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RG flows from (1,0) 6D SCFTs to $N = 1$ SCFTs in four and three dimensions

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Abstract: We study $AdS_5 \times \Sigma_2$ and $AdS_4 \times \Sigma_3$ solutions of $N = 2$, SO(4) gauged supergravity in seven dimensions with $\Sigma_{2,3}$ being $S^{2,3}$ or $H^{2,3}$. The SO(4) gauged supergravity is obtained from coupling three vector multiplets to the pure $N = 2$, SU(2) gauged supergravity. With a topological mass term for the 3-form field, the SO(4) $\sim$ SU(2) $\times$ SU(2) gauged supergravity admits two supersymmetric AdS_7 critical points, with SO(4) and SO(3) symmetries, provided that the two SU(2) gauge couplings are different. These vacua correspond to $N = (1, 0)$ superconformal field theories (SCFTs) in six dimensions. In the case of Σ_2, we find a class of $AdS_5 \times S^2$ and $AdS_5 \times H^2$ solutions preserving eight supercharges and SO(2) $\times$ SO(2) symmetry, but only $AdS_5 \times H^2$ solutions exist for SO(2) symmetry. These should correspond to some $N = 1$ four-dimensional SCFTs. We also give RG flow solutions from the $N = (1, 0)$ SCFTs in six dimensions to these four-dimensional fixed points including a two-step flow from the SO(4) $N = (1, 0)$ SCFT to the SO(3) $N = (1, 0)$ SCFT that eventually flows to the $N = 1$ SCFT in four dimensions. For $AdS_4 \times \Sigma_3$, we find a class of $AdS_4 \times S^3$ and $AdS_4 \times H^3$ solutions with four supercharges, corresponding to $N = 1$ SCFTs in three dimensions. When the two SU(2) gauge couplings are equal, only $AdS_4 \times H^3$ are possible. The uplifted solutions for equal SU(2) gauge couplings to eleven dimensions are also given.

Keywords: Gauge-gravity correspondence, AdS-CFT Correspondence, Supergravity Models

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Contents

1 Introduction

Six-dimensional superconformal field theories (SCFTs) are interesting in various aspects. In the context of M-theory, these SCFTs arise as a worldvolume theory of M5-branes in the near horizon limit. The correspondence between a six-dimensional $N = (2, 0)$ SCFT and M-theory on $AdS_7 \times S^4$ is one of the examples given in the AdS/CFT correspondence originally proposed in [1]. This AdS_7/CFT_6 correspondence has been explored in great details both from the M-theory point of view and the effective $N = 4$ SO(5) gauged supergravity in seven dimensions.

In this paper, we are interested in the half-maximal $N = (1, 0)$ SCFTs in six dimensions. It has been shown in [2] that $N = (1, 0)$ field theory possesses a non-trivial fixed point, and recently many $N = (1, 0)$ SCFTs have been classified in [3, 4] and [5]. The holographic study of this $N = (1, 0)$ theory has mainly been investigated by orbifolding the $AdS_7 \times S^4$ geometry of eleven-dimensional supergravity, see for example [6–8]. Recently, many new AdS_7 geometries from massive type IIA string theory have been found in [9], and the dual SCFTs of these AdS_7 vacua have been studied in [10].

We are particularly interested in studying $N = (1, 0)$ SCFTs within the framework of seven-dimensional gauged supergravity. These SCFTs should be dual to AdS_7 solutions of

$N = 2$ gauged supergravity in seven dimensions [11]. Pure $N = 2$ gauged supergravity with SU(2) gauge group admits both supersymmetric and non-supersymmetric AdS_7 vacua [12]. The two vacua can be interpreted as a supersymmetric and a non-supersymmetric CFT, respectively. A domain wall solution interpolating between these vacua has been studied in [13]. This solution describes a non-supersymmetric deformation of the UV $N = (1, 0)$ SCFT to another non-supersymmetric CFT in the IR.

When coupled to vector multiplets, the $N = 2$ gauged supergravity with many possible gauge groups can be obtained [14–16]. Although the resulting matter-coupled theory can support only a half-supersymmetric domain wall vacuum, supersymmetric AdS_7 vacua are possible if a topological mass term for the 3-form field, dual to the 2-form field in the gravity multiplet, is introduced. These supersymmetric AdS_7 critical points with SO(4) and SO(3) symmetries together with analytic RG flows interpolating between them have been studied in [17] in the case of SO(4) gauge group. And recently, AdS_7 vacua including compactifications to AdS_5 of non-compact gauge groups have been explored in [18]. The latter type of solutions generally describe twisted compactifications of $N = (1, 0)$ six-dimensional field theories to four dimensions.

In this paper, we are interested in holographic description of twisted compactifications of $N = (1, 0)$ SCFTs on two-manifolds $\Sigma_2 = (S^2, H^2)$ and three-manifold $\Sigma_3 = (S^3, H^3)$. The corresponding gravity solutions will take the form of $AdS_5 \times \Sigma_2$ and $AdS_4 \times \Sigma_3$, respectively. The dual field theories will be SCFTs in four or three dimensions. Gravity solutions interpolating between above mentioned AdS_7 vacua and these AdS_5 or AdS_4 geometries will describe RG flows from $N = (1, 0)$ SCFTs to lower dimensional SCFTs. Previously, this type of solutions has mainly been studied within the framework of the maximal $N = 4$ gauged supergravity. The solutions provide gravity duals of twisted compactifications of the $N = (2, 0)$ SCFTs. A number of these AdS_5 solutions together with the uplift to eleven-dimensional supergravity by using the reduction ansatz given in [19] and [20] have been studied previously in [21–24]. In addition, compactifications of $N = (1, 0)$ SCFT has recently been explored from the point of view of massive type IIA theory in [25].

We will give another new solution to this class from $N = 2$ SO(4) gauged supergravity. It has been pointed out in [22] that the $AdS_5 \times S^2$ solution preserving $\mathrm{SO}(2) \times \mathrm{SO}(2)$ symmetry and $N = 2$ supersymmetry in five dimensions, eight supercharges, cannot be obtained from pure minimal $N = 2$ gauged supergravity. We will show that this solution is a solution of $N = 2$ SO(4) gauged supergravity obtained from coupling pure $N = 2$ gauged supergravity to three vector multiplets. We will additionally give new $AdS_5 \times H^2$ solutions which are different from those given in [22] and [23] in the sense that the two SU(2) gauge couplings are different, and the residual symmetry is only the diagonal subgroup of $\mathrm{SO}(2) \times \mathrm{SO}(2)$. This case is not a truncation of the $N = 4$ SO(5) gauged supergravity, and the embedding of these solutions in higher dimensions are presently unknown. We will also study holographic RG flow solutions interpolating between AdS_7 vacua and these AdS_5 fixed points. The solutions describe deformations of $N = (1, 0)$ SCFTs in six dimensions to the IR $N = 1$ SCFT in four dimensions.

On AdS_4 solutions from seven-dimensional gauged supergravity, a class of $AdS_4 \times H^3$ and $AdS_4 \times S^3$ solutions have been obtained in [26]. A number of extensive studies of these solutions in terms of wrapped M5-branes on various supersymmetric cycles in special holonomy manifolds have been given in [27–29]. In particular, the solution studied in [29] has been obtained from the maximal gauged supergravity and preserves $N = 2$ superconformal symmetry in three dimensions. In this work, we will look for AdS_4 solutions in the $N = 2$ SO(4) gauged supergravity preserving only four supercharges. The corresponding solutions should then correspond to some $N = 1$ SCFTs in three dimensions. We will show that there exist $AdS_4 \times S^3$ and $AdS_4 \times H^3$ solutions in this SO(4) gauged supergravity with four supercharges when the two SU(2) gauge couplings are different. For equal SU(2) gauge couplings, only $AdS_4 \times H^3$ solutions exist and can be uplifted to eleven dimensions using the reduction ansatz given in [30].

The paper is organized as follow. In section 2, relevant information on $N = 2$ SO(4) gauged supergravity in seven dimensions and supersymmetric AdS_7 critical points are reviewed. $AdS_5 \times S^2$ and $AdS_5 \times H^2$ solutions together with holographic RG flows from AdS_7 critical points to these AdS_5 fixed points will be given in section 3. We present $AdS_4 \times S^3$ and $AdS_4 \times H^3$ solutions in section 4 and give the embedding of some $AdS_5 \times \Sigma_2$ and $AdS_4 \times \Sigma_3$ solutions in eleven dimensions in section 5. We finally give some comments and conclusions in section 6.

2 Seven-dimensional $N = 2$ SO(4) gauged supergravity and AdS_7 critical points

In this section, we give a description of the SO(4) $N = 2$ gauged supergravity in seven dimensions and the associated supersymmetric AdS_7 critical points. These critical points preserve $N = 2$ supersymmetry in seven dimensions and correspond to six-dimensional $N = (1, 0)$ SCFTs. All of the notations used throughout the paper are the same as those in [16] and [17].

2.1 SO(4) gauged supergravity

The SO(4) $N = 2$ gauged supergravity in seven dimensions is constructed by gauging the half-maximal $N = 2$ supergravity coupled to three vector multiplets. The supergravity multiplet $(e^m_\mu, \psi^A_\mu, A^i_\mu, \chi^A, B_{\mu\nu}, \sigma)$ consists of the graviton, two gravitini, three vectors, two spin-$\frac{1}{2}$ fields, a two-form field and the dilaton. We will use the convention that curved and flat space-time indices are denoted by $\mu, \nu, \ldots$ and $m, n, \ldots$, respectively. Each vector multiplet $(A_\mu, \lambda^A, \phi^i)$ contains a vector field, two gauginos and three scalars. The bosonic field content of the matter coupled supergravity then consists of the graviton, six vectors and ten scalars parametrized by the $\mathbb{R}^+ \times SO(3, 3)/SO(3) \times SO(3) \sim \mathbb{R}^+ \times SL(4, \mathbb{R})/SO(4)$ coset manifold. In the following, we will consider the supergravity theory in which the two-form field $B_{\mu\nu}$ is dualized to a three-form field $C_{\mu\nu\rho}$. The latter admits a topological mass term, so the resulting gauged supergravity admits an AdS_7 vacuum.

The SO(4) gauged supergravity is obtained by gauging the SO(4) $\sim$ SO(3) $\times$ SO(3) subgroup of the global symmetry group SO(3, 3). One of the SO(3) in the gauge group SO(3) $\times$ SO(3) is the $SO(3)_R \sim USp(2)_R \sim SU(2)_R$ R-symmetry. All spinor fields, including the supersymmetry parameter ϵ^A, are symplectic-Majorana spinors transforming as doublets of the $\mathrm{SU}(2)_R$ R-symmetry. From now on, the $\mathrm{SU}(2)_R$ douplet indices $A, B = 1, 2$ will not be shown explicitly. The $\mathrm{SU}(2)_R$ triplets are labeled by indices $i, j = 1, 2, 3$ while indices $r, s = 1, 2, 3$ are the triplet indices of the other $\mathrm{SO}(3)$ in $\mathrm{SO}(3)_R \times \mathrm{SO}(3)$.

The 9 scalar fields in the $\mathrm{SO}(3,3)/\mathrm{SO}(3)\times\mathrm{SO}(3)$ coset are parametrized by the coset representative $L = ({L_I}^i, {L_I}^r)$ which transforms under the global $\mathrm{SO}(3,3)$ and the local composite $\mathrm{SO}(3)\times\mathrm{SO}(3)$ by left and right multiplications, respectively. The inverse of L is denoted by $L^{-1} = ({L^I}_i, {L^I}_r)$ satisfying the relations ${L^I}_i = \eta^{IJ}L_{Ji}$ and ${L^I}_r = \eta^{IJ}L_{Jr}$.

The bosonic Lagrangian of the $N = 2$ gauged supergravity is given by

$$\begin{aligned} e^{-1}\mathcal{L} = &\frac{1}{2}R - \frac{1}{4}e^{\sigma}a_{IJ}F^I_{\mu\nu}F^{J\mu\nu} - \frac{1}{48}e^{-2\sigma}H_{\mu\nu\rho\sigma}H^{\mu\nu\rho\sigma} - \frac{5}{8}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}P^{ir}_\mu P^\mu_{ir} \\ &- \frac{1}{144\sqrt{2}}e^{-1}\epsilon^{\mu_1\ldots\mu_7}H_{\mu_1\ldots\mu_4}\omega_{\mu_5\ldots\mu_7} + \frac{1}{36}he^{-1}\epsilon^{\mu_1\ldots\mu_7}H_{\mu_1\ldots\mu_4}C_{\mu_5\ldots\mu_7} - V \end{aligned} \tag{2.1}$$

where the scalar potential and the Chern-Simons term are given by

$$V = \frac{1}{4}e^{-\sigma}\left(C^{ir}C_{ir} - \frac{1}{9}C^2\right) + 16h^2e^{4\sigma} - \frac{4\sqrt{2}}{3}he^{\frac{3\sigma}{2}}C, \tag{2.2}$$

$$\omega_{\mu\nu\rho} = 3\eta_{IJ}F^I_{[\mu\nu}A^J_{\rho]} - {f_{IJ}}^K A^I_\mu \wedge A^J_\nu \wedge A_{\rho K} \tag{2.3}$$

with the gauge field strength defined by $F^I_{\mu\nu} = 2\partial_{[\mu}A^I_{\nu]} + {f_{JK}}^I A^J_\mu A^K_\nu$. The structure constants ${f_{IJ}}^K$ of the gauge group include the gauge coupling associated to each simple factor in a general gauge group $G_0 \subset \mathrm{SO}(3,3)$.

We are mainly interested in supersymmetric solutions. Therefore, the supersymmetry transformations of fermions are necessary. However, we will not consider bosonic solutions with the three-form field turned on. We will accordingly set $C_{\mu\nu\rho} = 0$ throughout. The fermionic supersymmetry transformations, with all fermions and the three-form field vanishing, are given by

$$\delta\psi_\mu = 2D_\mu\epsilon - \frac{\sqrt{2}}{30}e^{-\frac{\sigma}{2}}C\gamma_\mu\epsilon - \frac{i}{20}e^{\frac{\sigma}{2}}F^i_{\rho\sigma}\sigma^i\left(3\gamma_\mu\gamma^{\rho\sigma} - 5\gamma^{\rho\sigma}\gamma_\mu\right)\epsilon - \frac{4}{5}he^{2\sigma}\gamma_\mu\epsilon, \tag{2.4}$$

$$\delta\chi = -\frac{1}{2}\gamma^\mu\partial_\mu\sigma\epsilon - \frac{i}{10}e^{\frac{\sigma}{2}}F^i_{\mu\nu}\sigma^i\gamma^{\mu\nu}\epsilon + \frac{\sqrt{2}}{30}e^{-\frac{\sigma}{2}}C\epsilon - \frac{16}{5}e^{2\sigma}h\epsilon, \tag{2.5}$$

$$\delta\lambda^r = -i\gamma^\mu P^{ir}_\mu\sigma^i\epsilon - \frac{1}{2}e^{\frac{\sigma}{2}}F^r_{\mu\nu}\gamma^{\mu\nu}\epsilon - \frac{i}{\sqrt{2}}e^{-\frac{\sigma}{2}}C^{ir}\sigma^i\epsilon\,. \tag{2.6}$$

Various quantities appearing in the Lagrangian and supersymmetry transformations are defined by the following relations

$$\begin{aligned} D_\mu\epsilon &= \partial_\mu\epsilon + \frac{1}{4}\omega^{mn}_\mu\gamma_{mn} + \frac{i}{4}\sigma^i\epsilon^{ijk}Q_{\mu jk}, \\ P^{ir}_\mu &= L^{Ir}\left(\delta^K_I\partial_\mu + {f_{IJ}}^K A^J_\mu\right){L^i}_K, & Q^{ij}_\mu &= L^{Ij}\left(\delta^K_I\partial_\mu + {f_{IJ}}^K A^J_\mu\right){L^i}_K, \\ C_{ir} &= \frac{1}{\sqrt{2}}{f_{IJ}}^K {L^I}_j {L^J}_k L_{Kr}\epsilon^{ijk}, & C &= -\frac{1}{\sqrt{2}}{f_{IJ}}^K {L^I}_i {L^J}_j L_{Kk}\epsilon^{ijk}, \\ C_{rsi} &= {f_{IJ}}^K {L^I}_r {L^J}_s L_{Ki}, & a_{IJ} &= {L^i}_I L_{iJ} + {L^r}_I L_{rJ}, \\ F^i_{\mu\nu} &= {L_I}^i F^I, & F^r_{\mu\nu} &= {L_I}^r F^I \end{aligned} \tag{2.7}$$

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where γ^m are space-time gamma matrices satisfying $\{\gamma^m, \gamma^n\} = 2\eta^{mn}$ with $\eta^{mn} = \text{diag}(-1, 1, 1, 1, 1, 1, 1)$.

2.2 Supersymmetric AdS_7 critical points

We will now briefly review supersymmetric AdS_7 critical points found in [17]. There are two critical points preserving the full $N = 2$ supersymmetry in seven dimensions. The two critical points however have different symmetries namely one critical point, at which all scalars vanishing, preserves the full SO(4) gauge symmetry while the other is only invariant under the diagonal subgroup $\text{SO}(3)_{\text{diag}} \subset \text{SO}(3) \times \text{SO}(3)$.

For SO(3) × SO(3) gauge group, the gauge structure constants can be written as [16]

$$f_{IJK} = (g_1\epsilon_{ijk}, -g_2\epsilon_{rst}). \tag{2.8}$$

Before discussing the detail of the two critical points, we give an explicit parametrization of the SO(3, 3)/SO(3) × SO(3) coset as follow. With the 36 basis elements of a general 6 × 6 matrix

$$(e_{IJ})_{KL} = \delta_{IK}\delta_{JL}, \qquad I, J, \ldots = 1, \ldots, 6 \tag{2.9}$$

the generators of the composite SO(3) × SO(3) symmetry are given by

$$\begin{aligned} \text{SO}(3)_R: \qquad & J^{(1)}_{ij} = e_{ji} - e_{ij}, && i, j = 1, 2, 3, \\ \text{SO}(3): \qquad & J^{(2)}_{rs} = e_{s+3,r+3} - e_{r+3,s+3}, && r, s = 1, 2, 3\,. \end{aligned} \tag{2.10}$$

The non-compact generators corresponding to 9 scalars take the form of

$$Y^{ir} = e_{i,r+3} + e_{r+3,i}\,. \tag{2.11}$$

Accordingly, the coset representative can be obtained by an exponentiation of the appropriate Y^{ir} generators. Y^{ir} generators and the 9 scalars transform as $(\mathbf{3}, \mathbf{3})$ under the SO(3) × SO(3) local symmetry.

The supersymmetric AdS_7 critical points preserve at least SO(3) symmetry. Therefore, we will consider only the coset representative invariant under SO(3) symmetry. The dilaton σ is an SO(3) × SO(3) singlet. From the 9 scalars in SO(3, 3)/SO(3) × SO(3), there is one $\text{SO}(3)_{\text{diag}}$ singlet from the decomposition $\mathbf{3} \times \mathbf{3} \to \mathbf{1} + \mathbf{3} + \mathbf{5}$. The singlet corresponds to the non-compact generator

$$Y_s = Y^{11} + Y^{22} + Y^{33}\,. \tag{2.12}$$

The coset representative is then given by

$$L = e^{\phi Y_s}\,. \tag{2.13}$$

The scalar potential for the dilaton σ and the $\text{SO}(3)_{\text{diag}}$ singlet scalar ϕ can be straightforwardly computed. Its explicit form reads [17]

$$\begin{aligned} V = \frac{1}{32}e^{-\sigma}\Big[&(g_1^2 + g_2^2)\left(\cosh(6\phi) - 9\cosh(2\phi)\right) + 8g_1g_2\sinh^3(2\phi) \\ &+8\left[g_2^2 - g_1^2 + 64h^2e^{5\sigma} + 32e^{\frac{5\sigma}{2}}h\left(g_1\cosh^2\phi + g_2\sinh^3\phi\right)\right]\Big]. \end{aligned} \tag{2.14}$$

There are two supersymmetric AdS_7 vacua given by

$$\text{SO(4)} - \text{critical point}: \qquad \sigma = \phi = 0, \qquad V_0 = -240h^2, \tag{2.15}$$

$$\text{SO(3)} - \text{critical point}: \qquad \sigma = -\frac{1}{5}\ln\left[\frac{g_2^2 - 256h^2}{g_2^2}\right],$$

$$\phi = \frac{1}{2}\ln\left[\frac{g_2 + 16h}{g_2 - 16h}\right], \quad V_0 = -\frac{240 g_2^{\frac{8}{5}} h^2}{(g_2^2 - 256h^2)^{\frac{4}{5}}} \tag{2.16}$$

where we have chosen $g_1 = -16h$ in order to make the SO(4) critical point occurs at $\sigma = 0$. This is achieved by shifting σ. The value of the cosmological constant has been denoted by V_0.

The two critical points correspond to $N = (1, 0)$ SCFTs in six dimensions with SO(4) and SO(3) symmetries, respectively. An RG flow solution interpolating between these two critical points has already been studied in [17]. In the next sections, we will study supersymmetric RG flows from these SCFTs to other SCFTs in four and three dimensions providing holographic descriptions of twisted compactifications of these $N = (1, 0)$ SCFTs.

3 Flows to $N = 1$ SCFTs in four dimensions

In this section, we look for solutions of the form $AdS_5 \times S^2$ or $AdS_5 \times H^2$ in which S^2 and H^2 are a two-sphere and a two-dimensional hyperbolic space, respectively.

In the case of S^2, we take the seven-dimensional metric to be

$$ds_7^2 = e^{2F(r)} dx_{1,3}^2 + dr^2 + e^{2G(r)}(d\theta^2 + \sin^2 d\phi^2) \tag{3.1}$$

with $dx_{1,3}^2$ being the flat metric on the four-dimensional spacetime. By using the vielbein

$$\begin{aligned} e^{\hat{\mu}} &= e^F dx^\mu, & e^{\hat{r}} &= dr, \\ e^{\hat{\theta}} &= e^G d\theta, & e^{\hat{\phi}} &= e^G \sin\theta d\phi, \end{aligned} \tag{3.2}$$

we can compute the following spin connections

$$\begin{aligned} \omega^{\hat{\phi}}{}_{\hat{\theta}} &= e^{-G}\cot\theta e^{\hat{\phi}}, & \omega^{\hat{\phi}}{}_{\hat{r}} &= G' e^{\hat{\phi}}, \\ \omega^{\hat{\theta}}{}_{\hat{r}} &= G' e^{\hat{\theta}}, & \omega^{\hat{\mu}}{}_{\hat{r}} &= F' e^{\hat{\mu}}. \end{aligned} \tag{3.3}$$

where $'$ denotes the r-derivative. Hatted indices are tangent space indices.

In the case of H^2, we take the matric to be

$$ds_7^2 = e^{2F(r)} dx_{1,3}^2 + dr^2 + \frac{e^{2G(r)}}{y^2}(dx^2 + dy^2). \tag{3.4}$$

With the vielbein

$$\begin{aligned} e^{\hat{\mu}} &= e^F dx^\mu, & e^{\hat{r}} &= dr, \\ e^{\hat{x}} &= \frac{e^G}{y} dx, & e^{\hat{y}} &= \frac{e^G}{y} dy, \end{aligned} \tag{3.5}$$

the spin connections are found to be

$$\omega^{\hat{x}}{}_{\hat{r}} = G' e^{\hat{x}}, \qquad \omega^{\hat{y}}{}_{\hat{r}} = G' e^{\hat{y}},$$
$$\omega^{\hat{\mu}}{}_{\hat{r}} = F' e^{\hat{\mu}}, \qquad \omega^{\hat{x}}{}_{\hat{y}} = -e^{-G(r)} e^{\hat{x}}\,. \tag{3.6}$$

3.1 AdS_5 solutions with SO(2) $\times$ SO(2) symmetry

We now construct the BPS equations from the supersymmetry transformations of fermions. We first consider the S^2 case. In order to preserve supersymmetry, we make a twist by turning on the SO(2) $\times$ SO(2) $\subset$ SO(4) gauge fields, among the six gauge fields A^I,

$$A^3 = a\cos\theta d\phi \qquad \text{and} \qquad A^6 = b\cos\theta d\phi \tag{3.7}$$

such that the spin connections on S^2 is cancelled by these gauge connections. The Killing spinor corresponding to the unbroken supersymmetry is then a constant spinor on S^2.

We begin with the solutions preserving the full SO(2)$\times$SO(2) residual gauge symmetry generated by $J^{(1)}_{12}$ and $J^{(2)}_{12}$. Scalars which are singlet under SO(2) $\times$ SO(2) are the dilaton and the scalar corresponding to the SO(3, 3) non-compact generators Y^{33}. We will denote this scalar by Φ. By considering the variation of the gravitino along S^2 directions, we find that the cancellation between the spin and gauge connections imposes the twist condition

$$a g_1 = 1\,. \tag{3.8}$$

Using the projection conditions

$$\gamma_r \epsilon = \epsilon, \qquad \text{and} \qquad i\sigma^3 \gamma^{\hat{\theta}\hat{\phi}} \epsilon = \epsilon, \tag{3.9}$$

we find the following BPS equations

$$\Phi' = \frac{1}{2} e^{-\frac{\sigma}{2}-\Phi-2G}\left[e^{2G} g_1 (e^{2\Phi}-1) - a e^{\sigma}(e^{2\Phi}-1) - b e^{\sigma}(e^{2\Phi}+1)\right], \tag{3.10}$$

$$\sigma' = \frac{1}{5} e^{-\frac{\sigma}{2}-\Phi-2G}\left[e^{\sigma}\left[a-b+(a+b)e^{2\Phi}\right] - e^{2G}\left(g_1 + g_1 e^{2\Phi} + 32 h e^{\frac{5\sigma}{2}+\Phi}\right)\right], \tag{3.11}$$

$$G' = -\frac{1}{10} e^{-\frac{\sigma}{2}-\Phi-2G}\left[4e^{\sigma}\left[a-b+(a+b)e^{2\Phi}\right] + e^{2G}\left(g_1 + g_1 e^{2\Phi} - 8 h e^{\frac{5\sigma}{2}+\Phi}\right)\right], \tag{3.12}$$

$$F' = \frac{1}{10} e^{-\frac{\sigma}{2}-\Phi-2G}\left[e^{\sigma}\left[a-b+(a+b)e^{2\Phi}\right] - e^{2G}\left(g_1 + g_1 e^{2\Phi} - 8 h e^{\frac{5\sigma}{2}+\Phi}\right)\right]. \tag{3.13}$$

In the H^2 case, we choose the gauge fields to be

$$A^3 = \frac{a}{y} dx \qquad \text{and} \qquad A^6 = \frac{b}{y} dx \tag{3.14}$$

which can be verified that the spin connection $\omega^{\hat{x}\hat{y}}$ in (3.6) is cancelled by virtue of the twist condition (3.8) and the projection conditions

$$\gamma_r \epsilon = \epsilon \qquad \text{and} \qquad i\sigma^3 \gamma^{\hat{x}\hat{y}} \epsilon = \epsilon\,. \tag{3.15}$$

By an analogous computation, we find a similar set of BPS equations as in (3.10), (3.11), (3.12) and (3.13) with (a, b) replaced by $(-a, -b)$.

At large r, solutions to the above BPS equations should approach the SO(4) AdS_7 critical point with $\Phi \sim \sigma \sim 0$ and $F \sim G \sim r$. This is the UV (1,0) SCFT. As $r \to -\infty$, we look for the solution of the form $AdS_5 \times S^2$ or $AdS_5 \times H^2$ such that $\phi' = \sigma' = G' = 0$ and $F' =$ constant. We find that there is an AdS_5 solution given by

$$\begin{aligned}
\Phi &= \frac{1}{2}\ln\left[\frac{b \pm \sqrt{4a^2-3b^2}}{2(a+b)}\right], \\
\sigma &= \frac{1}{5}\ln\left[\frac{g_1^2 b^2(b \pm \sqrt{4a^2-3b^2})}{32(a+b)h^2(3b-2a \pm \sqrt{4a^2-3b^2})}\right], \\
G &= \frac{1}{10}\ln\left[\frac{b^2(a+b)^4(b \pm \sqrt{4a^2-3b^2})(2a-3b \mp \sqrt{4a^2-3b^2})^3}{32 g_1^3 h^2(2a+b \mp \sqrt{4a^2-3b^2})^5}\right], \\
L_{\mathrm{AdS}_5} &= \left[\frac{(a+b)^2(2a-3b \pm \sqrt{4a^2-3b^2})^4}{b^4 g_1^4 h(b \mp \sqrt{4a^2-3b^2})^2}\right]^{\frac{1}{5}}.
\end{aligned} \tag{3.16}$$

This solution is given for $\Sigma_2 = S^2$. The solution in the H^2 case is given similarly by flipping the signs of a and b.

It should be noted that, in this fixed point solution with $\mathrm{SO}(2)\times\mathrm{SO}(2)$ symmetry, the coupling g_2 does not appear. The solution can then be taken as a solution of the gauged supergravity with $g_2 = g_1$. Therefore, the solution can be uplifted to eleven dimensions by using the reduction ansatz in [30]. This will be done in section 5. The uplifted solution is however not new since similar solutions have been found previously in [22, 23], and supergravity solutions interpolating between AdS_7 and $AdS_5 \times S^2$ or $AdS_5 \times H^2$ have also been investigated. The solutions have an interpretation in terms of RG flows from the UV SCFT in six dimensions to four-dimensional SCFTs with $\mathrm{SO}(2)\times\mathrm{SO}(2)$ symmetry.

Note also that, in this case, it is not possible to find an RG flow from the SO(3) AdS_7 point to any of these four-dimensional SCFTs since this AdS_7 critical point is not accessible from the BPS equations given above.

3.2 AdS_5 solutions with SO(2) symmetry

We now consider AdS_5 solutions with SO(2) symmetry. We will study two possibilities namely the $\mathrm{SO}(2)_{\mathrm{diag}} \subset \mathrm{SO}(2)\times\mathrm{SO}(2) \subset \mathrm{SO}(3)\times\mathrm{SO}(3)$ and $\mathrm{SO}(2)_R \subset \mathrm{SO}(3)_R$.

3.2.1 Flows with $\mathrm{SO}(2)_{\mathrm{diag}}$ symmetry

We begin with the $\mathrm{SO}(2)_{\mathrm{diag}}$ symmetry generated by $J^{(1)}_{12} + J^{(2)}_{12}$. Among the 9 scalars in $\mathrm{SO}(3,3)/\mathrm{SO}(3)\times\mathrm{SO}(3)$, there are three singlets under $\mathrm{SO}(2)_{\mathrm{diag}}$ corresponding to the following decomposition of $\mathrm{SO}(3)\times\mathrm{SO}(3)$ representations under $\mathrm{SO}(2)_{\mathrm{diag}}$

$$\mathbf{3}\times\mathbf{3} = (\mathbf{2}+\mathbf{1})\times(\mathbf{2}+\mathbf{1}) = \mathbf{1}+\mathbf{1}+\mathbf{2}+\mathbf{2}+\mathbf{2}+\mathbf{1}\,. \tag{3.17}$$

The three singlets correspond to the non-compact generators

$$Y^{11}+Y^{22}, \qquad Y^{33}, \qquad Y^{12}-Y^{21}\,. \tag{3.18}$$

The coset representative describing these singlets can be written as

$$L = e^{\Phi_1(Y^{11}+Y^{22})} e^{\Phi_2 Y^{33}} e^{\Phi_3(Y^{12}-Y^{21})} \,. \tag{3.19}$$

Since we have not found any $AdS_5 \times S^2$ solution, we will give only the result for the H^2 case. The $\mathrm{SO}(2)_{\mathrm{diag}}$ gauge field can be obtained from the $\mathrm{SO}(2) \times \mathrm{SO}(2)$ gauge fields in (3.7) with the condition that

$$b g_2 = a g_1 \,. \tag{3.20}$$

As in the previous case, the twist imposes the condition $g_1 a = 1$ which in the present case also implies $g_2 b = 1$.

Using the projection conditions (3.15), we find the following BPS equations

$$\Phi_1' = \frac{1}{8} e^{-\frac{\sigma}{2}-2\Phi_1-\Phi_2}(e^{4\Phi_1}-1)\left[g_1 - g_2 + (g_1+g_2)e^{2\Phi_2}\right], \tag{3.21}$$

$$\begin{aligned}\Phi_2' = \frac{1}{16 g_2} e^{-\frac{\sigma}{2}} \big[& 8 g_1 a \left[g_1 - g_2 + (g_1+g_2)e^{\Phi_2}\right] \\ & + g_2 \big[e^{-2\Phi_1-\Phi_2-2\Phi_3}(1+e^{4\Phi_1})(1+e^{4\Phi_3})\left[g_2 - g_1 + (g_1+g_2)e^{2\Phi_2}\right] \\ & +4(g_1-g_2)e^{\Phi_2} - (g_1+g_2)e^{-\Phi_2}\big]\big],\end{aligned} \tag{3.22}$$

$$\Phi_3' = \frac{1}{8} e^{-\frac{\sigma}{2}-\Phi_2-2\Phi_3}(e^{4\Phi_3}-1)\left[g_1 - g_2 + (g_1+g_2)e^{2\Phi_2}\right], \tag{3.23}$$

$$\begin{aligned}\sigma' = \frac{1}{40 g_2} e^{-\frac{\sigma}{2}-2\Phi_1-\Phi_2-2\Phi_3} \big[& 8 a e^{\sigma+2\Phi_1+2\Phi_3-2G}\left[g_1 - g_2 - (g_1+g_2)e^{2\Phi_2}\right] \\ & - g_2 \big[g_1(1+e^{2\Phi_2})(1+e^{4\Phi_1}+e^{4\Phi_3}+4e^{2\Phi_1+2\Phi_3}+e^{4\Phi_1+4\Phi_3}) \\ & + g_2(e^{2\Phi_2}-1)(1+e^{4\Phi_1}+e^{4\Phi_3}-4e^{2\Phi_1+2\Phi_3}+e^{4\Phi_1+4\Phi_3}) \\ & +256 h e^{\frac{5\sigma}{2}+2\Phi_1+\Phi_2+2\Phi_3}\big]\big],\end{aligned} \tag{3.24}$$

$$\begin{aligned}G' = \frac{1}{20} e^{-\frac{\sigma}{2}} \Big[& 16 h e^{\frac{5\sigma}{2}} - g_1(e^{\Phi_2}+e^{-\Phi_2}) + g_2(e^{\Phi_2}-e^{-\Phi_2}) \\ & -\frac{1}{4} e^{-2\Phi_1-\Phi_2-2\Phi_3}(1+e^{4\Phi_1})(1+e^{4\Phi_3})[g_1 - g_2 + (g_1+g_2)e^{2\Phi_2}] \\ & +\frac{8a}{g_2} e^{\sigma-\Phi_2-2G}[g_2 - g_1 + (g_1-g_2)e^{2\Phi_2}]\Big],\end{aligned} \tag{3.25}$$

$$\begin{aligned}F' = \frac{1}{20} e^{-\frac{\sigma}{2}} \Big[& 16 h e^{\frac{5\sigma}{2}} - g_1(e^{\Phi_2}+e^{-\Phi_2}) + g_2(e^{\Phi_2}-e^{-\Phi_2}) \\ & -\frac{1}{4} e^{-2\Phi_1-\Phi_2-2\Phi_3}(1+e^{4\Phi_1})(1+e^{4\Phi_3})[g_1 - g_2 + (g_1+g_2)e^{2\Phi_2}] \\ & -\frac{2a}{g_2} e^{\sigma-\Phi_2-2G}[g_2 - g_1 + (g_1-g_2)e^{2\Phi_2}]\Big].\end{aligned} \tag{3.26}$$

In this case, there are a number of possible AdS_5 fixed point solutions, and it is possible to have a solution interpolating between the $\mathrm{SO}(3)$ AdS_7 critical points and the AdS_5 in the IR. We will investigate each of them in the following discussion.

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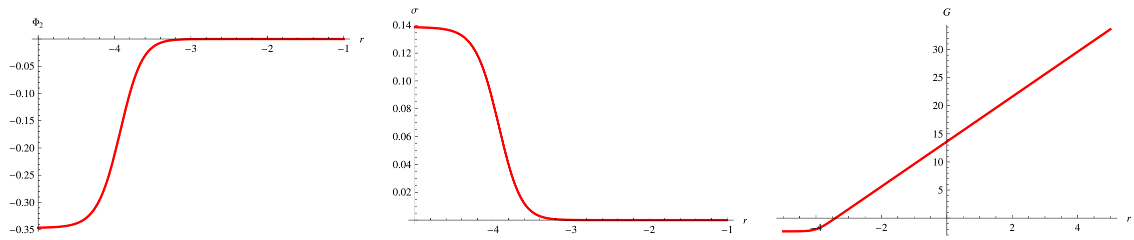


(a) Φ_2 solution. (b) σ solution. (c) G solution.

Figure 1. RG flows from SO(4) $N = (1, 0)$ SCFT in six dimensions to four-dimensional $N = 1$ SCFT with $\mathrm{SO}(2)_{\mathrm{diag}}$ symmetry for $g_1 = g_2$.

We first look at the $AdS_5 \times H^2$ critical point with $g_2 = g_1$ since this can be uplifted to eleven dimensions. When $g_2 = g_1$, the fixed point solution exists only for $\Phi_1 = \Phi_3 = 0$, and the corresponding solution is given by

$$\Phi_2 = -\frac{1}{2}\ln 2, \sigma = \frac{1}{5}\ln 2,$$
$$G = \frac{3}{5}\ln 2 - \frac{1}{2}\ln\left[\frac{g_1}{a}\right], \qquad L_{\mathrm{AdS}_5} = \frac{1}{2^{\frac{12}{5}}h} \tag{3.27}$$

The AdS_5 solution preserves eight supercharges corresponding to $N = 1$ superconformal field theory in four dimensions with SO(2) symmetry. A flow solution interpolating between this $AdS_5 \times H^2$ fixed point and the SO(4) AdS_7 given in (2.15) for $h = 1$ is shown in figure 1.

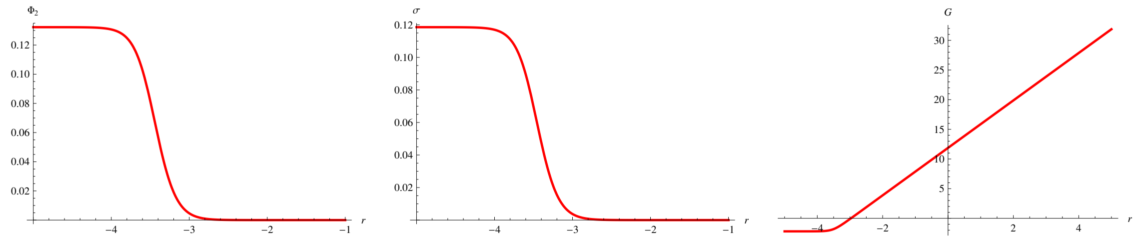
It should be noted here that this fixed point can be obtained from the $\mathrm{SO}(2) \times \mathrm{SO}(2)$ fixed points given in the previous section by setting the parameter $b = a$. It can be readily verified that, for $b = a$, solution in (3.16) is valid only for the upper sign and $\Sigma_2 = H^2$. The resulting solution is precisely that given in (3.27).

We now move to solutions with $g_2 \neq g_1$. The solution given in (3.27) is a special case of a more general solution, with $\Phi_1 = \Phi_3 = 0$ and $g_2 \neq g_1$, which is given by

$$\Phi_2 = \frac{1}{2}\ln\left[\frac{g_1 \pm \sqrt{4g_2^2 - 3g_1^2}}{2(g_1 + g_2)}\right], \qquad \Phi_1 = \Phi_3 = 0,$$
$$\sigma = \frac{1}{5}\left[\frac{1024h^2(\sqrt{g_2^2 - 192h^2} \mp 8h)}{(g_2 - 16h)(g_2 + 24h \mp \sqrt{g_2^2 - 192h^2})}\right],$$
$$G = \frac{1}{10}\ln\left[\frac{a^5(g_2 - 16h)^4(\sqrt{g_2^2 - 192h^2} \mp 8h)(g_2 + 24h \mp \sqrt{g_2^2 - 192h^2})^3}{1024g_2^5h^3(g_2 - 8h \mp \sqrt{g_2^2 - 192h^2})^5}\right],$$
$$L_{\mathrm{AdS}_5} = \frac{1}{2}\left[\frac{(g_2 - 16h)^2(g_2 + 24h \mp \sqrt{g_2^2 - 192h^2})^4}{2h^9(8h \mp \sqrt{g_2^2 - 192h^2})}\right]^{\frac{1}{5}} \tag{3.28}$$

where we have used the relation $g_1 = -16h$ in the solutions for σ and G to simplify the expressions. An example of the corresponding flow solutions from the UV $N = (1, 0)$ SO(4) SCFT to this critical point, with $g_2 = -2g_1$ and $h = 1$, is given in figure 2.

In all of the above solutions, it is not possible to have a flow from the SO(3) AdS_7 critical point (2.16). To find this type of flows, we look for AdS_5 fixed points with $\Phi_3 = 0$



(a) Φ_2 solution. (b) σ solution. (c) G solution.

Figure 2. RG flows from SO(4) $N = (1, 0)$ SCFT in six dimensions to four-dimensional $N = 1$ SCFT with $\mathrm{SO}(2)_{\mathrm{diag}}$ symmetry for $g_1 \neq g_2$.

but $\Phi_1 \neq 0$ and $\Phi_2 \neq 0$. In this case, the $AdS_5 \times H^2$ solution is given by

$$\begin{aligned}
\Phi_2 &= \frac{1}{2}\ln\left[\frac{g_2 - g_1}{g_2 + g_1}\right], \qquad \Phi_1 = \pm\Phi_2, \\
\sigma &= \frac{1}{5}\ln\left[\frac{g_1^2 g_2^2}{144h^2(g_2^2 - g_1^2)}\right], \qquad G = \frac{1}{2}\ln\left[\frac{2^{\frac{4}{5}}(3^{\frac{3}{5}})a(g_2^2 - 256h^2)^{\frac{4}{5}}}{g_2^{\frac{8}{5}} g_1}\right], \\
L_{\mathrm{AdS}_5} &= \frac{3^{\frac{4}{5}}(g_2^2 - 256h^2)^{\frac{2}{5}}}{2^{\frac{18}{5}} g_2^{\frac{4}{5}} h}\,. \qquad (3.29)
\end{aligned}$$

Note that at the values of Φ_1 and Φ_2 are the same as the SO(3) AdS_7 point. In equation (2.16), we have

$$\Phi_1 = \Phi_2 = \frac{1}{2}\ln\left[\frac{g_2 - g_1}{g_2 + g_1}\right] \equiv \Phi_0\,. \qquad (3.30)$$

Actually, there are two equivalent values of Φ_1 namely either $\Phi_1 = \Phi_0$ or $\Phi_1 = -\Phi_0$. The two choices are equivalent in the sense that they give rise to the same value of the cosmological constant and the same scalar masses. The difference between the two is the generators of SO(3) under which the SO(3) singlet scalar ϕ in (2.16) is invariant. For $\Phi_1 = \Phi_0$, we have $\Phi_1 = \Phi_2$ which is invariant under the SO(3) generated by $J^{(1)}_{ij} + J^{(2)}_{ij}$. The alternative value of $\Phi_1 = -\Phi_0$ gives $\Phi_1 = -\Phi_2$ which is invariant under SO(3) generators $J^{(1)}_{12} + J^{(2)}_{12}$, $J^{(1)}_{13} - J^{(2)}_{13}$ and $J^{(1)}_{23} - J^{(2)}_{23}$. This difference does not affect the result discussed here since, in both cases, the residual $\mathrm{SO}(2)_{\mathrm{diag}}$ is still generated by $J^{(1)}_{12} + J^{(2)}_{12}$.

The flow from SO(3) $N = (1, 0)$ SCFT would be driven only by the dilaton σ which has different values at the SO(3) AdS_7 and the AdS_5 fixed points. This is expected since at SO(3) AdS_7 critical point only σ corresponds to relevant operators, see the scalar masses in [17].

We now consider RG flows from $N = (1, 0)$ SCFTs in six dimensions to four-dimensional SCFTs identified with the critical point (3.29). In order to give some explicit examples, we choose particular values of the two couplings g_1 and g_2. In the following solutions, we will set $g_2 = -2g_1$ and $h = 1$. With these, the IR $AdS_5 \times H^2$ is given by

$$\Phi_1 = \Phi_2 = \frac{1}{2}\ln 3 \approx 0.5493, \qquad \sigma = \frac{1}{5}\ln\left[\frac{64}{27}\right] \approx 0.1726,$$
$$G = \frac{1}{10}\ln\left[\frac{3^7}{2^{44}}\right] \approx -2.2808\,. \tag{3.31}$$

The SO(4) UV point (2.15) is given by

$$\Phi_1 = \Phi_2 = \sigma = 0 \tag{3.32}$$

while the SO(3) AdS_7 point (2.16) occurs at

$$\sigma = \frac{1}{5}\ln\frac{4}{3} \approx 0.0575, \qquad \Phi_2 = \Phi_1 = \frac{1}{2}\ln 3 \approx 0.5493\,. \tag{3.33}$$

We have chosen $\Phi_1 = \Phi_2$ at the IR fixed points for definiteness.

There exist an RG flow from the SO(4) $N = (1, 0)$ SCFT in the UV to the $N = 1$ four-dimensional SCFT in the IR as shown in figure 3. With a particular boundary condition, we can find an RG flow from the SO(4) AdS_7 to the SO(3) AdS_7 critical points and then to the AdS_5 critical point as shown in figure 4. This solution is similar to the flow from SO(6) AdS_5 to Khavaev-Pilch-Warner (KPW) AdS_5 critical point and continue to a two-dimensional $N = (2, 0)$ SCFT in [31].

3.2.2 Flows with SO(2)$_R$ symmetry

We then move on and briefly look at the SO(2)$_R$ symmetry. There are three singlet scalars from the SO(3, 3)/SO(3) × SO(3) coset. These scalars will be denoted by Φ_1, Φ_2 and Φ_3 corresponding to the non-compact generators Y_{31}, Y_{32} and Y_{33}, respectively.

In this case, the gauge field corresponding the SO(2)$_R$ generator is given by

$$A^3 = a\cos\theta d\phi\,. \tag{3.34}$$

By using the same procedure, we find that, in order to have a fixed point, all of the Φ_i's must vanish, and only $AdS_5 \times H^2$ solutions exist. The solution again preserves eight supercharges corresponding to $N = 1$ superconformal symmetry in four dimensions. The fixed point solution is given by

$$\sigma = \frac{2}{5}\ln\frac{4}{3}, \qquad G = \frac{1}{5}\ln\frac{4}{3} - \frac{1}{2}\ln\frac{g_1}{3a}, \qquad F = \frac{16h}{9^{\frac{2}{5}}}r \tag{3.35}$$

There exist RG flows from the SO(4) $N = (1, 0)$ SCFT to these four-dimensional SCFTs. The BPS equations describing theses flows are given by

$$\sigma' = \frac{2}{5}e^{-\frac{\sigma}{2}}\left(ae^{\sigma-2G} - g_1 - 16he^{\frac{5\sigma}{2}}\right), \tag{3.36}$$
$$G' = \frac{1}{5}e^{-\frac{\sigma}{2}}\left(4he^{\frac{5\sigma}{2}} - g_1 - 4ae^{\sigma-2G}\right), \tag{3.37}$$
$$F' = \frac{1}{5}e^{-\frac{\sigma}{2}}\left(4he^{\frac{5\sigma}{2}} - g_1 + ae^{\sigma-2G}\right). \tag{3.38}$$

Examples of the solutions with some values of the parameter a are shown in figure 5. This critical point is also a solution of pure $N = 2$ gauged supergravity studied in [21].

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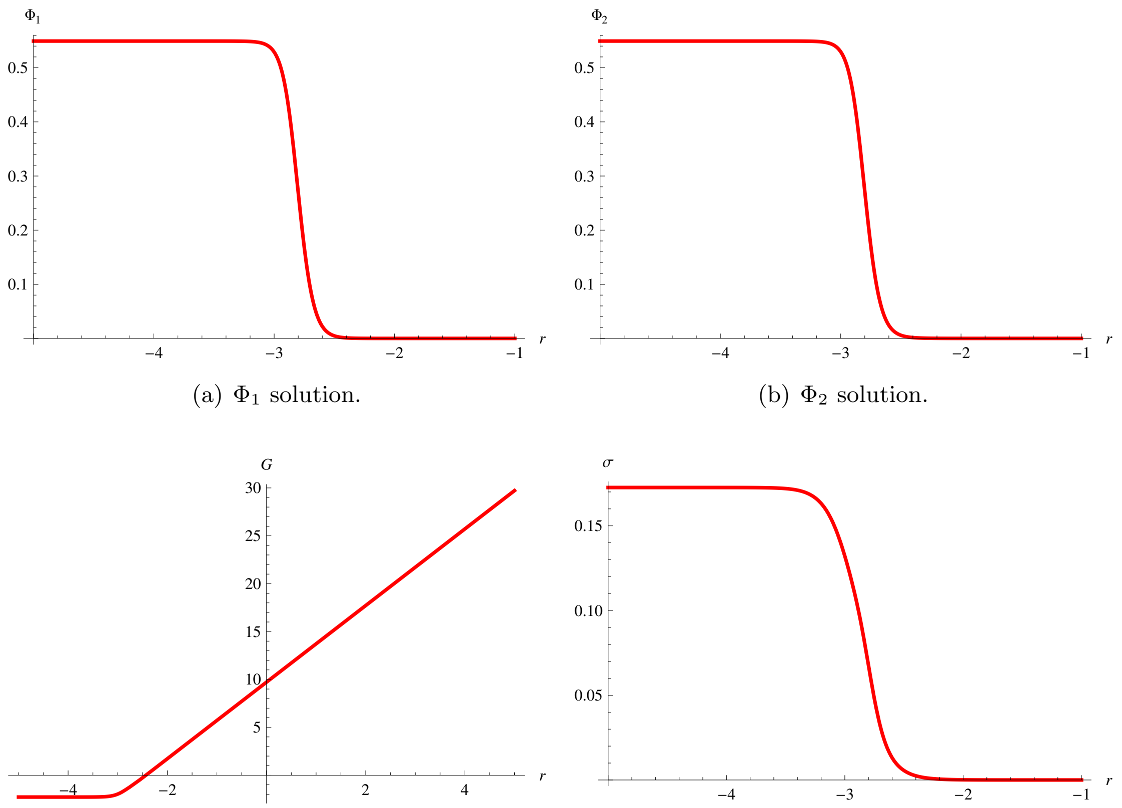

(a) Φ_1 solution.

(b) Φ_2 solution.

(c) G solution.

(d) σ solution.

Figure 3. An RG flow from SO(4) $N = (1, 0)$ SCFT in six dimensions to four-dimensional $N = 1$ SCFT with $\mathrm{SO}(2)_{\mathrm{diag}}$ symmetry.

4 Flows to $N = 1$ SCFTs in three dimensions

In this section, we look for AdS_4 vacua of the form $AdS_4 \times S^3$ or $AdS_4 \times H^3$ with S^3 and H^3 being a three-sphere and a three-dimensional hyperbolic space, respectively. These solutions will correspond to some SCFTs in three dimensions. In order to identify these AdS_4 vacua with the IR fixed points of the six-dimensional SCFTs corresponding to both of the AdS_7 vacua given in (2.15) and (2.16), we consider the scalars which are singlets under $\mathrm{SO}(3)_{\mathrm{diag}}$ subgroup of the full SO(4) gauge group. The relevant scalar from the $\mathrm{SO}(3,3)/\mathrm{SO}(3) \times \mathrm{SO}(3)$ coset is the one corresponding to the generator (2.12) with the coset representative given in (2.13).

In the S^3 case, we will take the metric ansatz to be

$$ds_7^2 = e^{2F} dx_{1,2}^2 + dr^2 + e^{2G} \left[d\psi^2 + \sin^2\psi (d\theta^2 + \sin^2\theta d\phi^2) \right] . \tag{4.1}$$

From the above metric, we find the spin connections

$$\begin{aligned}
&\omega^{\hat{\mu}}{}_{\hat{r}} = F' e^{\hat{\mu}}, && \omega^{\hat{\psi}}{}_{\hat{r}} = G' e^{\hat{\psi}}, && \omega^{\hat{\theta}}{}_{\hat{r}} = G' e^{\hat{\theta}}, \\
&\omega^{\hat{\phi}}{}_{\hat{r}} = G' e^{\hat{\phi}}, && \omega^{\hat{\phi}}{}_{\hat{\theta}} = e^{-G} \frac{\cot\theta}{\sin\psi} e^{\hat{\phi}}, && \\
&\omega^{\hat{\phi}}{}_{\hat{\psi}} = e^{-G} \cot\psi e^{\hat{\phi}}, && \omega^{\hat{\theta}}{}_{\hat{\psi}} = e^{-G} \cot\psi e^{\hat{\theta}} &&
\end{aligned} \tag{4.2}$$

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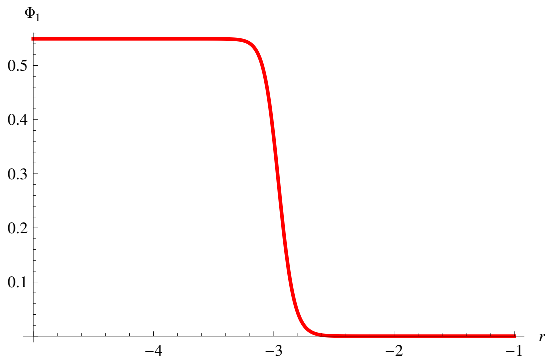
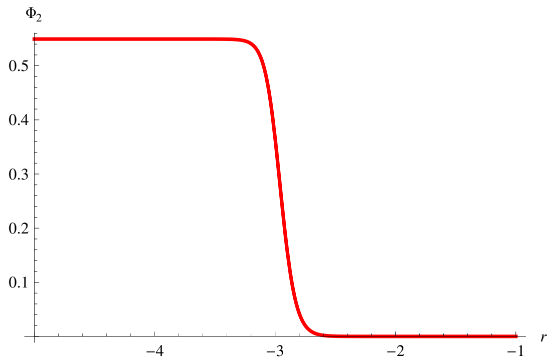
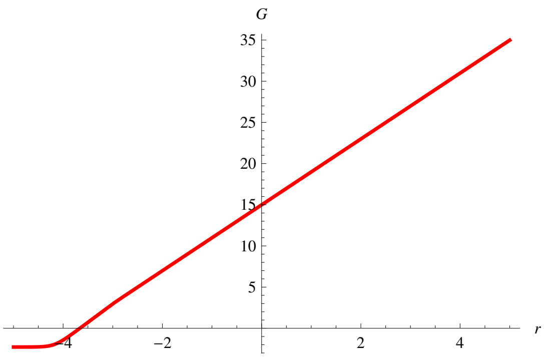


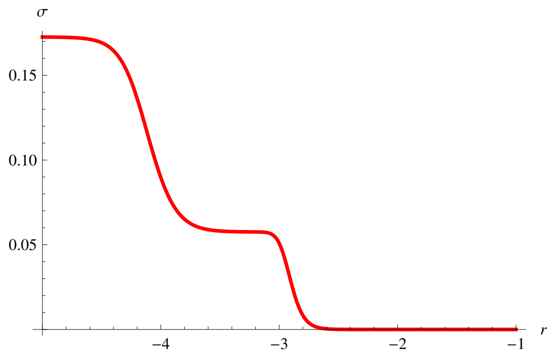
(a) Φ_1 solution.



(b) Φ_2 solution.

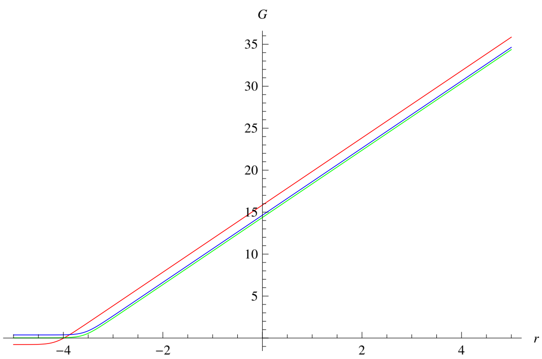


(c) G solution.

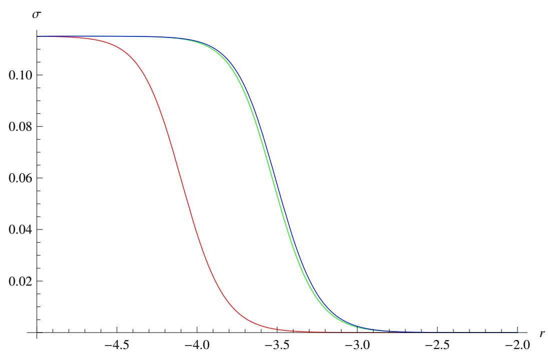


(d) σ solution.

Figure 4. An RG flow from SO(4) $N = (1, 0)$ SCFT to SO(3) $N = (1, 0)$ SCFT in six dimensions and then to $N = 1$ four-dimensional SCFT with $\mathrm{SO}(2)_{\mathrm{diag}}$ symmetry.



(a) Φ solution.



(b) σ solution.

Figure 5. RG flows from SO(4) $N = (1, 0)$ SCFT in six dimensions to four-dimensional $N = 1$ SCFT with $\mathrm{SO}(2)_R$ symmetry for $a = 1, 5, 10$ (red, green, blue).

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which accordingly suggest to turn on the following $\mathrm{SO}(3)_{\mathrm{diag}}$ gauge fields

$$\begin{aligned} A^1 &= \frac{g_2}{g_1}A^4 = a\cos\psi d\theta, \\ A^2 &= \frac{g_2}{g_1}A^5 = a\cos\theta d\phi, \\ A^3 &= \frac{g_2}{g_1}A^6 = a\cos\psi\sin\theta d\phi\,. \end{aligned} \tag{4.3}$$

Note that at the beginning, the parameter a of each gauge field needs not be equal. However, the twist condition

$$ag_1 = 1 \tag{4.4}$$

requires that all of the parameters in front of A^i must be equal. The corresponding field strengths are, after using (4.4),

$$\begin{aligned} F^1 &= -ae^{-2G}e^{\hat{\psi}}\wedge e^{\hat{\theta}}, \\ F^2 &= -ae^{-2G}e^{\hat{\theta}}\wedge e^{\hat{\phi}}, \\ F^3 &= -ae^{-2G}e^{\hat{\psi}}\wedge e^{\hat{\phi}}\,. \end{aligned} \tag{4.5}$$

To set up the BPS equations, we impose the projection conditions

$$\gamma_r\epsilon = \epsilon, \qquad i\sigma^1\gamma_{\hat{\theta}\hat{\psi}}\epsilon = \epsilon, \qquad i\sigma^2\gamma_{\hat{\phi}\hat{\theta}}\epsilon = \epsilon, \qquad i\sigma^3\gamma_{\hat{\phi}\hat{\psi}}\epsilon = \epsilon\,. \tag{4.6}$$

For the H^3 case, we take the metric to be

$$ds_7^2 = e^{2F}dx_{1,2}^2 + dr^2 + \frac{e^{2G}}{y^2}(dx^2+dy^2+dz^2) \tag{4.7}$$

with the spin connections given by

$$\begin{aligned} &\omega^{\hat{z}}{}_{\hat{r}} = G'e^{\hat{z}}, && \omega^{\hat{y}}{}_{\hat{r}} = G'e^{\hat{y}}, && \omega^{\hat{x}}{}_{\hat{r}} = G'e^{\hat{x}}, \\ &\omega^{\hat{x}}{}_{\hat{y}} = -\,e^{-G}e^{\hat{x}}, && \omega^{\hat{z}}{}_{\hat{y}} = -\,e^{-G}e^{\hat{z}}, && \omega^{\hat{\mu}}{}_{\hat{r}} = F'e^{\hat{\mu}}\,. \end{aligned} \tag{4.8}$$

We then turn on the following gauge fields, to cancel the above spin connections on H^3,

$$A^1 = \frac{a}{y}dx, \qquad A^2 = 0, \qquad A^3 = \frac{a}{y}dz \tag{4.9}$$

with $A^{i+3} = \frac{g_1}{g_2}A^i$, $i = 1,2,3$. These gauge fields then become $\mathrm{SO}(3)_{\mathrm{diag}}$ gauge fields.

We will also impose the projection conditions

$$\gamma_r\epsilon = \epsilon, \qquad i\sigma^1\gamma_{\hat{x}\hat{y}}\epsilon = -\epsilon, \qquad i\sigma^2\gamma_{\hat{x}\hat{z}}\epsilon = -\epsilon, \qquad i\sigma^3\gamma_{\hat{z}\hat{y}}\epsilon = -\epsilon\,. \tag{4.10}$$

The twist condition is still given by (4.4).

In both cases, the last projector in (4.6) and (4.10) is not independent from the second and the third ones, so the fixed point solution will preserve four supercharges corresponding to $N = 1$ superconformal symmetry in three dimensions.

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With all of the above conditions, we find the following BPS equations, for the H^3 case,

$$\phi' = -\frac{1}{8g_2}e^{-\frac{\sigma}{2}-3\phi-2G}\left[e^{2G}(e^{4\phi}-1)g_2-4ae^{\sigma+2\phi}\right]\left[g_1-g_2+(g_1+g_2)e^{2\phi}\right], \quad (4.11)$$

$$\begin{aligned}\sigma' = &-\frac{1}{20}e^{-\frac{\sigma}{2}-3\phi-2G}\left[\frac{12a}{g_2}e^{\sigma+2\phi}\left[(e^{2\phi}-1)g_1+(1+e^{2\phi})g_2\right]\right.\\ &\left.+e^{2G}g_2\left[g_2(e^{2\phi}-1)^3+g_1(e^{2\phi}+1)^3+128he^{\frac{5\sigma}{2}+3\phi}\right]\right], \quad (4.12)\end{aligned}$$

$$\begin{aligned}G' = &\frac{1}{40}e^{-\frac{\sigma}{2}-3\phi-2G}\left[\frac{28a}{g_2}e^{\sigma+2\phi}\left[(e^{2\phi}-1)g_1+(1+e^{2\phi})g_2\right]\right.\\ &\left.-e^{2G}g_2\left[g_2(e^{2\phi}-1)^3+g_1(e^{2\phi}+1)^3-32he^{\frac{5\sigma}{2}+3\phi}\right]\right], \quad (4.13)\end{aligned}$$

$$\begin{aligned}F' = &-\frac{1}{40}e^{-\frac{\sigma}{2}-3\phi-2G}\left[\frac{12a}{g_2}e^{\sigma+2\phi}\left[(e^{2\phi}-1)g_1+(1+e^{2\phi})g_2\right]\right.\\ &\left.+e^{2G}g_2\left[g_2(e^{2\phi}-1)^3+g_1(e^{2\phi}+1)^3-32he^{\frac{5\sigma}{2}+3\phi}\right]\right]. \quad (4.14)\end{aligned}$$

The corresponding equations for the S^3 case are similar with a replaced by $-a$.

We now look for a fixed point solution at which $G' = \phi' = \sigma' = 0$ and $F' =$ constant. For $g_2 = g_1$, only $AdS_4 \times H^3$ solutions exist and are given by

$$\begin{aligned}\phi &= \frac{1}{4}\ln 2, & \sigma &= \frac{3}{10}\ln 2,\\ G &= \frac{1}{10}\ln\left[\frac{64a^5}{g_1^3h^2}\right], & L_{\text{AdS}_5} &= \frac{1}{2^{\frac{13}{5}}h}. \end{aligned} \quad (4.15)$$

This solution can be uplifted to eleven dimensions using the ansatz of [30].

When $g_2 \neq g_1$, we also find $AdS_4 \times H^3$ solutions

$$\begin{aligned}\phi &= \frac{1}{2}\ln\left[\frac{g_2-g_1}{g_2+g_1}\right], \qquad \sigma = \frac{1}{5}\ln\left[\frac{g_1^2g_2^2}{100h^2(g_2^2-g_1^2)}\right],\\ G &= \frac{1}{2}\ln\left[\frac{5a(g_2^2-g_1^2)}{g_1g_2^2}\right]+\frac{1}{5}\ln\left[\frac{-g_1g_2}{10h\sqrt{g_2^2-g_1^2}}\right],\\ L_{\text{AdS}_4} &= \frac{1}{2^{\frac{6}{5}}h}\left[\frac{25h^2(g_2^2-g_1^2)}{g_1^2g_2^2}\right]^{\frac{2}{5}}. \end{aligned} \quad (4.16)$$

This solution can be connected to both AdS_7 critical points in (2.15) and (2.16) by some RG flows.

In this $g_2 \neq g_1$ case, there can be both $AdS_4 \times S^3$ and $AdS_4 \times H^3$ solutions. The solution however takes a more complicated form depending on the values of g_1 and g_2. The $AdS_4 \times H^3$ and $AdS_4 \times S^3$ solutions are given respectively by

$$G = \frac{1}{2}\ln\left[\frac{4ae^{\sigma+2\phi_0}}{g_2(e^{4\phi_0}-1)}\right], \quad (4.17)$$

$$\sigma = \frac{2}{5}\ln\left[\frac{e^{-3\phi_0}\left[g_2(1-e^{6\phi_0})-g_1(e^{6\phi_0}+1)\right]}{32h}\right] \quad (4.18)$$

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and

$$G = \frac{1}{2}\ln\left[\frac{4ae^{\sigma+2\phi_0}}{g_2(1-e^{4\phi_0})}\right], \tag{4.19}$$

$$\sigma = \frac{2}{5}\ln\left[\frac{e^{-3\phi_0}\left[g_2(1-e^{6\phi_0})-g_1(e^{6\phi_0}+1)\right]}{32h}\right]. \tag{4.20}$$

In both cases, the scalar ϕ_0 is a solution to the equation

$$g_1(1-2e^{2\phi_0}-2e^{4\phi_0}+e^{6\phi_0})-g_2(1+2e^{2\phi_0}-2e^{4\phi_0}-e^{6\phi_0})=0\,. \tag{4.21}$$

The explicit form of ϕ_0 can be obtained but will not be given here due to its complexity. There are many possible solutions for ϕ_0 depending on the values of g_1, g_2 and a. An example of $AdS_4\times S^3$ solutions is, for $g_2=\frac{1}{2}g_1$, given by

$$\phi=-0.9158,\qquad \sigma=0.5493,\qquad G=0.4116+\frac{1}{2}\ln\left[\frac{a}{g_1}\right]. \tag{4.22}$$

One of the $AdS_4\times H^3$ solutions is, for $g_2=\frac{1}{2}g_1$, given by

$$\phi=0.2706,\qquad \sigma=0.2351,\qquad G=1.0936+\frac{1}{2}\ln\left[\frac{a}{g_1}\right]. \tag{4.23}$$

Numerical solutions for RG flows from the UV $N=(1,0)$ SCFTs in six dimensions to these three-dimensional $N=1$ SCFTs can be found in the same way as those given in the previous section. And, with suitable boundary conditions, the flow from SO(4) AdS_7 point to the SO(3) AdS_7 point and then to $AdS_4\times S^3$ or $AdS_4\times H^3$ in the case of $g_2\neq g_1$ should be similarly obtained. We will however not give these solutions here.

5 Uplifting the solutions to eleven dimensions

In this section, we will uplift some of the AdS_5 and AdS_4 solutions found in the previous sections to eleven dimensions using a reduction ansatz given in [30]. Only solutions with equal SU(2) gauge couplings, $g_2=g_1$, can be uplifted by this ansatz. Therefore, we will consider only this case in the remaining of this section.

The reduction ansatz given in [30] is naturally written in terms of $SL(4,\mathbb{R})/SO(4)$ scalar manifold rather than the $SO(3,3)/SO(3)\times SO(3)$ we have considered throughout the previous sections. It is then useful to change the parametrization of scalars from the $SO(3,3)/SO(3)\times SO(3)$ to $SL(4,\mathbb{R})/SO(4)$ cosets. For convenience, we will repeat the supersymmetry transformations of fermions with the three-form field and fermions vanishing

$$
\begin{aligned}
\delta\psi_\mu &= D_\mu\epsilon - \frac{1}{20}gX\tilde{T}\gamma_\mu\epsilon - \frac{1}{20}X^{-4}\gamma_\mu\epsilon \\
&\quad + \frac{1}{40\sqrt{2}}X^{-1}\left(\gamma_\mu{}^{\nu\rho} - 8\delta^\nu_\mu\gamma^\rho\right)\Gamma_{RS}F^{RS}_{\nu\rho}\epsilon, \qquad (5.1)\\
\delta\chi &= -X^{-1}\gamma^\mu\partial_\mu X\epsilon - \frac{2}{5}gX^{-4}\epsilon + \frac{1}{10}gX\tilde{T} - \frac{1}{20\sqrt{2}}X^{-1}\gamma^{\mu\nu}\Gamma_{RS}F^{RS}_{\mu\nu}\epsilon, \qquad (5.2)\\
\delta\hat{\lambda}_R &= -\frac{1}{2}\gamma^\mu\Gamma^S P_{\mu RS}\epsilon - \frac{1}{8}gX\tilde{T}\Gamma_R\epsilon + \frac{1}{2}gX\tilde{T}_{RS}\Gamma^S\epsilon \\
&\quad - \frac{1}{8\sqrt{2}}X^{-1}\gamma^{\mu\nu}\Gamma_S\left(F^{RS}_{\mu\nu} + \frac{1}{2}\epsilon_{RSTU}F^{TU}_{\mu\nu}\right)\epsilon \qquad (5.3)
\end{aligned}
$$

where

$$
\begin{aligned}
P_{RS} &= (\mathcal{V}^{-1})^\alpha_{(R}\left(\delta^\beta_\alpha d + gA_{(1)\alpha}{}^\beta\right)\mathcal{V}_\beta{}^T\delta_{S)T}, \\
Q_{RS} &= (\mathcal{V}^{-1})^\alpha_{[R}\left(\delta^\beta_\alpha d + gA_{(1)\alpha}{}^\beta\right)\mathcal{V}_\beta{}^T\delta_{S]T}, \\
D\epsilon &= d\epsilon + \frac{1}{4}\omega_{ab}\gamma^{ab} + \frac{1}{4}Q_{RS}\Gamma^{RS} \\
\tilde{T}_{RS} &= (\mathcal{V}^{-1})_R^{\ \alpha}(\mathcal{V}^{-1})_S^{\ \beta}\delta_{\alpha\beta}, \qquad \tilde{T} = \tilde{T}_{RS}\delta^{RS}\,. \qquad (5.4)
\end{aligned}
$$

In the above equations, $\mathcal{V}^R_{\ \alpha}$ denotes the $SL(4,\mathbb{R})/SO(4)$ coset representative.

For the explicit form of the eleven-dimensional metric and the four-form field including the notations used in the above equations, we refer the reader to [30]. We now consider the AdS_5 and AdS_4 solutions separately.

5.1 Uplifting the AdS_5 solutions

For AdS_5 solutions, the seven-dimensional metric is given by (3.1) and (3.4). We will restrict ourselves to AdS_5 fixed points with $SO(2)\times SO(2)$ symmetry. The non-zero gauge fields are $A^{\alpha\beta} = (A^{12}, A^{34})$ whose explicit form is given by

$$
A^{12} = a\cos\theta d\phi \qquad \text{and} \qquad A^{34} = b\cos\theta d\phi\,. \qquad (5.5)
$$

The $U(1)\times U(1)$ singlet scalar from $SL(4,\mathbb{R})/SO(4)$ coset is parametrized by the coset representative

$$
\mathcal{V}^R_{\ \alpha} = \text{diag}(e^{\frac{\Phi}{2}}, e^{\frac{\Phi}{2}}, e^{-\frac{\Phi}{2}}, e^{-\frac{\Phi}{2}}) \qquad (5.6)
$$

from which the $\tilde{T}_{RS} = \text{diag}(e^{-\Phi}, e^{-\Phi}, e^{\Phi}, e^{\Phi})$ follows. Note that the parameter a and b here are different from those in section 3 since the gauge fields A^i and A^r correspond respectively to the anti-self-dual and self-dual parts of the $SO(4)$ gauge fields $A^{\alpha\beta}$.

Using the above supersymmetry transformations and imposing the projection conditions $\gamma_{\hat{r}}\epsilon = \epsilon$ and $\gamma^{\hat{\theta}\hat{\phi}}\Gamma_{12}\epsilon = \epsilon$, we obtain the BPS equations

$$X^{-1}X' - \frac{2}{5}gX^{-4} + \frac{1}{5}gX(e^{\Phi} + e^{-\Phi}) + \frac{1}{5\sqrt{2}}X^{-1}e^{-2G}(ae^{\Phi} - be^{-\Phi}) = 0, \tag{5.7}$$

$$-\Phi' - gX(e^{\Phi} - e^{-\Phi}) + \frac{1}{\sqrt{2}}X^{-1}e^{-2G}(ae^{\Phi} + be^{-\Phi}) = 0, \tag{5.8}$$

$$F' - \frac{1}{5}gX(e^{\Phi} + e^{-\Phi}) - \frac{1}{10}gX^{-4} - \frac{1}{10\sqrt{2}}X^{-1}e^{-2G}(ae^{\Phi} - be^{-\Phi}) = 0, \tag{5.9}$$

$$G' - \frac{1}{5}gX(e^{\Phi} + e^{-\Phi}) - \frac{1}{10}gX^{-4} + \frac{4}{5\sqrt{2}}X^{-1}e^{-2G}(ae^{\Phi} - be^{-\Phi}) = 0\,. \tag{5.10}$$

In the above equations, we have used $\Gamma_{34}\epsilon = -\Gamma_{12}\epsilon$ which follows from the condition $\Gamma_{1234}\epsilon = \epsilon$. The latter is part of the truncation from the maximal SO(5) gauged supergravity to the half-maximal SO(4) gauged supergravity studied in [30]. We have also used the twist condition given by

$$g(a - b) + 1 = 0\,. \tag{5.11}$$

which comes from the requirement that the gauge connection cancels the spin connection. Note that this condition differs from (3.8) since the gauge fields are different. In condition (3.8), the $SU(2)_R$ gauge fields are given by the A^I with $I = 1, 2, 3$, and the $SO(2)_R \subset SU(2)_R$ gauge field has been chosen to be A^3. On the other hand, the condition (5.11) involves $A^{12} - A^{34}$ corresponding to the $SO(2)_R$ subgroup of the $SU(2)_R$ R-Symmetry for which the corresponding gauge fields are identified with the anti-self-dual part of the SO(4) gauge fields $A^{\alpha\beta}$ in the convention of [30].

For large r, the solution should approach $X = 1$, $\Phi = 0$ and $F \sim G \sim r$ giving AdS_7 background with SO(4) symmetry. This corresponds to the UV $N = (1, 0)$ SCFT in six dimensions. In the IR with the boundary condition $F \sim r$ and $G, \Phi, \sigma \sim$ constant, there is a class of solutions given by

$$\begin{aligned}
\Phi &= \frac{1}{2}\ln\left[\frac{a + b \pm \sqrt{a^2 + ab + b^2}}{a}\right], \\
G &= \frac{1}{2}\ln\left[\frac{a\left(a + 2b \pm \sqrt{a^2 + ab + b^2}\right)}{\sqrt{2}gX^2\left(b \pm \sqrt{a^2 + ab + b^2}\right)}\right], \\
X^{10} &= \frac{a\left(a + 2b \pm \sqrt{a^2 + ab + b^2}\right)^2}{4(a + b)^2\left(a + b \pm \sqrt{a^2 + ab + b^2}\right)}, \\
L_{\text{AdS}_5} &= \frac{a2^{\frac{1}{5}}}{g}\left[\frac{a + 2b \pm \sqrt{a^2 + ab + b^2}}{(a + b)^2\left(a + b \pm \sqrt{a^2 + ab + b^2}\right)}\right]^{\frac{2}{5}}.
\end{aligned} \tag{5.12}$$

This gives $AdS_5 \times S^2$ background preserving $U(1) \times U(1)$ symmetry and eight supercharges since only the projector $\gamma^{\hat{\theta}\hat{\phi}}\Gamma_{12}\epsilon = \epsilon$ is needed at the fixed point. Therefore, this solution corresponds to $N = 1$ SCFT in four dimensions. This solution is the same as in [22] with the identification $(m_1, m_2) \to (-b, a)$ up to some field redefinitions. So, we conclude that the $AdS_5 \times \Sigma_2$ solutions found in [22] is a solution of the $N = 2$ SO(4) gauged supergravity.

For the H^2 case, the above analysis can be repeated in a similar manner. The resulting BPS equations are, as expected, given by (5.7), (5.8), (5.9) and (5.10) with (a, b) replaced by $(-a, -b)$. It can also be verified that for both $AdS_5 \times S^2$ and $AdS_5 \times H^2$ solutions given in (5.12), solutions with the positive sign are valid for $g > 0$ and $a > 0$ while solutions with the negative sign are valid for $g < 0$ and $a < 0$.

It should also be noted that we can truncate the above BPS equations to those of $\mathrm{SO}(2)_R$ symmetry, generated by the anti-selfdual gauge field $A^{12} - A^{34}$, by setting $b = -a$. Since the twist condition in this case becomes $2ga = -1$ which implies that $ga < 0$, only the $AdS_5 \times H^2$ exists. This precisely agrees with the result of section 3.2.2. The corresponding solution is given by

$$X = \left(\frac{3}{4}\right)^{\frac{1}{5}}, \qquad G = -\frac{1}{2}\ln\left[-\frac{g}{2^{\frac{3}{10}}3^{\frac{3}{5}}a}\right], \qquad L_{\mathrm{AdS}_5} = \frac{3^{\frac{4}{5}}}{2^{\frac{3}{5}}g}\,. \tag{5.13}$$

The $AdS_5 \times H^2$ with $\mathrm{SO}(2)_{\mathrm{diag}}$ symmetry found in section 3.2.1 for $g_2 = g_1$ can also be uplifted using the formulae given here by truncating the $\mathrm{SO}(2) \times \mathrm{SO}(2)$ symmetry to $\mathrm{SO}(2)_{\mathrm{diag}}$ as remarked previously in section 3.2.1. The $\mathrm{SO}(2)_{\mathrm{diag}}$ corresponds to the gauge field A^{12} since the A^3 and A^6, in section 3.2, are related to the anti-self-dual, $\frac{1}{2}(A^{12} - A^{34})$, and self-dual, $\frac{1}{2}(A^{12} + A^{34})$, fields, respectively. So, the $\mathrm{SO}(2)_{\mathrm{diag}}$ gauge field is given by A^{12}. As in section 3.2, only solutions with the upper sign in the solution (5.12) and $AdS_5 \times H^2$ are possible. The result is given by

$$\Phi = \frac{1}{2}\ln 2, \qquad X^{10} = \frac{1}{8}, \qquad G = \frac{1}{2}\ln\left[-\frac{a2^{\frac{11}{10}}}{g}\right]. \tag{5.14}$$

This is consistent with the twist condition (5.11) which, for $b = 0$, becomes $ga = -1$.

We now move to the uplift of these AdS_5 solutions. Both $AdS_5 \times S^2$ and $AdS_5 \times H^2$ solutions can be uplifted in a similar way. For definiteness, we will only give the uplifted $AdS_5 \times S^2$ solution. Using the reduction ansatz given in [30], we find the eleven-dimensional metric

$$\begin{aligned}
ds_{11}^2 = {}& \Delta^{\frac{1}{3}}\left[e^{\frac{2r}{L_{\mathrm{AdS}_5}}}dx_{1,3}^2 + dr^2 + e^{2G_0}(d\theta^2 + \sin^2\theta d\phi^2)\right] \\
& + \frac{2}{g}\Delta^{-\frac{2}{3}}X_0^3\left[X_0\cos^2\xi + X_0^{-4}\sin^2\xi\left(e^{-\Phi_0}\sin^2\psi + e^{\Phi_0}\cos^2\psi\right)\right]d\xi^2 \\
& + \frac{1}{2g^2}\Delta^{-\frac{2}{3}}X_0^{-1}\cos^2\xi\left[e^{-\Phi_0}\left[\cos^2\psi d\phi^2 + \sin^2\psi(d\alpha - ag\cos\theta d\phi)^2\right]\right. \\
& \qquad\qquad \left. + e^{\Phi_0}\left[\cos^2\psi d\phi^2 + \sin^2\psi(d\beta - bg\cos\theta d\phi)^2\right]\right] \\
& - \frac{1}{2g^2}\Delta^{-\frac{2}{3}}X_0^{-1}\sin\xi\sin(2\psi)\left(e^{-\Phi_0} - e^{\Phi_0}\right)d\xi d\psi
\end{aligned} \tag{5.15}$$

where we have used the coordinates μ^α, satisfying $\mu^\alpha\mu^\alpha = 1$, as follow

$$\begin{aligned}
\mu^1 &= \sin\psi\cos\alpha, \qquad \mu^2 = \sin\psi\sin\alpha, \\
\mu^3 &= \cos\psi\cos\beta, \qquad \mu^4 = \cos\psi\sin\beta\,.
\end{aligned} \tag{5.16}$$

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The quantities X_0, Φ_0 and G_0 are the values of the corresponding fields at the fixed point (5.12). The quantity Δ is defined by

$$\Delta = X^{-4}\sin^2\xi + X\tilde{T}_\alpha\mu^\alpha\mu^\beta\cos^2\xi \tag{5.17}$$

which, in the present case, gives

$$\Delta = X_0\cos^2\xi\left(e^{-\Phi_0}\sin^2\psi + e^{\Phi_0}\cos^2\psi\right) + X_0^{-4}\sin^2\xi\,. \tag{5.18}$$

The 4-form field, at the fixed point, is given by

$$\begin{aligned}\hat{F}_{(4)} =\ & \frac{1}{g^3}U\Delta^{-2}\cos^3\xi d\xi\wedge\epsilon_{(3)} + \frac{a}{g^2}\cos\theta\cos\xi\left[\sin\xi\cos\xi\sin\psi\cos\psi X_0^{-4}d\psi\right.\\ & \qquad\left.\cos^2\psi\left(X_0^{-4}\sin^2\xi + e^{\Phi_0}X_0^2\cos^2\xi\right)d\xi\right]\wedge d\beta\wedge d\theta\wedge d\phi\\ & -\frac{b}{g^2}\sin\theta\cos\xi\left[\sin\xi\cos\xi\sin\psi\cos\psi X_0^{-4}d\psi\right.\\ & \qquad\left.-\left(X_0^{-4}\sin^2\xi + X_0^2\cos^2\xi e^{-\Phi_0}\right)\sin^2\psi d\xi\right]\wedge d\alpha\wedge d\theta\wedge d\phi\end{aligned} \tag{5.19}$$

where

$$\begin{aligned}U =\ & \sin^2\xi\left[X_0^{-8} - 2X_0^{-3}\left(e^{\Phi_0} + e^{-\Phi_0}\right)\right]\\ & -\cos^2\xi\left[2X_0^2 + X_0^{-3}\left(e^{-\Phi_0}\sin^2\psi + e^{\Phi_0}\cos^2\psi\right)\right].\end{aligned} \tag{5.20}$$

The uplifted solutions for some particular values of a and b have already been given in [23].

5.2 Uplifting the AdS_4 solutions

We now consider the embedding of the $AdS_4\times H^3$ solution given in (4.15) in eleven dimensions. The $\mathrm{SL}(4,\mathbb{R})/\mathrm{SO}(4)$ coset representative, invariant under $\mathrm{SO}(3)_{\mathrm{diag}}$, is given by

$$\mathcal{V}_\alpha^R = (\delta_{ab}e^{\frac{\phi}{2}}, e^{-\frac{3\phi}{2}}) \tag{5.21}$$

which gives $\tilde{T}_{RS} = (\delta_{ab}e^{-\phi}, e^{3\phi})$. We have split the α index as follow $\alpha = (a,4)$, $a = 1,2,3$.

To set up the associated BPS equations, we use the seven-dimensional metric (4.7) and the following gauge fields

$$A^{12} = \frac{a}{y}dz, \qquad A^{31} = 0, \qquad A^{23} = \frac{a}{y}dx\,. \tag{5.22}$$

The twist condition is given by $ga = 1$. We will also impose the projection conditions

$$\Gamma_{23}\gamma_{\hat{x}\hat{y}}\epsilon = -\epsilon, \qquad \Gamma_{13}\gamma_{\hat{z}\hat{x}}\epsilon = -\epsilon, \qquad \Gamma_{12}\gamma_{\hat{z}\hat{y}}\epsilon = -\epsilon, \qquad \Gamma_{\hat{r}}\epsilon = \epsilon\,. \tag{5.23}$$

With all of the above conditions, we obtain the following BPS equations

$$-\phi' + \frac{1}{2}gX(e^{-\phi} - e^{3\phi}) + \sqrt{2}aX^{-1}e^{\phi-2G} = 0, \tag{5.24}$$

$$-X^{-1}X' - \frac{2}{5}gX^{-4} + \frac{1}{10}gX(3e^{-\phi} + e^{3\phi}) + \frac{3}{5\sqrt{2}}aX^{-1}e^{\phi-2G} = 0, \tag{5.25}$$

$$G' - \frac{1}{10}gX(3e^{-\phi} + e^{3\phi}) - \frac{1}{10}gX^{-4} + \frac{7}{5\sqrt{2}}aX^{-1}e^{\phi-2G} = 0, \tag{5.26}$$

$$F' - \frac{1}{10}gX(3e^{-\phi} + e^{3\phi}) - \frac{1}{10}gX^{-4} - \frac{3}{5\sqrt{2}}aX^{-1}e^{\phi-2G} = 0\,. \tag{5.27}$$

These equations admit a fixed point solution

$$\phi_0 = \frac{1}{4}\ln\frac{11}{3}, \qquad\qquad X_0^{20} = \frac{11(3^3)}{2^{12}},$$
$$G_0 = \frac{1}{10}\ln\left[\frac{3(11^2)}{2\sqrt{2}}\right] - \frac{1}{2}\ln\left[\frac{g}{a}\right], \qquad L_{\mathrm{AdS}_4} = \frac{1}{g}\left(\frac{11(3^3)}{2^7}\right)^{\frac{1}{5}}. \tag{5.28}$$

The parametrization of the μ^α coordinates can be chosen to be

$$\mu^\alpha = (\cos\Psi\hat{\mu}^a, \sin\Psi) \tag{5.29}$$

with $\hat{\mu}^a$ satisfying $\hat{\mu}^a\hat{\mu}^a = 1$. The $\mathrm{SO}(3)_{\mathrm{diag}}$ symmetry corresponds to the gauge fields A^{ab}. In the following, we accordingly set $A^{4a} = 0$ for $a = 1, 2, 3$ and find that

$$D\mu^a = \cos\Psi D\hat{\mu}^a - \sin\Psi\hat{\mu}^a d\Psi, \qquad D\mu^4 = \cos\Psi d\Psi \tag{5.30}$$

where

$$D\hat{\mu}^a = d\hat{\mu}^a + gA^{ab}\hat{\mu}^b\,. \tag{5.31}$$

With all these results, the eleven-dimensional metric is given by

$$\begin{aligned}
ds_{11}^2 = {} & \Delta^{\frac{1}{3}}\left[e^{\frac{r}{L_{\mathrm{AdS}_4}}}dx_{1,2}^2 + dr^2 + \frac{e^{2G_0}}{y^2}\left[dx^2+dy^2+dz^2\right]\right] \\
& + \frac{2}{g^2}\Delta^{-\frac{2}{3}}X_0^3\left[X_0\cos^2\xi + X_0^{-4}\sin^2\xi\left(\cos^2\Psi e^{\phi_0} + \sin^2\Psi e^{-3\phi_0}\right)\right]d\xi^2 \\
& + \frac{1}{2g^2}\Delta^{-\frac{2}{3}}X_0^{-1}\cos^2\xi\left[\cos^2\Psi e^{\phi_0}D\hat{\mu}^a D\hat{\mu}^a + \left(\sin^2\Psi e^{\phi_0} + \cos^2\Psi e^{-3\phi_0}\right)d\Psi^2\right] \\
& - \frac{1}{g^2}\Delta^{-\frac{2}{3}}X_0^{-1}\sin\xi\left(e^{-3\phi_0} - e^{\phi_0}\right)\sin\Psi\cos\Psi d\Psi d\xi\,.
\end{aligned} \tag{5.32}$$

The S^2 coordinates $\hat{\mu}^a$ can be parametrized by

$$\hat{\mu}^1 = \sin\beta\cos\alpha, \qquad \hat{\mu}^2 = \sin\beta\sin\alpha, \qquad \hat{\mu}^3 = \cos\beta\,. \tag{5.33}$$

The warped factor Δ is given by

$$\Delta = X_0^2 e^{-\phi_0}\cos^2\xi\cos^2\Psi + X_0^{-4}\sin^2\xi + X_0 e^{3\phi_0}\sin^2\Psi\cos^2\xi\,. \tag{5.34}$$

The four-form field on the $AdS_4 \times H^3$ background can be written as

$$\begin{aligned}
\hat{F}_{(4)} = {} & \frac{1}{g^3}U\cos^3\xi\cos^2\Psi d\xi\wedge d\Psi\wedge\epsilon_{(2)} \\
& + \frac{1}{2g^2}\cos\xi\epsilon_{abc}\left[\hat{\mu}^c\left[X_0^{-4}\sin^2\xi(\sin^2\Psi - \cos^2\Psi)\right.\right. \\
& \qquad\qquad \left. + X_0^2(e^{3\phi_0}\sin^2\Psi - e^{-\phi_0}\cos^2\Psi)\right]d\xi\wedge F^{ab}\wedge d\Psi \\
& - \left[(X_0^{-4}\sin^2\xi + X_0^2\cos^2\xi e^{3\phi_0})\sin\Psi\cos\Psi d\xi\right. \\
& \qquad\qquad \left.\left. + X_0^{-4}\cos\xi\sin\xi\cos^2\Psi d\Psi\right]\wedge F^{ab}\wedge D\hat{\mu}^c\right]
\end{aligned} \tag{5.35}$$

where

$$\begin{aligned}
\epsilon_{(2)} &= \frac{1}{2}\epsilon_{abc}\hat{\mu}^a D\hat{\mu}^b \wedge D\hat{\mu}^c, \\
U &= \cos^2\xi \left[X_0^2 \left[e^{6\phi_0}\sin^2\Psi - e^{-2\phi_0}\cos^2\Psi - e^{2\phi_0}(2\sin^2\Psi + 1)\right]\right. \\
&\qquad\qquad \left. -X_0^{-3}(e^{-\phi_0}\cos^2\Psi + e^{3\phi_0}\sin^2\Psi)\right] \\
&\quad + \sin^2\xi X_0^{-3}(X_0^{-5} - 3e^{-\phi_0} - e^{3\phi_0}).
\end{aligned} \tag{5.36}$$

6 Conclusions

We have studied $AdS_5 \times \Sigma_2$ and $AdS_4 \times \Sigma_3$ solutions of $N = 2$ gauged supergravity in seven dimensions with SO(4) gauge group. We have found that there exist both $AdS_5 \times S^2$ and $AdS_5 \times H^2$ solutions with the gauge fields for $\mathrm{SO}(2) \times \mathrm{SO}(2)$ turned on. With $\mathrm{SO}(2)_R$ or $\mathrm{SO}(2)_{\mathrm{diag}}$ gauge fields, only $AdS_5 \times H^2$ solution is possible. This is consistent with the results given in [21] and [23]. We recover $AdS_5 \times S^2$ and $AdS_5 \times H^2$ solutions studied in [22] and [23] with $\mathrm{SO}(2) \times \mathrm{SO}(2)$ symmetry. In the case of equal SU(2) gauge couplings, the solutions can be uplifted to eleven dimensions, and the uplifted solutions have explicitly given.

We have also considered RG flow solutions interpolating between supersymmetric AdS_7 critical points in the UV and these AdS_5 solutions in the IR. In the case of $\mathrm{SO}(2)_{\mathrm{diag}}$ symmetry, there exist flow solutions from SO(4) AdS_7 critical point to AdS_5 as well as flows from SO(4) AdS_7 to SO(3) AdS_7 and then continue to AdS_5 fixed points similar to the flows from four-dimensional SCFTs to two-dimensional $N = (2, 0)$ SCFTs studied in [31]. Other results of this paper are a number of new $AdS_4 \times S^3$ and $AdS_4 \times H^3$ solutions for unequal SU(2) gauge couplings. With equal SU(2) couplings, only $AdS_4 \times H^3$ geometry is possible, and the resulting solutions can be uplifted to eleven dimensions.

The results obtained in this paper should be relevant in the holographic study of $N = (1, 0)$ SCFTs in six dimensions. These would also provide new AdS_5 and AdS_4 solutions, corresponding to new SCFTs in four and three dimensions, within the framework of seven-dimensional gauged supergravity. The embedding of the solutions in the case of unequal SU(2) gauge couplings (if possible) would be interesting to explore. It would also be interesting to compare the AdS_5 and AdS_4 solutions obtained here and the solutions found recently in [32, 33] in the context of massive type IIA theory. Finally, it is of particular interest to find an interpretation of all these solutions in terms of wrapped M5-branes on Σ_2 and Σ_3. Along this line, it would also be useful to find an implication of the AdS_4 solutions in terms of the M2-brane worldvolume theories.

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