Published for SISSA by Springer

Received: April 13, 2017
Accepted: May 27, 2017
Published: June 7, 2017

Wilson lines and UV sensitivity in magnetic compactifications

JHEP06(2017)039

D.M. Ghilencea[a] and Hyun Min Lee[b]

[a] *Theoretical Physics Department, National Institute of Physics and Nuclear Engineering, Bucharest, MG-6 077125 Romania*

[b] *Department of Physics, Chung-Ang University, 06974 Seoul, Korea*

E-mail: dumitru.ghilencea@cern.ch, hminlee@cau.ac.kr

Abstract: We investigate the ultraviolet (UV) behaviour of 6D N=1 supersymmetric effective (Abelian) gauge theories compactified on a two-torus (T_2) with magnetic flux. To this purpose we compute offshell the one-loop correction to the Wilson line state self-energy. The offshell calculation is actually *necessary* to capture the usual effective field theory expansion in powers of (∂/Λ). Particular care is paid to the regularization of the (divergent) momentum integrals, which is relevant for identifying the corresponding counterterm(s). We find a counterterm which is a new higher dimensional effective operator of dimension d=6, that is enhanced for a larger compactification area (where the effective theory applies) and is consistent with the symmetries of the theory. Its consequences are briefly discussed and comparison is made with orbifold compactifications without flux.

Keywords: Effective Field Theories, Flux compactifications, Compactification and String Models, Supersymmetric Gauge Theory

ArXiv ePrint: 1703.10418

doi:10.1007/JHEP06(2017)039

Contents

1 Introduction

In this letter we explore the ultraviolet behaviour of supersymmetric models compactified to four dimensions on a two-torus T_2 in the presence of magnetic flux. Compactifications on tori with magnetic fluxes were investigated in string theory e.g. [1]–[6] (for a review [7, 8]) and are interesting because they can break supersymmetry and lead to chiral fermions [9]. This motivated the interest in effective theory approach to model building e.g. [10]–[24].

In this work we compute (offshell) the one-loop correction to a two-point Green function of the self-energy of a complex scalar field φ in a compactification to four dimensions of a 6D N=1 supersymmetric Abelian gauge theory on T_2 with magnetic flux. The scalar field φ is actually a Wilson line state, which is a fluctuation of a combination of components $A_{5,6}$ of the gauge fields A_M $(M = \mu, 5, 6)$. The motivation is two-fold: few quantum investigations exist for such compactification and the field φ may play the role of a higgs field in realistic models, which is interesting for model building and the hierarchy problem.

We pay particular attention to the regularization of the quantum corrections. Indeed, the one-loop integrals are divergent and call for a UV regularization consistent with the symmetries of the theory. The regularization ensures that (the series of) these integrals are well-defined and any divergences of the result in the limit of removing the regulator dictate the form of the corresponding counterterm operators. Since effective theories are non-renormalizable, the counterterms may be higher dimensional operators. The *offshell* calculation is important and is actually *necessary* in order to capture the behaviour of the effective theory which is an expansion in powers[1] of (∂/Λ) [32] where Λ is a high scale (e.g. compactification scale). We use dimensional regularization (DR), since it respects all symmetries, in particular gauge symmetry. We then compare the UV behaviour of our result for the quantum correction in the presence of magnetic flux against similar results in orbifold compactifications without flux such as effective theory on T_2/Z_2, etc.

[1] Most quantum corrections from extra dimensions are computed onshell. For offshell results see [25–31].

JHEP06(2017)039

2 Magnetic compactification on a torus

We begin our study with the relevant part of the action. Consider first the action of a 6D N=1 vector superfield and hypermultiplet compactified to 4D on a torus T_2 in the presence of magnetic flux. This can be described in 4D N=1 superfields language [33, 34]. For the details of this compactification we refer the reader to [10–13]. For the vector superfield

$$S_v = \int d^6x \Big\{ \int d^4\theta \Big[\partial V \overline{\partial} V + \Phi^\dagger \Phi + \sqrt{2}\, V\, (\overline{\partial}\Phi + \text{h.c.})\Big] + \frac{1}{4}\int d^2\theta\, W^\alpha W_\alpha + \text{h.c.} \Big\} \tag{2.1}$$

where $\partial \equiv \partial_5 - i\partial_6$. Only zero-modes of V (hereafter V_0), of gauge kinetic field-strength W (W_0) and of the superfield Φ (Φ_0) are relevant below. We have $\Phi_0|_{\theta=\overline{\theta}=0} = 1/\sqrt{2}(A_6 + iA_5) + \varphi$, where φ defines a complex continuous Wilson line state on T_2.

We also need a 6D N=1 hypermultiplet of chiral superfields Q, $\tilde{Q}$ of charges $\pm q_0$

$$\begin{aligned} S_h = \int d^6x \Big\{ \int d^4\theta \Big[Q^\dagger e^{2\, q_0\, g\, V}\, Q + \tilde{Q}^\dagger e^{-2\, q_0\, g\, V}\, \tilde{Q} \Big] \\ + \Big[\int d^2\theta\, \tilde{Q}\, (\partial + \sqrt{2} g\, q_0\, \Phi)\, Q + \text{h.c.} \Big] \Big\} \end{aligned} \tag{2.2}$$

with g the gauge coupling. One must integrate S_h, S_v over T_2 in the presence of magnetic flux [10], but a set of basis functions is required. First, we use a symmetric gauge choice with $A_5 = (-1/2) f x_6$ and $A_6 = (1/2) f x_5$ (f=constant), satisfying a constant field strength $F_{56} = \partial_5 A_6 - \partial_6 A_5 = f$. Its flux through T_2 closed surface is then quantised[2] $q_0 g/(2\pi) \int_{T^2} F_{56} dx_5 dx_6 = q_0 g f \mathcal{A}/(2\pi) \in Z$, ($\mathcal{A}$ is the area of the torus). The Kaluza-Klein (KK) spectrum of the charged fields will then resemble that of Landau levels [10–12, 15].

To find the basis set of functions, notice that covariant derivatives $D_k = \partial_k + i q_0 g A_k$ ($k = 5, 6$) satisfy $[iD_5, iD_6] = -i q_0 g f$. Assuming $f < 0$, one can construct a 1D harmonic oscillator Hamiltonian $H = p^2/(2m) + 1/2\, m\omega^2 x^2$ of $p \sim iD_6$ and $x \sim iD_5$, $m = 1/2$, $\omega = 2$. Its eigenfunctions define the basis set of functions $\psi_{n,j}$ [10–12]. The ladder operators are $a = (1/\sqrt{\alpha})\, (iD_5 - D_6)$, $a^\dagger = 1/\sqrt{\alpha}\, (iD_5 + D_6)$ with $[a, a^\dagger] = 1$, so $H = \alpha(a^\dagger a + 1/2)$ and

$$\alpha = -2 q_0 g f = \frac{4\pi N}{\mathcal{A}} > 0, \qquad (N \in Z_+) \tag{2.3}$$

The basis functions are $\psi_{n,j} = (a^\dagger)^n / \sqrt{n!}\, \psi_{0,j}$, where n refers to the Landau level and j reflects the N-fold degeneracy. These are orthonormal on T_2, and $a^\dagger \psi_{n,j} = \sqrt{n+1}\, \psi_{n,j}$, with $\psi_{0,j}$ as zero mode: $a\, \psi_{0,j} = 0$. Then $\partial + \sqrt{2} q_0 g\, \Phi_0 = -i\sqrt{\alpha}\, a^\dagger + \sqrt{2} q_0 g\, \varphi$, which is used in S_h, together with an expansion of superfields in the basis functions $\psi_{n,j}(x_m)$:

$$Q(x_M, \theta, \overline{\theta}) = \sum_{n,j} Q_{n,j}(x_\mu, \theta, \overline{\theta})\, \psi_{n,j}(x_m), \qquad M = \mu, 5, 6. \tag{2.4}$$

[2] Another way to see the quantisation condition is the following. We can make a gauge choice near $x_5 = 0$ and $x_5 = 2\pi R_5$: Region I ($-\pi R_5 < x_5 < \pi R_5$): $A_5 = 0, A_6 = f x_5$, and Region II ($\pi R_5 < x_5 < 2\pi R_5$): $A_5 = 0, A_6 = f(x_5 - 2\pi R_5)$. Then, two gauge potentials are connected by a gauge transformation in the overlapping region: $A_{II} - A_I = -2\pi f R_5 = \partial\Lambda$, with $\Lambda = -2\pi f R_5 x_6$. As a result, the wavefunctions of charged fields, ϕ, are connected in this overlapping region as $\phi_{II} = e^{-i q_0 g f 2\pi R_5 x_6} \phi_I$. Then, single-valuedness of wavefunctions along the x_6 direction requires $q_0 g f (2\pi) R_5 R_6 = N$ with N integer. The periodicity along the x_6 direction is guaranteed by the same quantisation condition.

JHEP06(2017)039

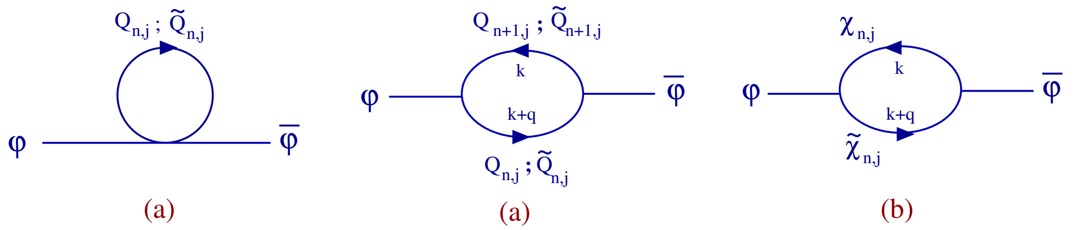

Figure 1. One-loop diagrams, at external momentum q, of bosonic (a) and fermionic (b) contributions, respectively, to the Wilson line scalar (φ) self-energy.

JHEP06(2017)039

A similar expansion exists for $\tilde{Q}(x_M,\theta,\overline{\theta})$ in this basis, with coefficients $\tilde{Q}_{\tilde{n},\tilde{j}}(x_\mu,\theta,\overline{\theta})$. One finds the relevant part of the 4D action [10]

$$\begin{aligned} S \supset \int d^4x \Big\{ \int d^4\theta \Big[\varphi^\dagger\varphi + \sum_{n,j} Q^\dagger_{n,j} e^{2\, q_0\, g\, V_0} Q_{n,j} + \sum_{n,j} \tilde{Q}^\dagger_{n,j} e^{-2\, q_0\, g\, V_0} \tilde{Q}_{n,j} + 2 f V_0 \Big] \\ + \int d^2\theta \Big[\frac{1}{4} W_0^\alpha W_{0,\alpha} - i \sum_{n,j} \sqrt{\alpha(n+1)}\, \tilde{Q}_{n+1,j} Q_{n,j} + \sum_{n,j} \sqrt{2}\, q_0\, g\, \tilde{Q}_{n,j}\, \varphi\, Q_{n,j} \Big] + \text{h.c.} \Big\} \end{aligned} \tag{2.5}$$

where we kept only the zero modes of the gauge kinetic term and of Wilson line scalar φ. After eliminating the auxiliary fields one identifies the scalar fields mass: $m^2_{\tilde{Q}_{n,j}} = m^2_{Q_{n,j}} = \alpha(n+1/2)$; for fermions their mass can be read from the last line of the above equation: $m^2_{\Psi_{n,j}} = \alpha(n+1)$ for a Dirac fermion composed of two Weyl spinors as in $\Psi_{n,j} \equiv (\tilde{\chi}_{n+1,j}, \chi_{n,j})^T$. The (onshell-SUSY) couplings of these fields, in components, are:

$$\begin{aligned} L =& -i\sqrt{2} q_0 g \sum_{n,j} \sqrt{\alpha(n+1)}\, \varphi \big[\tilde{Q}^\dagger_{n+1,j} \tilde{Q}_{n,j} - Q^\dagger_{n,j} Q_{n+1,j} \big] - \sqrt{2} q_0\, g \sum_{n,j} \varphi\, \tilde{\chi}_{n,j} \chi_{n,j} + \text{h.c.} \\ & -2 q_0^2\, g^2 \sum_{n,j} \big[|Q_{n,j}|^2 + |\tilde{Q}_{n,j}|^2 \big]\, |\varphi|^2 \end{aligned} \tag{2.6}$$

where the sums are over $n \geq 0$; $\tilde{\chi}$ (χ) are the Weyl spinors of $\tilde{Q}$ (Q) superfields. With this information we can investigate the quantum corrections to the mass of the scalar field φ.

3 One-loop corrections to Wilson line

With the above action, we compute the one-loop corrections to the Wilson line scalar, shown in figure 1 for non-vanishing external 4-momentum q. This allows us to investigate their UV behaviour under scaling of the momentum. Since the integrals are divergent, we use the DR scheme, in order to find the poles and identify their corresponding counterterms. This regularization preserves all symmetries of the theory. After performing a Wick rotation to the Euclidean space and with the DR subtraction scale μ introduced to ensure dimensionless coupling (g) in $d = 4 - 2\epsilon$ dimensions, we find for the bosonic contribution

$$\delta m_b^2(q^2) = 2\, q_0^2\, g^2\, N \mu^{2\epsilon} \sum_{n\geq 0} \int \frac{d^dk}{(2\pi)^d} \frac{2\, k^2 + \alpha}{\big[(q+k)^2 + \alpha\,(n+1/2)\big]\big[k^2 + \alpha\,(n+3/2)\big]}. \tag{3.1}$$

For the fermionic part

$$\delta m_f^2(q^2) = -2\, q_0^2\, g^2\, N \mu^{2\epsilon} \sum_{n\geq 0} \int \frac{d^d k}{(2\pi)^d} \frac{2\, k\, (q+k)}{\left[(q+k)^2 + \alpha\, n\right] \left[(k^2 + \alpha\,(n+1)\right]}. \tag{3.2}$$

Performing the integrals in the DR scheme (see the appendix) gives[3]

$$\begin{aligned}
\delta m_b^2(q^2) &= K_0\, (4\pi\mu^2/\alpha)^\epsilon \int_0^1 dx \Big[(2q^2\, x^2 + \alpha)\, \Gamma[\epsilon]\, \zeta[\epsilon, \rho_1] + d\, \alpha\, \Gamma[-1+\epsilon]\, \zeta[-1+\epsilon, \rho_1]\Big] \\
\delta m_f^2(q^2) &= -K_0\, (4\pi\mu^2/\alpha)^\epsilon \int_0^1 dx \Big[2q^2\, x(x-1)\, \Gamma[\epsilon]\, \zeta[\epsilon, \rho_2] + d\, \alpha\, \Gamma[-1+\epsilon]\, \zeta[-1+\epsilon, \rho_2]\Big]
\end{aligned} \tag{3.3}$$

with the notation

$$K_0 \equiv \frac{2 q_0^2 g^2 N}{(4\pi)^2}, \qquad \rho_2 = \rho_1 - \frac{1}{2} = (1-x)\left(1 + x\, \frac{q^2}{\alpha}\right) > 0 \tag{3.4}$$

where we introduced the Hurwitz zeta function $\zeta[s,a] = \sum_{n\geq 0} (n+a)^{-s}$ [35, 36]. The above bosonic and fermionic contributions have poles from Gamma functions, $\Gamma[\epsilon]$ and $\Gamma[-1+\epsilon]$. One could proceed in eqs. (3.3) to Taylor expand the zeta functions for small ϵ and isolate the poles from the finite part, however, one cannot then integrate the resulting terms involving $(d/dz\, \zeta[z,\rho])_{z=-1}$ since for this derivative only asymptotic expansions are known [35]. To avoid this, we integrate by parts the second term in both $\delta m_{b,f}^2$ and use

$$\frac{\partial \zeta[s,\rho]}{\partial \rho} = -s\, \zeta[s+1, \rho] \tag{3.5}$$

This gives

$$\begin{aligned}
\delta m_b^2(q^2) &= K_0 \left(\frac{4\pi\mu^2}{\alpha}\right)^\epsilon \left[d\, \alpha\, \Gamma[\epsilon-1]\, \zeta[-1+\epsilon, 1/2] + \Gamma[\epsilon] \int_0^1 dx\, \zeta[\epsilon, \rho_1]\, f_1(x)\right] \\
\delta m_f^2(q^2) &= -K_0 \left(\frac{4\pi\mu^2}{\alpha}\right)^\epsilon \left[d\, \alpha\, \Gamma[\epsilon-1]\, \zeta[-1+\epsilon, 0] + \Gamma[\epsilon] \int_0^1 dx\, \zeta[\epsilon, \rho_2]\, f_2(x)\right]
\end{aligned}$$

Here $f_1(x) = 2\, q^2\, x^2 + \alpha + x\, d\, \alpha \rho_1'(x)$ and $f_2(x) = 2\, q^2\, x(x-1) + x\, d\, \alpha \rho_2'(x)$ with a notation $\rho_j'(x) = (d/dx)\rho_j(x)$, $j = 1, 2$. Further

$$\begin{aligned}
\Gamma[\epsilon] &= \frac{1}{\epsilon} - \gamma_E + \mathcal{O}(\epsilon) \\
\zeta[\epsilon, \rho_j] &= \zeta[0, \rho_j] + \epsilon\, \zeta^{(1,0)}[0, \rho_j] + \mathcal{O}(\epsilon^2), \quad j = 1, 2. \\
\left(\frac{4\pi\mu^2}{\alpha}\right)^\epsilon &= 1 + \epsilon \ln \frac{4\pi\mu^2}{\alpha} + \mathcal{O}(\epsilon^2)
\end{aligned} \tag{3.6}$$

[3] Unlike in 6D orbifolds, in the present case only one KK sum is present, which would apparently make the result less UV divergent. This is however misleading because in the present case the (masses)^2 under the sum are linear rather than quadratic in the level (n), thus there is no UV improvement in this sense.

with the Euler constant $\gamma_E \approx 0.577216$. We then find[4]

$$\delta m_b^2(q^2) = K_0\Big[-\frac{q^4}{30\,\alpha}\Big(\frac{1}{\epsilon}+\ln\frac{4\pi\mu^2 e^{-\gamma_E}}{\alpha}\Big) - \frac{\alpha}{12}\ln\frac{e^3\,G^{24}}{4} - \frac{q^2}{6} - \frac{q^4}{30\,\alpha} + H_1(q)\Big] + \mathcal{O}(\epsilon)$$
$$\delta m_f^2(q^2) = -K_0\Big[\frac{4\,q^4}{30\,\alpha}\Big(\frac{1}{\epsilon}+\ln\frac{4\pi\mu^2 e^{-\gamma_E}}{\alpha}\Big) + \alpha\ln G^4 - \frac{q^4}{30\,\alpha} + H_2(q)\Big] + \mathcal{O}(\epsilon) \tag{3.7}$$

with $G = 1.28243$ the Glaisher constant[5,6]. Above we introduced the functions H_1, H_2

$$\begin{aligned} H_1(q) &= \int_0^1 dx\,(2\,q^2\,x^2+\alpha+4\,x\,\alpha\,\rho_1'(x))\ln\frac{\Gamma[\rho_1(x)]}{\sqrt{2\pi}} \\ &= \frac{\alpha}{12}\ln\frac{G^{24}\,e^3}{4} + \frac{q^2}{6} - \frac{9\,\zeta[3]}{8\pi^2}\,q^2 + c_b\,\frac{q^4}{\alpha} + \mathcal{O}((q^2/\alpha)^3) \end{aligned} \tag{3.8}$$

with $c_b = -109/720 - (1/15)\ln 2 + \ln G - 14\,\zeta'[-3]) \approx -0.02414$, and

$$\begin{aligned} H_2(q) &= \int_0^1 dx\,(2q^2\,x\,(x-1)+4\alpha\,x\,\rho_2'(x))\ln\frac{\Gamma[\rho_2(x)]}{\sqrt{2\pi}} \\ &= -\alpha\ln G^4 + \frac{3\zeta[3]}{2\pi^2}\,q^2 + c_f\,\frac{q^4}{\alpha} + \mathcal{O}((q^2/\alpha)^3) \end{aligned} \tag{3.9}$$

and $c_f = 11/45 + 16\,\zeta'[-3] \approx 0.3305$. Therefore, up to irrelevant $\mathcal{O}(\epsilon)$ terms

$$\begin{aligned} \delta m_b^2(q^2) &= K_0\Big[-\frac{q^4}{30\alpha\epsilon} + \mathcal{O}(q^2/\alpha)\Big] \\ \delta m_b^2(q^2) &= K_0\Big[-\frac{4q^4}{30\alpha\epsilon} + \mathcal{O}(q^2/\alpha)\Big] \end{aligned} \tag{3.10}$$

The sum of bosonic and fermionic contributions $\delta m^2(q^2) = \delta m_b^2(q^2) + \delta m_f^2(q^2)$, is found in general from eq. (3.7), but for small momenta $q^2 \ll \alpha$ it simplifies

$$\delta m^2(q^2) = K_0\Big[-\frac{q^4}{6\alpha}\Big(\frac{1}{\epsilon}+\ln\frac{4\pi\mu^2 e^{-\gamma_0}}{\alpha}\Big) - \frac{21\zeta[3]}{8\pi^2}q^2 + \mathcal{O}(q^6/\alpha^3)\Big]. \tag{3.11}$$

with $\gamma_0 = \gamma_E + 6(c_b - c_f)$. A pole is present in the two-point Green function,[7] reflecting the UV divergences of the theory and *the limits $q^2 \to 0$ and $\epsilon \to 0$ do not commute*, which shows the importance of this calculation. A finite quantum correction ($\propto q^2$) is also present.

4 Counterterms and symmetries

Eq. (3.11) shows that a counterterm is needed to cancel the pole q^4/ϵ. The counterterm involves the 2-point self-energy, so it has the form $L_{c.t.} = -K_0/(6\alpha)\varphi^\dagger\Box^2\varphi$. In superfield language, this operator has the form (λ is a new dimensionless coupling):

$$L = \frac{\lambda}{\alpha}\int d^4\theta\;\varphi^\dagger\Box\varphi = -\frac{\lambda}{\alpha}\varphi^\dagger\Box^2\varphi + \cdots \tag{4.1}$$

where we used the same notation for the superfield and its scalar component.

[4]One has $\zeta[0,\rho] = 1/2-\rho$, $\zeta^{(1,0)}[0,\rho] = \ln\Gamma[\rho] - \ln\sqrt{2\pi}$, and $\zeta^{(1,0)}[-1,\frac{1}{2}] = -\frac{1}{2}\zeta^{(1,0)}[-1,0] - \frac{1}{24}\ln 2$ [36].

[5]Glaisher constant is given by $\ln G = 1/12 - \zeta'[-1]$, with $\zeta[x]$ the Riemann zeta function.

[6]The poles in eqs. (3.7) are identical to those obtained if we Taylor expanded the expressions in eqs. (3.3) about $\epsilon = 0$, and used $\zeta[0,\rho] = 1/2-\rho$, and $\zeta[-1,\rho] = -1/2\,(\rho^2-\rho+1/6)$ [36].

[7]This is a genuine 6D divergence.

This operator respects all symmetries of the theory and its presence is a reminder that our theory, although supersymmetric, is nevertheless non-renormalizable. Indeed, such theories are an expansion in powers $(\partial/\Lambda)^n$ [32], so such counterterms are expected; here Λ is the scale of new physics (from a 4D perspective), in this case $\Lambda^2 \sim \alpha \sim 1/\mathcal{A}$. Higher loops will generate more operators of this type. This operator, often overlooked in similar quantum calculations due to technical difficulties, is not specific to compactification with fluxes - it was also seen in 5D and 6D orbifold models at one-loop [25–31]. The counterterm modifies the dispersion relations (the poles of the propagator) of the scalar φ, which acquires a new solution, ghost-like, due to the higher order derivative [37]. Eqs. (3.11), (4.1) show the propagator of φ has new pole at

$$m_{pole}^2 = \frac{\alpha}{\lambda}\left[1 + \frac{21\zeta[3]}{8\pi^2} K_0\right] \tag{4.2}$$

This mass state is of the order of the compactification scale $\sqrt{\alpha} \sim 1/\sqrt{\mathcal{A}}$ and corresponds to the ghost degree of freedom. Note that the effective theory approach is reliable for a large torus area/radii (or small flux[8] $\alpha \sim 1/\mathcal{A}$) but then also operator (4.1) is enhanced!

In applications it is useful to replace this operator by an equivalent polynomial one [38, 39]; this is done by a non-linear field-redefinition or, equivalently, by using the equation of motion (in superfields) for φ: $-1/4\overline{D}^2\varphi^\dagger + \sqrt{2}q_0 g \sum_{n,j} \tilde{Q}_{n,j} Q_{n,j} = O(\mathcal{A})$, where we used eq. (2.5). This is used back in the action and effectively integrates the ghost $(\overline{D}^2\varphi^\dagger)$ but leaves φ in the action; then operator (4.1) becomes (with $-16\varphi^\dagger\Box\varphi = \varphi^\dagger\overline{D}^2 D^2\varphi$)

$$L \propto \frac{\lambda}{\alpha}\int d^4\theta\, q_0^2 g^2 \Big\vert \sum_{n,j} \tilde{Q}_{n,j} Q_{n,j}\Big\vert^2 \tag{4.3}$$

This is a dimension-six polynomial effective operator, equivalent to L of (4.1) and brings many non-renormalizable operators in the action, suppressed by α.

Having identified the counterterm operator, we can now formally set $q^2 = 0$ in the one-loop correction $\delta m_{b,f}^2(q^2)$ of eqs. (3.7) and by using the exact relations

$$H_1(0) = (\alpha/12)\ln(G^{24}e^3/4), \qquad H_2(0) = -\alpha \ln G^4 \tag{4.4}$$

we find from eqs. (3.7)

$$\delta m_b^2(0) = 0, \qquad \delta m_f^2(0) = 0, \quad \Rightarrow \quad \delta m^2(0) \equiv \delta m_b^2(0) + \delta m_f^2(0) = 0. \tag{4.5}$$

Therefore the bosonic and fermionic contributions vanish separately for $q^2 \to 0$, as conjectured in [10]. This indicates that at one-loop φ is a flat direction of the corresponding potential which has a vanishing curvature: $\delta m^2(q^2{=}0) = 0$, as we showed. Beyond one-loop, any quantum calculation must include the one-loop counterterm of eq. (4.1).

Let us comment briefly on the result of eq. (4.5). In compactifications without flux the Wilson line φ changes the boundary conditions of the charged fields, giving a continuous shift of the KK levels masses [9]. Then φ acquires at one-loop a potential and a nonzero

[8] One cannot take $\alpha \to 0$ since the flux is quantised.

correction $\delta m^2(q^2=0)$ [40]. By contrast, in our compactification with flux, $\varphi = \varphi_1 + i\varphi_2$ only shifts [12] the argument $z = (x_5, x_6)$ of the wavefunction of the KK modes[9] by an amount $\propto (\varphi_1/f, \varphi_2/f)$ and so the Wilson line does not enter in the mass formulae of the KK modes (and of the potential). This explains why the momentum-independent correction $\delta m^2(0)$ vanished at one-loop, with φ a flat direction. This appears as a consequence of the continuous (classical) translation symmetry of T_2 which can "shift away" φ, so the KK spectrum (and the potential) does not depend on it.

The initial continuous translation symmetry of T_2 is broken however at the quantum level by non-local Wilson loops.[10] To see this, we put together the solution for the background gauge potential in the symmetric gauge and the constant Wilson line $\varphi = \varphi_1 + i\varphi_2$ in the following form:[11] $A_5 = -\frac{1}{2} f x_6 + \varphi_2$ and $A_6 = \frac{1}{2} f x_5 + \varphi_1$. Now, the Wilson loops integrals are: $w_1(x_6) = \exp[i\, q_0\, g \int_0^{a_5} dx_5 A_5] = \exp[i\, q_0\, g(-f x_6/2 + \varphi_2) a_5]$ and $w_2(x_5) = \exp[i\, q_0\, g \int_0^{a_6} dx_6 A_6] = \exp[i\, q_0\, g(f x_5/2 + \varphi_1) a_6]$ where $a_k = 2\pi R_k$, $k = 5, 6$. The Wilson loops $w_{1,2}$ must be invariant, in particular under translations $x_6 \to x_6 + \delta_6$, $x_5 \to x_5 + \delta_5$; this happens only if $\delta_5 = 4\pi k/(f a_6 q_0 g) = 2 a_5 k/N$ and $\delta_6 = 4\pi l/(f a_5 q_0 g) = 2 a_6 l/N$. Here $k, l, N \in \mathbb{Z}$ are integers and we used the flux quantisation condition $f a_5 a_6 q_0 g = 2\pi N$ (see section 2). As a result, the continuous translation symmetry of T_2 is broken by non-local Wilson loops to a discrete (accidental) translation symmetry $x_5 \to x_5 + a_5(2k/N)$, $x_6 \to x_6 + a_6(2l/N)$ [13, 41]. With this continuous symmetry broken, one must investigate at higher orders if the one-loop flat direction of φ can be maintained.

To complete our discussion, let us also examine what happens if the sum over the modes in the calculation of the quantum corrections to $\delta m_{b,f}(q^2)$ is truncated to a fixed number of levels. Truncating the summation to $0 \le n \le n_0 - 1$ for bosons (n_0 levels) and to $0 \le n \le n_0' - 1$ for fermions (n_0' levels) we find from eqs. (3.1), (3.2), after some algebra[12]

$$\begin{aligned} \delta m_b^2(q^2) &= -\frac{1}{\epsilon}\, K_0\, \alpha\, n_0\, (2n_0 + 1) + \mathcal{O}(\epsilon^0) \\ \delta m_f^2(q^2) &= \frac{1}{\epsilon}\, K_0\, \alpha\, n_0'\, (2n_0' + q^2/\alpha) + \mathcal{O}(\epsilon^0) \end{aligned} \tag{4.6}$$

Their sum becomes, for $n_0 = n_0'$ (by supersymmetry)

$$\delta m^2(q^2) = -\frac{1}{\epsilon}\, K_0\, \alpha\, n_0\, (1 - q^2/\alpha) + \mathcal{O}(\epsilon^0) \tag{4.7}$$

[9] Let us show this for the KK zero modes. The Wilson line changes the equation for the zero mode: $a\, \psi_0 = 0$: $(i\bar\partial + \frac{1}{2} i\, q_0\, g\, |f| z - q_0\, g\, \varphi^\dagger)\psi_0 = 0$, with $z = x_5 + i x_6$. Then, the solution for the zero mode becomes $\psi_0 = h(z)\, \exp[-\frac{1}{2}\, q_0\, g\, |f|\, (\bar z - \bar z_0)(z - z_0)]$, where $z_0 \equiv -2\sqrt{2}\, i \varphi^\dagger/|f|$ and $h(z)$ is a holomorphic function. Therefore, a constant Wilson line only shifts z by z_0, but the number of zero modes (equivalent to the number of the possible centers within the fundamental domain on a torus) remains fixed by the quantisation condition as it is for a vanishing Wilson line. The same conclusion can be drawn in an asymmetric gauge for the background, such as $A_5 = 0$ and $A_6 = f x_5$.

[10] In orbifolds (no flux) this translation symmetry is broken explicitly by the orbifold fixed points.

[11] The gauge $A_5 = -f x_6/2$, $A_6 = f x_5/2$ is not invariant under translations $x_j \to x_j + \delta_j$ ($j = 5, 6$), but a gauge transformation $\vec A \to \vec A - f/2 \vec\nabla(\delta_5 x_6 - \delta_6 x_5)$ with $\vec A = (A_5, A_6)$ restores the translation symmetry.

[12] This is done by writing the "truncated" sum as a difference of two infinite towers/sums, bringing in eq. (3.3) a difference of Hurwitz zeta functions for each zeta function present there, e.g. for bosons: $\zeta[\epsilon, \rho_1] \to \zeta[\epsilon, \rho_1] - \zeta[\epsilon, n_0 + \rho_1]$ and similar for fermions with $\rho_1 \to \rho_2$, $n_0 \to n_0'$. Similar for $\zeta[\epsilon - 1, \rho_{1,2}]$.

This shows that a truncation of the towers to a same finite level would bring in the action a wavefunction renormalization for the superfield φ (due to the term $\propto q^2/\epsilon$) familiar in softly broken supersymmetry and also a momentum-independent quadratic divergence[13] $\propto \alpha/\epsilon$ (due to broken supersymmetry), but no higher dimensional counterterm is present. The theory is exactly 4-dimensional and renormalizable. Summing instead the whole tower, as we did, changes these two divergences into a *worse* "quartic" divergence $\propto q^4/(\alpha\epsilon)$ discussed earlier; this demanded instead a higher dimensional counterterm operator (L) specific to non-renormalizable theories (n_0, n_0' being now infinite).

A situation similar to that above is expected for the quantum corrections to the gauge coupling in this theory, when the 6D Lorentz invariance "promotes" counterterm (4.1) for the Wilson line to $F^{MN}\Box F_{MN}$ which also contains $F^{\mu\nu}\Box F_{\mu\nu}$. This is similar to 6D orbifolds without flux where such a higher dimensional counterterm ($\mathcal{A}\int d^2\theta\, \mathrm{Tr} W^\alpha \Box W_\alpha + \text{h.c.}$, in superfield notation) is generated [28–31] and is actually the reason for the so-called "power-like" running near the compactification scale.

5 Conclusions

Compactifications of effective theories in the presence of magnetic flux are interesting for model building since they provide supersymmetry breaking and chiral fermions. However, very few quantum calculations exist in such cases and this motivated our study. We examined the one-loop offshell correction to the two-point Green function of the Wilson line self-energy, in 6D N=1 Abelian gauge theories compactified on T_2 with magnetic flux ($\propto \alpha$). The offshell calculation is important and *necessary* in order to capture the usual effective theory expansion in powers of ∂/Λ; (from a 4D view $\Lambda \sim 1/\sqrt{\mathcal{A}} \sim \sqrt{\alpha}$, $\mathcal{A} =$ torus area).

Since the one-loop momentum integrals are UV divergent, a regularization is needed. We used the DR scheme which preserves all the symmetries of the initial 6D gauge theory. The result shows that in the limit of removing the regulator, the two-point Green function has a pole which dictates the form of the counterterm. This is a higher dimensional (derivative) operator that was also seen in orbifold compactifications without flux. One consequence of this counterterm is that a ghost state is present of $(\text{mass})^2 \propto \alpha$. We showed that such operator is equivalent to an operator of the same dimension (six) that is actually *polynomial* (quartic) in the charged superfields and is obtained by integrating out (decoupling) the ghost state. This operator is enhanced by a larger compactification area (when effective theory is applicable) and a reminder that effective theories are non-renormalizable.

After identifying the counterterm for the one-loop offshell self-energy, one may also consider the momentum independent mass correction $\delta m^2(q^2=0)$, which is the curvature of the corresponding one-loop potential. Unlike in orbifold compactifications (without flux), this mass correction vanishes at one-loop and the Wilson line corresponds to a flat direction. The reason for this is a translation symmetry in internal dimensions which is

[13] This situation is worse than the case of eq.16 in the first paper in [25–27] of ordinary orbifolds (no flux) where only usual q^2/ϵ poles i.e. wavefunction renormalization existed for a "truncated" tower summation.

JHEP06(2017)039

broken however at the quantum level by non-local Wilson loops. It is worth investigating this issue beyond the one-loop order considered here.

A One-loop integrals

In the text we used the following integrals in Euclidean space

$$
\begin{aligned}
I_1 &\equiv \int \frac{d^d p}{(2\pi)^d} \frac{p_\mu}{((p+q)^2+m_2^2)(p^2+m_1^2)} \\
&= \frac{-q_\mu}{(4\pi)^{\frac{d}{2}}} \int_0^1 dx\, x\, \Gamma\big[2-d/2\big] \Big[L(x,q^2,m_{1,2})\Big]^{\frac{d}{2}-2} \\
I_2 &\equiv \int \frac{d^d p}{(2\pi)^d} \frac{1}{((p+q)^2+m_2^2)(p^2+m_1^2)} \\
&= \frac{1}{(4\pi)^{\frac{d}{2}}} \int_0^1 dx\;\; \Gamma\big[2-d/2\big] \Big[L(x,q^2,m_{1,2})\Big]^{\frac{d}{2}-2} \\
I_3 &\equiv \int \frac{d^d p}{(2\pi)^d} \frac{p_\mu p_\nu}{((p+q)^2+m_2^2)(p^2+m_1^2)} \\
&= \frac{1}{(4\pi)^{\frac{d}{2}}} \frac{\delta_{\mu\nu}}{2} \int_0^1 dx\, \Gamma\big[1-d/2\big] \Big[L(x,q^2,m_{1,2})\Big]^{\frac{d}{2}-1} \\
&\quad + \frac{1}{(4\pi)^{\frac{d}{2}}}\, q_\mu q_\nu \int_0^1 dx\, x^2\, \Gamma\big[2-d/2\big] \Big[L(x,q^2,m_{1,2})\Big]^{\frac{d}{2}-2}
\end{aligned}
$$

where

$$
L(x,q^2,m_{1,2}) \equiv x\,(1-x)\,q^2 + x\,m_2^2 + (1-x)m_1^2 \tag{A.1}
$$

and $\sum_\mu \delta_{\mu\mu} = d$, $(d = 4-2\,\epsilon)$.

Acknowledgments

The work of HML is supported in part by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (NRF-2016R1A2B4008759).

References

[1] M. Berkooz, M.R. Douglas and R.G. Leigh, *Branes intersecting at angles*, Nucl. Phys. **B 480** (1996) 265 [hep-th/9606139] [INSPIRE].

[2] C. Angelantonj, I. Antoniadis, E. Dudas and A. Sagnotti, *Type I strings on magnetized orbifolds and brane transmutation*, Phys. Lett. **B 489** (2000) 223 [hep-th/0007090] [INSPIRE].

[3] R. Blumenhagen, L. Görlich, B. Körs and D. Lüst, *Noncommutative compactifications of type-I strings on tori with magnetic background flux*, *JHEP* **10** (2000) 006 [hep-th/0007024] [INSPIRE].

[4] P. Anastasopoulos, I. Antoniadis, K. Benakli, M.D. Goodsell and A. Vichi, *One-loop adjoint masses for non-supersymmetric intersecting branes*, *JHEP* **08** (2011) 120 [arXiv:1105.0591] [INSPIRE].

[5] G. Aldazabal, S. Franco, L.E. Ibáñez, R. Rabadán and A.M. Uranga, *Intersecting brane worlds*, *JHEP* **02** (2001) 047 [hep-ph/0011132] [INSPIRE].

[6] D. Cremades, L.E. Ibáñez and F. Marchesano, *Computing Yukawa couplings from magnetized extra dimensions*, *JHEP* **05** (2004) 079 [hep-th/0404229] [INSPIRE].

[7] R. Blumenhagen, B. Körs, D. Lüst and S. Stieberger, *Four-dimensional string compactifications with D-branes, orientifolds and fluxes*, *Phys. Rept.* **445** (2007) 1 [hep-th/0610327] [INSPIRE].

[8] C. Angelantonj and A. Sagnotti, *Open strings*, *Phys. Rept.* **371** (2002) 1 [hep-th/0204089] [INSPIRE].

[9] C. Bachas, *A way to break supersymmetry*, hep-th/9503030 [INSPIRE].

[10] W. Buchmüller, M. Dierigl, E. Dudas and J. Schweizer, *Effective field theory for magnetic compactifications*, *JHEP* **04** (2017) 052 [arXiv:1611.03798] [INSPIRE].

[11] M. Ishida, K. Nishiwaki and Y. Tatsuta, *Brane-localized masses in magnetic compactifications*, arXiv:1702.08226 [INSPIRE].

[12] Y. Hamada and T. Kobayashi, *Massive modes in magnetized brane models*, *Prog. Theor. Phys.* **128** (2012) 903 [arXiv:1207.6867] [INSPIRE].

[13] M.H. Al-Hashimi and U.J. Wiese, *Discrete accidental symmetry for a particle in a constant magnetic field on a torus*, *Annals Phys.* **324** (2009) 343 [arXiv:0807.0630] [INSPIRE].

[14] W. Buchmüller, M. Dierigl, F. Ruehle and J. Schweizer, *De Sitter vacua and supersymmetry breaking in six-dimensional flux compactifications*, *Phys. Rev.* **D 94** (2016) 025025 [arXiv:1606.05653] [INSPIRE].

[15] L.D. Landau and E.M. Lifshitz, *Quantum mechanics (Non-relativistic theory)*, third Edition, Betterworth-Heiniemann, U.S.A. (1977).

[16] W. Buchmüller and J. Schweizer, *Flavor mixings in flux compactifications*, *Phys. Rev.* **D 95** (2017) 075024 [arXiv:1701.06935] [INSPIRE].

[17] T.-H. Abe, Y. Fujimoto, T. Kobayashi, T. Miura, K. Nishiwaki and M. Sakamoto, *Z_N twisted orbifold models with magnetic flux*, *JHEP* **01** (2014) 065 [arXiv:1309.4925] [INSPIRE].

[18] T.-h. Abe, Y. Fujimoto, T. Kobayashi, T. Miura, K. Nishiwaki and M. Sakamoto, *Operator analysis of physical states on magnetized T^2/Z_N orbifolds*, *Nucl. Phys.* **B 890** (2014) 442 [arXiv:1409.5421] [INSPIRE].

[19] T.-h. Abe, Y. Fujimoto, T. Kobayashi, T. Miura, K. Nishiwaki, M. Sakamoto et al., *Classification of three-generation models on magnetized orbifolds*, *Nucl. Phys.* **B 894** (2015) 374 [arXiv:1501.02787] [INSPIRE].

[20] Y. Fujimoto, T. Kobayashi, K. Nishiwaki, M. Sakamoto and Y. Tatsuta, *Comprehensive analysis of Yukawa hierarchies on T^2/Z_N with magnetic fluxes*, *Phys. Rev.* **D 94** (2016) 035031 [arXiv:1605.00140] [INSPIRE].

JHEP06(2017)039

[21] T. Kobayashi, K. Nishiwaki and Y. Tatsuta, *CP-violating phase on magnetized toroidal orbifolds*, JHEP **04** (2017) 080 [arXiv:1609.08608] [INSPIRE].

[22] W. Buchmüller, M. Dierigl, F. Ruehle and J. Schweizer, *Split symmetries*, Phys. Lett. **B 750** (2015) 615 [arXiv:1507.06819] [INSPIRE].

[23] H. Abe, T. Kobayashi, K. Sumita and Y. Tatsuta, *Supersymmetric models on magnetized orbifolds with flux-induced Fayet-Iliopoulos terms*, Phys. Rev. **D 95** (2017) 015005 [arXiv:1610.07730] [INSPIRE].

[24] Y. Fujimoto, T. Kobayashi, T. Miura, K. Nishiwaki and M. Sakamoto, *Shifted orbifold models with magnetic flux*, Phys. Rev. **D 87** (2013) 086001 [arXiv:1302.5768] [INSPIRE].

[25] D.M. Ghilencea, *Higher derivative operators as loop counterterms in one-dimensional field theory orbifolds*, JHEP **03** (2005) 009 [hep-ph/0409214] [INSPIRE].

[26] D.M. Ghilencea and H.M. Lee, *Higher derivative operators from transmission of supersymmetry breaking on S^1/Z_2*, JHEP **09** (2005) 024 [hep-ph/0505187] [INSPIRE].

[27] D.M. Ghilencea and H.M. Lee, *Higher derivative operators from Scherk-Schwarz supersymmetry breaking on T^2/Z_2*, JHEP **12** (2005) 039 [hep-ph/0508221] [INSPIRE].

[28] S. Groot Nibbelink and M. Hillenbach, *Quantum corrections to non-Abelian SUSY theories on orbifolds*, Nucl. Phys. **B 748** (2006) 60 [hep-th/0602155] [INSPIRE].

[29] S. Groot Nibbelink and M. Hillenbach, *Renormalization of supersymmetric gauge theories on orbifolds: brane gauge couplings and higher derivative operators*, Phys. Lett. **B 616** (2005) 125 [hep-th/0503153] [INSPIRE].

[30] D.M. Ghilencea, H.M. Lee and K. Schmidt-Hoberg, *Higher derivatives and brane-localised kinetic terms in gauge theories on orbifolds*, JHEP **08** (2006) 009 [hep-ph/0604215] [INSPIRE].

[31] D.M. Ghilencea, *Compact dimensions and their radiative mixing*, Phys. Rev. **D 70** (2004) 045018 [hep-ph/0311264] [INSPIRE].

[32] H. Georgi, *Effective field theory*, Ann. Rev. Nucl. Part. Sci. **43** (1993) 209 [INSPIRE].

[33] N. Marcus, A. Sagnotti and W. Siegel, *Ten-dimensional supersymmetric Yang-Mills theory in terms of four-dimensional superfields*, Nucl. Phys. **B 224** (1983) 159 [INSPIRE].

[34] N. Arkani-Hamed, T. Gregoire and J.G. Wacker, *Higher dimensional supersymmetry in $4-D$ superspace*, JHEP **03** (2002) 055 [hep-th/0101233] [INSPIRE].

[35] E. Elizalde, *Ten physical applications of spectral zeta functions*, second edition, Berlin, Springer, Germany (2012).

[36] I. Gradshteyn and I. Ryzhik, *Table of integrals, series and products*, 7th edition, Academic Press, U.S.A. (2007).

[37] I. Antoniadis, E. Dudas and D.M. Ghilencea, *Living with ghosts and their radiative corrections*, Nucl. Phys. **B 767** (2007) 29 [hep-th/0608094] [INSPIRE].

[38] I. Antoniadis, E. Dudas and D.M. Ghilencea, *Supersymmetric Models with Higher Dimensional Operators*, JHEP **03** (2008) 045 [arXiv:0708.0383] [INSPIRE].

[39] E. Dudas and D.M. Ghilencea, *Effective operators in SUSY, superfield constraints and searches for a UV completion*, JHEP **06** (2015) 124 [arXiv:1503.08319] [INSPIRE].

[40] D.M. Ghilencea, D. Hoover, C.P. Burgess and F. Quevedo, *Casimir energies for* $6D$ *supergravities compactified on* $T^2/Z(N)$ *with Wilson lines*, *JHEP* **09** (2005) 050 [hep-th/0506164] [INSPIRE].

[41] E. Onofri, *Landau levels on a torus*, *Int. J. Theor. Phys.* **40** (2001) 537 [quant-ph/0007055] [INSPIRE].