and ζ_7 at five loops) do not occur, for a discussion of this issue see refs. [21, 35, 75]. In contrast, one cannot expect their absence in the MiniMOM scheme. Indeed eq. (4.11) includes terms with ζ_3^2 and ζ_7, and for $\xi^{\rm MM} \neq 0$ also ζ_6 occurs.

The numerical values of the above beta function for three to five quark flavours are

$$\begin{aligned}
\tilde\beta^{\rm MM}(n_f=3) &= 1 + 0.5658842421\alpha_{\rm s}^{\rm MM} \;\; + 0.9419859046(\alpha_{\rm s}^{\rm MM})^2 \\
&\quad + 2.304494526(\alpha_{\rm s}^{\rm MM})^3 + 6.647485913(\alpha_{\rm s}^{\rm MM})^4, \\
\tilde\beta^{\rm MM}(n_f=4) &= 1 + 0.4901972247\alpha_{\rm s}^{\rm MM} \;\; + 0.6452147391(\alpha_{\rm s}^{\rm MM})^2 \\
&\quad + 1.638457168(\alpha_{\rm s}^{\rm MM})^3 + 3.466865543(\alpha_{\rm s}^{\rm MM})^4, \\
\tilde\beta^{\rm MM}(n_f=5) &= 1 + 0.4013472477\alpha_{\rm s}^{\rm MM} \;\; + 0.3288519562(\alpha_{\rm s}^{\rm MM})^2 \\
&\quad + 1.026892491(\alpha_{\rm s}^{\rm MM})^3 + 0.8417657296(\alpha_{\rm s}^{\rm MM})^4,
\end{aligned} \tag{4.12}$$

where $\tilde\beta \equiv \beta(a)/(-\beta_0 a^2)$ has been re-expanded in powers of $\alpha_{\rm s} = 4\pi a$. These values may be compared with those in the $\overline{\rm MS}$ scheme [16, 19] reading

$$\begin{aligned}
\tilde\beta^{\overline{\rm MS}}(n_f=3) &= 1 + 0.5658842421\alpha_{\rm s}^{\overline{\rm MS}} \;\; + 0.4530135791(\alpha_{\rm s}^{\overline{\rm MS}})^2 \\
&\quad + 0.6769674420(\alpha_{\rm s}^{\overline{\rm MS}})^3 + 0.5809276379(\alpha_{\rm s}^{\overline{\rm MS}})^4, \\
\tilde\beta^{\overline{\rm MS}}(n_f=4) &= 1 + 0.4901972247\alpha_{\rm s}^{\overline{\rm MS}} \;\; + 0.3087903795(\alpha_{\rm s}^{\overline{\rm MS}})^2 \\
&\quad + 0.4859007965(\alpha_{\rm s}^{\overline{\rm MS}})^3 + 0.2806008338(\alpha_{\rm s}^{\overline{\rm MS}})^4, \\
\tilde\beta^{\overline{\rm MS}}(n_f=5) &= 1 + 0.4013472477\alpha_{\rm s}^{\overline{\rm MS}} \;\; + 0.1494273313(\alpha_{\rm s}^{\overline{\rm MS}})^2 \\
&\quad + 0.3172233974(\alpha_{\rm s}^{\overline{\rm MS}})^3 + 0.08092104151(\alpha_{\rm s}^{\overline{\rm MS}})^4.
\end{aligned} \tag{4.13}$$

Obviously, the MiniMOM coefficients in eqs. (4.12) are (much) larger than their $\overline{\rm MS}$ counterparts in eqs. (4.13) starting from the second order; moreover, they exhibit a definite growth with the order that is absent in the $\overline{\rm MS}$ case. One may expect that this behaviour, and the larger value of $\alpha_{\rm s}^{\rm MM}$, is more than compensated by smaller expansion coefficients for observables, leading to a better overall convergence in MOM-like schemes. However, this issue has been studied up to four loops in some detail for the R-ratio in electron-positron annihilation, without arriving at such a clear-cut conclusion [59].

5 Summary and outlook

We have computed the four-loop three-particle vertices with one vanishing momentum and the four-loop self-energies for QCD-like theories in a manner that is as general as presently possible. Our results, explicitly presented in the appendix of this article for the $D = 4$ renormalized quantities in the $\overline{\rm MS}$ scheme, and available online in terms of their more lengthy bare counterparts, should be useful for precision studies of QCD-like theories with any simple compact gauge group, in any linear covariant gauge, for any MS-like or $\widetilde{\text{MOM}}$-like renormalization scheme (see also ref. [40]), and in any number of space-time dimensions D. The latter requires replacing the master integrals, which is the only component that is not known exactly in D.

As an example application, we have determined the five-loop beta function in the MiniMOM scheme of ref. [56], i.e., we have extended the result of refs. [56, 57] by one order in the coupling constant α_{s}. This function appears to have a higher-order structure quite different from that in the $\overline{\text{MS}}$ scheme, thus inviting further studies especially for the physical case of QCD in four dimensions.

Our computations have been made possible by the construction of FORCER [37–39], a four-loop generalization of the well-known MINCER program [62, 63] for the parametric reduction of three-loop massless self-energy integrals. We have verified that, except for an issue in the transition from SU(N) to a general gauge group that was also noted in ref. [20], our renormalized self-energies and vertices up to three loops agree with the results of ref. [40]. Furthermore, we have verified to four loops that the ghost-ghost-gluon vertex is unrenormalized in the Landau gauge, i.e., its anomalous dimension is proportional to ξ and that the gluon propagator is transverse. Earlier, we had checked the four-loop beta function and the four-loop SU(N) [74] renormalization constants and anomalous dimensions of ref. [13].

Performing a similar calculation at five loops is a formidable challenge: so far most methods used for computations at five loops use infrared rearrangement, which modifies the finite terms. A direct computation, as has been performed in this article, would require a five-loop FORCER equivalent. This is hard for at least two reasons: the number of topologies that need a manually designed step-by-step IBP reduction is larger than 200, and the number of parameters increases from 14 at four loops to 20 at five loops. Additionally, given the size of the step from three to four loops, the required computer time could be an issue.

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A Four-loop $\overline{\text{MS}}$ results for self-energies and vertices

We expand the results for the scalar 'form factors' of the self-energies and vertices addressed in eqs. (2.1)–(2.16) at the point $q^2 = -\mu^2$ in the $\overline{\text{MS}}$ scheme as

$$\Pi^{\overline{\text{MS}}}(-\mu^2) = \sum_{l=1}^{4} a^l \Pi^{\overline{\text{MS}},(l)} + \mathcal{O}(a^5), \tag{A.1}$$

$$\tilde{\Pi}^{\overline{\text{MS}}}(-\mu^2) = \sum_{l=1}^{4} a^l \tilde{\Pi}^{\overline{\text{MS}},(l)} + \mathcal{O}(a^5), \tag{A.2}$$

$$\Sigma_V^{\overline{\text{MS}}}(-\mu^2) = \sum_{l=1}^{4} a^l \Sigma_V^{\overline{\text{MS}},(l)} + \mathcal{O}(a^5), \tag{A.3}$$

$$T_1^{\overline{\text{MS}}}(-\mu^2) = 1 + \sum_{l=1}^{4} a^l T_1^{\overline{\text{MS}},(l)} + \mathcal{O}(a^5), \tag{A.4}$$

$$T_2^{\overline{\text{MS}}}(-\mu^2) = \sum_{l=1}^{4} a^l T_2^{\overline{\text{MS}},(l)} + \mathcal{O}(a^5), \tag{A.5}$$

$$\tilde{\Gamma}_i^{\overline{\text{MS}}}(-\mu^2) = 1 + \sum_{l=1}^{4} a^l \tilde{\Gamma}_i^{\overline{\text{MS}},(l)} + \mathcal{O}(a^5), \qquad i = h, g, \tag{A.6}$$

$$\Lambda_i^{\overline{\text{MS}}}(-\mu^2) = 1 + \sum_{l=1}^{4} a^l \Lambda_i^{\overline{\text{MS}},(l)} + \mathcal{O}(a^5), \qquad i = q, g, \tag{A.7}$$

$$\Lambda_i^{T,\overline{\text{MS}}}(-\mu^2) = \sum_{l=1}^{4} a^l \Lambda_i^{T,\overline{\text{MS}},(l)} + \mathcal{O}(a^5), \qquad i = q, g. \tag{A.8}$$

Here the coupling constant $a = \alpha_s/(4\pi) = g^2/(16\pi^2)$ and the gauge parameter ξ are the renormalized ones in the $\overline{\text{MS}}$ scheme, i.e., $a^{\overline{\text{MS}}}(\mu^2)$ and $\xi^{\overline{\text{MS}}}(\mu^2)$. The Landau gauge corresponds to $\xi = 0$. Recall that this differs from the convention in Mincer and Forcer, where $\xi = 0$ corresponds to the Feynman gauge and $\xi = 1$ to the Landau gauge.

A.1 Gluon self-energy

$$\Pi^{\overline{\text{MS}},(1)} = \left(-\frac{97}{36}C_A + \frac{20}{9}T_F n_f\right) - \frac{1}{2}C_A\xi - \frac{1}{4}C_A\xi^2,$$

$$\begin{aligned}\Pi^{\overline{\text{MS}},(2)} = &\left[C_A^2\left(-\frac{2381}{96} + 3\zeta_3\right) + C_A T_F n_f\left(\frac{59}{4} + 8\zeta_3\right) + C_F T_F n_f\left(\frac{55}{3} - 16\zeta_3\right)\right] \\ &+ \left[C_A^2\left(\frac{463}{288} - 2\zeta_3\right) - \frac{10}{9}C_A T_F n_f\right]\xi + \left(\frac{95}{144}C_A^2 - \frac{10}{9}C_A T_F n_f\right)\xi^2 - \frac{1}{16}C_A^2\xi^3 \\ &+ \frac{1}{16}C_A^2\xi^4,\end{aligned}$$

$$\begin{aligned}\Pi^{\overline{\text{MS}},(3)} = &\left[C_A^3\left(-\frac{10221367}{31104} + \frac{1549}{24}\zeta_3 + \frac{9}{32}\zeta_4 + \frac{7025}{192}\zeta_5\right) + C_A^2 T_F n_f\left(\frac{1154561}{3888} + \frac{871}{18}\zeta_3\right.\right. \\ &\left. - 9\zeta_4 - \frac{160}{3}\zeta_5\right) + C_A C_F T_F n_f\left(\frac{96809}{324} - \frac{1492}{9}\zeta_3 + 12\zeta_4 - 80\zeta_5\right) + C_F^2 T_F n_f\left(-\frac{286}{9}\right.\end{aligned}$$

$$
-\frac{296}{3}\zeta_3+160\zeta_5\Big)+C_AT_F^2n_f^2\Big(-\frac{10499}{243}-\frac{256}{9}\zeta_3\Big)+C_FT_F^2n_f^2\Big(-\frac{7402}{81}+\frac{608}{9}\zeta_3\Big)\Big]
$$

$$
+\Big[C_A^3\Big(\frac{13141}{1152}-\frac{12071}{288}\zeta_3+\frac{3}{8}\zeta_4+\frac{115}{8}\zeta_5\Big)+C_A^2T_Fn_f\Big(-\frac{241}{24}+\frac{202}{9}\zeta_3\Big)
$$

$$
+C_AC_FT_Fn_f\Big(-\frac{55}{6}+8\zeta_3\Big)+C_AT_F^2n_f^2\Big(\frac{16}{9}-\frac{64}{9}\zeta_3\Big)\Big]\xi+\Big[C_A^3\Big(\frac{30835}{10368}-\frac{161}{96}\zeta_3
$$

$$
+\frac{3}{32}\zeta_4+\frac{385}{96}\zeta_5\Big)+C_A^2T_Fn_f\Big(-\frac{6137}{1296}-\frac{25}{6}\zeta_3\Big)+C_AC_FT_Fn_f\Big(-\frac{55}{6}+8\zeta_3\Big)
$$

$$
-\frac{100}{81}C_AT_F^2n_f^2\Big]\xi^2+\Big[C_A^3\Big(-\frac{2813}{1152}+\frac{149}{96}\zeta_3-\frac{5}{24}\zeta_5\Big)-\frac{5}{18}C_A^2T_Fn_f\Big]\xi^3+\Big[C_A^3\Big(-\frac{29}{48}
$$

$$
+\frac{13}{96}\zeta_3-\frac{35}{192}\zeta_5\Big)+\frac{5}{12}C_A^2T_Fn_f\Big]\xi^4+\frac{1}{16}C_A^3\xi^5-\frac{1}{64}C_A^3\xi^6,
$$

$$
\Pi^{\overline{\rm MS},(4)}=\Big[C_A^4\Big(-\frac{4698915983}{746496}+\frac{122449741}{82944}\zeta_3+\frac{49117}{2048}\zeta_4+\frac{338738527}{110592}\zeta_5-\frac{238325}{4096}\zeta_6
$$

$$
-\frac{2360029}{18432}\zeta_3^2-\frac{9266411}{6144}\zeta_7\Big)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\Big(\frac{3319}{3456}-\frac{311113}{768}\zeta_3-\frac{989}{32}\zeta_4-\frac{226975}{512}\zeta_5
$$

$$
-\frac{50925}{512}\zeta_6+\frac{753337}{768}\zeta_3^2-\frac{99757}{128}\zeta_7\Big)+C_A^3T_Fn_f\Big(\frac{42950657}{5832}-\frac{232351}{1296}\zeta_3-\frac{2767}{32}\zeta_4
$$

$$
-\frac{1650287}{432}\zeta_5+\frac{275}{4}\zeta_6-\frac{557}{18}\zeta_3^2+\frac{9471}{4}\zeta_7\Big)+C_A^2C_FT_Fn_f\Big(\frac{20572427}{2916}-\frac{324935}{162}\zeta_3
$$

$$
+61\zeta_4-\frac{36166}{9}\zeta_5+75\zeta_6+410\zeta_3^2-\frac{2800}{3}\zeta_7\Big)+C_AC_F^2T_Fn_f\Big(-\frac{576533}{648}-\frac{192157}{54}\zeta_3
$$

$$
+\frac{245}{6}\zeta_4+\frac{29210}{9}\zeta_5-150\zeta_6-60\zeta_3^2+2240\zeta_7\Big)+C_F^3T_Fn_f\Big(-\frac{31}{3}+104\zeta_3+1960\zeta_5
$$

$$
-2240\zeta_7\Big)+\frac{d_F^{abcd}d_A^{abcd}}{N_A}n_f\Big(-\frac{4544}{27}+1964\zeta_3+172\zeta_4+1005\zeta_5+75\zeta_6-\frac{5350}{3}\zeta_3^2
$$

$$
+\frac{49}{2}\zeta_7\Big)+C_A^2T_F^2n_f^2\Big(-\frac{44177531}{23328}-\frac{156817}{162}\zeta_3-\frac{41}{6}\zeta_4+\frac{16586}{27}\zeta_5-\frac{704}{9}\zeta_3^2\Big)
$$

$$
+C_AC_FT_F^2n_f^2\Big(-\frac{13241219}{2916}+\frac{124210}{81}\zeta_3-22\zeta_4+\frac{6008}{3}\zeta_5+256\zeta_3^2\Big)
$$

$$
+C_F^2T_F^2n_f^2\Big(-\frac{7505}{162}+\frac{49696}{27}\zeta_3+\frac{88}{3}\zeta_4-\frac{16000}{9}\zeta_5-256\zeta_3^2\Big)+\frac{d_F^{abcd}d_F^{abcd}}{N_A}n_f^2\Big(\frac{6896}{27}
$$

$$
-336\zeta_3-64\zeta_4+320\zeta_5-\frac{512}{3}\zeta_3^2\Big)+C_AT_F^3n_f^3\Big(\frac{113743}{1458}+\frac{4280}{27}\zeta_3+\frac{8}{3}\zeta_4-\frac{640}{27}\zeta_5\Big)
$$

$$
+C_FT_F^3n_f^3\Big(\frac{393026}{729}-\frac{25888}{81}\zeta_3-\frac{1280}{9}\zeta_5\Big)\Big]+\Big[C_A^4\Big(\frac{129316433}{1492992}-\frac{51101257}{82944}\zeta_3
$$

$$
+\frac{8411}{1536}\zeta_4+\frac{1847129}{6144}\zeta_5-\frac{51175}{6144}\zeta_6+\frac{63007}{3072}\zeta_3^2-\frac{7806659}{36864}\zeta_7\Big)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\Big(-\frac{385}{96}
$$

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$$
\begin{aligned}
&-\frac{33515}{192}\zeta_3+\frac{1347}{128}\zeta_4+\frac{322195}{768}\zeta_5-\frac{2375}{256}\zeta_6-\frac{17449}{128}\zeta_3^2-\frac{3997}{384}\zeta_7\Bigg)\\
&+C_A^3T_Fn_f\left(-\frac{21636697}{186624}+\frac{260717}{432}\zeta_3+\frac{5}{6}\zeta_4-\frac{22703}{144}\zeta_5+\frac{29}{6}\zeta_3^2+\frac{539}{8}\zeta_7\right)\\
&+C_A^2C_FT_Fn_f\left(-\frac{485489}{2592}+\frac{1625}{9}\zeta_3-\frac{3}{2}\zeta_4+67\zeta_5\right)+C_AC_F^2T_Fn_f\Bigg(\frac{143}{9}+\frac{148}{3}\zeta_3\\
&-80\zeta_5\Bigg)+\frac{d_F^{abcd}d_A^{abcd}}{N_A}n_f\left(\frac{992}{3}\zeta_3-340\zeta_5+16\zeta_3^2-\frac{147}{2}\zeta_7\right)+C_A^2T_F^2n_f^2\Bigg(\frac{552001}{11664}\\
&-\frac{5452}{27}\zeta_3+\zeta_4+\frac{230}{9}\zeta_5\Bigg)+C_AC_FT_F^2n_f^2\left(\frac{4781}{81}-\frac{784}{9}\zeta_3\right)+C_AT_F^3n_f^3\Bigg(-\frac{320}{81}\\
&+\frac{1280}{81}\zeta_3\Bigg)\Bigg]\xi+\Bigg[C_A^4\Bigg(\frac{882095}{27648}-\frac{193639}{4608}\zeta_3+\frac{365}{1536}\zeta_4+\frac{69395}{1152}\zeta_5-\frac{175}{512}\zeta_6-\frac{147}{256}\zeta_3^2\\
&-\frac{147553}{6144}\zeta_7\Bigg)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\Bigg(-\frac{487}{384}+\frac{1313}{96}\zeta_3+\frac{9}{8}\zeta_4-\frac{5195}{128}\zeta_5+\frac{375}{128}\zeta_6-\frac{1501}{64}\zeta_3^2\\
&+\frac{2625}{64}\zeta_7\Bigg)+C_A^3T_Fn_f\left(-\frac{298517}{3456}-\frac{8369}{288}\zeta_3+\frac{41}{6}\zeta_4+\frac{1397}{48}\zeta_5+\frac{5}{6}\zeta_3^2\right)\\
&+C_A^2C_FT_Fn_f\left(-\frac{82181}{648}+\frac{413}{6}\zeta_3-9\zeta_4+40\zeta_5\right)+C_AC_F^2T_Fn_f\left(\frac{143}{9}+\frac{148}{3}\zeta_3-80\zeta_5\right)\\
&+C_A^2T_F^2n_f^2\left(\frac{227}{36}+\frac{50}{9}\zeta_3-\frac{10}{9}\zeta_5\right)+C_AC_FT_F^2n_f^2\left(\frac{2051}{81}-16\zeta_3\right)\Bigg]\xi^2\\
&+\Bigg[C_A^4\left(-\frac{1100549}{41472}+\frac{651137}{27648}\zeta_3-\frac{73}{256}\zeta_4-\frac{515119}{55296}\zeta_5+\frac{475}{6144}\zeta_6-\frac{985}{3072}\zeta_3^2+\frac{15785}{9216}\zeta_7\right)\\
&+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\left(\frac{283}{64}\zeta_3+\frac{3}{128}\zeta_4-\frac{3255}{256}\zeta_5+\frac{275}{256}\zeta_6-\frac{633}{128}\zeta_3^2+\frac{2737}{384}\zeta_7\right)\\
&+C_A^3T_Fn_f\left(-\frac{4751}{1296}-\frac{1213}{108}\zeta_3+\frac{595}{432}\zeta_5+\frac{1}{6}\zeta_3^2\right)+C_A^2C_FT_Fn_f\left(-\frac{55}{24}+2\zeta_3\right)\\
&+C_A^2T_F^2n_f^2\left(-\frac{97}{81}+\frac{32}{9}\zeta_3\right)\Bigg]\xi^3+\Bigg[C_A^4\Bigg(-\frac{14803}{2048}+\frac{90341}{27648}\zeta_3-\frac{181}{2048}\zeta_4-\frac{310055}{110592}\zeta_5\\
&+\frac{575}{12288}\zeta_6+\frac{1279}{6144}\zeta_3^2-\frac{189}{256}\zeta_7\Bigg)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\Bigg(-\frac{1195}{768}\zeta_3+\frac{3}{64}\zeta_4+\frac{4505}{1536}\zeta_5+\frac{25}{512}\zeta_6\\
&+\frac{329}{256}\zeta_3^2-\frac{567}{128}\zeta_7\Bigg)+C_A^3T_Fn_f\left(-\frac{875}{1728}+\frac{59}{27}\zeta_3-\frac{175}{216}\zeta_5\right)+C_A^2C_FT_Fn_f\left(\frac{55}{16}-3\zeta_3\right)\\
&+\frac{25}{27}C_A^2T_F^2n_f^2\Bigg]\xi^4+\Bigg[C_A^4\left(-\frac{121}{1536}-\frac{157}{384}\zeta_3-\frac{4565}{9216}\zeta_5+\frac{21}{512}\zeta_3^2+\frac{4823}{36864}\zeta_7\right)\\
&+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\left(-\frac{5}{64}\zeta_3+\frac{35}{384}\zeta_5+\frac{15}{64}\zeta_3^2-\frac{91}{192}\zeta_7\right)+\frac{5}{12}C_A^3T_Fn_f\Bigg]\xi^5+\Bigg[C_A^4\Bigg(\frac{341}{1152}\\
&-\frac{13}{192}\zeta_3+\frac{35}{384}\zeta_5\Bigg)-\frac{5}{36}C_A^3T_Fn_f\Bigg]\xi^6-\frac{7}{256}C_A^4\xi^7+\frac{1}{256}C_A^4\xi^8. \qquad \text{(A.9)}
\end{aligned}
$$

A.2 Ghost self-energy

$$
\begin{aligned}
\tilde{\Pi}^{\overline{\mathrm{MS}},(1)} &= -C_A, \\
\tilde{\Pi}^{\overline{\mathrm{MS}},(2)} &= \left[C_A^2\left(-\frac{1751}{192}+\frac{15}{16}\zeta_3\right)+\frac{95}{24}C_A T_F n_f\right]+C_A^2\left(\frac{7}{64}-\frac{3}{8}\zeta_3\right)\xi+C_A^2\left(-\frac{3}{8}+\frac{3}{16}\zeta_3\right)\xi^2, \\
\tilde{\Pi}^{\overline{\mathrm{MS}},(3)} &= \left[C_A^3\left(-\frac{466373}{3888}+\frac{12403}{576}\zeta_3-\frac{9}{64}\zeta_4+\frac{65}{32}\zeta_5\right)+C_A^2 T_F n_f\left(\frac{150247}{1944}+\frac{29}{9}\zeta_3+\frac{9}{2}\zeta_4\right)\right. \\
&\quad \left.+C_A C_F T_F n_f\left(\frac{899}{24}-22\zeta_3-6\zeta_4\right)+C_A T_F^2 n_f^2\left(-\frac{5161}{486}-\frac{8}{9}\zeta_3\right)\right]+\left[C_A^3\left(-\frac{61}{576}\right.\right. \\
&\quad \left.\left.-\frac{1429}{192}\zeta_3-\frac{3}{16}\zeta_4+\frac{65}{32}\zeta_5\right)+C_A^2 T_F n_f\left(-\frac{199}{144}+\frac{13}{6}\zeta_3\right)\right]\xi+C_A^3\left(-\frac{327}{128}+\frac{93}{64}\zeta_3-\frac{3}{64}\zeta_4\right. \\
&\quad \left.-\frac{35}{32}\zeta_5\right)\xi^2+C_A^3\left(-\frac{361}{384}+\frac{23}{64}\zeta_3+\frac{5}{32}\zeta_5\right)\xi^3, \\
\tilde{\Pi}^{\overline{\mathrm{MS}},(4)} &= \left[C_A^4\left(-\frac{13485064225}{5971968}+\frac{26382331}{55296}\zeta_3-\frac{139015}{12288}\zeta_4+\frac{9816437}{24576}\zeta_5+\frac{238325}{8192}\zeta_6\right.\right. \\
&\quad \left.-\frac{83463}{4096}\zeta_3^2-\frac{711739}{4096}\zeta_7\right)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\left(\frac{3259}{256}-\frac{202049}{512}\zeta_3-\frac{1827}{64}\zeta_4+\frac{363115}{1024}\zeta_5\right. \\
&\quad \left.+\frac{50925}{1024}\zeta_6-\frac{15879}{512}\zeta_3^2+\frac{106827}{1024}\zeta_7\right)+C_A^3 T_F n_f\left(\frac{267238975}{124416}+\frac{51919}{1152}\zeta_3+\frac{2731}{64}\zeta_4\right. \\
&\quad \left.-\frac{21887}{96}\zeta_5-\frac{275}{8}\zeta_6-\frac{121}{8}\zeta_3^2+\frac{441}{8}\zeta_7\right)+C_A^2 C_F T_F n_f\left(\frac{1805161}{1728}-\frac{13667}{24}\zeta_3-27\zeta_4\right. \\
&\quad \left.-213\zeta_5-\frac{75}{2}\zeta_6+9\zeta_3^2\right)+C_A C_F^2 T_F n_f\left(-\frac{4393}{48}-\frac{947}{4}\zeta_3-\frac{111}{4}\zeta_4+285\zeta_5+75\zeta_6\right. \\
&\quad \left.+30\zeta_3^2\right)+\frac{d_F^{abcd}d_A^{abcd}}{N_A}n_f\left(318\zeta_3+18\zeta_4-\frac{665}{2}\zeta_5-\frac{75}{2}\zeta_6-15\zeta_3^2\right) \\
&\quad +C_A^2 T_F^2 n_f^2\left(-\frac{2051269}{3456}-\frac{3469}{36}\zeta_3-\frac{45}{4}\zeta_4+\frac{67}{2}\zeta_5\right)+C_A C_F T_F^2 n_f^2\left(-\frac{36485}{108}+\frac{689}{3}\zeta_3\right. \\
&\quad \left.\left.+15\zeta_4+12\zeta_5\right)+C_A T_F^3 n_f^3\left(\frac{150979}{2916}+\frac{20}{27}\zeta_3-\frac{4}{3}\zeta_4\right)\right]+\left[C_A^4\left(\frac{24242833}{1492992}-\frac{315153}{2048}\zeta_3\right.\right. \\
&\quad \left.-\frac{1195}{1024}\zeta_4+\frac{1747691}{36864}\zeta_5+\frac{7675}{12288}\zeta_6+\frac{20387}{6144}\zeta_3^2+\frac{105217}{12288}\zeta_7\right)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\left(\frac{139}{64}+8\zeta_3\right. \\
&\quad \left.-\frac{15}{256}\zeta_4-\frac{17735}{512}\zeta_5-\frac{2425}{512}\zeta_6+\frac{1807}{256}\zeta_3^2+\frac{119}{8}\zeta_7\right)+C_A^3 T_F n_f\left(-\frac{5301809}{186624}+\frac{14683}{192}\zeta_3\right. \\
&\quad \left.-\frac{15}{32}\zeta_4-\frac{2459}{288}\zeta_5+\frac{7}{2}\zeta_3^2\right)+C_A^2 C_F T_F n_f\left(-\frac{1321}{96}+\frac{143}{8}\zeta_3+\frac{3}{4}\zeta_4+\frac{7}{2}\zeta_5-6\zeta_3^2\right) \\
&\quad \left.+C_A^2 T_F^2 n_f^2\left(\frac{173743}{23328}-\frac{17}{2}\zeta_3+\frac{1}{2}\zeta_4-\frac{10}{3}\zeta_5\right)\right]\xi+\left[C_A^4\left(-\frac{9026467}{221184}+\frac{33203}{1536}\zeta_3-\frac{75}{512}\zeta_4\right.\right. \\
&\quad \left.-\frac{199099}{9216}\zeta_5-\frac{225}{1024}\zeta_6-\frac{257}{256}\zeta_3^2+\frac{100037}{12288}\zeta_7\right)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\left(\frac{125}{256}-\frac{205}{32}\zeta_3+\frac{27}{128}\zeta_4\right.
\end{aligned}
$$

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$$+\frac{3005}{256}\zeta_5-\frac{525}{256}\zeta_6-\frac{51}{128}\zeta_3^2-\frac{2639}{512}\zeta_7\bigg)+C_A^3T_Fn_f\bigg(\frac{71593}{6912}-\frac{827}{128}\zeta_3-\frac{3}{32}\zeta_4+\frac{211}{72}\zeta_5$$

$$+\frac{5}{8}\zeta_3^2\bigg)\bigg]\xi^2+\bigg[C_A^4\bigg(-\frac{13949}{1024}+\frac{13575}{2048}\zeta_3+\frac{25}{512}\zeta_4+\frac{11203}{12288}\zeta_5-\frac{1975}{12288}\zeta_6-\frac{527}{6144}\zeta_3^2$$

$$-\frac{30961}{12288}\zeta_7\bigg)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\bigg(\frac{435}{128}\zeta_3+\frac{141}{256}\zeta_4-\frac{1765}{512}\zeta_5-\frac{875}{512}\zeta_6+\frac{317}{256}\zeta_3^2-\frac{7}{4}\zeta_7\bigg)\bigg]\xi^3$$

$$+\bigg[C_A^4\bigg(-\frac{35579}{12288}+\frac{6319}{6144}\zeta_3+\frac{41}{4096}\zeta_4+\frac{2985}{8192}\zeta_5-\frac{175}{24576}\zeta_6-\frac{515}{12288}\zeta_3^2+\frac{231}{1024}\zeta_7\bigg)$$

$$+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\bigg(-\frac{523}{512}\zeta_3-\frac{3}{16}\zeta_4+\frac{345}{1024}\zeta_5+\frac{175}{1024}\zeta_6+\frac{155}{512}\zeta_3^2+\frac{483}{1024}\zeta_7\bigg)\bigg]\xi^4. \quad (A.10)$$

A.3 Quark self-energy

$$\Sigma_V^{\overline{\mathrm{MS}},(1)}=C_F\xi,$$

$$\Sigma_V^{\overline{\mathrm{MS}},(2)}=\bigg[C_AC_F\bigg(\frac{41}{4}-3\zeta_3\bigg)-\frac{5}{8}C_F^2-\frac{7}{2}C_FT_Fn_f\bigg]+C_AC_F\bigg(\frac{13}{2}-3\zeta_3\bigg)\xi+\frac{9}{8}C_AC_F\xi^2,$$

$$\Sigma_V^{\overline{\mathrm{MS}},(3)}=\bigg[C_A^2C_F\bigg(\frac{159257}{648}-\frac{3139}{24}\zeta_3-\frac{69}{16}\zeta_4+\frac{165}{4}\zeta_5\bigg)+C_AC_F^2\bigg(-\frac{997}{24}+44\zeta_3+6\zeta_4$$

$$-20\zeta_5\bigg)-\frac{73}{12}C_F^3+C_AC_FT_Fn_f\bigg(-\frac{11887}{81}+\frac{52}{3}\zeta_3\bigg)+C_F^2T_Fn_f\bigg(-\frac{79}{6}+16\zeta_3\bigg)$$

$$+\frac{1570}{81}C_FT_F^2n_f^2\bigg]+\bigg[C_A^2C_F\bigg(\frac{39799}{576}-35\zeta_3+\frac{3}{8}\zeta_4+\frac{5}{2}\zeta_5\bigg)+C_AC_F^2(4-17\zeta_3+20\zeta_5)$$

$$+\frac{7}{8}C_F^3+C_AC_FT_Fn_f\bigg(-\frac{1723}{72}+8\zeta_3\bigg)-\frac{3}{2}C_F^2T_Fn_f\bigg]\xi+\bigg[C_A^2C_F\bigg(\frac{787}{64}-\frac{39}{8}\zeta_3+\frac{3}{16}\zeta_4$$

$$+\frac{5}{4}\zeta_5\bigg)+\frac{3}{2}C_AC_F^2\bigg]\xi^2+\bigg[C_A^2C_F\bigg(\frac{55}{24}-\frac{1}{3}\zeta_3\bigg)+C_AC_F^2\bigg(-\frac{1}{8}+\zeta_3\bigg)-\frac{2}{3}C_F^3\zeta_3\bigg]\xi^3,$$

$$\Sigma_V^{\overline{\mathrm{MS}},(4)}=\bigg[C_A^3C_F\bigg(\frac{164005363}{31104}-\frac{1073621}{288}\zeta_3-\frac{18627}{512}\zeta_4+\frac{2256031}{1152}\zeta_5-\frac{20725}{256}\zeta_6$$

$$+\frac{10795}{64}\zeta_3^2-\frac{264467}{384}\zeta_7\bigg)+C_A^2C_F^2\bigg(-\frac{1278281}{864}-\frac{2263}{3}\zeta_3-\frac{1263}{16}\zeta_4+\frac{1669}{8}\zeta_5$$

$$+\frac{3925}{8}\zeta_6-\frac{351}{4}\zeta_3^2+\frac{27895}{16}\zeta_7\bigg)+C_AC_F^3\bigg(\frac{278975}{288}+\frac{32359}{8}\zeta_3+318\zeta_4-\frac{1225}{3}\zeta_5$$

$$-900\zeta_6+156\zeta_3^2-\frac{10185}{2}\zeta_7\bigg)+C_F^4\bigg(-\frac{42945}{128}-\frac{3889}{2}\zeta_3-150\zeta_4-970\zeta_5+400\zeta_6$$

$$-80\zeta_3^2+3528\zeta_7\bigg)+\frac{d_F^{abcd}d_A^{abcd}}{N_R}\bigg(-\frac{455}{2}+\frac{613}{16}\zeta_3+\frac{3339}{64}\zeta_4+\frac{12415}{32}\zeta_5-\frac{4275}{64}\zeta_6$$

$$+\frac{4929}{32}\zeta_3^2-\frac{4697}{8}\zeta_7\bigg)+C_A^2C_FT_Fn_f\bigg(-\frac{5660951}{1296}+\frac{46643}{24}\zeta_3-\frac{9}{16}\zeta_4-\frac{63769}{72}\zeta_5-50\zeta_6$$

$$
\begin{aligned}
&+84\zeta_3^2-\frac{441}{2}\zeta_7\bigg)+C_AC_F^2T_Fn_f\bigg(-\frac{447197}{432}+\frac{779}{6}\zeta_3+118\zeta_5+100\zeta_6-160\zeta_3^2\\
&+882\zeta_7\bigg)+C_F^3T_Fn_f\bigg(\frac{21775}{144}-103\zeta_3-24\zeta_4+\frac{20}{3}\zeta_5\bigg)+\frac{d_F^{abcd}d_F^{abcd}}{N_R}n_f\big(474+1124\zeta_3\\
&-1240\zeta_5-384\zeta_3^2\big)+C_AC_FT_F^2n_f^2\bigg(\frac{124783}{108}-\frac{280}{3}\zeta_3+12\zeta_4+\frac{440}{9}\zeta_5\bigg)\\
&+C_F^2T_F^2n_f^2\bigg(\frac{79531}{216}-\frac{796}{3}\zeta_3-12\zeta_4\bigg)+C_FT_F^3n_f^3\bigg(-\frac{21391}{243}-\frac{16}{9}\zeta_3\bigg)\bigg]\\
&+\bigg[C_A^3C_F\bigg(\frac{842973089}{746496}-\frac{536887}{864}\zeta_3+\frac{2799}{256}\zeta_4-\frac{51577}{576}\zeta_5-\frac{2875}{192}\zeta_6+\frac{8659}{96}\zeta_3^2\\
&-\frac{791}{24}\zeta_7\bigg)+C_A^2C_F^2\bigg(\frac{72335}{648}-\frac{7471}{36}\zeta_3-\frac{45}{8}\zeta_4+\frac{19865}{72}\zeta_5+\frac{25}{8}\zeta_6-\frac{165}{4}\zeta_3^2+\frac{707}{16}\zeta_7\bigg)\\
&+C_AC_F^3\bigg(\frac{635}{48}-\frac{405}{8}\zeta_3+6\zeta_4+230\zeta_5-\frac{399}{2}\zeta_7\bigg)-\frac{91}{12}C_F^4+\frac{d_F^{abcd}d_A^{abcd}}{N_R}\bigg(\frac{9}{2}+252\zeta_3\\
&+\frac{303}{16}\zeta_4-240\zeta_5-\frac{275}{8}\zeta_6-\frac{7}{4}\zeta_3^2-\frac{1547}{32}\zeta_7\bigg)+C_A^2C_FT_Fn_f\bigg(-\frac{62286421}{93312}+\frac{4826}{27}\zeta_3\\
&-\frac{183}{8}\zeta_4+\frac{2227}{18}\zeta_5+14\zeta_3^2\bigg)+C_AC_F^2T_Fn_f\bigg(-\frac{716687}{2592}+\frac{4303}{18}\zeta_3+\frac{57}{2}\zeta_4-\frac{1034}{9}\zeta_5\\
&-32\zeta_3^2\bigg)+C_F^3T_Fn_f\bigg(-\frac{131}{6}+24\zeta_3\bigg)+C_AC_FT_F^2n_f^2\bigg(\frac{118117}{1458}-\frac{356}{27}\zeta_3+2\zeta_4-\frac{80}{3}\zeta_5\bigg)\\
&+C_F^2T_F^2n_f^2\bigg(\frac{202}{81}+\frac{32}{3}\zeta_3\bigg)\bigg]\xi+\bigg[C_A^3C_F\bigg(\frac{5085305}{27648}-\frac{103229}{1152}\zeta_3+\frac{663}{256}\zeta_4+\frac{1387}{72}\zeta_5\\
&-\frac{475}{384}\zeta_6-\frac{181}{96}\zeta_3^2+\frac{7}{12}\zeta_7\bigg)+C_A^2C_F^2\bigg(\frac{2743}{576}+\frac{101}{24}\zeta_3+\frac{3}{8}\zeta_4+30\zeta_5+\frac{9}{2}\zeta_3^2-\frac{35}{4}\zeta_7\bigg)\\
&+C_AC_F^3\bigg(\frac{75}{64}-28\zeta_3+20\zeta_5\bigg)+3C_F^4\zeta_3+\frac{d_F^{abcd}d_A^{abcd}}{N_R}\bigg(\frac{23}{8}\zeta_3+\frac{33}{32}\zeta_4-\frac{735}{16}\zeta_5-\frac{175}{32}\zeta_6\\
&-\frac{35}{16}\zeta_3^2+\frac{679}{16}\zeta_7\bigg)+C_A^2C_FT_Fn_f\bigg(-\frac{78361}{1728}+\frac{1009}{72}\zeta_3-\frac{7}{16}\zeta_4-\frac{149}{72}\zeta_5+\zeta_3^2\bigg)\\
&+C_AC_F^2T_Fn_f\bigg(-\frac{353}{144}-\frac{32}{3}\zeta_3\bigg)+4C_F^3T_Fn_f\zeta_3\bigg]\xi^2+\bigg[C_A^3C_F\bigg(\frac{19907}{512}-\frac{1379}{96}\zeta_3+\frac{87}{256}\zeta_4\\
&+\frac{153}{64}\zeta_5-\frac{1}{16}\zeta_3^2+\frac{147}{128}\zeta_7\bigg)+C_A^2C_F^2\bigg(-\frac{271}{64}+\frac{75}{8}\zeta_3+\frac{3}{16}\zeta_4+\frac{5}{4}\zeta_5\bigg)+C_AC_F^3\bigg(\frac{3}{2}\\
&-4\zeta_3\bigg)+\frac{d_F^{abcd}d_A^{abcd}}{N_R}\bigg(-4\zeta_3-\frac{9}{16}\zeta_4-\frac{15}{2}\zeta_5-3\zeta_3^2+\frac{441}{32}\zeta_7\bigg)\bigg]\xi^3+\bigg[C_A^3C_F\bigg(\frac{2263}{384}\\
&-\frac{389}{384}\zeta_3+\frac{1}{512}\zeta_4+\frac{35}{384}\zeta_5+\frac{25}{768}\zeta_6-\frac{5}{192}\zeta_3^2\bigg)+C_A^2C_F^2\bigg(-\frac{449}{384}+\frac{29}{12}\zeta_3\bigg)+C_AC_F^3\bigg(\frac{1}{2}\\
&-\frac{1}{2}\zeta_3\bigg)-\frac{2}{3}C_F^4\zeta_3+\frac{d_F^{abcd}d_A^{abcd}}{N_R}\bigg(-\frac{35}{16}\zeta_3-\frac{21}{64}\zeta_4+\frac{95}{32}\zeta_5+\frac{25}{64}\zeta_6-\frac{19}{32}\zeta_3^2\bigg)\bigg]\xi^4. \qquad \text{(A.11)}
\end{aligned}
$$

A.4 Triple-gluon vertex

$$
\begin{aligned}
T_1^{\overline{\mathrm{MS}},(1)} &= \left(-\frac{61}{36}C_A + \frac{20}{9}T_F n_f\right) - \frac{1}{4}C_A \xi^2, \\
T_1^{\overline{\mathrm{MS}},(2)} &= \left[C_A^2\left(-\frac{9907}{576} + \frac{13}{8}\zeta_3\right) + C_A T_F n_f\left(\frac{955}{72} + 8\zeta_3\right) + C_F T_F n_f\left(\frac{55}{3} - 16\zeta_3\right)\right] \\
&\quad + C_A^2\left(\frac{153}{64} - \frac{15}{8}\zeta_3\right)\xi + \left(\frac{35}{36}C_A^2 - \frac{10}{9}C_A T_F n_f\right)\xi^2 - \frac{3}{16}C_A^2\xi^3 + \frac{1}{16}C_A^2\xi^4, \\
T_1^{\overline{\mathrm{MS}},(3)} &= \left[C_A^3\left(-\frac{156221}{648} + \frac{81685}{2304}\zeta_3 + \frac{27}{64}\zeta_4 + \frac{33295}{768}\zeta_5\right) + \frac{d_A^{abcd}d_A^{abcd}}{N_A C_A}\left(\frac{2}{3} + \frac{499}{32}\zeta_3\right.\right. \\
&\quad \left. - \frac{565}{32}\zeta_5\right) + C_A^2 T_F n_f\left(\frac{682607}{2592} + \frac{3733}{72}\zeta_3 - \frac{27}{2}\zeta_4 - \frac{160}{3}\zeta_5\right) + C_A C_F T_F n_f\left(\frac{181711}{648}\right. \\
&\quad \left. - \frac{1438}{9}\zeta_3 + 18\zeta_4 - 80\zeta_5\right) + C_F^2 T_F n_f\left(-\frac{286}{9} - \frac{296}{3}\zeta_3 + 160\zeta_5\right) + C_A T_F^2 n_f^2\left(-\frac{3415}{81}\right. \\
&\quad \left.\left. - \frac{248}{9}\zeta_3\right) + C_F T_F^2 n_f^2\left(-\frac{7402}{81} + \frac{608}{9}\zeta_3\right)\right] + \left[C_A^3\left(\frac{43963}{2304} - \frac{31501}{768}\zeta_3 + \frac{9}{16}\zeta_4\right.\right. \\
&\quad \left. + \frac{30185}{2304}\zeta_5\right) + \frac{d_A^{abcd}d_A^{abcd}}{N_A C_A}\left(\frac{1}{6} - \frac{43}{96}\zeta_3 + \frac{325}{96}\zeta_5\right) + C_A^2 T_F n_f\left(-\frac{521}{96} + \frac{1889}{72}\zeta_3\right) \\
&\quad \left. + C_A T_F^2 n_f^2\left(\frac{16}{9} - \frac{64}{9}\zeta_3\right)\right]\xi + \left[C_A^3\left(\frac{130151}{20736} - \frac{5149}{2304}\zeta_3 + \frac{9}{64}\zeta_4 + \frac{10015}{2304}\zeta_5\right)\right. \\
&\quad + \frac{d_A^{abcd}d_A^{abcd}}{N_A C_A}\left(-\frac{1}{12} - \frac{101}{96}\zeta_3 + \frac{215}{96}\zeta_5\right) + C_A^2 T_F n_f\left(-\frac{9511}{2592} - \frac{25}{6}\zeta_3\right) \\
&\quad \left. + C_A C_F T_F n_f\left(-\frac{55}{6} + 8\zeta_3\right) - \frac{100}{81}C_A T_F^2 n_f^2\right]\xi^2 + \left[C_A^3\left(-\frac{271}{192} + \frac{3239}{2304}\zeta_3 - \frac{305}{2304}\zeta_5\right)\right. \\
&\quad \left. + \frac{d_A^{abcd}d_A^{abcd}}{N_A C_A}\left(-\frac{17}{96}\zeta_3 + \frac{35}{96}\zeta_5\right) - \frac{5}{6}C_A^2 T_F n_f\right]\xi^3 + \left[C_A^3\left(-\frac{149}{192} + \frac{13}{96}\zeta_3 - \frac{35}{192}\zeta_5\right)\right. \\
&\quad \left. + \frac{5}{12}C_A^2 T_F n_f\right]\xi^4 + \frac{3}{32}C_A^3\xi^5 - \frac{1}{64}C_A^3\xi^6, \\
T_1^{\overline{\mathrm{MS}},(4)} &= \left[C_A^4\left(-\frac{83311536457}{17915904} + \frac{277163107}{331776}\zeta_3 + \frac{435973}{12288}\zeta_4 + \frac{697315451}{221184}\zeta_5 - \frac{714975}{8192}\zeta_6\right.\right. \\
&\quad \left. - \frac{3674197}{36864}\zeta_3^2 - \frac{228033463}{147456}\zeta_7\right) + \frac{d_A^{abcd}d_A^{abcd}}{N_A}\left(\frac{32405}{3456} + \frac{7355}{1152}\zeta_3 - \frac{151}{64}\zeta_4 - \frac{2825035}{3072}\zeta_5\right. \\
&\quad \left. - \frac{152775}{1024}\zeta_6 + \frac{1424413}{1536}\zeta_3^2 - \frac{3981719}{6144}\zeta_7\right) + C_A^3 T_F n_f\left(\frac{2301578797}{373248} - \frac{25253}{864}\zeta_3\right. \\
&\quad \left. - \frac{8621}{64}\zeta_4 - \frac{13206259}{3456}\zeta_5 + \frac{825}{8}\zeta_6 - \frac{2677}{144}\zeta_3^2 + \frac{904463}{384}\zeta_7\right) + C_A^2 C_F T_F n_f\left(\frac{152575171}{23328}\right. \\
&\quad \left. - \frac{292085}{162}\zeta_3 + \frac{575}{6}\zeta_4 - \frac{34879}{9}\zeta_5 + \frac{225}{2}\zeta_6 + 409\zeta_3^2 - \frac{2800}{3}\zeta_7\right) \\
&\quad + C_A C_F^2 T_F n_f\left(-\frac{1076125}{1296} - \frac{369401}{108}\zeta_3 + \frac{823}{12}\zeta_4 + \frac{28085}{9}\zeta_5 - 225\zeta_6 - 90\zeta_3^2 + 2240\zeta_7\right)
\end{aligned}
$$

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$$+C_F^3T_Fn_f\Big(-\frac{31}{3}+104\zeta_3+1960\zeta_5-2240\zeta_7\Big)+\frac{d_F^{abcd}d_A^{abcd}}{N_A}n_f\Big(-\frac{4577}{27}+\frac{5288}{3}\zeta_3$$

$$+154\zeta_4+\frac{6985}{6}\zeta_5+\frac{225}{2}\zeta_6-\frac{5383}{3}\zeta_3^2+98\zeta_7\Big)+\frac{d_A^{abcd}d_A^{abcd}}{N_AC_A}T_Fn_f\Big(-\frac{12383}{864}-\frac{437}{48}\zeta_3$$

$$-\frac{5255}{72}\zeta_5+\frac{37}{2}\zeta_3^2+\frac{931}{16}\zeta_7\Big)+C_A^2T_F^2n_f^2\Big(-\frac{50721673}{31104}-\frac{68971}{72}\zeta_3-\frac{67}{12}\zeta_4+\frac{7139}{12}\zeta_5$$

$$-\frac{704}{9}\zeta_3^2\Big)+C_AC_FT_F^2n_f^2\Big(-\frac{26032123}{5832}+\frac{120817}{81}\zeta_3-\frac{71}{3}\zeta_4+\frac{5972}{3}\zeta_5+256\zeta_3^2\Big)$$

$$+C_F^2T_F^2n_f^2\Big(-\frac{7505}{162}+\frac{49696}{27}\zeta_3+\frac{88}{3}\zeta_4-\frac{16000}{9}\zeta_5-256\zeta_3^2\Big)+\frac{d_F^{abcd}d_F^{abcd}}{N_A}n_f^2\Big(\frac{6896}{27}$$

$$-336\zeta_3-64\zeta_4+320\zeta_5-\frac{512}{3}\zeta_3^2\Big)+C_AT_F^3n_f^3\Big(\frac{484561}{8748}+\frac{12940}{81}\zeta_3+4\zeta_4-\frac{640}{27}\zeta_5\Big)$$

$$+C_FT_F^3n_f^3\Big(\frac{393026}{729}-\frac{25888}{81}\zeta_3-\frac{1280}{9}\zeta_5\Big)\Big]+\Big[C_A^4\Big(\frac{100057939}{497664}-\frac{23173583}{36864}\zeta_3$$

$$+\frac{9419}{1024}\zeta_4+\frac{49667617}{165888}\zeta_5-\frac{51175}{4096}\zeta_6+\frac{353353}{18432}\zeta_3^2-\frac{42985019}{221184}\zeta_7\Big)$$

$$+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\Big(-\frac{3739}{6912}-\frac{2207275}{13824}\zeta_3+\frac{4041}{256}\zeta_4+\frac{8197825}{13824}\zeta_5-\frac{7125}{512}\zeta_6-\frac{117481}{768}\zeta_3^2$$

$$-\frac{1229389}{9216}\zeta_7\Big)+C_A^3T_Fn_f\Big(-\frac{9160777}{124416}+\frac{6646841}{10368}\zeta_3-\frac{49}{4}\zeta_4-\frac{1875569}{10368}\zeta_5+\frac{185}{32}\zeta_3^2$$

$$+\frac{8575}{128}\zeta_7\Big)+C_A^2C_FT_Fn_f\Big(-\frac{4865}{64}+\frac{2605}{24}\zeta_3+\frac{63}{4}\zeta_4+\frac{61}{2}\zeta_5\Big)+\frac{d_F^{abcd}d_A^{abcd}}{N_A}n_f\Big(\frac{992}{3}\zeta_3$$

$$-340\zeta_5+16\zeta_3^2-\frac{147}{2}\zeta_7\Big)+\frac{d_A^{abcd}d_A^{abcd}}{N_AC_A}T_Fn_f\Big(\frac{893}{432}-\frac{185}{432}\zeta_3-\frac{8575}{432}\zeta_5-\frac{13}{4}\zeta_3^2$$

$$+\frac{147}{16}\zeta_7\Big)+C_A^2T_F^2n_f^2\Big(\frac{255649}{7776}-\frac{5932}{27}\zeta_3+\frac{3}{2}\zeta_4+\frac{680}{27}\zeta_5\Big)+C_AC_FT_F^2n_f^2\Big(\frac{40}{3}$$

$$-\frac{160}{3}\zeta_3\Big)+C_AT_F^3n_f^3\Big(-\frac{320}{81}+\frac{1280}{81}\zeta_3\Big)\Big]\xi+\Big[C_A^4\Big(\frac{146690135}{1990656}-\frac{6310895}{110592}\zeta_3$$

$$+\frac{1157}{1024}\zeta_4+\frac{2693255}{36864}\zeta_5-\frac{525}{1024}\zeta_6-\frac{5069}{4608}\zeta_3^2-\frac{1990975}{73728}\zeta_7\Big)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\Big(-\frac{1759}{576}$$

$$+\frac{120743}{4608}\zeta_3+\frac{27}{16}\zeta_4-\frac{5485}{384}\zeta_5+\frac{1125}{256}\zeta_6-\frac{7165}{192}\zeta_3^2+\frac{115423}{3072}\zeta_7\Big)$$

$$+C_A^3T_Fn_f\Big(-\frac{10823585}{124416}-\frac{24167}{1728}\zeta_3+8\zeta_4+\frac{25679}{864}\zeta_5+\frac{275}{288}\zeta_3^2\Big)$$

$$+C_A^2C_FT_Fn_f\Big(-\frac{301967}{2592}+\frac{187}{3}\zeta_3-\frac{21}{2}\zeta_4+40\zeta_5\Big)+C_AC_F^2T_Fn_f\Big(\frac{143}{9}+\frac{148}{3}\zeta_3$$

$$-80\zeta_5\Big)+\frac{d_A^{abcd}d_A^{abcd}}{N_AC_A}T_Fn_f\Big(\frac{1}{32}-\frac{199}{72}\zeta_3+\frac{325}{144}\zeta_5+\frac{1}{12}\zeta_3^2\Big)+C_A^2T_F^2n_f^2\Big(\frac{15083}{1944}$$

$$+\frac{16}{9}\zeta_3-\frac{10}{9}\zeta_5\Big)+C_AC_FT_F^2n_f^2\Big(\frac{2051}{81}-16\zeta_3\Big)\Big]\xi^2+\Big[C_A^4\Big(-\frac{255695}{18432}+\frac{7085227}{331776}\zeta_3$$

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$$
\begin{aligned}
&-\frac{99}{512}\zeta_4-\frac{868733}{165888}\zeta_5+\frac{475}{4096}\zeta_6-\frac{8873}{18432}\zeta_3^2+\frac{452221}{221184}\zeta_7\bigg)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\bigg(-\frac{233}{2304}\\
&+\frac{13163}{13824}\zeta_3+\frac{9}{256}\zeta_4-\frac{169265}{13824}\zeta_5+\frac{825}{512}\zeta_6-\frac{6055}{768}\zeta_3^2+\frac{167321}{9216}\zeta_7\bigg)+C_A^3T_Fn_f\bigg(-\frac{3031}{864}\\
&-\frac{70439}{5184}\zeta_3+\frac{8015}{5184}\zeta_5+\frac{1}{6}\zeta_3^2\bigg)+C_A^2C_FT_Fn_f\bigg(-\frac{55}{8}+6\zeta_3\bigg)+\frac{d_A^{abcd}d_A^{abcd}}{N_AC_A}T_Fn_f\bigg(-\frac{85}{216}\zeta_3\\
&+\frac{175}{216}\zeta_5\bigg)+C_A^2T_F^2n_f^2\bigg(-\frac{49}{27}+\frac{32}{9}\zeta_3\bigg)\bigg]\xi^3+\bigg[C_A^4\bigg(-\frac{218989}{36864}+\frac{198031}{55296}\zeta_3-\frac{447}{4096}\zeta_4\\
&-\frac{203575}{73728}\zeta_5+\frac{575}{8192}\zeta_6+\frac{5845}{36864}\zeta_3^2-\frac{120883}{147456}\zeta_7\bigg)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\bigg(\frac{1}{48}-\frac{911}{512}\zeta_3+\frac{9}{128}\zeta_4\\
&+\frac{22505}{9216}\zeta_5+\frac{75}{1024}\zeta_6+\frac{1571}{1536}\zeta_3^2-\frac{18095}{6144}\zeta_7\bigg)+C_A^3T_Fn_f\bigg(-\frac{4711}{3456}+\frac{59}{27}\zeta_3-\frac{175}{216}\zeta_5\bigg)\\
&+C_A^2C_FT_Fn_f\bigg(\frac{55}{16}-3\zeta_3\bigg)+\frac{25}{27}C_A^2T_F^2n_f^2\bigg]\xi^4+\bigg[C_A^4\bigg(-\frac{127}{192}-\frac{2807}{9216}\zeta_3-\frac{155}{256}\zeta_5\\
&+\frac{21}{512}\zeta_3^2+\frac{4823}{36864}\zeta_7\bigg)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\bigg(-\frac{13}{384}\zeta_3+\frac{15}{64}\zeta_3^2-\frac{91}{192}\zeta_7\bigg)+\frac{5}{8}C_A^3T_Fn_f\bigg]\xi^5\\
&+\bigg[C_A^4\bigg(\frac{835}{2304}-\frac{13}{192}\zeta_3+\frac{35}{384}\zeta_5\bigg)-\frac{5}{36}C_A^3T_Fn_f\bigg]\xi^6-\frac{9}{256}C_A^4\xi^7+\frac{1}{256}C_A^4\xi^8. \qquad \text{(A.12)}
\end{aligned}
$$

$$
\begin{aligned}
T_2^{\overline{\rm MS},(1)} &= \bigg(-\frac{37}{12}C_A+\frac{8}{3}T_Fn_f\bigg)+\frac{3}{2}C_A\xi+\frac{1}{4}C_A\xi^2,\\
T_2^{\overline{\rm MS},(2)} &= \bigg[C_A^2\bigg(-\frac{443}{24}-\frac{3}{2}\zeta_3\bigg)+\frac{91}{6}C_AT_Fn_f+8C_FT_Fn_f\bigg]+\bigg[C_A^2\bigg(\frac{31}{48}+\frac{1}{2}\zeta_3\bigg)\\
&\quad+\frac{5}{2}C_AT_Fn_f\bigg]\xi+\bigg(\frac{59}{72}C_A^2-\frac{2}{9}C_AT_Fn_f\bigg)\xi^2-\frac{11}{16}C_A^2\xi^3-\frac{1}{8}C_A^2\xi^4,\\
T_2^{\overline{\rm MS},(3)} &= \bigg[C_A^3\bigg(-\frac{2014735}{6912}+\frac{37153}{1152}\zeta_3-\frac{10745}{576}\zeta_5\bigg)+\frac{d_A^{abcd}d_A^{abcd}}{N_AC_A}\bigg(-\frac{53}{6}-\frac{7249}{48}\zeta_3\\
&\quad+\frac{1025}{6}\zeta_5\bigg)+C_A^2T_Fn_f\bigg(\frac{277013}{864}+\frac{343}{36}\zeta_3+\frac{230}{9}\zeta_5\bigg)+C_AC_FT_Fn_f\bigg(\frac{379}{2}-\frac{352}{3}\zeta_3\bigg)\\
&\quad-4C_F^2T_Fn_f+\frac{d_F^{abcd}d_A^{abcd}}{N_AC_A}n_f\bigg(\frac{32}{3}+\frac{544}{3}\zeta_3-\frac{640}{3}\zeta_5\bigg)+C_AT_F^2n_f^2\bigg(-\frac{1741}{27}-16\zeta_3\bigg)\\
&\quad+C_FT_F^2n_f^2\bigg(-\frac{176}{3}+\frac{128}{3}\zeta_3\bigg)\bigg]+\bigg[C_A^3\bigg(-\frac{2729}{768}-\frac{3527}{576}\zeta_3+\frac{2165}{288}\zeta_5\bigg)\\
&\quad+\frac{d_A^{abcd}d_A^{abcd}}{N_AC_A}\bigg(\frac{61}{12}-\frac{245}{24}\zeta_3+\frac{125}{6}\zeta_5\bigg)+C_A^2T_Fn_f\bigg(\frac{197}{12}+\frac{5}{18}\zeta_3+\frac{10}{3}\zeta_5\bigg)\\
&\quad+C_AC_FT_Fn_f(25-24\zeta_3)+C_AT_F^2n_f^2\bigg(-\frac{14}{9}+\frac{32}{9}\zeta_3\bigg)\bigg]\xi+\bigg[C_A^3\bigg(\frac{178183}{20736}+\frac{347}{288}\zeta_3
\end{aligned}
$$

$$
\begin{aligned}
& - \frac{1165}{144}\zeta_5\Big) + \frac{d_A^{abcd}d_A^{abcd}}{N_A C_A}\Big(-\frac{49}{6}\zeta_3 + \frac{50}{3}\zeta_5\Big) + C_A^2 T_F n_f\Big(\frac{3637}{2592} + \frac{19}{4}\zeta_3\Big) \\
& + C_A C_F T_F n_f\Big(\frac{31}{6} - 8\zeta_3\Big) - \frac{140}{81} C_A T_F^2 n_f^2\Big]\xi^2 + \Big[C_A^3\Big(\frac{7927}{2304} - \frac{1325}{576}\zeta_3 + \frac{325}{288}\zeta_5\Big) \\
& + \frac{d_A^{abcd}d_A^{abcd}}{N_A C_A}\Big(-\frac{1}{4} + \frac{5}{24}\zeta_3 + \frac{5}{3}\zeta_5\Big) - \frac{131}{36} C_A^2 T_F n_f\Big]\xi^3 + \Big[C_A^3\Big(-\frac{57}{128} - \frac{55}{384}\zeta_3 \\
& + \frac{35}{192}\zeta_5\Big) + \frac{3}{16}\frac{d_A^{abcd}d_A^{abcd}}{N_A C_A}\zeta_3 - \frac{1}{3} C_A^2 T_F n_f\Big]\xi^4 + \frac{5}{32} C_A^3 \xi^5 + \frac{3}{64} C_A^3 \xi^6,
\end{aligned}
$$

$$
\begin{aligned}
T_2^{\overline{\mathrm{MS}},(4)} = & \Big[C_A^4\Big(-\frac{36253501}{6144} + \frac{142598795}{165888}\zeta_3 - \frac{333}{256}\zeta_4 - \frac{58734965}{165888}\zeta_5 + \frac{24983}{512}\zeta_3^2 \\
& + \frac{2036881}{2304}\zeta_7\Big) + \frac{d_A^{abcd}d_A^{abcd}}{N_A}\Big(-\frac{57299}{3456} - \frac{33416963}{6912}\zeta_3 + \frac{13980415}{3456}\zeta_5 - \frac{38569}{64}\zeta_3^2 \\
& + \frac{1241527}{768}\zeta_7\Big) + C_A^3 T_F n_f\Big(\frac{76607999}{10368} + \frac{86143}{576}\zeta_3 + \frac{171}{4}\zeta_4 + \frac{1303235}{1728}\zeta_5 - \frac{2251}{72}\zeta_3^2 \\
& - \frac{522277}{384}\zeta_7\Big) + C_A^2 C_F T_F n_f\Big(\frac{15197363}{2592} - \frac{336827}{108}\zeta_3 - \frac{111}{2}\zeta_4 - \frac{9190}{9}\zeta_5 - \frac{40}{3}\zeta_3^2 \\
& - \frac{959}{9}\zeta_7\Big) + C_A C_F^2 T_F n_f\Big(-\frac{2009}{12} - \frac{4576}{3}\zeta_3 + \frac{7040}{3}\zeta_5\Big) - 92 C_F^3 T_F n_f \\
& + \frac{d_F^{abcd}d_A^{abcd}}{N_A} n_f\Big(-\frac{818}{27} + \frac{174668}{27}\zeta_3 - \frac{252080}{27}\zeta_5 + 720\zeta_3^2 + \frac{5474}{3}\zeta_7\Big) \\
& + C_F\frac{d_F^{abcd}d_A^{abcd}}{N_A C_A} n_f\Big(\frac{2752}{3}\zeta_3 - \frac{4720}{3}\zeta_5 + 128\zeta_3^2 + \frac{952}{3}\zeta_7\Big) + \frac{d_A^{abcd}d_A^{abcd}}{N_A C_A} T_F n_f\Big(\frac{10925}{432} \\
& + \frac{27970}{27}\zeta_3 + \frac{232205}{108}\zeta_5 + \frac{1565}{6}\zeta_3^2 - \frac{84595}{24}\zeta_7\Big) + C_A^2 T_F^2 n_f^2\Big(-\frac{343361}{144} - \frac{200851}{324}\zeta_3 \\
& - 36\zeta_4 + \frac{17735}{162}\zeta_5 + 28\zeta_3^2 - \frac{147}{2}\zeta_7\Big) + C_A C_F T_F^2 n_f^2\Big(-\frac{632765}{162} + \frac{58448}{27}\zeta_3 + 48\zeta_4 \\
& + \frac{1280}{3}\zeta_5\Big) + C_F^2 T_F^2 n_f^2\Big(-\frac{232}{3} + \frac{2432}{3}\zeta_3 - \frac{2560}{3}\zeta_5\Big) + \frac{d_F^{abcd}d_F^{abcd}}{N_A} n_f^2\Big(\frac{1408}{9} \\
& - \frac{1024}{3}\zeta_3\Big) + \frac{d_F^{abcd}d_A^{abcd}}{N_A C_A} T_F n_f^2\Big(-\frac{1664}{27} - \frac{27520}{27}\zeta_3 + \frac{42880}{27}\zeta_5 - 256\zeta_3^2\Big) \\
& + C_A T_F^3 n_f^3\Big(\frac{14720}{81} + \frac{1216}{9}\zeta_3\Big) + C_F T_F^3 n_f^3\Big(\frac{38656}{81} - \frac{9728}{27}\zeta_3\Big)\Big] + \Big[C_A^4\Big(-\frac{31295251}{165888} \\
& - \frac{23663381}{82944}\zeta_3 - \frac{87}{128}\zeta_4 + \frac{28034275}{82944}\zeta_5 - \frac{63067}{4608}\zeta_3^2 - \frac{2580725}{36864}\zeta_7\Big) + \frac{d_A^{abcd}d_A^{abcd}}{N_A}\Big(-\frac{359}{192} \\
& - \frac{235835}{3456}\zeta_3 - \frac{974095}{3456}\zeta_5 + \frac{8555}{96}\zeta_3^2 + \frac{544901}{1536}\zeta_7\Big) + C_A^3 T_F n_f\Big(\frac{7274947}{20736} + \frac{296669}{1296}\zeta_3 \\
& - \frac{129}{4}\zeta_4 - \frac{99055}{2592}\zeta_5 - \frac{7}{18}\zeta_3^2 - \frac{27013}{192}\zeta_7\Big) + C_A^2 C_F T_F n_f\Big(\frac{73247}{216} - \frac{635}{3}\zeta_3 + 45\zeta_4 \\
& - \frac{440}{3}\zeta_5\Big) + C_A C_F^2 T_F n_f\Big(-\frac{557}{12} - 148\zeta_3 + 240\zeta_5\Big) + \frac{d_F^{abcd}d_A^{abcd}}{N_A} n_f\Big(\frac{50}{9} - \frac{2164}{3}\zeta_3
\end{aligned}
$$

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$$
\begin{aligned}
&+540\zeta_5+24\zeta_3^2+\frac{1561}{6}\zeta_7\Big)+\frac{d_A^{abcd}d_A^{abcd}}{N_A C_A}T_F n_f\Big(-\frac{14513}{432}+\frac{78277}{216}\zeta_3-\frac{48185}{108}\zeta_5\\
&-\frac{187}{6}\zeta_3^2+\frac{2177}{24}\zeta_7\Big)+C_A^2T_F^2n_f^2\Big(-\frac{39503}{432}-\frac{5239}{108}\zeta_3+\frac{1475}{54}\zeta_5+\frac{8}{3}\zeta_3^2\Big)\\
&+C_AC_FT_F^2n_f^2\Big(-\frac{3521}{27}+\frac{344}{3}\zeta_3\Big)+C_AT_F^3n_f^3\Big(\frac{364}{81}-\frac{256}{81}\zeta_3\Big)\Big]\xi+\Big[C_A^4\Big(\frac{30754063}{497664}\\
&-\frac{3037327}{27648}\zeta_3+\frac{39}{32}\zeta_4-\frac{2112595}{27648}\zeta_5+\frac{9367}{1536}\zeta_3^2+\frac{4086299}{36864}\zeta_7\Big)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\Big(\frac{46199}{1728}\\
&+\frac{1111}{1152}\zeta_3+\frac{162835}{288}\zeta_5-\frac{103}{64}\zeta_3^2-\frac{811475}{1536}\zeta_7\Big)+C_A^3T_Fn_f\Big(-\frac{10529}{3888}+\frac{7741}{72}\zeta_3-\frac{15}{2}\zeta_4\\
&-\frac{6265}{192}\zeta_5-\frac{29}{12}\zeta_3^2-\frac{343}{128}\zeta_7\Big)+C_A^2C_FT_Fn_f\Big(\frac{151901}{2592}-\frac{367}{12}\zeta_3+\frac{21}{2}\zeta_4-40\zeta_5\Big)\\
&+C_AC_F^2T_Fn_f\Big(-\frac{125}{9}-\frac{148}{3}\zeta_3+80\zeta_5\Big)+\frac{d_F^{abcd}d_A^{abcd}}{N_A}n_f\Big(-\frac{16}{3}-\frac{272}{3}\zeta_3+\frac{320}{3}\zeta_5\Big)\\
&+\frac{d_A^{abcd}d_A^{abcd}}{N_A C_A}T_Fn_f\Big(\frac{227}{144}-\frac{269}{12}\zeta_3-\frac{85}{6}\zeta_5+\frac{5}{2}\zeta_3^2+\frac{147}{4}\zeta_7\Big)+C_A^2T_F^2n_f^2\Big(-\frac{11987}{1944}\\
&-\frac{224}{9}\zeta_3+\frac{5}{9}\zeta_5\Big)+C_AC_FT_F^2n_f^2\Big(-\frac{2375}{81}+16\zeta_3\Big)\Big]\xi^2+\Big[C_A^4\Big(\frac{1729109}{55296}-\frac{905531}{41472}\zeta_3\\
&+\frac{87}{128}\zeta_4+\frac{827555}{41472}\zeta_5+\frac{5965}{4608}\zeta_3^2-\frac{1395121}{110592}\zeta_7\Big)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\Big(-\frac{12031}{1728}+\frac{6211}{216}\zeta_3\\
&+\frac{11665}{1728}\zeta_5-\frac{1759}{192}\zeta_3^2-\frac{16835}{4608}\zeta_7\Big)+C_A^3T_Fn_f\Big(-\frac{481057}{20736}+\frac{35687}{5184}\zeta_3-\frac{65}{81}\zeta_5-\frac{1}{24}\zeta_3^2\Big)\\
&+C_A^2C_FT_Fn_f\Big(-\frac{647}{24}+22\zeta_3\Big)+\frac{d_A^{abcd}d_A^{abcd}}{N_A C_A}T_Fn_f\Big(-\frac{67}{144}+\frac{589}{216}\zeta_3+\frac{35}{54}\zeta_5+\frac{1}{2}\zeta_3^2\Big)\\
&+C_A^2T_F^2n_f^2\Big(-\frac{245}{81}-\frac{16}{3}\zeta_3\Big)\Big]\xi^3+\Big[C_A^4\Big(-\frac{339797}{82944}+\frac{1159}{18432}\zeta_3+\frac{21}{256}\zeta_4+\frac{406165}{55296}\zeta_5\\
&-\frac{65}{128}\zeta_3^2+\frac{1813}{4096}\zeta_7\Big)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\Big(\frac{377}{3456}+\frac{1643}{2304}\zeta_3-\frac{6455}{384}\zeta_5-\frac{21}{8}\zeta_3^2+\frac{11487}{512}\zeta_7\Big)\\
&+C_A^3T_Fn_f\Big(-\frac{1249}{192}-\frac{1457}{432}\zeta_3-\frac{35}{216}\zeta_5\Big)+C_A^2C_FT_Fn_f\Big(-\frac{43}{8}+6\zeta_3\Big)\\
&+\frac{5}{6}\frac{d_A^{abcd}d_A^{abcd}}{N_A C_A}T_Fn_f\zeta_3+\frac{10}{27}C_A^2T_F^2n_f^2\Big]\xi^4+\Big[C_A^4\Big(-\frac{4729}{1152}+\frac{17743}{9216}\zeta_3-\frac{20335}{27648}\zeta_5\\
&-\frac{89}{2304}\zeta_3^2-\frac{539}{4608}\zeta_7\Big)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\Big(\frac{1}{8}+\frac{371}{384}\zeta_3-\frac{2165}{1152}\zeta_5-\frac{47}{192}\zeta_3^2+\frac{385}{384}\zeta_7\Big)\\
&+\frac{157}{96}C_A^3T_Fn_f\Big]\xi^5+\Big[C_A^4\Big(\frac{17}{768}+\frac{107}{768}\zeta_3-\frac{35}{192}\zeta_5\Big)-\frac{3}{32}\frac{d_A^{abcd}d_A^{abcd}}{N_A}\zeta_3+\frac{1}{4}C_A^3T_Fn_f\Big]\xi^6\\
&-\frac{3}{256}C_A^4\xi^7-\frac{1}{64}C_A^4\xi^8.
\end{aligned}
\tag{A.13}
$$

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A.5 Ghost-gluon vertex

$$
\begin{aligned}
\tilde{\Gamma}_h^{\overline{\text{MS}},(1)} &= \frac{1}{2} C_A \xi , \\
\tilde{\Gamma}_h^{\overline{\text{MS}},(2)} &= C_A^2 \left(\frac{43}{16} - \frac{9}{16}\zeta_3 \right) \xi + C_A^2 \left(\frac{7}{16} + \frac{3}{16}\zeta_3 \right) \xi^2 , \\
\tilde{\Gamma}_h^{\overline{\text{MS}},(3)} &= \left[C_A^3 \left(\frac{3631}{128} - \frac{1939}{192}\zeta_3 + \frac{85}{64}\zeta_5 \right) + \frac{d_A^{abcd} d_A^{abcd}}{N_A C_A} \left(9\zeta_3 - \frac{75}{8}\zeta_5 \right) + C_A^2 T_F n_f \left(-\frac{493}{48} \right.\right. \\
&\quad \left.\left. + \frac{29}{12}\zeta_3 \right) \right] \xi + \left[C_A^3 \left(\frac{2089}{384} + \frac{209}{192}\zeta_3 - \frac{215}{192}\zeta_5 \right) + \frac{d_A^{abcd} d_A^{abcd}}{N_A C_A} \left(2\zeta_3 - \frac{25}{8}\zeta_5 \right) \right] \xi^2 \\
&\quad + \left[C_A^3 \left(\frac{17}{24} + \frac{5}{16}\zeta_3 + \frac{25}{96}\zeta_5 \right) + \frac{d_A^{abcd} d_A^{abcd}}{N_A C_A} \left(-\frac{3}{2}\zeta_3 + \frac{5}{4}\zeta_5 \right) \right] \xi^3 , \\
\tilde{\Gamma}_h^{\overline{\text{MS}},(4)} &= \left[C_A^4 \left(\frac{357972595}{746496} - \frac{1471145}{6912}\zeta_3 + \frac{1111}{512}\zeta_4 + \frac{148387}{9216}\zeta_5 - \frac{3625}{1024}\zeta_6 + \frac{2351}{256}\zeta_3^2 \right.\right. \\
&\quad \left. - \frac{121513}{73728}\zeta_7 \right) + \frac{d_A^{abcd} d_A^{abcd}}{N_A} \left(\frac{16069}{192}\zeta_3 + \frac{333}{64}\zeta_4 - \frac{5595}{128}\zeta_5 - \frac{75}{8}\zeta_6 - \frac{21}{8}\zeta_3^2 - \frac{3521}{192}\zeta_7 \right) \\
&\quad + C_A^3 T_F n_f \left(-\frac{26691869}{93312} + \frac{27587}{432}\zeta_3 - \frac{303}{32}\zeta_4 + \frac{191}{36}\zeta_5 + \frac{55}{12}\zeta_3^2 \right) + C_A^2 C_F T_F n_f \left(-\frac{301}{3} \right. \\
&\quad \left. + \frac{1177}{16}\zeta_3 + 12\zeta_4 + 7\zeta_5 - 9\zeta_3^2 \right) + \frac{d_A^{abcd} d_A^{abcd}}{N_A C_A} T_F n_f \left(\frac{13}{3}\zeta_3 - \frac{145}{8}\zeta_5 + \frac{5}{2}\zeta_3^2 \right) \\
&\quad \left. + C_A^2 T_F^2 n_f^2 \left(\frac{105805}{2916} - \frac{155}{27}\zeta_3 + \zeta_4 - \frac{50}{9}\zeta_5 \right) \right] \xi + \left[C_A^4 \left(\frac{43117}{576} + \frac{4411}{864}\zeta_3 + \frac{545}{1024}\zeta_4 \right.\right. \\
&\quad \left. - \frac{209731}{13824}\zeta_5 - \frac{25}{64}\zeta_6 - \frac{397}{384}\zeta_3^2 + \frac{44121}{8192}\zeta_7 \right) + \frac{d_A^{abcd} d_A^{abcd}}{N_A} \left(\frac{2903}{576}\zeta_3 + \frac{99}{128}\zeta_4 - \frac{9295}{1152}\zeta_5 \right. \\
&\quad \left. - \frac{75}{128}\zeta_6 - \frac{557}{64}\zeta_3^2 + \frac{6951}{512}\zeta_7 \right) + C_A^3 T_F n_f \left(-\frac{21889}{1152} - \frac{8281}{1728}\zeta_3 - \frac{5}{32}\zeta_4 + \frac{5575}{1728}\zeta_5 \right. \\
&\quad \left.\left. + \frac{35}{48}\zeta_3^2 \right) + \frac{d_A^{abcd} d_A^{abcd}}{N_A C_A} T_F n_f \left(-\frac{61}{9}\zeta_3 + \frac{575}{72}\zeta_5 + \frac{1}{2}\zeta_3^2 \right) \right] \xi^2 + \left[C_A^4 \left(\frac{13279}{1024} + \frac{5011}{1536}\zeta_3 \right.\right. \\
&\quad \left. + \frac{9}{64}\zeta_4 + \frac{6463}{3072}\zeta_5 - \frac{125}{1024}\zeta_6 - \frac{33}{128}\zeta_3^2 - \frac{12761}{24576}\zeta_7 \right) + \frac{d_A^{abcd} d_A^{abcd}}{N_A} \left(\frac{221}{64}\zeta_3 + \frac{9}{16}\zeta_4 \right. \\
&\quad \left.\left. - \frac{715}{128}\zeta_5 - \frac{75}{64}\zeta_6 - \frac{15}{16}\zeta_3^2 - \frac{1295}{512}\zeta_7 \right) \right] \xi^3 + \left[C_A^4 \left(\frac{2023}{1536} + \frac{1189}{1536}\zeta_3 + \frac{1}{1024}\zeta_4 + \frac{5065}{4608}\zeta_5 \right.\right. \\
&\quad \left. + \frac{25}{1536}\zeta_6 - \frac{31}{256}\zeta_3^2 + \frac{819}{8192}\zeta_7 \right) + \frac{d_A^{abcd} d_A^{abcd}}{N_A} \left(-\frac{387}{64}\zeta_3 - \frac{21}{128}\zeta_4 + \frac{1435}{384}\zeta_5 + \frac{25}{128}\zeta_6 \right. \\
&\quad \left.\left. + \frac{21}{64}\zeta_3^2 + \frac{231}{256}\zeta_7 \right) \right] \xi^4 . \qquad \text{(A.14)}
\end{aligned}
$$

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$$\tilde{\Gamma}_g^{\overline{\rm MS},(1)} = \frac{3}{4}C_A + \frac{1}{4}C_A\xi,$$

$$\tilde{\Gamma}_g^{\overline{\rm MS},(2)} = \left(\frac{599}{96}C_A^2 - \frac{29}{12}C_AT_Fn_f\right) + C_A^2\left(\frac{97}{32} - \frac{9}{16}\zeta_3\right)\xi + C_A^2\left(\frac{1}{4} + \frac{3}{16}\zeta_3\right)\xi^2,$$

$$\begin{aligned}
\tilde{\Gamma}_g^{\overline{\rm MS},(3)} = &\left[C_A^3\left(\frac{89543}{864} - \frac{1061}{128}\zeta_3 + \frac{25}{32}\zeta_5\right) + \frac{d_A^{abcd}d_A^{abcd}}{N_AC_A}\left(-\frac{9}{4} + \frac{213}{16}\zeta_3 - \frac{75}{8}\zeta_5\right)\right. \\
&\left. + C_A^2T_Fn_f\left(-\frac{15143}{216} - \frac{49}{8}\zeta_3\right) + C_AC_FT_Fn_f(-16 + 12\zeta_3) + \frac{280}{27}C_AT_F^2n_f^2\right] \\
&+ \left[C_A^3\left(\frac{25967}{768} - \frac{4705}{384}\zeta_3 + \frac{1015}{192}\zeta_5\right) + \frac{d_A^{abcd}d_A^{abcd}}{N_AC_A}\left(\frac{7}{4} + \frac{83}{16}\zeta_3 - \frac{55}{8}\zeta_5\right)\right. \\
&\left. + C_A^2T_Fn_f\left(-\frac{357}{32} + \frac{55}{24}\zeta_3\right)\right]\xi + \left[C_A^3\left(\frac{4387}{768} + \frac{389}{384}\zeta_3 - \frac{235}{192}\zeta_5\right) + \frac{d_A^{abcd}d_A^{abcd}}{N_AC_A}\left(-\frac{1}{4}\right.\right. \\
&\left.\left. + \frac{37}{16}\zeta_3 - \frac{5}{2}\zeta_5\right)\right]\xi^2 + \left[C_A^3\left(\frac{31}{48} + \frac{55}{384}\zeta_3 + \frac{5}{16}\zeta_5\right) - \frac{1}{16}\frac{d_A^{abcd}d_A^{abcd}}{N_AC_A}\zeta_3\right]\xi^3,
\end{aligned}$$

$$\begin{aligned}
\tilde{\Gamma}_g^{\overline{\rm MS},(4)} = &\left[C_A^4\left(\frac{2073522751}{995328} - \frac{12627131}{36864}\zeta_3 - \frac{167275}{36864}\zeta_5 + \frac{10405}{3072}\zeta_3^2 - \frac{1230425}{73728}\zeta_7\right)\right. \\
&+ \frac{d_A^{abcd}d_A^{abcd}}{N_A}\left(-\frac{22805}{768} - \frac{332267}{1536}\zeta_3 + \frac{169855}{192}\zeta_5 + \frac{269}{8}\zeta_3^2 - \frac{1600291}{3072}\zeta_7\right) \\
&+ C_A^3T_Fn_f\left(-\frac{21234067}{10368} - \frac{59819}{1152}\zeta_3 + \frac{15955}{288}\zeta_5 - \frac{5}{8}\zeta_3^2 + \frac{147}{4}\zeta_7\right) \\
&+ C_A^2C_FT_Fn_f\left(-\frac{270353}{432} + \frac{2927}{8}\zeta_3 + 65\zeta_5\right) + C_AC_F^2T_Fn_f\left(\frac{599}{24} + 74\zeta_3 - 120\zeta_5\right) \\
&+ \frac{d_F^{abcd}d_A^{abcd}}{N_A}n_f\left(\frac{11}{3} + 66\zeta_3 - 40\zeta_5 - 30\zeta_3^2\right) + \frac{d_A^{abcd}d_A^{abcd}}{N_AC_A}T_Fn_f\left(\frac{8857}{288} + \frac{1343}{24}\zeta_3\right. \\
&\left. - \frac{4125}{16}\zeta_5 - \frac{21}{4}\zeta_3^2 + \frac{441}{4}\zeta_7\right) + C_A^2T_F^2n_f^2\left(\frac{387845}{648} + \frac{4333}{72}\zeta_3 - \frac{485}{36}\zeta_5\right) \\
&\left. + C_AC_FT_F^2n_f^2\left(\frac{11267}{54} - 140\zeta_3\right) - \frac{12530}{243}C_AT_F^3n_f^3\right] + \left[C_A^4\left(\frac{459546241}{746496} - \frac{22565215}{110592}\zeta_3\right.\right. \\
&\left. + \frac{1075}{512}\zeta_4 + \frac{11158619}{110592}\zeta_5 - \frac{3625}{1024}\zeta_6 + \frac{1299}{512}\zeta_3^2 - \frac{1371559}{24576}\zeta_7\right) + \frac{d_A^{abcd}d_A^{abcd}}{N_A}\left(-\frac{30703}{2304}\right. \\
&\left. + \frac{599507}{4608}\zeta_3 + \frac{333}{64}\zeta_4 - \frac{297785}{1152}\zeta_5 - \frac{75}{8}\zeta_6 + \frac{1185}{64}\zeta_3^2 + \frac{185395}{1024}\zeta_7\right) \\
&+ C_A^3T_Fn_f\left(-\frac{133267097}{373248} + \frac{7001}{128}\zeta_3 - \frac{231}{32}\zeta_4 - \frac{35129}{3456}\zeta_5 + \frac{203}{32}\zeta_3^2\right) \\
&+ C_A^2C_FT_Fn_f\left(-\frac{2401}{24} + \frac{1211}{16}\zeta_3 + 9\zeta_4 + 7\zeta_5 - 9\zeta_3^2\right) + \frac{d_A^{abcd}d_A^{abcd}}{N_AC_A}T_Fn_f\left(-\frac{1207}{144}\right. \\
&\left.\left. - \frac{1613}{144}\zeta_3 + \frac{875}{144}\zeta_5 - \frac{15}{4}\zeta_3^2\right) + C_A^2T_F^2n_f^2\left(\frac{256109}{5832} - \frac{223}{54}\zeta_3 + \zeta_4 - \frac{50}{9}\zeta_5\right)\right]\xi
\end{aligned}$$

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$$
\begin{aligned}
&+\Bigg[C_A^4\bigg(\frac{10731167}{110592}-\frac{1457435}{110592}\zeta_3+\frac{449}{1024}\zeta_4-\frac{611293}{110592}\zeta_5-\frac{25}{64}\zeta_6-\frac{509}{1024}\zeta_3^2\\
&+\frac{133357}{9216}\zeta_7\bigg)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\bigg(\frac{22457}{2304}+\frac{55715}{4608}\zeta_3+\frac{99}{128}\zeta_4-\frac{39565}{576}\zeta_5-\frac{75}{128}\zeta_6+\frac{249}{128}\zeta_3^2\\
&+\frac{86807}{1536}\zeta_7\bigg)+C_A^3T_Fn_f\bigg(-\frac{290183}{13824}-\frac{8983}{3456}\zeta_3-\frac{5}{32}\zeta_4+\frac{5215}{3456}\zeta_5+\frac{13}{32}\zeta_3^2\bigg)\\
&+\frac{d_A^{abcd}d_A^{abcd}}{N_AC_A}T_Fn_f\bigg(\frac{55}{288}-\frac{1235}{144}\zeta_3+\frac{625}{72}\zeta_5\bigg)\Bigg]\xi^2+\Bigg[C_A^4\bigg(\frac{1002679}{55296}-\frac{25127}{36864}\zeta_3+\frac{15}{128}\zeta_4\\
&+\frac{218411}{36864}\zeta_5-\frac{125}{1024}\zeta_6-\frac{95}{1536}\zeta_3^2-\frac{235571}{73728}\zeta_7\bigg)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\bigg(-\frac{1807}{2304}+\frac{19943}{1536}\zeta_3\\
&+\frac{9}{16}\zeta_4+\frac{395}{192}\zeta_5-\frac{75}{64}\zeta_6+\frac{53}{64}\zeta_3^2-\frac{50323}{3072}\zeta_7\bigg)\Bigg]\xi^3+\Bigg[C_A^4\bigg(\frac{6247}{3072}+\frac{131}{1536}\zeta_3+\frac{1}{1024}\zeta_4\\
&+\frac{20815}{18432}\zeta_5+\frac{25}{1536}\zeta_6-\frac{47}{512}\zeta_3^2-\frac{245}{24576}\zeta_7\bigg)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\bigg(-\frac{493}{256}\zeta_3-\frac{21}{128}\zeta_4+\frac{775}{384}\zeta_5\\
&+\frac{25}{128}\zeta_6+\frac{51}{128}\zeta_3^2-\frac{343}{1024}\zeta_7\bigg)\Bigg]\xi^4. \qquad\qquad (A.15)
\end{aligned}
$$

A.6 Quark-gluon vertex

$$
\begin{aligned}
\Lambda_q^{\overline{\text{MS}},(1)} &= C_A+\left(\frac{1}{2}C_A+C_F\right)\xi,\\
\Lambda_q^{\overline{\text{MS}},(2)} &= \left[C_A^2\left(\frac{2015}{192}-\frac{3}{2}\zeta_3\right)+C_AC_F\left(\frac{41}{4}-3\zeta_3\right)-\frac{5}{8}C_F^2-\frac{95}{24}C_AT_Fn_f-\frac{7}{2}C_FT_Fn_f\right]\\
&+\left[\frac{181}{64}C_A^2+C_AC_F\left(\frac{15}{2}-3\zeta_3\right)\right]\xi+\left(\frac{13}{16}C_A^2+\frac{13}{8}C_AC_F\right)\xi^2,\\
\Lambda_q^{\overline{\text{MS}},(3)} &= \Bigg[C_A^3\left(\frac{1127389}{7776}-\frac{2615}{72}\zeta_3+\frac{9}{64}\zeta_4+\frac{185}{96}\zeta_5\right)+C_A^2C_F\bigg(\frac{165737}{648}-\frac{3175}{24}\zeta_3-\frac{69}{16}\zeta_4\\
&+\frac{165}{4}\zeta_5\bigg)+C_AC_F^2\left(-\frac{253}{6}+44\zeta_3+6\zeta_4-20\zeta_5\right)-\frac{73}{12}C_F^3+\frac{d_F^{abcd}d_A^{abcd}}{N_RC_F}\bigg(\frac{1}{8}+38\zeta_3\\
&-\frac{85}{2}\zeta_5\bigg)+C_A^2T_Fn_f\left(-\frac{169525}{1944}-\frac{2}{9}\zeta_3-\frac{9}{2}\zeta_4\right)+C_AC_FT_Fn_f\bigg(-\frac{121637}{648}+\frac{118}{3}\zeta_3\\
&+6\zeta_4\bigg)+C_F^2T_Fn_f\left(-\frac{79}{6}+16\zeta_3\right)+C_AT_F^2n_f^2\left(\frac{5161}{486}+\frac{8}{9}\zeta_3\right)+\frac{1570}{81}C_FT_F^2n_f^2\Bigg]\\
&+\Bigg[C_A^3\left(\frac{6497}{192}-\frac{119}{192}\zeta_3+\frac{3}{16}\zeta_4-\frac{35}{16}\zeta_5\right)+C_A^2C_F\left(\frac{13135}{144}-41\zeta_3+\frac{3}{8}\zeta_4+\frac{5}{2}\zeta_5\right)\\
&+C_AC_F^2\left(\frac{59}{16}-17\zeta_3+20\zeta_5\right)+\frac{7}{8}C_F^3+\frac{d_F^{abcd}d_A^{abcd}}{N_RC_F}\left(\frac{1}{4}-\frac{15}{4}\zeta_5\right)+C_A^2T_Fn_f\bigg(-\frac{491}{48}\\
&-\frac{1}{6}\zeta_3\bigg)+C_AC_FT_Fn_f\left(-\frac{1067}{36}+8\zeta_3\right)-\frac{3}{2}C_F^2T_Fn_f\Bigg]\xi+\Bigg[C_A^3\bigg(\frac{3281}{384}-\frac{5}{32}\zeta_3+\frac{3}{64}\zeta_4
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{5}{32}\zeta_5\Big)+C_A^2C_F\Big(\frac{39}{2}-\frac{51}{8}\zeta_3+\frac{3}{16}\zeta_4+\frac{5}{4}\zeta_5\Big)+\frac{3}{2}C_AC_F^2+\frac{d_F^{abcd}d_A^{abcd}}{N_RC_F}\Big(\frac{1}{8}-\frac{3}{2}\zeta_3\Big)\Big]\xi^2\\
&+\Big[C_A^3\Big(\frac{235}{128}-\frac{9}{64}\zeta_3+\frac{5}{48}\zeta_5\Big)+C_A^2C_F\Big(\frac{11}{3}-\frac{1}{3}\zeta_3\Big)+C_AC_F^2\Big(-\frac{1}{8}+\zeta_3\Big)-\frac{2}{3}C_F^3\zeta_3\\
&+\frac{d_F^{abcd}d_A^{abcd}}{N_RC_F}\Big(-\frac{3}{2}\zeta_3+\frac{5}{4}\zeta_5\Big)\Big]\xi^3,
\end{aligned}
$$

$$
\begin{aligned}
\Lambda_q^{\overline{\mathrm{MS}},(4)}=&\Big[C_A^4\Big(\frac{16476486067}{5971968}-\frac{38745445}{55296}\zeta_3+\frac{142471}{12288}\zeta_4-\frac{22095031}{73728}\zeta_5-\frac{238325}{8192}\zeta_6\\
&+\frac{139287}{4096}\zeta_3^2-\frac{2194031}{73728}\zeta_7\Big)+C_A^3C_F\Big(\frac{348927845}{62208}-\frac{2396227}{576}\zeta_3-\frac{20835}{512}\zeta_4\\
&+\frac{2317921}{1152}\zeta_5-\frac{20725}{256}\zeta_6+\frac{11063}{64}\zeta_3^2-\frac{131299}{384}\zeta_7\Big)+C_A^2C_F^2\Big(-\frac{21153731}{13824}-\frac{26755}{48}\zeta_3\\
&-\frac{1167}{16}\zeta_4+\frac{1769}{8}\zeta_5+\frac{3925}{8}\zeta_6-\frac{351}{4}\zeta_3^2+\frac{24367}{16}\zeta_7\Big)+C_AC_F^3\Big(\frac{277223}{288}+\frac{32359}{8}\zeta_3\\
&+318\zeta_4-\frac{1225}{3}\zeta_5-900\zeta_6+156\zeta_3^2-\frac{10185}{2}\zeta_7\Big)+C_F^4\Big(-\frac{42945}{128}-\frac{3889}{2}\zeta_3-150\zeta_4\\
&-970\zeta_5+400\zeta_6-80\zeta_3^2+3528\zeta_7\Big)+\frac{d_F^{abcd}d_A^{abcd}}{N_R}\Big(-\frac{933}{4}+\frac{6357}{16}\zeta_3+\frac{14955}{32}\zeta_5\\
&-\frac{4275}{64}\zeta_6-1043\zeta_7\Big)+\frac{d_F^{abcd}d_A^{abcd}}{N_R}\Big(\frac{3339}{64}\zeta_4+\frac{3585}{32}\zeta_3^2\Big)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\Big(-\frac{3359}{256}\\
&+\frac{1827}{64}\zeta_4\Big)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\Big(\frac{611963}{1536}\zeta_3-\frac{382995}{1024}\zeta_5-\frac{50925}{1024}\zeta_6+\frac{10407}{512}\zeta_3^2-\frac{200389}{3072}\zeta_7\Big)\\
&+C_A\frac{d_F^{abcd}d_A^{abcd}}{N_RC_F}\Big(\frac{1775}{96}+\frac{6577}{24}\zeta_3-\frac{156275}{192}\zeta_5+\frac{6279}{32}\zeta_3^2+\frac{35665}{128}\zeta_7\Big)\\
&+C_A^3T_Fn_f\Big(-\frac{103135751}{41472}+\frac{50161}{576}\zeta_3-\frac{3307}{64}\zeta_4+\frac{23627}{96}\zeta_5+\frac{275}{8}\zeta_6+\frac{305}{12}\zeta_3^2-\frac{931}{16}\zeta_7\Big)\\
&+C_A^2C_FT_Fn_f\Big(-\frac{19779479}{3456}+\frac{5263}{2}\zeta_3+\frac{615}{16}\zeta_4-\frac{49873}{72}\zeta_5-\frac{25}{2}\zeta_6+57\zeta_3^2-\frac{441}{2}\zeta_7\Big)\\
&+C_AC_F^2T_Fn_f\Big(-\frac{1649117}{1728}+\frac{4591}{12}\zeta_3+\frac{111}{4}\zeta_4-167\zeta_5+25\zeta_6-190\zeta_3^2+882\zeta_7\Big)\\
&+C_F^3T_Fn_f\Big(\frac{21775}{144}-103\zeta_3-24\zeta_4+\frac{20}{3}\zeta_5\Big)+\frac{d_F^{abcd}d_F^{abcd}}{N_R}n_f\big(474+1124\zeta_3-1240\zeta_5\\
&-384\zeta_3^2\big)+\frac{d_F^{abcd}d_A^{abcd}}{N_A}n_f\Big(-\frac{37}{6}-\frac{3103}{4}\zeta_3-18\zeta_4+\frac{1975}{6}\zeta_5+\frac{75}{2}\zeta_6-17\zeta_3^2+588\zeta_7\Big)\\
&+C_A\frac{d_F^{abcd}d_F^{abcd}}{N_RC_F}n_f\Big(-\frac{248}{3}\zeta_3-\frac{20}{3}\zeta_5-32\zeta_3^2+147\zeta_7\Big)+C_A^2T_F^2n_f^2\Big(\frac{20100131}{31104}\\
&+\frac{3053}{36}\zeta_3+\frac{45}{4}\zeta_4-\frac{133}{3}\zeta_5\Big)+C_AC_FT_F^2n_f^2\Big(\frac{1978291}{1296}-323\zeta_3-3\zeta_4+\frac{332}{9}\zeta_5\Big)\\
&+C_F^2T_F^2n_f^2\Big(\frac{79531}{216}-\frac{796}{3}\zeta_3-12\zeta_4\Big)+C_AT_F^3n_f^3\Big(-\frac{150979}{2916}-\frac{20}{27}\zeta_3+\frac{4}{3}\zeta_4\Big)
\end{aligned}
$$

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$$
\begin{aligned}
&+C_F T_F^3 n_f^3\bigg(-\frac{21391}{243}-\frac{16}{9}\zeta_3\bigg)\Bigg]+\Bigg[C_A^4\bigg(\frac{202830145}{373248}-\frac{1358927}{55296}\zeta_3+\frac{3873}{1024}\zeta_4\\
&-\frac{2675863}{36864}\zeta_5-\frac{51175}{12288}\zeta_6+\frac{2281}{6144}\zeta_3^2+\frac{142807}{18432}\zeta_7\bigg)+C_A^3 C_F\bigg(\frac{1166182445}{746496}-\frac{700441}{864}\zeta_3\\
&+\frac{2379}{256}\zeta_4-\frac{18971}{288}\zeta_5-\frac{2875}{192}\zeta_6+\frac{9061}{96}\zeta_3^2-\frac{791}{32}\zeta_7\bigg)+C_A^2 C_F^2\bigg(\frac{3850247}{41472}-\frac{7237}{36}\zeta_3\\
&-\frac{21}{8}\zeta_4+\frac{20585}{72}\zeta_5+\frac{25}{8}\zeta_6-\frac{165}{4}\zeta_3^2+\frac{707}{16}\zeta_7\bigg)+C_A C_F^3\bigg(\frac{177}{16}-\frac{405}{8}\zeta_3+6\zeta_4+230\zeta_5\\
&-\frac{399}{2}\zeta_7\bigg)-\frac{91}{12}C_F^4+\frac{d_F^{abcd}d_A^{abcd}}{N_R}\bigg(\frac{39}{8}+286\zeta_3+\frac{303}{16}\zeta_4-\frac{1905}{8}\zeta_5-\frac{275}{8}\zeta_6+\frac{5}{4}\zeta_3^2\\
&-\frac{2835}{32}\zeta_7\bigg)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\bigg(-\frac{385}{192}+\frac{655}{16}\zeta_3+\frac{1347}{256}\zeta_4-\frac{12275}{1536}\zeta_5-\frac{2375}{512}\zeta_6+\frac{481}{256}\zeta_3^2\\
&-\frac{50687}{1536}\zeta_7\bigg)+C_A\frac{d_F^{abcd}d_A^{abcd}}{N_R C_F}\bigg(-\frac{233}{288}-\frac{151}{8}\zeta_3+\frac{145}{12}\zeta_5+\frac{373}{16}\zeta_3^2-\frac{22911}{256}\zeta_7\bigg)\\
&+C_A^3 T_F n_f\bigg(-\frac{6775939}{23328}-\frac{30169}{864}\zeta_3-\frac{45}{4}\zeta_4+\frac{2591}{144}\zeta_5+\frac{13}{12}\zeta_3^2\bigg)\\
&+C_A^2 C_F T_F n_f\bigg(-\frac{23050741}{23328}+\frac{57523}{216}\zeta_3-\frac{105}{8}\zeta_4+\frac{1145}{9}\zeta_5+14\zeta_3^2\bigg)\\
&+C_A C_F^2 T_F n_f\bigg(-\frac{834731}{2592}+\frac{4843}{18}\zeta_3+\frac{69}{2}\zeta_4-\frac{1034}{9}\zeta_5-32\zeta_3^2\bigg)+C_F^3 T_F n_f\bigg(-\frac{131}{6}\\
&+24\zeta_3\bigg)+\frac{d_F^{abcd}d_A^{abcd}}{N_A}n_f\bigg(-\frac{1}{9}+27\zeta_3-\frac{5}{3}\zeta_5-8\zeta_3^2\bigg)+C_A^2 T_F^2 n_f^2\bigg(\frac{717649}{23328}+\frac{235}{54}\zeta_3\\
&+\frac{1}{2}\zeta_4-\frac{10}{9}\zeta_5\bigg)+C_A C_F T_F^2 n_f^2\bigg(\frac{73865}{729}-\frac{332}{27}\zeta_3+2\zeta_4-\frac{80}{3}\zeta_5\bigg)+C_F^2 T_F^2 n_f^2\bigg(\frac{202}{81}\\
&+\frac{32}{3}\zeta_3\bigg)\Bigg]\xi+\Bigg[C_A^4\bigg(\frac{28395497}{221184}-\frac{88345}{9216}\zeta_3+\frac{887}{1024}\zeta_4+\frac{1145}{768}\zeta_5-\frac{175}{1024}\zeta_6+\frac{179}{384}\zeta_3^2\\
&-\frac{4501}{9216}\zeta_7\bigg)+C_A^3 C_F\bigg(\frac{8380967}{27648}-\frac{144245}{1152}\zeta_3+\frac{807}{256}\zeta_4+\frac{2819}{144}\zeta_5-\frac{475}{384}\zeta_6-\frac{181}{96}\zeta_3^2\\
&+\frac{7}{12}\zeta_7\bigg)+C_A^2 C_F^2\bigg(\frac{8933}{1152}-\frac{103}{24}\zeta_3+\frac{3}{8}\zeta_4+40\zeta_5+\frac{9}{2}\zeta_3^2-\frac{35}{4}\zeta_7\bigg)+C_A C_F^3\bigg(\frac{103}{64}-28\zeta_3\\
&+20\zeta_5\bigg)+3C_F^4\zeta_3+\frac{d_F^{abcd}d_A^{abcd}}{N_R}\bigg(\frac{1}{4}+\frac{23}{8}\zeta_3+\frac{33}{32}\zeta_4-\frac{795}{16}\zeta_5-\frac{175}{32}\zeta_6-\frac{35}{16}\zeta_3^2\\
&+\frac{679}{16}\zeta_7\bigg)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\bigg(-\frac{113}{256}+\frac{283}{48}\zeta_3+\frac{9}{16}\zeta_4-\frac{1855}{768}\zeta_5+\frac{375}{256}\zeta_6-\frac{247}{128}\zeta_3^2\\
&-\frac{665}{1536}\zeta_7\bigg)+C_A\frac{d_F^{abcd}d_A^{abcd}}{N_R C_F}\bigg(\frac{277}{288}-\frac{227}{12}\zeta_3+\frac{245}{64}\zeta_5-\frac{3}{16}\zeta_3^2-\frac{1799}{128}\zeta_7\bigg)\\
&+C_A^3 T_F n_f\bigg(-\frac{216713}{6912}+\frac{53}{72}\zeta_3-\frac{1}{16}\zeta_4-\frac{49}{288}\zeta_5\bigg)+C_A^2 C_F T_F n_f\bigg(-\frac{64661}{864}+\frac{1285}{72}\zeta_3\\
&-\frac{7}{16}\zeta_4-\frac{149}{72}\zeta_5+\zeta_3^2\bigg)+C_A C_F^2 T_F n_f\bigg(-\frac{461}{144}-\frac{32}{3}\zeta_3\bigg)+4C_F^3 T_F n_f\zeta_3
\end{aligned}
$$

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$$+\frac{d_F^{abcd}d_A^{abcd}}{N_A}n_f\left(-\frac{5}{18}+\frac{37}{12}\zeta_3\right)\Bigg]\xi^2+\Bigg[C_A^4\left(\frac{276155}{9216}-\frac{66505}{18432}\zeta_3+\frac{59}{512}\zeta_4+\frac{9643}{4096}\zeta_5\right.$$
$$+\frac{475}{12288}\zeta_6-\frac{805}{6144}\zeta_3^2+\left.\frac{4333}{4608}\zeta_7\right)+C_A^3C_F\left(\frac{4117}{64}-\frac{947}{48}\zeta_3+\frac{123}{256}\zeta_4+\frac{203}{64}\zeta_5-\frac{1}{16}\zeta_3^2\right.$$
$$\left.+\frac{147}{128}\zeta_7\right)+C_A^2C_F^2\left(-\frac{231}{64}+\frac{83}{8}\zeta_3+\frac{3}{16}\zeta_4+\frac{5}{4}\zeta_5\right)+C_AC_F^3\left(\frac{3}{2}-\frac{14}{3}\zeta_3\right)$$
$$+\frac{d_F^{abcd}d_A^{abcd}}{N_R}\left(\frac{1}{8}-\frac{11}{2}\zeta_3-\frac{9}{16}\zeta_4-\frac{15}{2}\zeta_5-3\zeta_3^2+\frac{441}{32}\zeta_7\right)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\left(\frac{1}{96}+\frac{1079}{384}\zeta_3\right.$$
$$\left.+\frac{3}{256}\zeta_4-\frac{3715}{512}\zeta_5+\frac{275}{512}\zeta_6-\frac{197}{256}\zeta_3^2+\frac{8449}{1536}\zeta_7\right)+C_A\frac{d_F^{abcd}d_A^{abcd}}{N_RC_F}\left(\frac{7}{32}-\frac{55}{4}\zeta_3+\frac{125}{16}\zeta_5\right.$$
$$\left.-\frac{27}{16}\zeta_3^2-\frac{77}{256}\zeta_7\right)\Bigg]\xi^3+\Bigg[C_A^4\left(\frac{61283}{12288}-\frac{1201}{2048}\zeta_3-\frac{37}{4096}\zeta_4+\frac{48415}{73728}\zeta_5+\frac{575}{24576}\zeta_6\right.$$
$$\left.-\frac{973}{12288}\zeta_3^2-\frac{1029}{8192}\zeta_7\right)+C_A^3C_F\left(\frac{1253}{128}-\frac{169}{128}\zeta_3+\frac{1}{512}\zeta_4+\frac{25}{128}\zeta_5+\frac{25}{768}\zeta_6-\frac{5}{192}\zeta_3^2\right)$$
$$+C_A^2C_F^2\left(-\frac{473}{384}+\frac{35}{12}\zeta_3\right)+C_AC_F^3\left(\frac{1}{2}-\frac{5}{6}\zeta_3\right)-\frac{2}{3}C_F^4\zeta_3+\frac{d_F^{abcd}d_A^{abcd}}{N_R}\left(-\frac{59}{16}\zeta_3-\frac{21}{64}\zeta_4\right.$$
$$\left.+\frac{135}{32}\zeta_5+\frac{25}{64}\zeta_6-\frac{19}{32}\zeta_3^2\right)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\left(\frac{51}{512}\zeta_3+\frac{3}{128}\zeta_4-\frac{1075}{3072}\zeta_5+\frac{25}{1024}\zeta_6-\frac{35}{512}\zeta_3^2\right.$$
$$\left.+\frac{441}{1024}\zeta_7\right)+C_A\frac{d_F^{abcd}d_A^{abcd}}{N_RC_F}\left(-\frac{41}{8}\zeta_3+\frac{15}{4}\zeta_5+\frac{3}{32}\zeta_3^2\right)\Bigg]\xi^4. \tag{A.16}$$

$$\Lambda_q^{T,\overline{\rm MS},(1)}=\left(\frac{9}{4}C_A-2C_F\right)-C_A\xi-\frac{1}{4}C_A\xi^2,$$
$$\Lambda_q^{T,\overline{\rm MS},(2)}=\Bigg[C_A^2\left(\frac{523}{24}+\frac{11}{16}\zeta_3\right)-\frac{505}{18}C_AC_F+9C_F^2+C_AT_Fn_f\left(-\frac{16}{3}-4\zeta_3\right)+\frac{52}{9}C_FT_Fn_f\Bigg]$$
$$+\Bigg[C_A^2\left(-\frac{145}{48}-\frac{11}{8}\zeta_3\right)+\frac{13}{4}C_AC_F-2C_F^2-\frac{2}{3}C_AT_Fn_f\Bigg]\xi+\Bigg[C_A^2\left(-\frac{215}{144}-\frac{1}{16}\zeta_3\right)$$
$$-\frac{1}{2}C_AC_F-\frac{5}{9}C_AT_Fn_f\Bigg]\xi^2+\left(-\frac{1}{16}C_A^2-\frac{1}{4}C_AC_F\right)\xi^3+\frac{1}{16}C_A^2\xi^4,$$
$$\Lambda_q^{T,\overline{\rm MS},(3)}=\Bigg[C_A^3\left(\frac{2876719}{6912}+\frac{337}{6}\zeta_3-\frac{74015}{576}\zeta_5\right)+C_A^2C_F\left(-\frac{1470739}{2592}-\frac{5545}{24}\zeta_3\right.$$
$$\left.+\frac{2225}{6}\zeta_5\right)+C_AC_F^2\left(\frac{76339}{288}+316\zeta_3-\frac{1240}{3}\zeta_5\right)+C_F^3\left(-\frac{973}{12}-\frac{496}{3}\zeta_3+\frac{640}{3}\zeta_5\right)$$
$$+\frac{d_F^{abcd}d_A^{abcd}}{N_RC_F}\left(-\frac{383}{24}+\frac{68}{3}\zeta_3+\frac{235}{12}\zeta_5\right)+C_A^2T_Fn_f\left(-\frac{186119}{864}-92\zeta_3+\frac{320}{9}\zeta_5\right)$$
$$+C_AC_FT_Fn_f\left(\frac{130465}{648}+\frac{116}{3}\zeta_3\right)+C_F^2T_Fn_f\left(-\frac{43}{9}-\frac{64}{3}\zeta_3\right)+\frac{d_F^{abcd}d_F^{abcd}}{N_RC_F}n_f\left(32\right.$$
$$\left.-\frac{32}{3}\zeta_3-\frac{160}{3}\zeta_5\right)+C_AT_F^2n_f^2\left(\frac{700}{27}+\frac{112}{9}\zeta_3\right)-\frac{2000}{81}C_FT_F^2n_f^2\Bigg]+\Bigg[C_A^3\left(-\frac{265453}{6912}\right.$$

$$
\begin{aligned}
&-\frac{1801}{96}\zeta_3+\frac{925}{96}\zeta_5\Big)+C_A^2C_F\left(\frac{10223}{288}+\frac{29}{48}\zeta_3-\frac{25}{6}\zeta_5\right)+C_AC_F^2\left(-\frac{3235}{72}+6\zeta_3\right)\\
&+9C_F^3+\frac{d_F^{abcd}d_A^{abcd}}{N_RC_F}\left(-\frac{41}{24}-15\zeta_3+\frac{55}{4}\zeta_5\right)+C_A^2T_Fn_f\left(\frac{415}{72}+\frac{275}{18}\zeta_3\right)\\
&+C_AC_FT_Fn_f\left(-\frac{73}{18}-4\zeta_3\right)+\frac{52}{9}C_F^2T_Fn_f+C_AT_F^2n_f^2\left(\frac{40}{27}-\frac{32}{9}\zeta_3\right)\Big]\xi+\Big[C_A^3\Big(-\frac{40909}{2304}\\
&-\frac{469}{576}\zeta_3+\frac{595}{144}\zeta_5\Big)+C_A^2C_F\left(-\frac{269}{96}+\frac{5}{2}\zeta_3\right)-\frac{107}{32}C_AC_F^2+\frac{d_F^{abcd}d_A^{abcd}}{N_RC_F}\Big(\frac{5}{8}-\frac{5}{6}\zeta_3\\
&-\frac{5}{12}\zeta_5\Big)+C_A^2T_Fn_f\left(-\frac{575}{288}-\frac{3}{2}\zeta_3\right)+C_AC_FT_Fn_f\left(-\frac{113}{24}+4\zeta_3\right)\Big]\xi^2+\Big[C_A^3\Big(-\frac{3491}{768}\\
&+\frac{17}{12}\zeta_3-\frac{35}{96}\zeta_5\Big)+C_A^2C_F\left(-\frac{593}{144}+\frac{11}{16}\zeta_3\right)+\frac{1}{2}C_AC_F^2+\frac{d_F^{abcd}d_A^{abcd}}{N_RC_F}\left(\frac{1}{24}+\frac{3}{2}\zeta_3-\frac{5}{4}\zeta_5\right)\\
&+\frac{1}{36}C_A^2T_Fn_f-\frac{5}{9}C_AC_FT_Fn_f\Big]\xi^3+\Big[C_A^3\left(-\frac{155}{576}+\frac{29}{192}\zeta_3-\frac{35}{192}\zeta_5\right)-\frac{15}{32}C_A^2C_F\\
&+\frac{5}{18}C_A^2T_Fn_f\Big]\xi^4+\left(\frac{1}{16}C_A^3+\frac{1}{16}C_A^2C_F\right)\xi^5-\frac{1}{64}C_A^3\xi^6,
\end{aligned}
$$

$$
\begin{aligned}
\Lambda_q^{T,\overline{\rm MS},(4)}=&\Big[C_A^4\Big(\frac{127725515}{13824}+\frac{5736041}{3072}\zeta_3+\frac{81}{256}\zeta_4-\frac{4370125}{1728}\zeta_5+\frac{1723933}{2304}\zeta_3^2\\
&-\frac{92359883}{24576}\zeta_7\Big)+C_A^3C_F\Big(-\frac{1347318649}{93312}-\frac{26965903}{1728}\zeta_3-\frac{639}{64}\zeta_4+\frac{3102505}{432}\zeta_5\\
&-\frac{118177}{48}\zeta_3^2+\frac{7820533}{384}\zeta_7\Big)+C_A^2C_F^2\Big(\frac{24977213}{2592}+\frac{35295913}{1152}\zeta_3+\frac{177}{8}\zeta_4-\frac{120955}{36}\zeta_5\\
&+\frac{49427}{12}\zeta_3^2-\frac{4165903}{96}\zeta_7\Big)+C_AC_F^3\Big(-\frac{94547}{24}-\frac{205459}{9}\zeta_3-12\zeta_4-\frac{76475}{9}\zeta_5-3508\zeta_3^2\\
&+42658\zeta_7\Big)+C_F^4\left(\frac{10435}{24}+\frac{16256}{3}\zeta_3+8280\zeta_5+1120\zeta_3^2-16464\zeta_7\right)\\
&+\frac{12399}{2}\frac{d_F^{abcd}d_A^{abcd}}{N_R}\zeta_3+\frac{d_F^{abcd}d_A^{abcd}}{N_R}\left(\frac{2899}{36}+\frac{145}{24}\zeta_5-\frac{1281}{2}\zeta_3^2-\frac{26831}{4}\zeta_7\right)\\
&+\frac{490987}{768}\frac{d_A^{abcd}d_A^{abcd}}{N_A}\zeta_7+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\left(-\frac{5419}{576}+\frac{9131}{144}\zeta_3-\frac{104605}{96}\zeta_5+\frac{92479}{192}\zeta_3^2\right)\\
&+C_A\frac{d_F^{abcd}d_A^{abcd}}{N_RC_F}\left(-\frac{132143}{144}-\frac{429431}{96}\zeta_3+\frac{238265}{192}\zeta_5-\frac{2199}{32}\zeta_3^2+\frac{8572963}{1536}\zeta_7\right)\\
&+C_A^3T_Fn_f\left(-\frac{3008095}{432}-\frac{1333327}{3456}\zeta_3-\frac{81}{8}\zeta_4+\frac{633175}{864}\zeta_5-\frac{27743}{144}\zeta_3^2-\frac{982177}{1728}\zeta_7\right)\\
&+C_A^2C_FT_Fn_f\left(\frac{56186995}{7776}+\frac{187493}{864}\zeta_3+\frac{45}{2}\zeta_4-\frac{216905}{54}\zeta_5+270\zeta_3^2+\frac{130907}{36}\zeta_7\right)\\
&+C_AC_F^2T_Fn_f\left(-\frac{195383}{108}-\frac{47639}{18}\zeta_3-12\zeta_4+\frac{58340}{9}\zeta_5+\frac{80}{3}\zeta_3^2-3528\zeta_7\right)\\
&+C_F^3T_Fn_f\left(\frac{5305}{9}+\frac{16064}{9}\zeta_3-\frac{21920}{9}\zeta_5\right)+\frac{d_F^{abcd}d_F^{abcd}}{N_R}n_f\Big(-\frac{4352}{9}-\frac{12544}{3}\zeta_3
\end{aligned}
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\begin{aligned}
&+6400\zeta_5+1600\zeta_3^2-\frac{9016}{3}\zeta_7\bigg)+\frac{d_F^{abcd}d_A^{abcd}}{N_A}n_f\bigg(\frac{1979}{6}-\frac{13331}{18}\zeta_3-\frac{14255}{9}\zeta_5\\
&-\frac{1078}{3}\zeta_3^2+\frac{66619}{36}\zeta_7\bigg)+C_A\frac{d_F^{abcd}d_F^{abcd}}{N_RC_F}n_f\bigg(\frac{19144}{9}+\frac{28304}{9}\zeta_3-\frac{65780}{9}\zeta_5-\frac{4384}{3}\zeta_3^2\\
&+\frac{28840}{9}\zeta_7\bigg)+C_A^2T_F^2n_f^2\bigg(\frac{363235}{216}+\frac{1745}{6}\zeta_3+\frac{3455}{9}\zeta_5+\frac{484}{3}\zeta_3^2-\frac{441}{4}\zeta_7\bigg)\\
&+C_AC_FT_F^2n_f^2\bigg(-\frac{70051}{54}+\frac{1034}{27}\zeta_3-\frac{8480}{27}\zeta_5-192\zeta_3^2\bigg)+C_F^2T_F^2n_f^2\bigg(-\frac{16066}{81}\\
&+\frac{832}{3}\zeta_3\bigg)+\frac{d_F^{abcd}d_F^{abcd}}{N_A}n_f^2\bigg(-\frac{1792}{3}+\frac{256}{3}\zeta_3+\frac{1280}{3}\zeta_5\bigg)+C_AT_F^3n_f^3\bigg(-\frac{3592}{27}-\frac{128}{27}\zeta_3\\
&-\frac{1600}{27}\zeta_5\bigg)+\frac{86560}{729}C_FT_F^3n_f^3\bigg]+\bigg[C_A^4\bigg(-\frac{147243139}{248832}-\frac{12532529}{82944}\zeta_3+\frac{167885}{1024}\zeta_5\\
&+\frac{13089}{512}\zeta_3^2-\frac{9099041}{110592}\zeta_7\bigg)+C_A^3C_F\bigg(\frac{2296813}{3456}+\frac{42353}{864}\zeta_3+\frac{153}{32}\zeta_4-\frac{295855}{1728}\zeta_5\\
&+\frac{1279}{96}\zeta_3^2-\frac{50953}{256}\zeta_7\bigg)+C_A^2C_F^2\bigg(-\frac{9925495}{10368}-\frac{62251}{192}\zeta_3-\frac{27}{4}\zeta_4+510\zeta_5-\frac{5}{4}\zeta_3^2\\
&+\frac{5005}{32}\zeta_7\bigg)+C_AC_F^3\bigg(\frac{111187}{288}+\frac{1199}{3}\zeta_3-560\zeta_5\bigg)+C_F^4\bigg(-\frac{1009}{12}-\frac{496}{3}\zeta_3+\frac{640}{3}\zeta_5\bigg)\\
&+\frac{d_F^{abcd}d_A^{abcd}}{N_R}\bigg(-\frac{509}{24}+\frac{314}{3}\zeta_3-\frac{3515}{12}\zeta_5+\frac{87}{2}\zeta_3^2+\frac{3059}{16}\zeta_7\bigg)-\frac{25}{192}\frac{d_A^{abcd}d_A^{abcd}}{N_A}\\
&+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\bigg(-\frac{155347}{576}\zeta_3+\frac{14515}{48}\zeta_5-\frac{5825}{64}\zeta_3^2+\frac{232981}{2304}\zeta_7\bigg)+C_A\frac{d_F^{abcd}d_A^{abcd}}{N_RC_F}\bigg(\frac{10283}{864}\\
&-\frac{5659}{18}\zeta_3+\frac{435805}{576}\zeta_5+\frac{607}{16}\zeta_3^2-\frac{82341}{128}\zeta_7\bigg)+C_A^3T_Fn_f\bigg(\frac{5521495}{31104}+\frac{1595761}{5184}\zeta_3\\
&+\frac{27}{2}\zeta_4-\frac{83185}{864}\zeta_5+\frac{2027}{144}\zeta_3^2+\frac{5047}{64}\zeta_7\bigg)+C_A^2C_FT_Fn_f\bigg(-\frac{48115}{288}-\frac{5855}{432}\zeta_3-18\zeta_4\\
&+\frac{1235}{27}\zeta_5-\frac{40}{3}\zeta_3^2\bigg)+\frac{193249}{648}C_AC_F^2T_Fn_f+C_F^3T_Fn_f\bigg(-\frac{79}{9}-\frac{64}{3}\zeta_3\bigg)\\
&+\frac{d_F^{abcd}d_F^{abcd}}{N_R}n_f\bigg(32-\frac{32}{3}\zeta_3-\frac{160}{3}\zeta_5\bigg)+\frac{d_F^{abcd}d_A^{abcd}}{N_A}n_f\bigg(-\frac{91}{54}+\frac{3481}{9}\zeta_3-\frac{55}{3}\zeta_5+\frac{40}{3}\zeta_3^2\\
&-\frac{1617}{4}\zeta_7\bigg)+C_A\frac{d_F^{abcd}d_F^{abcd}}{N_RC_F}n_f\bigg(-16+\frac{8}{3}\zeta_3+\frac{20}{9}\zeta_5+\frac{64}{3}\zeta_3^2\bigg)+C_A^2T_F^2n_f^2\bigg(\frac{49687}{972}\\
&-\frac{18971}{162}\zeta_3-10\zeta_5\bigg)+C_AC_FT_F^2n_f^2\bigg(\frac{1219}{27}-\frac{128}{3}\zeta_3\bigg)-\frac{2000}{81}C_F^2T_F^2n_f^2\\
&+C_AT_F^3n_f^3\bigg(-\frac{2528}{243}+\frac{1280}{81}\zeta_3\bigg)\bigg]\xi+\bigg[C_A^4\bigg(-\frac{17515951}{62208}-\frac{1072369}{82944}\zeta_3-\frac{81}{128}\zeta_4\\
&+\frac{4375235}{41472}\zeta_5-\frac{3289}{2304}\zeta_3^2-\frac{889357}{55296}\zeta_7\bigg)+C_A^3C_F\bigg(-\frac{35429}{10368}+\frac{112277}{1152}\zeta_3+\frac{33}{32}\zeta_4\\
&-\frac{27445}{288}\zeta_5+\frac{147}{32}\zeta_3^2-\frac{4207}{768}\zeta_7\bigg)+C_A^2C_F^2\bigg(-\frac{109513}{1152}-\frac{24233}{384}\zeta_3-\frac{15}{8}\zeta_4+\frac{245}{3}\zeta_5\bigg)
\end{aligned}
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\begin{aligned}
&+ C_A C_F^3\Big(\frac{565}{24} + \frac{124}{3}\zeta_3 - \frac{160}{3}\zeta_5\Big) + \frac{d_F^{abcd}d_A^{abcd}}{N_R}\Big(-\frac{35}{24} + \frac{61}{6}\zeta_3 + \frac{605}{24}\zeta_5 - 6\zeta_3^2 \\
&- \frac{441}{16}\zeta_7\Big) - \frac{7}{576}\frac{d_A^{abcd}d_A^{abcd}}{N_A} + \frac{d_A^{abcd}d_A^{abcd}}{N_A}\Big(\frac{749}{96}\zeta_3 - \frac{47905}{1152}\zeta_5 - \frac{415}{24}\zeta_3^2 + \frac{160475}{4608}\zeta_7\Big) \\
&+ C_A\frac{d_F^{abcd}d_A^{abcd}}{N_R C_F}\Big(\frac{11}{4} - \frac{1717}{432}\zeta_3 - \frac{134585}{1728}\zeta_5 - \frac{26}{3}\zeta_3^2 + \frac{15421}{384}\zeta_7\Big) + C_A^3 T_F n_f\Big(\frac{1181573}{31104} \\
&- \frac{169241}{10368}\zeta_3 + \frac{45}{8}\zeta_4 + \frac{6205}{1296}\zeta_5 + \frac{10}{9}\zeta_3^2\Big) + C_A^2 C_F T_F n_f\Big(-\frac{14611}{324} + \frac{12215}{288}\zeta_3 - \frac{15}{2}\zeta_4 \\
&+ 20\zeta_5\Big) + C_A C_F^2 T_F n_f\Big(\frac{409}{18} + 18\zeta_3 - 40\zeta_5\Big) + \frac{d_F^{abcd}d_A^{abcd}}{N_A} n_f\Big(\frac{23}{18} - \frac{35}{54}\zeta_3 + \frac{310}{27}\zeta_5 \\
&+ \frac{4}{3}\zeta_3^2\Big) + C_A\frac{d_F^{abcd}d_F^{abcd}}{N_R C_F} n_f\Big(-8 + \frac{8}{3}\zeta_3 + \frac{40}{3}\zeta_5\Big) + C_A^2 T_F^2 n_f^2\Big(\frac{2797}{486} + \frac{74}{9}\zeta_3 - \frac{10}{9}\zeta_5\Big) \\
&+ C_A C_F T_F^2 n_f^2\Big(\frac{439}{18} - \frac{184}{9}\zeta_3\Big)\Big]\xi^2 + \Big[C_A^4\Big(-\frac{925339}{13824} + \frac{794815}{27648}\zeta_3 - \frac{3}{8}\zeta_4 - \frac{14645}{1152}\zeta_5 \\
&- \frac{499}{2304}\zeta_3^2 + \frac{371}{2048}\zeta_7\Big) + C_A^3 C_F\Big(-\frac{28045}{576} + \frac{1727}{144}\zeta_3 - \frac{9}{32}\zeta_4 + \frac{275}{72}\zeta_5 + \frac{3}{16}\zeta_3^2\Big) \\
&+ C_A^2 C_F^2\Big(\frac{1243}{384} + \frac{139}{24}\zeta_3 - 5\zeta_5\Big) + C_A C_F^3\Big(-\frac{71}{32} - \frac{7}{2}\zeta_3\Big) + \frac{4}{3}C_F^4\zeta_3 + \frac{d_F^{abcd}d_A^{abcd}}{N_R}\Big(\frac{13}{24} \\
&- \frac{5}{6}\zeta_3 - \frac{5}{12}\zeta_5\Big) + \frac{d_A^{abcd}d_A^{abcd}}{N_A}\Big(\frac{11}{576} + \frac{103}{144}\zeta_3 - \frac{1775}{384}\zeta_5 - \frac{365}{96}\zeta_3^2 + \frac{231}{128}\zeta_7\Big) \\
&+ C_A\frac{d_F^{abcd}d_A^{abcd}}{N_R C_F}\Big(\frac{11}{144} + \frac{39}{4}\zeta_3 - \frac{1165}{64}\zeta_5 + \frac{25}{16}\zeta_3^2 + \frac{161}{16}\zeta_7\Big) + C_A^3 T_F n_f\Big(\frac{10855}{3456} \\
&- \frac{1507}{144}\zeta_3 + \frac{55}{32}\zeta_5 + \frac{1}{6}\zeta_3^2\Big) + C_A^2 C_F T_F n_f\Big(-\frac{149}{96} - \frac{5}{6}\zeta_3\Big) + C_A C_F^2 T_F n_f\Big(-\frac{109}{24} + 4\zeta_3\Big) \\
&+ \frac{d_F^{abcd}d_A^{abcd}}{N_A} n_f\Big(\frac{1}{18} + \frac{1}{3}\zeta_3\Big) + C_A^2 T_F^2 n_f^2\Big(-\frac{4}{9} + \frac{8}{3}\zeta_3\Big)\Big]\xi^3 + \Big[C_A^4\Big(-\frac{431095}{41472} \\
&+ \frac{18239}{4608}\zeta_3 - \frac{15}{256}\zeta_4 - \frac{66985}{13824}\zeta_5 + \frac{21}{64}\zeta_3^2 - \frac{151613}{221184}\zeta_7\Big) + C_A^3 C_F\Big(-\frac{14015}{1152} + \frac{997}{384}\zeta_3 \\
&- \frac{3}{64}\zeta_4 - \frac{5}{16}\zeta_5\Big) + C_A^2 C_F^2\Big(\frac{123}{128} - \zeta_3\Big) + \frac{2}{3}C_A C_F^3\zeta_3 + \frac{d_F^{abcd}d_A^{abcd}}{N_R}\Big(\frac{1}{24} + \frac{3}{2}\zeta_3 - \frac{5}{4}\zeta_5\Big) \\
&+ \frac{d_A^{abcd}d_A^{abcd}}{N_A}\Big(\frac{1}{288} - \frac{587}{288}\zeta_3 + \frac{4585}{1152}\zeta_5 + \frac{83}{64}\zeta_3^2 - \frac{22631}{4608}\zeta_7\Big) + C_A\frac{d_F^{abcd}d_A^{abcd}}{N_R C_F}\Big(-\frac{3}{16} \\
&+ \frac{529}{96}\zeta_3 - \frac{5}{2}\zeta_5 - \frac{1}{32}\zeta_3^2 - \frac{721}{512}\zeta_7\Big) + C_A^3 T_F n_f\Big(\frac{6787}{10368} + \frac{541}{432}\zeta_3 - \frac{175}{432}\zeta_5\Big) \\
&+ C_A^2 C_F T_F n_f\Big(\frac{41}{32} - 2\zeta_3\Big) + \frac{25}{81}C_A^2 T_F^2 n_f^2\Big]\xi^4 + \Big[C_A^4\Big(\frac{2207}{9216} - \frac{21}{64}\zeta_3 - \frac{4445}{9216}\zeta_5 + \frac{21}{512}\zeta_3^2 \\
&+ \frac{4823}{36864}\zeta_7\Big) + C_A^3 C_F\Big(-\frac{727}{1152} + \frac{3}{64}\zeta_3 - \frac{35}{192}\zeta_5\Big) + C_A^2 C_F^2\Big(-\frac{3}{32} - \frac{1}{4}\zeta_3\Big) + \frac{1}{6}C_A C_F^3\zeta_3 \\
&+ \frac{d_A^{abcd}d_A^{abcd}}{N_A}\Big(-\frac{5}{64}\zeta_3 + \frac{35}{384}\zeta_5 + \frac{15}{64}\zeta_3^2 - \frac{91}{192}\zeta_7\Big) - \frac{1}{96}C_A\frac{d_F^{abcd}d_A^{abcd}}{N_R C_F} + \frac{17}{72}C_A^3 T_F n_f
\end{aligned}
$$

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$$+\frac{5}{18}C_A^2C_FT_Fn_f\Bigg]\xi^5+\Bigg[C_A^4\left(\frac{163}{768}-\frac{55}{768}\zeta_3+\frac{35}{384}\zeta_5\right)+\frac{21}{128}C_A^3C_F-\frac{5}{48}C_A^3T_Fn_f\Bigg]\xi^6$$
$$+\left(-\frac{7}{256}C_A^4-\frac{1}{64}C_A^3C_F\right)\xi^7+\frac{1}{256}C_A^4\xi^8. \tag{A.17}$$

$$\Lambda_g^{\overline{\text{MS}},(1)}=\frac{1}{4}C_A+\left(\frac{3}{4}C_A-C_F\right)\xi,$$
$$\Lambda_g^{\overline{\text{MS}},(2)}=\Bigg[C_A^2\left(\frac{1075}{192}-3\zeta_3\right)+C_AC_F\left(\frac{3}{4}-3\zeta_3\right)+\frac{19}{8}C_F^2-\frac{55}{24}C_AT_Fn_f+\frac{1}{2}C_FT_Fn_f\Bigg]$$
$$+\Bigg[\frac{181}{64}C_A^2+C_AC_F\left(-\frac{3}{2}-3\zeta_3\right)\Bigg]\xi+\left(C_A^2-\frac{3}{8}C_AC_F-2C_F^2\right)\xi^2,$$
$$\Lambda_g^{\overline{\text{MS}},(3)}=\Bigg[C_A^3\left(\frac{234157}{7776}-\frac{5461}{144}\zeta_3+\frac{9}{64}\zeta_4+\frac{335}{12}\zeta_5\right)+C_A^2C_F\left(\frac{5675}{162}-\frac{4105}{24}\zeta_3-\frac{69}{16}\zeta_4\right.$$
$$\left.+\frac{525}{4}\zeta_5\right)+C_AC_F^2\left(\frac{1259}{32}+82\zeta_3+6\zeta_4-100\zeta_5\right)-\frac{109}{12}C_F^3+\frac{d_F^{abcd}d_A^{abcd}}{N_RC_F}\left(\frac{9}{4}+\frac{37}{4}\zeta_3\right.$$
$$\left.-\frac{35}{2}\zeta_5\right)+C_A^2T_Fn_f\left(-\frac{89777}{3888}+\frac{359}{18}\zeta_3-\frac{9}{2}\zeta_4-20\zeta_5\right)+C_AC_FT_Fn_f\left(-\frac{6823}{324}+\frac{58}{3}\zeta_3\right.$$
$$\left.+6\zeta_4\right)+C_F^2T_Fn_f\left(-\frac{45}{2}+16\zeta_3\right)+C_AT_F^2n_f^2\left(\frac{1741}{486}+\frac{8}{9}\zeta_3\right)-\frac{302}{81}C_FT_F^2n_f^2\Bigg]$$
$$+\Bigg[C_A^3\left(\frac{17309}{384}-\frac{485}{32}\zeta_3+\frac{3}{16}\zeta_4-\frac{35}{12}\zeta_5\right)+C_A^2C_F\left(-\frac{13405}{288}+\frac{75}{8}\zeta_3+\frac{3}{8}\zeta_4+\frac{5}{2}\zeta_5\right)$$
$$+C_AC_F^2\left(-\frac{747}{32}-17\zeta_3+20\zeta_5\right)+\frac{41}{8}C_F^3+\frac{d_F^{abcd}d_A^{abcd}}{N_RC_F}\left(-2-\frac{145}{4}\zeta_3+40\zeta_5\right)$$
$$+C_A^2T_Fn_f\left(-14+\frac{17}{6}\zeta_3\right)+C_AC_FT_Fn_f\left(\frac{761}{72}-2\zeta_3\right)+\frac{19}{2}C_F^2T_Fn_f\Bigg]\xi+\Bigg[C_A^3\left(\frac{4061}{384}\right.$$
$$\left.-\frac{113}{48}\zeta_3+\frac{3}{64}\zeta_4+\frac{5}{12}\zeta_5\right)+C_A^2C_F\left(-\frac{161}{16}-\frac{9}{8}\zeta_3+\frac{3}{16}\zeta_4+\frac{5}{4}\zeta_5\right)+C_AC_F^2\left(-\frac{29}{2}\right.$$
$$\left.+\frac{9}{2}\zeta_3\right)+\frac{d_F^{abcd}d_A^{abcd}}{N_RC_F}\left(-\frac{1}{4}+\frac{1}{4}\zeta_3-\frac{5}{2}\zeta_5\right)\Bigg]\xi^2+\Bigg[C_A^3\left(\frac{141}{64}-\frac{17}{96}\zeta_3\right)+C_A^2C_F\left(-\frac{173}{96}\right.$$
$$\left.+\frac{7}{24}\zeta_3\right)+C_AC_F^2\left(-\frac{23}{8}+\frac{1}{2}\zeta_3\right)-\frac{2}{3}C_F^3\zeta_3-\frac{1}{4}\frac{d_F^{abcd}d_A^{abcd}}{N_RC_F}\zeta_3\Bigg]\xi^3,$$
$$\Lambda_g^{\overline{\text{MS}},(4)}=\Bigg[C_A^4\left(\frac{1411848247}{5971968}+\frac{3054371}{55296}\zeta_3+\frac{141175}{12288}\zeta_4+\frac{584781}{8192}\zeta_5-\frac{238325}{8192}\zeta_6\right.$$
$$\left.+\frac{4257169}{12288}\zeta_3^2-\frac{13021043}{18432}\zeta_7\right)+C_A^3C_F\left(\frac{3435791}{62208}-\frac{3103177}{576}\zeta_3-\frac{19179}{512}\zeta_4\right.$$
$$\left.+\frac{6046291}{1152}\zeta_5-\frac{20725}{256}\zeta_6-\frac{70617}{64}\zeta_3^2+\frac{218155}{128}\zeta_7\right)+C_A^2C_F^2\left(\frac{8967169}{13824}+\frac{178705}{48}\zeta_3\right.$$

$$-\frac{1239}{16}\zeta_4-\frac{49193}{24}\zeta_5+\frac{3925}{8}\zeta_6+\frac{6137}{4}\zeta_3^2-\frac{88109}{16}\zeta_7\bigg)+C_AC_F^3\bigg(\frac{106883}{288}+\frac{6239}{8}\zeta_3$$

$$+318\zeta_4+\frac{1115}{3}\zeta_5-900\zeta_6-492\zeta_3^2-\frac{315}{2}\zeta_7\bigg)+C_F^4\bigg(-\frac{10321}{128}-\frac{2289}{2}\zeta_3-150\zeta_4$$

$$-2250\zeta_5+400\zeta_6-80\zeta_3^2+3528\zeta_7\bigg)+\frac{d_F^{abcd}d_A^{abcd}}{N_R}\bigg(-\frac{2075}{12}-\frac{3391}{16}\zeta_3+\frac{3339}{64}\zeta_4$$

$$-\frac{5185}{32}\zeta_5-\frac{4275}{64}\zeta_6+\frac{4881}{32}\zeta_3^2+\frac{903}{4}\zeta_7\bigg)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\bigg(-\frac{4163}{2304}+\frac{143047}{1536}\zeta_3+\frac{1827}{64}\zeta_4$$

$$+\frac{1110235}{3072}\zeta_5-\frac{50925}{1024}\zeta_6+\frac{37611}{512}\zeta_3^2-\frac{106477}{192}\zeta_7\bigg)+C_A\frac{d_F^{abcd}d_A^{abcd}}{N_RC_F}\bigg(\frac{5225}{96}+\frac{7193}{192}\zeta_3$$

$$-\frac{31405}{64}\zeta_5-\frac{30149}{64}\zeta_3^2+\frac{111391}{128}\zeta_7\bigg)+C_A^3T_Fn_f\bigg(-\frac{4654519}{13824}+\frac{160463}{576}\zeta_3-\frac{3091}{64}\zeta_4$$

$$-\frac{404653}{576}\zeta_5+\frac{275}{8}\zeta_6+\frac{314}{3}\zeta_3^2+\frac{47915}{576}\zeta_7\bigg)+C_A^2C_FT_Fn_f\bigg(\frac{1065853}{3456}+\frac{4513}{3}\zeta_3+\frac{543}{16}\zeta_4$$

$$-\frac{100033}{72}\zeta_5-\frac{25}{2}\zeta_6+97\zeta_3^2+581\zeta_7\bigg)+C_AC_F^2T_Fn_f\bigg(-\frac{1649381}{1728}-\frac{831}{4}\zeta_3+\frac{111}{4}\zeta_4$$

$$+\frac{779}{3}\zeta_5+25\zeta_6-254\zeta_3^2+882\zeta_7\bigg)+C_F^3T_Fn_f\bigg(\frac{4159}{144}+25\zeta_3-24\zeta_4+\frac{20}{3}\zeta_5\bigg)$$

$$+\frac{d_F^{abcd}d_F^{abcd}}{N_R}n_f\big(218+1124\zeta_3-1240\zeta_5-384\zeta_3^2\big)+\frac{d_F^{abcd}d_A^{abcd}}{N_A}n_f\bigg(\frac{73}{6}-18\zeta_4+\frac{75}{2}\zeta_6$$

$$+48\zeta_3^2-\frac{385}{6}\zeta_7\bigg)+\frac{d_F^{abcd}d_A^{abcd}}{N_A}n_f\bigg(-\frac{597}{2}\zeta_3+\frac{4955}{12}\zeta_5\bigg)+C_A\frac{d_F^{abcd}d_F^{abcd}}{N_RC_F}n_f\bigg(-\frac{80}{3}$$

$$+\frac{7312}{3}\zeta_3-\frac{4960}{3}\zeta_5-400\zeta_3^2-\frac{1631}{3}\zeta_7\bigg)+C_A^2T_F^2n_f^2\bigg(\frac{4931027}{31104}+\frac{1075}{36}\zeta_3+\frac{45}{4}\zeta_4$$

$$+\frac{551}{9}\zeta_5-16\zeta_3^2\bigg)+C_AC_FT_F^2n_f^2\bigg(-\frac{370733}{1296}-207\zeta_3-3\zeta_4+\frac{332}{9}\zeta_5\bigg)$$

$$+C_F^2T_F^2n_f^2\bigg(\frac{27067}{216}-\frac{220}{3}\zeta_3-12\zeta_4\bigg)+C_AT_F^3n_f^3\bigg(-\frac{62179}{2916}-\frac{20}{27}\zeta_3+\frac{4}{3}\zeta_4\bigg)$$

$$+C_FT_F^3n_f^3\bigg(\frac{14609}{243}-\frac{16}{9}\zeta_3\bigg)\bigg]+\bigg[C_A^4\bigg(\frac{491609003}{746496}-\frac{5711371}{18432}\zeta_3+\frac{3837}{1024}\zeta_4$$

$$+\frac{6419771}{110592}\zeta_5-\frac{51175}{12288}\zeta_6-\frac{101843}{6144}\zeta_3^2-\frac{94661}{6144}\zeta_7\bigg)+C_A^3C_F\bigg(-\frac{653068915}{746496}+\frac{34507}{108}\zeta_3$$

$$+\frac{1815}{256}\zeta_4-\frac{29587}{576}\zeta_5-\frac{2875}{192}\zeta_6+\frac{8893}{96}\zeta_3^2-\frac{3115}{128}\zeta_7\bigg)+C_A^2C_F^2\bigg(-\frac{22760417}{41472}+\frac{5915}{36}\zeta_3$$

$$+\frac{15}{2}\zeta_4-\frac{5395}{72}\zeta_5+\frac{25}{8}\zeta_6-\frac{117}{4}\zeta_3^2+\frac{707}{16}\zeta_7\bigg)+C_AC_F^3\bigg(\frac{3631}{24}-\frac{809}{8}\zeta_3-6\zeta_4+210\zeta_5$$

$$-\frac{399}{2}\zeta_7\bigg)+\frac{19}{12}C_F^4+\frac{d_F^{abcd}d_A^{abcd}}{N_R}\bigg(\frac{85}{3}+\frac{275}{4}\zeta_3+\frac{303}{16}\zeta_4+\frac{605}{4}\zeta_5-\frac{275}{8}\zeta_6+\frac{47}{4}\zeta_3^2$$

$$-\frac{9177}{32}\zeta_7\bigg)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\bigg(-\frac{859}{192}+\frac{515}{12}\zeta_3-\frac{27125}{1536}\zeta_5-\frac{637}{128}\zeta_7\bigg)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\bigg(\frac{1347}{256}\zeta_4$$

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$$
-\frac{2375}{512}\zeta_6-\frac{2951}{256}\zeta_3^2\Big)+C_A\frac{d_F^{abcd}d_A^{abcd}}{N_RC_F}\Big(-\frac{2003}{288}-\frac{477905}{576}\zeta_3+\frac{74365}{288}\zeta_5+\frac{3619}{64}\zeta_3^2
$$

$$
+\frac{66815}{128}\zeta_7\Big)+C_A^3T_Fn_f\Big(-\frac{69331229}{186624}+\frac{188165}{1728}\zeta_3-\frac{117}{8}\zeta_4-\frac{4237}{432}\zeta_5+\frac{25}{12}\zeta_3^2
$$

$$
-\frac{441}{64}\zeta_7\Big)+C_A^2C_FT_Fn_f\Big(\frac{7973609}{23328}-\frac{22637}{216}\zeta_3+\frac{147}{8}\zeta_4+\frac{845}{9}\zeta_5+14\zeta_3^2\Big)
$$

$$
+C_AC_F^2T_Fn_f\Big(\frac{1003913}{2592}-\frac{569}{18}\zeta_3-\frac{3}{2}\zeta_4-\frac{74}{9}\zeta_5-32\zeta_3^2\Big)+C_F^3T_Fn_f\Big(\frac{19}{6}-8\zeta_3\Big)
$$

$$
+\frac{d_F^{abcd}d_A^{abcd}}{N_A}n_f\Big(\frac{181}{18}+\frac{898}{9}\zeta_3-\frac{3295}{36}\zeta_5+19\zeta_3^2\Big)+C_A^2T_F^2n_f^2\Big(\frac{985201}{23328}-\frac{455}{54}\zeta_3
$$

$$
+\frac{1}{2}\zeta_4\Big)+C_AC_FT_F^2n_f^2\Big(-\frac{74507}{1458}+\frac{724}{27}\zeta_3+2\zeta_4-\frac{80}{3}\zeta_5\Big)+C_F^2T_F^2n_f^2\Big(-\frac{3946}{81}
$$

$$
+\frac{32}{3}\zeta_3\Big)\Big]\xi+\Big[C_A^4\Big(\frac{38637757}{221184}-\frac{584339}{6912}\zeta_3+\frac{995}{1024}\zeta_4+\frac{284285}{27648}\zeta_5-\frac{175}{1024}\zeta_6-\frac{965}{768}\zeta_3^2
$$

$$
+\frac{136465}{18432}\zeta_7\Big)+C_A^3C_F\Big(-\frac{6427921}{27648}+\frac{111637}{1152}\zeta_3+\frac{507}{256}\zeta_4+\frac{1027}{72}\zeta_5-\frac{475}{384}\zeta_6-\frac{157}{96}\zeta_3^2
$$

$$
-\frac{259}{128}\zeta_7\Big)+C_A^2C_F^2\Big(-\frac{116905}{576}+\frac{3029}{48}\zeta_3-\frac{3}{8}\zeta_4+15\zeta_5+\frac{9}{2}\zeta_3^2-\frac{35}{4}\zeta_7\Big)
$$

$$
+C_AC_F^3\Big(-\frac{129}{64}+\frac{3}{2}\zeta_3-20\zeta_5\Big)+C_F^4\Big(-\frac{7}{4}+3\zeta_3\Big)+\frac{33}{32}\frac{d_F^{abcd}d_A^{abcd}}{N_R}\zeta_4
$$

$$
+\frac{d_F^{abcd}d_A^{abcd}}{N_R}\Big(-\frac{7}{4}-\frac{131}{8}\zeta_3-\frac{55}{16}\zeta_5-\frac{175}{32}\zeta_6-\frac{131}{16}\zeta_3^2+\frac{105}{2}\zeta_7\Big)+\frac{9}{16}\frac{d_A^{abcd}d_A^{abcd}}{N_A}\zeta_4
$$

$$
+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\Big(-\frac{1757}{2304}+\frac{181}{12}\zeta_3-\frac{695}{96}\zeta_5+\frac{375}{256}\zeta_6-\frac{537}{128}\zeta_3^2-\frac{161}{384}\zeta_7\Big)
$$

$$
+C_A\frac{d_F^{abcd}d_A^{abcd}}{N_RC_F}\Big(-\frac{2711}{288}-\frac{30833}{576}\zeta_3+\frac{31595}{576}\zeta_5-\frac{347}{64}\zeta_3^2-\frac{3507}{128}\zeta_7\Big)
$$

$$
+C_A^3T_Fn_f\Big(-\frac{301121}{6912}+\frac{26191}{1728}\zeta_3-\frac{1}{16}\zeta_4-\frac{4969}{1728}\zeta_5+\frac{1}{12}\zeta_3^2\Big)+C_A^2C_FT_Fn_f\Big(\frac{19991}{432}
$$

$$
-\frac{1661}{72}\zeta_3-\frac{7}{16}\zeta_4+\frac{91}{72}\zeta_5+\zeta_3^2\Big)+C_AC_F^2T_Fn_f\Big(\frac{9635}{144}-\frac{61}{3}\zeta_3\Big)+C_F^3T_Fn_f(3+4\zeta_3)
$$

$$
+\frac{20}{9}\frac{d_F^{abcd}d_A^{abcd}}{N_A}n_f\zeta_5+\frac{d_F^{abcd}d_A^{abcd}}{N_A}n_f\Big(\frac{14}{9}+\frac{35}{18}\zeta_3+4\zeta_3^2\Big)\Big]\xi^2+\Big[C_A^4\Big(\frac{59059}{1536}
$$

$$
-\frac{255379}{18432}\zeta_3+\frac{77}{512}\zeta_4+\frac{90227}{36864}\zeta_5+\frac{475}{12288}\zeta_6+\frac{29}{2048}\zeta_3^2+\frac{9821}{18432}\zeta_7\Big)+C_A^3C_F\Big(-\frac{4825}{96}
$$

$$
+\frac{707}{32}\zeta_3+\frac{63}{256}\zeta_4+\frac{79}{192}\zeta_5-\frac{1}{16}\zeta_3^2+\frac{147}{128}\zeta_7\Big)+C_A^2C_F^2\Big(-\frac{2099}{64}-\frac{11}{2}\zeta_3-\frac{3}{16}\zeta_4
$$

$$
-\frac{5}{4}\zeta_5\Big)+C_AC_F^3\Big(-\frac{3}{2}+\frac{10}{3}\zeta_3\Big)+\frac{d_F^{abcd}d_A^{abcd}}{N_R}\Big(-\frac{5}{4}\zeta_3-\frac{9}{16}\zeta_4-5\zeta_5-3\zeta_3^2+\frac{441}{32}\zeta_7\Big)
$$

$$
+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\Big(\frac{239}{384}\zeta_3+\frac{3}{256}\zeta_4-\frac{215}{1536}\zeta_5+\frac{275}{512}\zeta_6-\frac{141}{256}\zeta_3^2+\frac{35}{24}\zeta_7\Big)
$$

$$+ C_A\frac{d_F^{abcd}d_A^{abcd}}{N_R C_F}\left(-\frac{17}{32}-\frac{167}{64}\zeta_3-\frac{265}{32}\zeta_5-\frac{75}{64}\zeta_3^2+\frac{161}{128}\zeta_7\right)\Bigg]\xi^3+\Bigg[C_A^4\bigg(\frac{75971}{12288}$$

$$-\frac{8441}{6144}\zeta_3-\frac{37}{4096}\zeta_4+\frac{8635}{73728}\zeta_5+\frac{575}{24576}\zeta_6+\frac{23}{12288}\zeta_3^2\bigg)+C_A^3C_F\bigg(-\frac{2795}{384}+\frac{1595}{384}\zeta_3$$

$$+\frac{1}{512}\zeta_4-\frac{5}{384}\zeta_5+\frac{25}{768}\zeta_6-\frac{5}{192}\zeta_3^2\bigg)+C_A^2C_F^2\left(-\frac{2065}{384}-\frac{35}{12}\zeta_3\right)+C_AC_F^3\left(\frac{3}{4}-\frac{1}{2}\zeta_3\right)$$

$$+\frac{2}{3}C_F^4\zeta_3+\frac{d_F^{abcd}d_A^{abcd}}{N_R}\left(-\frac{3}{16}\zeta_3-\frac{21}{64}\zeta_4+\frac{55}{32}\zeta_5+\frac{25}{64}\zeta_6-\frac{19}{32}\zeta_3^2\right)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\bigg(\frac{79}{512}\zeta_3$$

$$+\frac{3}{128}\zeta_4+\frac{125}{3072}\zeta_5+\frac{25}{1024}\zeta_6+\frac{1}{512}\zeta_3^2\bigg)+C_A\frac{d_F^{abcd}d_A^{abcd}}{N_RC_F}\left(-\frac{53}{32}\zeta_3+\frac{25}{32}\zeta_5-\frac{3}{8}\zeta_3^2\right)\Bigg]\xi^4. \tag{A.18}$$

$$\Lambda_g^{T,\overline{\text{MS}},(1)}=\frac{1}{2}C_A+\left(-\frac{1}{2}C_A+2C_F\right)\xi,$$

$$\Lambda_g^{T,\overline{\text{MS}},(2)}=\left[C_A^2\left(\frac{167}{36}-\frac{5}{2}\zeta_3\right)+C_AC_F\left(\frac{43}{6}+4\zeta_3\right)-3C_F^2-\frac{7}{9}C_AT_Fn_f-4C_FT_Fn_f\right]$$

$$+\left[C_A^2\left(-\frac{29}{24}-\frac{1}{2}\zeta_3\right)+9C_AC_F\right]\xi+\left(-\frac{5}{8}C_A^2+\frac{3}{2}C_AC_F+2C_F^2\right)\xi^2,$$

$$\Lambda_g^{T,\overline{\text{MS}},(3)}=\Bigg[C_A^3\left(\frac{1044265}{10368}-\frac{10229}{144}\zeta_3+\frac{10}{3}\zeta_5\right)+C_A^2C_F\left(\frac{38057}{216}+\frac{5141}{36}\zeta_3-\frac{410}{3}\zeta_5\right)$$

$$+C_AC_F^2\left(-\frac{3515}{48}-\frac{184}{3}\zeta_3+\frac{280}{3}\zeta_5\right)+3C_F^3+\frac{d_F^{abcd}d_A^{abcd}}{N_RC_F}\left(-\frac{53}{6}+68\zeta_3-70\zeta_5\right)$$

$$+C_A^2T_Fn_f\left(-\frac{61085}{1296}-\frac{14}{9}\zeta_3+\frac{80}{3}\zeta_5\right)+C_AC_FT_Fn_f\left(-\frac{15827}{108}-\frac{40}{9}\zeta_3\right)+\frac{28}{3}C_F^2T_Fn_f$$

$$+\frac{260}{81}C_AT_F^2n_f^2+\frac{208}{9}C_FT_F^2n_f^2\Bigg]+\Bigg[C_A^3\left(-\frac{38071}{1728}+\frac{157}{72}\zeta_3+\frac{55}{18}\zeta_5\right)+C_A^2C_F\bigg(\frac{12401}{96}$$

$$-\frac{139}{3}\zeta_3+\frac{10}{3}\zeta_5\bigg)+C_AC_F^2\left(\frac{361}{16}+6\zeta_3\right)-\frac{17}{4}C_F^3+\frac{d_F^{abcd}d_A^{abcd}}{N_RC_F}\left(\frac{8}{3}+\frac{100}{3}\zeta_3-\frac{130}{3}\zeta_5\right)$$

$$+C_A^2T_Fn_f\left(\frac{3197}{432}-\frac{2}{9}\zeta_3\right)+C_AC_FT_Fn_f\left(-\frac{112}{3}+8\zeta_3\right)-11C_F^2T_Fn_f\Bigg]\xi+\Bigg[C_A^3\bigg(-\frac{6983}{1152}$$

$$+\frac{7}{16}\zeta_3\bigg)+C_A^2C_F\left(\frac{757}{32}-\frac{9}{4}\zeta_3\right)+C_AC_F^2(17-6\zeta_3)+\frac{1}{6}\frac{d_F^{abcd}d_A^{abcd}}{N_RC_F}\Bigg]\xi^2+\bigg(-\frac{43}{32}C_A^3$$

$$+\frac{63}{16}C_A^2C_F+\frac{11}{4}C_AC_F^2\bigg)\xi^3,$$

$$\Lambda_g^{T,\overline{\text{MS}},(4)}=\Bigg[C_A^4\bigg(\frac{876642067}{373248}-\frac{16108271}{6912}\zeta_3+\frac{9}{128}\zeta_4+\frac{3803705}{13824}\zeta_5-\frac{35587}{192}\zeta_3^2$$

$$+\frac{5847541}{9216}\zeta_7\bigg)+C_A^3C_F\bigg(\frac{45193697}{10368}+\frac{3280259}{864}\zeta_3-\frac{69}{32}\zeta_4-\frac{108155}{24}\zeta_5+\frac{5777}{4}\zeta_3^2$$

$$
\begin{aligned}
&-\frac{1379777}{576}\zeta_7\bigg)+C_A^2C_F^2\bigg(-\frac{1056145}{576}-\frac{76895}{16}\zeta_3+3\zeta_4+\frac{22525}{9}\zeta_5-\frac{6172}{3}\zeta_3^2\\
&+\frac{91945}{12}\zeta_7\bigg)+C_AC_F^3\bigg(\frac{26735}{48}+\frac{6285}{2}\zeta_3-\frac{1730}{3}\zeta_5+832\zeta_3^2-\frac{15617}{3}\zeta_7\bigg)+C_F^4\bigg(-\frac{2039}{8}\\
&-800\zeta_3+1280\zeta_5\bigg)+\frac{d_F^{abcd}d_A^{abcd}}{N_R}\bigg(\frac{1}{3}+\frac{1933}{4}\zeta_3+\frac{2785}{12}\zeta_5-383\zeta_3^2-\frac{1603}{6}\zeta_7\bigg)\\
&+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\bigg(-\frac{145}{18}+\frac{767}{16}\zeta_3-\frac{983}{24}\zeta_3^2\bigg)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\bigg(-\frac{288785}{576}\zeta_5+\frac{198443}{384}\zeta_7\bigg)\\
&+C_A\frac{d_F^{abcd}d_A^{abcd}}{N_RC_F}\bigg(-\frac{2869}{18}+\frac{9201}{8}\zeta_3-\frac{105965}{72}\zeta_5+\frac{17069}{24}\zeta_3^2-\frac{84399}{128}\zeta_7\bigg)\\
&+C_A^3T_Fn_f\bigg(-\frac{13701067}{7776}+\frac{250429}{432}\zeta_3-\frac{9}{4}\zeta_4+\frac{465295}{432}\zeta_5-\frac{1985}{36}\zeta_3^2-\frac{92015}{432}\zeta_7\bigg)\\
&+C_A^2C_FT_Fn_f\bigg(-\frac{6677069}{1296}+\frac{12107}{108}\zeta_3+3\zeta_4+\frac{8090}{9}\zeta_5-\frac{688}{3}\zeta_3^2-\frac{2324}{3}\zeta_7\bigg)\\
&+C_AC_F^2T_Fn_f\bigg(-\frac{127}{12}+\frac{1534}{3}\zeta_3-\frac{4520}{9}\zeta_5+\frac{608}{3}\zeta_3^2\bigg)+C_F^3T_Fn_f\bigg(\frac{367}{3}-128\zeta_3\bigg)\\
&+256\frac{d_F^{abcd}d_F^{abcd}}{N_R}n_f+\frac{d_F^{abcd}d_A^{abcd}}{N_A}n_f\bigg(\frac{85}{3}-\frac{2156}{9}\zeta_3+\frac{395}{9}\zeta_5-\frac{212}{3}\zeta_3^2+\frac{2534}{9}\zeta_7\bigg)\\
&+C_A\frac{d_F^{abcd}d_F^{abcd}}{N_RC_F}n_f\bigg(\frac{128}{3}-\frac{22688}{9}\zeta_3+\frac{5200}{3}\zeta_5+544\zeta_3^2+\frac{2996}{9}\zeta_7\bigg)\\
&+C_A^2T_F^2n_f^2\bigg(\frac{208127}{648}-\frac{1004}{27}\zeta_3-\frac{5300}{27}\zeta_5+\frac{64}{3}\zeta_3^2\bigg)+C_AC_FT_F^2n_f^2\bigg(\frac{44135}{27}+\frac{640}{27}\zeta_3\\
&+\frac{160}{3}\zeta_5\bigg)+C_F^2T_F^2n_f^2\bigg(\frac{2186}{9}-192\zeta_3\bigg)-\frac{7960}{729}C_AT_F^3n_f^3-\frac{4000}{27}C_FT_F^3n_f^3\bigg]\\
&+\bigg[C_A^4\bigg(-\frac{2397059}{7776}+\frac{20759}{864}\zeta_3-\frac{15}{128}\zeta_4+\frac{21815}{2592}\zeta_5+\frac{10415}{576}\zeta_3^2+\frac{97895}{9216}\zeta_7\bigg)\\
&+C_A^3C_F\bigg(\frac{23173339}{10368}-\frac{52135}{54}\zeta_3+\frac{51}{16}\zeta_4-\frac{14375}{216}\zeta_5+\frac{265}{8}\zeta_3^2+\frac{7021}{288}\zeta_7\bigg)\\
&+C_A^2C_F^2\bigg(\frac{363343}{648}-\frac{8425}{48}\zeta_3-\frac{93}{8}\zeta_4+\frac{625}{2}\zeta_5-40\zeta_3^2-\frac{133}{4}\zeta_7\bigg)+C_AC_F^3\bigg(-\frac{1543}{12}\\
&-11\zeta_3+12\zeta_4+80\zeta_5\bigg)-\frac{55}{6}C_F^4+\frac{d_F^{abcd}d_A^{abcd}}{N_R}\bigg(-\frac{106}{3}+\frac{1951}{6}\zeta_3-\frac{1760}{3}\zeta_5-32\zeta_3^2\\
&+\frac{3647}{12}\zeta_7\bigg)-\frac{2583}{64}\frac{d_A^{abcd}d_A^{abcd}}{N_A}\zeta_7+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\bigg(\frac{577}{144}-\frac{241}{72}\zeta_3+\frac{135}{8}\zeta_5+\frac{283}{24}\zeta_3^2\bigg)\\
&+C_A\frac{d_F^{abcd}d_A^{abcd}}{N_RC_F}\bigg(-\frac{1045}{54}+\frac{169645}{216}\zeta_3-\frac{69595}{216}\zeta_5-16\zeta_3^2-\frac{24493}{48}\zeta_7\bigg)\\
&+C_A^3T_Fn_f\bigg(\frac{1469045}{7776}-\frac{22199}{1296}\zeta_3+\frac{27}{4}\zeta_4+\frac{20365}{1296}\zeta_5+\frac{1}{4}\zeta_3^2+\frac{147}{16}\zeta_7\bigg)\\
&+C_A^2C_FT_Fn_f\bigg(-\frac{1519595}{1296}+\frac{28531}{108}\zeta_3-36\zeta_4+\frac{1040}{27}\zeta_5-\frac{16}{3}\zeta_3^2\bigg)
\end{aligned}
$$

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$$
\begin{aligned}
&+C_A C_F^2 T_F n_f\left(-\frac{54716}{81}+264\zeta_3+36\zeta_4-\frac{320}{3}\zeta_5\right)+C_F^3 T_F n_f\left(-25+32\zeta_3\right)\\
&+\frac{d_F^{abcd}d_A^{abcd}}{N_A}n_f\left(-\frac{209}{27}-\frac{1924}{27}\zeta_3+\frac{2170}{27}\zeta_5-12\zeta_3^2\right)+C_A^2 T_F^2 n_f^2\left(-\frac{22861}{972}+\frac{100}{81}\zeta_3\right.\\
&\left.-\frac{40}{9}\zeta_5\right)+C_A C_F T_F^2 n_f^2\left(\frac{11108}{81}-\frac{80}{3}\zeta_3\right)+\frac{4148}{81}C_F^2 T_F^2 n_f^2\Bigg]\xi+\Bigg[C_A^4\left(-\frac{1409761}{13824}\right.\\
&+\frac{16565}{384}\zeta_3-\frac{33}{128}\zeta_4-\frac{30715}{6912}\zeta_5-\frac{53}{576}\zeta_3^2-\frac{209629}{27648}\zeta_7\Bigg)+C_A^3 C_F\left(\frac{491945}{1152}-\frac{1969}{12}\zeta_3\right.\\
&\left.+\frac{33}{32}\zeta_4+\frac{2665}{144}\zeta_5+\frac{53}{24}\zeta_3^2-\frac{1001}{576}\zeta_7\right)+C_A^2 C_F^2\left(\frac{41879}{192}-\frac{481}{6}\zeta_3+\frac{3}{4}\zeta_4+25\zeta_5\right)\\
&+C_A C_F^3\left(\frac{25}{48}-26\zeta_3+40\zeta_5\right)+\frac{7}{4}C_F^4+\frac{d_F^{abcd}d_A^{abcd}}{N_R}\left(3+31\zeta_3-\frac{185}{3}\zeta_5+7\zeta_3^2-\frac{161}{12}\zeta_7\right)\\
&+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\left(\frac{61}{108}+\frac{35}{24}\zeta_3^2\right)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\left(-\frac{565}{48}\zeta_3+\frac{2035}{288}\zeta_5+\frac{1225}{1152}\zeta_7\right)\\
&+C_A\frac{d_F^{abcd}d_A^{abcd}}{N_R C_F}\left(\frac{995}{108}+\frac{106}{3}\zeta_3-\frac{675}{8}\zeta_5+\frac{39}{8}\zeta_3^2+\frac{6601}{128}\zeta_7\right)+C_A^3 T_F n_f\left(\frac{68671}{2592}-\frac{91}{12}\zeta_3\right.\\
&\left.+\frac{125}{72}\zeta_5+\frac{1}{6}\zeta_3^2\right)+C_A^2 C_F T_F n_f\left(-\frac{373}{4}+\frac{82}{3}\zeta_3-\frac{10}{3}\zeta_5\right)+C_A C_F^2 T_F n_f\left(-\frac{285}{4}\right.\\
&\left.+\frac{40}{3}\zeta_3\right)-3C_F^3 T_F n_f+\frac{d_F^{abcd}d_A^{abcd}}{N_A}n_f\left(-\frac{34}{27}-4\zeta_3+5\zeta_5-4\zeta_3^2\right)\Bigg]\xi^2+\Bigg[C_A^4\left(-\frac{10015}{432}\right.\\
&+\frac{10829}{1728}\zeta_3-\frac{9}{128}\zeta_4-\frac{2095}{1728}\zeta_5-\frac{17}{576}\zeta_3^2-\frac{1883}{27648}\zeta_7\Bigg)+C_A^3 C_F\left(\frac{32903}{384}-\frac{53}{2}\zeta_3+\frac{3}{16}\zeta_4\right.\\
&\left.+\frac{65}{24}\zeta_5\right)+C_A^2 C_F^2\left(\frac{1573}{48}+\frac{21}{2}\zeta_3+\frac{3}{8}\zeta_4+\frac{5}{2}\zeta_5\right)+C_A C_F^3\left(3-\frac{28}{3}\zeta_3\right)\\
&+\frac{d_F^{abcd}d_A^{abcd}}{N_R}\left(-\frac{5}{2}\zeta_3-5\zeta_5\right)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\left(-\frac{1}{144}-\frac{25}{36}\zeta_3+\frac{5}{36}\zeta_5+\frac{5}{24}\zeta_3^2-\frac{35}{36}\zeta_7\right)\\
&+C_A\frac{d_F^{abcd}d_A^{abcd}}{N_R C_F}\left(\frac{7}{12}+\frac{115}{24}\zeta_3+\frac{35}{24}\zeta_5-\frac{1}{4}\zeta_3^2-\frac{161}{192}\zeta_7\right)\Bigg]\xi^3+\Bigg[C_A^4\left(-\frac{31}{8}+\frac{171}{256}\zeta_3\right.\\
&\left.-\frac{115}{1536}\zeta_5\right)+C_A^3 C_F\left(\frac{2327}{192}-\frac{323}{96}\zeta_3+\frac{5}{48}\zeta_5\right)+C_A^2 C_F^2\left(\frac{223}{48}+\frac{29}{6}\zeta_3\right)+C_A C_F^3\left(-\frac{1}{4}\right.\\
&\left.-\frac{2}{3}\zeta_3\right)-\frac{4}{3}C_F^4\zeta_3+\frac{d_F^{abcd}d_A^{abcd}}{N_R}\left(-\frac{9}{4}\zeta_3+\frac{5}{4}\zeta_5\right)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\left(-\frac{1}{8}\zeta_3-\frac{5}{64}\zeta_5\right)\\
&+\frac{7}{8}C_A\frac{d_F^{abcd}d_A^{abcd}}{N_R C_F}\zeta_3\Bigg]\xi^4. \qquad \text{(A.19)}
\end{aligned}
$$

B General five-loop MiniMOM coupling constant and beta function

The right-hand side of eq. (4.5) is expanded as

$$
\frac{a^{\mathrm{MM}}(\mu^2)}{a^{\overline{\mathrm{MS}}}(\mu^2)}=1+\sum_{l=1}^{4}a^l c^{(l)}+\mathcal{O}(a^5) \qquad \text{(B.1)}
$$

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where a and ξ are understood as $a^{\overline{\mathrm{MS}}}(\mu^2)$ and $\xi^{\overline{\mathrm{MS}}}(\mu^2)$. The coefficients $c^{(l)}$ are given by

$$
\begin{aligned}
c^{(1)} &= \left(\frac{169}{36}C_A - \frac{20}{9}T_F n_f\right) + \frac{1}{2}C_A\xi + \frac{1}{4}C_A\xi^2, \\
c^{(2)} &= \left[C_A^2\left(\frac{76063}{1296} - \frac{39}{8}\zeta_3\right) + C_A T_F n_f\left(-\frac{3166}{81} - 8\zeta_3\right) + C_F T_F n_f\left(-\frac{55}{3} + 16\zeta_3\right)\right. \\
&\quad \left. + \frac{400}{81}T_F^2 n_f^2\right] + \left[C_A^2\left(\frac{269}{144} + \frac{11}{4}\zeta_3\right) - \frac{10}{9}C_A T_F n_f\right]\xi + C_A^2\left(\frac{35}{16} - \frac{3}{8}\zeta_3\right)\xi^2 + \frac{5}{16}C_A^2\xi^3, \\
c^{(3)} &= \left[C_A^3\left(\frac{42074947}{46656} - \frac{20225}{144}\zeta_3 - \frac{7805}{192}\zeta_5\right) + C_A^2 T_F n_f\left(-\frac{202997}{243} - \frac{868}{9}\zeta_3 + \frac{160}{3}\zeta_5\right)\right. \\
&\quad + C_A C_F T_F n_f\left(-\frac{41243}{81} + 328\zeta_3 + 80\zeta_5\right) + C_F^2 T_F n_f\left(\frac{286}{9} + \frac{296}{3}\zeta_3 - 160\zeta_5\right) \\
&\quad \left. + C_A T_F^2 n_f^2\left(\frac{47965}{243} + \frac{592}{9}\zeta_3\right) + C_F T_F^2 n_f^2\left(\frac{14002}{81} - \frac{416}{3}\zeta_3\right) - \frac{8000}{729}T_F^3 n_f^3\right] \\
&\quad + \left[C_A^3\left(\frac{17743}{648} + \frac{10355}{144}\zeta_3 - \frac{295}{16}\zeta_5\right) + C_A^2 T_F n_f\left(-\frac{1010}{81} - \frac{136}{3}\zeta_3\right)\right. \\
&\quad \left. + C_A C_F T_F n_f\left(-\frac{55}{6} + 8\zeta_3\right) + C_A T_F^2 n_f^2\left(\frac{56}{81} + \frac{64}{9}\zeta_3\right)\right]\xi + \left[C_A^3\left(\frac{16235}{576} - \frac{71}{24}\zeta_3\right.\right. \\
&\quad \left.\left. - \frac{175}{96}\zeta_5\right) + C_A^2 T_F n_f\left(-\frac{497}{72} + \zeta_3\right)\right]\xi^2 + C_A^3\left(\frac{1207}{192} - \frac{61}{48}\zeta_3 - \frac{5}{48}\zeta_5\right)\xi^3 + C_A^3\left(\frac{169}{192}\right. \\
&\quad \left. - \frac{11}{48}\zeta_3 + \frac{35}{192}\zeta_5\right)\xi^4, \\
c^{(4)} &= \left[C_A^4\left(\frac{14547680453}{839808} - \frac{2765315}{768}\zeta_3 - \frac{11}{6}\zeta_4 - \frac{114885815}{27648}\zeta_5 + \frac{857341}{4608}\zeta_3^2 + \frac{2850407}{1536}\zeta_7\right)\right. \\
&\quad + \frac{d_A^{abcd}d_A^{abcd}}{N_A}\left(-\frac{5707}{216} + \frac{229315}{192}\zeta_3 + 88\zeta_4 - \frac{34035}{128}\zeta_5 - \frac{176425}{192}\zeta_3^2 + \frac{292201}{512}\zeta_7\right) \\
&\quad + C_A^3 T_F n_f\left(-\frac{2162787959}{104976} - \frac{30109}{72}\zeta_3 + 17\zeta_4 + \frac{174305}{36}\zeta_5 + \frac{4471}{36}\zeta_3^2 - 2478\zeta_7\right) \\
&\quad + C_A^2 C_F T_F n_f\left(-\frac{53165981}{3888} + \frac{154055}{24}\zeta_3 - \frac{82}{3}\zeta_4 + \frac{45320}{9}\zeta_5 - 554\zeta_3^2 + \frac{2800}{3}\zeta_7\right) \\
&\quad + C_A C_F^2 T_F n_f\left(\frac{11768}{9} + 4761\zeta_3 + \frac{44}{3}\zeta_4 - \frac{44980}{9}\zeta_5 - 2240\zeta_7\right) + C_F^3 T_F n_f\left(\frac{31}{3} - 104\zeta_3\right. \\
&\quad \left. - 1960\zeta_5 + 2240\zeta_7\right) + \frac{d_F^{abcd}d_A^{abcd}}{N_A}n_f\left(\frac{4544}{27} - 2600\zeta_3 - 208\zeta_4 - 340\zeta_5 + \frac{5440}{3}\zeta_3^2\right. \\
&\quad \left. - \frac{49}{2}\zeta_7\right) + C_A^2 T_F^2 n_f^2\left(\frac{31237051}{4374} + \frac{19931}{9}\zeta_3 + \frac{28}{3}\zeta_4 - \frac{2755}{3}\zeta_5 + \frac{1280}{9}\zeta_3^2\right) \\
&\quad + C_A C_F T_F^2 n_f^2\left(\frac{80044}{9} - \frac{39140}{9}\zeta_3 + \frac{56}{3}\zeta_4 - \frac{21440}{9}\zeta_5 - 512\zeta_3^2\right) + C_F^2 T_F^2 n_f^2\left(\frac{13025}{54}\right. \\
&\quad \left. - \frac{25792}{9}\zeta_3 - \frac{88}{3}\zeta_4 + \frac{22400}{9}\zeta_5 + 512\zeta_3^2\right) + \frac{d_F^{abcd}d_F^{abcd}}{N_A}n_f^2\left(-\frac{6896}{27} + 336\zeta_3 + 64\zeta_4\right.
\end{aligned}
$$

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$$
\begin{aligned}
&-320\zeta_5+\frac{512}{3}\zeta_3^2\bigg)+C_AT_F^3n_f^3\bigg(-\frac{5370989}{6561}-\frac{3680}{9}\zeta_3+\frac{640}{27}\zeta_5\bigg)+C_FT_F^3n_f^3\bigg(-\frac{295702}{243}\\
&+\frac{7712}{9}\zeta_3+\frac{1280}{9}\zeta_5\bigg)+\frac{160000}{6561}T_F^4n_f^4\bigg]+\bigg[C_A^4\bigg(\frac{7179859}{15552}+\frac{1693429}{1152}\zeta_3-\frac{1003}{256}\zeta_4\\
&-\frac{5192539}{9216}\zeta_5+\frac{3625}{512}\zeta_6-\frac{24195}{512}\zeta_3^2+\frac{7175357}{36864}\zeta_7\bigg)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\bigg(-\frac{1}{3}+\frac{30443}{192}\zeta_3\\
&-\frac{333}{32}\zeta_4-\frac{134495}{384}\zeta_5+\frac{75}{4}\zeta_6+\frac{7821}{64}\zeta_3^2-\frac{7427}{384}\zeta_7\bigg)+C_A^3T_Fn_f\bigg(-\frac{7734173}{23328}-\frac{628187}{432}\zeta_3\\
&+\frac{87}{16}\zeta_4+\frac{21671}{72}\zeta_5-\frac{299}{6}\zeta_3^2-\frac{539}{8}\zeta_7\bigg)+C_A^2C_FT_Fn_f\bigg(-\frac{113789}{648}-\frac{205}{3}\zeta_3-6\zeta_4+6\zeta_5\\
&+88\zeta_3^2\bigg)+C_AC_F^2T_Fn_f\bigg(\frac{143}{9}+\frac{148}{3}\zeta_3-80\zeta_5\bigg)+\frac{d_F^{abcd}d_A^{abcd}}{N_A}n_f\bigg(-\frac{992}{3}\zeta_3+340\zeta_5\\
&-16\zeta_3^2+\frac{147}{2}\zeta_7\bigg)+C_A^2T_F^2n_f^2\bigg(\frac{32087}{1458}+\frac{12935}{27}\zeta_3-2\zeta_4-\frac{170}{9}\zeta_5\bigg)\\
&+C_AC_FT_F^2n_f^2\bigg(\frac{5921}{81}-16\zeta_3\bigg)+C_AT_F^3n_f^3\bigg(\frac{4640}{729}-\frac{1280}{27}\zeta_3\bigg)\bigg]\xi+\bigg[C_A^4\bigg(\frac{59745655}{124416}\\
&-\frac{37511}{1152}\zeta_3-\frac{185}{512}\zeta_4-\frac{966623}{13824}\zeta_5+\frac{25}{32}\zeta_6+\frac{2735}{256}\zeta_3^2+\frac{11879}{1536}\zeta_7\bigg)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\bigg(\frac{7}{24}\\
&-\frac{83}{96}\zeta_3-\frac{99}{64}\zeta_4+\frac{1095}{64}\zeta_5+\frac{75}{64}\zeta_6+\frac{97}{4}\zeta_3^2-\frac{7861}{256}\zeta_7\bigg)+C_A^3T_Fn_f\bigg(-\frac{3380129}{15552}-\frac{4249}{432}\zeta_3\\
&-\frac{67}{16}\zeta_4+\frac{2015}{432}\zeta_5+\frac{11}{12}\zeta_3^2\bigg)+C_A^2C_FT_Fn_f\bigg(-\frac{2887}{48}+\frac{387}{8}\zeta_3+6\zeta_4-6\zeta_3^2\bigg)\\
&+C_A^2T_F^2n_f^2\bigg(\frac{7771}{486}+\frac{128}{27}\zeta_3+\frac{10}{9}\zeta_5\bigg)\bigg]\xi^2+\bigg[C_A^4\bigg(\frac{766171}{6912}-\frac{16391}{864}\zeta_3+\frac{3}{64}\zeta_4\\
&-\frac{53467}{13824}\zeta_5+\frac{125}{512}\zeta_6-\frac{87}{128}\zeta_3^2+\frac{61313}{18432}\zeta_7\bigg)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\bigg(-\frac{359}{32}\zeta_3-\frac{9}{8}\zeta_4+\frac{1255}{64}\zeta_5\\
&+\frac{75}{32}\zeta_6+\frac{79}{32}\zeta_3^2-\frac{1393}{384}\zeta_7\bigg)+C_A^3T_Fn_f\bigg(-\frac{30983}{1728}+\frac{601}{216}\zeta_3-\frac{695}{432}\zeta_5-\frac{1}{6}\zeta_3^2\bigg)\bigg]\xi^3\\
&+\bigg[C_A^4\bigg(\frac{109757}{4608}-\frac{17009}{2304}\zeta_3+\frac{23}{512}\zeta_4+\frac{18575}{9216}\zeta_5-\frac{25}{768}\zeta_6-\frac{29}{1536}\zeta_3^2+\frac{147}{512}\zeta_7\bigg)\\
&+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\bigg(\frac{691}{192}\zeta_3+\frac{21}{64}\zeta_4-\frac{1385}{384}\zeta_5-\frac{25}{64}\zeta_6-\frac{121}{64}\zeta_3^2+\frac{1785}{512}\zeta_7\bigg)\bigg]\xi^4+\bigg[C_A^4\bigg(\frac{323}{128}\\
&-\frac{259}{384}\zeta_3+\frac{6485}{9216}\zeta_5-\frac{21}{512}\zeta_3^2-\frac{4823}{36864}\zeta_7\bigg)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\bigg(\frac{5}{64}\zeta_3-\frac{35}{384}\zeta_5-\frac{15}{64}\zeta_3^2\\
&+\frac{91}{192}\zeta_7\bigg)\bigg]\xi^5. \qquad \text{(B.2)}
\end{aligned}
$$

In terms of $a^{\rm MM}(\mu^2)$ and $\xi^{\rm MM}(\mu^2)$, the beta function in eq. (4.7) is expanded as

$$\beta^{\rm MM}(a^{\rm MM})=\mu^2\frac{da^{\rm MM}(\mu^2)}{d\mu^2}=-\sum_{l=0}^{4}a^{l+2}\beta_l^{\rm MM}+\mathcal{O}(a^7) \qquad \text{(B.3)}$$

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with

$$\beta_0^{\mathrm{MM}} = \frac{11}{3}C_A - \frac{4}{3}T_F n_f,$$

$$\beta_1^{\mathrm{MM}} = \left(\frac{34}{3}C_A^2 - \frac{20}{3}C_A T_F n_f - 4C_F T_F n_f\right) + \left(-\frac{13}{12}C_A^2 + \frac{2}{3}C_A T_F n_f\right)\xi + \bigg(-\frac{5}{6}C_A^2$$
$$+ \frac{2}{3}C_A T_F n_f\bigg)\xi^2 + \frac{1}{4}C_A^2 \xi^3,$$

$$\beta_2^{\mathrm{MM}} = \bigg[C_A^3\left(\frac{9655}{72} - \frac{143}{8}\zeta_3\right) + C_A^2 T_F n_f\left(-\frac{2009}{18} - \frac{137}{6}\zeta_3\right) + C_A C_F T_F n_f\bigg(-\frac{641}{9}$$
$$+ \frac{176}{3}\zeta_3\bigg) + 2C_F^2 T_F n_f + C_A T_F^2 n_f^2\left(\frac{46}{3} + \frac{32}{3}\zeta_3\right) + C_F T_F^2 n_f^2\left(\frac{184}{9} - \frac{64}{3}\zeta_3\right)\bigg]$$
$$+ \left[C_A^3\left(-\frac{1097}{96} + \frac{33}{8}\zeta_3\right) + \frac{37}{6}C_A^2 T_F n_f + 4C_A C_F T_F n_f\right]\xi + \bigg[C_A^3\left(-\frac{725}{96} + \frac{13}{8}\zeta_3\right)$$
$$+ C_A^2 T_F n_f\left(\frac{23}{4} - \frac{1}{2}\zeta_3\right) + 3C_A C_F T_F n_f\bigg]\xi^2 + \left[C_A^3\left(\frac{21}{32} - \frac{3}{8}\zeta_3\right) + \frac{1}{2}C_A^2 T_F n_f\right]\xi^3$$
$$+ \left(\frac{55}{96}C_A^3 - \frac{1}{12}C_A^2 T_F n_f\right)\xi^4,$$

$$\beta_3^{\mathrm{MM}} = \bigg[C_A^4\left(\frac{1381669}{648} - \frac{229559}{432}\zeta_3 - \frac{85855}{288}\zeta_5\right) + \frac{d_A^{abcd}d_A^{abcd}}{N_A}\left(-\frac{80}{9} + \frac{704}{3}\zeta_3\right)$$
$$+ C_A^3 T_F n_f\left(-\frac{244805}{108} + \frac{1409}{12}\zeta_3 + \frac{35965}{72}\zeta_5\right) + C_A^2 C_F T_F n_f\bigg(-\frac{60685}{36} + 680\zeta_3$$
$$+ \frac{1760}{3}\zeta_5\bigg) + C_A C_F^2 T_F n_f\left(\frac{527}{9} + \frac{2288}{3}\zeta_3 - \frac{3520}{3}\zeta_5\right) + 46C_F^3 T_F n_f$$
$$+ \frac{d_F^{abcd}d_A^{abcd}}{N_A}n_f\left(\frac{512}{9} - \frac{1664}{3}\zeta_3\right) + C_A^2 T_F^2 n_f^2\left(\frac{1655}{3} + \frac{1436}{9}\zeta_3 - \frac{1280}{9}\zeta_5\right)$$
$$+ C_A C_F T_F^2 n_f^2\left(\frac{9428}{9} - \frac{1376}{3}\zeta_3 - \frac{640}{3}\zeta_5\right) + C_F^2 T_F^2 n_f^2\left(-\frac{232}{9} - \frac{1024}{3}\zeta_3 + \frac{1280}{3}\zeta_5\right)$$
$$+ \frac{d_F^{abcd}d_F^{abcd}}{N_A}n_f^2\left(-\frac{704}{9} + \frac{512}{3}\zeta_3\right) + C_A T_F^3 n_f^3\left(-\frac{1792}{81} - \frac{896}{27}\zeta_3\right) + C_F T_F^3 n_f^3\bigg(-128$$
$$+ \frac{256}{3}\zeta_3\bigg)\bigg] + \bigg[C_A^4\left(-\frac{191047}{1152} + \frac{200059}{864}\zeta_3 - \frac{9145}{96}\zeta_5\right) + C_A^3 T_F n_f\bigg(\frac{75745}{432} - \frac{5749}{36}\zeta_3$$
$$+ \frac{295}{12}\zeta_5\bigg) + C_A^2 C_F T_F n_f\left(\frac{1193}{12} - 77\zeta_3\right) - 3C_A C_F^2 T_F n_f + C_A^2 T_F^2 n_f^2\left(-\frac{2039}{54} + \frac{536}{9}\zeta_3\right)$$
$$+ C_A C_F T_F^2 n_f^2\left(-\frac{92}{3} + 32\zeta_3\right) + C_A T_F^3 n_f^3\left(\frac{64}{27} - \frac{256}{27}\zeta_3\right)\bigg]\xi + \bigg[C_A^4\bigg(-\frac{57161}{576} + \frac{5519}{144}\zeta_3$$
$$- \frac{235}{16}\zeta_5\bigg) + C_A^3 T_F n_f\left(\frac{15263}{144} + \frac{127}{36}\zeta_3\right) + C_A^2 C_F T_F n_f\left(\frac{686}{9} - \frac{185}{3}\zeta_3\right) - 2C_A C_F^2 T_F n_f$$
$$+ C_A^2 T_F^2 n_f^2\left(-\frac{116}{9} - \frac{64}{9}\zeta_3\right) + C_A C_F T_F^2 n_f^2\left(-\frac{184}{9} + \frac{64}{3}\zeta_3\right)\bigg]\xi^2 + \bigg[C_A^4\bigg(\frac{9097}{1152}$$

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$$
\begin{aligned}
&-\frac{1307}{288}\zeta_3-\frac{275}{144}\zeta_5\biggr)+C_A^3T_Fn_f\biggl(\frac{941}{144}-\frac{16}{9}\zeta_3-\frac{5}{36}\zeta_5\biggr)+\frac{3}{4}C_A^2C_FT_Fn_f\biggr]\xi^3\\
&+\biggl[C_A^4\biggl(\frac{9061}{1152}-\frac{451}{144}\zeta_3-\frac{115}{288}\zeta_5\biggr)+C_A^3T_Fn_f\biggl(-\frac{25}{36}+\frac{5}{36}\zeta_3+\frac{35}{72}\zeta_5\biggr)\\
&-\frac{3}{4}C_A^2C_FT_Fn_f\biggr]\xi^4+\biggl[C_A^4\biggl(\frac{145}{128}-\frac{1}{12}\zeta_3+\frac{35}{96}\zeta_5\biggr)-\frac{1}{6}C_A^3T_Fn_f\biggr]\xi^5,
\end{aligned}
$$

$$
\begin{aligned}
\beta_4^{\mathrm{MM}}=&\biggl[C_A^5\biggl(\frac{361398305}{6912}-\frac{82264805}{6912}\zeta_3-\frac{349180015}{9216}\zeta_5+\frac{7423031}{4608}\zeta_3^2+\frac{31354477}{1536}\zeta_7\biggr)\\
&+C_A\frac{d_A^{abcd}d_A^{abcd}}{N_A}\biggl(-\frac{72745}{216}+\frac{9257251}{576}\zeta_3-\frac{3094355}{384}\zeta_5-\frac{1940675}{192}\zeta_3^2+\frac{3214211}{512}\zeta_7\biggr)\\
&+C_A^4T_Fn_f\biggl(-\frac{370399597}{5184}+\frac{13787695}{1728}\zeta_3+\frac{372847895}{6912}\zeta_5-\frac{249463}{384}\zeta_3^2-\frac{13317479}{384}\zeta_7\biggr)\\
&+C_A^3C_FT_Fn_f\biggl(-\frac{20916689}{432}+\frac{142513}{36}\zeta_3+\frac{1777405}{48}\zeta_5-3234\zeta_3^2+\frac{30800}{3}\zeta_7\biggr)\\
&+C_A^2C_F^2T_Fn_f\biggl(\frac{789839}{216}+\frac{1105007}{36}\zeta_3-\frac{58400}{3}\zeta_5-24640\zeta_7\biggr)+C_AC_F^3T_Fn_f\biggl(\frac{19097}{6}\\
&+\frac{2264}{3}\zeta_3-\frac{72160}{3}\zeta_5+24640\zeta_7\biggr)+C_F^4T_Fn_f\biggl(-\frac{4157}{6}-128\zeta_3\biggr)\\
&+C_A\frac{d_F^{abcd}d_A^{abcd}}{N_A}n_f\biggl(\frac{62288}{27}-\frac{104944}{3}\zeta_3+\frac{33860}{9}\zeta_5+\frac{59840}{3}\zeta_3^2-\frac{539}{2}\zeta_7\biggr)\\
&+C_F\frac{d_F^{abcd}d_A^{abcd}}{N_A}n_f\biggl(-320+\frac{1280}{3}\zeta_3+\frac{6400}{3}\zeta_5\biggr)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}T_Fn_f\biggl(\frac{7931}{54}-\frac{264899}{48}\zeta_3\\
&+\frac{434315}{288}\zeta_5+\frac{176425}{48}\zeta_3^2-\frac{292201}{128}\zeta_7\biggr)+C_A^3T_F^2n_f^2\biggl(\frac{6019289}{216}+\frac{19145}{4}\zeta_3-\frac{59675}{3}\zeta_5\\
&+\frac{1243}{3}\zeta_3^2+9912\zeta_7\biggr)+C_A^2C_FT_F^2n_f^2\biggl(\frac{829223}{18}-\frac{50771}{9}\zeta_3-\frac{294560}{9}\zeta_5+\frac{712}{3}\zeta_3^2\\
&-\frac{11200}{3}\zeta_7\biggr)+C_AC_F^2T_F^2n_f^2\biggl(-\frac{70298}{27}-23440\zeta_3+\frac{56960}{3}\zeta_5+\frac{2816}{3}\zeta_3^2+8960\zeta_7\biggr)\\
&+C_F^3T_F^2n_f^2\biggl(-\frac{1148}{3}-\frac{2176}{3}\zeta_3+\frac{30080}{3}\zeta_5-8960\zeta_7\biggr)+C_A\frac{d_F^{abcd}d_F^{abcd}}{N_A}n_f^2\biggl(-\frac{110768}{27}\\
&+\frac{51968}{9}\zeta_3-\frac{29440}{9}\zeta_5+\frac{5632}{3}\zeta_3^2\biggr)+C_F\frac{d_F^{abcd}d_F^{abcd}}{N_A}n_f^2\biggl(\frac{4160}{3}+\frac{5120}{3}\zeta_3-\frac{12800}{3}\zeta_5\biggr)\\
&+\frac{d_F^{abcd}d_A^{abcd}}{N_A}T_Fn_f^2\biggl(-\frac{18976}{27}+\frac{100480}{9}\zeta_3+\frac{10960}{9}\zeta_5-\frac{21760}{3}\zeta_3^2+98\zeta_7\biggr)\\
&+C_A^2T_F^3n_f^3\biggl(-\frac{94906}{27}-\frac{74396}{27}\zeta_3+\frac{6220}{3}\zeta_5-\frac{1280}{9}\zeta_3^2\biggr)+C_AC_FT_F^3n_f^3\biggl(-\frac{330676}{27}\\
&+\frac{25888}{9}\zeta_3+\frac{74240}{9}\zeta_5+\frac{1024}{3}\zeta_3^2\biggr)+C_F^2T_F^3n_f^3\biggl(\frac{1000}{9}+\frac{35968}{9}\zeta_3-\frac{12800}{3}\zeta_5\\
&-\frac{1024}{3}\zeta_3^2\biggr)+\frac{d_F^{abcd}d_F^{abcd}}{N_A}T_Fn_f^3\biggl(\frac{24064}{27}-\frac{9728}{9}\zeta_3+\frac{5120}{3}\zeta_5-\frac{2048}{3}\zeta_3^2\biggr)
\end{aligned}
$$

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$$+C_A T_F^4 n_f^4\left(\frac{7568}{81}+\frac{6400}{27}\zeta_3-\frac{2560}{27}\zeta_5\right)+C_F T_F^4 n_f^4\left(\frac{8576}{9}-\frac{3584}{9}\zeta_3-\frac{5120}{9}\zeta_5\right)\Bigg]$$

$$+\Bigg[C_A^5\left(-\frac{7426894751}{2239488}+\frac{5670451}{1536}\zeta_3-\frac{55067}{1536}\zeta_4-\frac{92929997}{55296}\zeta_5+\frac{192125}{3072}\zeta_6-\frac{54149}{1024}\zeta_3^2\right.$$

$$\left.+\frac{380293921}{221184}\zeta_7\right)+C_A\frac{d_A^{abcd}d_A^{abcd}}{N_A}\left(\frac{1217}{96}+\frac{1160503}{1152}\zeta_3-\frac{5883}{64}\zeta_4-\frac{7311565}{2304}\zeta_5\right.$$

$$\left.+\frac{1325}{8}\zeta_6+\frac{138171}{128}\zeta_3^2-\frac{393631}{2304}\zeta_7\right)+C_A^4 T_F n_f\left(\frac{336999223}{93312}-\frac{1678727}{432}\zeta_3+\frac{4733}{48}\zeta_4\right.$$

$$\left.+\frac{1188881}{1152}\zeta_5-\frac{3625}{192}\zeta_6+\frac{64553}{576}\zeta_3^2-\frac{15402653}{13824}\zeta_7\right)+C_A^3 C_F T_F n_f\left(\frac{817505}{216}-\frac{33455}{18}\zeta_3\right.$$

$$\left.-106\zeta_4-\frac{4421}{3}\zeta_5-\frac{968}{3}\zeta_3^2\right)+C_A^2 C_F^2 T_F n_f\left(-\frac{994}{9}-\frac{4609}{3}\zeta_3+\frac{7040}{3}\zeta_5\right)$$

$$-92 C_A C_F^3 T_F n_f+C_A\frac{d_F^{abcd}d_A^{abcd}}{N_A}n_f\left(-\frac{1024}{9}-\frac{16736}{9}\zeta_3+\frac{9190}{3}\zeta_5-\frac{424}{3}\zeta_3^2+\frac{2597}{4}\zeta_7\right)$$

$$+\frac{d_A^{abcd}d_A^{abcd}}{N_A}T_F n_f\left(\frac{8}{9}-\frac{30443}{72}\zeta_3+\frac{111}{4}\zeta_4+\frac{134495}{144}\zeta_5-50\zeta_6-\frac{2607}{8}\zeta_3^2+\frac{7427}{144}\zeta_7\right)$$

$$+C_A^3 T_F^2 n_f^2\left(-\frac{233570}{243}+\frac{26189}{18}\zeta_3-\frac{265}{6}\zeta_4-\frac{7726}{27}\zeta_5-\frac{388}{9}\zeta_3^2+\frac{539}{3}\zeta_7\right)$$

$$+C_A^2 C_F T_F^2 n_f^2\left(-\frac{63032}{27}+\frac{4592}{3}\zeta_3+32\zeta_4+\frac{1552}{3}\zeta_5+\frac{352}{3}\zeta_3^2\right)+C_A C_F^2 T_F^2 n_f^2\left(\frac{464}{9}\right.$$

$$\left.+\frac{2048}{3}\zeta_3-\frac{2560}{3}\zeta_5\right)+C_A\frac{d_F^{abcd}d_F^{abcd}}{N_A}n_f^2\left(\frac{1408}{9}-\frac{1024}{3}\zeta_3\right)$$

$$+\frac{d_F^{abcd}d_A^{abcd}}{N_A}T_F n_f^2\left(\frac{7936}{9}\zeta_3-\frac{2720}{3}\zeta_5+\frac{128}{3}\zeta_3^2-196\zeta_7\right)+C_A^2 T_F^3 n_f^3\left(\frac{121048}{2187}\right.$$

$$\left.-\frac{5320}{27}\zeta_3+\frac{16}{3}\zeta_4+\frac{1360}{27}\zeta_5\right)+C_A C_F T_F^3 n_f^3\left(\frac{2624}{9}-\frac{2816}{9}\zeta_3\right)\Bigg]\xi$$

$$+\Bigg[C_A^5\left(-\frac{1477213633}{746496}+\frac{10039865}{13824}\zeta_3-\frac{10177}{1536}\zeta_4+\frac{7102253}{55296}\zeta_5+\frac{26875}{3072}\zeta_6-\frac{104137}{3072}\zeta_3^2\right.$$

$$\left.+\frac{3658879}{24576}\zeta_7\right)+C_A\frac{d_A^{abcd}d_A^{abcd}}{N_A}\left(\frac{2983}{288}-\frac{149371}{1152}\zeta_3-\frac{993}{64}\zeta_4-\frac{113705}{768}\zeta_5+\frac{275}{16}\zeta_6\right.$$

$$\left.+\frac{85543}{384}\zeta_3^2-\frac{164647}{768}\zeta_7\right)+C_A^4 T_F n_f\left(\frac{224764001}{93312}-\frac{1727}{18}\zeta_3+\frac{3061}{384}\zeta_4-\frac{1885135}{3456}\zeta_5\right.$$

$$\left.-\frac{25}{24}\zeta_6-\frac{23957}{576}\zeta_3^2-\frac{50687}{1152}\zeta_7\right)+C_A^3 C_F T_F n_f\left(\frac{321707}{144}-\frac{6619}{6}\zeta_3-6\zeta_4-\frac{6061}{8}\zeta_5\right.$$

$$\left.+66\zeta_3^2\right)+C_A^2 C_F^2 T_F n_f\left(-\frac{5513}{72}-\frac{11413}{12}\zeta_3+\frac{4400}{3}\zeta_5\right)-\frac{115}{2}C_A C_F^3 T_F n_f$$

$$+C_A\frac{d_F^{abcd}d_A^{abcd}}{N_A}n_f\left(-\frac{640}{9}+480\zeta_3+230\zeta_5-8\zeta_3^2+\frac{147}{4}\zeta_7\right)+\frac{d_A^{abcd}d_A^{abcd}}{N_A}T_F n_f\left(-\frac{7}{18}\right.$$

$$\left.+\frac{83}{72}\zeta_3+\frac{33}{16}\zeta_4-\frac{365}{16}\zeta_5-\frac{25}{16}\zeta_6-\frac{97}{3}\zeta_3^2+\frac{7861}{192}\zeta_7\right)+C_A^3 T_F^2 n_f^2\left(-\frac{6644311}{11664}\right.$$

$$
\begin{aligned}
&-\frac{27881}{108}\zeta_3-\frac{17}{12}\zeta_4+\frac{19475}{108}\zeta_5+\frac{133}{9}\zeta_3^2\bigg)+C_A^2C_FT_F^2n_f^2\bigg(-\frac{23861}{18}+659\zeta_3+\frac{800}{3}\zeta_5\\
&-24\zeta_3^2\bigg)+C_AC_F^2T_F^2n_f^2\bigg(\frac{290}{9}+\frac{1280}{3}\zeta_3-\frac{1600}{3}\zeta_5\bigg)+C_A\frac{d_F^{abcd}d_F^{abcd}}{N_A}n_f^2\bigg(\frac{880}{9}-\frac{640}{3}\zeta_3\bigg)\\
&+C_A^2T_F^3n_f^3\bigg(\frac{1516}{81}+\frac{1400}{27}\zeta_3-\frac{40}{27}\zeta_5\bigg)+C_AC_FT_F^3n_f^3\bigg(160-\frac{320}{3}\zeta_3\bigg)\bigg]\xi^2\\
&+\bigg[C_A^5\bigg(\frac{5211575}{27648}-\frac{1711553}{6912}\zeta_3-\frac{785}{512}\zeta_4+\frac{206809}{3072}\zeta_5+\frac{1925}{1024}\zeta_6+\frac{385}{64}\zeta_3^2+\frac{278971}{12288}\zeta_7\bigg)\\
&+C_A\frac{d_A^{abcd}d_A^{abcd}}{N_A}\bigg(-\frac{17}{96}-\frac{6877}{192}\zeta_3-\frac{423}{64}\zeta_4+\frac{12835}{128}\zeta_5+\frac{375}{32}\zeta_6+\frac{2263}{64}\zeta_3^2-\frac{1505}{32}\zeta_7\bigg)\\
&+C_A^4T_Fn_f\bigg(-\frac{15245}{864}+\frac{2839}{16}\zeta_3+\frac{5}{16}\zeta_4-\frac{9625}{288}\zeta_5-\frac{25}{12}\zeta_3^2\bigg)+C_A^3C_FT_Fn_f\bigg(\frac{1969}{144}-\frac{9}{2}\zeta_3\\
&-\frac{5}{6}\zeta_5\bigg)+\frac{1}{2}C_A^2C_F^2T_Fn_f+C_A^3T_F^2n_f^2\bigg(\frac{1763}{108}-\frac{218}{3}\zeta_3+\frac{10}{9}\zeta_5\bigg)+C_A^2C_FT_F^2n_f^2\bigg(\frac{46}{9}\\
&-\frac{16}{3}\zeta_3\bigg)+C_A^2T_F^3n_f^3\bigg(-\frac{64}{27}+\frac{256}{27}\zeta_3\bigg)\bigg]\xi^3+\bigg[C_A^5\bigg(\frac{2993563}{27648}-\frac{221371}{3456}\zeta_3-\frac{439}{1536}\zeta_4\\
&+\frac{194767}{13824}\zeta_5+\frac{2675}{9216}\zeta_6+\frac{11947}{4608}\zeta_3^2+\frac{69545}{12288}\zeta_7\bigg)+C_A\frac{d_A^{abcd}d_A^{abcd}}{N_A}\bigg(-\frac{3}{32}-\frac{1987}{288}\zeta_3\\
&-\frac{59}{64}\zeta_4+\frac{3485}{144}\zeta_5+\frac{125}{48}\zeta_6-\frac{17}{24}\zeta_3^2+\frac{1379}{512}\zeta_7\bigg)+C_A^4T_Fn_f\bigg(-\frac{21649}{864}+\frac{9943}{1728}\zeta_3-\frac{1}{384}\zeta_4\\
&-\frac{11545}{6912}\zeta_5-\frac{25}{576}\zeta_6-\frac{965}{1152}\zeta_3^2+\frac{49}{128}\zeta_7\bigg)+C_A^3C_FT_Fn_f\bigg(-\frac{1351}{48}+\frac{95}{4}\zeta_3+\frac{35}{16}\zeta_5\bigg)\\
&+\frac{3}{4}C_A^2C_F^2T_Fn_f+\frac{d_A^{abcd}d_A^{abcd}}{N_A}T_Fn_f\bigg(\frac{691}{144}\zeta_3+\frac{7}{16}\zeta_4-\frac{1385}{288}\zeta_5-\frac{25}{48}\zeta_6-\frac{121}{48}\zeta_3^2\\
&+\frac{595}{128}\zeta_7\bigg)+C_A^3T_F^2n_f^2\bigg(\frac{47}{12}+\frac{4}{3}\zeta_3\bigg)+C_A^2C_FT_F^2n_f^2\bigg(\frac{23}{3}-8\zeta_3\bigg)\bigg]\xi^4+\bigg[C_A^5\bigg(\frac{60875}{2304}\\
&-\frac{36863}{4608}\zeta_3-\frac{1}{256}\zeta_4+\frac{19355}{55296}\zeta_5-\frac{25}{384}\zeta_6-\frac{1865}{3072}\zeta_3^2+\frac{122185}{221184}\zeta_7\bigg)\\
&+C_A\frac{d_A^{abcd}d_A^{abcd}}{N_A}\bigg(\frac{935}{128}\zeta_3+\frac{21}{32}\zeta_4-\frac{14585}{2304}\zeta_5-\frac{25}{32}\zeta_6-\frac{489}{128}\zeta_3^2+\frac{16247}{2304}\zeta_7\bigg)\\
&+C_A^4T_Fn_f\bigg(-\frac{103}{32}+\frac{65}{48}\zeta_3+\frac{7325}{3456}\zeta_5-\frac{7}{64}\zeta_3^2-\frac{4823}{13824}\zeta_7\bigg)-\frac{9}{8}C_A^3C_FT_Fn_f\\
&+\frac{d_A^{abcd}d_A^{abcd}}{N_A}T_Fn_f\bigg(\frac{5}{24}\zeta_3-\frac{35}{144}\zeta_5-\frac{5}{8}\zeta_3^2+\frac{91}{72}\zeta_7\bigg)\bigg]\xi^5+\bigg[C_A^5\bigg(\frac{2015}{1152}+\frac{1523}{4608}\zeta_3\\
&+\frac{39275}{18432}\zeta_5-\frac{105}{1024}\zeta_3^2-\frac{24115}{73728}\zeta_7\bigg)+C_A\frac{d_A^{abcd}d_A^{abcd}}{N_A}\bigg(\frac{33}{128}\zeta_3-\frac{145}{768}\zeta_5-\frac{75}{128}\zeta_3^2\\
&+\frac{455}{384}\zeta_7\bigg)+C_A^4T_Fn_f\bigg(-\frac{61}{288}-\frac{1}{288}\zeta_3-\frac{35}{288}\zeta_5\bigg)+\frac{1}{16}C_A^3C_FT_Fn_f\bigg]\xi^6. \qquad \text{(B.4)}
\end{aligned}
$$

C Notations in the result files

The notation of the result file, which is distributed along with this article as supplementary material, is displayed in three tables below. We define the colour factors in table 1 (see ref. [65] for the definitions), and the names of the unrenormalized and renormalized results in table 2 and table 3 respectively. We also provide transition functions from the calculationally convenient G-scheme [77], where a certain universal function is set to one, to the standard convention used for calculations in $\overline{\text{MS}}$, as well as values for the master integrals in $4-2\varepsilon$ dimensions, which were adapted for use by FORCER from the results in refs. [35, 36].

		SU(N)	QCD	QED
T_F	`tf`	$\frac{1}{2}$	$\frac{1}{2}$	1
C_A	`ca`	N	3	0
C_F	`cf`	$\frac{N^2-1}{2N}$	$\frac{4}{3}$	1
$\frac{d_A^{abcd}d_A^{abcd}}{N_A}$	`[d4AA/na]`	$\frac{N^2(N^2+36)}{24}$	$\frac{135}{8}$	0
$\frac{d_F^{abcd}d_A^{abcd}}{N_A}$	`[d4FA/na]`	$\frac{N(N^2+6)}{48}$	$\frac{15}{16}$	0
$\frac{d_F^{abcd}d_F^{abcd}}{N_A}$	`[d4FF/na]`	$\frac{N^4-6N^2+18}{96N^2}$	$\frac{5}{96}$	1
$\frac{d_F^{abcd}d_A^{abcd}}{N_R}$	`[d4FA/nr]`	$\frac{(N^2+6)(N^2-1)}{48}$	$\frac{5}{2}$	0
$\frac{d_F^{abcd}d_F^{abcd}}{N_R}$	`[d4FF/nr]`	$\frac{(N^4-6N^2+18)(N^2-1)}{96N^3}$	$\frac{5}{36}$	1

Table 1. Symbols for colour factors.

Π^B	`gluonB`		a^B	`a`
$\tilde{\Pi}^B$	`ghostB`		ξ^B	`xi`
Σ_V^B	`quarkB`			
T_i^B	`glglgl‘i’B`	`i` = 1 or 2		
$\tilde{\Gamma}_i^B$	`ghghgl‘i’B`	`i` = `h` or `g`		
Λ_i^B	`ququgl‘i’B`	`i` = `q` or `g`		
$\Lambda_i^{T,B}$	`ququglT‘i’B`	`i` = `q` or `g`		

Table 2. Symbols for unrenormalized results.

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$\Pi^{\overline{\rm MS}}$	`gluonMSb`		$a^{\overline{\rm MS}}$	`a`
$\tilde{\Pi}^{\overline{\rm MS}}$	`ghostMSb`		$\xi^{\overline{\rm MS}}$	`xi`
$\Sigma_V^{\overline{\rm MS}}$	`quarkMSb`			
$T_i^{\overline{\rm MS}}$	`glglgl‘i’MSb`	`i` = `1` or `2`		
$\tilde{\Gamma}_i^{\overline{\rm MS}}$	`ghghgl‘i’MSb`	`i` = `h` or `g`		
$\Lambda_i^{\overline{\rm MS}}$	`ququgl‘i’MSb`	`i` = `q` or `g`		
$\Lambda_i^{T,\overline{\rm MS}}$	`ququglT‘i’MSb`	`i` = `q` or `g`		

Table 3. Symbols for renormalized results.

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